Lattice determinations of α_s

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ALPHA Collaboration

Overview

Motivation

Lattice QCD

Dedicated approaches

Conclusions

MOTIVATION

Computing the strength of fundamental interactions

- Take some experimental observable $O(\mu; p)$.
- Work hard to get

$$O(\mu; p) = A(p)\alpha_{\overline{\rm MS}}(\mu) + B(p)\alpha^2_{\overline{\rm MS}}(\mu) + \dots$$

- Determine $\alpha_{\overline{\rm MS}}(\mu)$ by comparing experiment and theory computation

$$\begin{array}{rcl} g_e - 2 : \alpha_{em} & = & 7.297\,352\,5698(24) \times 10^{-3} \\ \text{recoil} : \alpha_{em} & = & 7.297\,352\,585(48) \times 10^{-3} \end{array} \qquad \begin{array}{rcl} \tau : \alpha_s(M_Z) & = & 0.1198(15) \\ e^+e^- : \alpha_s(M_Z) & = & 0.1172(37) \end{array}$$

Caveats

- In strong interactions asymptotic states are not quarks/gluons.
  - Hadronization?
- At what energy scale μ we match the experimental result?
  - What about higher orders in PT?. Resummation?
  - What about non-perturbative contributions? Renormalons, $\delta_{\rm NP}$, ...

The Lattice QCD approach

The main points

- Last 10 years: tremendous progress $\Rightarrow$ Solid determinations at the 1-2% level
- LQCD "hot" topics: QED, charm effects **not** very relevant for α_s
- Challenges for α_s: Reach high energies with precision **and** controlling $a \to 0$
- Improvement will come from dedicated approaches
  - Finite size scaling
  - Decoupling of heavy quarks
  - ...
- Different approaches to α_s on the lattice: At least as different as τ decay and electroweak precision fits

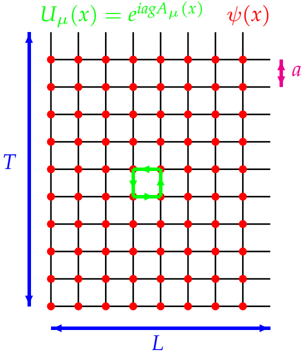
This talk: focus on the ideeas

- What are the limitations of current determinations of α_s?
- How do different approaches differ?
- Main ideas behind dedicated approaches

Overview

Computing path integrals: Lattice field theory

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



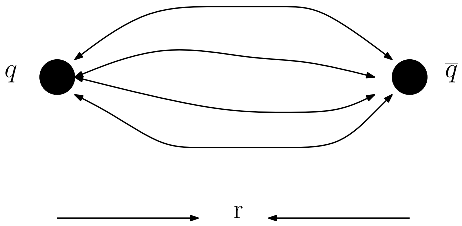
- Discretize space-time in an hyper-cubic lattice (spacing a)
- Path integral $\longrightarrow$ multiple integral (one variable for each field at each point)
- Compute the integral numerically $\rightarrow$ Monte Carlo sampling.

$$\langle O \rangle = \frac{1}{N_{\text{conf}}} \sum_{i=1}^{N_{\text{conf}}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\text{conf}}})$$

  Observable computed averaging over samples
- This works both in the perturbative and non-perturbative regimes!

$$S_G[U] = \frac{\beta}{6} \sum_{p \in \text{Plaquettes}} Tr(1 - U_p - U_p^+) \xrightarrow[a \to 0]{} -\frac{1}{2} \int d^4x \, \text{Tr}(F_{\mu\nu} F_{\mu\nu})$$

The strength of YM



- Take $O(\mu) = \frac{3r^2}{4} F(r)\Big|_{\mu=1/r}$
- This defines the "potential scheme". Non-perturbative coupling definition.

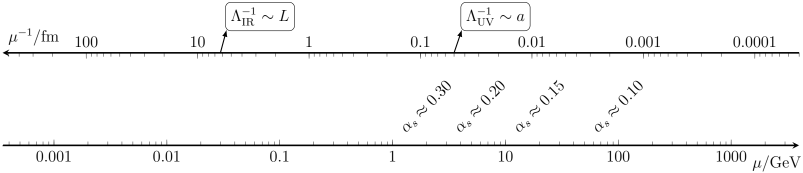
$$\alpha_{qq}(\mu) = \frac{3r^2}{4} F(r)\Big|_{\mu=1/r}$$

- Only $\alpha_{\overline{\rm MS}}(M_Z)$ useful for pheno

$$\alpha_{qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1 \alpha^2_{\overline{\rm MS}}(\mu) + \dots$$

- Again: What about truncation, NP effects, …

Lattice QCD typical scales



CLS ensembles ($N_f = 3$ QCD) **[Bruno et al. '15]**

Lattice sp. a	UV cutoff a^{-1}	IR cutoff L^{-1}	M_π	M_K
0.086 fm	2.3 GeV	35 – 70 MeV	130 – 420 MeV	420 – 480 MeV
0.064 fm	3.1 GeV	50 – 64 MeV	200 – 420 MeV	420 – 480 MeV
0.05 fm	3.9 GeV	60 MeV	260 – 420 MeV	420 – 470 MeV
0.04 fm	4.97 GeV	75 MeV	420 MeV	420 MeV

- $\alpha(\mu)$ not **that** small at scales reached in large volume LQCD
- Reducing α **exponentially** difficult problem!

Current status

*L. Del Debbio, A. Ramos. In preparation

Observable	l	$\alpha_{\overline{\mathrm{MS}}}(\mu_{\mathrm{PT}})$	μ_{PT} [GeV]	Power corrections
QCD vertices	3	$0.20 - 0.30$	$2 - 6$	$\sim 1/\mu^2, 1/\mu^{6}{}^{+}$
Static Potential	3	$0.19 - 0.36^{\star}$	$1.5 - 8^{\star}$	-*
HQ correlators	2	$0.20 - 0.36^{\dagger}$	$\bar{m}_c^{\dagger} - 4\bar{m}_c^{\dagger}$	-
Wilson loops	2	$0.22 - 0.40$	$1/a = 1.1 - 4.4$	$\sim 1/\mu^{2}{}^{\ddagger}$
Vacuum polarization	4	$0.22 - 0.31$	$1.6 - 4$	$\sim 1/p^{2k}\ (k = 1, ..., 4)$
Finite size scaling	2	$0.11 - 0.23$	$4 - 140$	-

Current status

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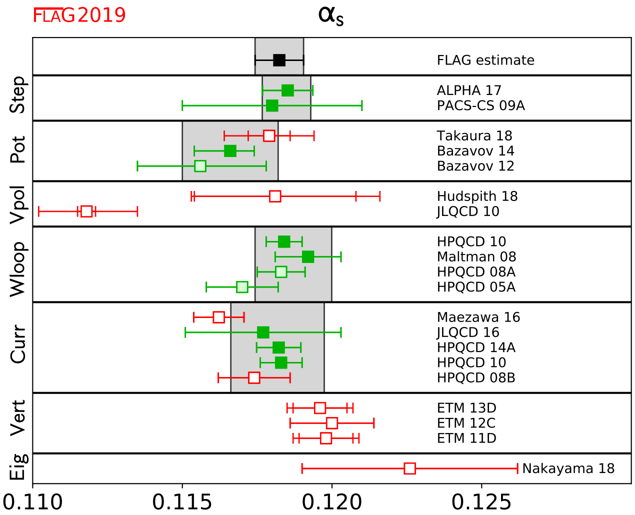

Current status

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Perturbative uncertainties dominates error in most cases: Example

- Moments of heavy quark correlators. Extraction at $Q \approx m_c$.
- JLQCD [Phys.Rev.D 94 (2016) 054507].

$$\alpha_{\overline{\rm MS}}(M_Z) = 0.1177(26)\,, \qquad [2.2\%]\,,$$

  Not using shortest distance moment because large cutoff effects.
  Scale variation method.

- Petreczky, Weber [Phys.Rev.D. 100 (2019) 3, 034519]

$$\alpha_{\overline{\rm MS}}(M_Z) = 0.1159(12)\,, \qquad [1.0\%]\,,$$

  Estimate of first missing PT coefficient.

- HPQCD [Phys.Rev.D 91 (2015) 5, 054508]

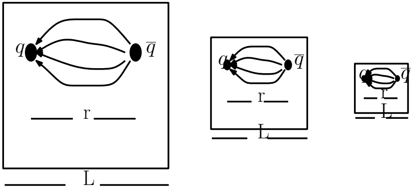
$$\alpha_{\overline{\rm MS}}(M_Z) = 0.11822(74)\,, \qquad [0.6\%]\,,$$

  Fit up to $\mathcal{O}(\alpha^{15})$ constrained by Bayesian priors.
  PT uncertainty estimated by varying number of fit terms.

- FLAG estimate: **1.3**% from different computations. Similar result to scale variation.

Overview

The solution: Finite size scaling [Lüscher, Weisz, Wolff '91]



Finite volume renormalization schemes: fix $\mu L = $ constant

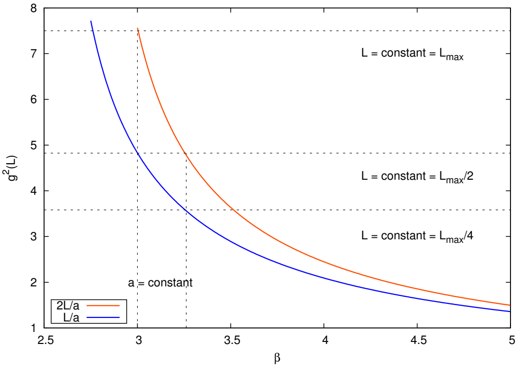
- Coupling $\alpha(\mu)$ depends on no other scale but L (Notation: $\alpha(L), \alpha(1/L)$).
- Small $L \Longrightarrow$ small $\alpha(L)$
- $a \ll 1/\mu$ easily achieved: $L/a \sim 10 - 40$
- Step scaling function: How much changes the coupling when we change the renormalization scale:

$$\sigma(u) = g^2(\mu/2)\Big|_{g^2(\mu)=u}$$

  achieved by simple changing $L/a \to 2L/a$!
- $1/L$ is a IR cutoff $\Rightarrow$ simulate directly $m_q = 0$
- We need dedicated simulations of the **femto-universe**

The solution: Finite size scaling [Lüscher, Weisz, Wolff '91]

$$\beta \Longleftrightarrow a; \qquad g^2(L) \Longleftrightarrow L \Longleftrightarrow \mu$$



Step scaling function

$$\Sigma(u, a/L) = g^2(2L)\Big|_{g^2(L)=u}$$

Continuum limit

$$\sigma(u) = \lim_{a/L \to 0} \Sigma(u, a/L)$$

Simulate several pair of lattices

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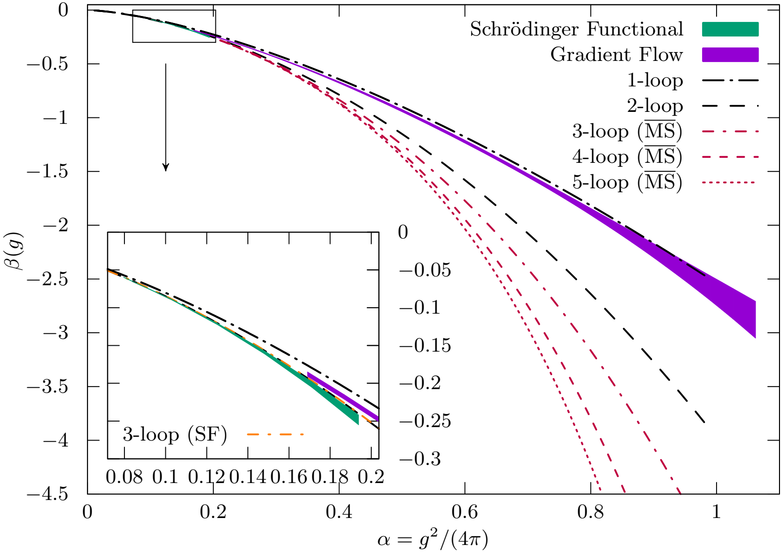
[M. Dalla Brida et al. Phys.Rev.D (2017) no.95, 014507]



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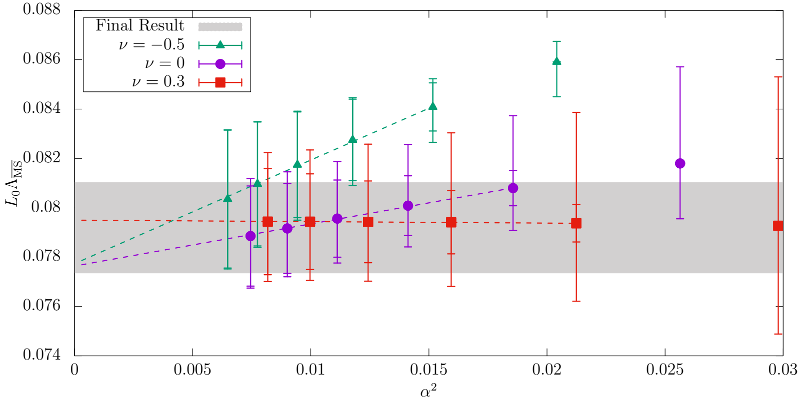

- Comparison of PT and NPT for scales $4 - 140$ GeV.
- At the electroweak scale, nice consistency

3M: A universe with three heavy degenerate quarks ($M \gg \Lambda$)

Alice uses fundamental theory

$$S_{\text{fund}}[A_\mu, \psi, \bar{\psi}] = \int \mathrm{d}^4x \left\{ -\frac{1}{2g^2} \mathrm{Tr}\left(F_{\mu\nu}F_{\mu\nu}\right) + \sum_{i=1}^{3} \bar{\psi}_i(\gamma_\mu D_\mu + M)\psi_i \right\}$$

Bob uses effective theory

$$S_{\text{eff}}[A_\mu] = -\frac{1}{2g_{\text{eff}}^2} \int \mathrm{d}^4x \left\{\mathrm{Tr}\left(F_{\mu\nu}F_{\mu\nu}\right)\right\} + \frac{1}{M^2} \sum_k \omega_k \int \mathrm{d}^4x \, \mathcal{L}_k^{(6)} + \ldots$$

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Decoupling

- Dimensionless ratios of "low energy scales" (i.e. string tension) can be computed in the effective theory

$$\frac{\mu_1^{\text{fund}}(M)}{\mu_2^{\text{fund}}(M)} = \frac{\mu_1^{\text{eff}}}{\mu_2^{\text{eff}}} + \mathcal{O}\left(\frac{\mu^2}{M^2}\right)$$

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Decoupling

- Dimensionless ratios of "low energy scales" (i.e. string tension) can be computed in the effective theory

$$\lim_{M \to \infty} \frac{\mu_1^{\text{fund}}(M)}{\mu_2^{\text{fund}}(M)} = \frac{\mu_1^{\text{eff}}}{\mu_2^{\text{eff}}}$$

3M: A universe with three heavy degenerate quarks ($M \gg \Lambda$)

Similar expression for Λ

Bob can also compute the fundamental parameters of 3M: $\Lambda^{(3)}$ from $\Lambda^{(0)}$

$$\Lambda^{(3)} = \lim_{M\to\infty} \mu_{\rm dec}(M) \times \frac{\Lambda^{(0)}}{\mu_{\rm dec}} \times \frac{1}{P(\Lambda/M)}$$

We need

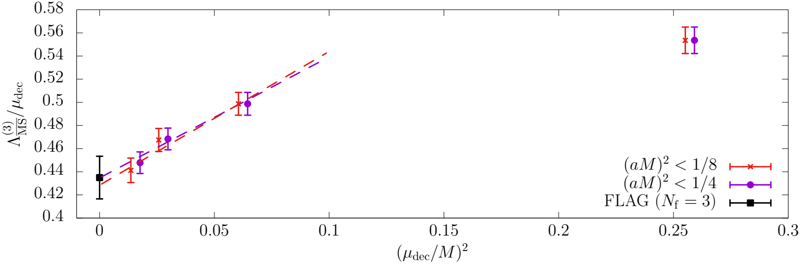
- Running in pure gauge: $\frac{\Lambda^{(0)}}{\mu_{\rm dec}}$
- A scale in a world with degenerate massive quarks: $\mu_{\rm dec}(M)$ in fm/MeV.
- Perturbative crossing across thresholds (at scale M): $P(\Lambda/M)$

Lattice QCD can simulate unphysical worlds

$$\mu_{\rm dec}(M) = \mu_{\rm phys} \times \frac{\mu_{\rm dec}(M)}{\mu_{\rm phys}}$$

Determination of $\Lambda^{(3)}$ from decoupling

ALPHA **[Phys.Lett.B 807 (2020) 135571]**



Tremendous advantage

- The hard multi-scale problem (i.e. Λ/μ) can be solved in pure gauge
- Compatible with both "large volume" approaches and "finite size scaling"

Overview

Conclusions

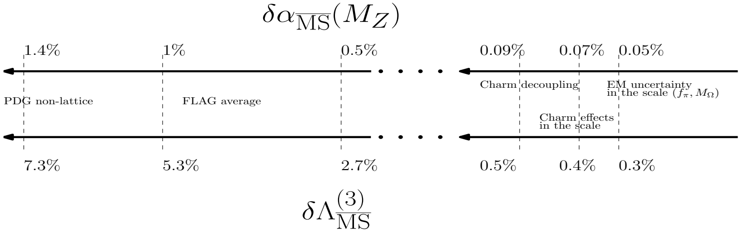
Present: LQCD dominates world average for α_s

- Euclidean field theory, non-perturbative framework
- **Several different** LQCD techniques
- Several "Large volume" approaches: Static potential, Wilson loops, HQ correlators
  - Same simulations as to determine other low energy QCD observables.
  - Main source of uncertainty: Missing orders in PT, dependence on physics at a few GeV.
  - State of the art determinations reach 1-2% precision.
  - Difficult to substantially reduce this uncertainty.
- Dedicated approach: Finite size scaling
  - Reach arbitrary high energies non-perturbatively
  - Allows to check matching with PT in large energy ranges (4-140 GeV).
  - Challenge is to reduce statistical uncertainties
- Excellent work by FLAG:
  - **Only** works that can control systematic effects enter in the average
  - Detailed analysis of the perturbative uncertainties

FLAG average

$$\alpha_s(M_Z) = 0.1182(8)\,.$$

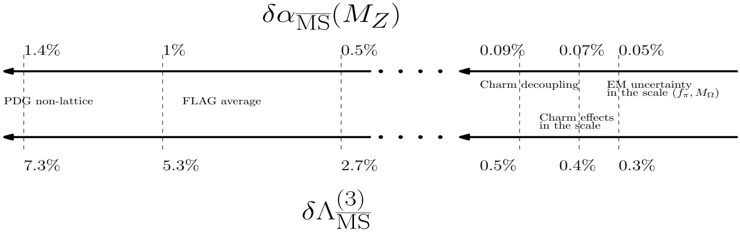
CONCLUSIONS



Future: Can LQCD half the error in α_s

- I would say: YES...
- ...But **only** using dedicated approaches
- Progress in large volume QCD will not reduce the uncertainty in α_s
  - Electromagnetic corrections
  - Charm effects (i.e. $N_\mathrm{f} = 2 + 1 + 1$ vs. $N_\mathrm{f} = 2 + 1$)
  - Connecting hadronic physics with EW scale **without assumptions** on low scale physics (i.e. perturbation theory)

Conclusions



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