

BERNHARD MISTLBERGER



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**COLOUR-SINGLET PRODUCTION AT N3LO**

# PRECISION LHC PHYSICS

- ▶ The large amount of LHC data allows us to measure some observables to astounding precision.
- ▶ Typically, such observables are characterised by clearly identifiable final states and involve very large final state particle momenta.

## Higgs, Z, W, top quark:

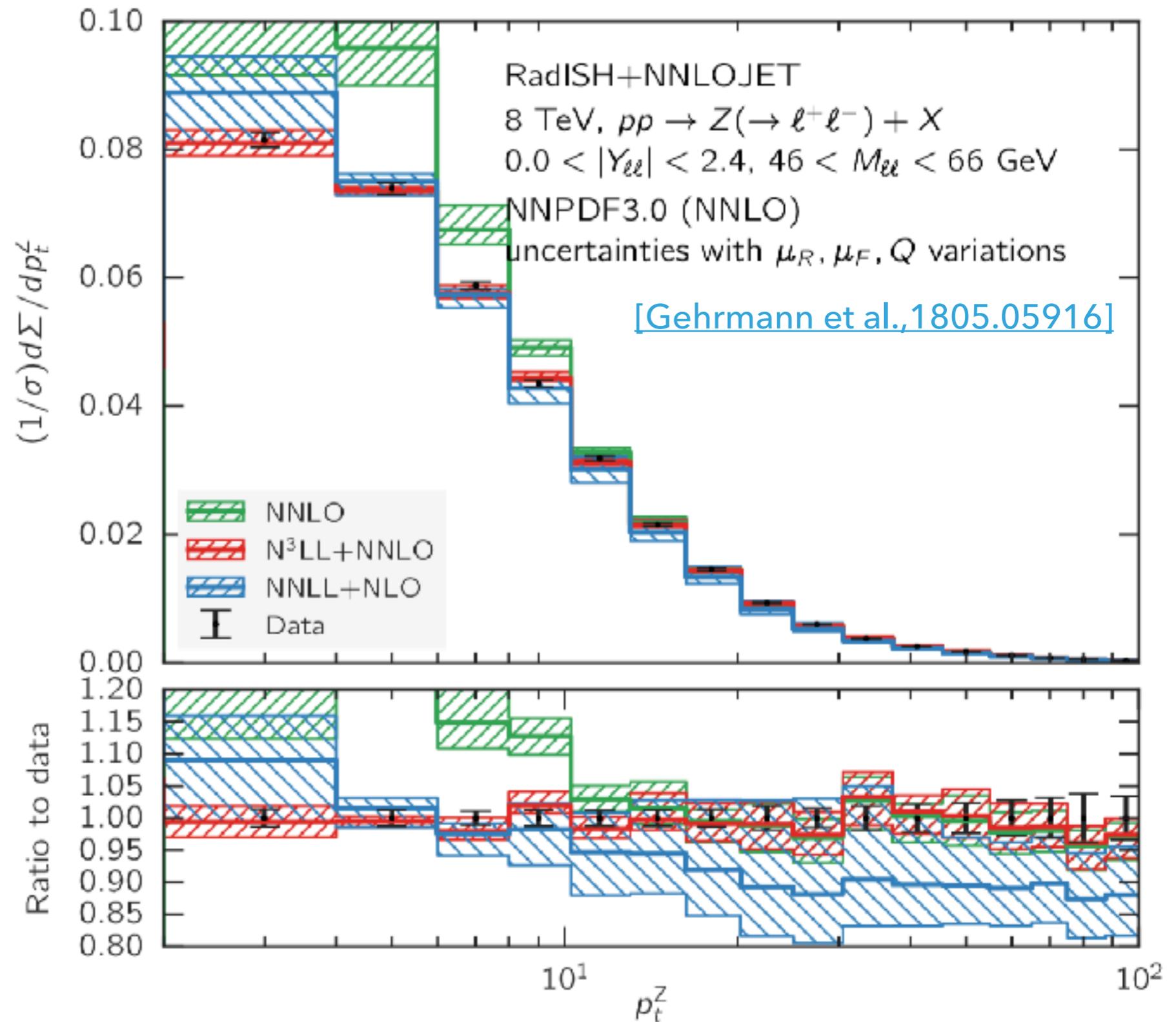


## Higgs, Z, W, top quark:

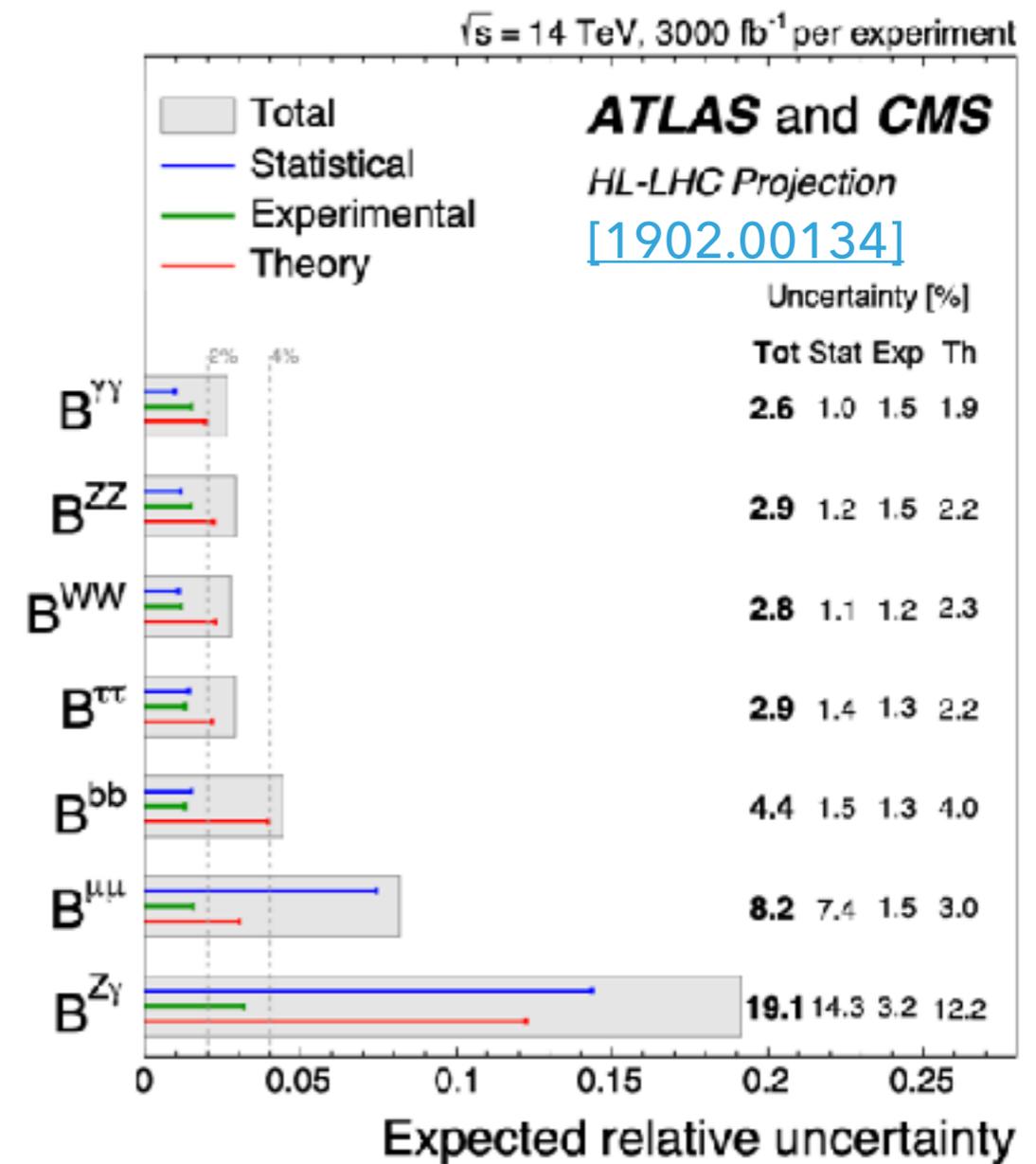
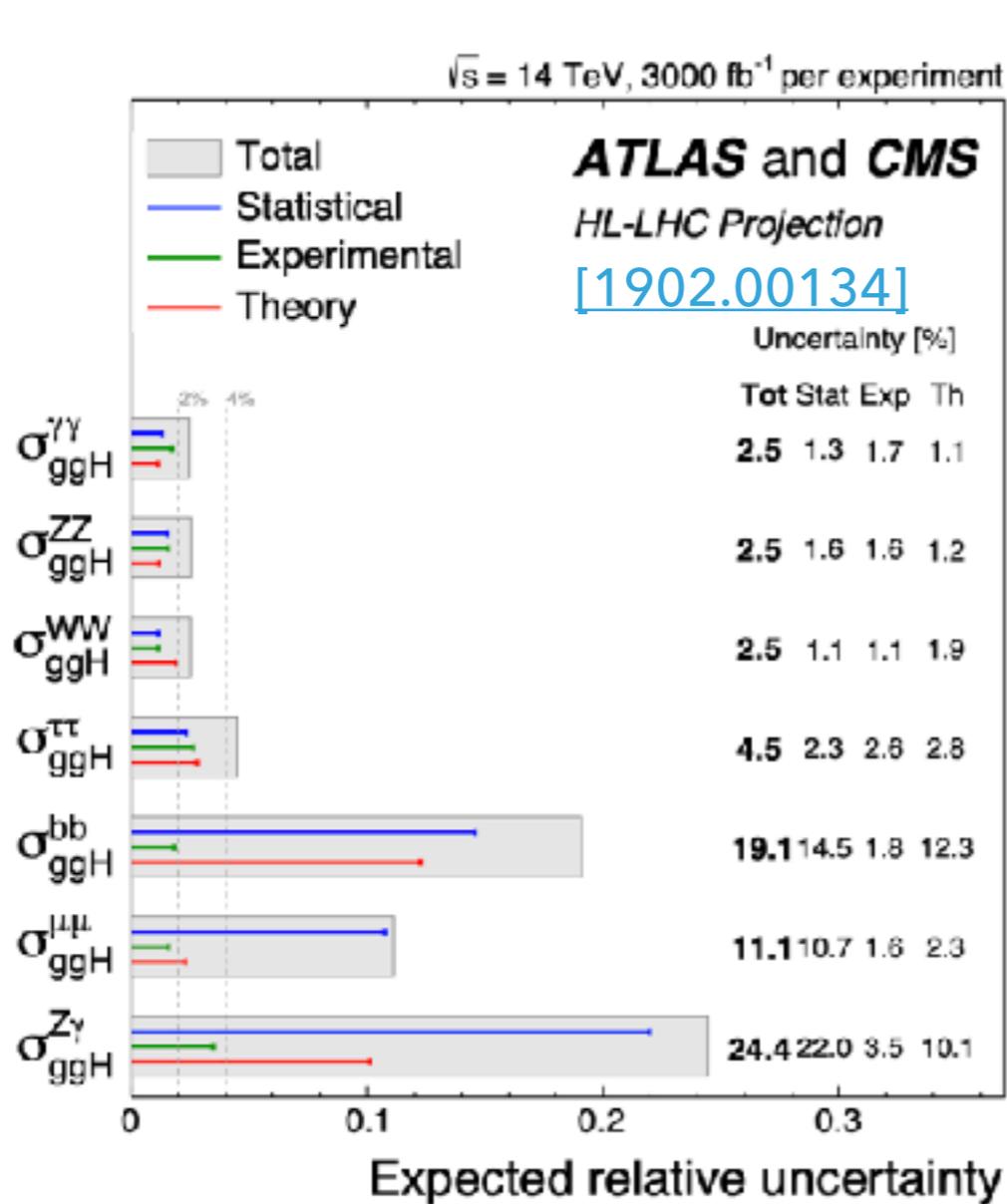


- ▶ **The quest nr 1:** What is the Higgs potential? What are the properties of the Higgs?
- ▶ Measuring their properties will allow us to systematically search for deviations from Standard Model Physics (with EFTs, with bumps).
- ▶ New physics aside: We can simple determine masses, couplings, parton distribution functions; study hadronisation, jets and interactions of highly energetic particles in general.

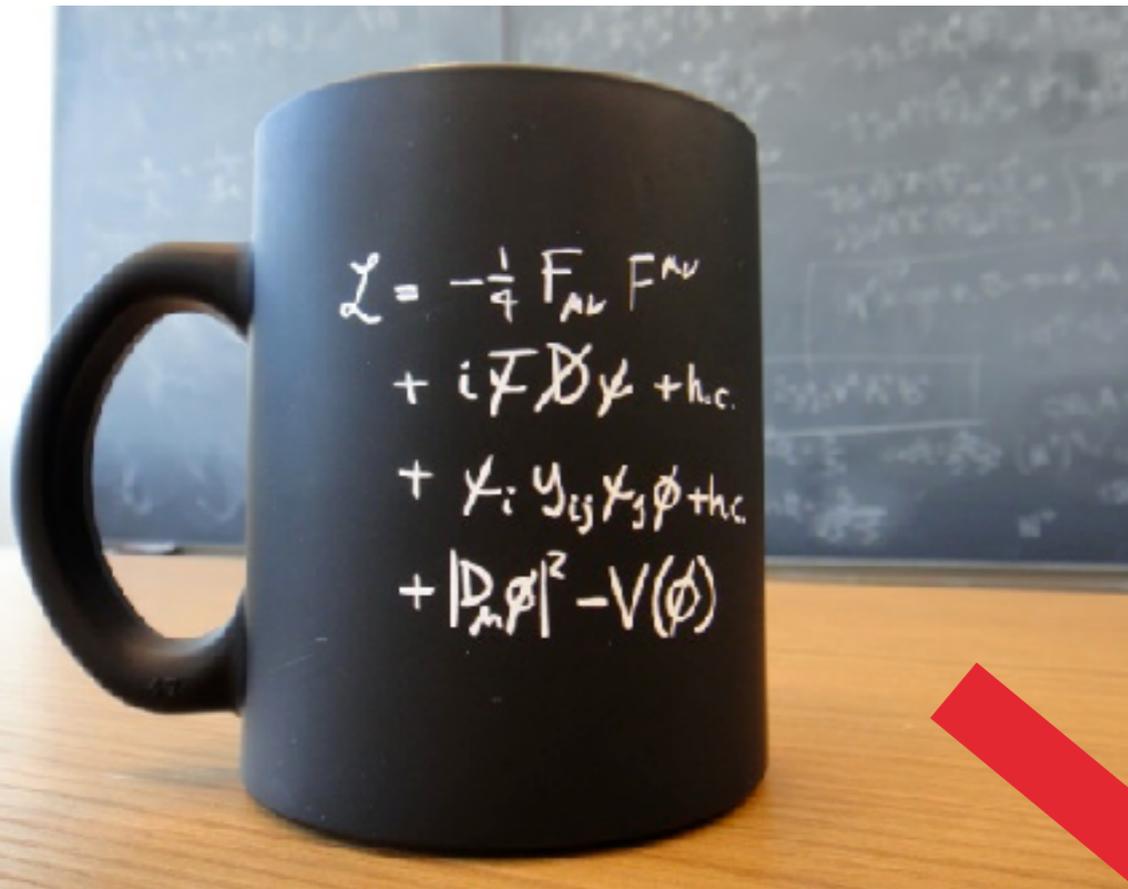
- ▶ Huge Cross Sections
- ▶ 2 - lepton final state
- ▶ Extremely Precise Measurements
- ▶ Here: 8 TeV Data.



- ▶ 3000 fb<sup>-1</sup>
- ▶ Determining Higgs properties at few percent!

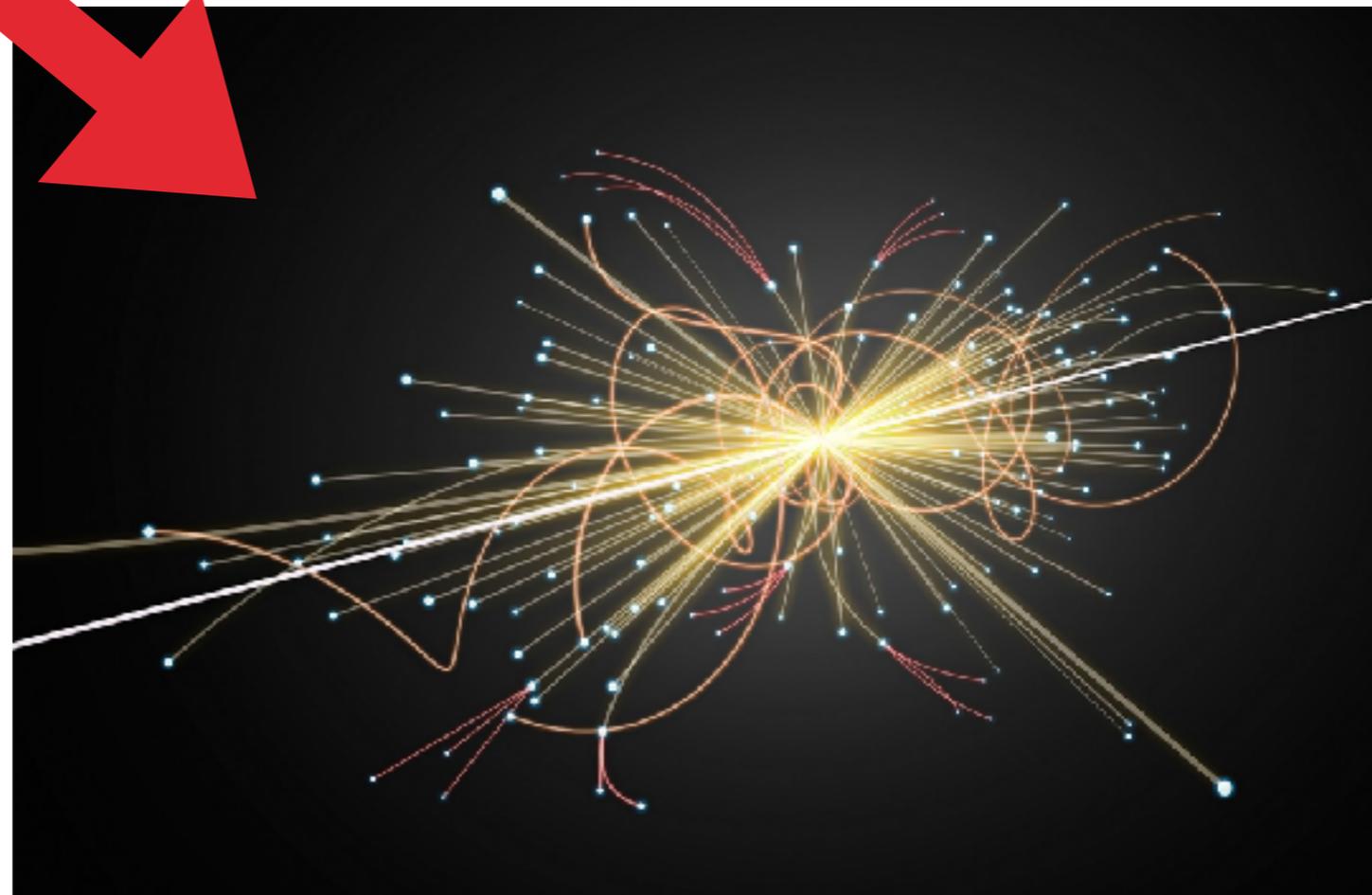


# COMPUTING AT N3LO

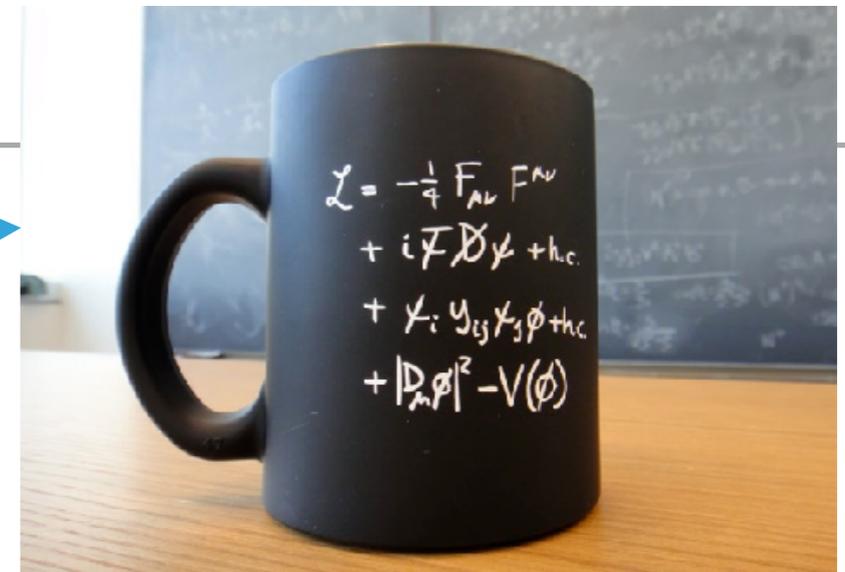


**To learn about nature  
we need to connect**

**Theory and Experiment  
at the same level of precision.**



$$\sigma \sim \int dx dy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$



- ▶ Perturbative partonic cross sections
- ▶ QCD perturbation theory is dominant  $\alpha_S = 0.118$

▶ Naively:

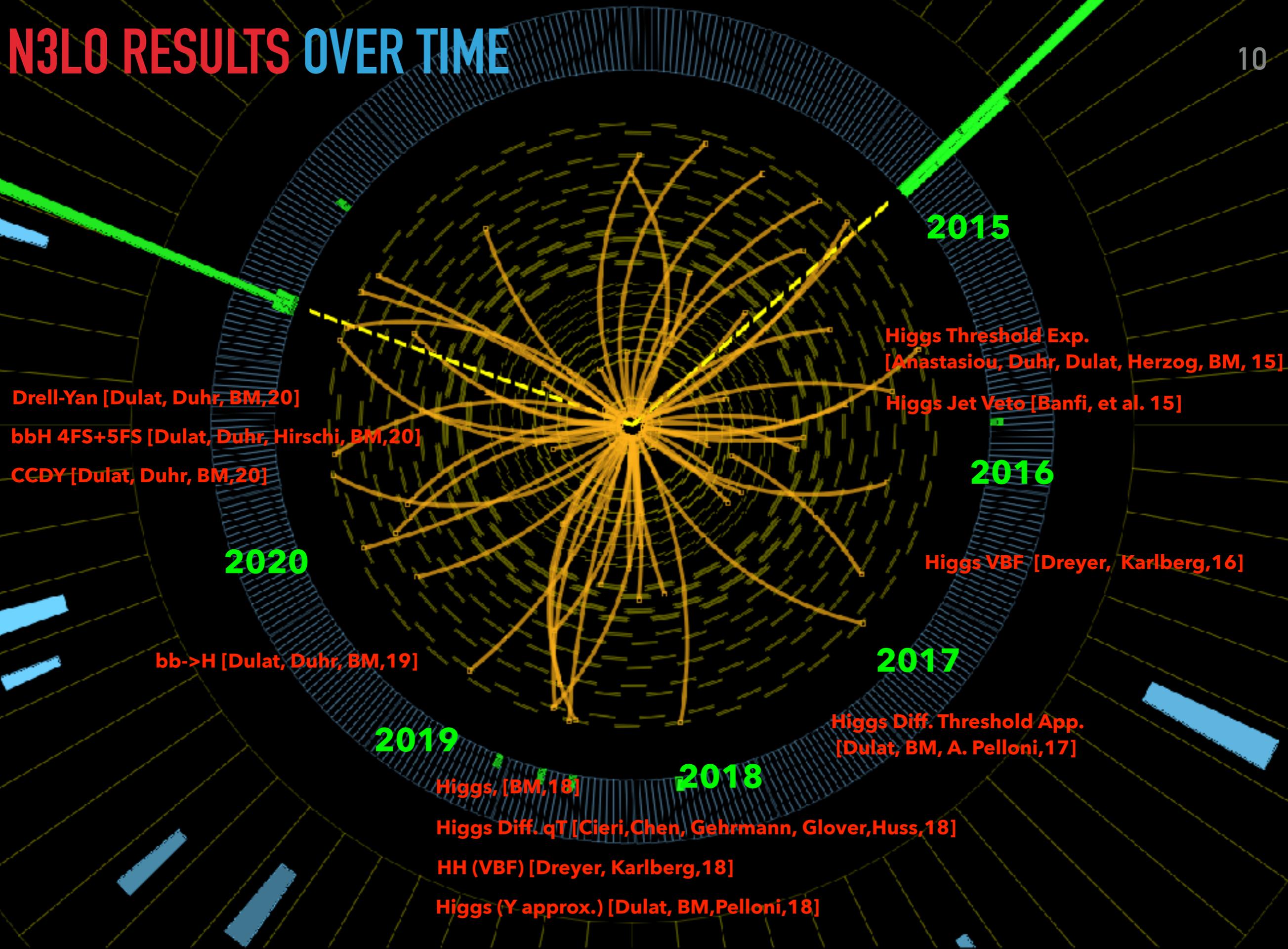
$$\hat{\sigma} = \text{LO } \hat{\sigma}^{(0)} + \alpha_S^1 \text{NLO } \hat{\sigma}^{(1)} + \alpha_S^2 \text{NNLO } \hat{\sigma}^{(2)} + \alpha_S^3 \text{N3LO } \hat{\sigma}^{(3)} + \dots$$

- ▶ Resum, where this doesn't work!



# N3LO RESULTS OVER TIME

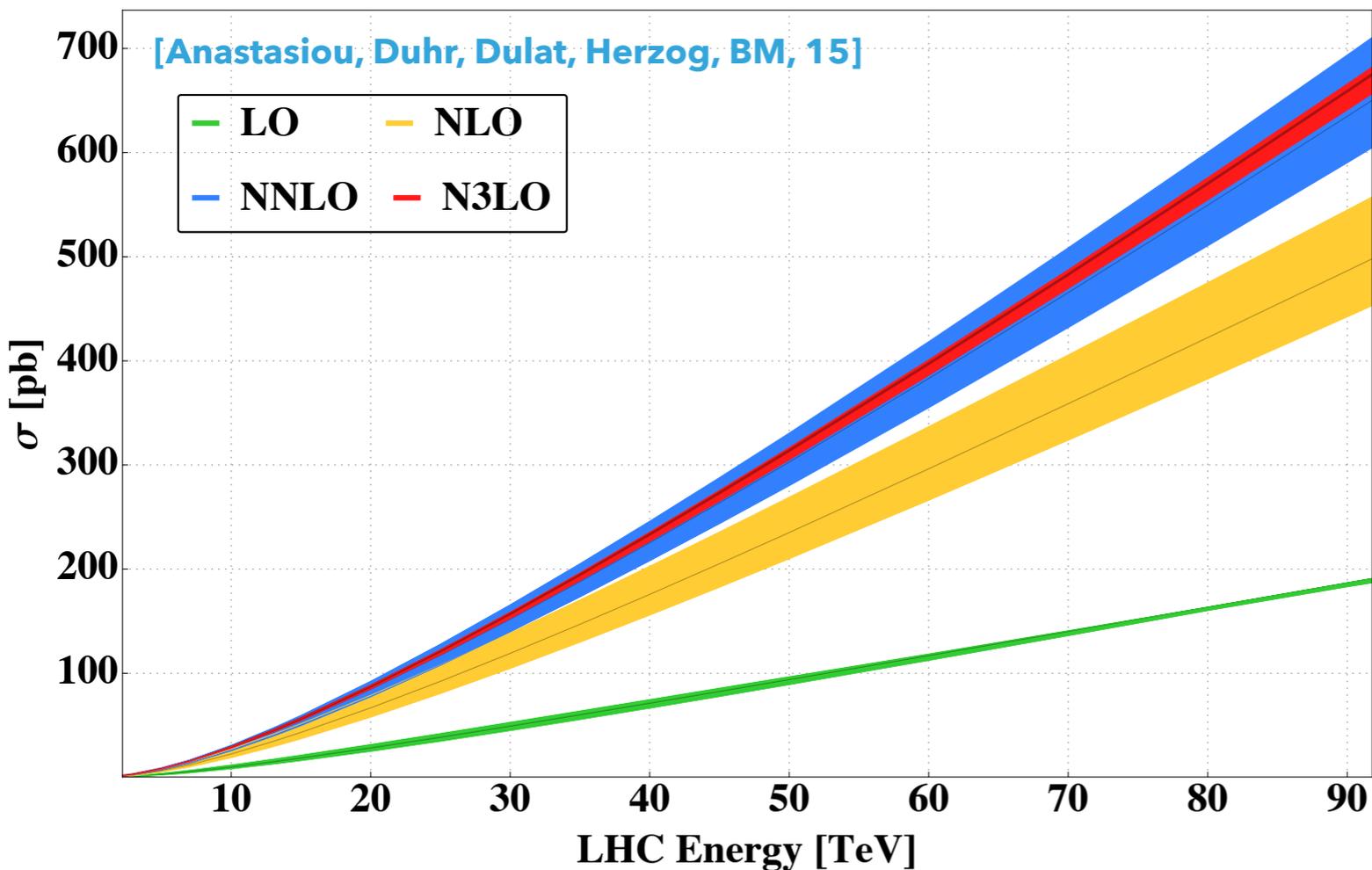
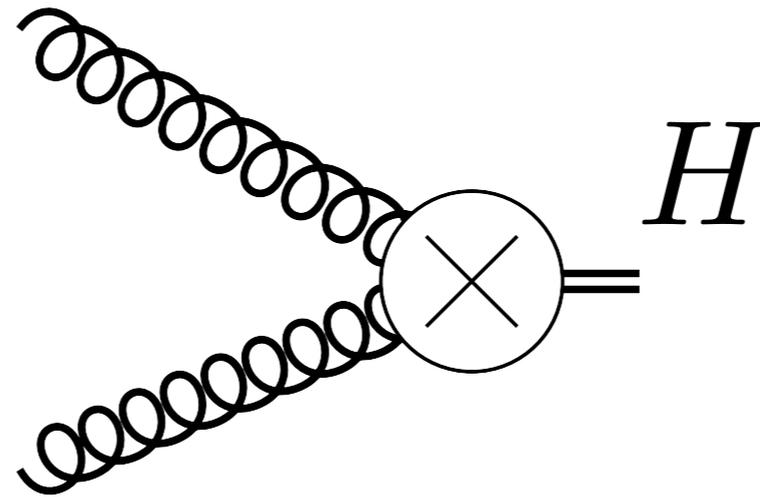
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# INCLUSIVE CROSS SECTIONS AT N3LO

$$P P \rightarrow H$$

# HIGGS PRODUCTION



- ▶ Observable plagued by large corrections.
- ▶ N3LO stabilises the expansion.
- ▶ Here: Only QCD corrections with bands = scale variation

**Much** more than QCD corrections

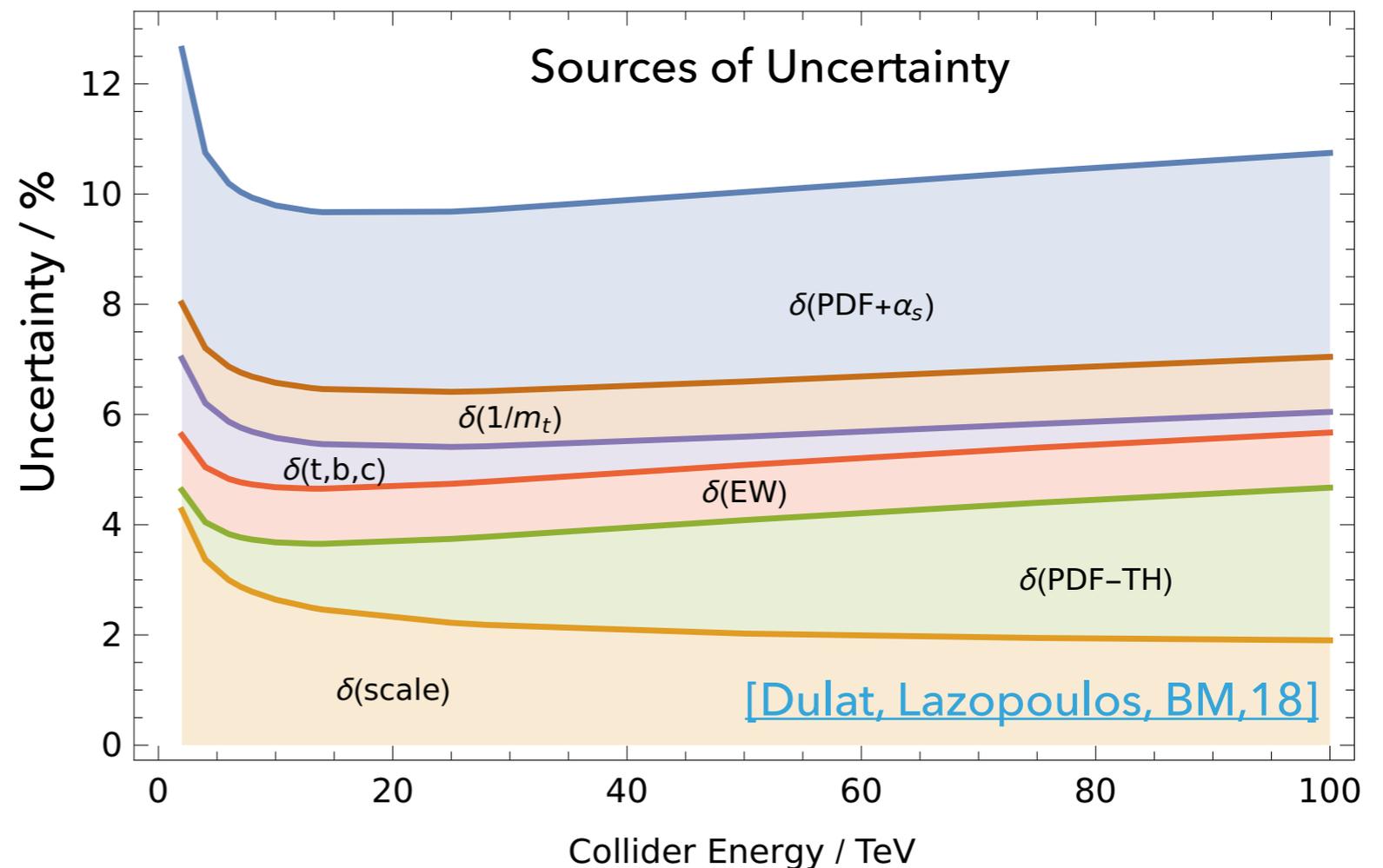
- ▶ Electro-weak corrections.
- ▶ Neglected quark mass effects.
- ▶ Coupling to bottom, charm quarks.

- ▶ Estimate uncertainties.

Truncation of perturbative series

PDF,  $\alpha_S$

- ▶ ...



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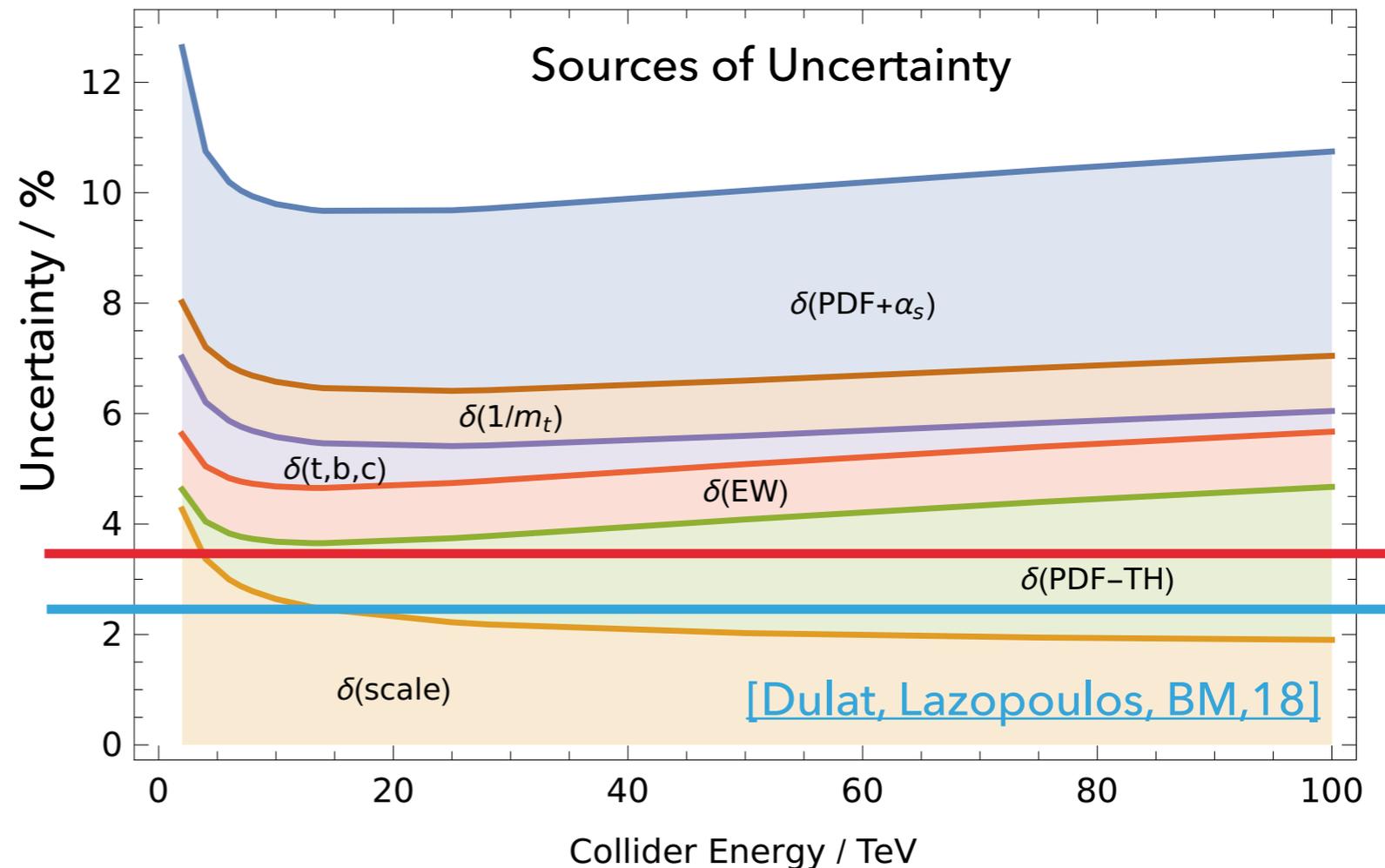
▶ ...

HL-LHC

Pessimistic scenario

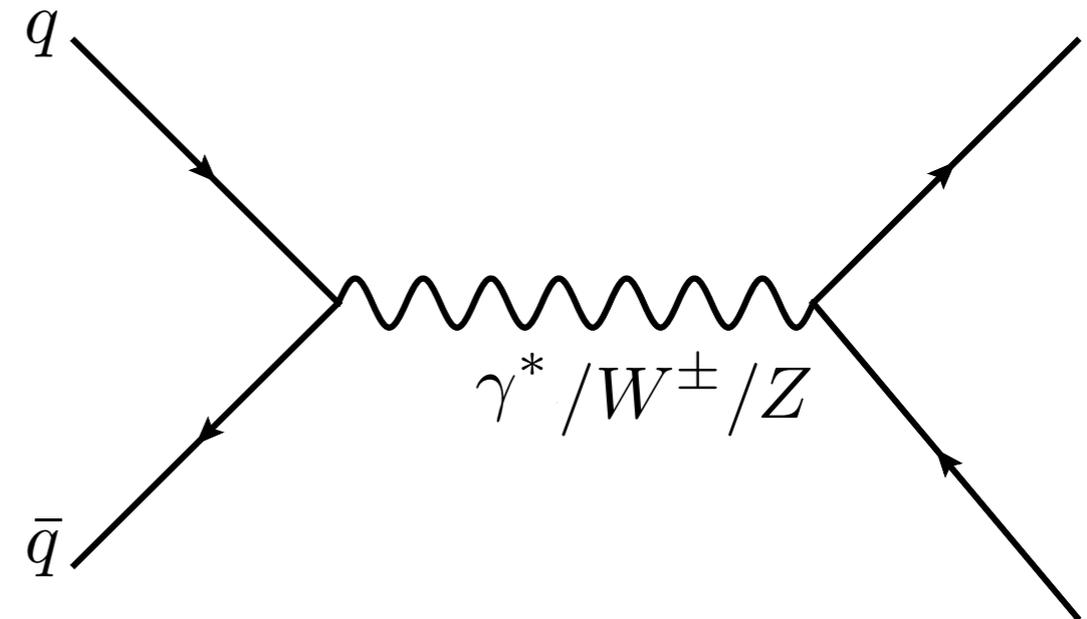
Optimistic scenario

$3000 \text{ fb}^{-1}$



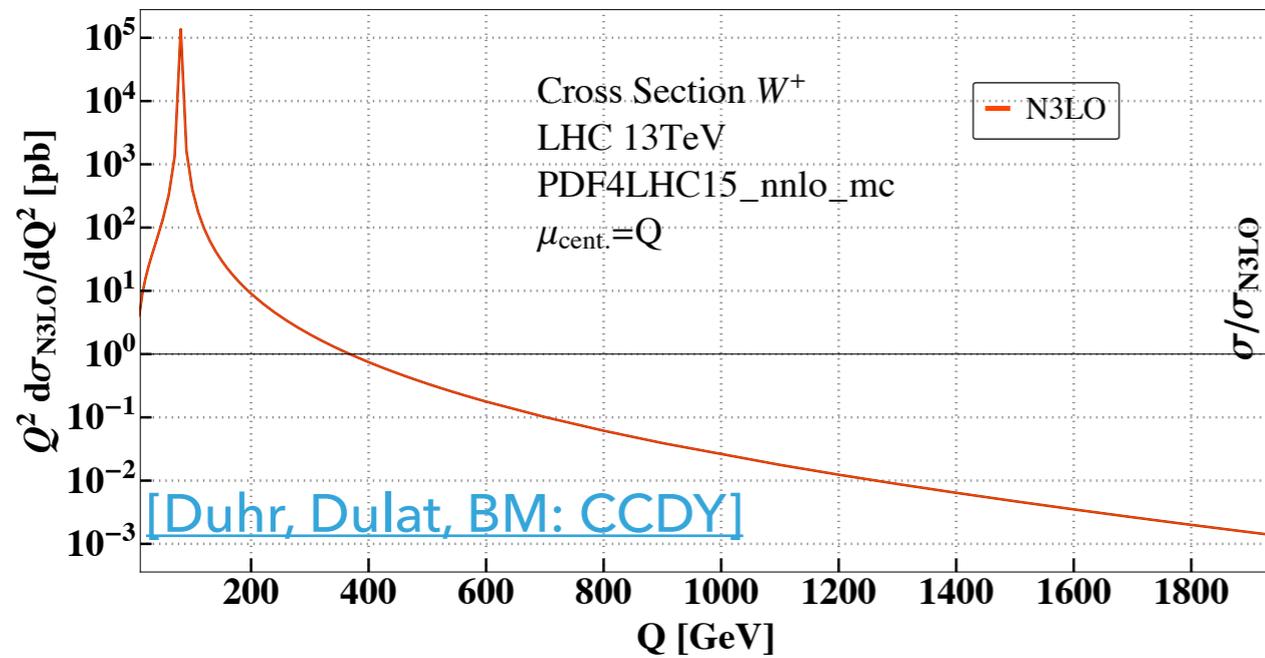
$$P P \rightarrow \gamma^* / W^\pm / Z$$

## DRELL-YAN PRODUCTION

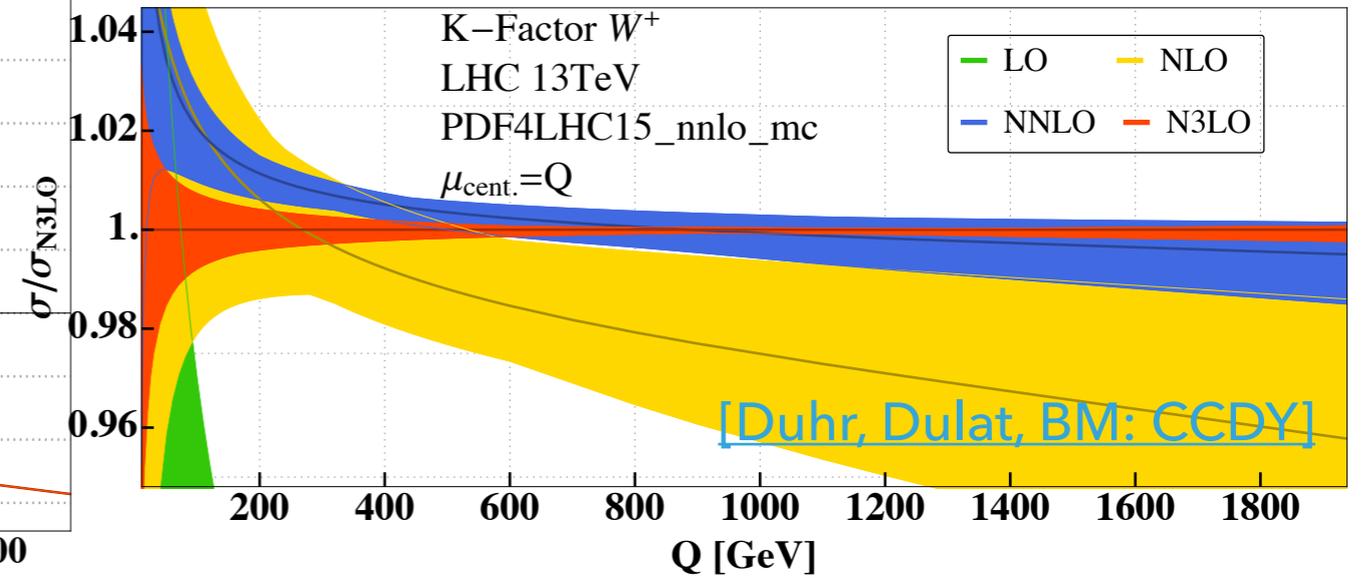


- ▶ Inclusive probability to produce a virtual vector boson that can decay into leptons or jets.
- ▶ Very clean final state measured with incredible statistics.
- ▶ Recently computed to **N3LO** in QCD perturbation theory
  - ★ K - Factors
  - ★ Learn about the progression of perturbation theory
  - ★ Learn about the precision achievable for such processes

## Nominal XS:

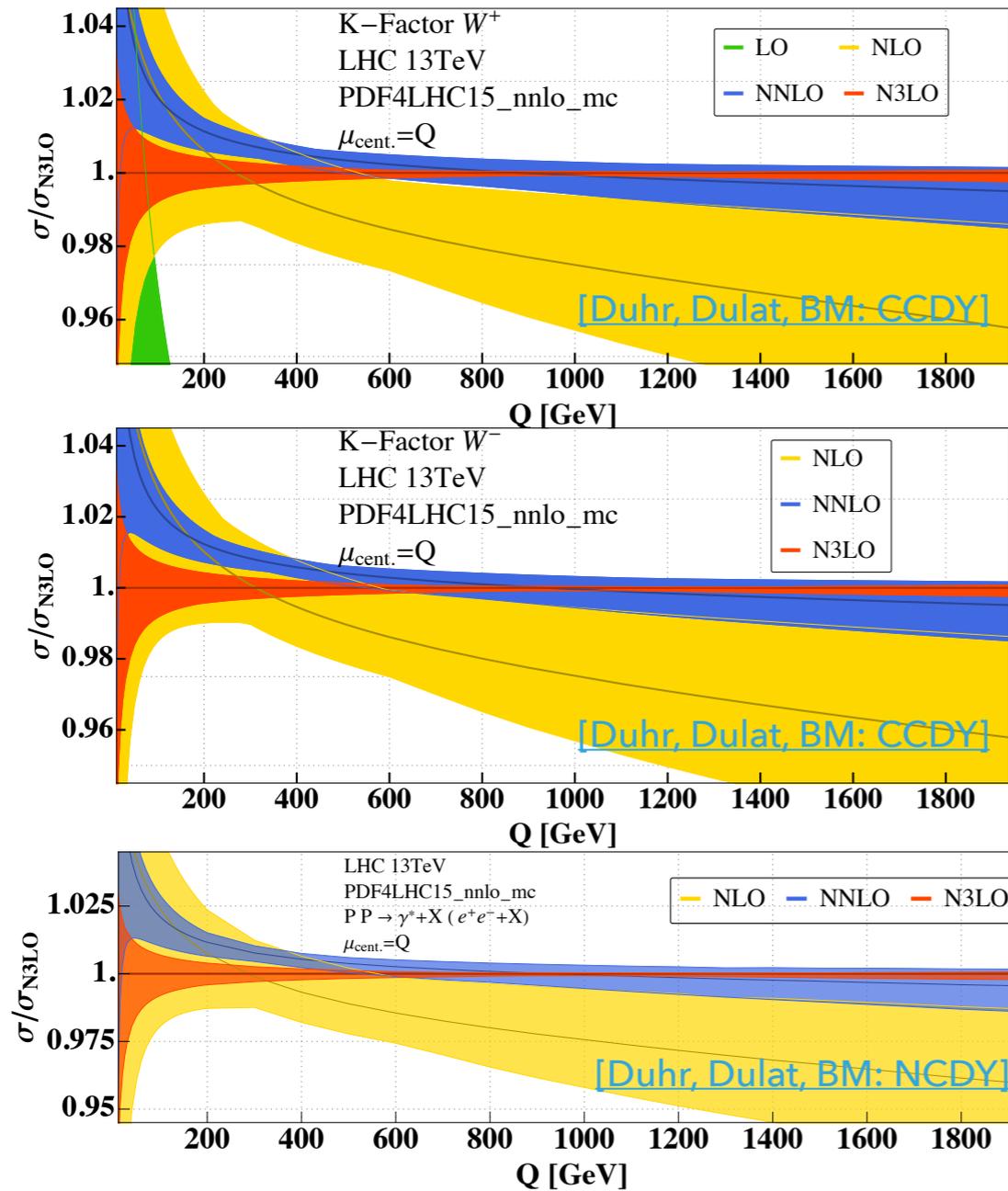


## K-Factor



- ▶ Percent level corrections.
- ▶ Stable at large scales.
- ▶ Per mille scale variations.

## K-Factor



**W<sup>+</sup>**

- ▶ Very similar corrections for CC and NC DY cross sections.

**W<sup>-</sup>**

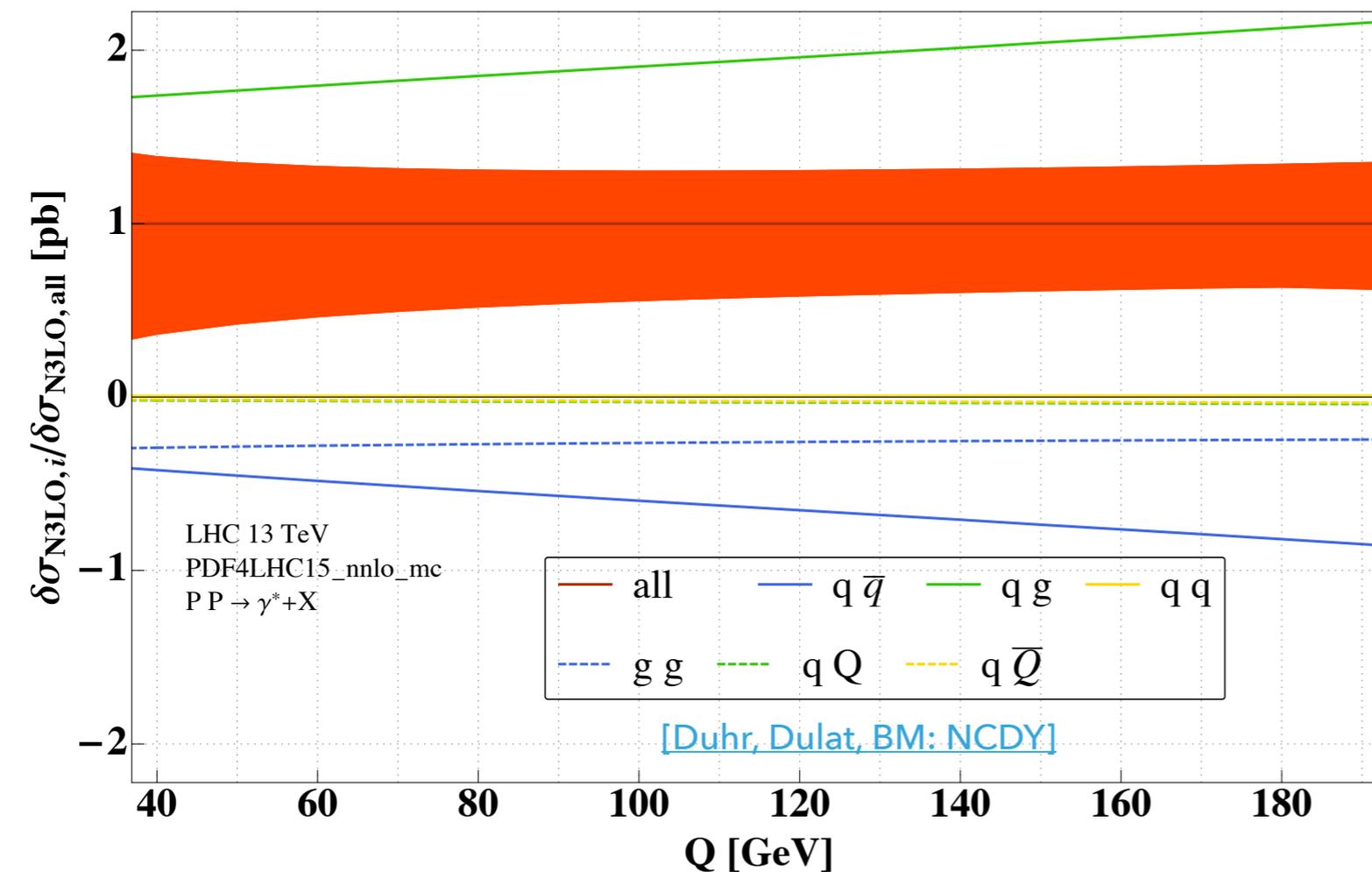
- ▶ Gap at  $\sim 100$  GeV of NNLO and N3LO bands.

**Photon**

$$P P \rightarrow \gamma^*$$

## Non-overlapping scale bands: N3LO vs NNLO

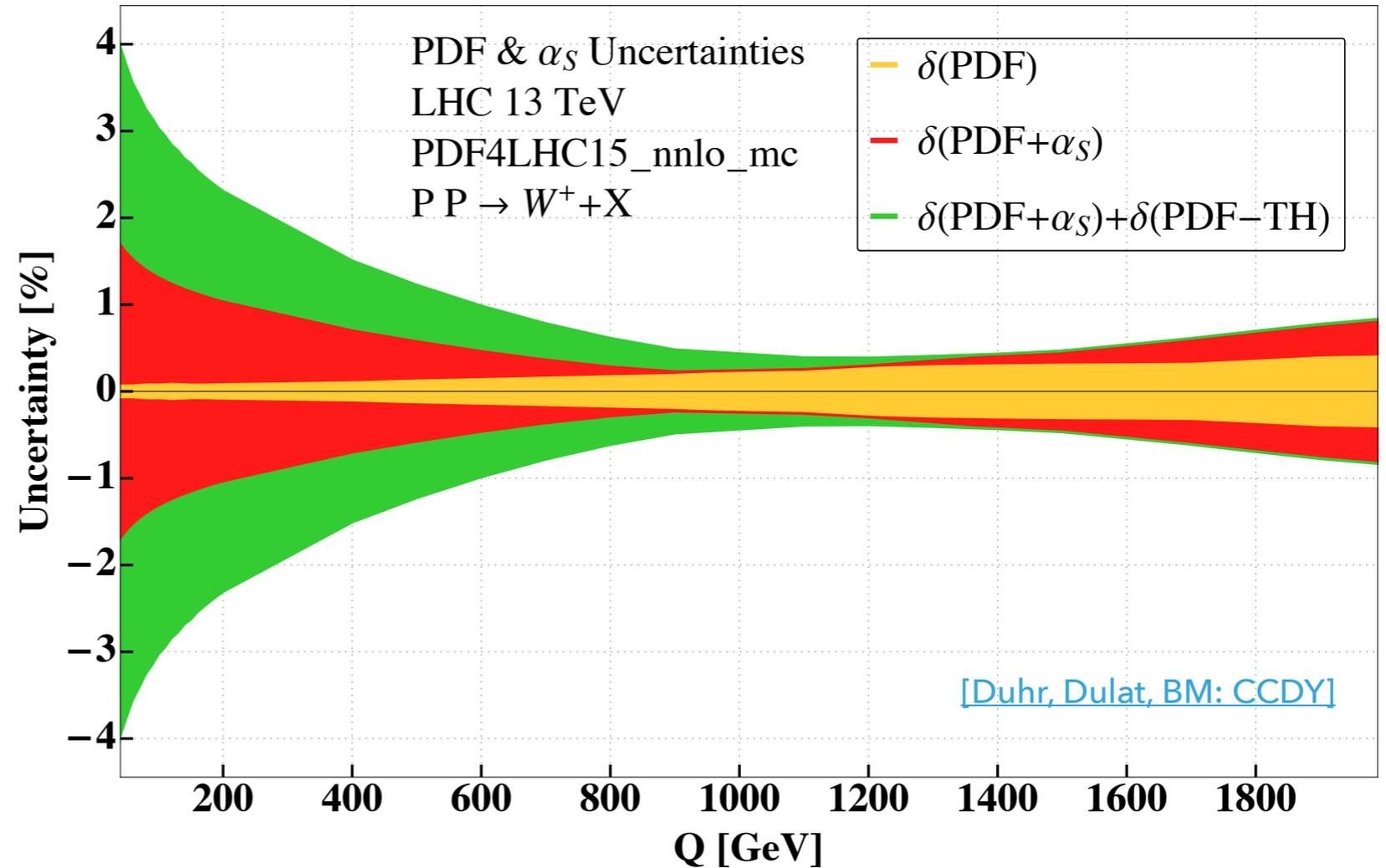
### Decompose partonic initial states:



- ▶ Large cancellation of quark and gluon initial state.
- ▶ Compensated by N3LO PDFs?? - currently missing.
- ▶ Similar for W bosons.

## Large uncertainties related to PDFs:

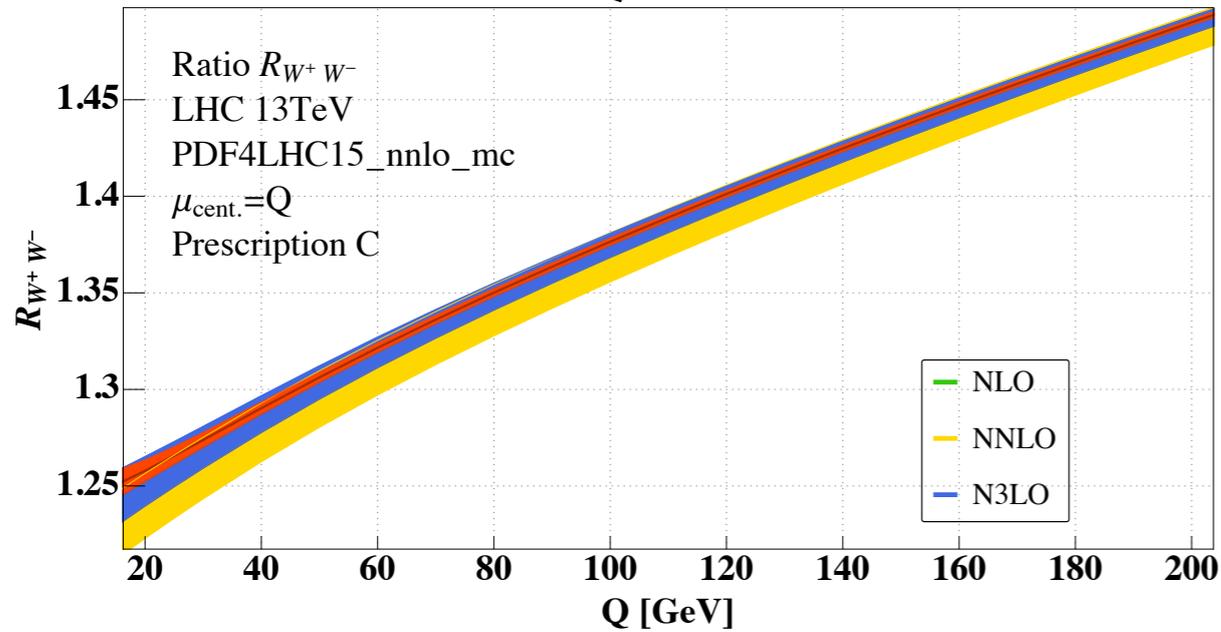
- ▶ Currently: no N3LO PDFs
- ▶ Strong Coupling Constant
- ▶ PDFs themselves are not known to the level of QCD predictions



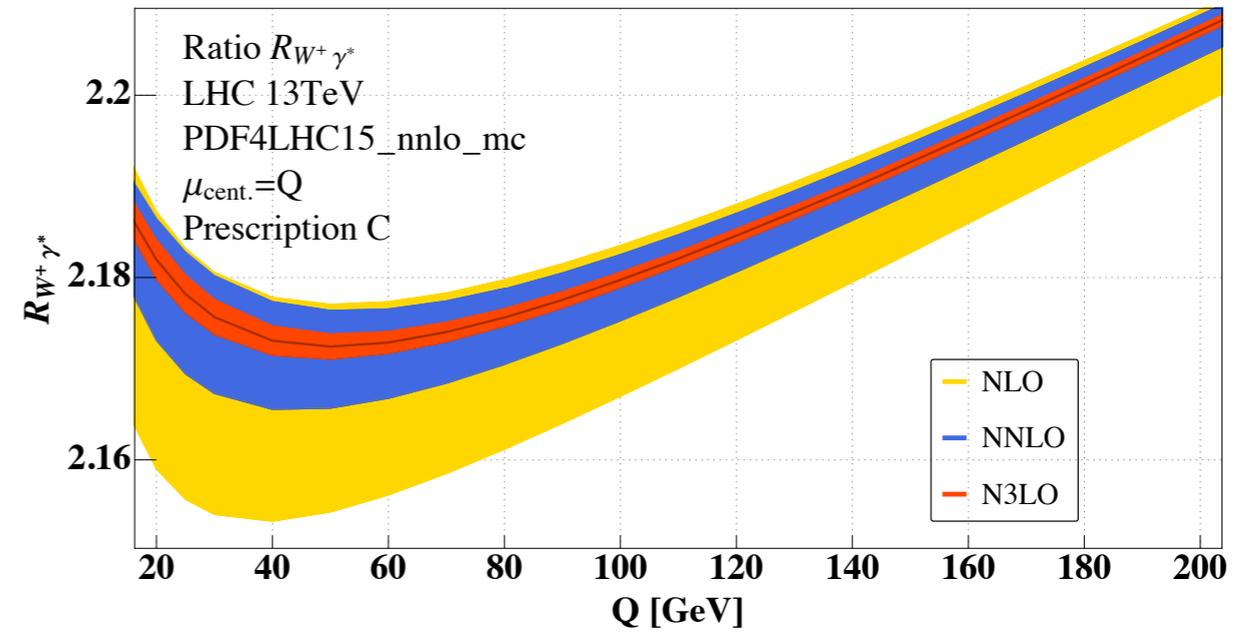
**Opportunity to push for better PDFs.**

**Per-Cent Level**

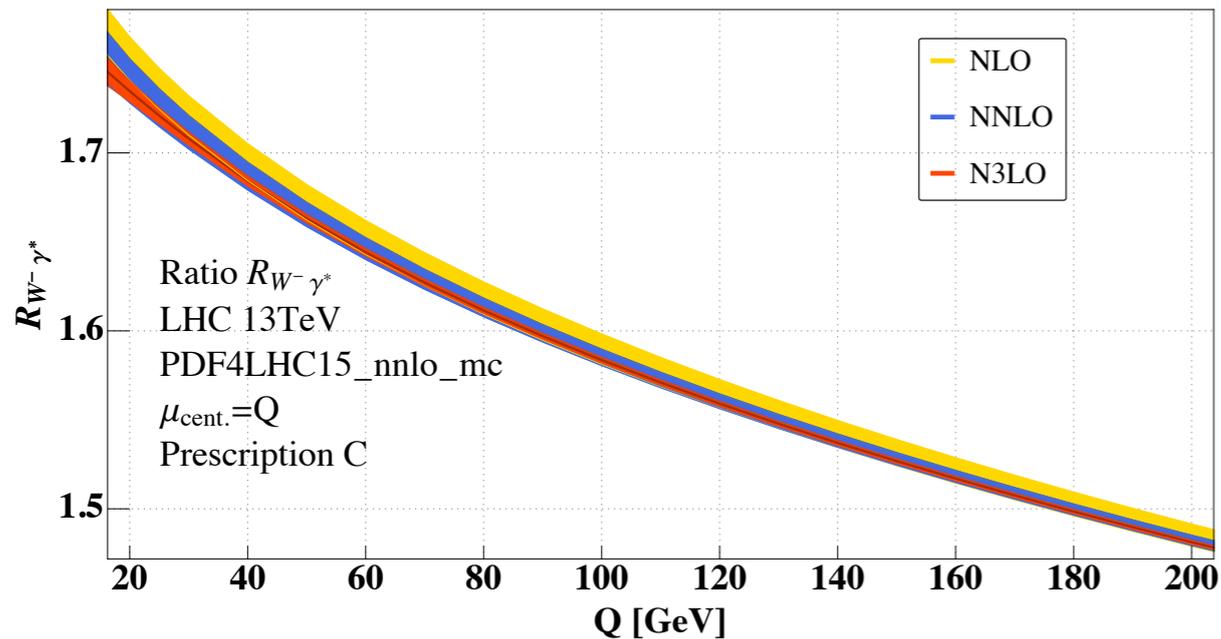
## $W^+ / W^-$



## $W^+ / \text{Photon}$



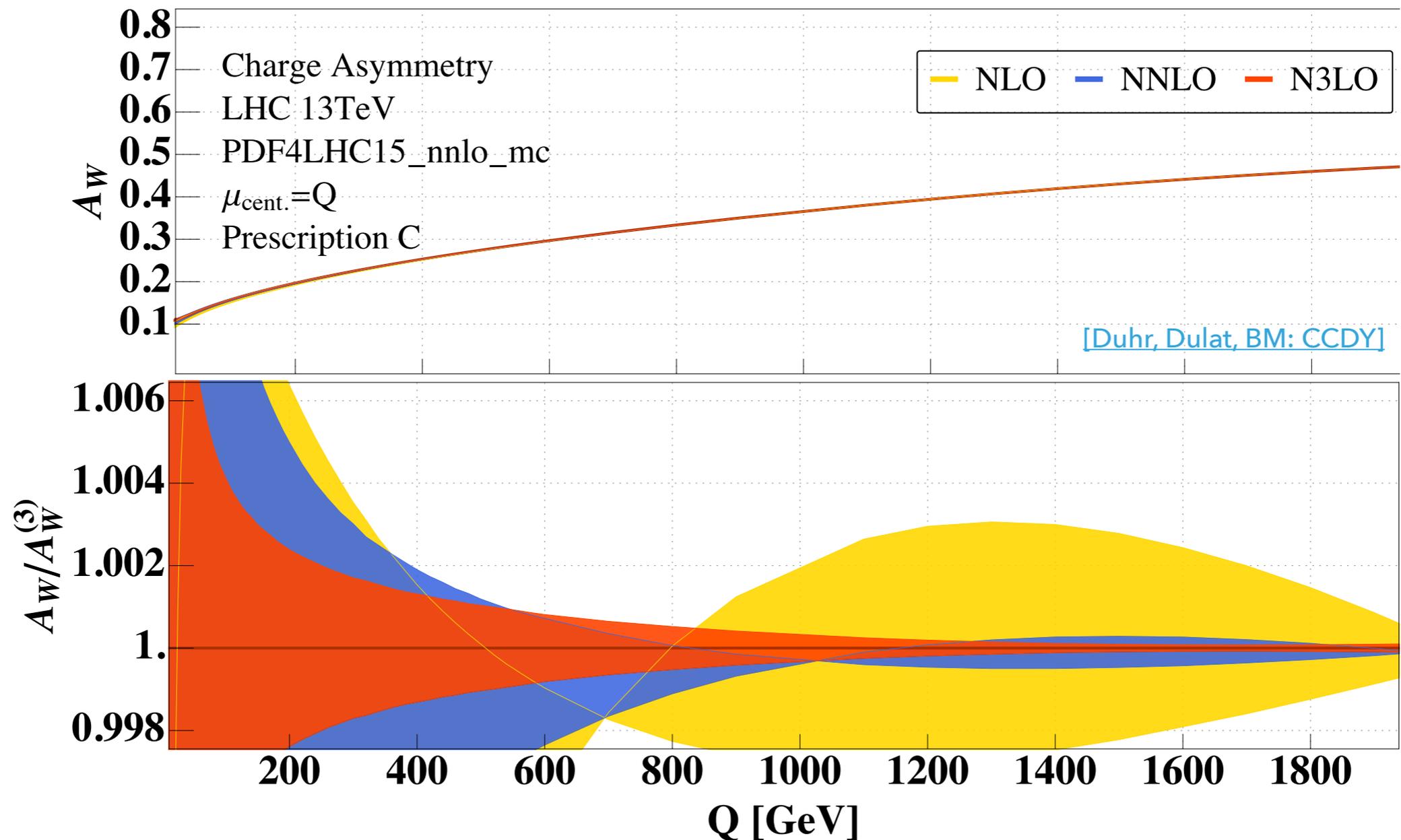
## $W^- / \text{Photon}$



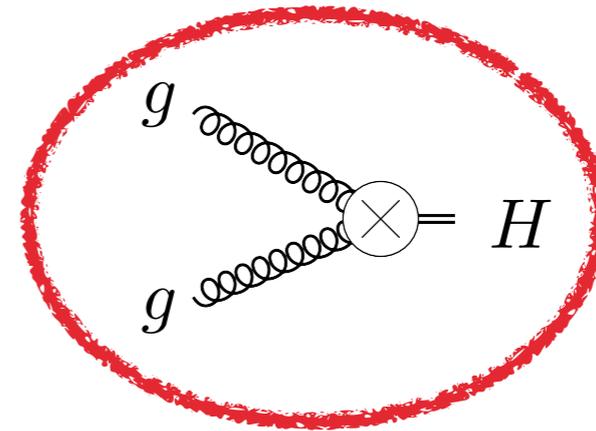
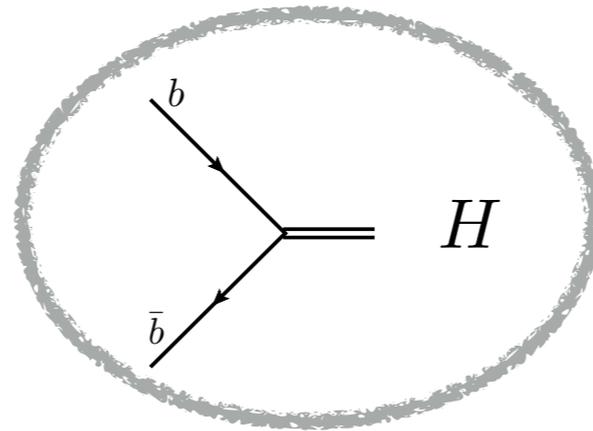
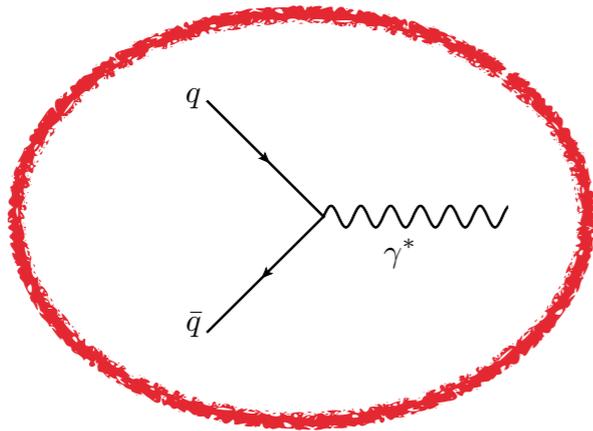
- ▶ Ratios of Cross Sections can be predicted very well.
- ▶ Reduce systematics: Lumi-Uncertainty, etc.

- ▶ One nice observable:

$$A_c = \left| \frac{\sigma_{W^+} - \sigma_{W^-}}{\sigma_{W^+} + \sigma_{W^-}} \right|$$



## Processes at N3LO:



$$\frac{\delta\sigma_{N3LO}}{\sigma_{NNLO}}$$

$-1.4\%$

$-2.3\%$

$3.5\%$

- ▶ Corrections are at the order of a few percent.
- ▶ N3LO stabilises the perturbative expansion.
- ▶ Dependence on perturbative scales is reduced.
- ▶ Let's not forget other uncertainties of comparable size: (PDF / EWK / AlphaS/ ...).
- ▶ Interpretation of MHOU?

## What would we wish for?

- ▶ Make theoretical uncertainty irrelevant.
- ▶ Easy framework to produce realistic predictions for any LHC observable with cutting edge precision.
- ▶ N3LO accurate perturbative predictions matched to parton showers and resummed in every possible corner.



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We need better techniques, better algorithms, ... , better everything!

**TOWARDS DIFFERENTIAL**

**N3LO**

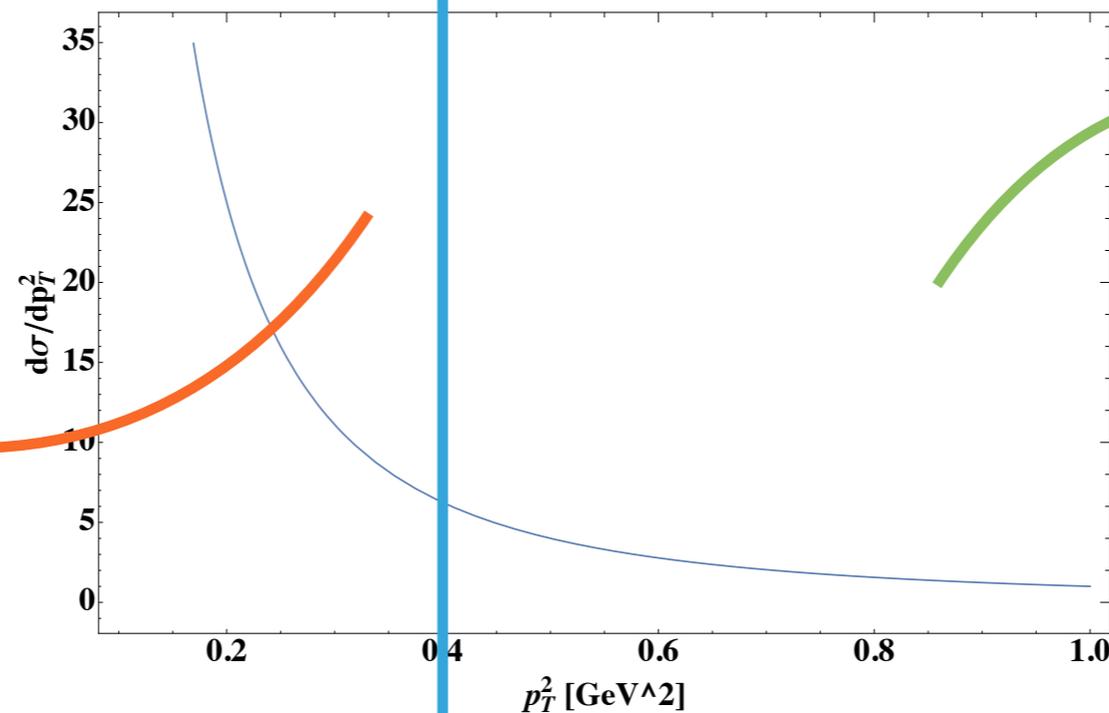
- ▶ qT - Subtraction / Slicing: Algorithm to handle infrared singularities when integrating over final state partons.
- ▶ Relatively "simple".
- ▶ Applied to colourless (+ ttbar) final states.

$\gamma^* W Z H \quad bbH \quad H^* G \quad WH ZH \quad ZZ WW$

- ▶ Successful at NNLO: [MATRIX](#), etc.

Approximate NNLO

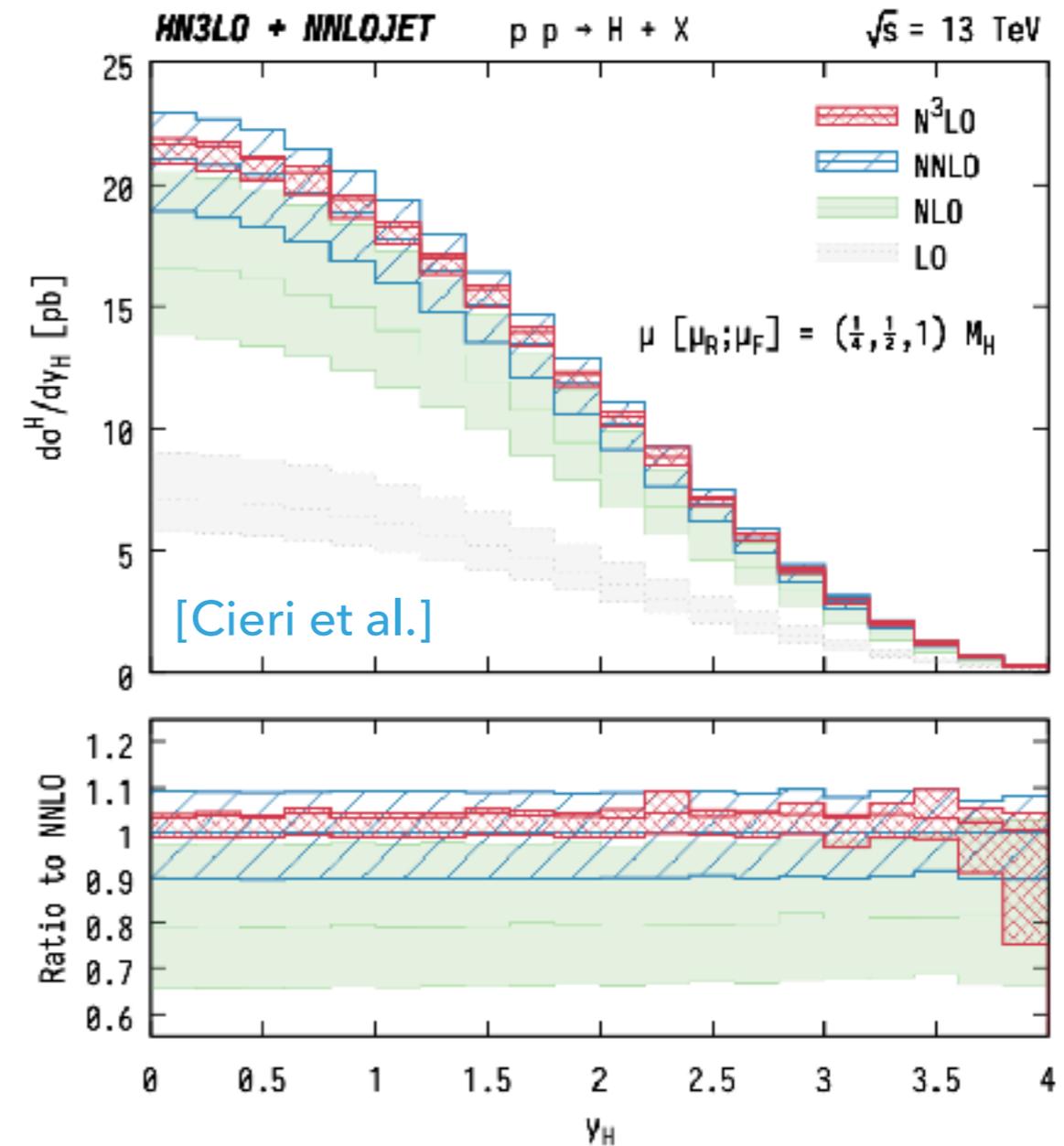
$$+\mathcal{O}(p_T^2)$$



Anyhow NLO

- ▶ Application to the Higgs Rapidity distribution in ggF!
- ▶ H+J at NNLO + approximate approximation below cut.
- ▶ Numerically very challenging!
- ▶ Nice perturbative progression at differential **N3LO**

$$Y = \frac{1}{2} \log \left( \frac{2P_1 p_h}{2P_2 p_h} \right)$$

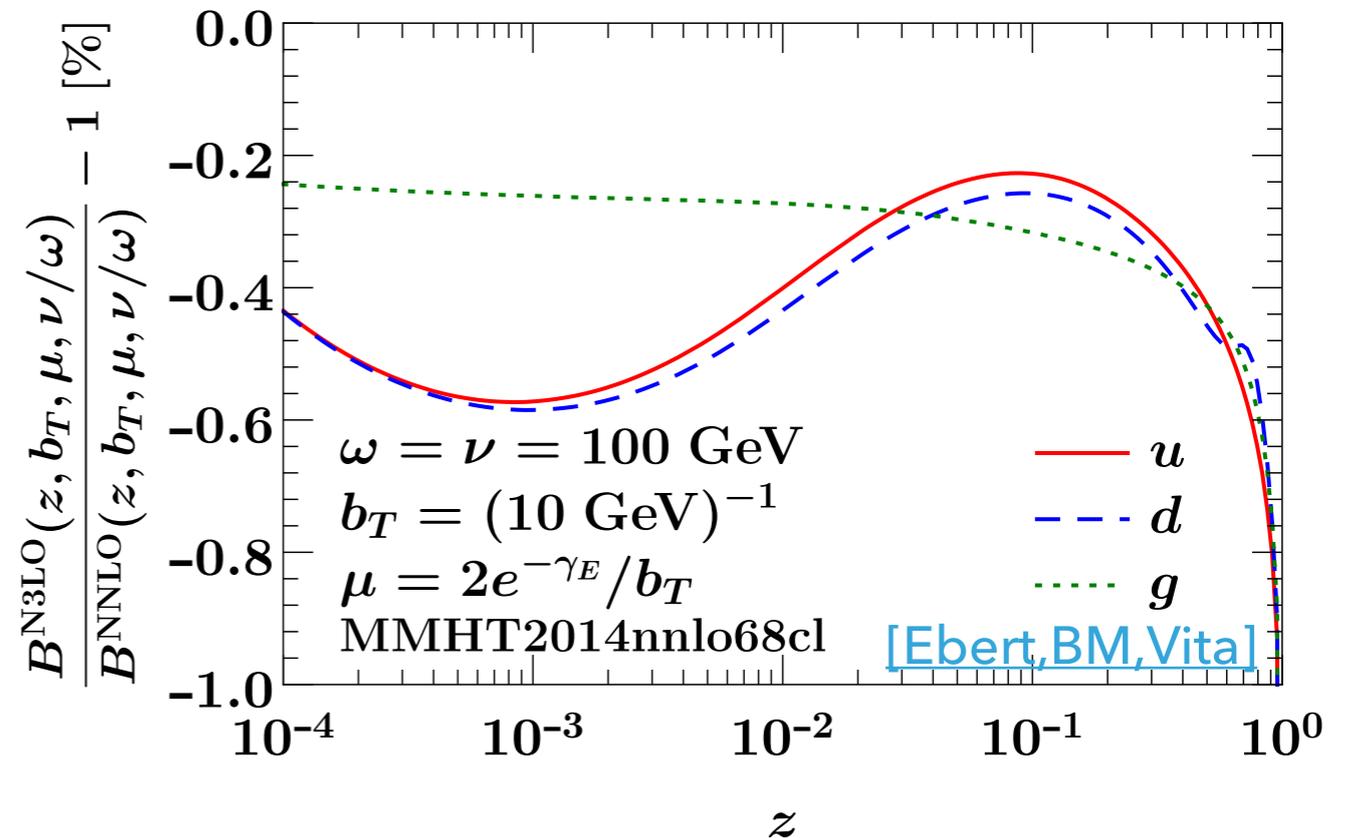


- ▶ Last missing universal ingredient for  $q_T$  subtraction at **N3LO**.

- ▶ All channels, fully analytic.

[See also \[Luo et al.,1912.05778\]](#)

- ▶ Can be used to create fully differential predictions for arbitrary colour singlet processes at **N3LO**.



$\gamma^* W Z H bbH H^* G \quad WH ZH ZZ WW$

In other news: We also computed the

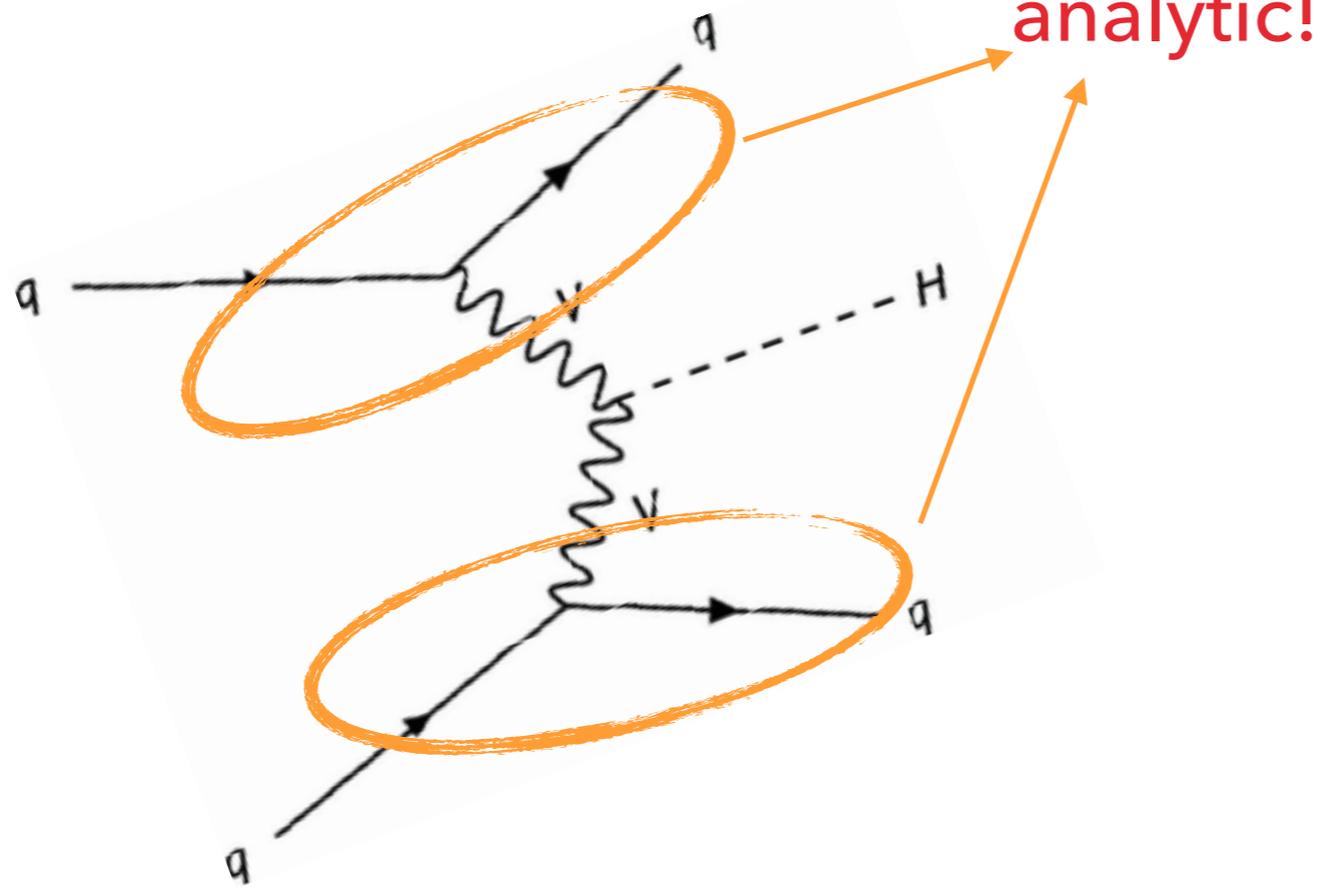
N-Jettiness Beam Functions at **N3LO**.

[\[Ebert,BM,Vita\]](#)

- ▶ Inclusive VBF Higgs production: [Dreyer, Karlberg; 16]

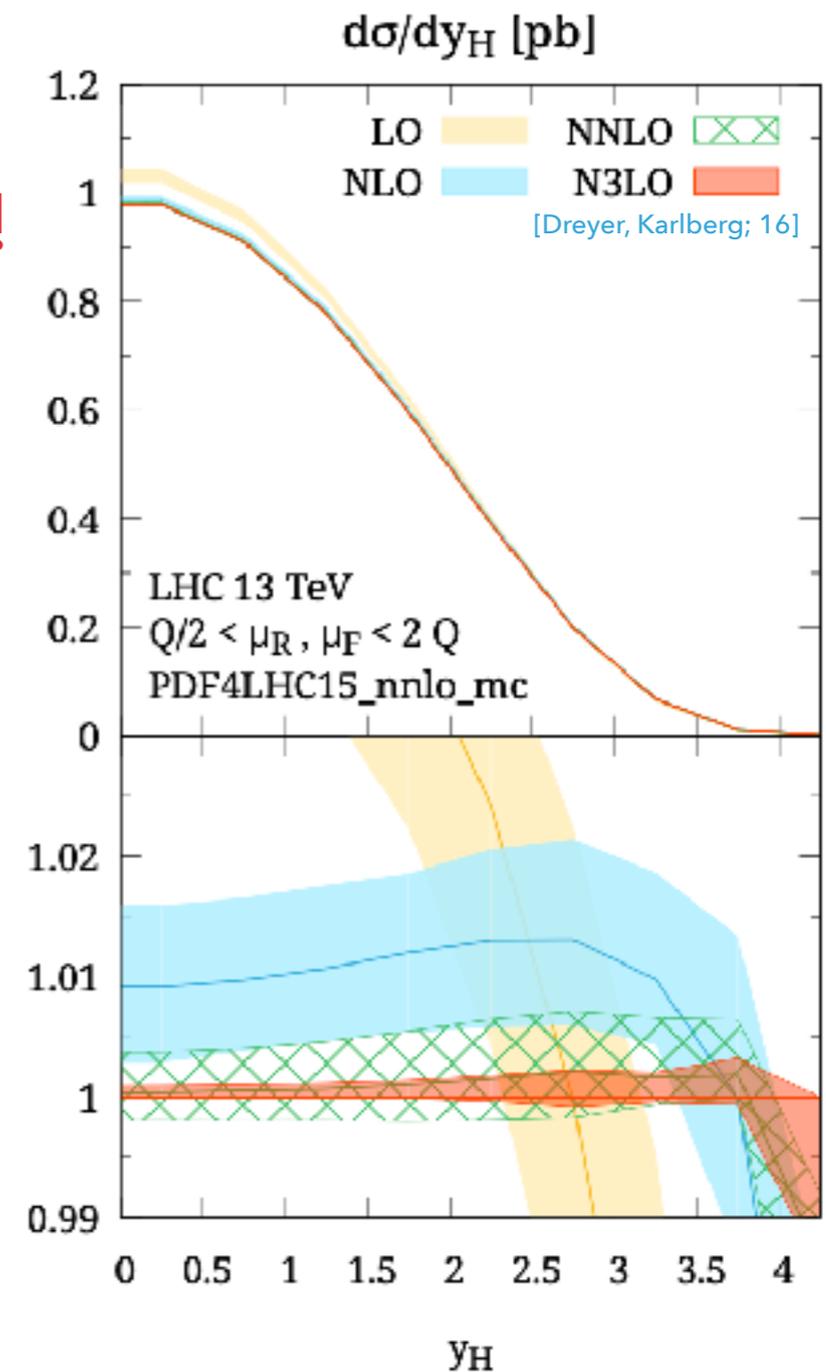
Still differential in Higgs momentum!

- ▶ Approximate N3LO cross section:  
Structure functions!



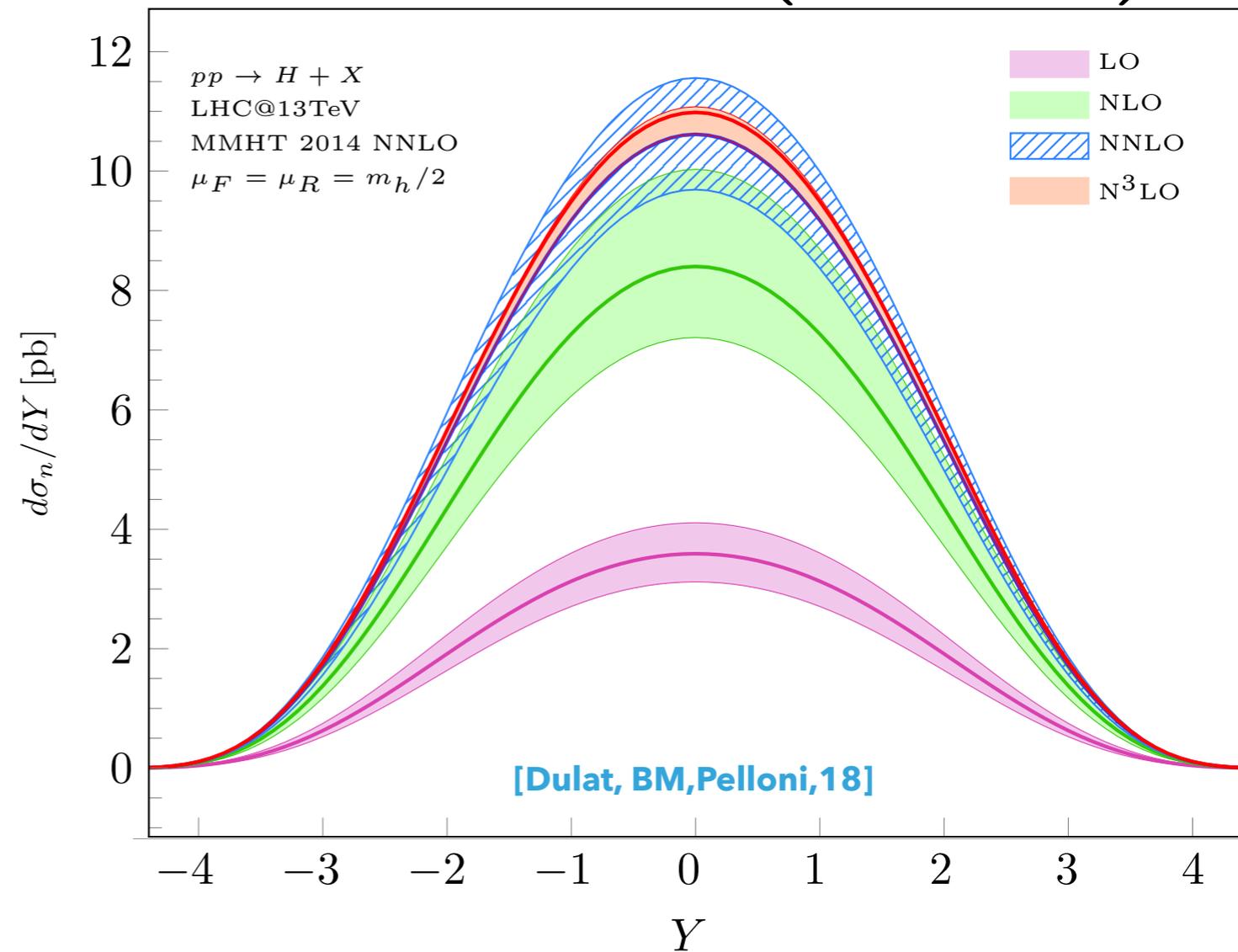
- ▶ Even VBF HH production at **N3LO!**

[Dreyer, Karlberg; 18]



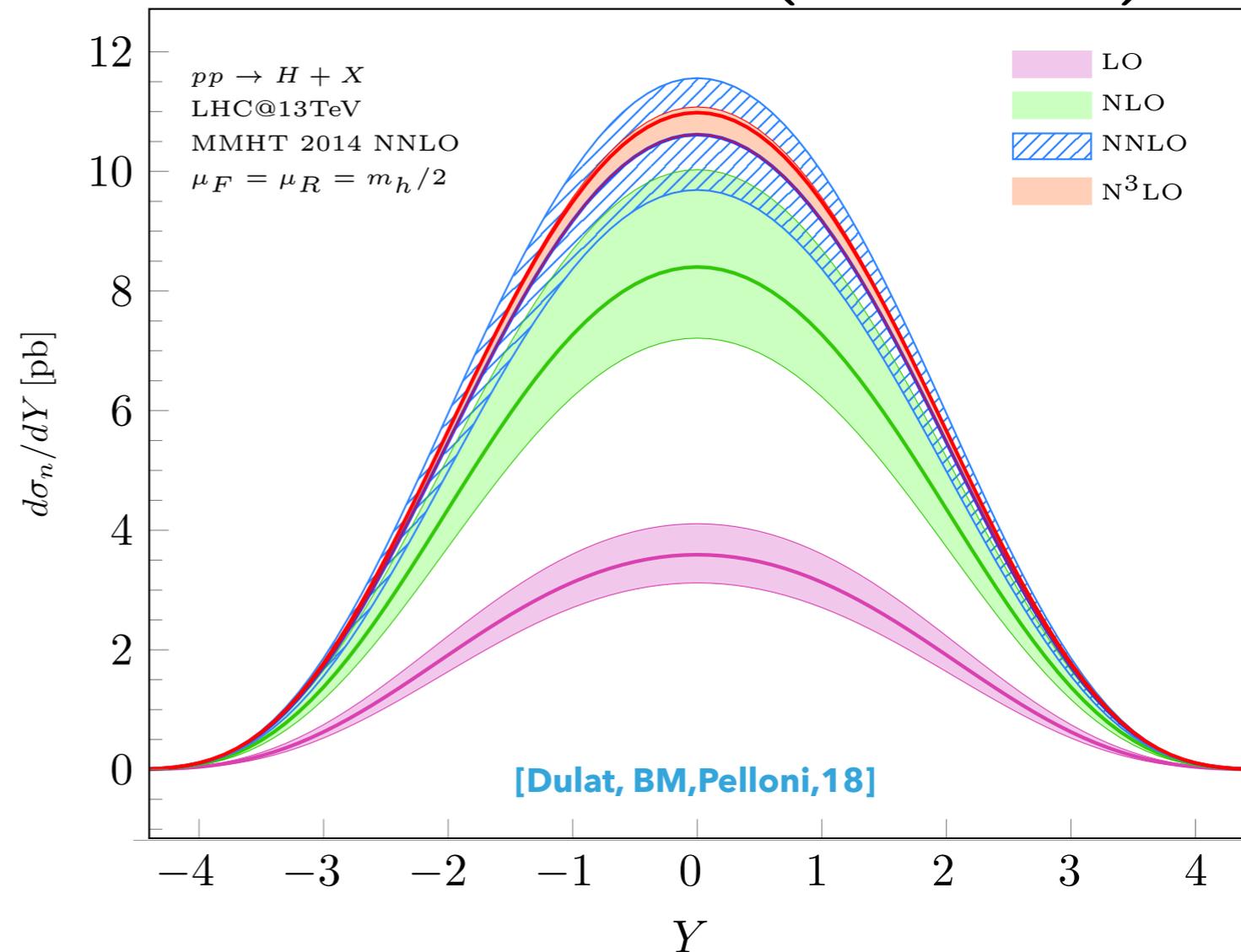
$$Y = \frac{1}{2} \log \left( \frac{2P_1 p_h}{2P_2 p_h} \right)$$

- ▶ Higgs boson rapidity distribution.
- ▶ Gluon - Fusion
- ▶ Analytic calculation: tailored for this observable.
- ▶ Differential K-factor is very similar to inclusive K-factor!



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**Evaluates in minutes!**

## PROJECTION TO BORN

[Cacciari,Dreyer,Karlberg,Salam,Zanderighi; 15]

$$\sigma \sim \int_0 \frac{dx}{x} F(x) [\mathcal{O}(x) - \mathcal{O}(0)] + \mathcal{O}(0)\sigma_{\text{integrated}}$$

### The perfect subtraction method!

(Subtraction term always identically cancels divergences if applicable.)

## N3LO Cross Section to produce F - Required ingredients:

- ★ Fully differential calculation of F+J at NNLO.
- ★ Inclusive N3LO F cross section differential in Born variables.

## PROJECTION TO BORN

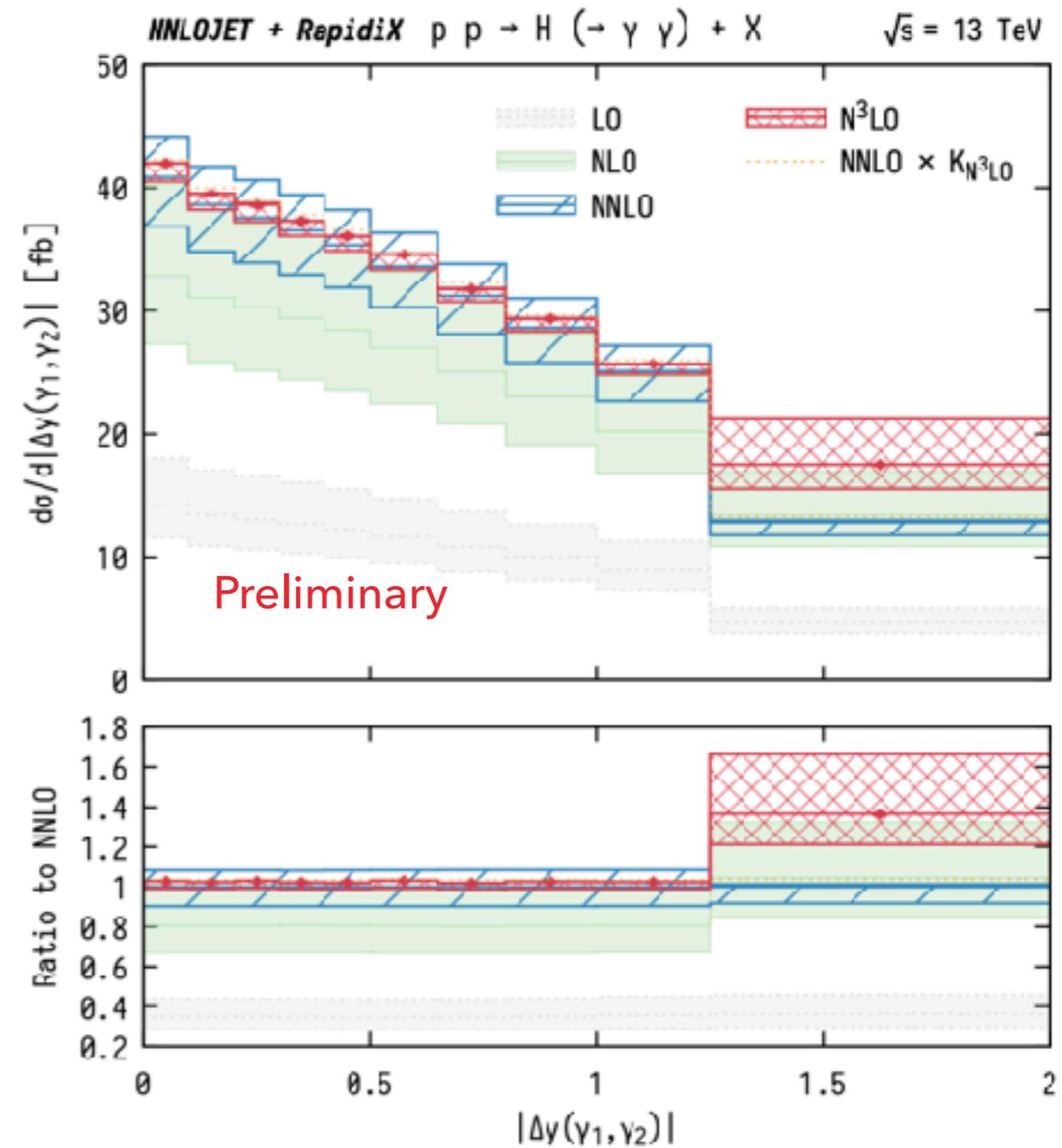
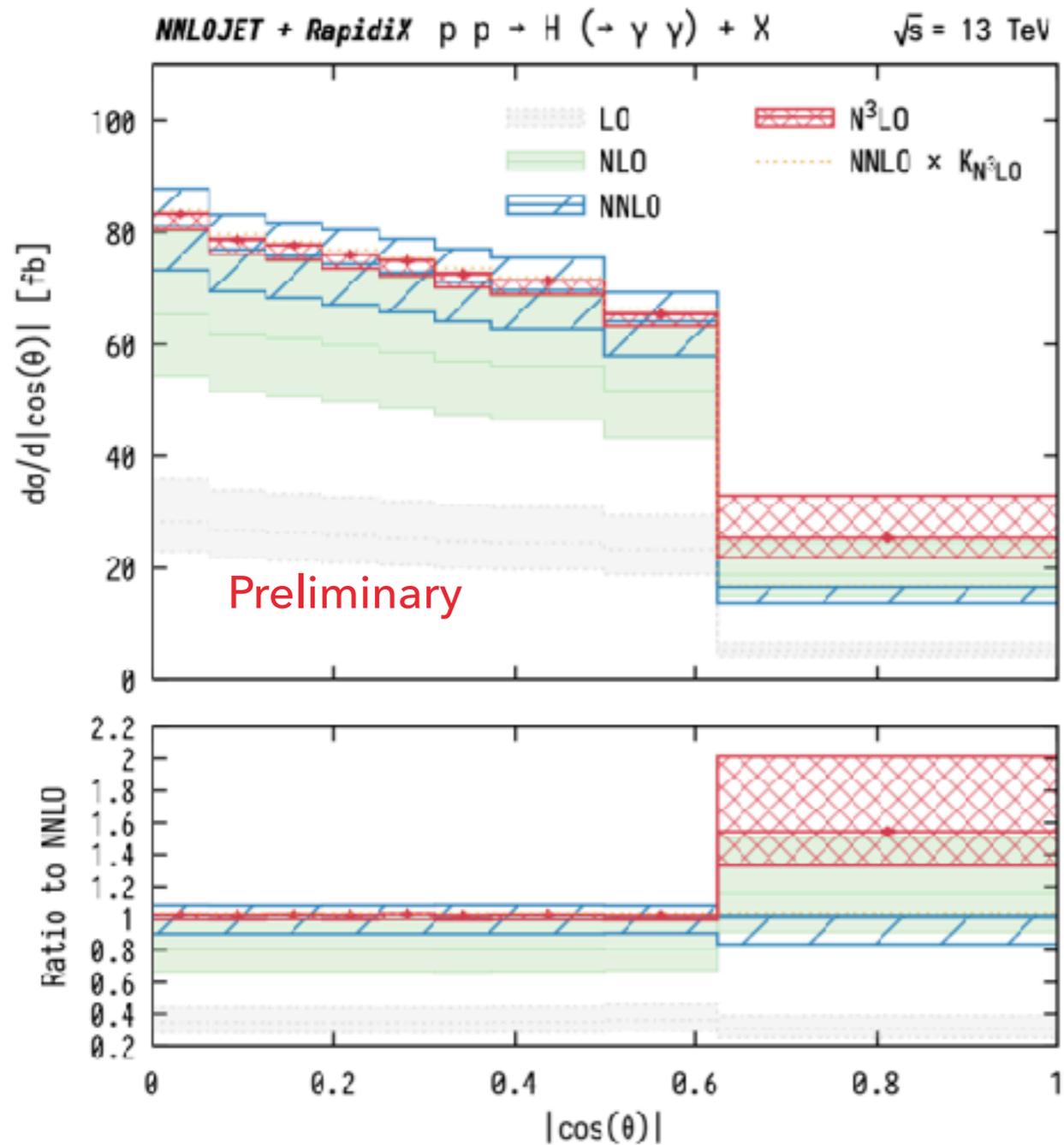
$$Y_H \propto H + J$$

$$P P \rightarrow H + X \rightarrow \gamma\gamma + X$$

- ▶ Define a fiducial volume

$$p_T^{\gamma_1} > 0.35 \times m_{\gamma\gamma}, \quad p_T^{\gamma_2} > 0.25 \times m_{\gamma\gamma},$$
$$|\eta^\gamma| < 2.37 \text{ excluding } 1.37 < |\eta^\gamma| < 1.52,$$

+Photon Isolation



# CROSS SECTIONS AT N3LO

- ▶ **N3LO**  
is the precision frontier

- ▶ **Universal Picture:**  
Percent level corrections at N3LO

- ▶ **First steps to differential N3LO**

- ▶ **Many challenges ahead!**

**THANK YOU!**