Emittance Exchange in MICE

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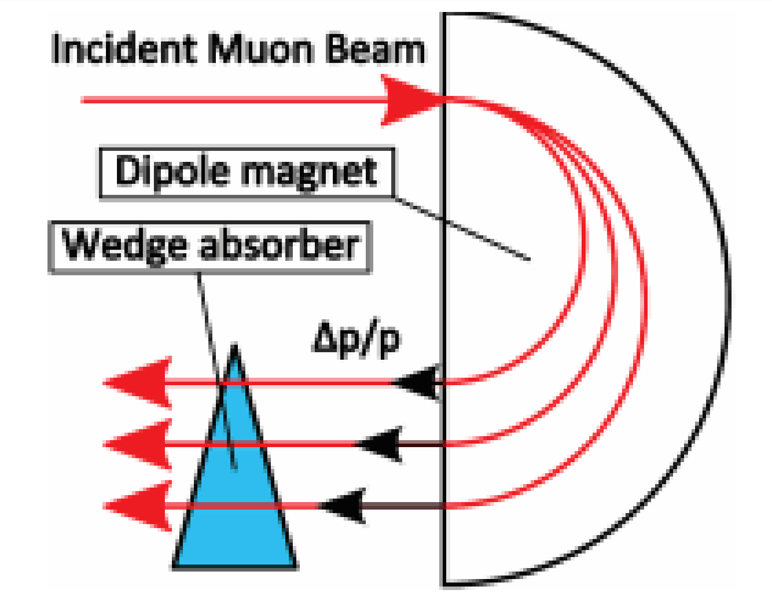
Aims

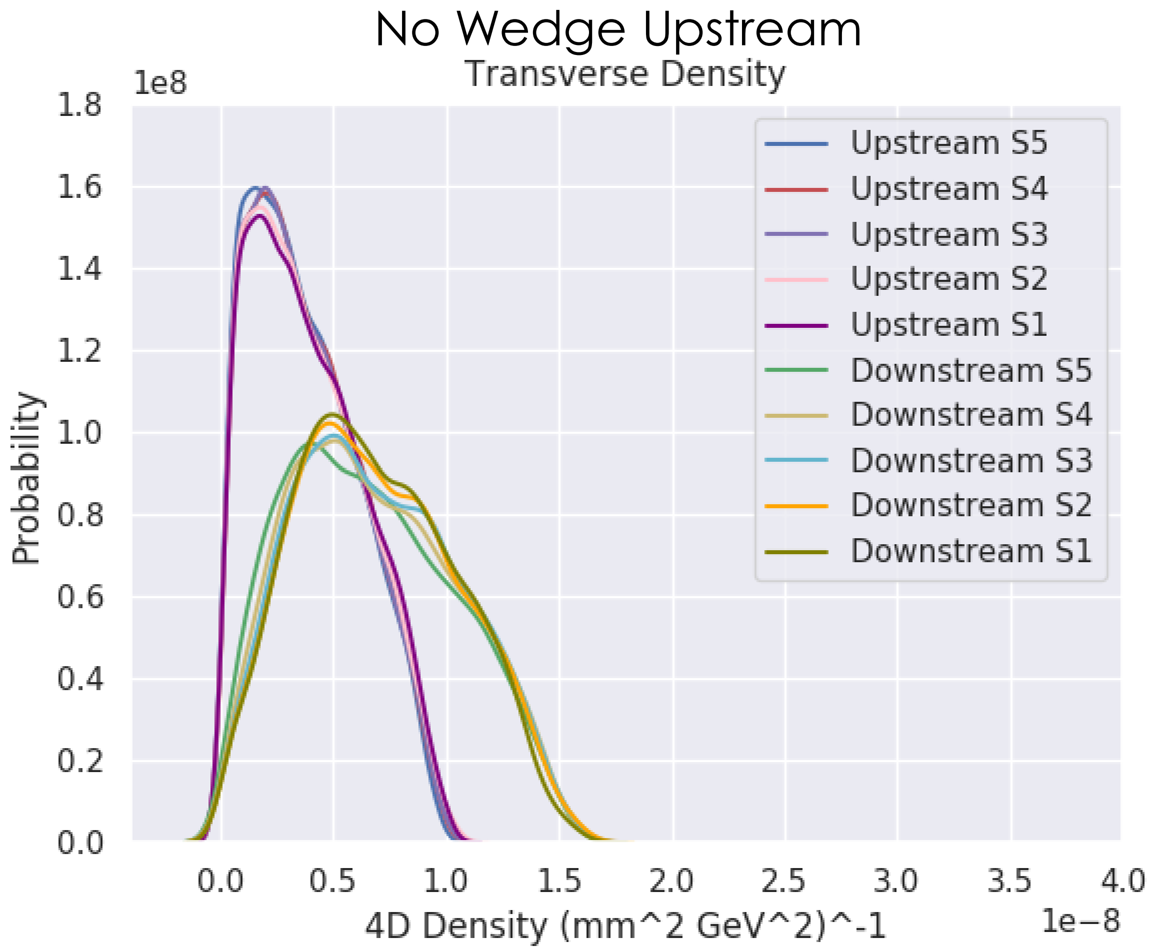


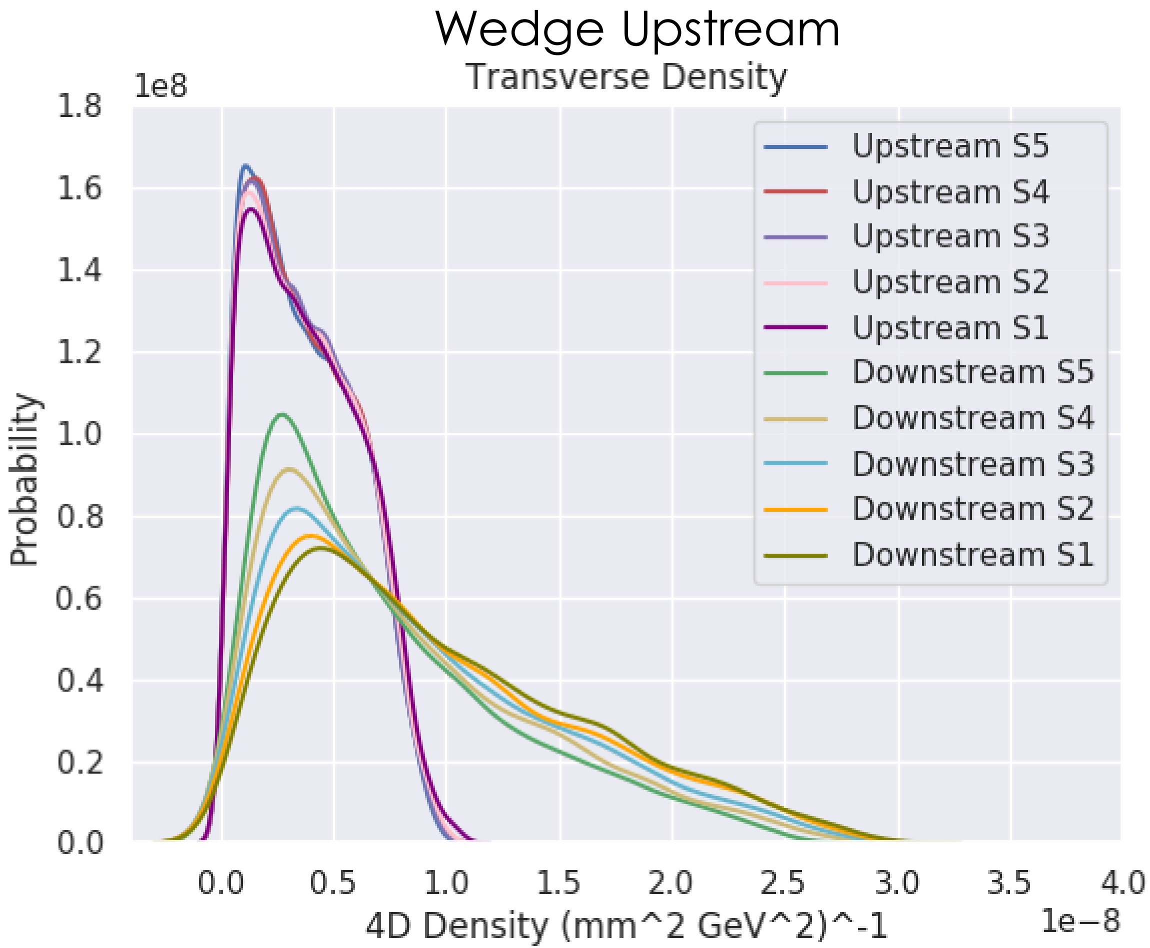
- Demonstrate Emittance Exchange and Reverse Emittance Exchange in the Wedge using MICE data
- Emittance Exchange can be demonstrated by looking at the change in phase space density of the particle selection before and after having passed through a Wedge absorber
- Emittance Exchange is shown by a decreased transverse phase space density (x, px, y, py) and increased longitudinal phase space density (z, pz), (and vice versa for Reverse Emittance Exchange)
- Can use a number of techniques to calculate phase space density: KDE, KNN, Voronoi Tessellations, etc.
- MICE beam only has a small natural dispersion → Use beam reweighing techniques to select beams with desired dispersion

Previously

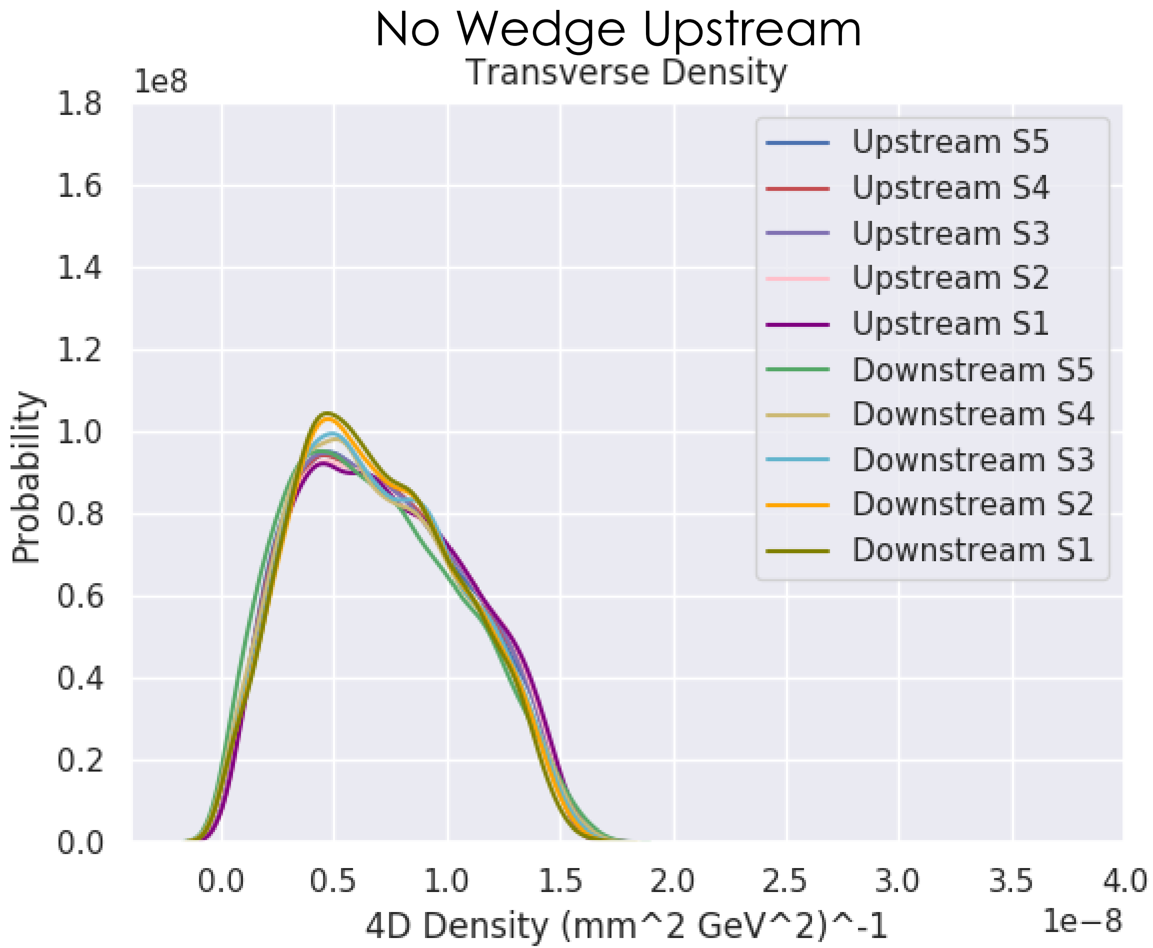
- Liouville -> 6D phase space density conserved (when no dissipative forces)
- MICE can be approximately split between 4D Transverse and 2D Longitudinal, only small amounts of scattering, Energy Loss and misalignments
- Looked at Transverse phase space density
  - Conserved for the case of single tracks
  - Transmission Losses spoil result due to change in particle distribution function
  - Not enough to normalize for number of particles, but must also account for change in covariance matrix
- Looked at Longitudinal phase space density
  - Appears to show wide variability
  - Time co-ordinate based on Pz, which in turn is based on Pt, which in turn is based on x and y positions as well as the modelled Magnetic field
  - Pz to x,y is not a 1:1 relationship -> affects 6D conservation
  - Errors and biases in any of these terms, may spoil the terms which depend on them
  - They may be different in each tracker, making phase space density, Emittance, Energy loss comparisons subject to these biases

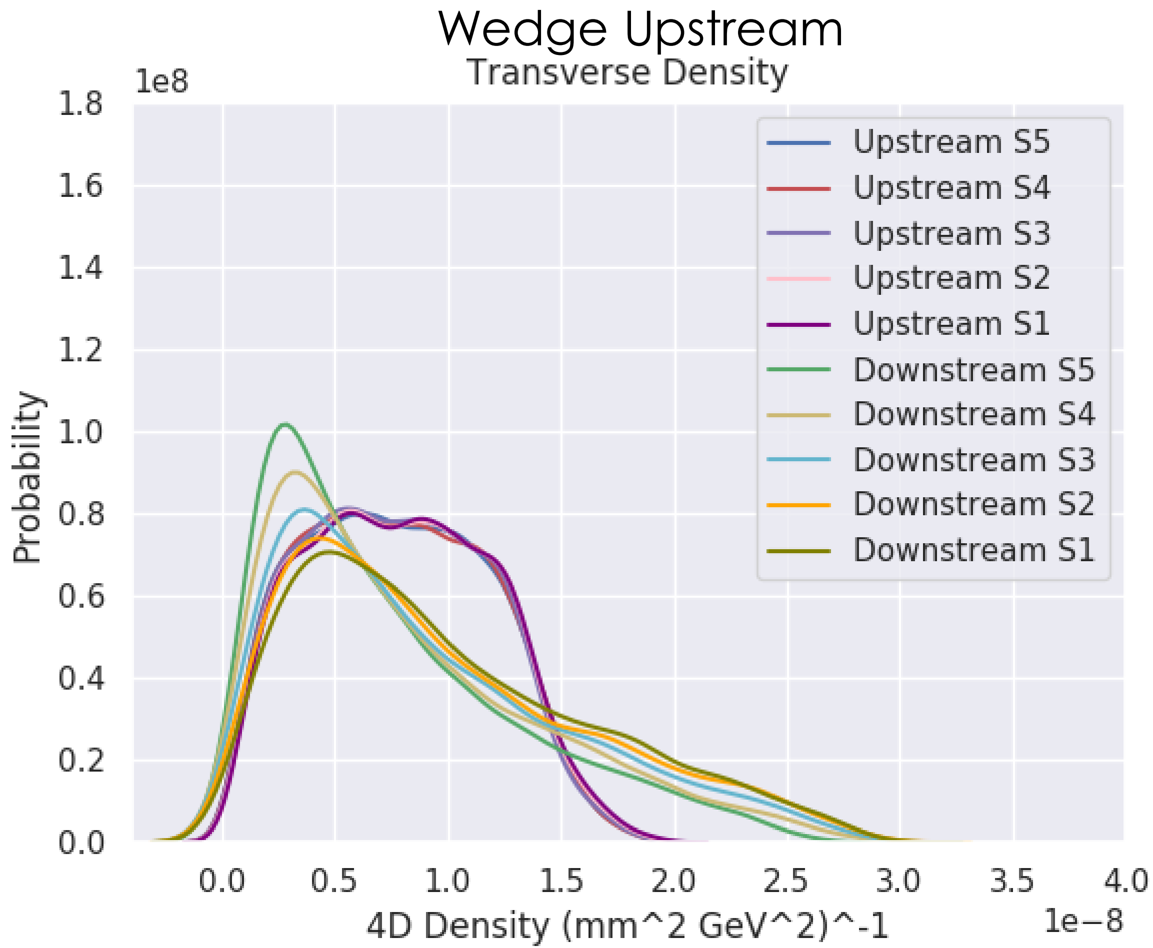
Transverse Phase Space Density Evolution
Full sample





Transverse Phase Space Density Evolution Only sample which makes it downstream

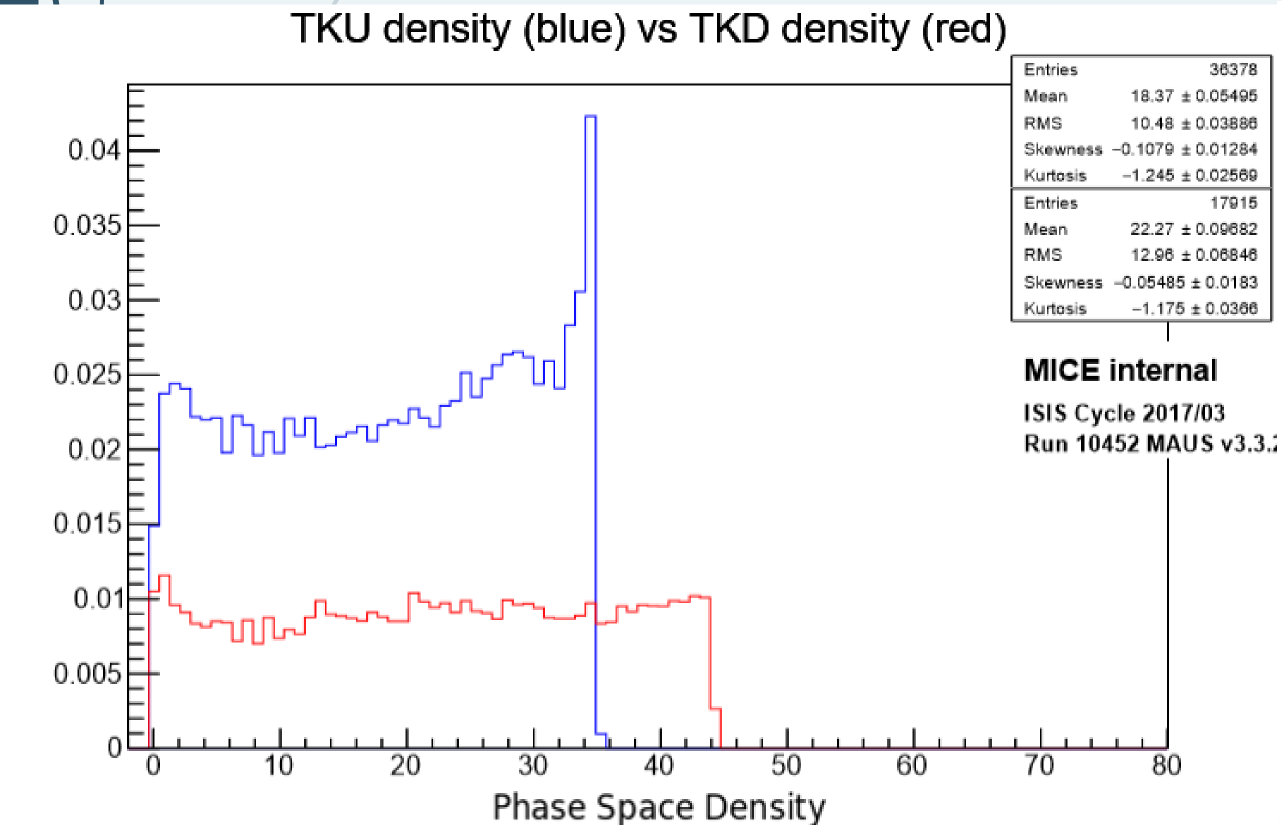




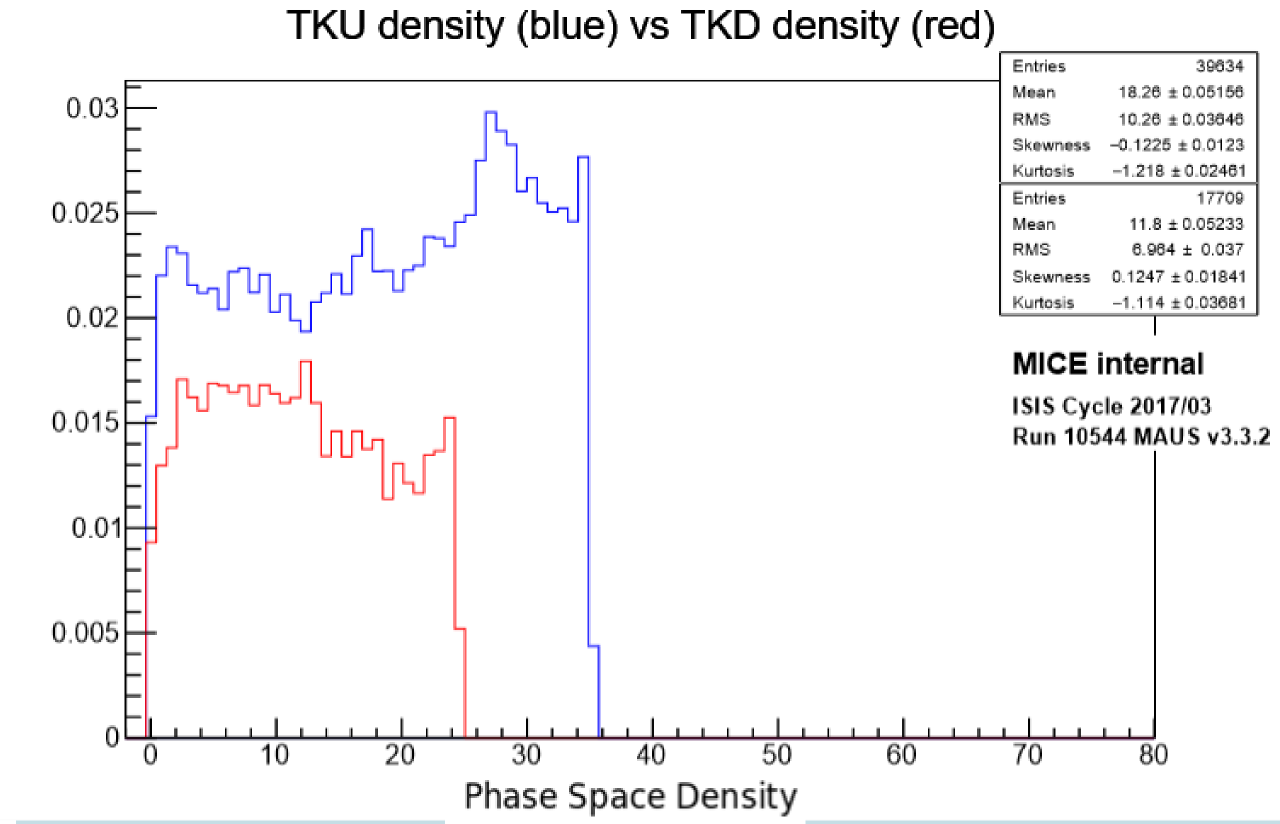
10-140 2D Longitudinal phase space density

- Single track that has gone both through Upstream and Downstream Tracker
- And single track that has only gone through Upstream Tracker
- Introduced Gaussian Time relation

No Wedge



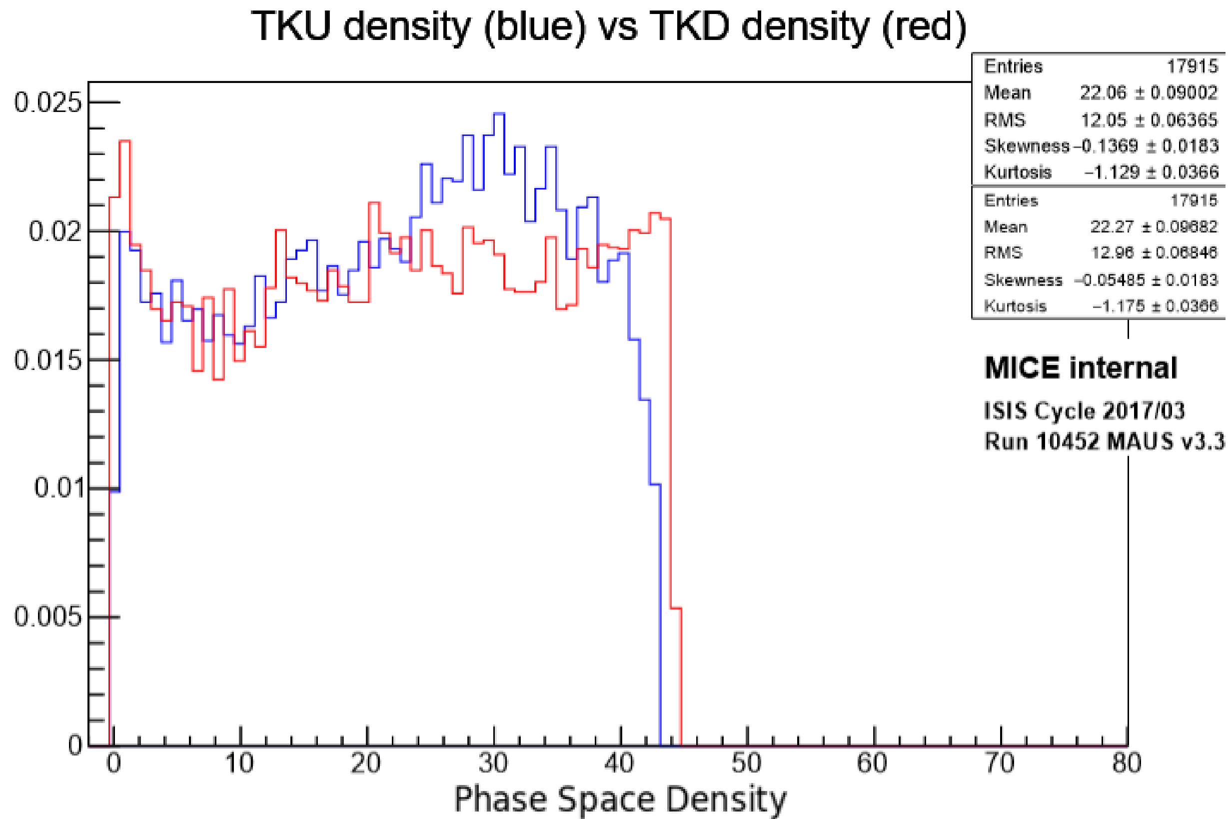
Wedge



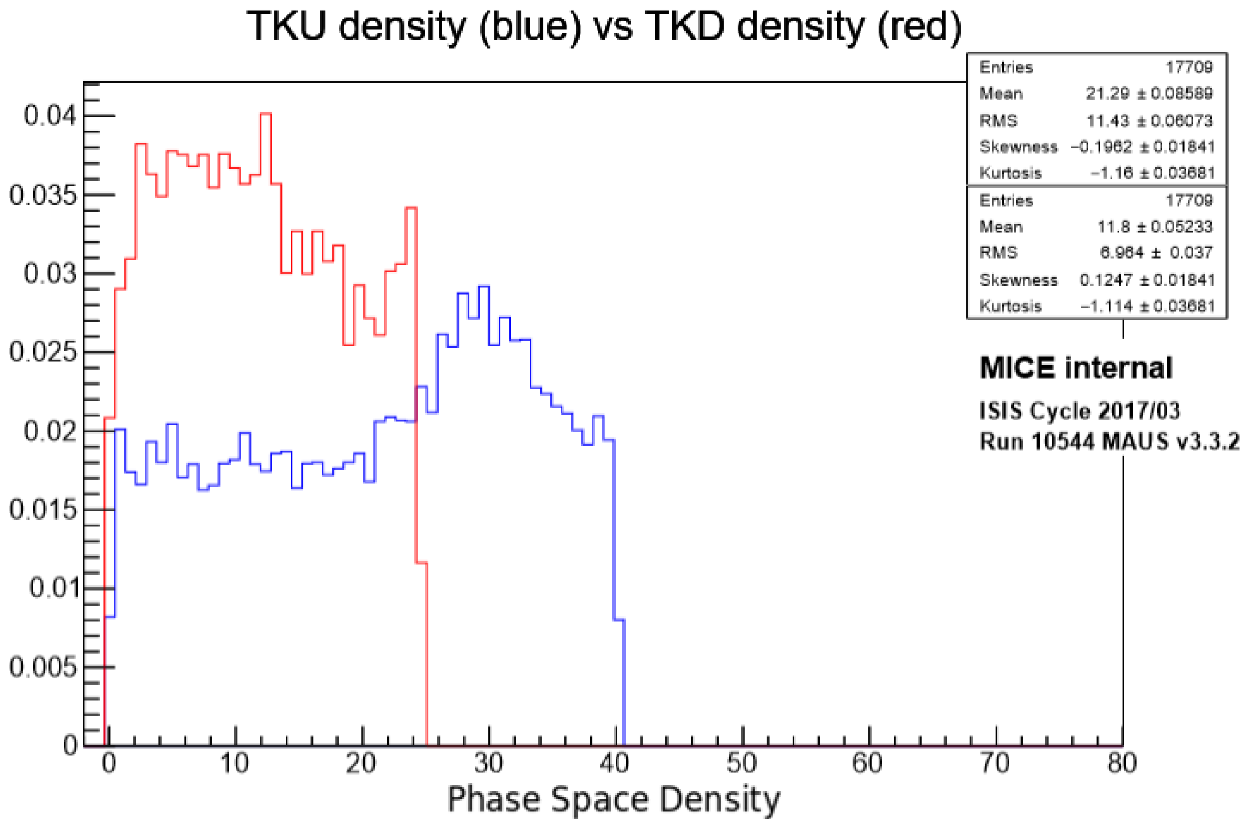
10-140 2D Longitudinal phase space density

- Single track that has gone both through Upstream and Downstream Tracker
- Introduced Gaussian Time relation

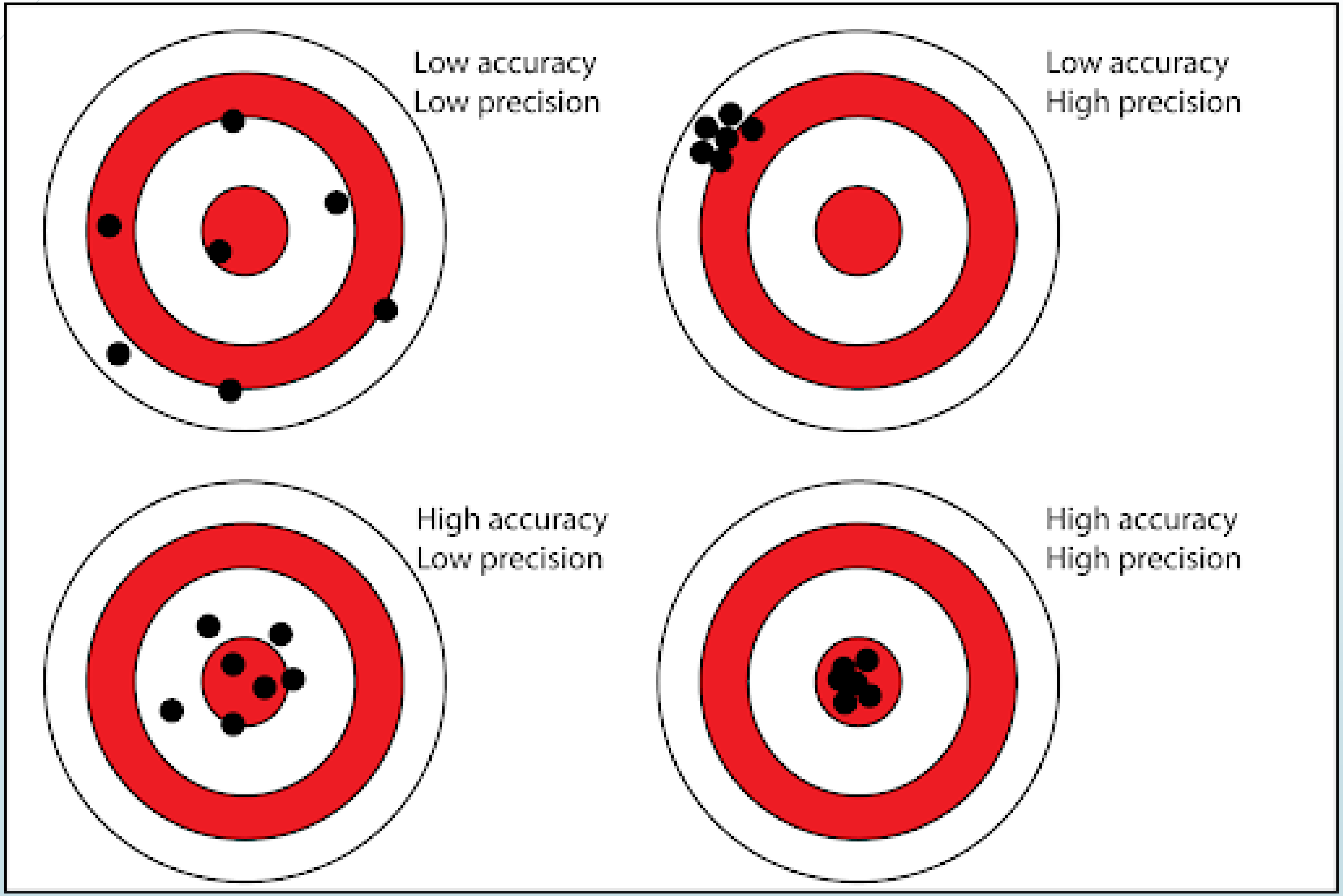
No Wedge



Wedge



Accuracy and Precision



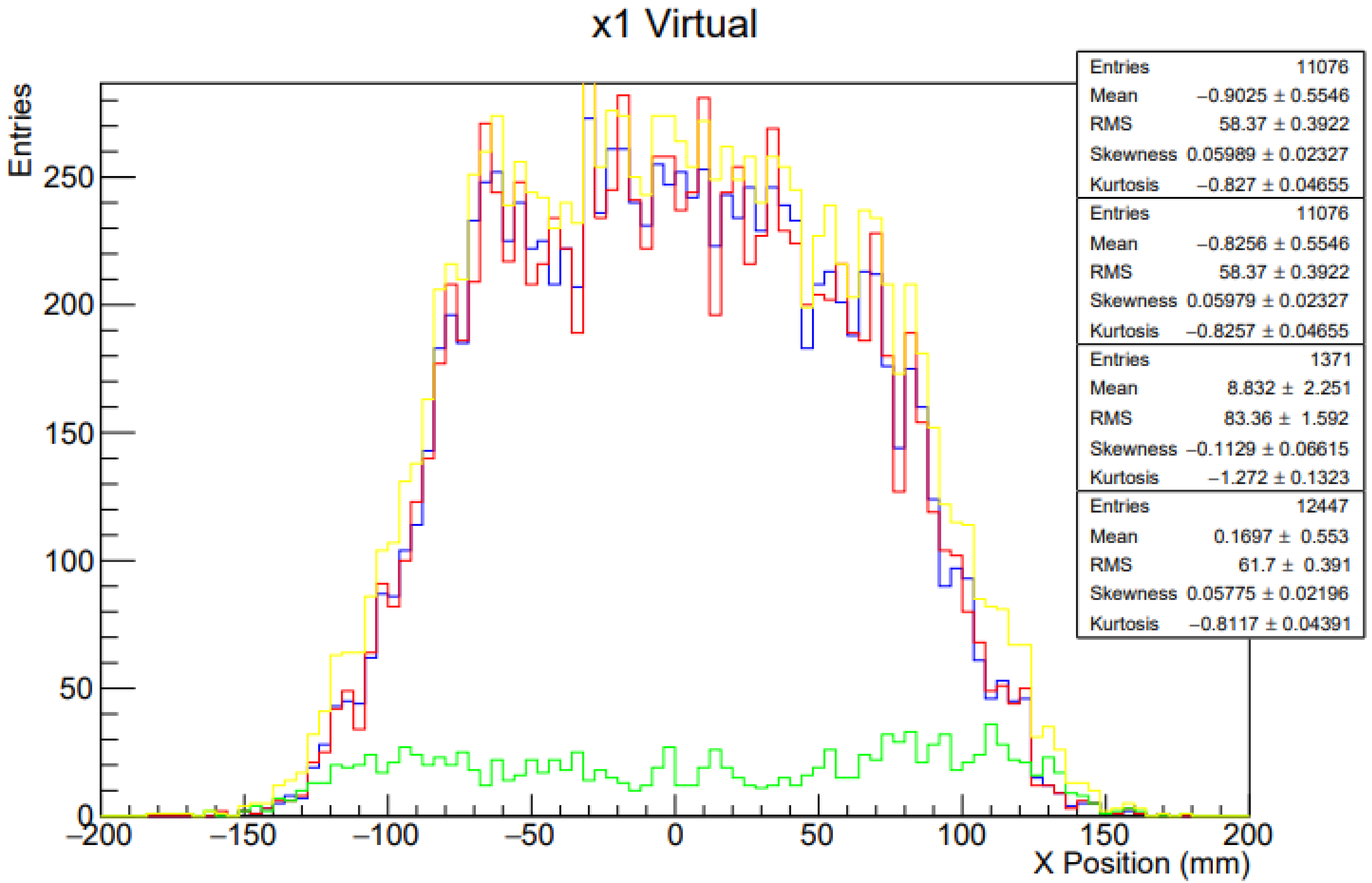
MC Truth and MC Recon

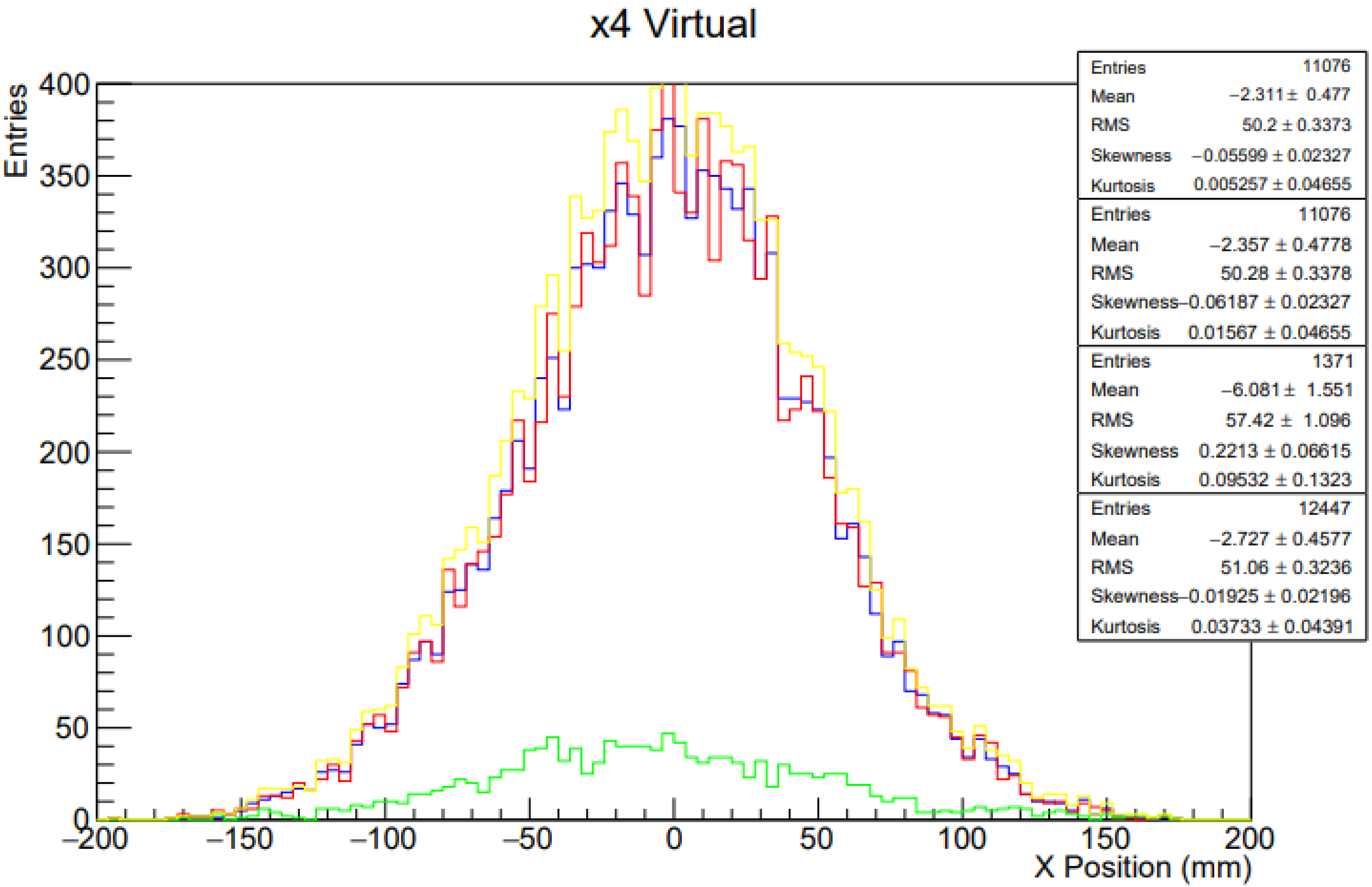
- Quantify discrepancy and compare bias between TKU and TKD
- Check Recon Efficiency
- Check Unreconstructed tracks have same distribution as Reconstructed tracks
  - i.e. make sure Reconstructed Events aren't accidentally gerrymandered and we only see what we want to see

MC Truth and MC Recon

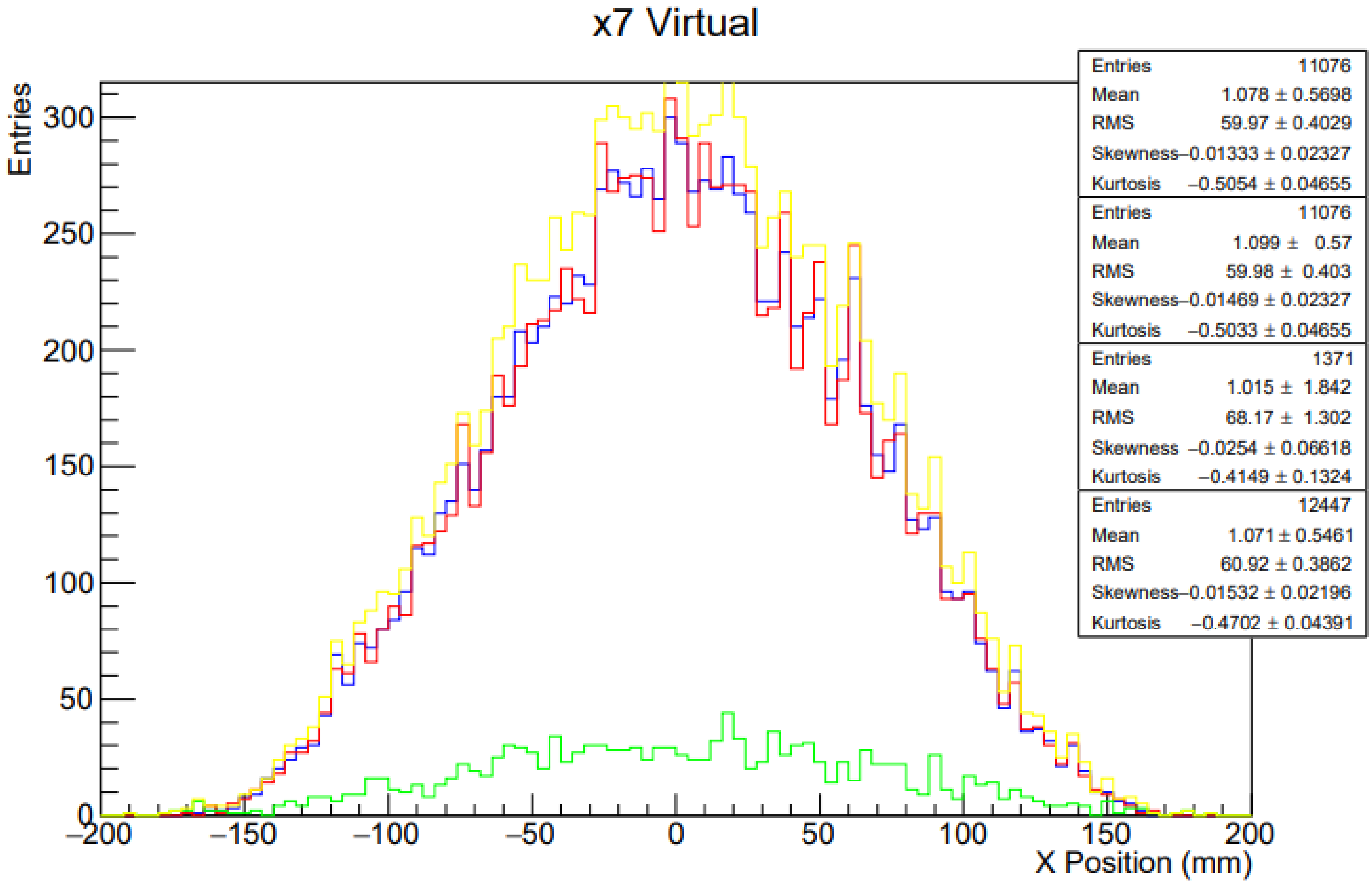
- Ran into bugs, so only have 10-140 solenoid Wedge data here
- No Absorber and Data Recon to be added in future
- Buyer Beware: Downstream Data will have asymmetry due to Wedge
- No cuts, except to insist the track is a Muon throughout in tracker

- 2 Trackers of 5 station each with 3 planes.
- In each of the following go from plane 1 which is most upstream plane, station and tracker all the way through to plane 30, the most downstream plane, station and Tracker
- Blue: MC Truth (That was Reconstructed)
- Red: MC Recon
- Green: MC Truth (Not Reconstructed)
- Yellow: MC Truth (Both Reconstructed and Not Reconstructed)

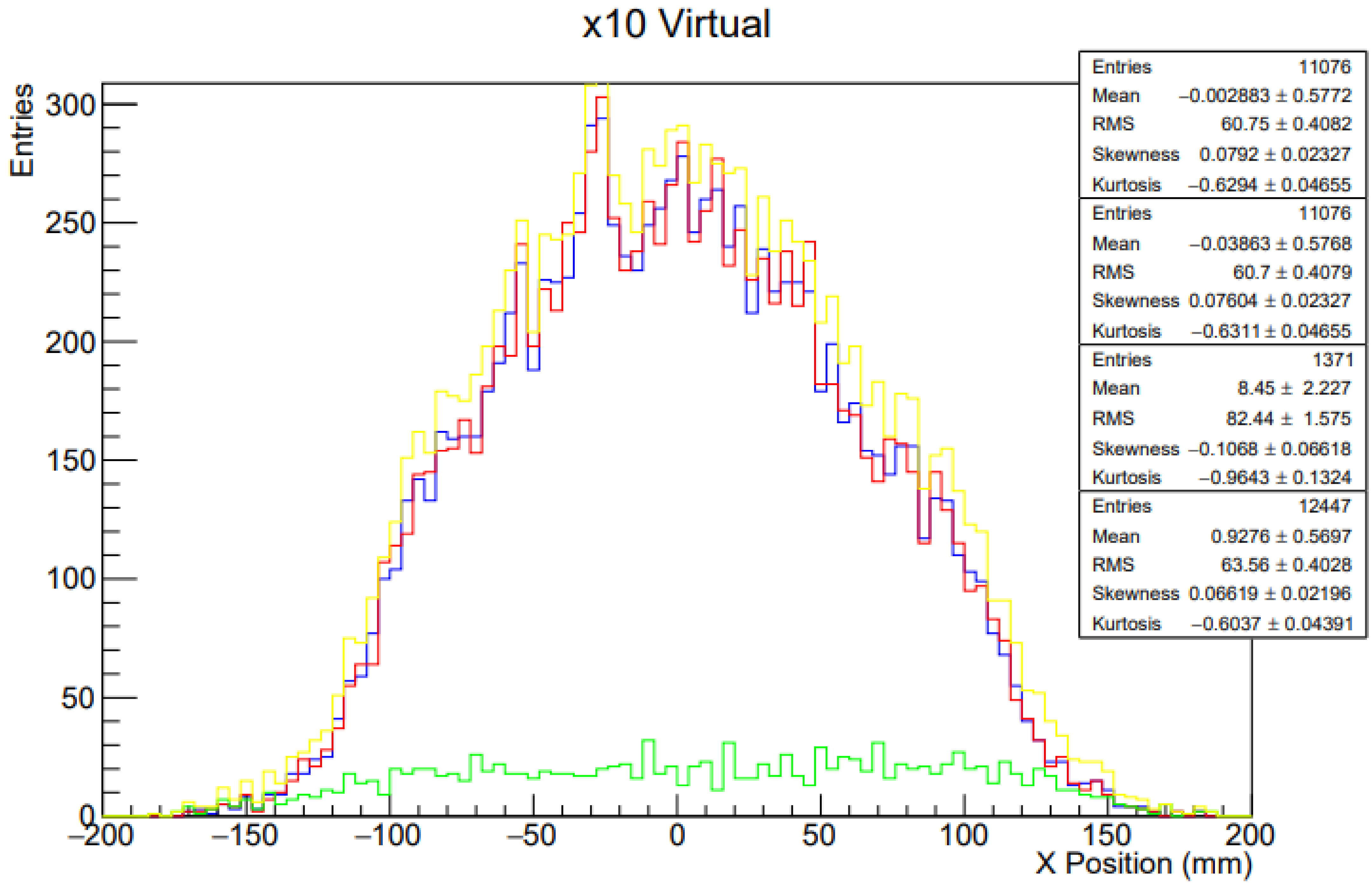

x1 Virtual
Entries
X Position (mm)
−200 −150 −100 −50 0 50 100 150 200
0 50 100 150 200 250
Entries 11076
Mean −0.9025 ± 0.5546
RMS 58.37 ± 0.3922
Skewness 0.05989 ± 0.02327
Kurtosis −0.827 ± 0.04655
Entries 11076
Mean −0.8256 ± 0.5546
RMS 58.37 ± 0.3922
Skewness 0.05979 ± 0.02327
Kurtosis −0.8257 ± 0.04655
Entries 1371
Mean 8.832 ± 2.251
RMS 83.36 ± 1.592
Skewness −0.1129 ± 0.06615
Kurtosis −1.272 ± 0.1323
Entries 12447
Mean 0.1697 ± 0.553
RMS 61.7 ± 0.391
Skewness 0.05775 ± 0.02196
Kurtosis −0.8117 ± 0.04391

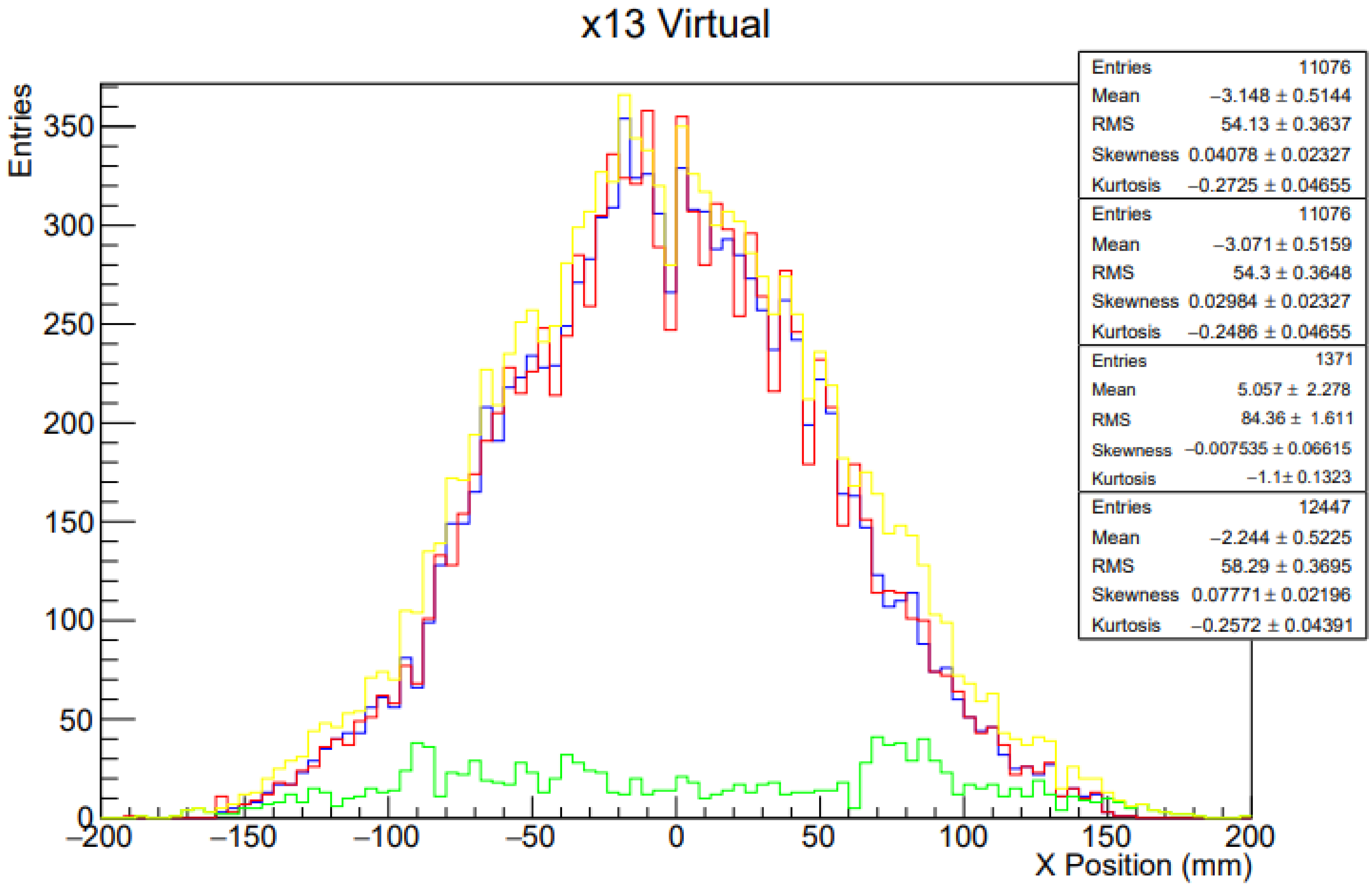


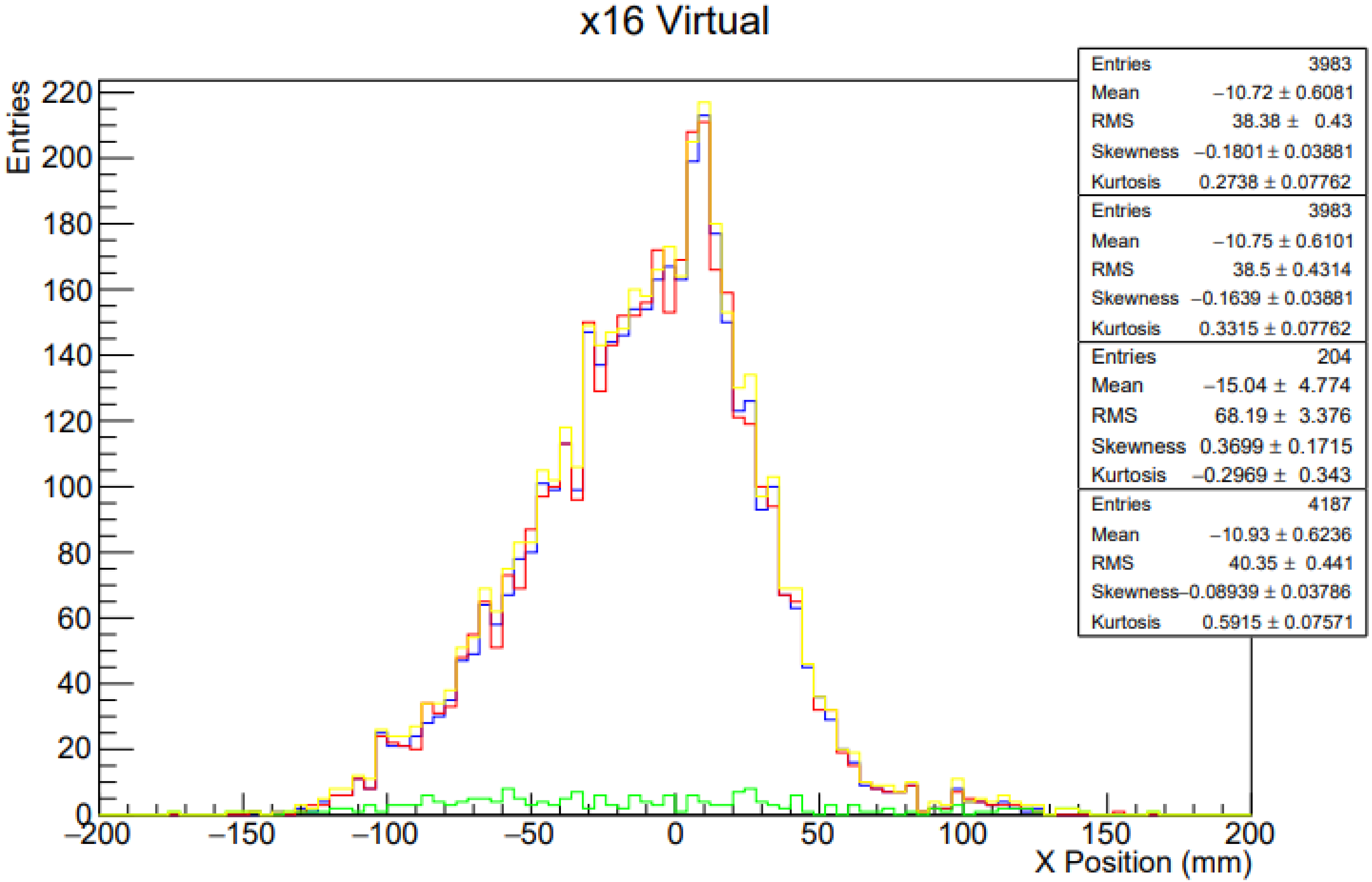
x4 Virtual
Entries
X Position (mm)
400
350
300
250
200
150
100
50
0
−200
−150
−100
−50
0
50
100
150
200
Entries 11076
Mean −2.311 ± 0.477
RMS 50.2 ± 0.3373
Skewness −0.05599 ± 0.02327
Kurtosis 0.005257 ± 0.04655
Entries 11076
Mean −2.357 ± 0.4778
RMS 50.28 ± 0.3378
Skewness −0.06187 ± 0.02327
Kurtosis 0.01567 ± 0.04655
Entries 1371
Mean −6.081 ± 1.551
RMS 57.42 ± 1.096
Skewness 0.2213 ± 0.06615
Kurtosis 0.09532 ± 0.1323
Entries 12447
Mean −2.727 ± 0.4577
RMS 51.06 ± 0.3236
Skewness −0.01925 ± 0.02196
Kurtosis 0.03733 ± 0.04391

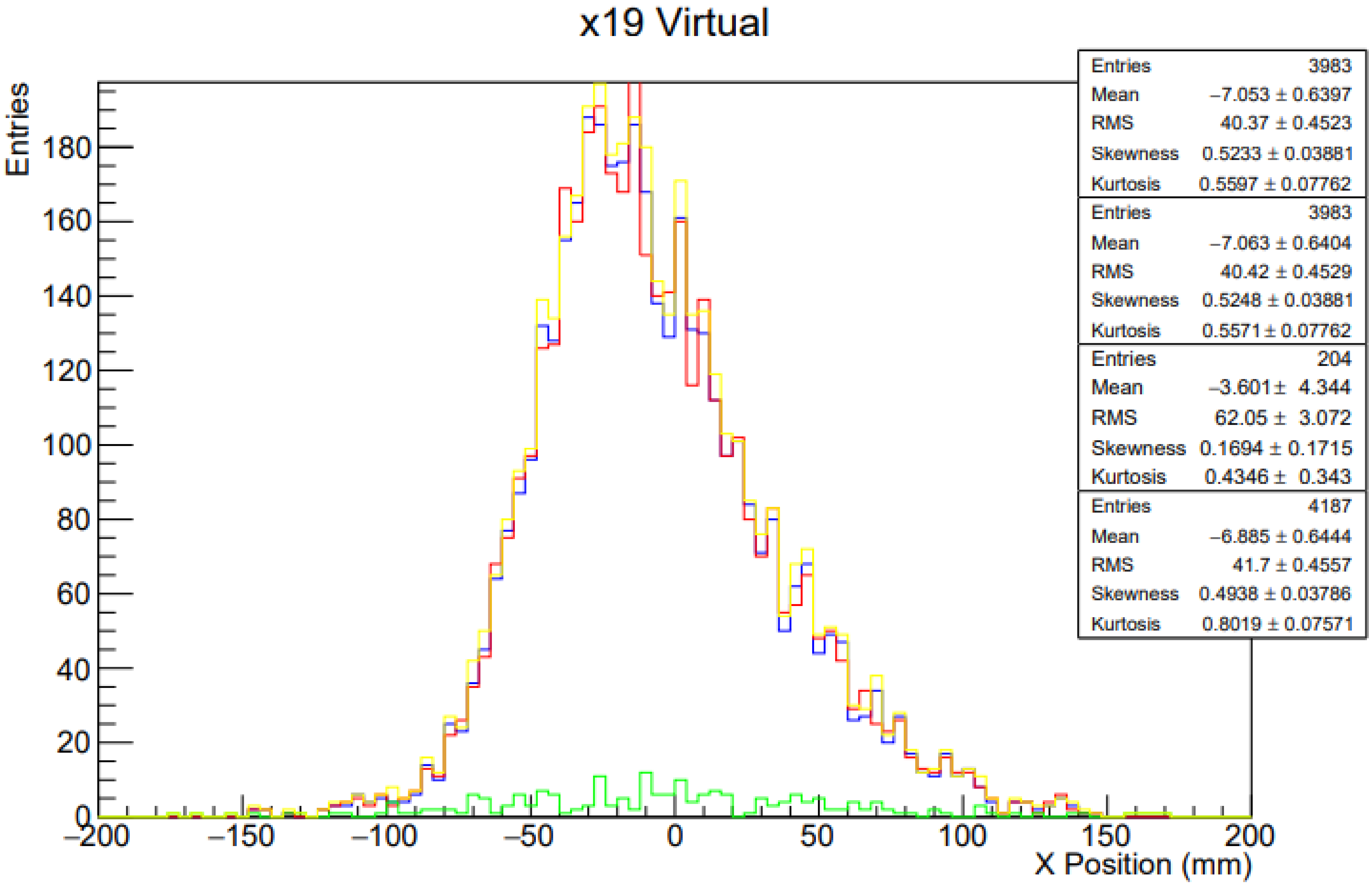


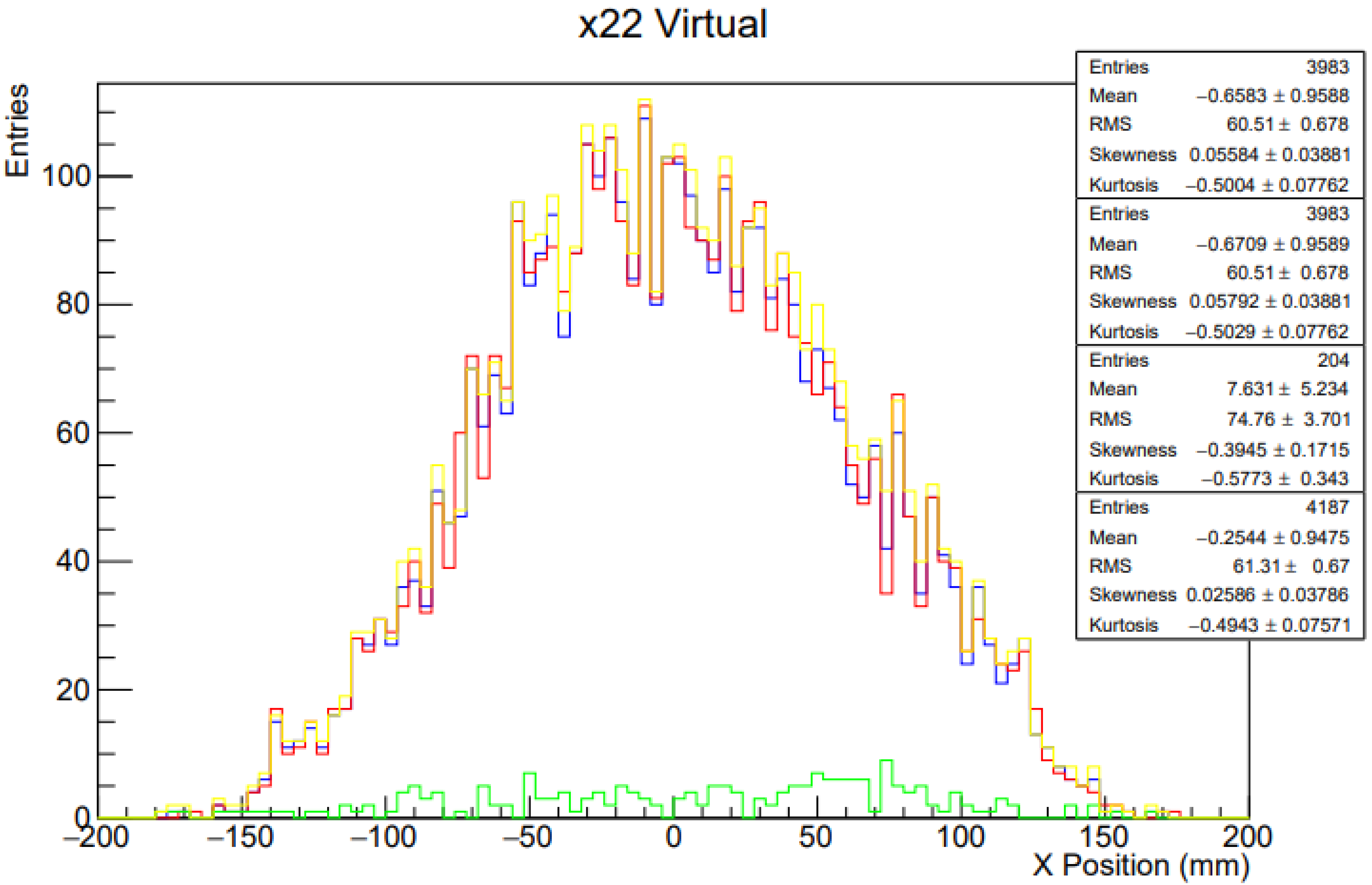
x7 Virtual
Entries
X Position (mm)
Entries 11076
Mean 1.078 ± 0.5698
RMS 59.97 ± 0.4029
Skewness −0.01333 ± 0.02327
Kurtosis −0.5054 ± 0.04655
Entries 11076
Mean 1.099 ± 0.57
RMS 59.98 ± 0.403
Skewness −0.01469 ± 0.02327
Kurtosis −0.5033 ± 0.04655
Entries 1371
Mean 1.015 ± 1.842
RMS 68.17 ± 1.302
Skewness −0.0254 ± 0.06618
Kurtosis −0.4149 ± 0.1324
Entries 12447
Mean 1.071 ± 0.5461
RMS 60.92 ± 0.3862
Skewness −0.01532 ± 0.02196
Kurtosis −0.4702 ± 0.04391


x10 Virtual
Entries
X Position (mm)
Entries 11076
Mean −0.002883 ± 0.5772
RMS 60.75 ± 0.4082
Skewness 0.0792 ± 0.02327
Kurtosis −0.6294 ± 0.04655
Entries 11076
Mean −0.03863 ± 0.5768
RMS 60.7 ± 0.4079
Skewness 0.07604 ± 0.02327
Kurtosis −0.6311 ± 0.04655
Entries 1371
Mean 8.45 ± 2.227
RMS 82.44 ± 1.575
Skewness −0.1068 ± 0.06618
Kurtosis −0.9643 ± 0.1324
Entries 12447
Mean 0.9276 ± 0.5697
RMS 63.56 ± 0.4028
Skewness 0.06619 ± 0.02196
Kurtosis −0.6037 ± 0.04391


x13 Virtual
Entries
X Position (mm)
Entries 11076
Mean −3.148 ± 0.5144
RMS 54.13 ± 0.3637
Skewness 0.04078 ± 0.02327
Kurtosis −0.2725 ± 0.04655
Entries 11076
Mean −3.071 ± 0.5159
RMS 54.3 ± 0.3648
Skewness 0.02984 ± 0.02327
Kurtosis −0.2486 ± 0.04655
Entries 1371
Mean 5.057 ± 2.278
RMS 84.36 ± 1.611
Skewness −0.007535 ± 0.06615
Kurtosis −1.1 ± 0.1323
Entries 12447
Mean −2.244 ± 0.5225
RMS 58.29 ± 0.3695
Skewness 0.07771 ± 0.02196
Kurtosis −0.2572 ± 0.04391

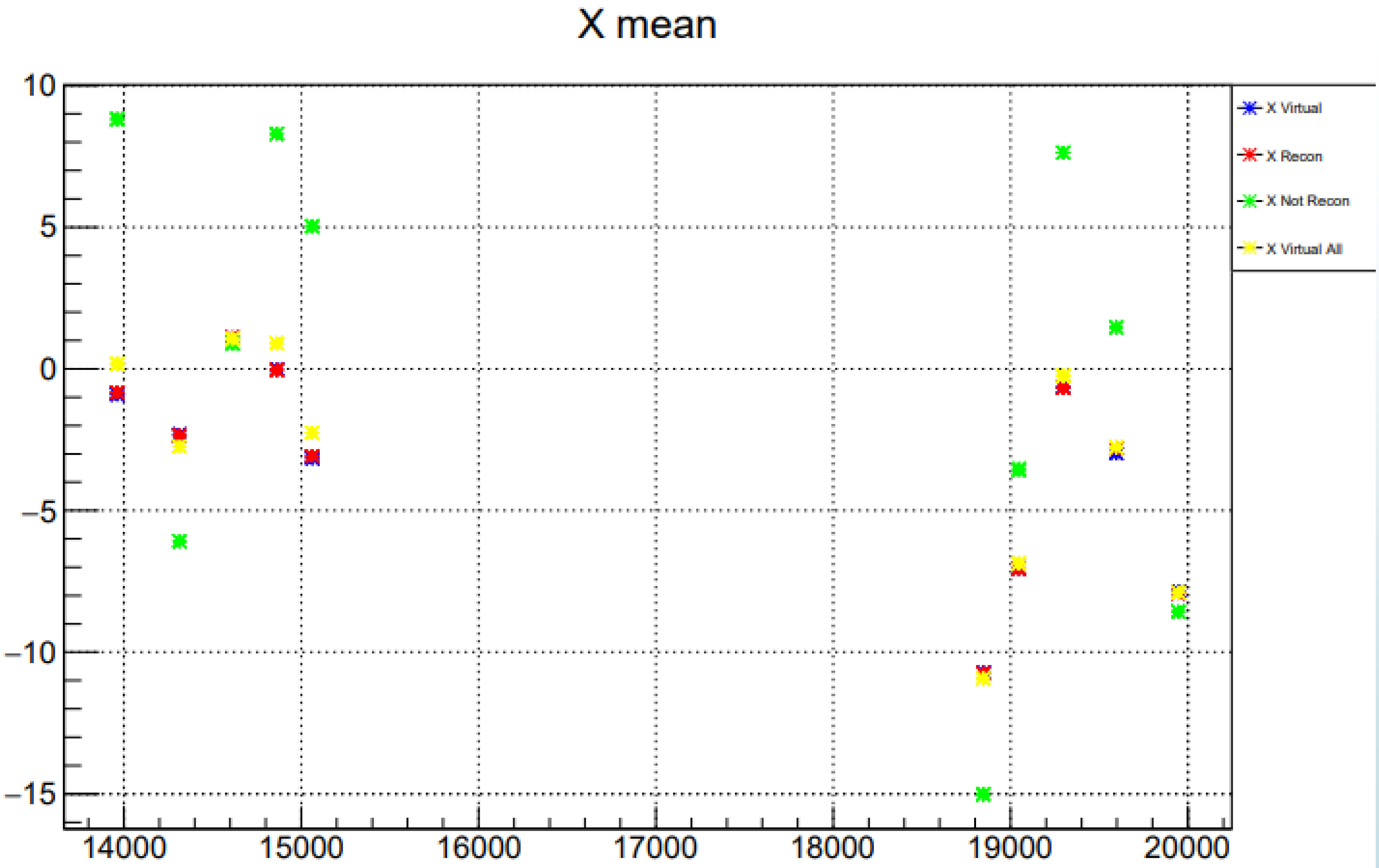

x16 Virtual
Entries
X Position (mm)
Entries 3983
Mean −10.72 ± 0.6081
RMS 38.38 ± 0.43
Skewness −0.1801 ± 0.03881
Kurtosis 0.2738 ± 0.07762
Entries 3983
Mean −10.75 ± 0.6101
RMS 38.5 ± 0.4314
Skewness −0.1639 ± 0.03881
Kurtosis 0.3315 ± 0.07762
Entries 204
Mean −15.04 ± 4.774
RMS 68.19 ± 3.376
Skewness 0.3699 ± 0.1715
Kurtosis −0.2969 ± 0.343
Entries 4187
Mean −10.93 ± 0.6236
RMS 40.35 ± 0.441
Skewness −0.08939 ± 0.03786
Kurtosis 0.5915 ± 0.07571

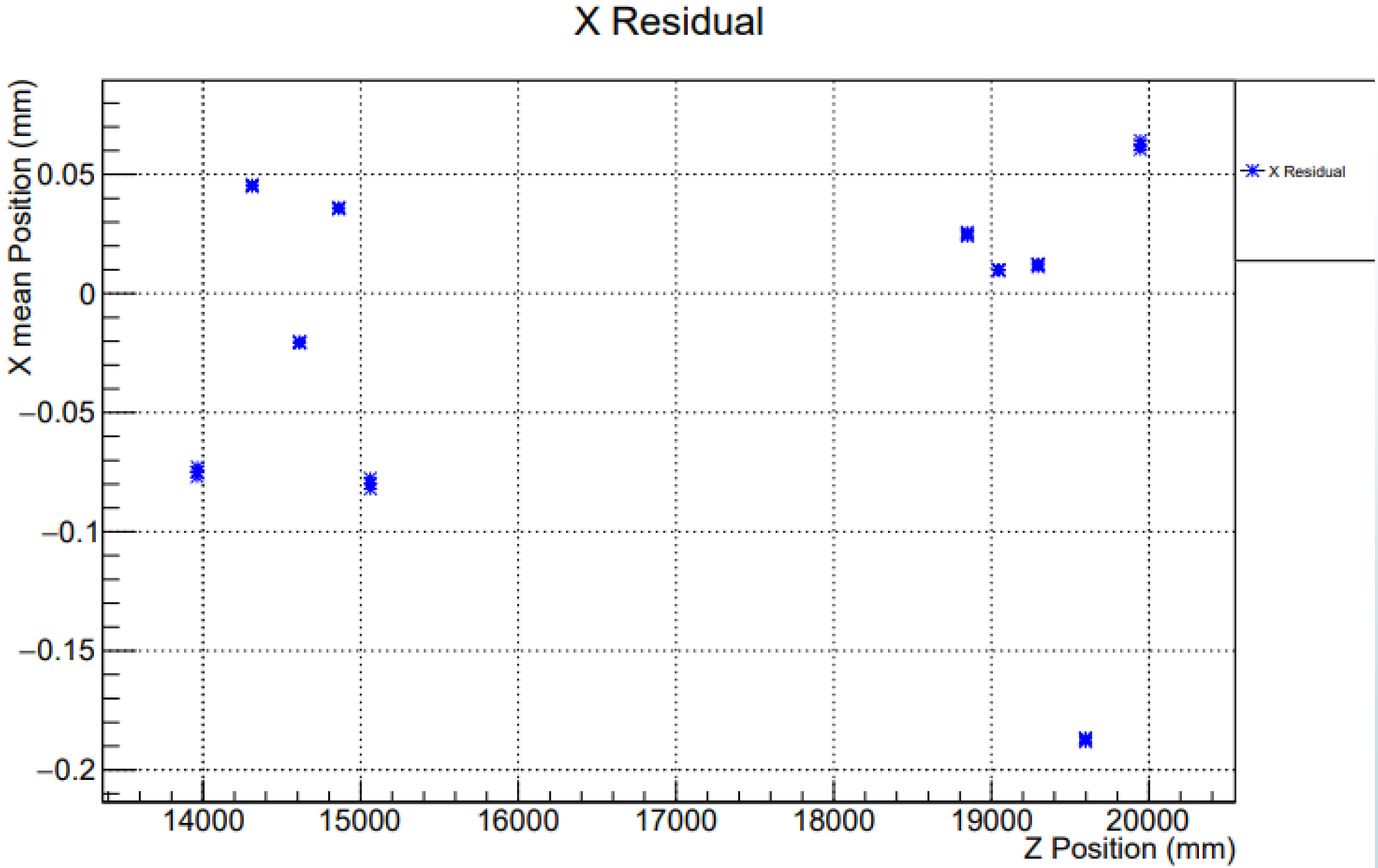

x19 Virtual
Entries
X Position (mm)
0
20
40
60
80
100
120
140
160
180
−200
−150
−100
−50
0
50
100
150
200
Entries 3983
Mean −7.053 ± 0.6397
RMS 40.37 ± 0.4523
Skewness 0.5233 ± 0.03881
Kurtosis 0.5597 ± 0.07762
Entries 3983
Mean −7.063 ± 0.6404
RMS 40.42 ± 0.4529
Skewness 0.5248 ± 0.03881
Kurtosis 0.5571 ± 0.07762
Entries 204
Mean −3.601 ± 4.344
RMS 62.05 ± 3.072
Skewness 0.1694 ± 0.1715
Kurtosis 0.4346 ± 0.343
Entries 4187
Mean −6.885 ± 0.6444
RMS 41.7 ± 0.4557
Skewness 0.4938 ± 0.03786
Kurtosis 0.8019 ± 0.07571


x22 Virtual
Entries
X Position (mm)
0
20
40
60
80
100
−200
−150
−100
−50
0
50
100
150
200
Entries 3983
Mean −0.6583 ± 0.9588
RMS 60.51 ± 0.678
Skewness 0.05584 ± 0.03881
Kurtosis −0.5004 ± 0.07762
Entries 3983
Mean −0.6709 ± 0.9589
RMS 60.51 ± 0.678
Skewness 0.05792 ± 0.03881
Kurtosis −0.5029 ± 0.07762
Entries 204
Mean 7.631 ± 5.234
RMS 74.76 ± 3.701
Skewness −0.3945 ± 0.1715
Kurtosis −0.5773 ± 0.343
Entries 4187
Mean −0.2544 ± 0.9475
RMS 61.31 ± 0.67
Skewness 0.02586 ± 0.03786
Kurtosis −0.4943 ± 0.07571

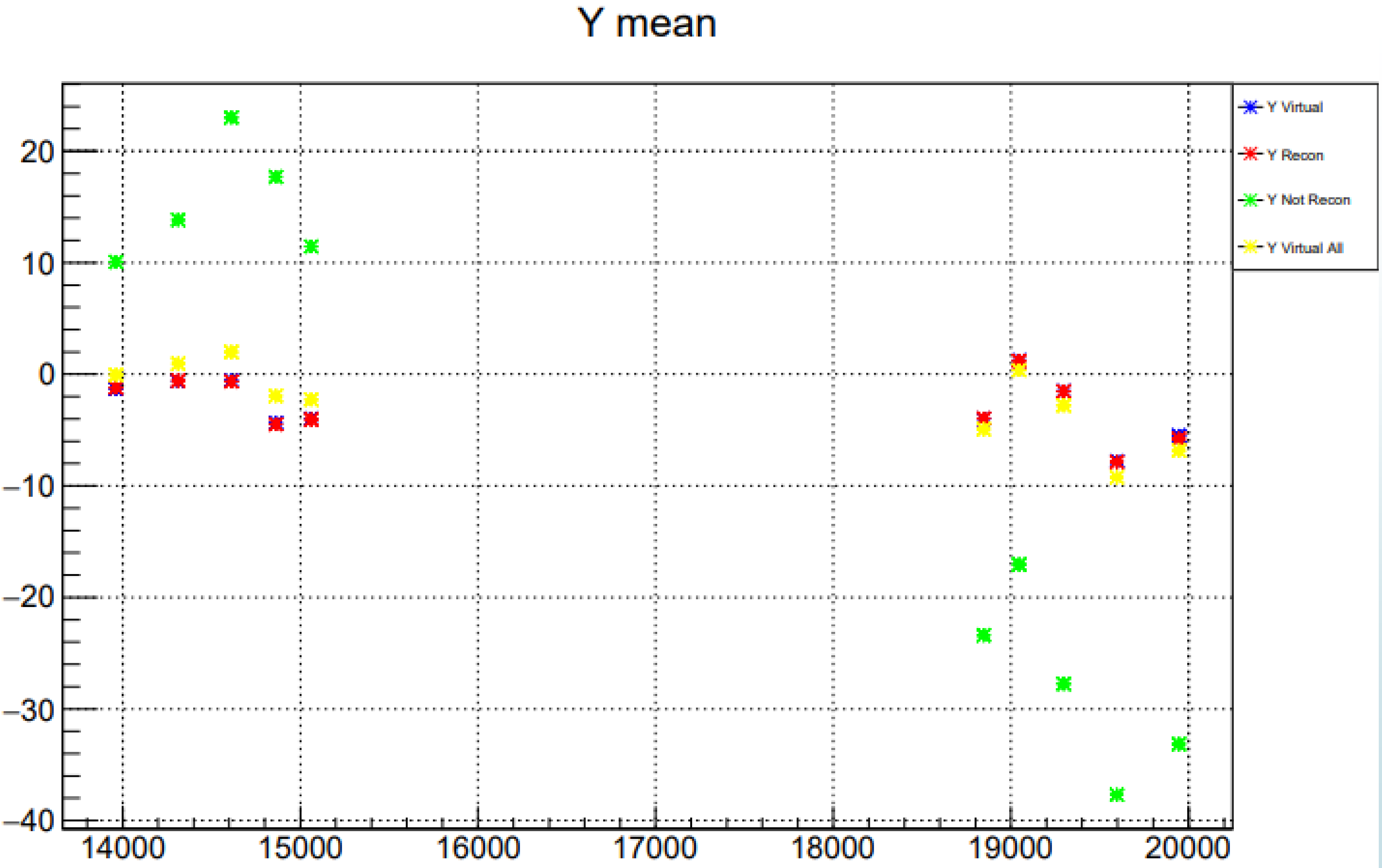
And so on

- Can be more useful to look at the change in moments (will only look at mean here) as well as residual for bias
- Plots are over Z Position
- Note, no cuts, so certain particles will drastically pull means
- Still useful to look at Distributions for certain cases to see where bias kicks in
- Effects are different between Upstream and Downstream as noted by different Recon Efficiency


X mean
X Virtual
X Recon
X Not Recon
X Virtual All
10
5
0
−5
−10
−15
14000
15000
16000
17000
18000
19000
20000

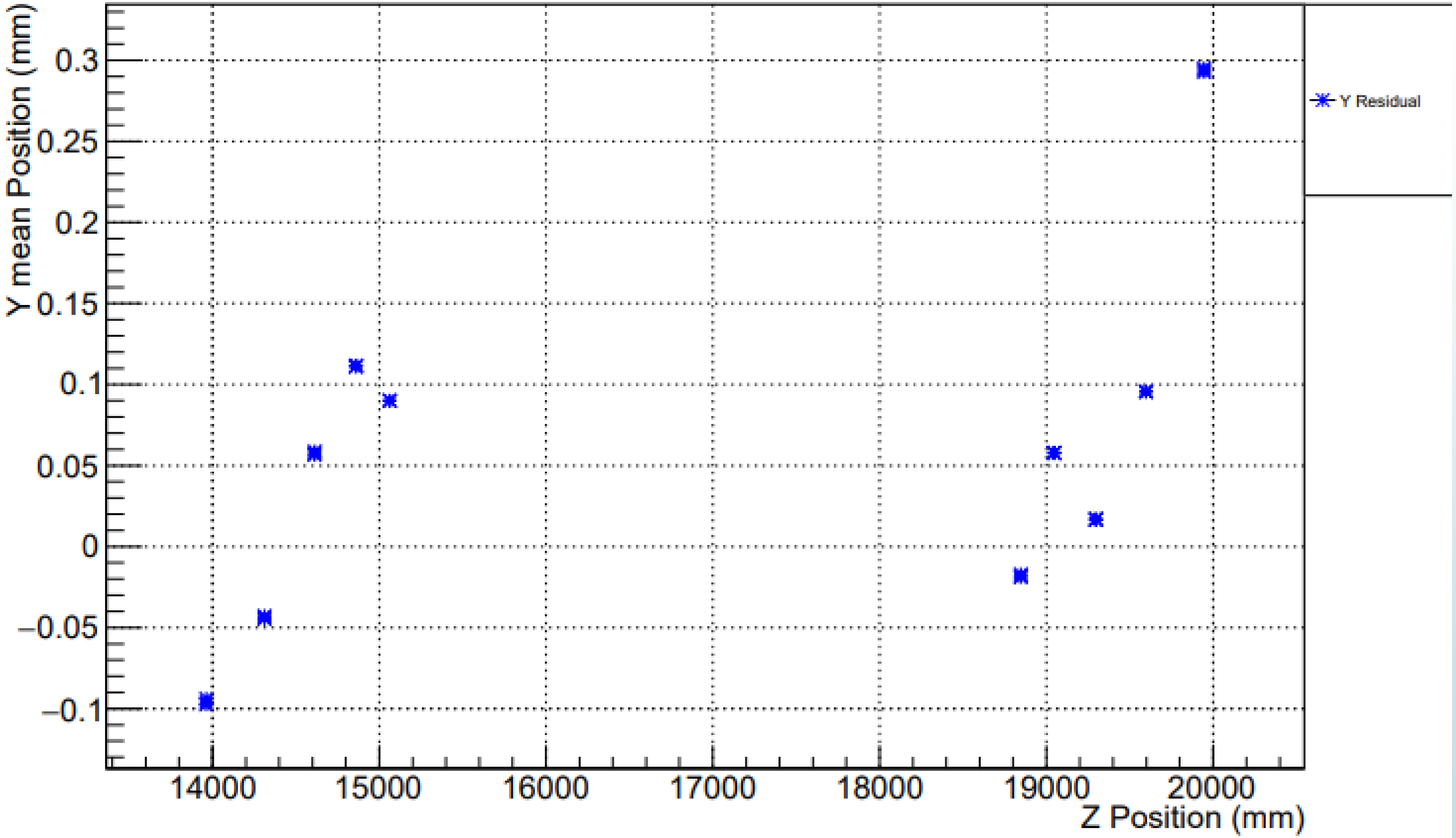


X Residual
X mean Position (mm)
0.05
0
−0.05
−0.1
−0.15
−0.2
14000
15000
16000
17000
18000
19000
20000
Z Position (mm)
X Residual



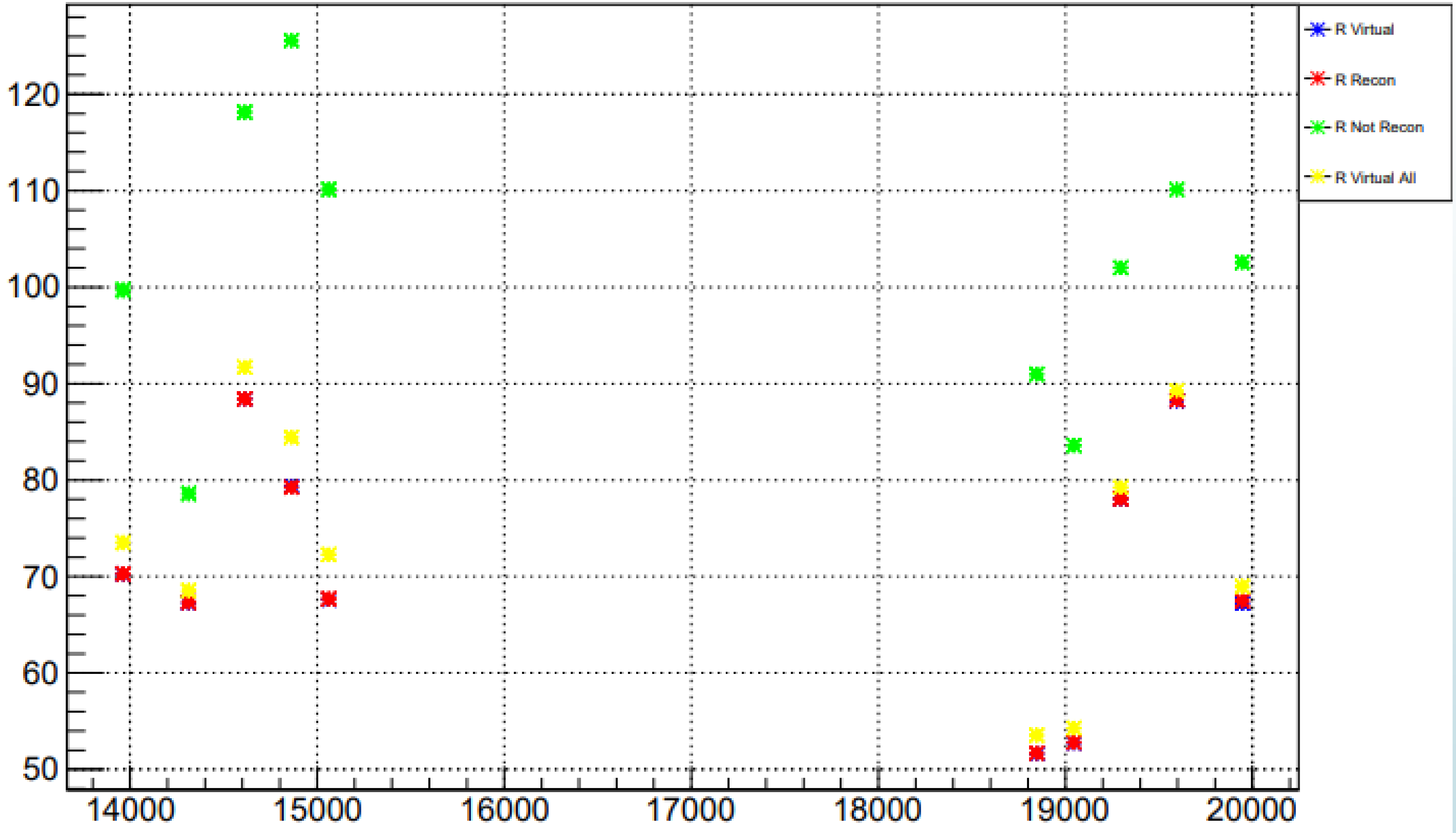
Y mean
Y Virtual
Y Recon
Y Not Recon
Y Virtual All
20
10
0
−10
−20
−30
−40
14000
15000
16000
17000
18000
19000
20000

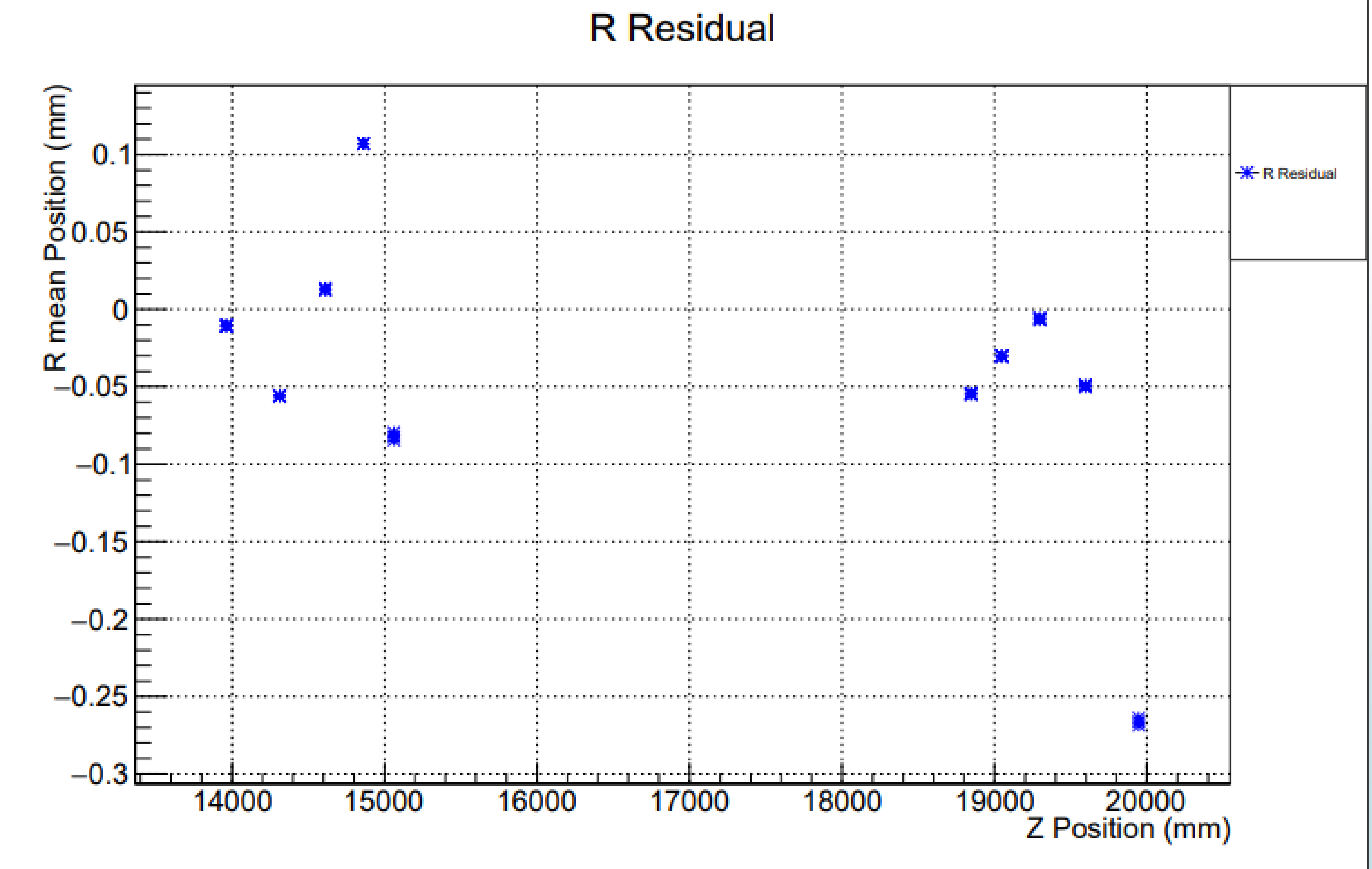
Y Residual



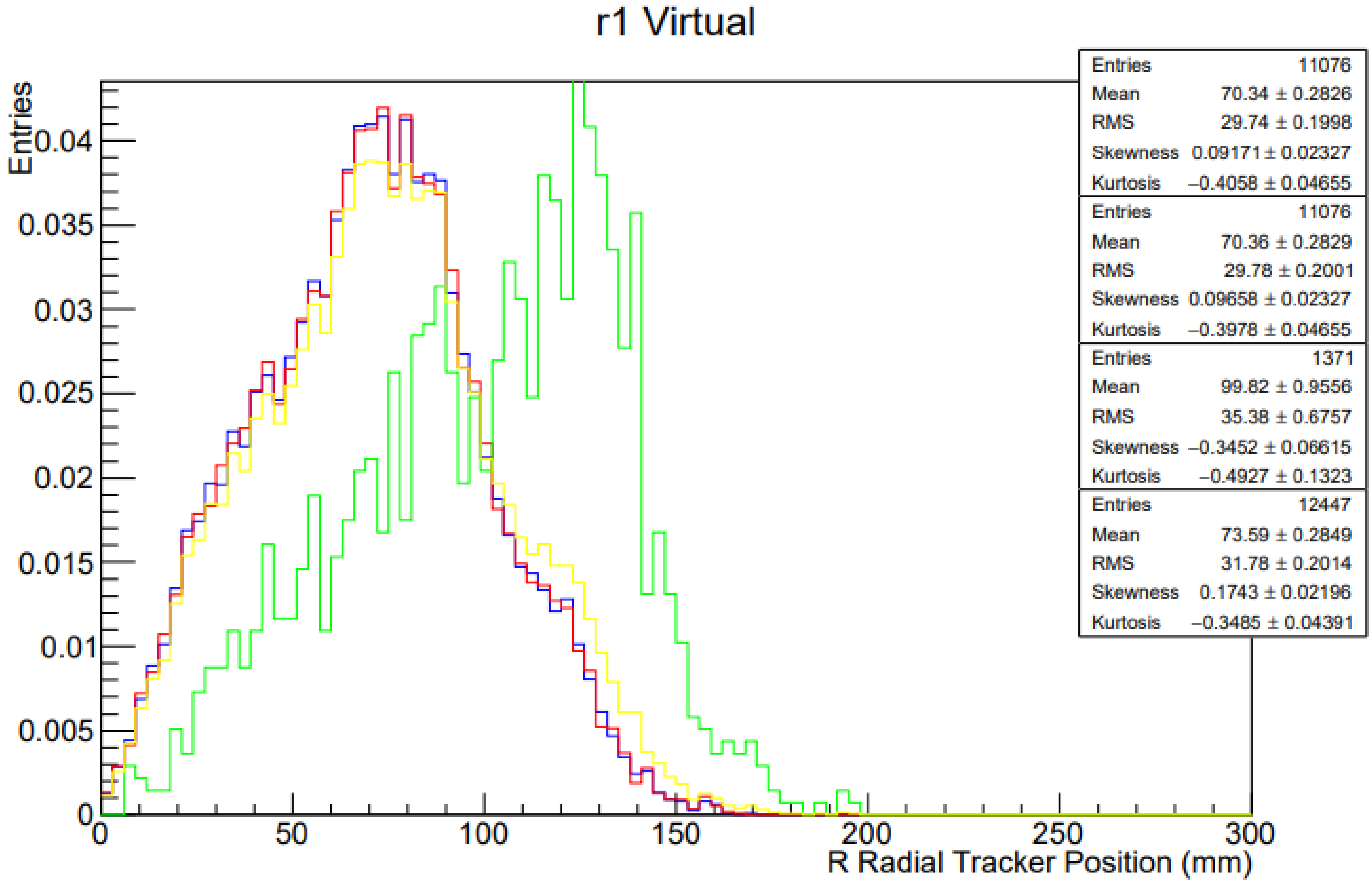
Y mean Position (mm)
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
14000
15000
16000
17000
18000
19000
20000
Z Position (mm)
Y Residual

R mean


120
110
100
90
80
70
60
50
14000
15000
16000
17000
18000
19000
20000
R Virtual
R Recon
R Not Recon
R Virtual All

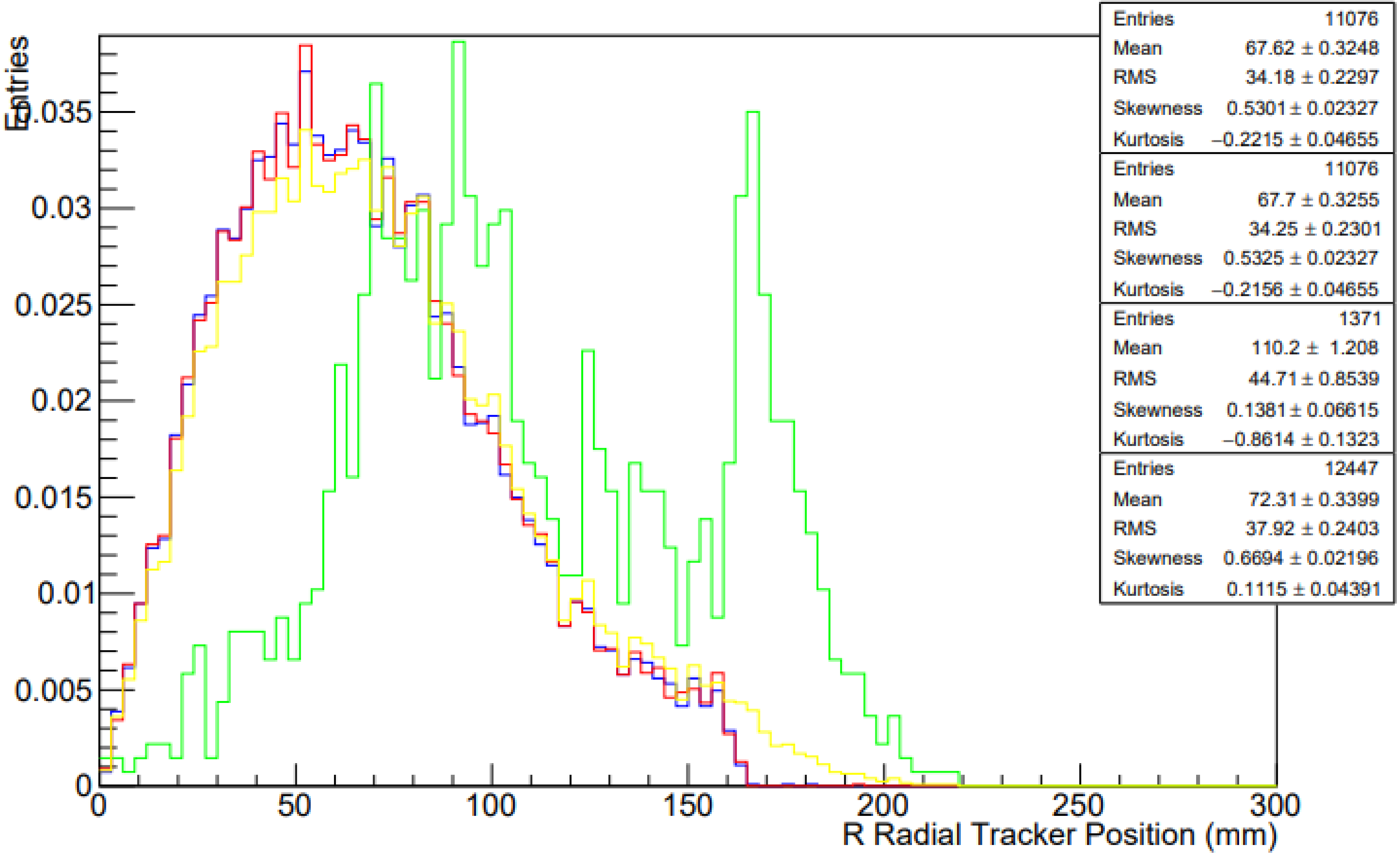


R Residual
R mean Position (mm)
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
−0.3
14000
15000
16000
17000
18000
19000
20000
Z Position (mm)
R Residual



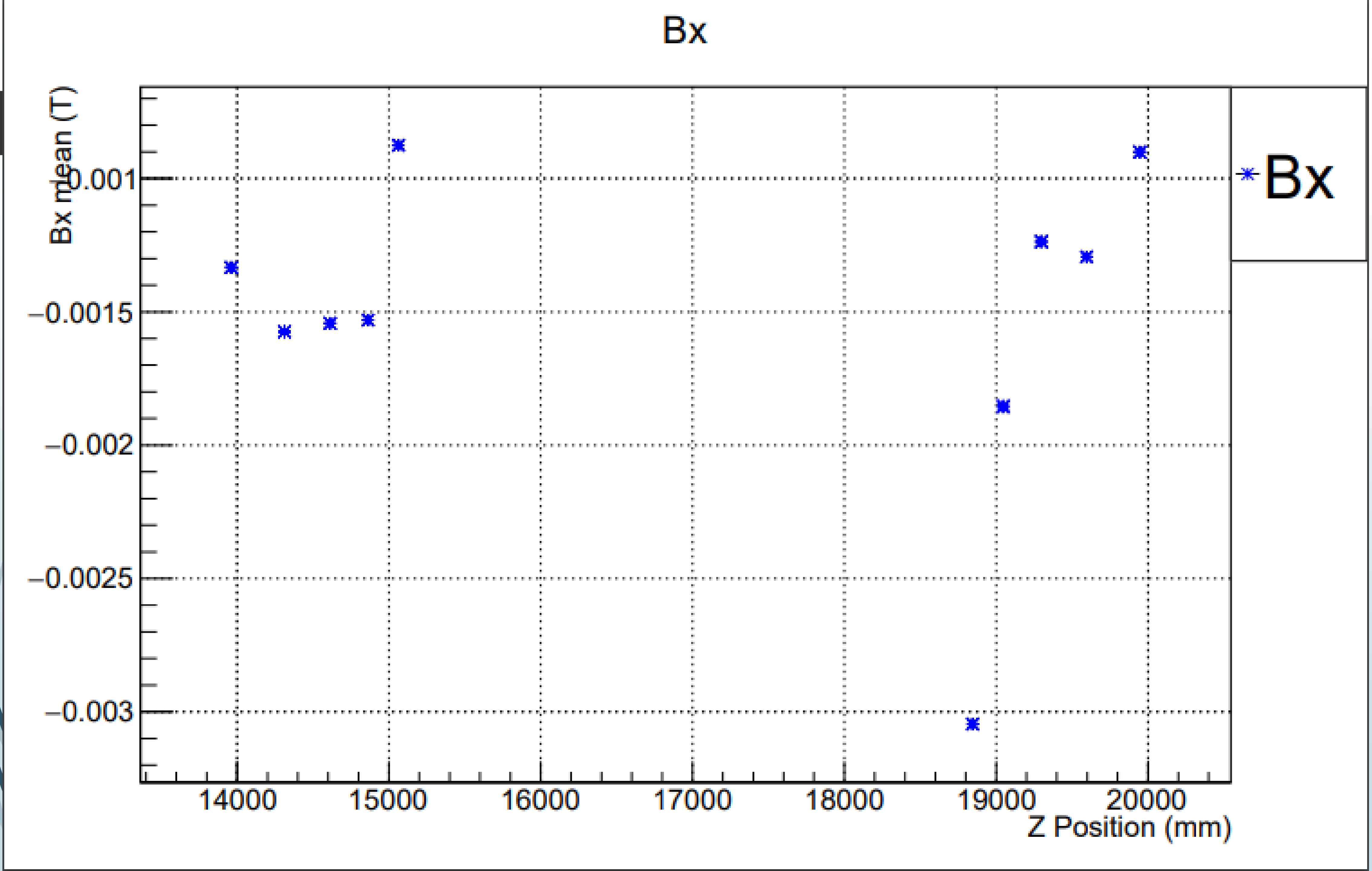
r1 Virtual
Entries
R Radial Tracker Position (mm)
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0
50
100
150
200
250
300
Entries 11076
Mean 70.34 ± 0.2826
RMS 29.74 ± 0.1998
Skewness 0.09171 ± 0.02327
Kurtosis −0.4058 ± 0.04655
Entries 11076
Mean 70.36 ± 0.2829
RMS 29.78 ± 0.2001
Skewness 0.09658 ± 0.02327
Kurtosis −0.3978 ± 0.04655
Entries 1371
Mean 99.82 ± 0.9556
RMS 35.38 ± 0.6757
Skewness −0.3452 ± 0.06615
Kurtosis −0.4927 ± 0.1323
Entries 12447
Mean 73.59 ± 0.2849
RMS 31.78 ± 0.2014
Skewness 0.1743 ± 0.02196
Kurtosis −0.3485 ± 0.04391

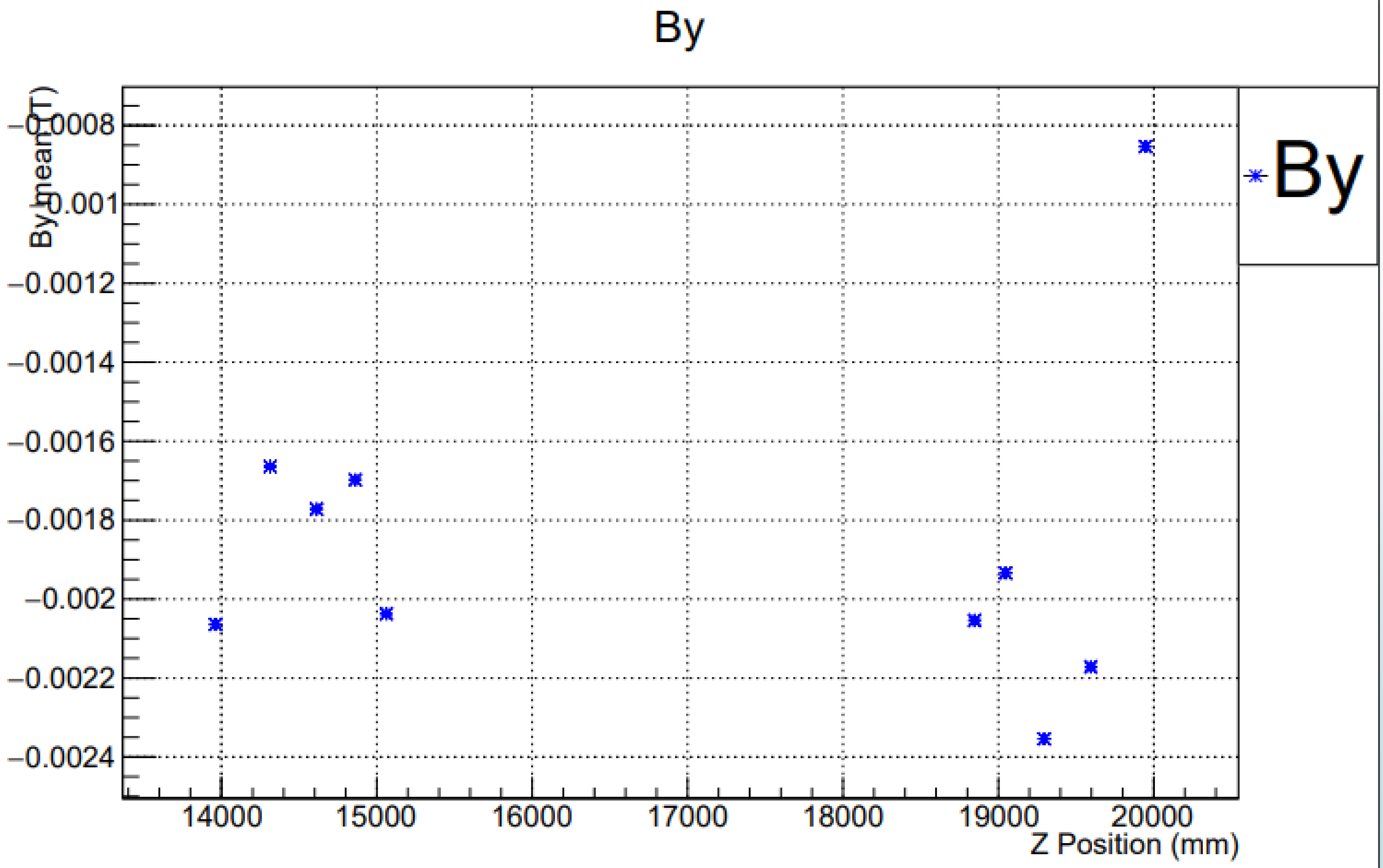
r13 Virtual

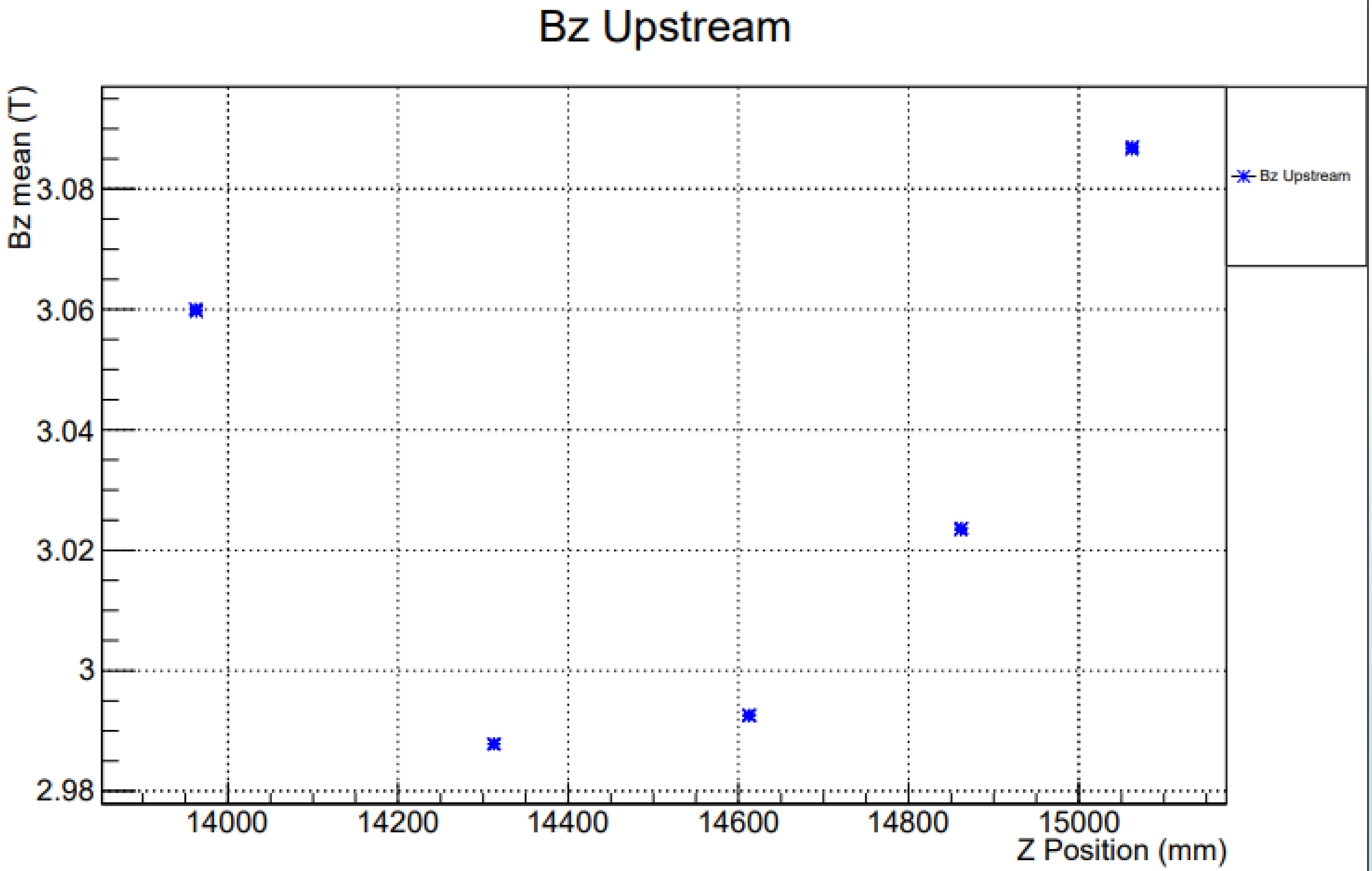

Entries
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
50
100
150
200
250
300
R Radial Tracker Position (mm)
Entries 11076
Mean 67.62 ± 0.3248
RMS 34.18 ± 0.2297
Skewness 0.5301 ± 0.02327
Kurtosis −0.2215 ± 0.04655
Entries 11076
Mean 67.7 ± 0.3255
RMS 34.25 ± 0.2301
Skewness 0.5325 ± 0.02327
Kurtosis −0.2156 ± 0.04655
Entries 1371
Mean 110.2 ± 1.208
RMS 44.71 ± 0.8539
Skewness 0.1381 ± 0.06615
Kurtosis −0.8614 ± 0.1323
Entries 12447
Mean 72.31 ± 0.3399
RMS 37.92 ± 0.2403
Skewness 0.6694 ± 0.02196
Kurtosis 0.1115 ± 0.04391

Radial Distribution

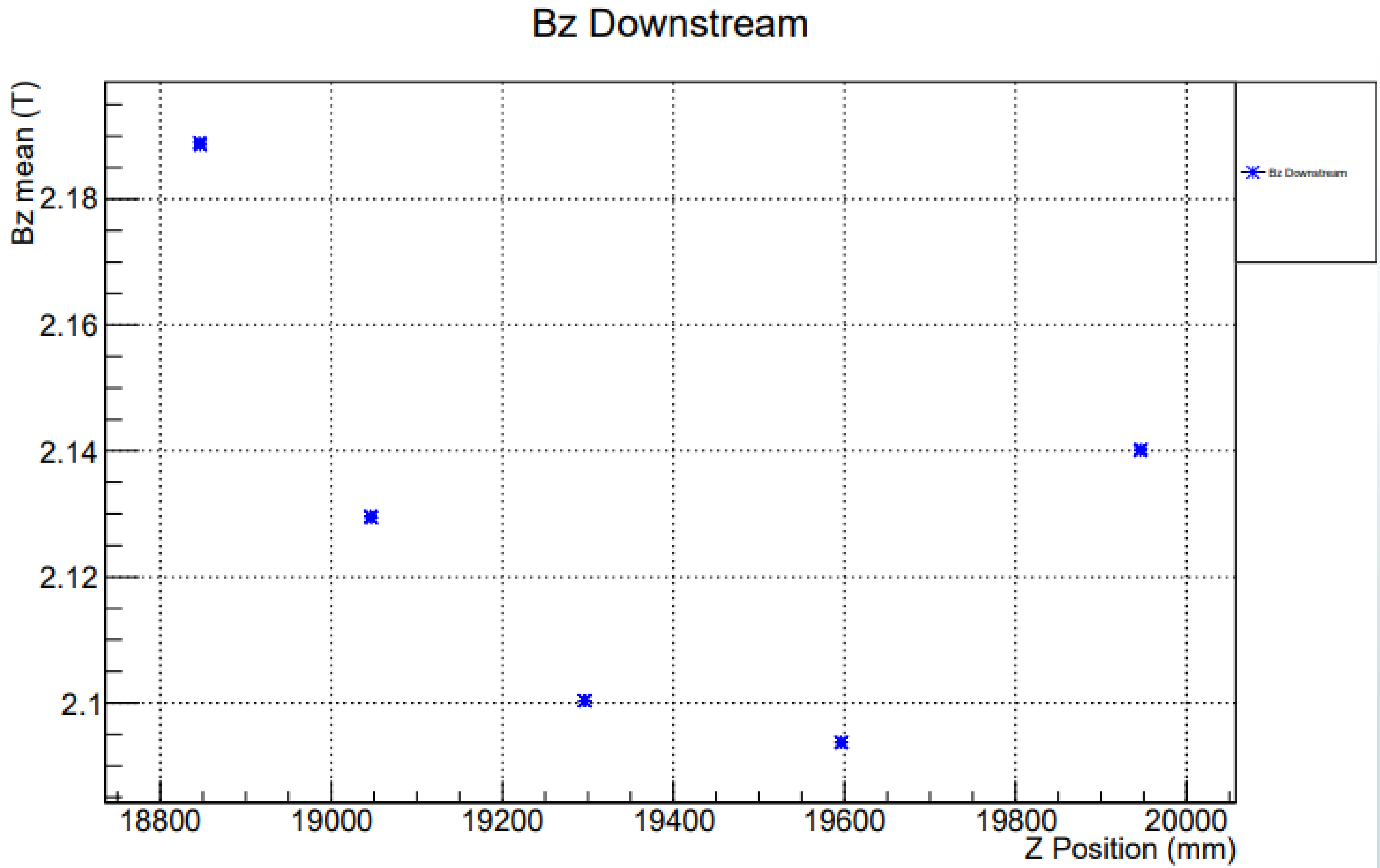
- Last 2 plots are normalised
- Particles at a higher Radial Position in the tracker are more likely not to be Reconstructed
- This is not all bad as many of those particles will be outside the radial acceptance of the tracker and will be eliminated through cuts in the future
- On the other hand, assume constant Magnetic field, while it is true that the magnets were kept at the same currents and so there is little variability in the field, there is variability with Z position that the Reconstruction doesn't take account of, as it uses a mean field


Bx
Bx mean (T)
-0.001
−0.0015
−0.002
−0.0025
−0.003
14000
15000
16000
17000
18000
19000
20000
Z Position (mm)
Bx

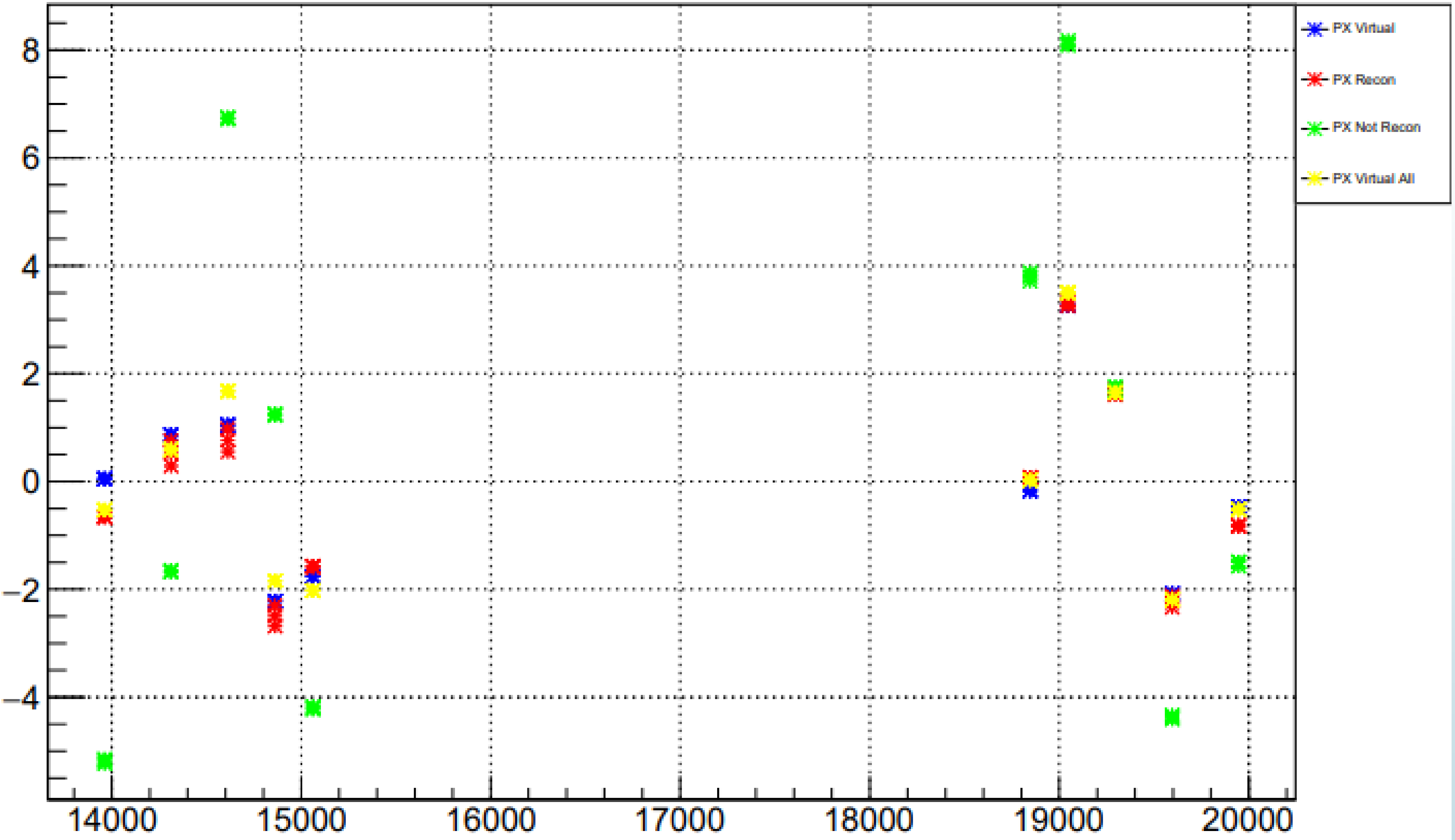

By
By
By (mean T)
−0.0008
−0.001
−0.0012
−0.0014
−0.0016
−0.0018
−0.002
−0.0022
−0.0024
14000
15000
16000
17000
18000
19000
20000
Z Position (mm)



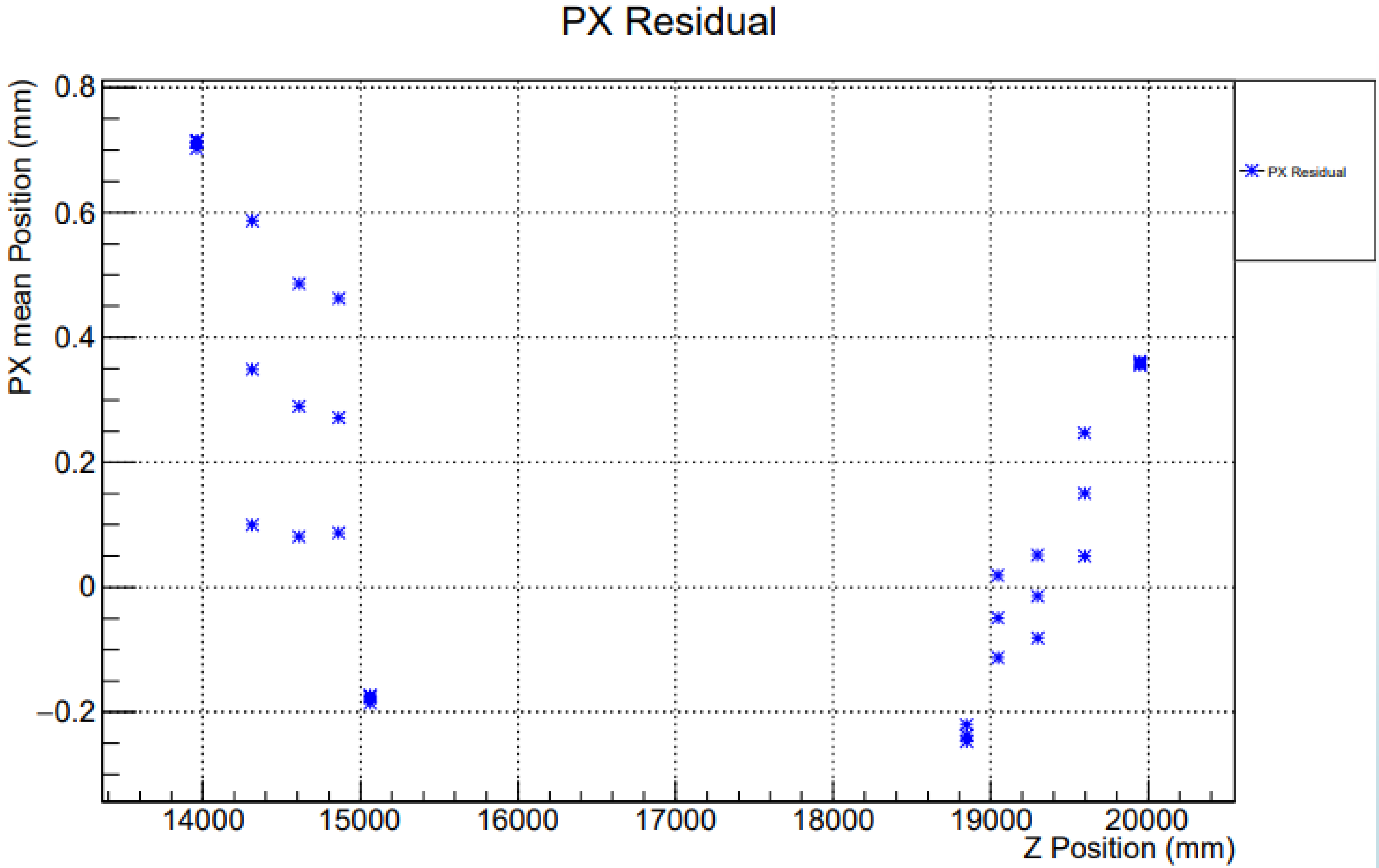
Bz Upstream
Bz mean (T)
3.08
3.06
3.04
3.02
3
2.98
14000
14200
14400
14600
14800
15000
Z Position (mm)
Bz Upstream


Bz Downstream
Bz mean (T)
2.18
2.16
2.14
2.12
2.1
18800
19000
19200
19400
19600
19800
20000
Z Position (mm)
Bz Downstream

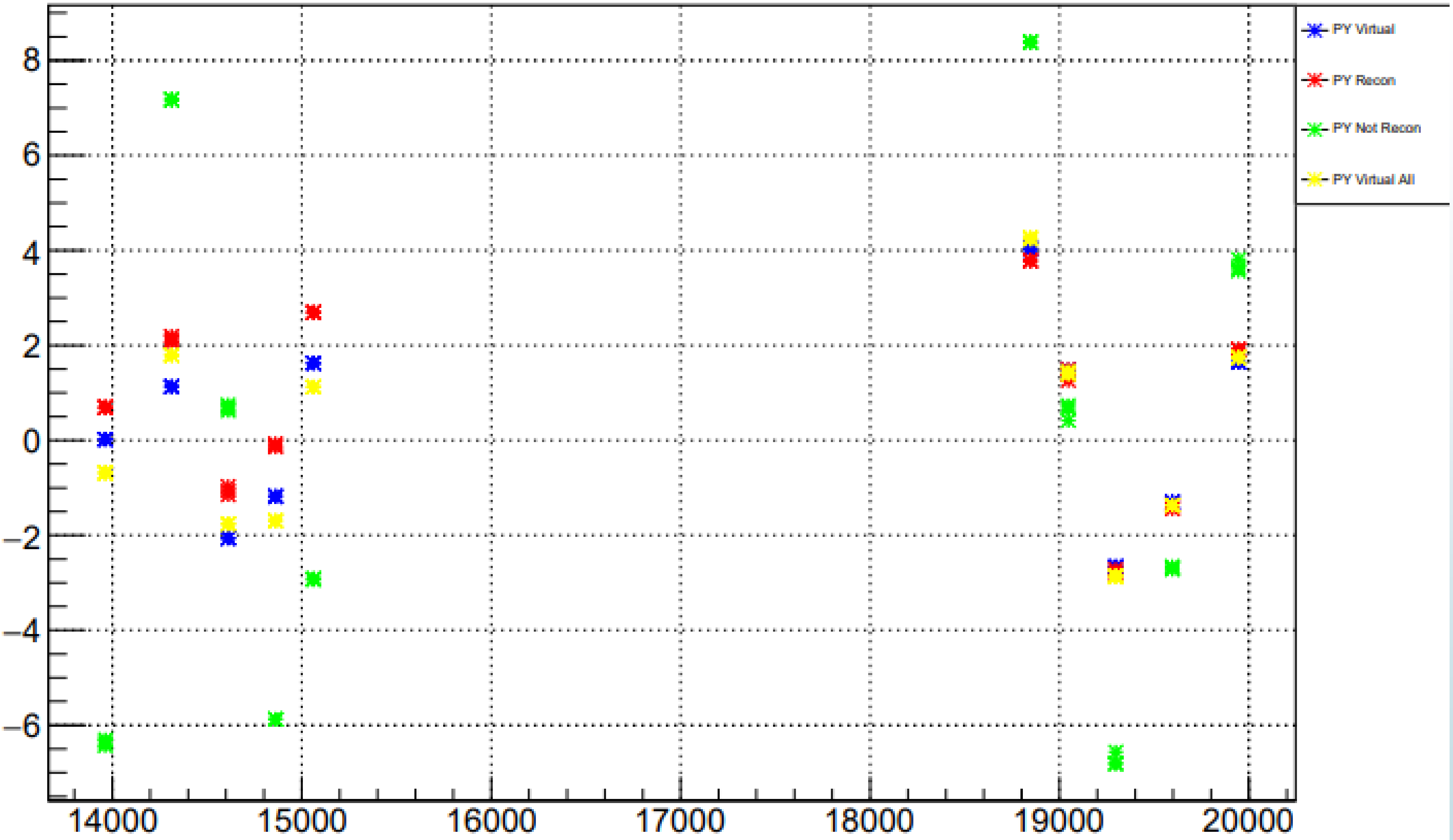
PX mean



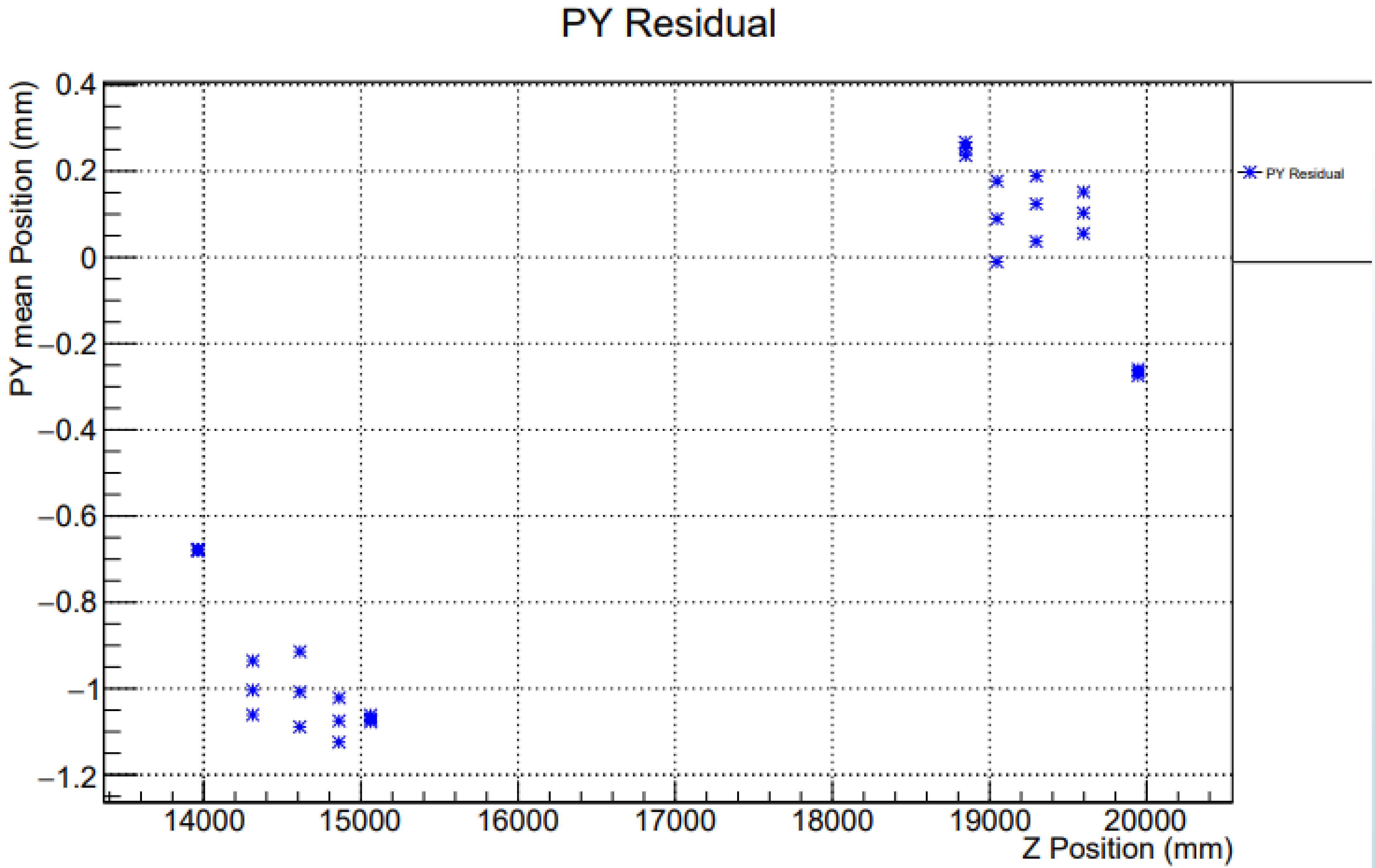
PX Virtual
PX Recon
PX Not Recon
PX Virtual All
8
6
4
2
0
−2
−4
14000
15000
16000
17000
18000
19000
20000

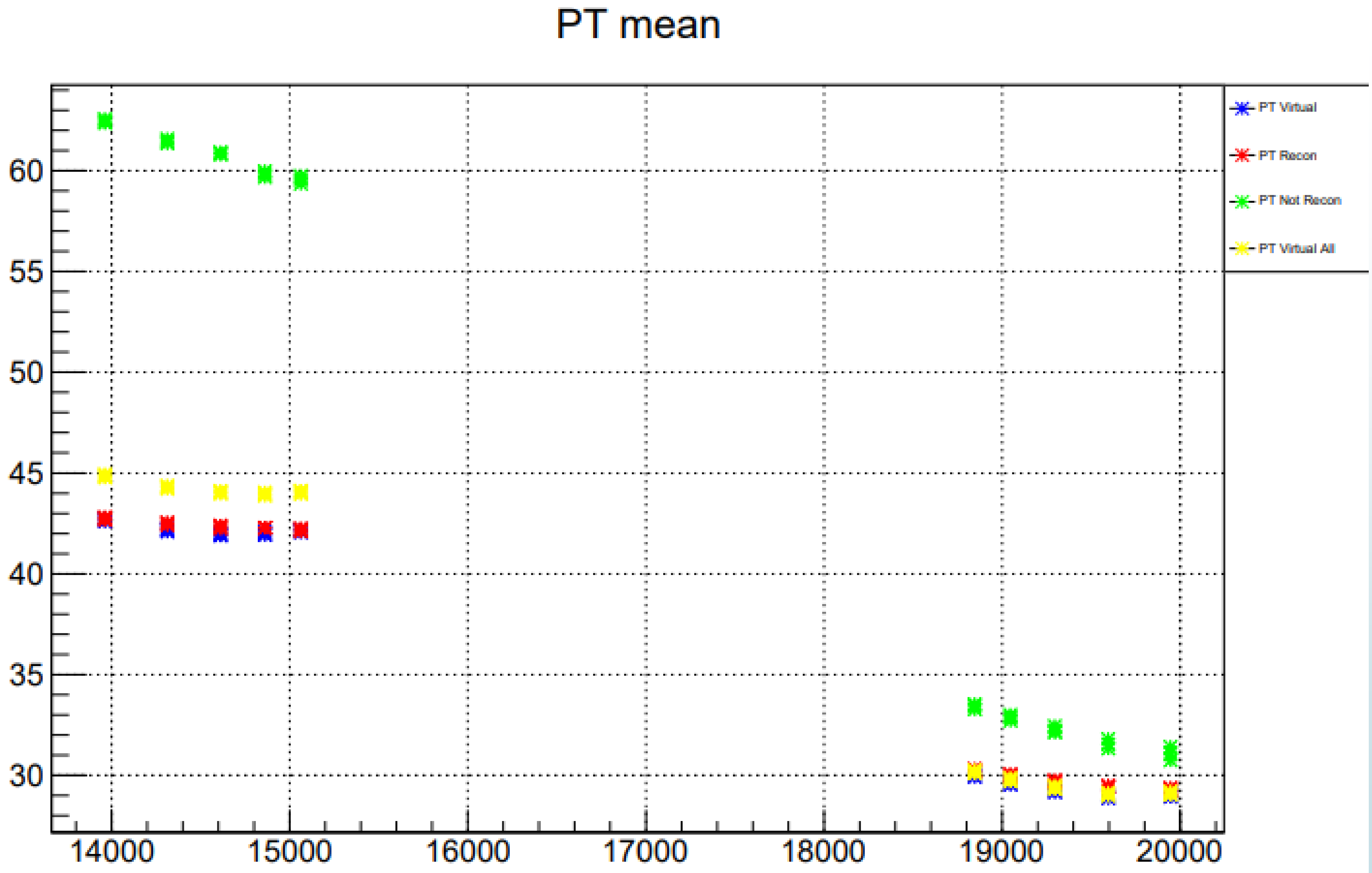

PX Residual
PX mean Position (mm)
0.8
0.6
0.4
0.2
0
−0.2
14000
15000
16000
17000
18000
19000
20000
Z Position (mm)
PX Residual

PY mean

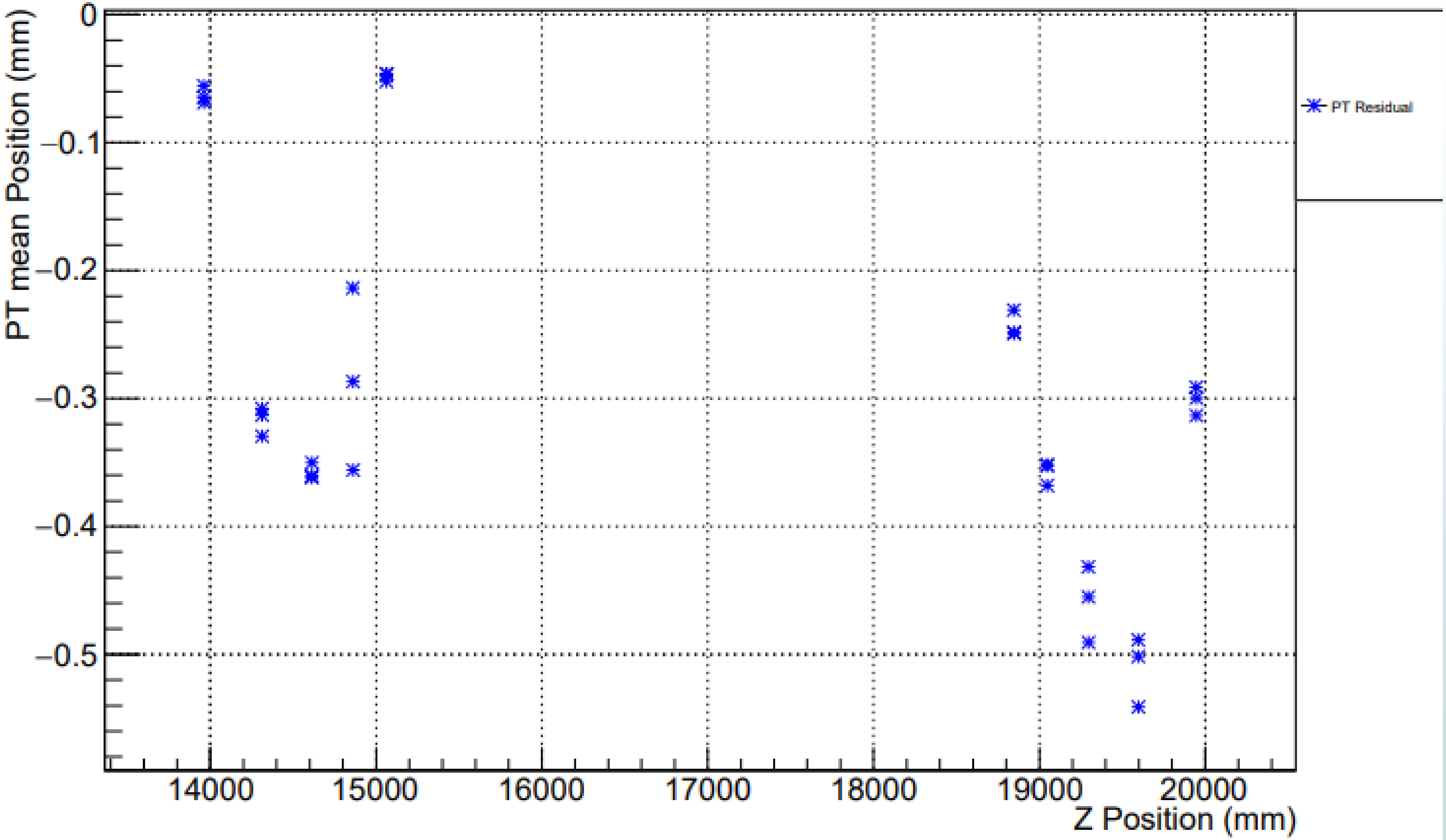


8
6
4
2
0
−2
−4
−6
14000
15000
16000
17000
18000
19000
20000
PY Virtual
PY Recon
PY Not Recon
PY Virtual All


PY Residual
PY mean Position (mm)
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
14000
15000
16000
17000
18000
19000
20000
Z Position (mm)
PY Residual

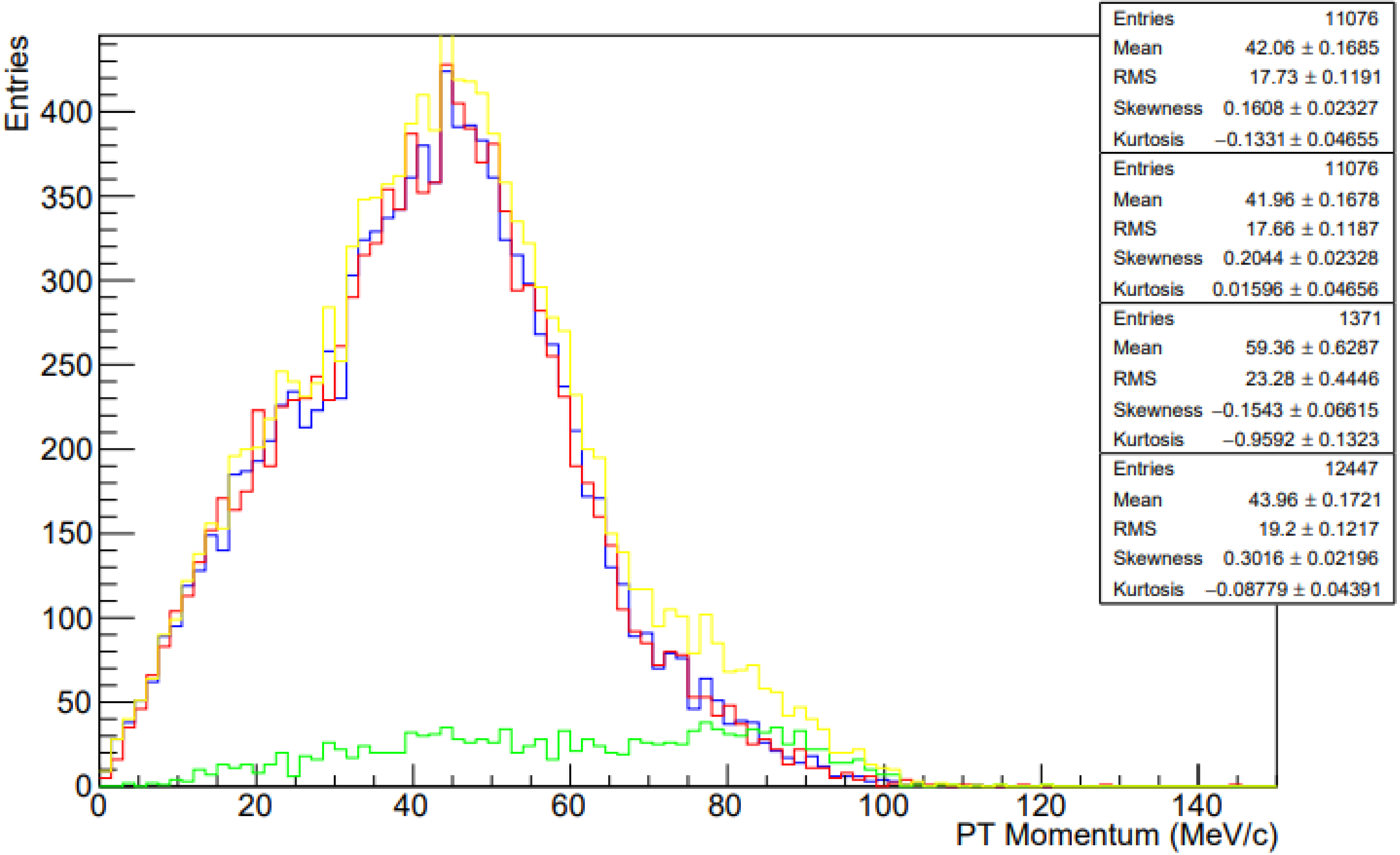

PT mean
PT Virtual
PT Recon
PT Not Recon
PT Virtual All
60
55
50
45
40
35
30
14000
15000
16000
17000
18000
19000
20000

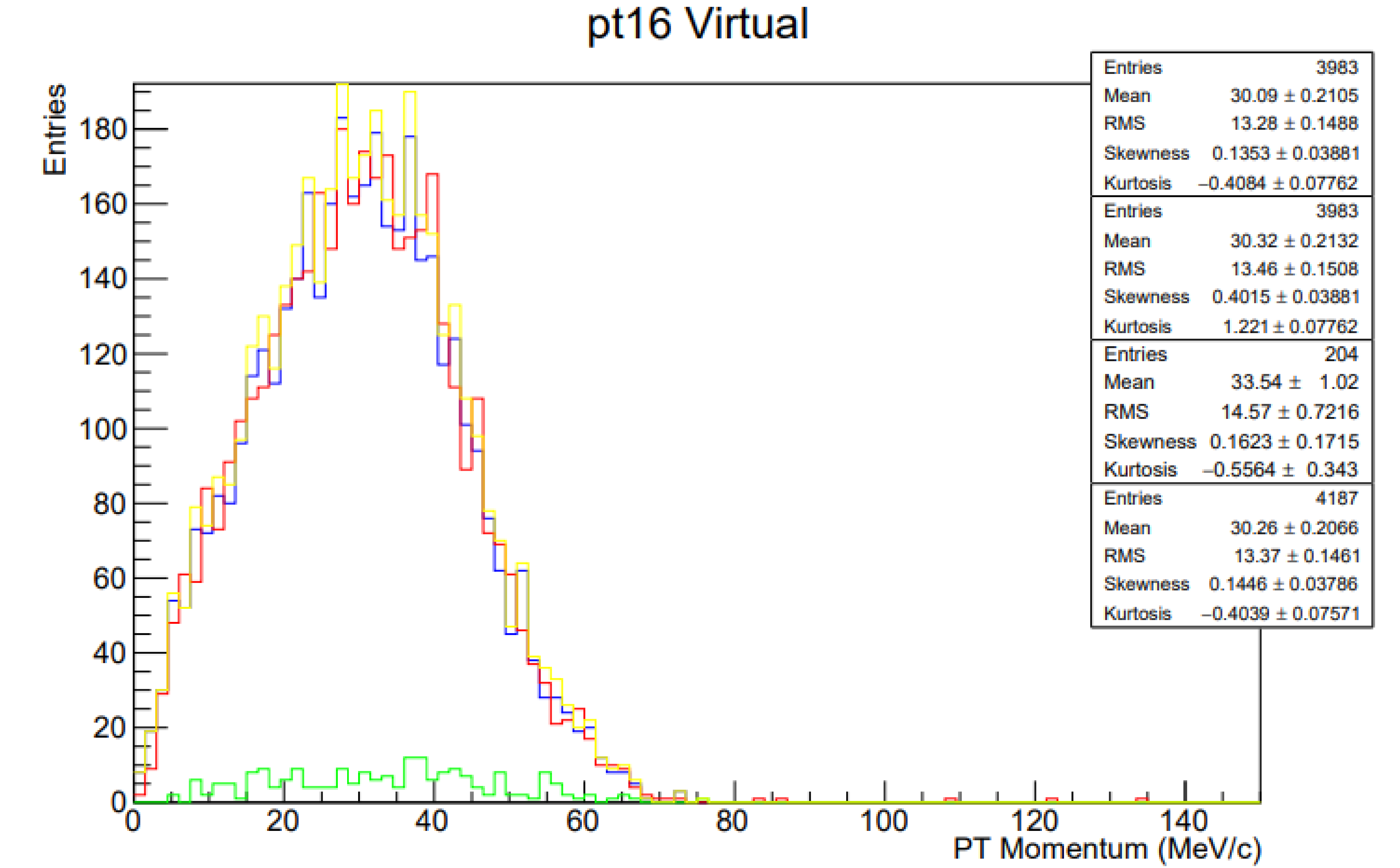
PT Residual



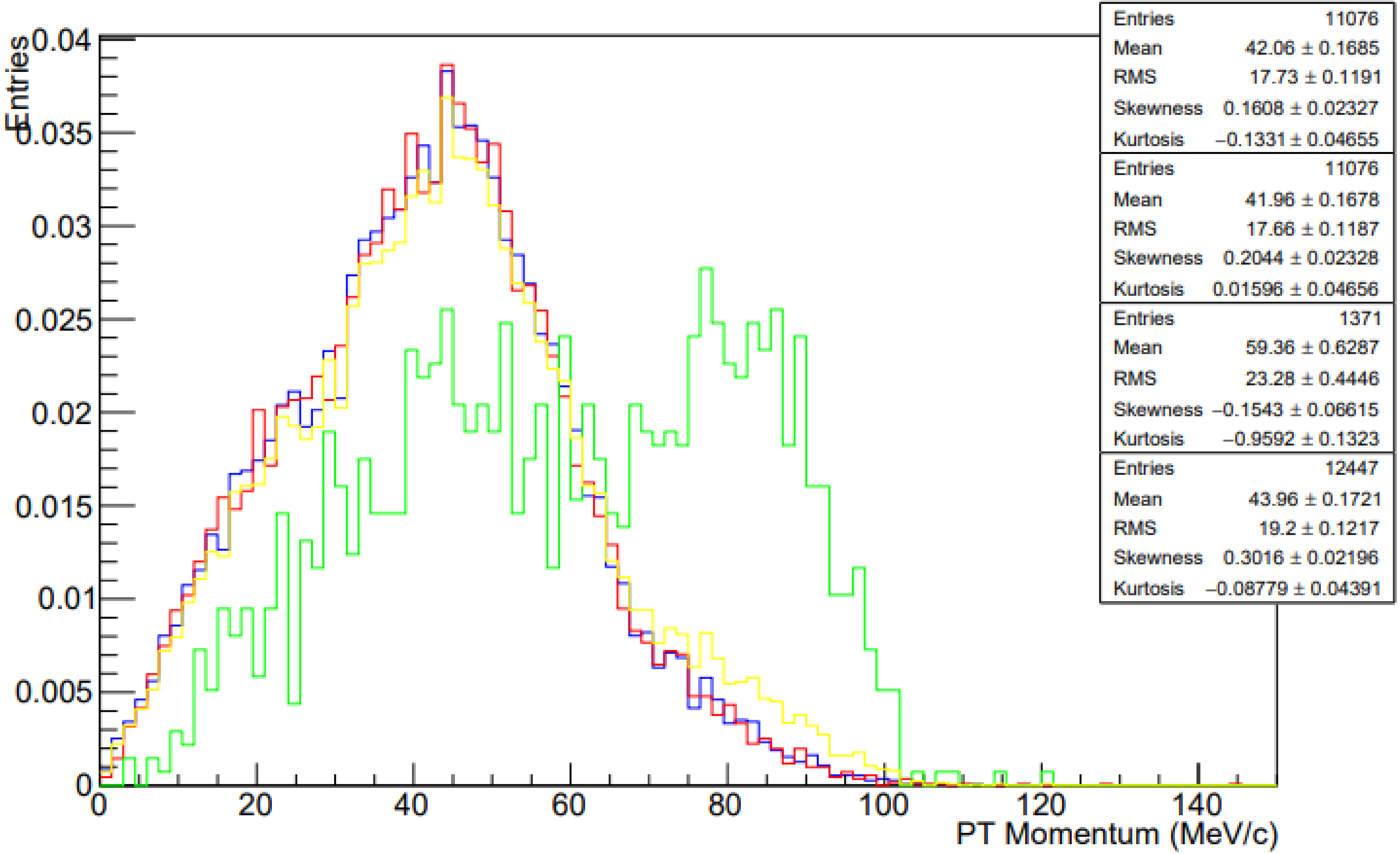
PT mean Position (mm)
0
−0.1
−0.2
−0.3
−0.4
−0.5
14000
15000
16000
17000
18000
19000
20000
Z Position (mm)
PT Residual

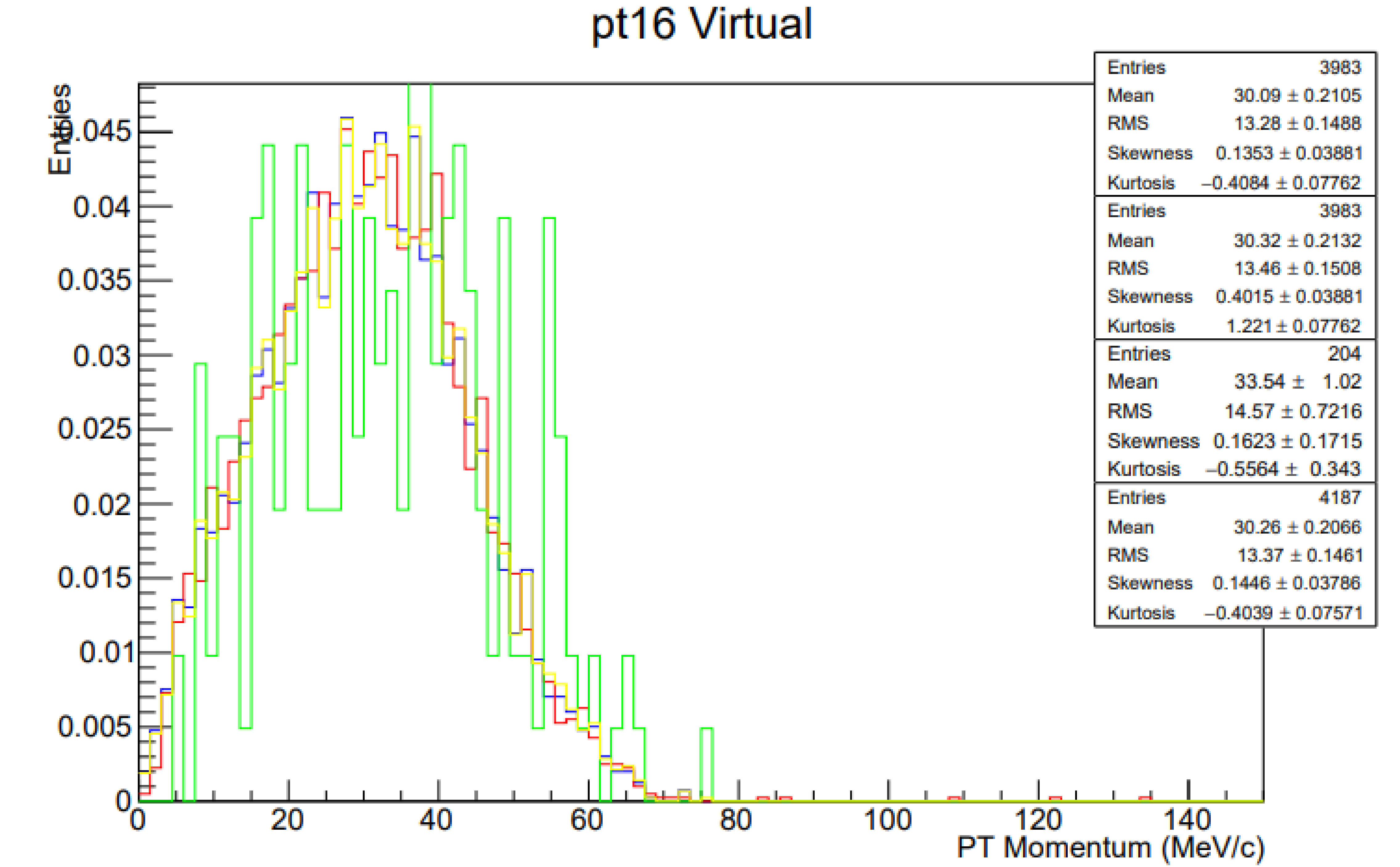
pt15 Virtual




pt16 Virtual
Entries
PT Momentum (MeV/c)
0
20
40
60
80
100
120
140
160
180
0
20
40
60
80
100
120
140
Entries 3983
Mean 30.09 ± 0.2105
RMS 13.28 ± 0.1488
Skewness 0.1353 ± 0.03881
Kurtosis −0.4084 ± 0.07762
Entries 3983
Mean 30.32 ± 0.2132
RMS 13.46 ± 0.1508
Skewness 0.4015 ± 0.03881
Kurtosis 1.221 ± 0.07762
Entries 204
Mean 33.54 ± 1.02
RMS 14.57 ± 0.7216
Skewness 0.1623 ± 0.1715
Kurtosis −0.5564 ± 0.343
Entries 4187
Mean 30.26 ± 0.2066
RMS 13.37 ± 0.1461
Skewness 0.1446 ± 0.03786
Kurtosis −0.4039 ± 0.07571

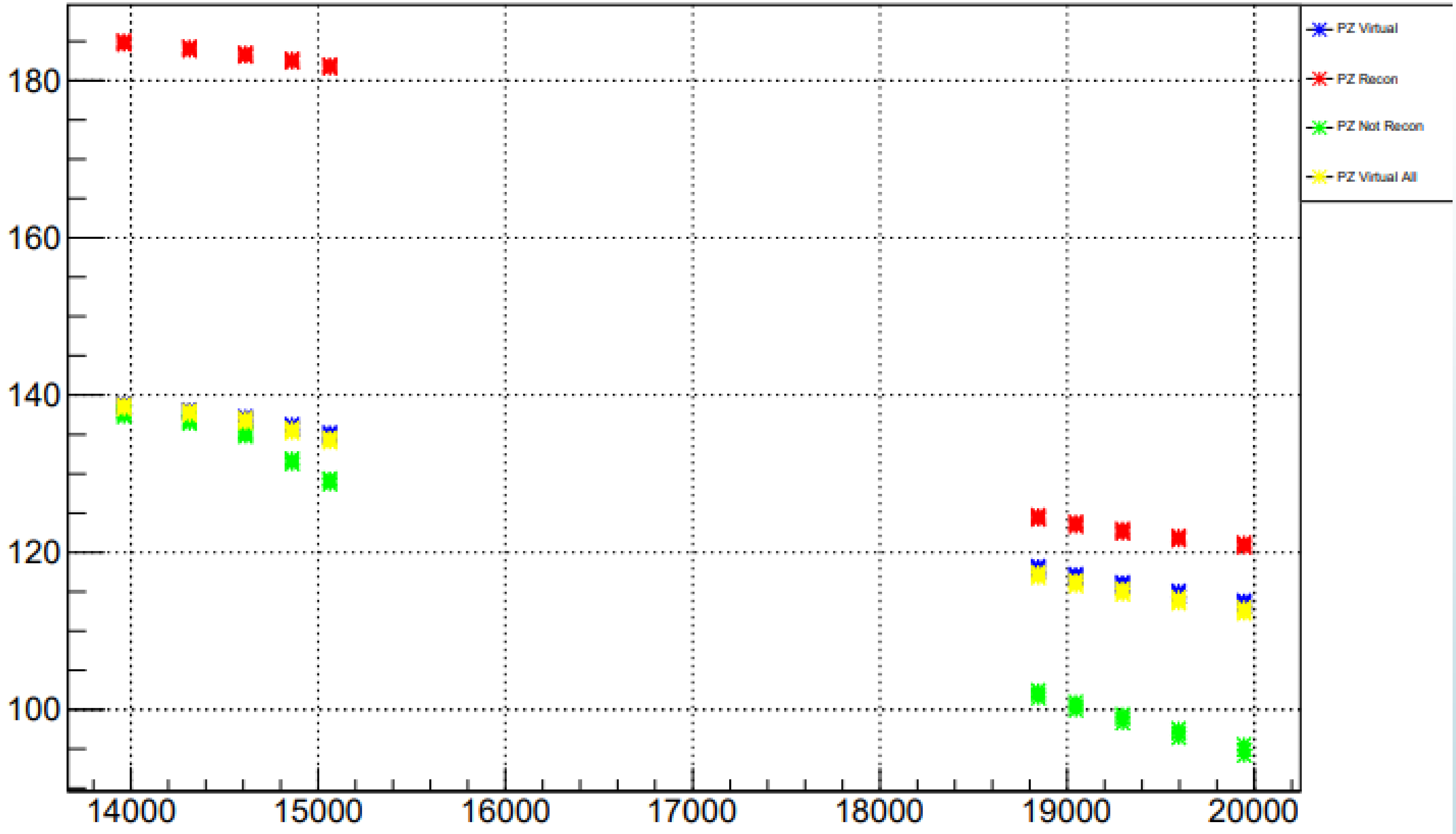
pt15 Virtual



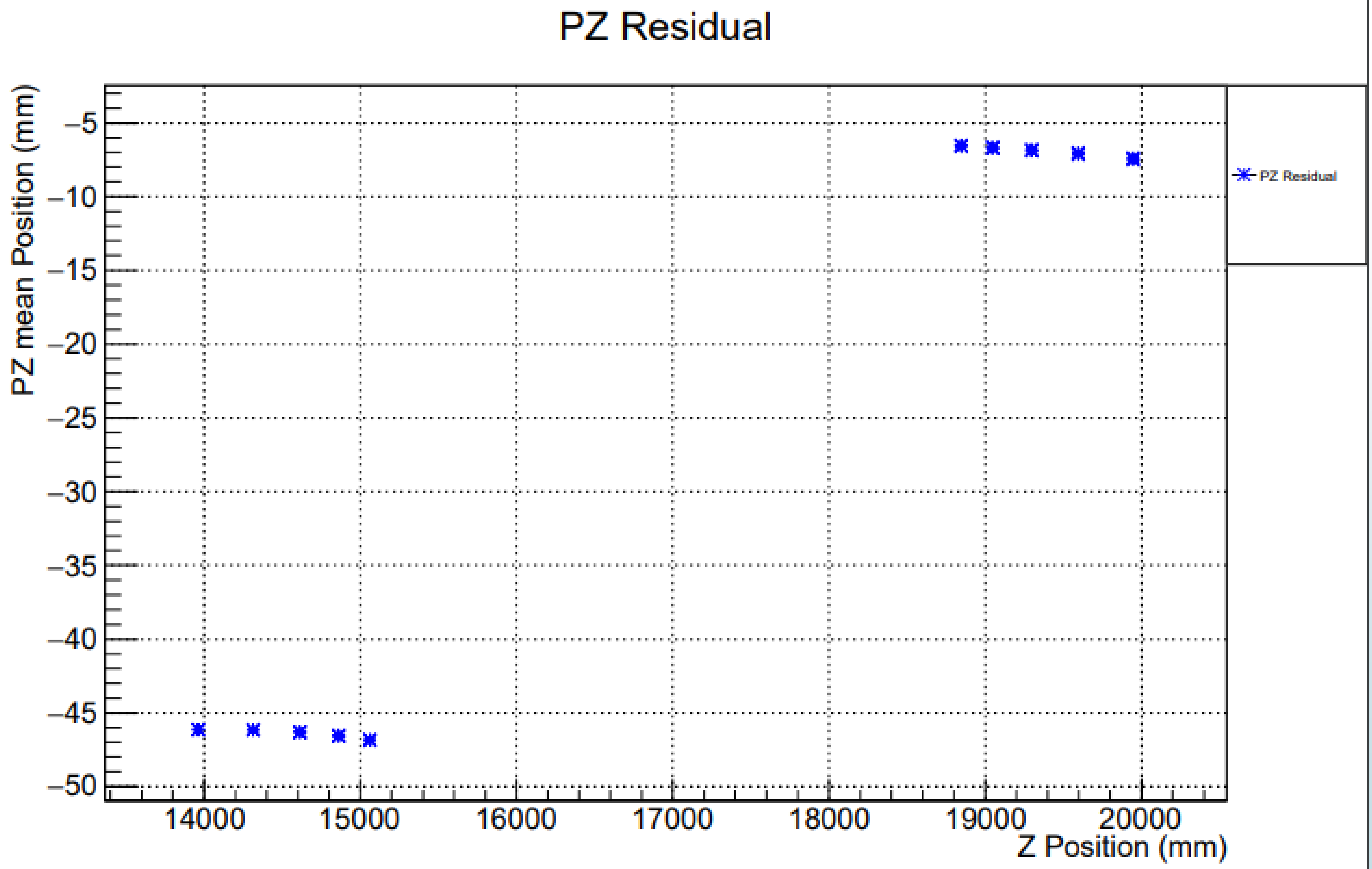


pt16 Virtual
Entries
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
20
40
60
80
100
120
140
PT Momentum (MeV/c)
Entries 3983
Mean 30.09 ± 0.2105
RMS 13.28 ± 0.1488
Skewness 0.1353 ± 0.03881
Kurtosis −0.4084 ± 0.07762
Entries 3983
Mean 30.32 ± 0.2132
RMS 13.46 ± 0.1508
Skewness 0.4015 ± 0.03881
Kurtosis 1.221 ± 0.07762
Entries 204
Mean 33.54 ± 1.02
RMS 14.57 ± 0.7216
Skewness 0.1623 ± 0.1715
Kurtosis −0.5564 ± 0.343
Entries 4187
Mean 30.26 ± 0.2066
RMS 13.37 ± 0.1461
Skewness 0.1446 ± 0.03786
Kurtosis −0.4039 ± 0.07571

PZ mean

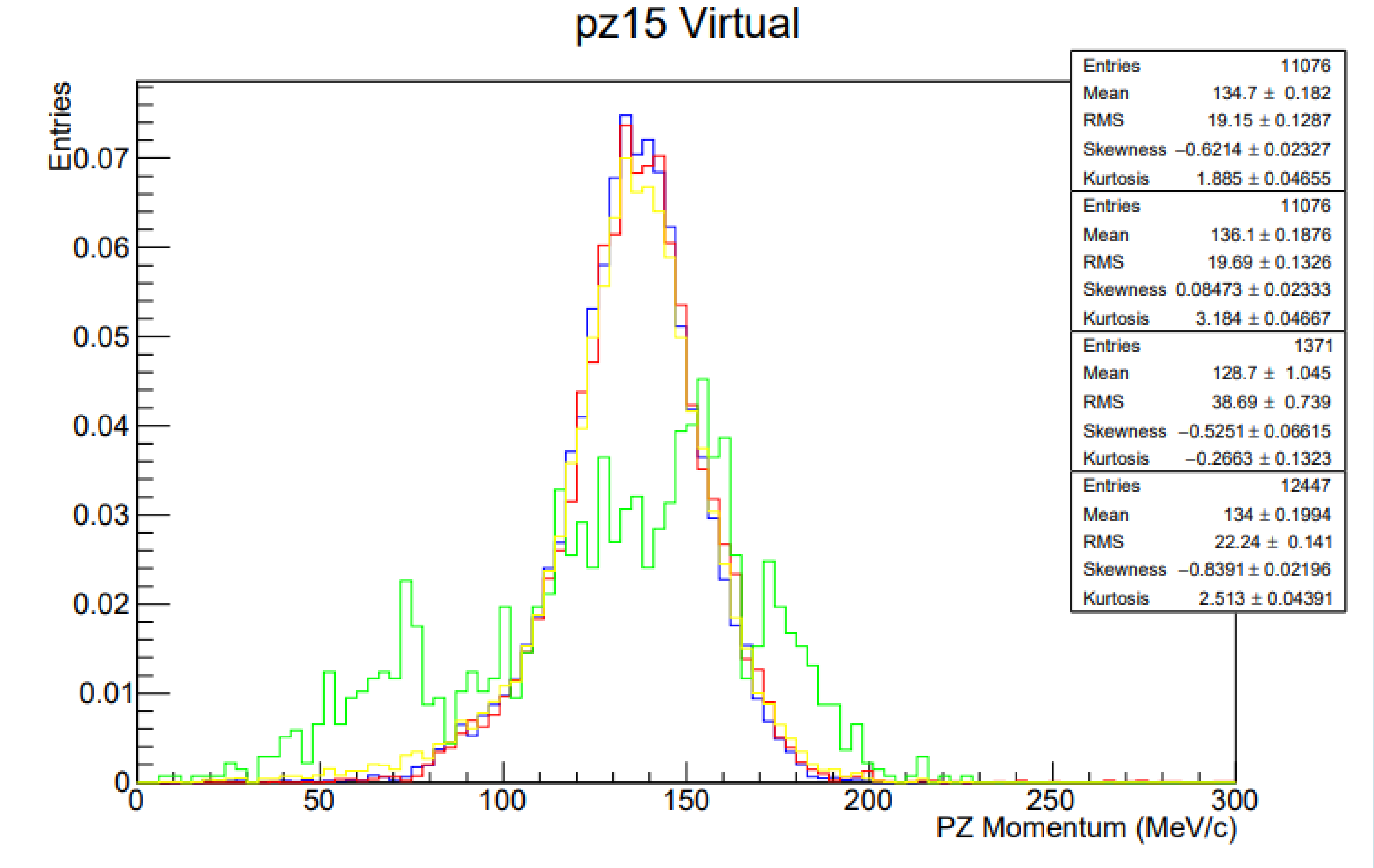


180
160
140
120
100
14000
15000
16000
17000
18000
19000
20000
PZ Virtual
PZ Recon
PZ Not Recon
PZ Virtual All


PZ Residual
PZ mean Position (mm)
−5
−10
−15
−20
−25
−30
−35
−40
−45
−50
14000
15000
16000
17000
18000
19000
20000
Z Position (mm)
PZ Residual

Pz

- Pz Residual mean to be taken with a pinch of salt until cuts are applied, as some particles are drastically mis-constructed skewing Pz Recon
- Can still look at Distribution though

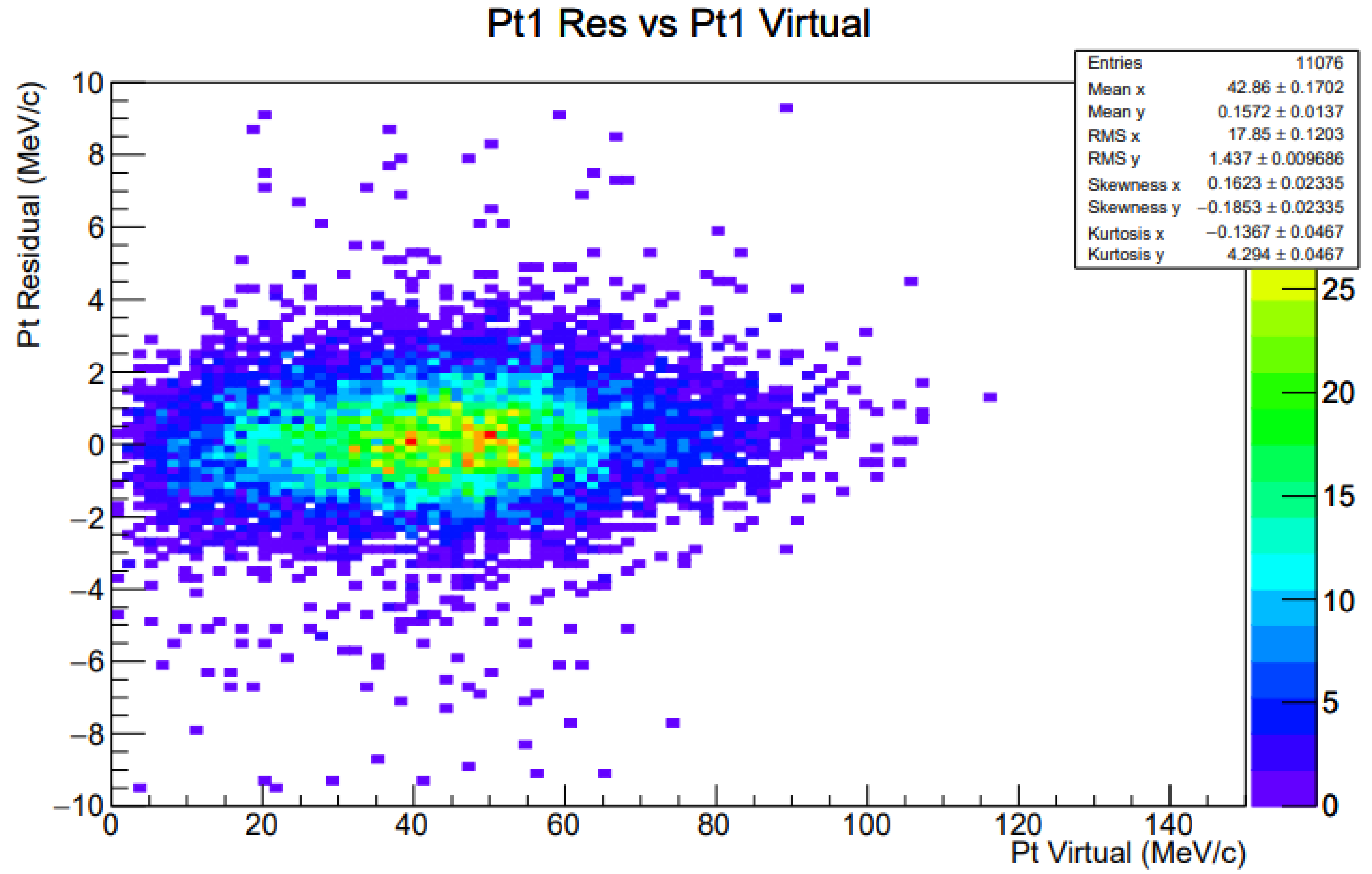

pz15 Virtual
Entries
PZ Momentum (MeV/c)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0
50
100
150
200
250
300
Entries 11076
Mean 134.7 ± 0.182
RMS 19.15 ± 0.1287
Skewness −0.6214 ± 0.02327
Kurtosis 1.885 ± 0.04655
Entries 11076
Mean 136.1 ± 0.1876
RMS 19.69 ± 0.1326
Skewness 0.08473 ± 0.02333
Kurtosis 3.184 ± 0.04667
Entries 1371
Mean 128.7 ± 1.045
RMS 38.69 ± 0.739
Skewness −0.5251 ± 0.06615
Kurtosis −0.2663 ± 0.1323
Entries 12447
Mean 134 ± 0.1994
RMS 22.24 ± 0.141
Skewness −0.8391 ± 0.02196
Kurtosis 2.513 ± 0.04391

Future

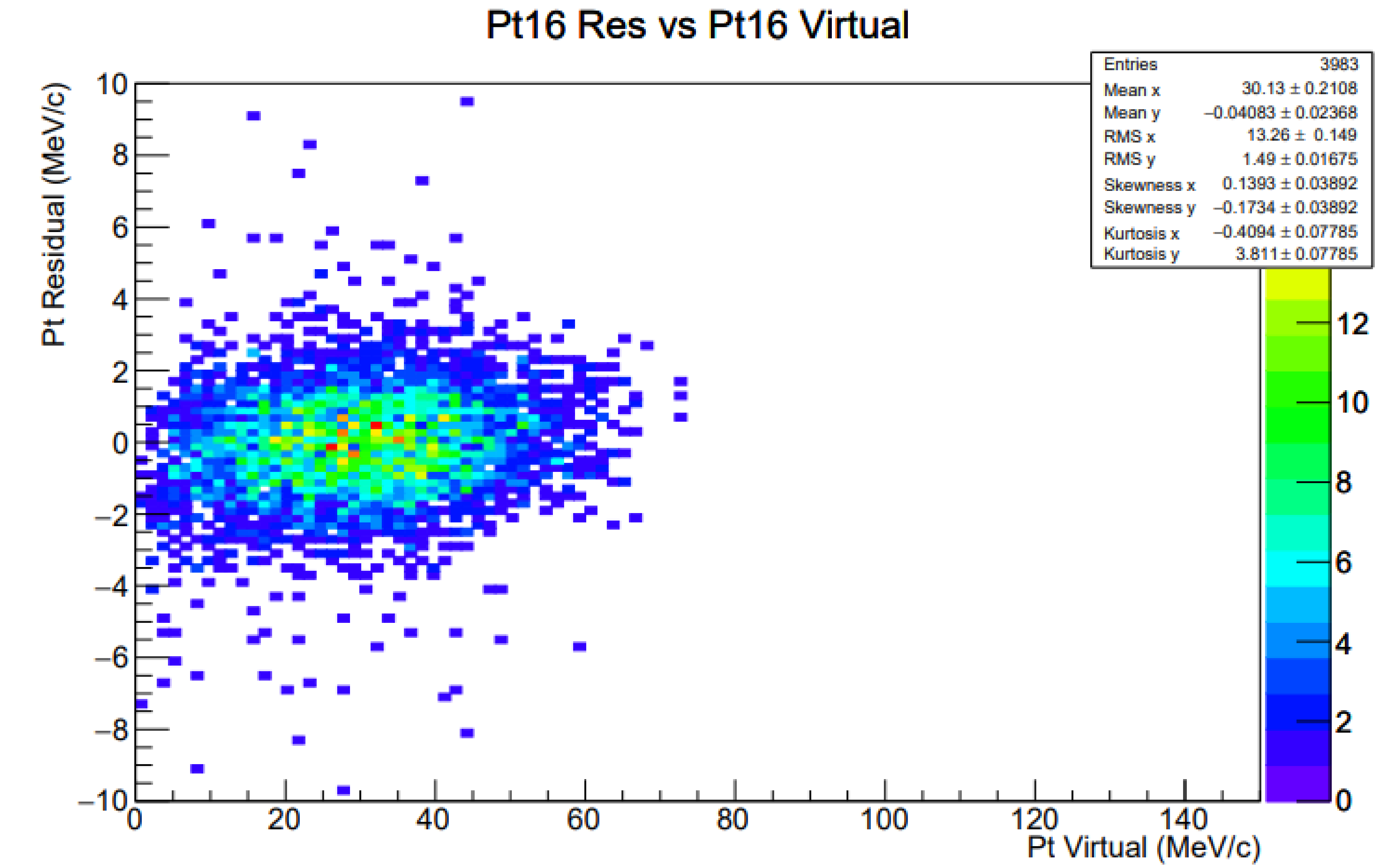
- Add cuts
- Quantify biases as well as differences between TKU and TKD
- Check for different beamline settings
- Make adjustments to phase space density results as required/increase errors

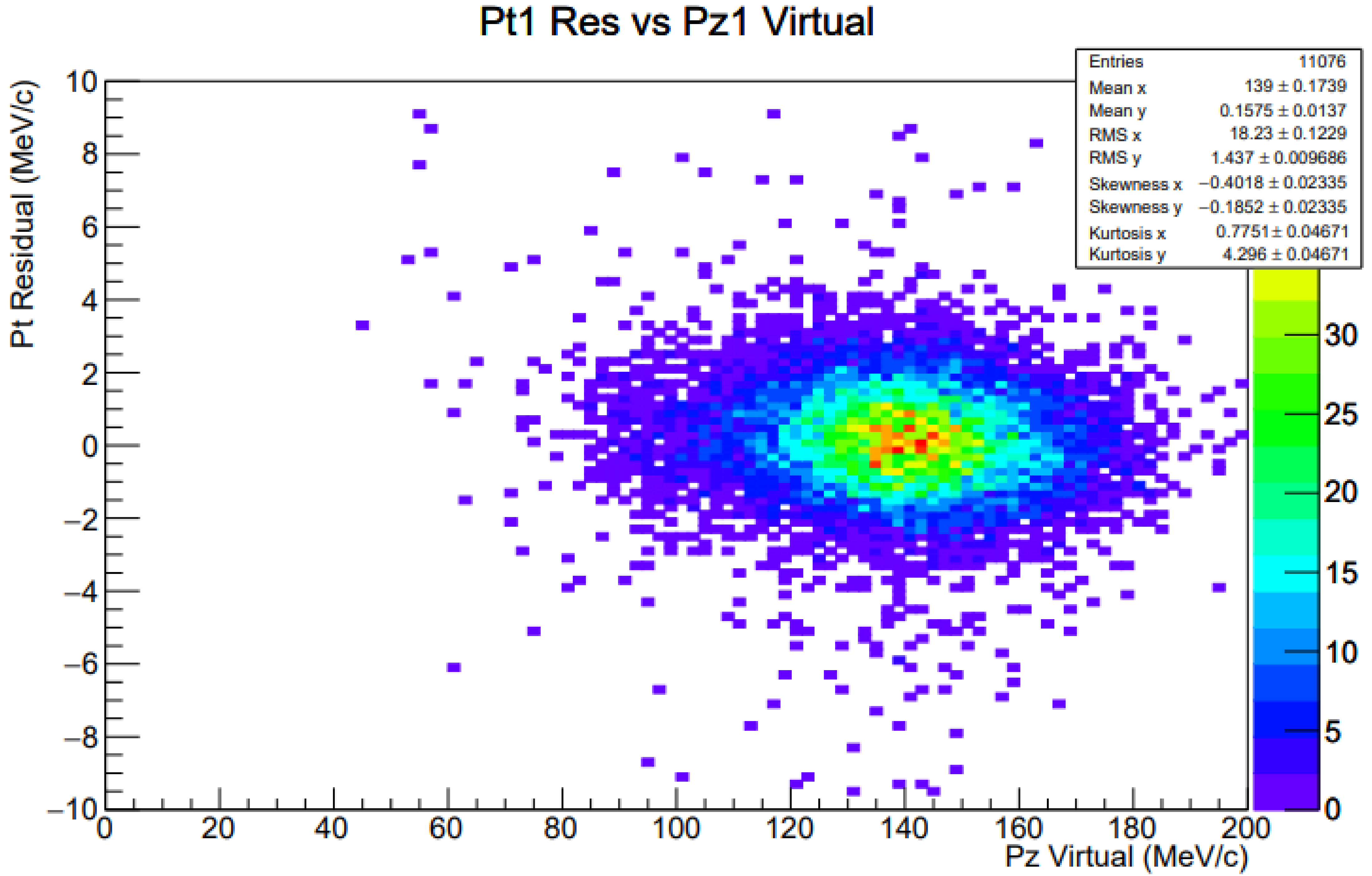
THE END

Extra Slides

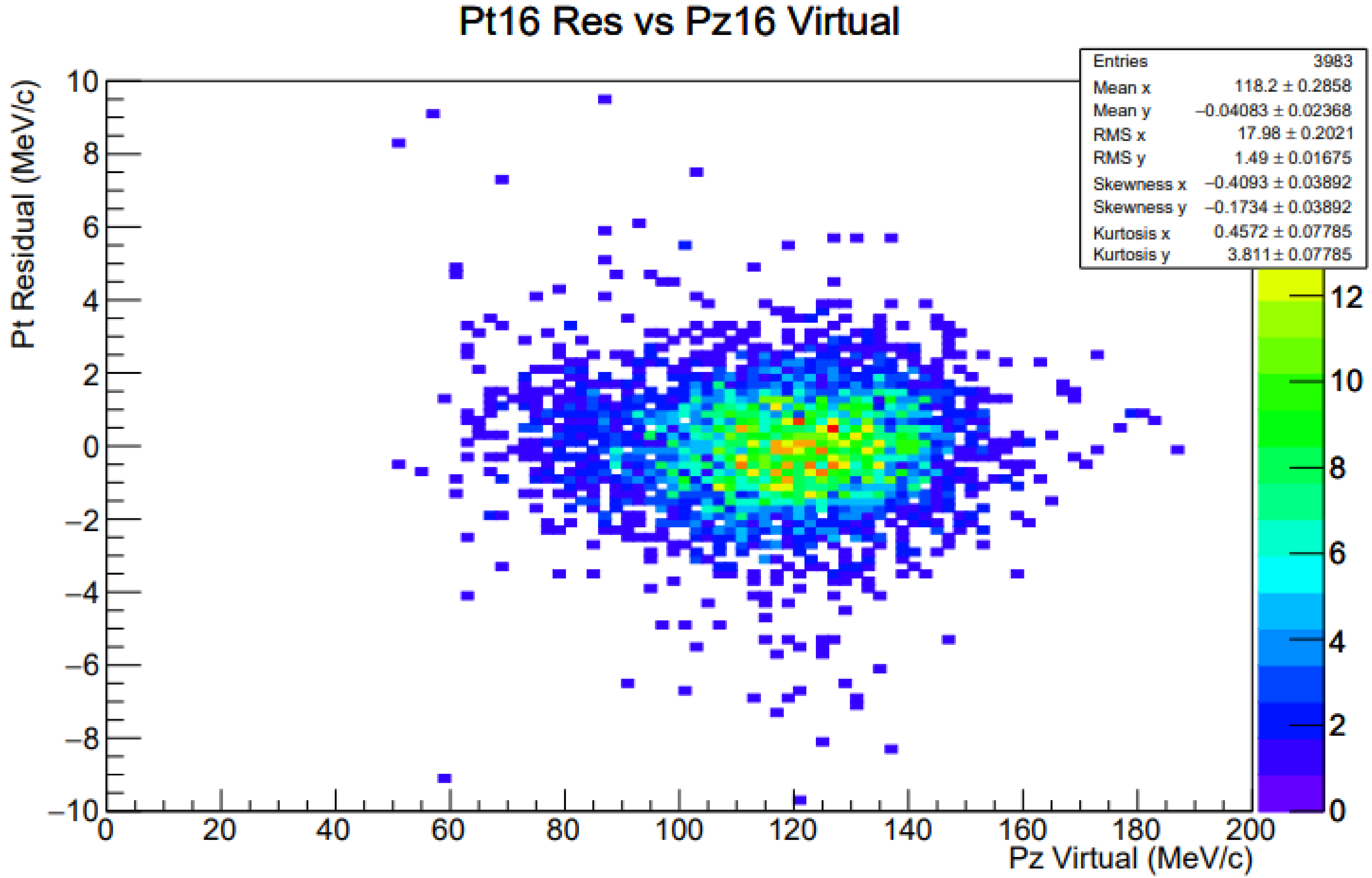


Pt1 Res vs Pt1 Virtual
Entries 11076
Mean x 42.86 ± 0.1702
Mean y 0.1572 ± 0.0137
RMS x 17.85 ± 0.1203
RMS y 1.437 ± 0.009686
Skewness x 0.1623 ± 0.02335
Skewness y −0.1853 ± 0.02335
Kurtosis x −0.1367 ± 0.0467
Kurtosis y 4.294 ± 0.0467
Pt Residual (MeV/c)
Pt Virtual (MeV/c)


Pt16 Res vs Pt16 Virtual
Entries 3983
Mean x 30.13 ± 0.2108
Mean y −0.04083 ± 0.02368
RMS x 13.26 ± 0.149
RMS y 1.49 ± 0.01675
Skewness x 0.1393 ± 0.03892
Skewness y −0.1734 ± 0.03892
Kurtosis x −0.4094 ± 0.07785
Kurtosis y 3.811 ± 0.07785
Pt Residual (MeV/c)
Pt Virtual (MeV/c)
10
8
6
4
2
0
−2
−4
−6
−8
−10
0
20
40
60
80
100
120
140
12
10
8
6
4
2
0



Pt1 Res vs Pz1 Virtual
Entries 11076
Mean x 139 ± 0.1739
Mean y 0.1575 ± 0.0137
RMS x 18.23 ± 0.1229
RMS y 1.437 ± 0.009686
Skewness x −0.4018 ± 0.02335
Skewness y −0.1852 ± 0.02335
Kurtosis x 0.7751 ± 0.04671
Kurtosis y 4.296 ± 0.04671
Pt Residual (MeV/c)
Pz Virtual (MeV/c)
10
8
6
4
2
0
−2
−4
−6
−8
−10
0
20
40
60
80
100
120
140
160
180
200
30
25
20
15
10
5
0



Pt16 Res vs Pz16 Virtual
Entries 3983
Mean x 118.2 ± 0.2858
Mean y −0.04083 ± 0.02368
RMS x 17.98 ± 0.2021
RMS y 1.49 ± 0.01675
Skewness x −0.4093 ± 0.03892
Skewness y −0.1734 ± 0.03892
Kurtosis x 0.4572 ± 0.07785
Kurtosis y 3.811 ± 0.07785
Pt Residual (MeV/c)
Pz Virtual (MeV/c)
10
8
6
4
2
0
−2
−4
−6
−8
−10
0
20
40
60
80
100
120
140
160
180
200
12
10
8
6
4
2
0

Transmission losses (Recall)

- Liouville's theorem only applies to the same particles (or to system with the same particle distribution function). I.e the volume remains the same and the change in the covariance matrix can be described in a conserved manner.
- Transmission losses and subsequent change in particle distribution function can be described by the change it has on the covariance matrix (subscript 1: Full Upstream sample, 2: Upstream which makes it downstream, 3: Upstream which goes missing)

$$\Sigma_1$$
$$= \frac{N_2^3}{(N_2+N_3)^3}\Sigma_2 + \frac{N_3^3}{(N_2+N_3)^3}\Sigma_3$$
$$+ \sum_1^{N_2}\left(N_2N_3(P_i-\bar{P}_2)(P_i-\bar{P}_3) + N_2N_3(P_i-\bar{P}_3)(P_i-\bar{P}_2) + N_3^2(P_i-\bar{P}_3)(P_i-\bar{P}_3)\right)/(N_2+N_3)^3$$
$$+ \sum_1^{N_3}\left(N_2N_3(P_i-\bar{P}_2)(P_i-\bar{P}_3) + N_2N_3(P_i-\bar{P}_3)(P_i-\bar{P}_2) + N_2^2(P_i-\bar{P}_2)(P_i-\bar{P}_2)\right)/(N_2+N_3)^3$$

- For the case of a symmetric absorber this can be simplified to

$$N_1\Sigma_1 = N_2\Sigma_2 + N_3\Sigma_3$$

The determinant of a matrix (Recall)

- The determinant of a matrix can be separated into parts using:

$$|\Sigma_1| = \sum_{i=0}^{n} \Gamma_n^i \left| {}^{\Sigma_2}\!/_{{\Sigma_3}^i} \right| = |\Sigma_2| + |\Sigma_3| + \sum_{i=1}^{n-1} \Gamma_n^i \left| {}^{\Sigma_2}\!/_{{\Sigma_3}^i} \right|$$

Where Γ_n^i represents substituting all combinations of i^{th} lines from Σ_2 by the same lines in Σ_3 and taking the subsequent determinant of the new matrix

- For the symmetric case (LiH, LH2 and no absorber) the previous and above substitutions could be made to compare the upstream and downstream densities. Due to the asymmetry this cannot be done for the wedge and requires further derivation for the asymmetric case.

Potential next step (Recall)

- The missing data downstream is inaccessible, however the upstream sample which makes it downstream can be compared to the downstream sample
- The transport, M, of a covariance matrix from upstream to downstream can be given by:

$$\Sigma_{down} = \langle X_{down}\tilde{X}_{down}\rangle = \langle M X_{up}\tilde{M}\tilde{X}_{up}\rangle = M\langle X_{up}\tilde{X}_{up}\rangle\tilde{M} = M\Sigma_{up}\tilde{M}$$

- The determinant is given by:

$$|\Sigma_{down}| = \left|M\Sigma_{up}\tilde{M}\right| = |M|^2\left|\Sigma_{up}\right| = \left|\Sigma_{up}\right|$$

- The transfer matrix M has been previously investigated by Sophie Middleton and Chris Rogers
- A potential investigation would be to investigate the change in R for different fraction sizes of the beam. If stable it could be used to investigate the missing data downstream to see if it is due to scraping and magnet misalignment affects and nothing else

What affect does it have on Pt and Pz

- $p_t = cBQR$
- c, B and Q are constant (should be), so transverse momentum changes by radius loss
- A particle loses approximately 0.6 MeV per station, so ~ 3 MeV per tracker, which for a 140 MeV particle is ~2%
- Therefore the radius from start to finish reduces by 2%
- For a high radius particle, e.g. 100mm, this radius reduction would be more than a few widths of fibres, leading to a poor qui-squared value for the circle fit and thus being excluded

What effect does it have on Pt and Pz

- z-s plane
- Another qui-squared cut is made in the z-s plane, if the fit in the z-s plane fits a straight line.
- $z = \frac{dz}{ds} s - s_0 \ with \ s = R\varphi$, however if the radius is not constant, or not the appropriate radius (wrong circle centre), then the phase advance will be wrong.
- Should have straight line between stations in s-z plane, however a small deviation at each station. That deviation should be similar at each station (i.e. angle change)
- A too strict straight line qui-squared cut may exclude valid particles, but more importantly:

$$^{p_z}/_{p_t} = {}^{\Delta z}/_{R\Delta\varphi}$$

- The p_t to R ratio should be fairly constant and thus p_z is heavily influenced by the phase advance.
- If the movement of circle centre isn't accounted for, then will have the wrong phase advance angle

Emittance in Experiments

- Emittance measurements can be biased
- The scraping of the beam on the aperture can give a false cooling effect
- Non-linearities can give rise to a false heating effect. The emittance of the beam has increased due to the non-linearities but the phase space volume hasn't changed size
- To see cooling, one can look at the change in phase-space volume or the change in density of that volume before and after it has gone through some material

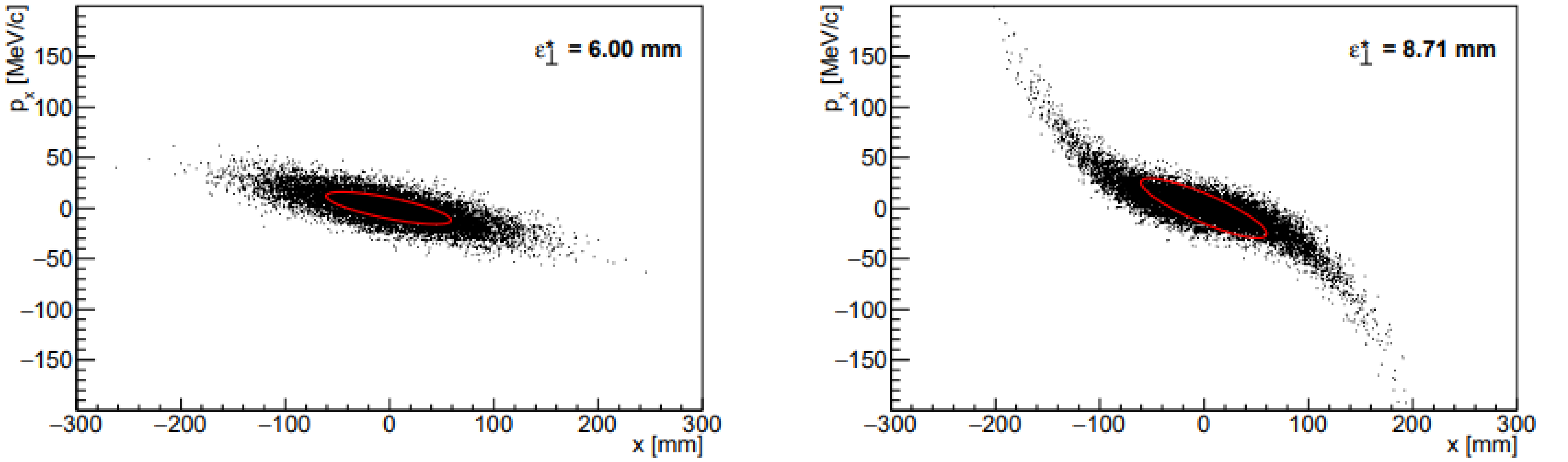
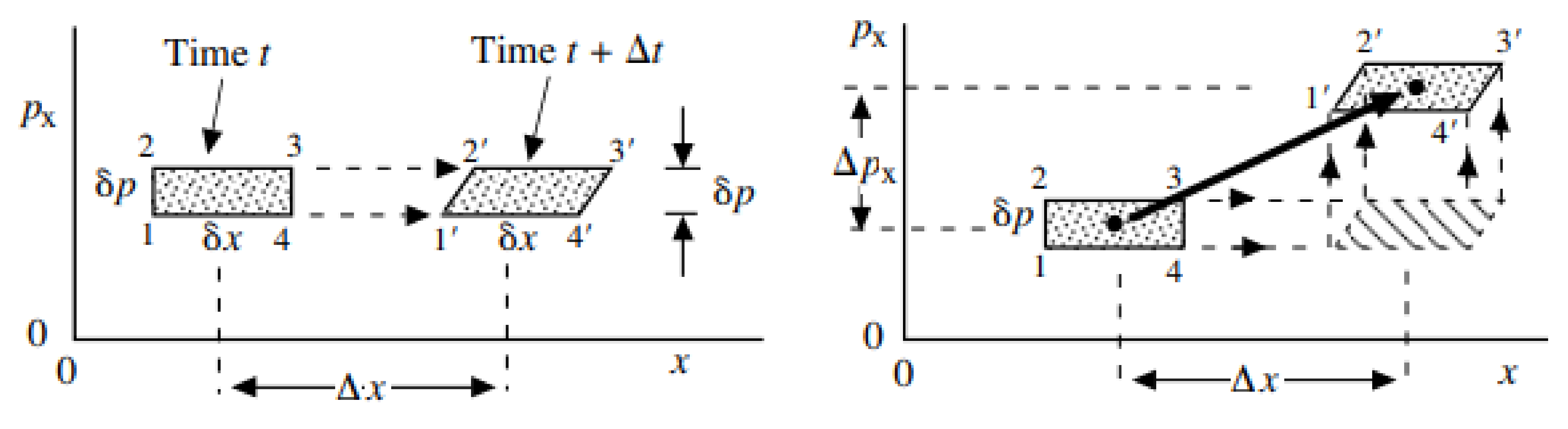


Figure 6.6: Scatter plot of a beam ($\epsilon_i = 6\,\mathrm{mm}$, $\langle p_z \rangle = 140\,\mathrm{MeV}/c$ and $\beta_\perp = 800\,\mathrm{mm}$) after transport through a linear focusing lens of $f = 5\,\mathrm{mm}^{-1}$ (left) and a similar nonlinear lens with $C_\alpha = 10^{-4}\,\mathrm{mm}^{-2}$ (right). The red curve is the RMS ellipse.

Phase Space Volume and Density

- Take an arbitrary phase space volume upstream of the absorber and count the number of particles in that volume. Take the same volume downstream and count the number of particles in that volume. If it has changed then heating or cooling has taken place
- The problem is what does that phase space volume actually look like downstream as it has changed in shape due to differing momenta of particles in the beam and the magnetic forces of the cooling channel
- Transmission losses also need to be accounted for in an unbiased way



Liouville's theorem

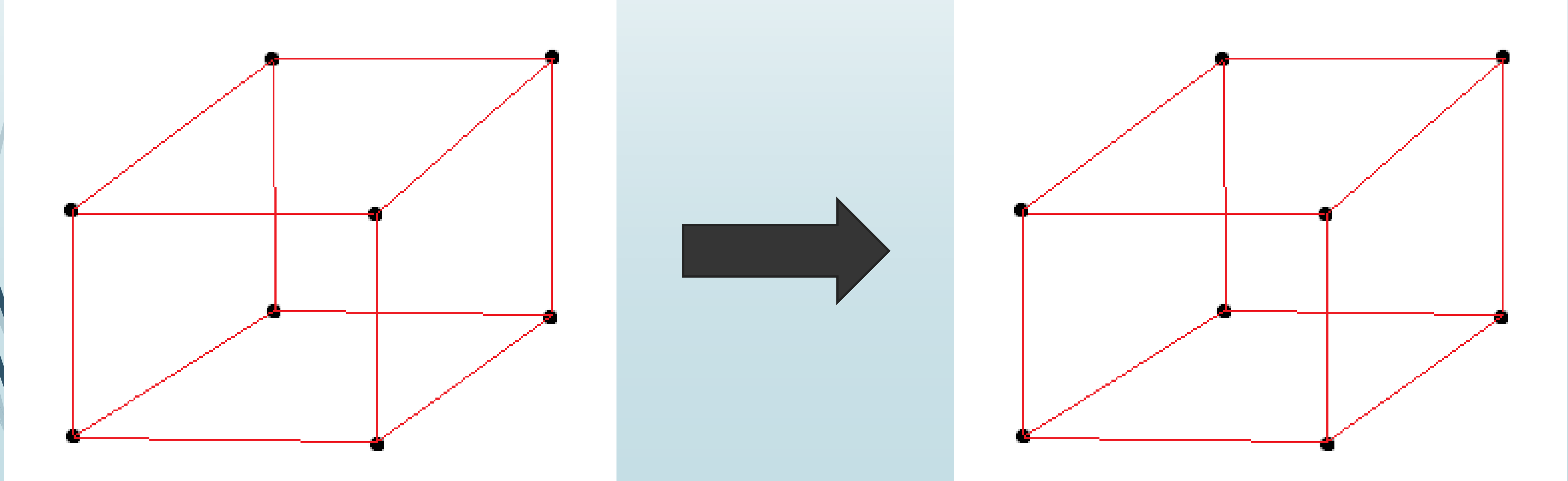
- A particle beam can be described by the distribution of the particles in the beam also known as the phase space density $\rho(x, y, z, p_x, p_y, p_z)$.
- Liouville's theorem states that the density of particles in phase space is a constant i.e. ${}^{d\rho}/_{dt} = 0$ (providing there are no dissipative forces)
- The number of particles in a phase-space volume is then given by:

$$N = \int \rho\big(x, y, z, p_x, p_y, p_z\big) dx dy dz dp_x dp_y dp_z = \int \rho dV$$

- The phase-space density is directly related to the phase space volume
- The phase-space density can be calculated in a number of ways using density estimation techniques such as Kernel Density Estimation (KDE), the k-Nearest Neighbour Approach (KNN) plus many more
- Phase Space Density Estimation is a non-parametric technique to estimate the underlying probability density, the probability that a particle will be realized at a particular phase space density

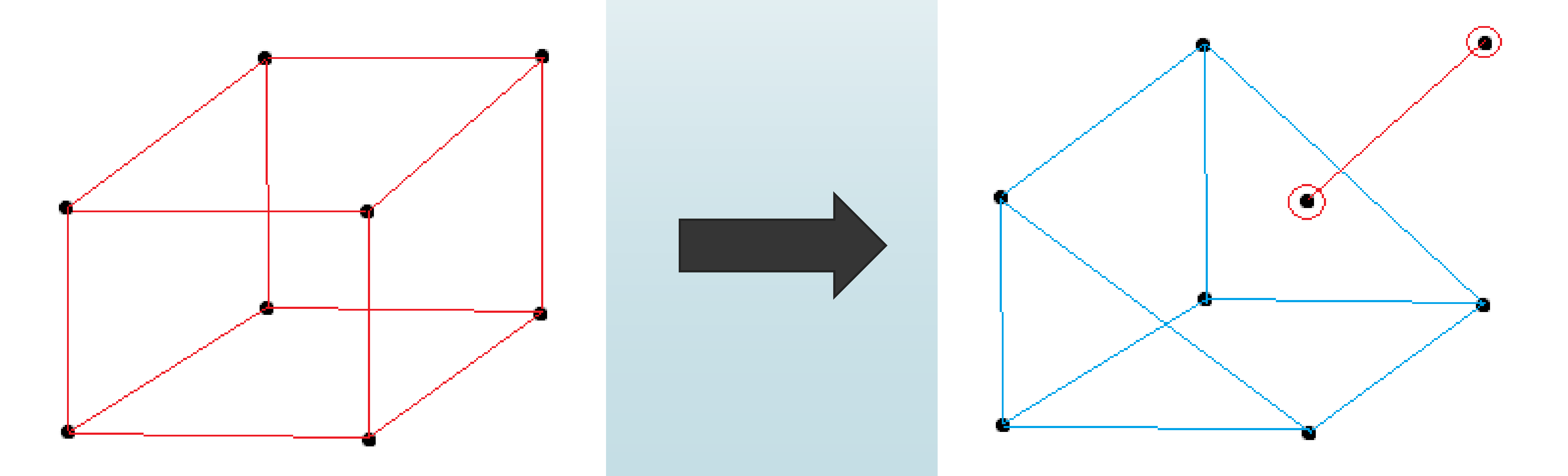
Transmission effects – extreme example

- Imagine phase space distribution given by 8 points arranged in a cube separated by a 1 unit distance, giving a 1 unit volume.
- The system is sent through a magnetic system with no dissipative forces. The points may have changed location, but the 1 unit volume is preserved.



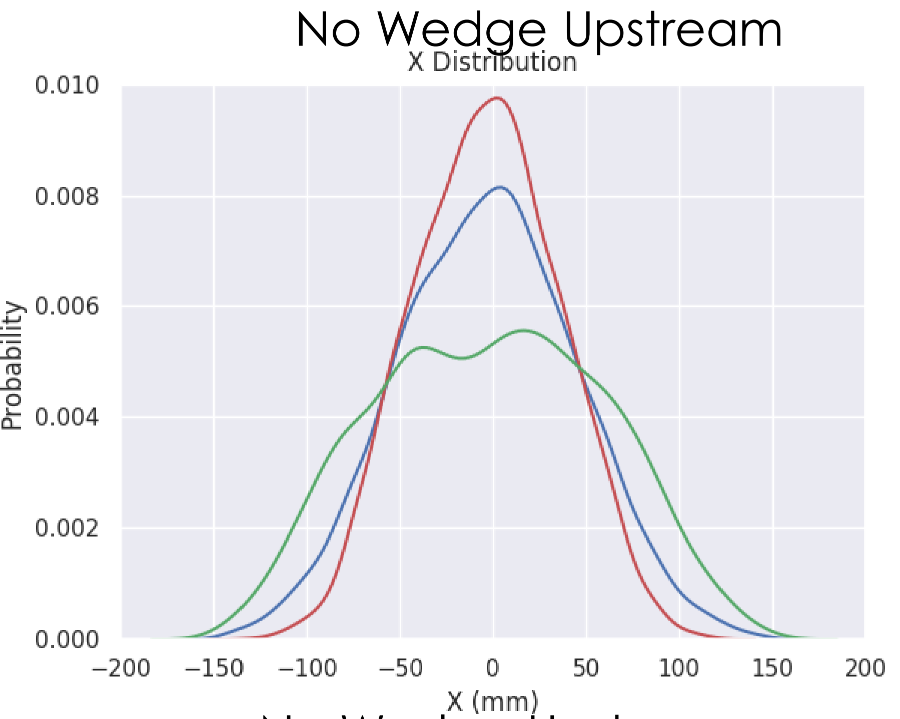
Transmission effects – extreme example

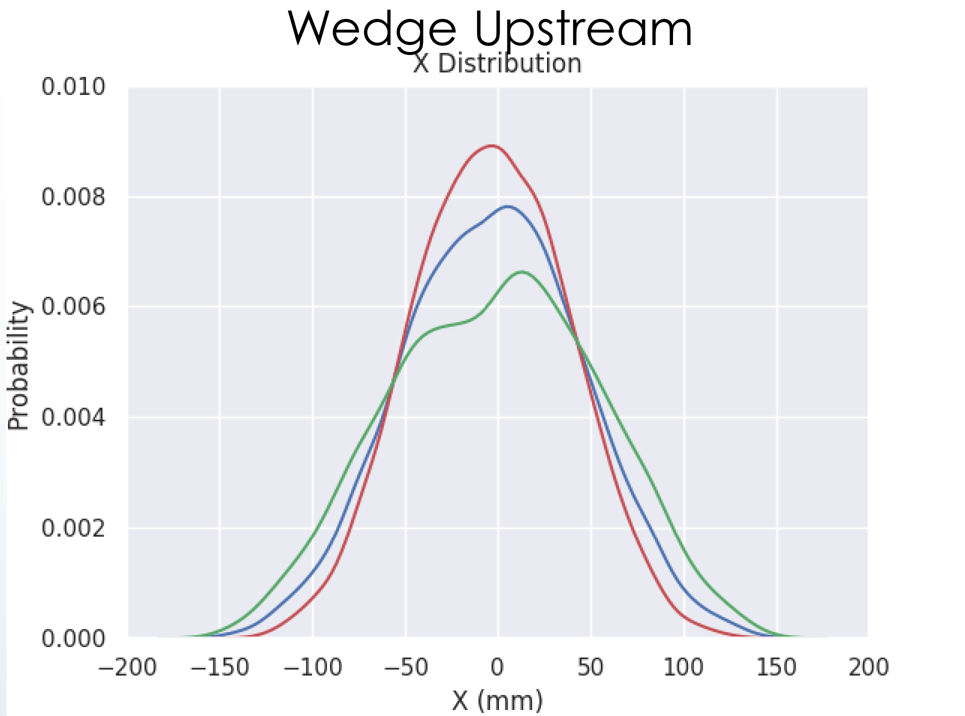
- The eight particles are again put through a magnetic system which has an aperture (acts as a dissipative force), resulting in a loss of two particles.
- The volume of the remaining 6 particles is 0.5 unit volume.
- If one were to normalize the downstream sample by the sample size, one would artificially increase the density (which is wrong). For transmission losses, the change is particle distribution is important.



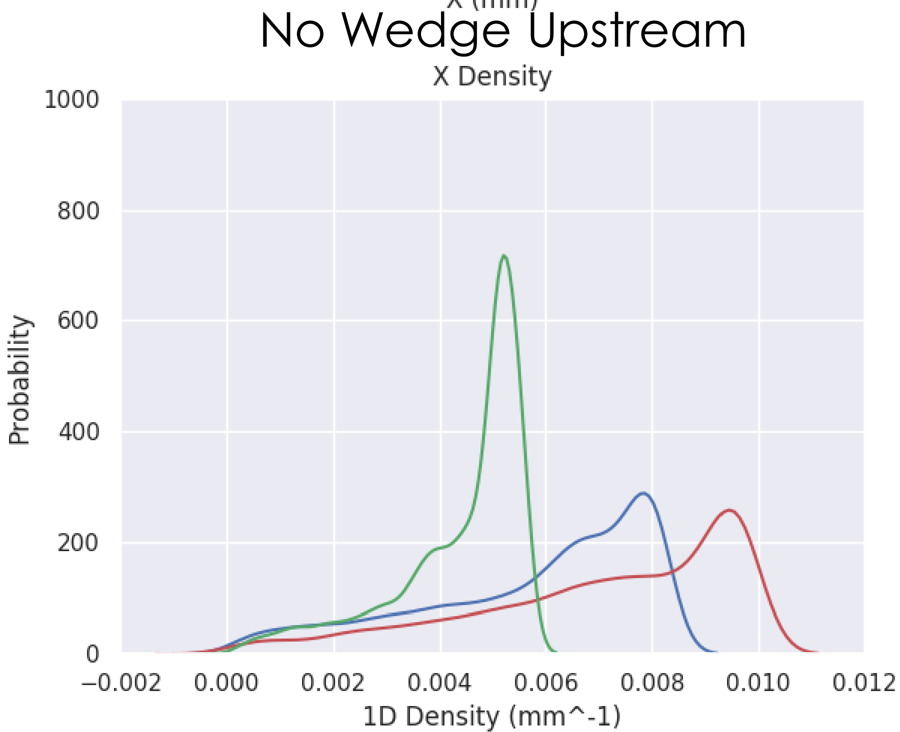
No Wedge (left) and Wedge (right)

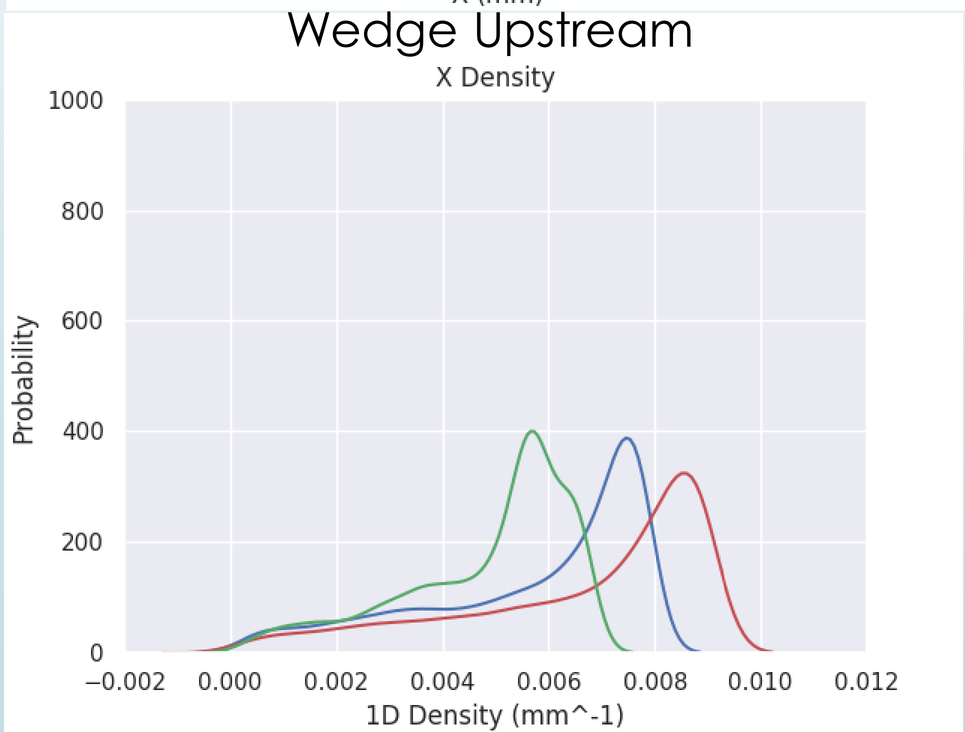
X Distribution (Top) and Density (Bottom)





Blue – Full Upstream Sample
Red – Upstream Sample which makes it Downstream
Green – Upstream Sample which does not make it Downstream





Small preference for larger magnitude x not to make it downstream

Wedge case shows slight directional bias as well. The Wedge does not transmit up to 15% of particles that would have made it downstream otherwise.

Tanaz (left) vs Francois (right) 6-140 LiH analysis

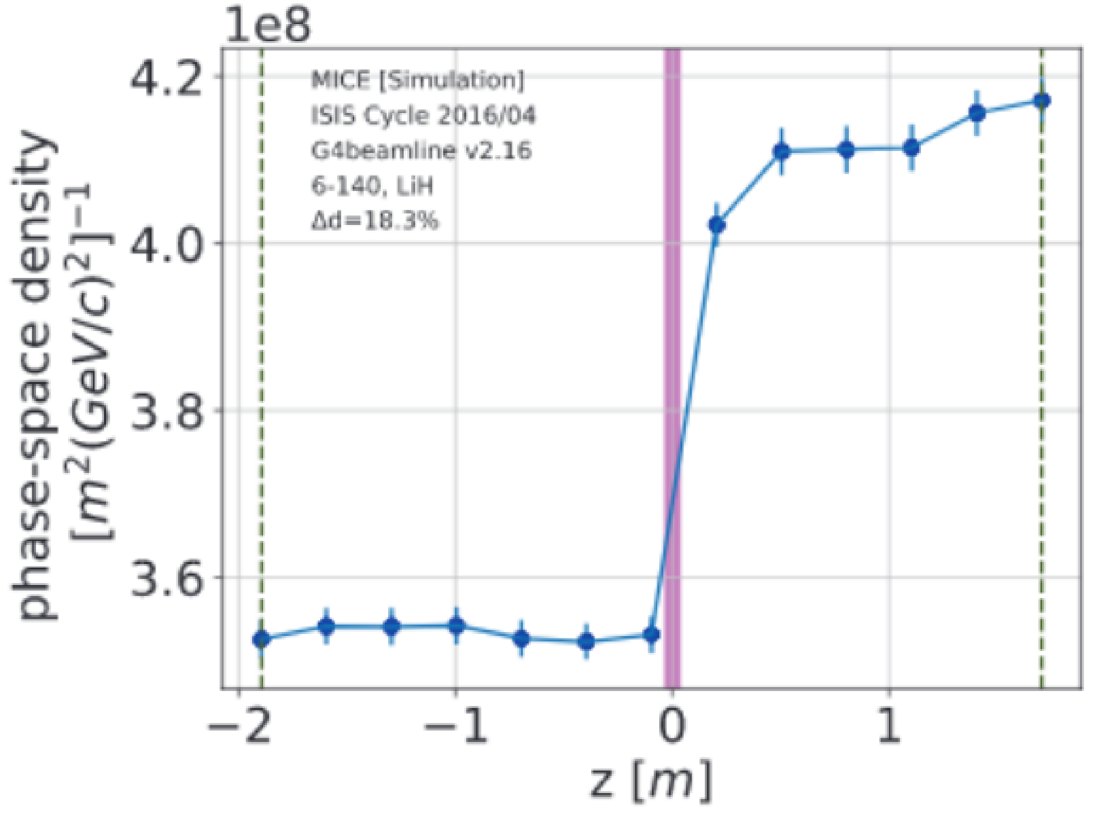
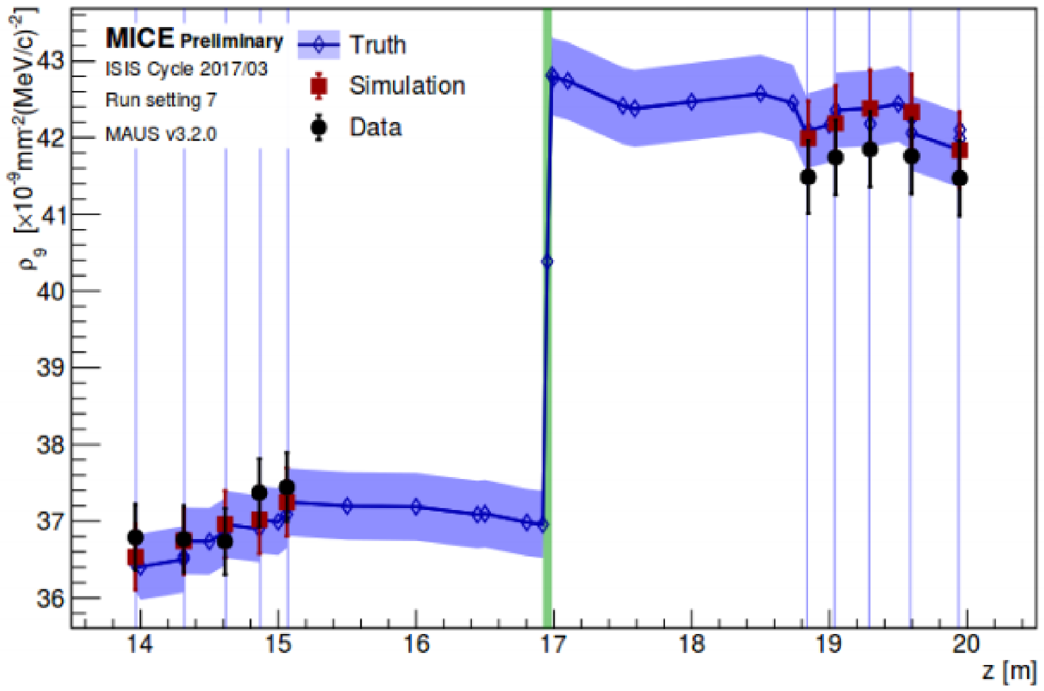
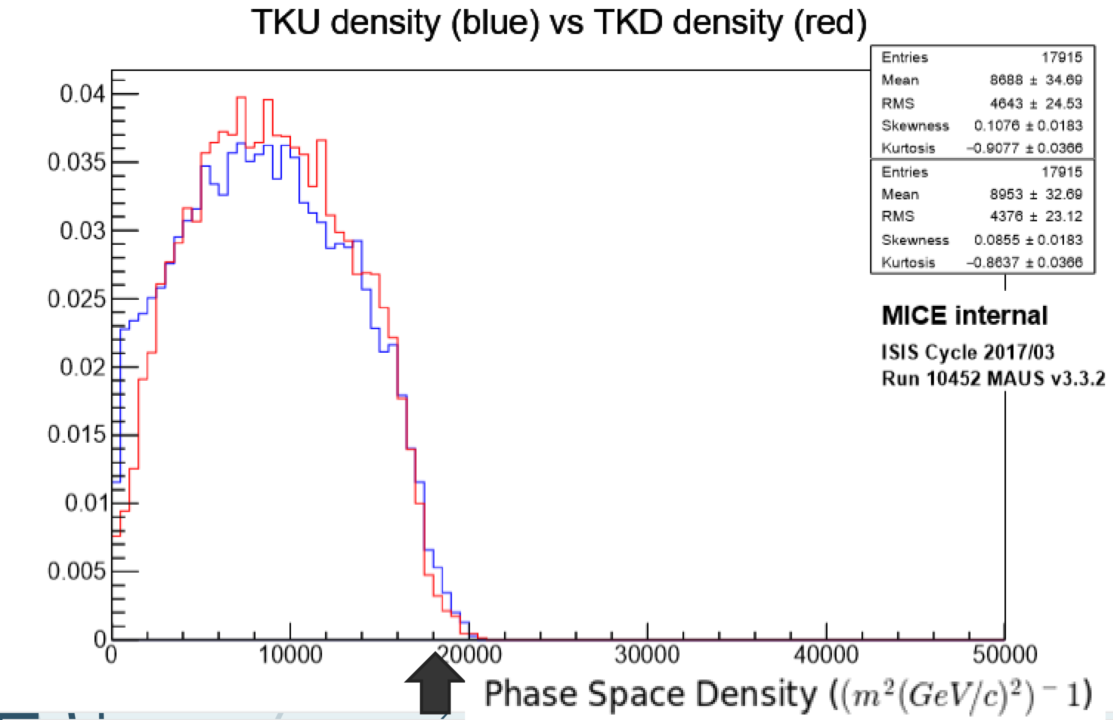


Figure 4: Evolution of the core phase-space density for the 6 – 140 beam setting.

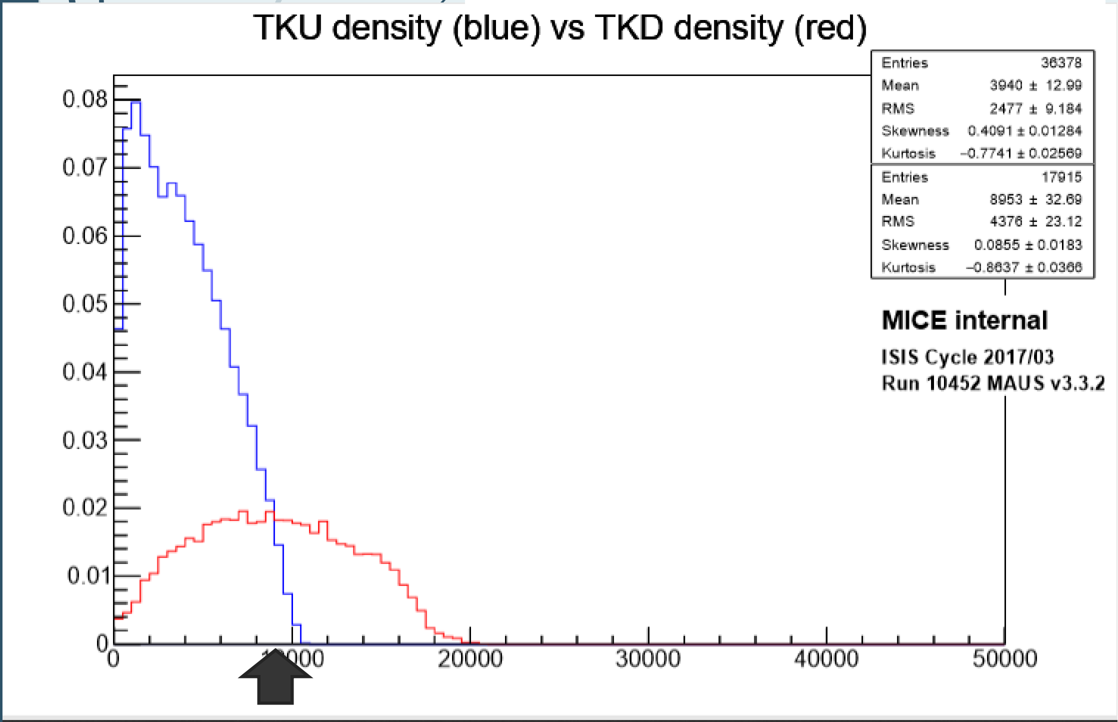
(9%) Contour density evolution (kNN)
6-mm 140-MeV/c beam – LiH – flip

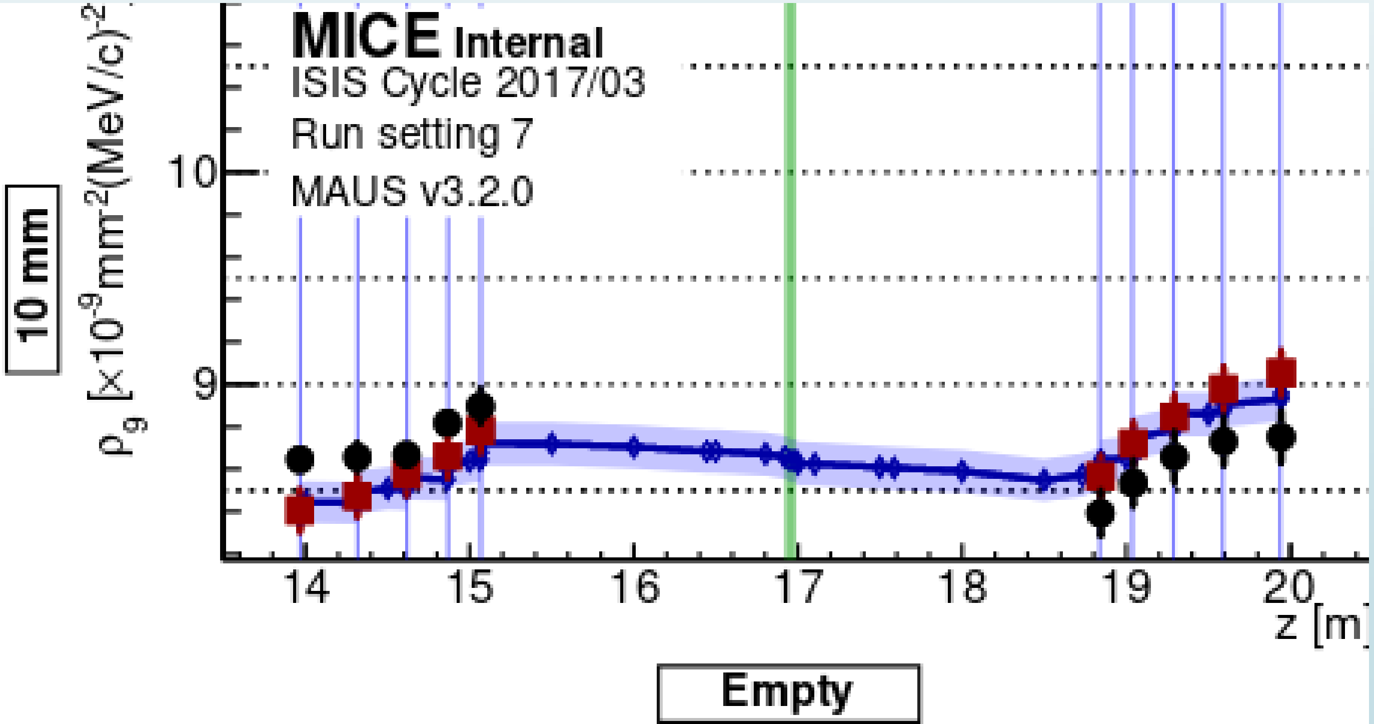


Me (left) vs Francois (right) 10-140 No absorber



- Bottom Right: Change in density through cooling channel
- Top left: Upstream (blue) which made it downstream (red) at reference planes (100% Transmission, biased sample)
- Bottom left: Full Upstream sample (blue) vs downstream (red) (Unbiased Upstream sample, ~50-60% Transmission)



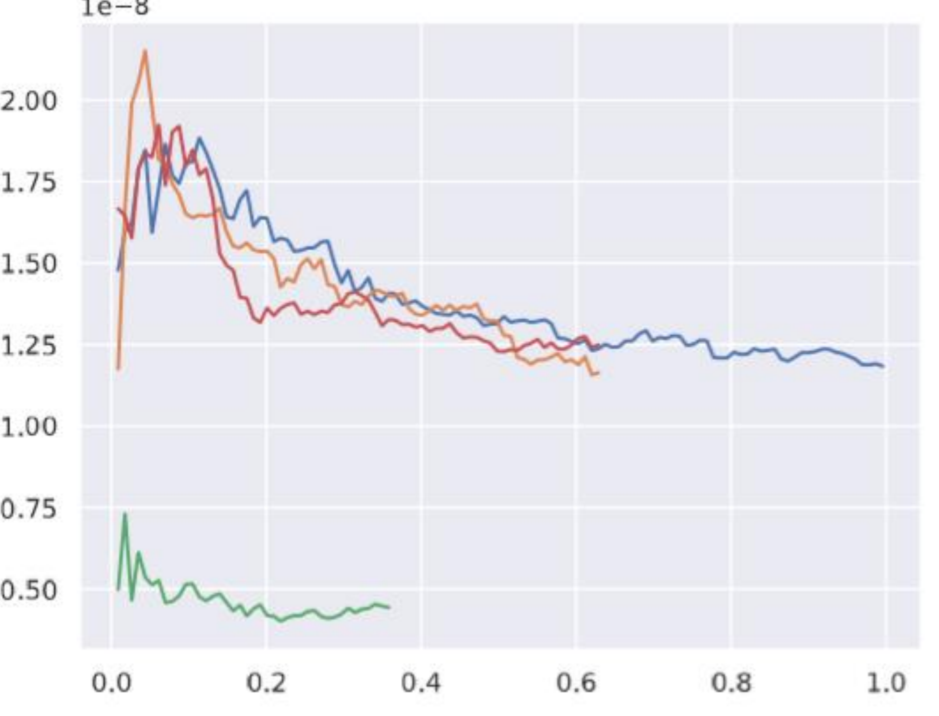


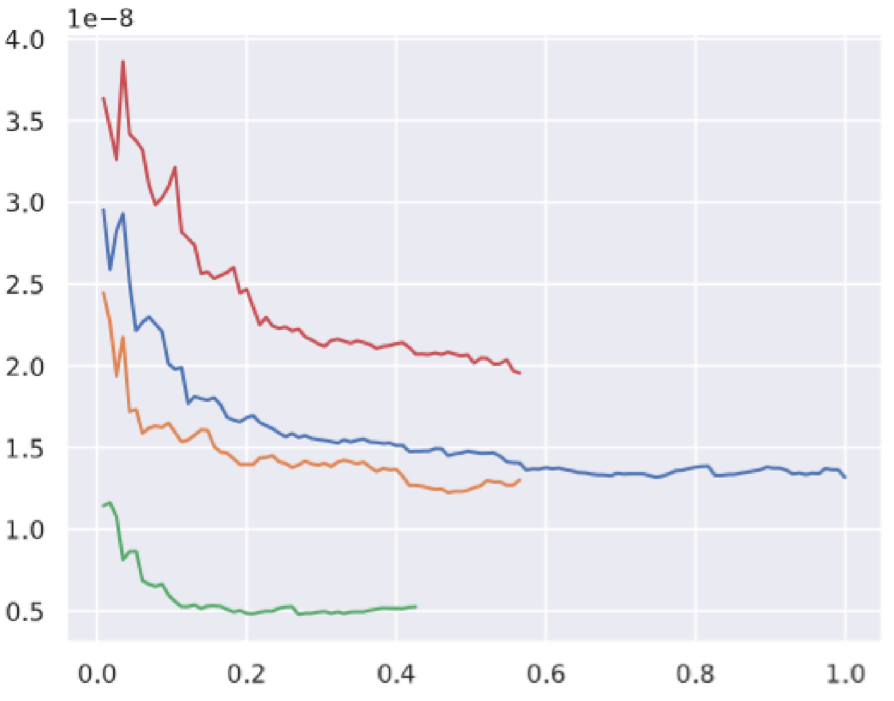
Tanaz and Francois analysis (why the numbers don't match)

To produce the core density evolution plot, the kernel density estimator is used to (the process of summing the kernel functions centered at each data point) re-estimate the density over the core muons, once a core contour is found. The idea is to first estimate the density everywhere (not just at the core of the beam) by summing over kernel functions of fixed widths centered at each muon. The widths of the kernel functions are selected such that the resulting estimated distribution has the smallest deviation from the true density (true density is assumed to be Gaussian). Such kernel width, known as optimal bandwidth parameter (explained in detail in Section 3.2),[27] ensures that the resulting estimated density is not overly smooth or noisy. Once the core contour is found, the transverse phase-space coordinates of core muons (muons with densities higher the density of the core contour density) are saved, and the Gaussian kernel functions are re-evaluated over them. However, this time, because the core has higher occupancy (data points are more closely spaced) than the tail, the optimal kernel width is now smaller than when the tail of the distribution was included in the density estimation process; this leads to an estimated distribution that has, on average higher density than when the density is estimated everywhere in the distribution. A comparison between the evolution plots (Figs. 4.6 and 4.7) and

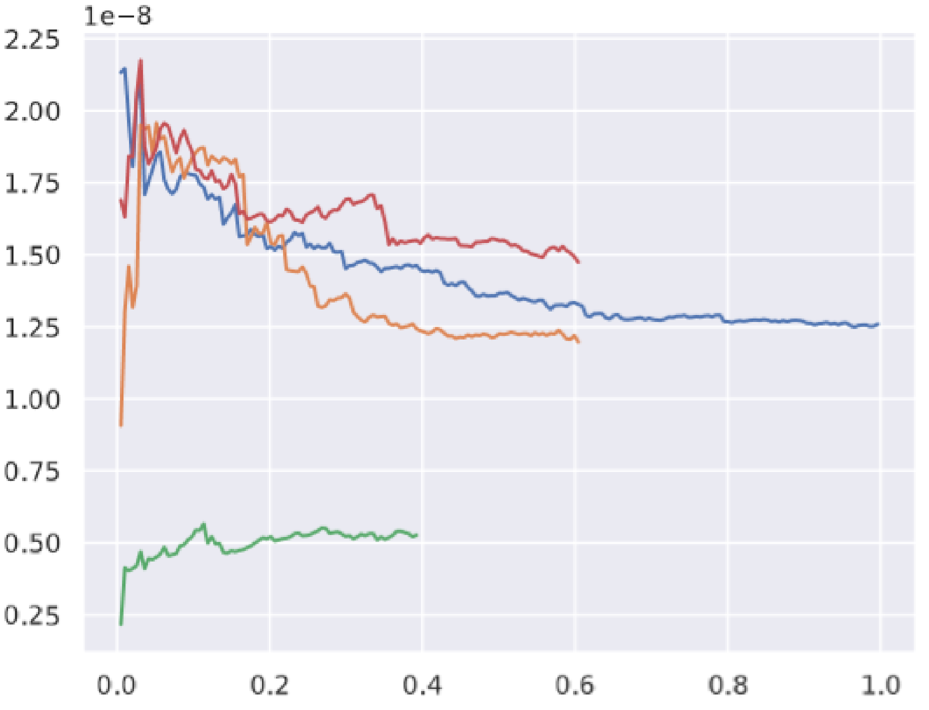
- I had agreement with Francois, difference with Tanaz
- Accounting for change in units, factor of 10,000 difference
- Tanaz and Francois results look similar bar the 10,000 difference, however, she actually does calculate the density differently:
- Tanaz finds the 9% core and isolates those particles. From those particles she recalculates the density with the remaining sample. This has changed the particle distribution, as well as the volume over which it has been calculated.
- Isolating the core can be advantageous to aid with transmission, however it appears the 9-th percentile density is calculated on the 9% core.
- ~10% for each of four dimensions would give a factor of ~10,000
- Effectively < 1% of particles are chosen, which can result in significant statistical fluctuations
- It also doesn't deal with transmission losses and if the same particles are being compared

Change in Peak density vs beam fraction

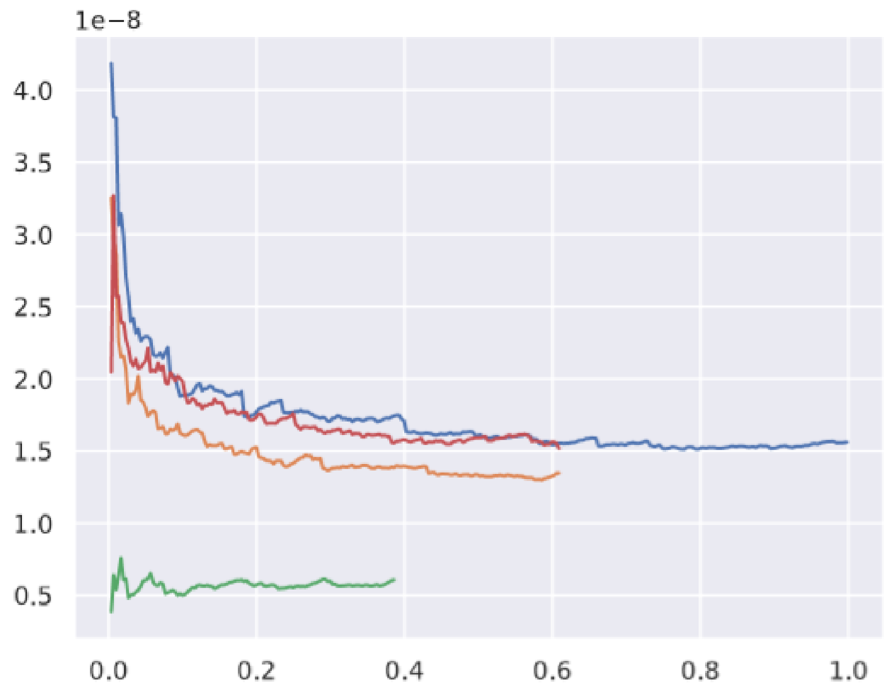




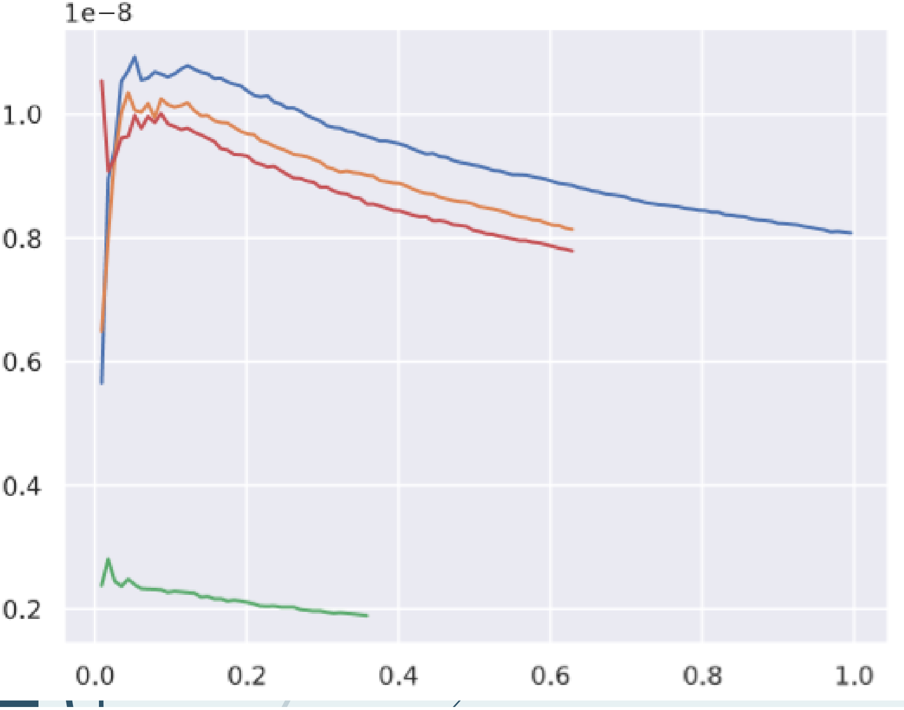
Top Left: No absorber
Top Right: Wedge
Bottom Left: LiH
Bottom Right: LH2

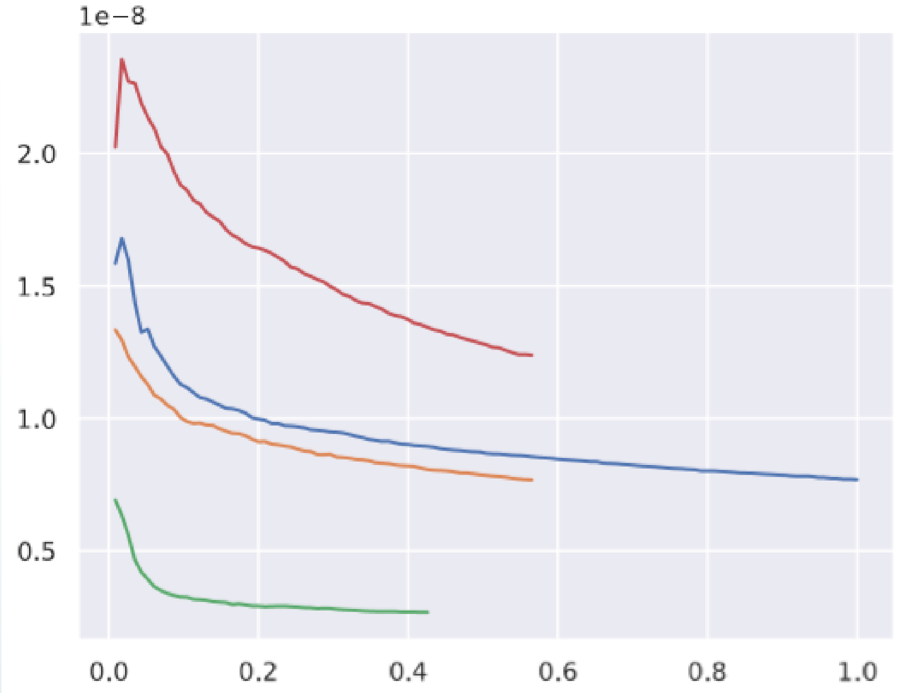


Blue – Full Upstream Sample
Red – Full Downstream Sample
Orange – Upstream Sample which made it Downstream
Green – Upstream Sample which doesn't make it downstream

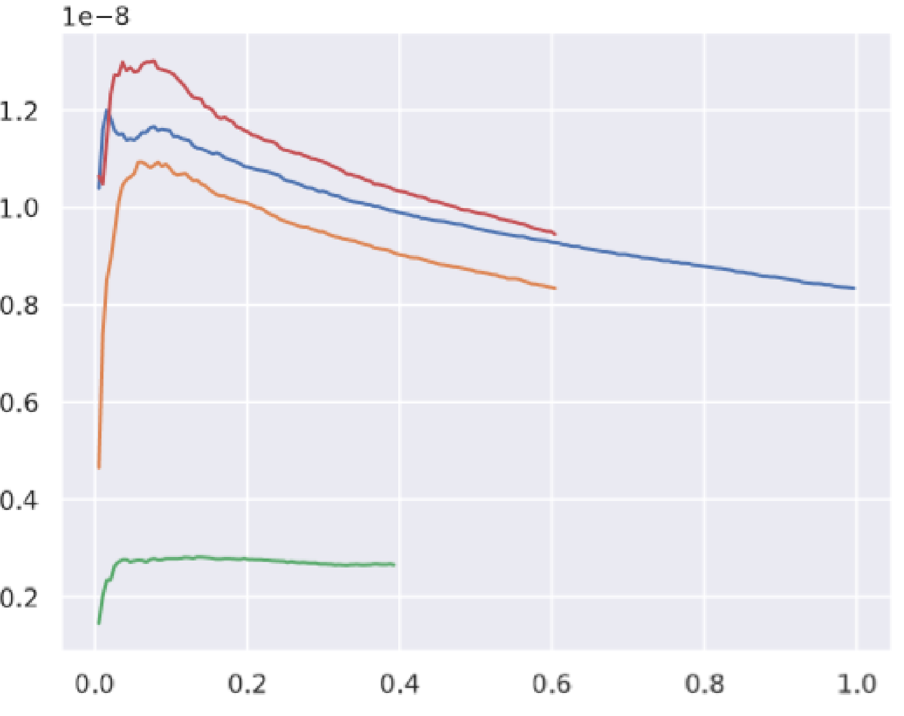


Change in 9th percentile density vs beam fraction

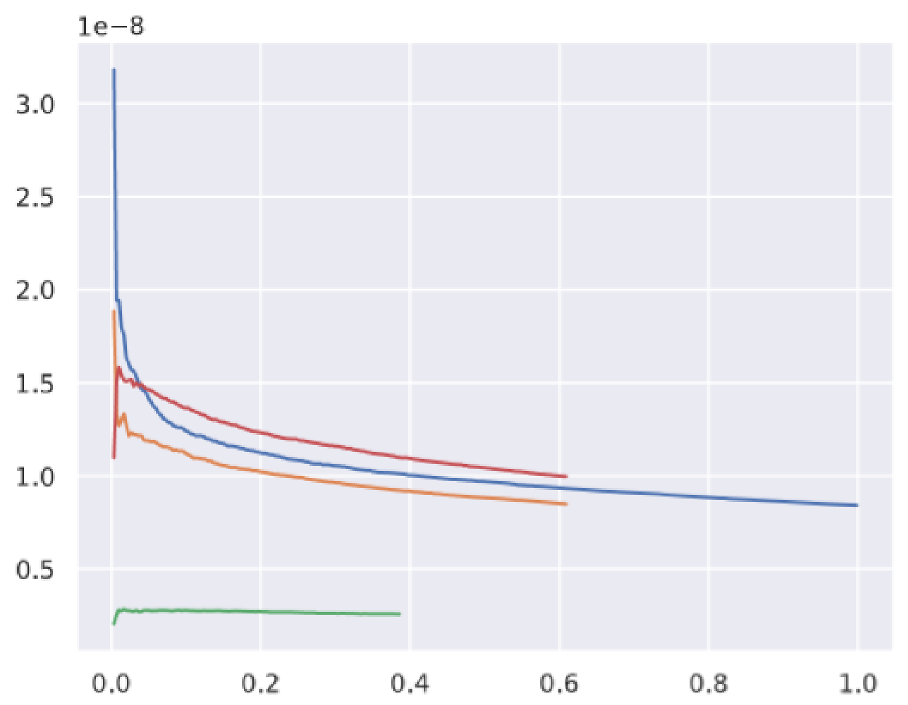




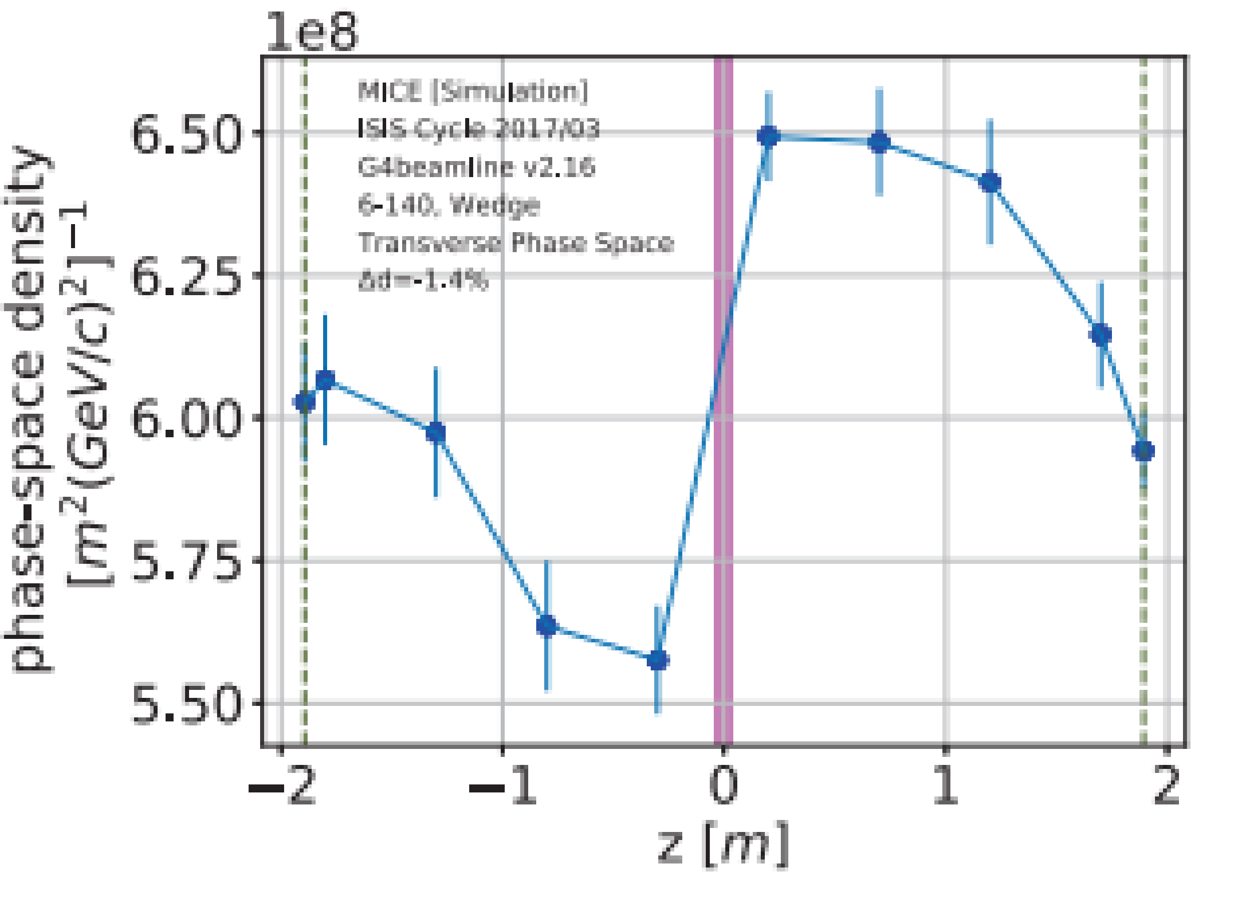
Top Left: No absorber
Top Right: Wedge
Bottom Left: LiH
Bottom Right: LH2



Blue – Full Upstream Sample
Red – Full Downstream Sample
Orange – Upstream Sample which made it Downstream
Green – Upstream Sample which doesn't make it downstream



Tanaz's 6-140 transverse 4D results – IPAC2018



- Tanaz 6-140 Wedge plot
- Analysis is based on comparing the reference planes where it claims a decrease in density.
- Liouville – change in density only through dissipative forces, therefore change in density should only occur across the absorber (the wedge in this case)
- Before and after the density should remain constant (for the case where transverse components can be isolated from the longitudinal components)
- However a change is seen (something has gone wrong)
- Either the transmission losses are heavily biasing the results, or the statistical errors of choosing too small a sample size haven't been accounted for.
- In either case, Emittance Exchange can't be claimed here

Not only low density particles are eliminated

Blue – Full Upstream Sample
Orange – Upstream Sample which makes it Downstream
Green – Upstream Sample which doesn't make it Downstream

The full upstream distribution (blue) can be divided into the upstream distribution which makes it downstream (orange) and upstream distribution which doesn't make it downstream (green) calculated over the full Upstream distribution volume.