QCD+QED q_T factorization for W/Z production and decay.

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[in preparation]

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Motivation for $pp \to Z/W \to L+X$.

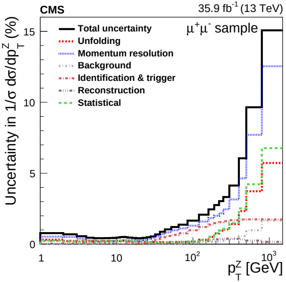
- Experimental measurement of $\mathrm{d}\sigma/\mathrm{d}p_T^Z$ yield $\lesssim 0.5\%$ uncertainty

- Thorough understanding of Z is necessary for analyses related to M_W measurement [ATLAS, 1701.07240]

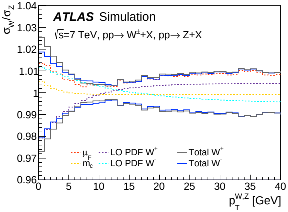
- Also, small $p_T^W < 40\,\mathrm{GeV}$ region is relevant for M_W determination

- QCD resummation for p_T known to high order $\Rightarrow$ See talks by Markus and Tobias

- *It is time* to start thinking about often neglected contributions like **QED**



[CMS 1909.04133]



[ATL-PHYS-PUB-2017-021]

Overview of QED effects.

How much **QCD** + **QED** complicates things?

- Pertubative quantities $\mathcal{F} = \sum \mathcal{F}^{(n,m)} \alpha_s^n \alpha_e^m$
- Coupled β functions
- α_s, α_e & RGE running is affected [GB, F.J. Tackmann, J. Talbert; '19]

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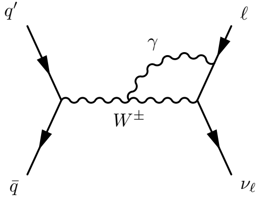
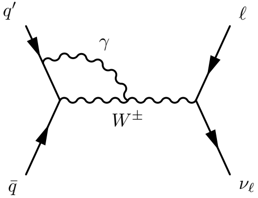
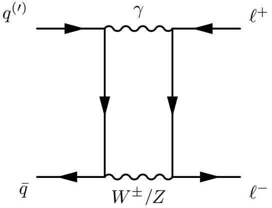
- α_s, α_e & RGE running is affected [GB, F.J. Tackmann, J. Talbert; '19]
- Quarks and leptons interact with each other via γ



Intermediate – Final (FI)



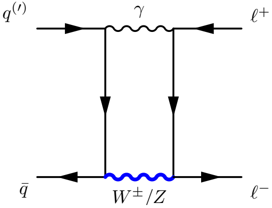
Initial – Intermediate (InI)



Initial – Final (IFI)

- Cannot separately investigate radiative corrections in $W_{\mu\nu}$ and $L^{\mu\nu}$
- q_T factorization is broken $\rightarrow$ is this the end of it?

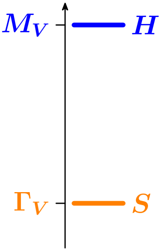
Heavy Vector Effective Theory (HVET).



- Work in the vicinity of the $V = Z, W$ resonance

$$\frac{P^2 - M_V^2}{M_V} \sim \Gamma_V \ll Q \sim M_V$$

- **Unstable particle EFT** [Beneke, Chapovsky, Signer, Zanderighi; '04]



- Hard modes $\sim \mathcal{O}(M_V)$
- Soft modes $\sim (\Gamma_V, \Gamma_V, \Gamma_V)$
- Collinear modes $\sim (M_V, \Gamma_V, \sqrt{M_V \Gamma_V})$

- Like HQET but with a heavy vector boson V_v^μ instead of Q_v

Heavy Vector Effective Theory (HVET).

Decompose vector boson momenta in label and residuals

$$P^\mu = M_V v^\mu + k^\mu$$

$$\frac{1}{P^2 - M_V^2 + \mathrm{i} M_V \Gamma_V} \sim \frac{1}{2M_V} \frac{1}{v \cdot k + \mathrm{i}\Gamma_V/2}$$

- Interactions with only $k \sim \mathcal{O}(\Gamma_V)$ keep V near mass shell

$$\mathcal{L}_{\mathsf{HVET}} = 2M_V V_v^{\mu\dagger}\Big(v \cdot D_s - \frac{\Delta}{2}\Big) V_\mu^v$$

$$\mathcal{L}_{\mathsf{EFT}} = \mathcal{L}_{\mathsf{HVET}} + \mathcal{L}_s^{\mathsf{QED}} + \mathcal{L}_{\mathsf{SCET}}$$

$$V_\mu^v(x) = e^{\mathrm{i}Mv\cdot x}(-g_{\mu\nu} + v_\mu v_\nu)\big[\mathcal{P}_+ V^\nu(x)\big]$$

- $\mathrm{Im}\Delta = -\Gamma_V$ but otherwise equivalent to the residual mass in HQET

Protofactorization.

- Full theory considerations for a generic measurement $\mathcal{O}$

$$
\begin{aligned}
\frac{d\sigma}{d\mathcal{O}} &\sim \sum_X \int d\Phi_{L,X} \int d^4x\, d^4y\;\delta^{(4)}(p_a + p_b - p_1 - p_2) \\
&\qquad \times \langle pp|\bar{\mathrm{T}}[J_H^\dagger(0) J_L(y)]\hat{\mathcal{O}}|LX\rangle\langle LX|\mathrm{T}[J_L^\dagger(x) J_H(0)]|pp\rangle \\
&\sim \int d^4x\, d^4y\, d^4z\,\langle pp|\bar{\mathrm{T}}[J_H^\dagger(z) J_L(y+z)]\hat{\mathcal{O}}\mathrm{T}[J_L^\dagger(x) J_H(0)]|pp\rangle
\end{aligned}
$$

! Cannot disentangle J_H and J_L

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$$\frac{d\sigma}{d\mathcal{O}} \sim \sum_X \int d\Phi_{L,X} \int d^4x\, d^4y\; \delta^{(4)}(p_a + p_b - p_1 - p_2)$$
$$\times \langle pp|\bar{\mathrm{T}}[J_H^\dagger(0) J_L(y)]\hat{\mathcal{O}}|LX\rangle \langle LX|\mathrm{T}[J_L^\dagger(x) J_H(0)]|pp\rangle$$
$$\sim \int d^4x\, d^4y\, d^4z\, \langle pp|\bar{\mathrm{T}}[J_H^\dagger(z) J_L(y+z)]\hat{\mathcal{O}}\mathrm{T}[J_L^\dagger(x) J_H(0)]|pp\rangle$$

! Cannot disentangle J_H and J_L

- Match $J_{H,L}$ to collinear quarks & leptons

$$J_{H,L}(x) \to \sum_{\{n_i\}} \int \{d\omega_i\}\, C_{H,L}(\{\omega_i\})\, e^{-i\mathcal{P}\cdot x} \left[V_\mu^{v\dagger} \bar{\chi}_{n_i,-\omega_i} \Gamma^\mu \chi_{n_j,\omega_j}\right](x)$$

Follow the standard procedure

→ Recombine label & residual momenta

→ Get explicit phases for residual momenta that couldn't be recombined

→ $\int d^4z \to \delta^{(4)}(\tilde{p}_a + \tilde{p}_b - \tilde{p}_1 - \tilde{p}_2)$ label momentum conservation

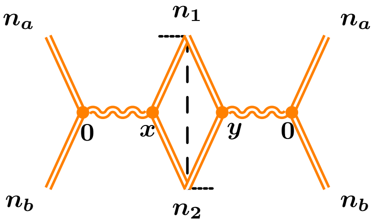
Protofactorization.

- EFT cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}} \sim \int \mathrm{d}\omega_{a,b} \int \mathrm{d}^4 p_{1,2}\, \delta^{(4)}(\tilde{p}_a + \tilde{p}_b - \tilde{p}_1 - \tilde{p}_2)$$
$$\times H^{\mu\nu}(\omega_{a,b}) L_{\mu\nu}(\omega_{1,2}) B_{n_a}(x_a) B_{n_b}(x_b)$$
$$\times \int \mathrm{d}^4 x\, \mathrm{d}^4 y\, J_{n_1}(p_1) J_{n_2}(p_2) e^{\mathrm{i}(p_1+p_2-Mv)\cdot(x-y)}$$
$$\times \left\langle [\mathcal{S}^\dagger_{ab} V_v](0)[V_v^\dagger \mathcal{S}^\dagger_{12}](y)\hat{\mathcal{O}}_s[\mathcal{S}_{12}V_v](x)[V_v^\dagger \mathcal{S}_{ab}](0)\right\rangle$$

With $\mathcal{S}_{ij} = Y^\dagger_{n_j} Y_{n_i}$

- Hadronic $H(\omega_{a,b}) = |C_H|^2$
- Leptonic $L(\omega_{1,2}) = |C_L|^2$
- No hard matching coeff. that 'sees' all 4 legs
- Inclusive lepton jet functions
- Non-local soft function
  - (n_a, n_b, n_1, n_2) Wilson lines
  - Connected with HVET Wilson lines



Protofactorization.

- EFT cross-section

$$\frac{d\sigma}{d\mathcal{O}} \sim \int d\omega_{a,b}\, \delta^{(4)}(\tilde{p}_a + \tilde{p}_b - \tilde{p}_1 - \tilde{p}_2)$$
$$\times H^{\mu\nu}(\omega_{a,b}) L_{\mu\nu}(\omega_{1,2}) B_{n_a}(x_a) B_{n_b}(x_b)$$
$$\times \int d^4 r\, d^4 p_{1,2}\; S(r) J_{n_1}(p_1) J_{n_2}(p_2) \delta^{(4)}(\underbrace{p_1 + p_2 - Mv}_{\mathcal{O}(\Gamma_V)} - r)$$

$$S(r) = \begin{cases} \text{Contains the line shape} \\ \text{Encodes soft radiation off the resonance} \\ \text{Captures IFI \& FSR radiation} \end{cases}$$

$$S^{(0)}(r) = \left| \frac{1}{v \cdot r - \Delta/2} \right|^2 \to \text{Not a } \delta(r) \text{ at LO!}$$

- Convolution between soft & jet function

  Anticipated by [Beneke, Chapovsky, Signer, Zanderighi; '04]

→ Final state radiation deforms the line-shape

- Consistency on the decay side suggests that $Z_s = Z_s^{\text{thrust}}$!
- Checked at $\mathcal{O}(\alpha_e)$ ✔

'Naive' Measurement & LO line-shape.

Let's measure $\mathcal{O} = \{Q^2, Y, \Delta y\} \equiv \Phi_L^{\text{naive}}$ defined by Born kinematics

$$\omega_{a,b} = Qe^{\pm Y}\,, \quad \omega_a \frac{n_a^\mu}{2} + \omega_b \frac{n_b^\mu}{2} = \omega_1 \frac{n_1^\mu}{2} + \omega_2 \frac{n_2^\mu}{2}$$

The EFT cross section at LO

$$\frac{d\sigma}{d\Phi_L^{\text{naive}}} \sim f_a\Big(\frac{Qe^Y}{E_{\rm cm}}\Big) f_b\Big(\frac{Qe^{-Y}}{E_{\rm cm}}\Big) \frac{1}{4M_V^2} \left| \frac{1}{v\cdot\big(Qe^Y \frac{n_a}{2} + Qe^{-Y}\frac{n_b}{2} - M_V v\big) - \frac{\Delta}{2}} \right|^2$$

- Set $v^\mu = (\cosh Y, 0_\perp, \sinh Y)$ $\rightarrow$ location of the propagator's pole
- $\mathrm{Im}\Delta = -\Gamma_V$ $\rightarrow$ keeps the propagator finite
- Recover correctly the familiar Breit-Wigner distribution

$$\frac{d\sigma}{d\Phi_L^{\text{naive}}} \sim f_a\Big(\frac{Qe^Y}{E_{\rm cm}}\Big) f_b\Big(\frac{Qe^{-Y}}{E_{\rm cm}}\Big) \frac{1}{(Q^2 - M_V^2)^2 + M_V^2\Gamma_V^2}$$

Lepton & (realistic) Measurement definitions.

Typical lepton definition used by expreriments

- **Bare:** after QED radiation
- **Born:** before (inclusive over) QED radiation
- **Dressed:** clustering of QED radiation with a cone/jet algorithm

→ Define suitable Born leptons

$$p_i^\mu = \bar{n}_i \cdot \Big[\omega_i \tfrac{n_i}{2} + k_i^+ \tfrac{\bar{n}_i}{2} + \underbrace{\sum_j \Theta_{ij} l_{s,j}}_{l_1^-, l_2^-}\Big] \frac{n_i^\mu}{2}$$

Measure $\mathcal{O} = \{Q^2, Y, \Delta y\} \equiv \Phi_L$

- Close to resonance Q^2 is constrained → sensitivity to residual momenta
- Otherwise $\{Y, \Delta y\}$ are unconstrained → defined by Born level kinematics

$$\delta\big(Q^2 - (p_1+p_2)^2\big)\,\delta\Big(Y - \tfrac{1}{2}\ln\big[\tfrac{n_b\cdot(p_1+p_2)}{n_a\cdot(p_1+p_2)}\big]\Big)$$
$$= \delta\big(\omega_{a,b} - e^{\pm Y}(Q - l)\big)\Big[1 + \mathcal{O}\big(\tfrac{\Gamma_V}{Q}\big)\Big]$$

with $l = l_1^- \frac{v\cdot n_1}{2} + l_2^- \frac{v\cdot n_2}{2} \sim \mathcal{O}(\Gamma_V)$

Realistic measurement.

- The EFT cross section now looks like this

$$\frac{d\sigma}{d\Phi_L} \sim H^{\mu\nu}(Q) L_{\mu\nu}(Q) B_{n_a}\Big(\frac{Qe^{Y}}{E_{\rm cm}}\Big) B_{n_b}\Big(\frac{Qe^{-Y}}{E_{\rm cm}}\Big)$$
$$\times \int d(v\cdot r)\, dl^-_{1,2} \int dk^+_{1,2}\; S\big(v\cdot r, l_1^-, l_2^-\big) J_{n_1}(\omega_1 k_1^+) J_{n_2}(\omega_2 k_2^+)$$
$$\times\, \delta\Big(\underbrace{Q - M_V}_{\mathcal{O}(\Gamma_V)} - k_1^+ - k_2^+ - \frac{l_1^-}{2} - \frac{l_2^-}{2} - v\cdot r\Big)$$

  - The fact that Q is *restricted* to be near M_V induces an $\mathcal{O}(\Gamma_V)$ sensitivity which modifies the soft function
  - Anywhere else (B, J, H, L) expand l away
  - Physical picture is that the parts of the radiation recovered by clustering with the leptons *should* be 'known' to the propagator
  - Define a new soft function

$$S(v\cdot\bar r) \equiv \int d(v\cdot r)\, dl^-_{1,2}\; \delta\Big((v\cdot\bar r) - (v\cdot r) - \frac{l_1^-}{2} - \frac{l_2^-}{2}\Big) S(v\cdot r, l_1^-, l_2^-)$$

Realistic measurement.

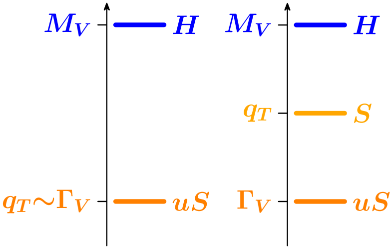
- Putting everything together and setting $\Delta Q \equiv Q - M_V$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_L} \sim H^{\mu\nu}(Q) L_{\mu\nu}(Q)\, B_{n_a}\Big(\frac{Qe^{Y}}{E_{\rm cm}}\Big) B_{n_b}\Big(\frac{Qe^{-Y}}{E_{\rm cm}}\Big)$$
$$\times \int \mathrm{d}k_1^+\, \mathrm{d}k_2^+ \; S(\Delta Q - k_1^+ - k_2^+) J_{n_1}(\omega_1 k_1^+) J_{n_2}(\omega_2 k_2^+)$$

- Redefinition of S results in the same convolution as for the 'naive' measurement
- Since J_{n_1}, J_{n_2} and $L_{\mu\nu}$ didn't change, then $S(v \cdot \bar{r})$ must renormalize same way as $S(v \cdot r)$
- Therefore, also $S(v \cdot \bar{r})$ renormalizes like thrust!
- Explicit check at $\mathcal{O}(\alpha_e)$ ✓
- Note that $S(v \cdot \bar{r})$, $S(v \cdot r)$ differ in their finite pieces
- IFI/FI/InI/II/ISR: $R + V = \mathcal{O}(\epsilon^0) \Rightarrow \mathrm{NLL}'$ effects

Measuring q_T: Regimes, Modes & Measurement.

- Let's measure in addition q_T



- **Regime I**: $q_T \sim \Gamma_V \ll M_V \sim Q$
  - uSofts $(\Gamma_V, \Gamma_V, \Gamma_V) \sim (q_T, q_T, q_T)$

- **Regime II**: $\Gamma_V \ll q_T \ll M_V \sim Q$
  - Softs $\sim (q_T, q_T, q_T)$
  - uSofts $\sim (\Gamma_V, \Gamma_V, \Gamma_V)$

! In Regime II only uSoft interactions keep the V_v^μ near its mass shell

Measuring q_T: Regimes, Modes & Measurement.

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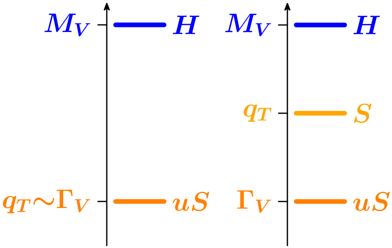

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  - uSofts $\sim (\Gamma_V, \Gamma_V, \Gamma_V)$

! In Regime II only uSoft interactions keep the V_v^μ near its mass shell

- Complete measurement

$$\mathcal{O} = \Theta_1 \big[\delta(l_1^- - k_1^-)\theta(k_2^- - k_1^-)\delta(l_2^-)\delta^{(2)}(q_T)\big] + (1 \leftrightarrow 2)$$
$$+ \Theta_{ab}\big[\delta(l_1^-)\delta(l_2^-)\delta^{(2)}(q_T - k_T)\big]$$
$$\text{with} \quad \Theta_{ab} + \Theta_1 + \Theta_2 = 1$$

→ Added Θ_{ab} region to cover the whole phase space

Factorization in **Regime 1**: $\Gamma_V \sim q_T \ll M_V$.

$$\frac{d\sigma}{d\Phi_L dq_T} \sim H_{\mu\nu}(Q) L^{\mu\nu}(Q) \int d^2k_{T_a}\, d^2k_{T_b}\, dk_1^+ dk_2^+$$
$$\times B(k_{T_a}, \omega_a) B(k_{T_b}, \omega_b)\, J_{n_1}(\omega_1 k_1^+) J_{n_2}(\omega_2 k_2^+)$$
$$\times S_I\Big(q_T - k_{T_a} - k_{T_b}, \Delta Q - k_1^+ - k_2^+\Big)$$

- Production: q_T convolution between S_I and q_T beam func'
- Decay: ΔQ convolution between S_I and inclusive jet func'

For the Z:

- Consistency on the decay side still holds as previously (thrust-like)
- Consistency on the production side is like $2 \to 0$ QCD Drell-Yan
- IFI don't contribute to the pole structure

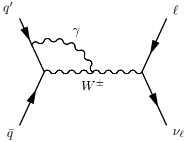
Factorization in **Regime 1**: $\Gamma_V \sim q_T \ll M_V$.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_L \mathrm{d}q_T} \sim H_{\mu\nu}(Q) L^{\mu\nu}(Q) \int \mathrm{d}^2 k_{T_a}\, \mathrm{d}^2 k_{T_b}\, \mathrm{d}k_1^+ \mathrm{d}k_2^+$$
$$\times\, B(k_{T_a}, \omega_a) B(k_{T_b}, \omega_b)\, J_{n_1}(\omega_1 k_1^+) J_{n_2}(\omega_2 k_2^+)$$
$$\times\, S_I\left(q_T - k_{T_a} - k_{T_b}, \Delta Q - k_1^+ - k_2^+\right)$$

- Production: q_T convolution between S_I and q_T beam func'
- Decay: ΔQ convolution between S_I and inclusive jet func'

For the W:

- IFI/FI don't contribute to the pole structure
- Naively we would expect (abelianization) that $\Gamma_0 \sim Q_q Q_{q'}$
- But InI modify it with an extra term $\rightarrow \Gamma_0 \sim Q_q^2 + Q_{q'}^2$
- Also rapidity anomalous dim. get modified $\gamma_\nu^0, \Gamma_0 \sim Q_q^2 + Q_{q'}^2$
- Dictated from collinear gauge invariance, nontrivial!

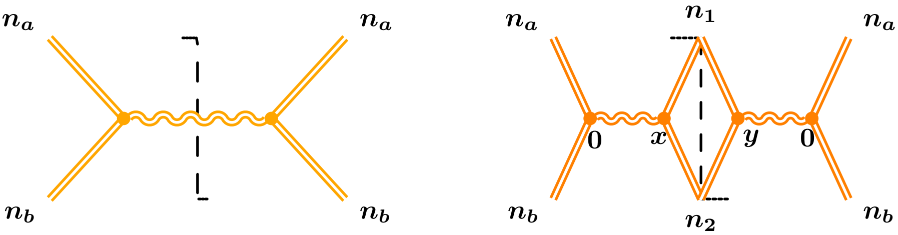


Factorization in **Regime 2**: $\Gamma_V \ll q_T \ll M_V$

- Assume the stronger limit $\Gamma_V \ll q_T$ and use that hierarchy to re-factorize
- The requirement to be around the peak $Q \sim M_V$ still holds

This implies that at the scale q_T

$$S_I(q_T, \bar{r}) = C_{II}(q_T) \times S_{II}(\bar{r}) \left[1 + \mathcal{O}\left(\frac{\Gamma_V}{q_T}\right)\right]$$



- **Physical picture?**
  Since softs $\sim \mathcal{O}(q_T)$ take V_v^μ far off shell, it can only interact with usofts $\sim \mathcal{O}(\Gamma_V)$ therefore any radiation on the decay side is power suppressed

The EFT cross-section in Regime 2 $\sim \Gamma_V \ll q_T \ll M_V \sim Q$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_L \mathrm{d}q_T} = W(Q, Y, q_T) L(Y, \Delta y, \Delta Q)$$

with

$$W = H(Q) \int \mathrm{d}^2 k_{T_a} \mathrm{d}^2 k_{T_b}\, C_{II}(q_T - k_{T_a} - k_{T_b}) B_{n_a}(k_{T_a}, \omega_a) B_{n_b}(k_{T_b}, \omega_b)$$

$$L = H_L(Q) \int \mathrm{d}k_1^+ \mathrm{d}k_2^+\, S_{II}(\Delta Q - k_1^+ - k_2^+) J_{n_1}\left(\omega_1 k_1^+\right) J_{n_2}\left(\omega_2 k_2^+\right)$$

- C_{II} is a Wilson coeff. at the scale q_T
  - describes soft ISR and II that contributes to q_T
  - For W: Not the usual $S_{\mathrm{DY}} \Rightarrow$ three-prong soft function [See Yannis' talk]
  - IFI/FI/FSR are power suppressed
- S_{II} is the same as in 'protofactorization' ($S(v \cdot \bar{r})$)
  - contains the line shape and describes usoft interactions with V_v
  - convoluted with lepton jet functions $\Rightarrow$ FSR modifies line shape

Factorization in **Regime 2**: $\Gamma_V \ll q_T \ll M_V$

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What about consistency?

- From Regime I to Regime II, only $S_I = C_{II} \times S_{II}$ changed
- That implies that the poles of C_{II} and S_{II} are those of S_I
- Consistency works separately for W and L

Recap.

We saw...

- A QCD-QED factorization for $\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_L \mathrm{d}q_T}$ for explicit production and decay
  - **Regime 1:** $\Gamma_V \sim q_T \ll M_V$
    - Non local soft function $S(k_T, v \cdot \bar{r})$ that contains the line shape
    - convoluted on the production side with q_T beam functions
    - and on the decay side with inclusive lepton jet functions (Born leptons) $\Rightarrow$ line shape modification!
  - **Regime 2:** $\Gamma_V \ll q_T \ll M_V$
    - We recover q_T factorization!
    - W captures all the q_T dependence mainly from ISR
    - L contains V^μ line-shape and decay captures FSR
    - $W^\pm$ RGE running is modified by $\gamma_\nu^0, \Gamma_0 \sim Q_q^2 + Q_{q'}^2$

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Thank you!