

# Machine Learning techniques for Monte Carlo generation

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arXiv:1907.03764, 1912.08824, 1912.00477, 2006.06685, and 2008.06545

with Armand Rousselot, Marco Bellagente, Sascha Diefenbacher, Gregor Kasieczka, Benjamin Nachman, Tilman Plehn, and  
Ramon Winterhalder



# The HEP trinity

## Theory

### Fundamental Lagrangian

- Perturbative QFT

### Standard Model vs. new physics

- Matrix elements, loop integrals

## Experiment

### Complex detector

- ATLAS, CMS, LHCb, ALICE, ...

### Reconstruction of individual events

- Big data: jet images, tracks, ...

## Precision simulations

### First-principle Monte Carlo generators

- Simulation of parton/particle-level events
- HERWIG, PYTHIA, SHERPA, MADGRAPH, ...

### Detector simulation

- Geant4, PGS, Delphes, ...

⇒ **Unweighted event samples**

# Neural networks for precision simulations

## Problems in MC simulations

- Event generation:
  - High-dimensional phase space
  - Low unweighting efficiency
  - Higher order: exponential growth in computing time
- Highly complex full detector simulation → very slow
- HL-LHC: factor 25 in data
  - reduction of statistical uncertainties by  $\sim 5$
  - need for percent level precision
- Limited resources: Precision vs. computing time

## Advantages of neural networks

- Flexible parametrisation
- Interpolation properties
- Fast evaluation
- Multiple generative models: GAN, VAE, normalizing flow

# Possibilities for ML in event generation

## Event generation

- Generating 4-momenta
- $Z > ll, pp > jj, pp > t\bar{t}$ +decay

[1901.00875] Otten et al. **VAE & GAN**

[1901.05282] Hashemi et al. **GAN**

[1903.02433] Di Sipio et al. **GAN**

[1903.02556] Lin et al. **GAN**

[1907.03764, 1912.08824] Butter et al. **GAN**

[1912.02748] Martinez et al. **GAN**

[2001.11103] Alanazi et al. **GAN**

## Monte Carlo integration

- Estimating matrix element
- Neural importance sampling

[1707.00028] Bendavid, **Regression & GAN**

[1810.11509] Klimek and Perelstein

[1912.11055] Bishara and Montull **Regression**

[2001.05478] Bothmann et al. **NF**

[2001.05486, 2001.10028] Gao et al. **NF**

[2002.07516] Badger and Bullock **Regression**

## Detector simulation

- Jet images
- Fast shower simulation in calorimeters

[1701.05927] de Oliveira et al. **GAN**

[1705.02355, 1712.10321] Paganini et al. **GAN**

[1802.03325, 1807.01954] Erdmann et al. **GAN**

[1805.00850] Musella et al. **GAN**

[ATL-SOFT-PUB-2018-001,  
ATL-SOFT-PROC-2019-007] ATLAS **VAE & GAN**

[1909.01359] Carazza and Dreyer **GAN**

[2005.05334] Buhmann et al. **VAE**

## Unfolding

- Detector to parton/particle level distributions

[1806.00433] Datta et al. **GAN**

[1911.09107] Andreassen et al.

[1912.0047] Bellagente et al. **GAN**

[2006.06685] Bellagente et al. **NF**

NO claim to completeness! *Review: Generative Networks for LHC events [2008.08558]*

## Boosting standard event generation...

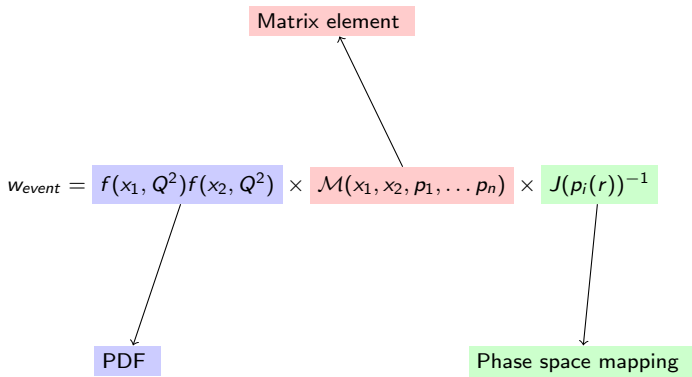
1. Generate phase space points

2. Calculate event weight

$$w_{event} = f(x_1, Q^2)f(x_2, Q^2) \times \mathcal{M}(x_1, x_2, p_1, \dots, p_n) \times J(p_i(r))^{-1}$$

3. Unweighting via importance sampling  
→ optimal for  $w \approx 1$

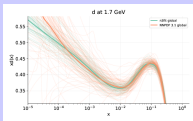
## Boosting standard event generation...



# Boosting standard event generation...

Matrix element

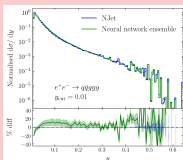
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- NNPDF since 2002(!)
- genetic algorithm
- n3fit: determ. gradient descent
- S. Carrazza, J. Cruz-Martinez [1907.05075]

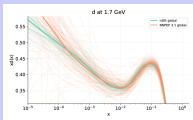
Phase space mapping

# Boosting standard event generation...



- regression network
- learn amplitude or K factor
- S. Badger, J. Bullock [2002.07516]
- J. Bendavid [1707.00028]

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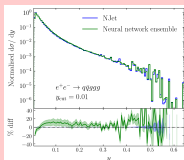


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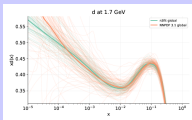


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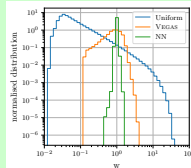


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- NNPDF since 2002(!)
- genetic algorithm
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- learn efficient phase space mapping ( $\rightarrow w \approx 1$ )
- normalizing flow
- Gao et al. [2001.10028]
- Bothmann et al. [2001.05478]

## ... or train generative network directly on events

Possible gains:

- Generate more statistics
  - Use network to sample phase space (similar to NF)
  - Ship model instead of samples (profit from fast evaluation of NN)
  - Replace fast detector simulations
  - Use conditional generative network to interpolate between measured samples
  - Use invertible architectures for unfolding
- Evaluate performance

## A generative model

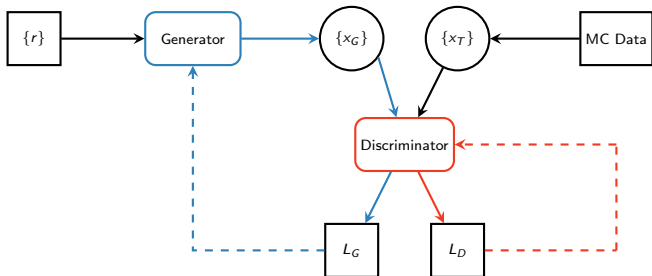
- Generative Adversarial Networks (GAN)
- Training data: true events  $\{x_T\}$   
Output data: generated events  $\{x_G\}$
- **Discriminator** distinguishes  $\{x_T\}, \{x_G\}$   $[D(x_r) \rightarrow 1, D(x_G) \rightarrow 0]$

$$L_D = \langle -\log D(x) \rangle_{x \sim P_T} + \langle -\log(1 - D(x)) \rangle_{x \sim P_G} \xrightarrow{D(x) \rightarrow 0.5} -2 \log 0.5$$

- **Generator** fools discriminator  $[D(x_G) \rightarrow 1]$

$$L_G = \langle -\log D(x) \rangle_{x \sim P_G}$$

⇒ **New statistically independent samples**



# Why GANs? Features, problems and solutions

- + Generate better samples than VAE
- + Large community working on GANs
- Unstable training

## Solutions

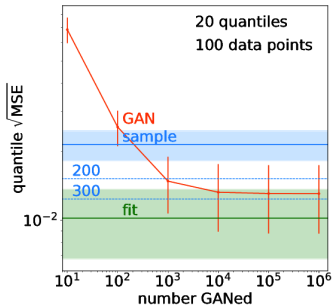
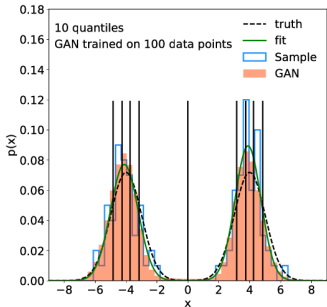
- Regularization of the discriminator, eg. gradient penalty [Ghosh, Butter et al., ...]
- Modified training objective:
  - Wasserstein GAN (incl. gradient penalty) [Lin et al., Erdmann et al., ...]
  - Least square GAN (LSGAN) [Martinez et al., ...]
  - MMD-GAN [Otten et al., ...]
  - MSGAN [Datta et al., ...]
  - Cycle GAN [Carazza et al., ...]
- Use of symmetries [Hashemi et al., ...]
- Whitening of data [Di Sipio et al., ...]
- Feature augmentation [Alanazi et al., ...]

## What is the statistical value of GANned events?

- Example: 1D camel function
- Compare Sample vs. GAN vs. 5 param.-fit (mean, width, relative height)
- Evaluation on quantiles

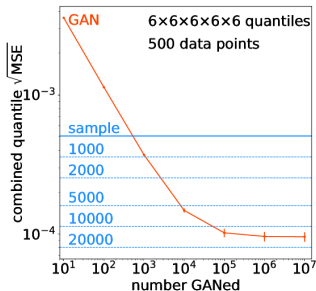
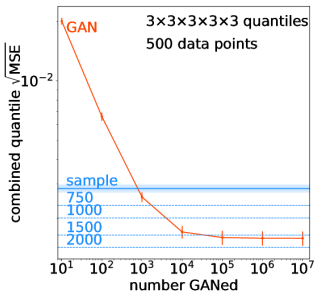
$$\text{MSE} = \frac{1}{N_{\text{quant}}} \sum_{j=1}^{N_{\text{quant}}} \left( x_j - \frac{1}{N_{\text{quant}}} \right)^2$$

- Convergence to amplification factor 2.5 for 20 quantiles



# What is the statistical value of GANned events?

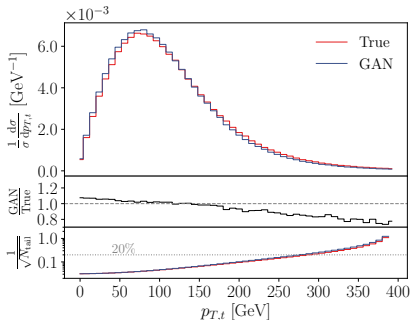
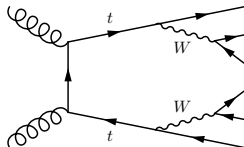
- Example: Sphere in 5 dimensions
- Sum over 5-dimensional quantiles
- Amplification factor increases for sparse quantiles:
  - 3 for  $3^5$  quantiles
  - 15 for  $6^5$  quantiles



# How to GAN LHC events

## Idea: generate hard process

- Realistic LHC final state  $t\bar{t} \rightarrow 6$  jets [1907.03764]
- 18 dim output [fix external mass, no mom. cons.]
- Flat observables precise
- Systematic undershoot in tails [10-20% deviation]

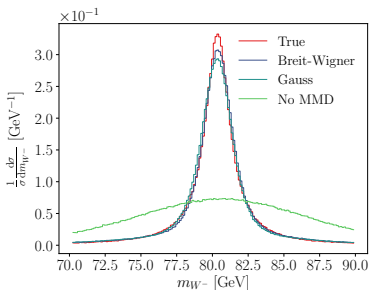
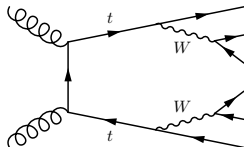


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- Sharp phase-space structures, not using  $\Gamma_W$

$$\text{MMD}^2(P_T, P_G) = \langle k(x, x') \rangle_{x, x' \sim P_T} + \langle k(y, y') \rangle_{y, y' \sim P_G} - 2\langle k(x, y) \rangle_{x \sim P_T, y \sim P_G}$$

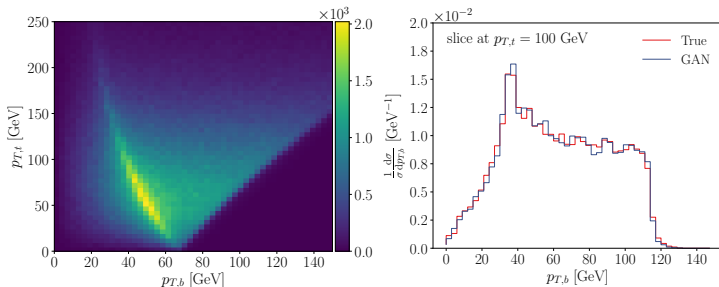
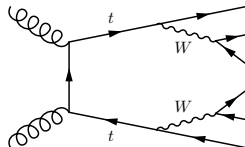




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- 18 dim output
- Flat observables precise
- Systematic undershoot in tails [10-20% deviation]
- Sharp phase-space structures, not using  $\Gamma_W$  [MMD-loss]
- 2D correlations



## How to GAN event subtraction

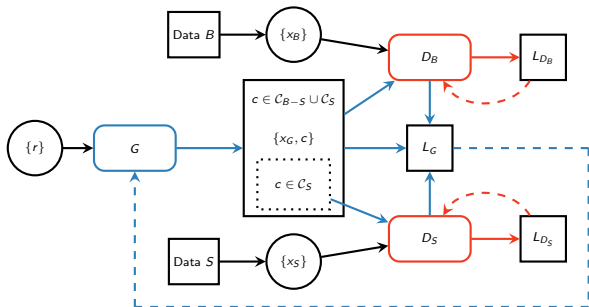
Idea: sample based subtraction of distributions [1912.08824]

- 1 Consistent multidimensional difference between two distributions
- 2 Beat bin-induced statistical uncertainty [interpolation of distributions]

$$\Delta_{B-S} = \sqrt{n_B^2 N_B + n_S^2 N_S} > \max(\Delta_B, \Delta_S)$$

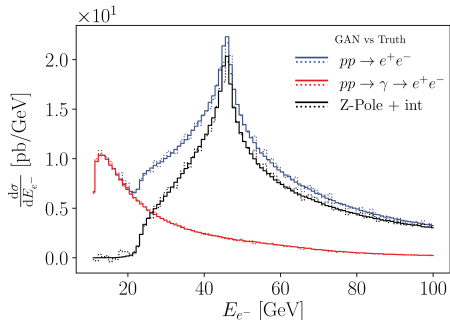
- Many applications:

- Soft-collinear subtraction, multi-jet merging, on-shell subtraction
- Background subtraction [4-body decays  $\rightarrow$  preserves correlations]



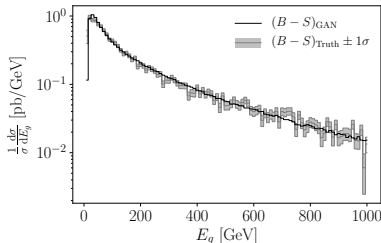
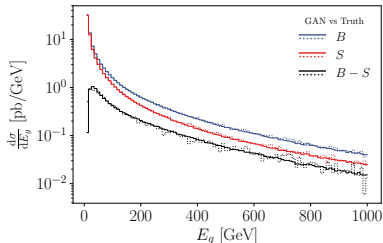
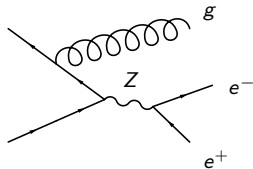
## Example I: Z pole

- Training data:
  - $pp \rightarrow e^+e^-$
  - $pp \rightarrow \gamma \rightarrow e^+e^-$
  - 1 M events per dataset, MadGraph5
- Generated events: Z-Pole + interference



## Example II: Dipole subtraction

- Theory uncertainties  $\rightarrow$  limiting factor for HL-LHC
- Higher order: Subtract diverging Catany Seymour Dipole from real emission term
- 1 M events per dataset, SHERPA



# How to GAN away detector effects

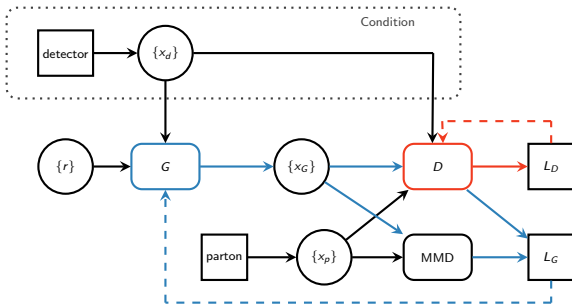
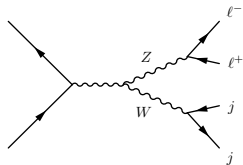
Idea: invert Markov process [1912.00477]

## Detector simulation

- Typical Markov process
- Prior dependent inversion possible [Datta et al.]
- Aim: unfolding multidimensional phase space

Reconstruct parton level  $pp \rightarrow ZW \rightarrow (\ell\ell)(jj)$

- GAN: no connection between input and discr.  
→ use **fully conditional** GAN (FCGAN)

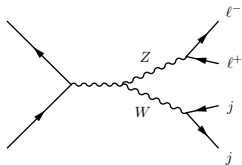


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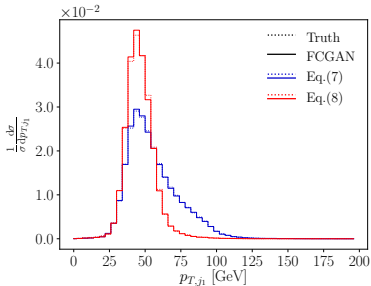
Reconstruct parton level  $pp \rightarrow ZW \rightarrow (\ell\ell)(jj)$

- Use **fully conditional** GAN (FCGAN)
- Inversion works ✓



$$\text{Eq.(7)} : p_{T,j_1} = 30 \dots 100 \text{ GeV} \quad (\sim 88\%)$$

$$\text{Eq.(8)} : p_{T,j_1} = 30 \dots 60 \text{ GeV} \quad \text{and} \quad p_{T,j_2} = 30 \dots 50 \text{ GeV} \quad (\sim 38\%)$$

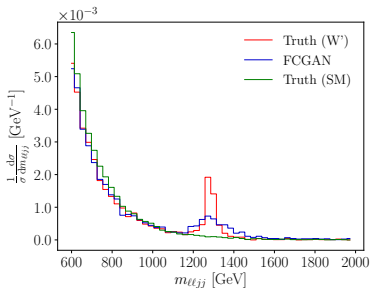
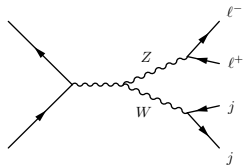


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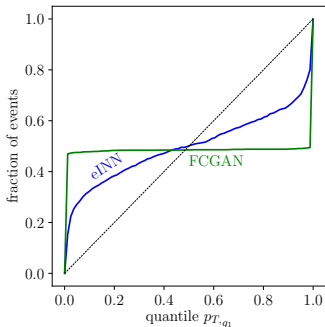
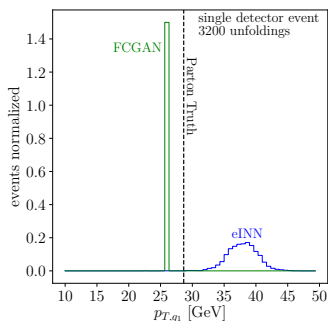
- Use **fully conditional** GAN (FCGAN)
- Inversion works ✓
- BSM injection ✓
  - train: SM events
  - test: 10% events with  $W'$  in s-channel



## Curing shortcomings with invertible structure

- cGAN calibration curves: mean correct, distribution too narrow
- INN: Normalizing flow with fast evaluation in both directions

$$\begin{array}{ccc} & \text{PYTHIA, DELPHES: } g \rightarrow & \\ \left( \begin{array}{c} x_p \\ r_p \end{array} \right) & \longleftrightarrow & \left( \begin{array}{c} x_d \\ r_d \end{array} \right) \\ & \longleftarrow \text{unfolding: } \bar{g} & \end{array}$$



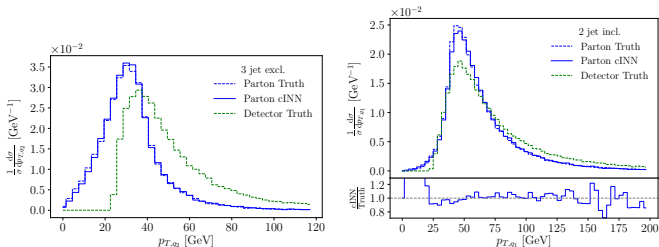


# Conditional invertible neural networks

## Condition INN on detector data [2006.06685]

$$x_p \xleftarrow{g(x_p, f(x_d)) \rightarrow} r \xrightarrow{\leftarrow \text{unfolding: } \bar{g}(r, f(x_d))}$$

- Use detector information  $x_d$  of arbitrary dimension
- Cross check 2/3/4 jet exclusive channels



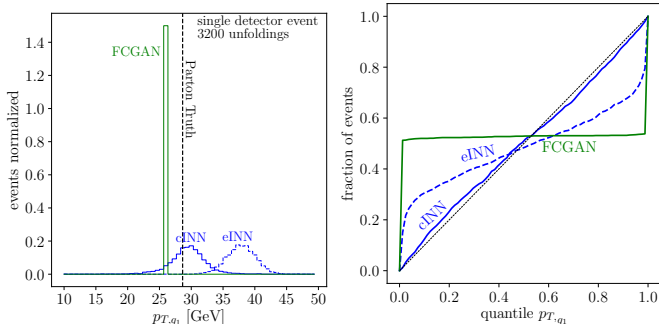
# Conditional invertible neural networks

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$$x_p \xleftarrow{\begin{array}{c} g(x_p, f(x_d)) \rightarrow \\ \leftarrow \text{unfolding: } \bar{g}(r, f(x_d)) \end{array}} r$$

$$\text{Minimizing } L = \left\langle \frac{\|g(x_p, f(x_d))\|_2^2}{2} - \log \left| \frac{\partial g(x_p, f(x_d))}{\partial x_p} \right| \right\rangle_{x_p \sim P_p, x_d \sim P_d} - \log p(\theta)$$

→ calibrated parton level distributions



## Summary

- We can **boost standard event generation** using ML
- **GANs** can learn **underlying distributions** from event samples
- Possibilities to stabilize GAN training: gradient penalty, WGAN-GP, LSGAN,...
- MMD improves performance for special features
- Successful **sample based subtraction** implemented
- Applications: background subtraction, soft-collinear subtraction, ...
- **Unfold high-dimensional** detector level distributions with cGANs and INN
- Stable under insertion of new data, proper calibration achieved by **cINN**



## Important next steps

1. Quantify uncertainties (eg. Bayesian networks)
  - including correlations
2. High precision
3. Automization
  - move away from hand engineered networks