AAMQs

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CENTRA/IST (Lisbon) & CERN PH-TH

[Javier Albacete, Néstor Armesto, JGM, Paloma Quiroga-Arias, and Carlos Salgado]

[also with J. Rojo]

Recent QCD Advances at the LHC
Les Houches, 15 Feb 2011

⥁ lore ::

:: a body of traditions and knowledge on a subject or held by a particular group, typically passed from person to person by word of mouth

- simple physical arguments require the inclusion of non-linearities in the evolution; the C(olour)G(lass)C(ondensate) is the correct framework in which to address small-x physics
  - ↪ how sizeable are the effects ?
  - ↪ what is the relevant kinematical domain ?
  - ↪ can observables be computed from 'first principles' ?

- DGLAP provides extremely accurate description of ALL available experimental data
  - ↪ can properties of the evolution be disentangled from ingenious choices of initial conditions ?
  - ↪ how uncertain are extrapolations into the unmeasured small-x region ?
  - ↪ can results from non-linear approaches be accommodated in the description by simply tuning initial conditions?

- [how] can these questions be answered ?

⟳ how to test the CGC?

- what is the CGC [in this talk]?
  - ↪ effective theory for description of small-x glue
  - ↪ well established non-linear evolution equations [B-JIMWLK]
  - ↪ large N_c approximation [BK] for suitable observables
  - ↪ NOT [at least in this talk] phenomenological models encoding 'saturation physics'

- what to do
  - ↪ extract universal unintegrated gluon distribution from cleanest process [DIS]
  - ↪ use to compute observables [cf. talks by C. Marquet & J. Albacete]

  - ↪ devise a set-up in which to compare DGLAP and CGC evolutions

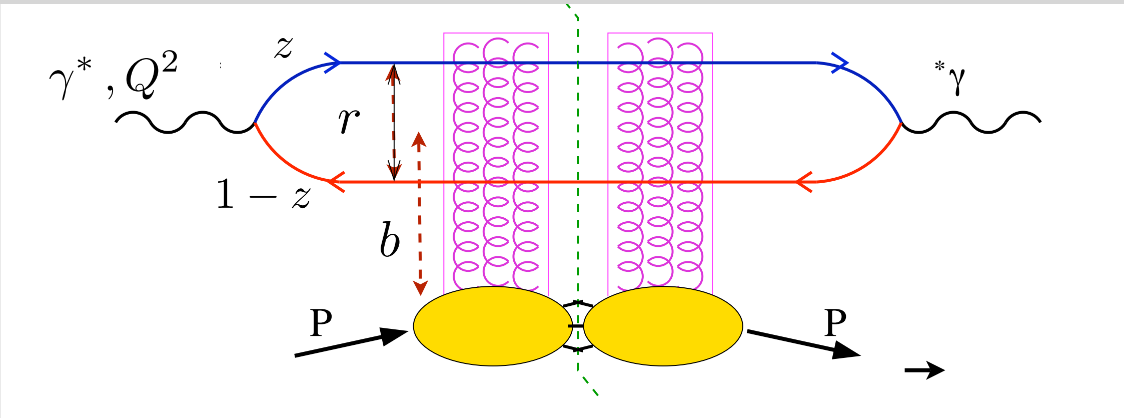
dipole formulation of QCD

- at high energy [x << 1] the coherence length of the virtual photon fluctuation

$$l_c \sim (2m_N x)^{-1} \simeq 0.1/x \text{ fm} \gg R_N$$

- total virtual photon-proton cross section can be factorized as

$$\sigma_{T,L}(x,Q^2) = 2\sum_f \int_0^1 dz \int d\mathbf{b}\, d\mathbf{r}\, |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2\, \mathcal{N}(\mathbf{b}, \mathbf{r}, x)$$

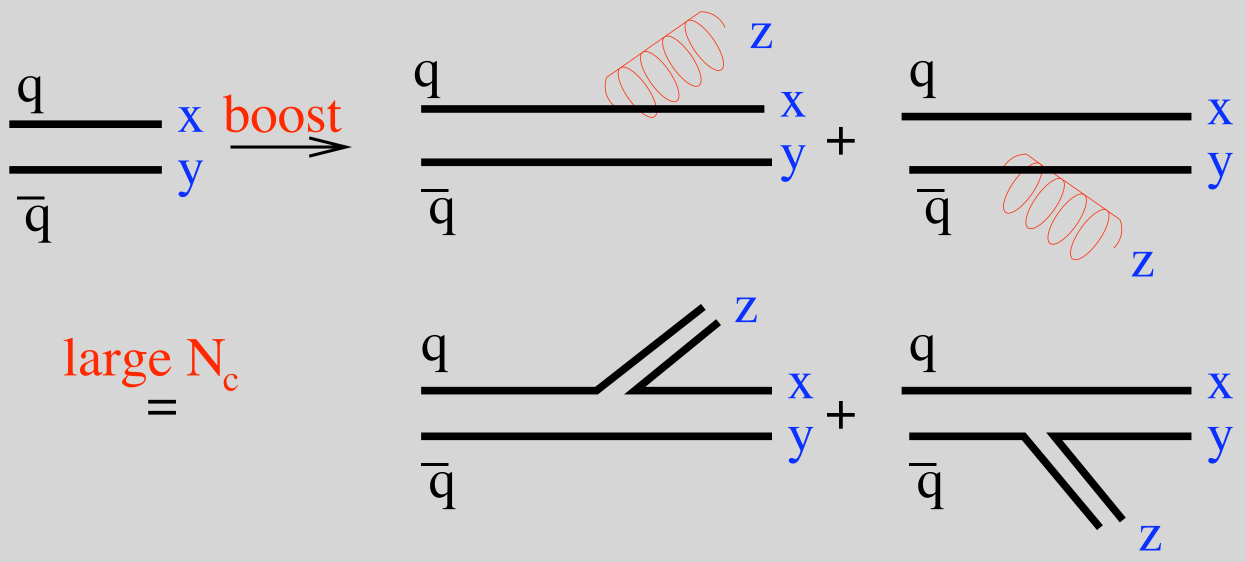


QED calculation

[imaginary part of]
dipole-target scattering amplitude
:: all QCD information
:: all x dependence
:: non-perturbative, but x-evolution computable from first principles [rcBK]

⟲ BK equation [simplest derivation]

[rapidity evolution of scattering probability $N(x,y;Y)$ of $q\bar{q}$ dipole with hadronic target]



$$\vec{r} = \vec{x} - \vec{y}$$
$$\vec{r}_1 = \vec{x} - \vec{z}$$
$$\vec{r}_2 = \vec{y} - \vec{z}$$

homogeneous target with radius much larger than any dipole size

↪ neglect impact parameter dependence (2-dim into 1-dim)

non-linear effect [double scattering]

$$\frac{\partial N(r,Y)}{\partial Y} = \int \frac{d^2z}{2\pi} K(\vec{r},\vec{r}_1,\vec{r}_2)\Big[N(r_1,Y)+N(r_2,Y)-N(r,Y)-N(r_1,Y)N(r_2,Y)\Big]$$

$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \bar{\alpha}_s \frac{r^2}{r_1^2 r_2^2}, \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

BFKL kernel:
probability of gluon (two dipoles) emission

◌ NLO-BK [evolution of dipole scattering amplitude]

NLO-BK = [all orders in $\alpha_s N_f$]+ [other conformal]

[running coupling] + [subtraction]

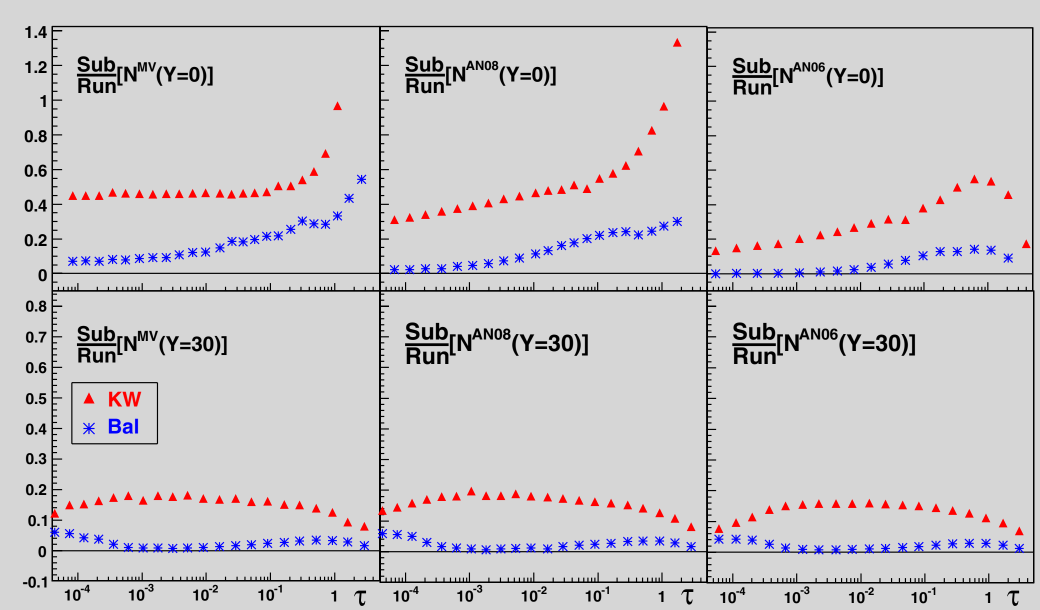
:: numerically challenging ::

numerically demanding, but

contribution minimized in Balitsky's subtraction scheme

[rcBK]

AAMQ$_s$ implementation



[Albacete, Kovchegov]

rcBK [Bal scheme]

- running coupling BK [rcBK]
  - fully compatible with DIS data [the point of this talk]
  - best, numerically implementable, incarnation of non-linear QCD

$$\frac{\partial \mathcal{N}(r,x)}{\partial \ln(x_0/x)} = \int d\mathbf{r_1}\, K^{\rm run}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) \;\; [\mathcal{N}(r_1,x)+\mathcal{N}(r_2,x)-\mathcal{N}(r,x)-\mathcal{N}(r_1,x)\mathcal{N}(r_2,x)]$$

modified kernel

same structure as LO-BK

$$K^{\rm run}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\,r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right]$$

LO-BK :: BFKL kernel

$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \bar{\alpha}_s\,\frac{r^2}{r_1^2 r_2^2}, \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

AAMQ$_s$ setup

- DIS reduced cross section

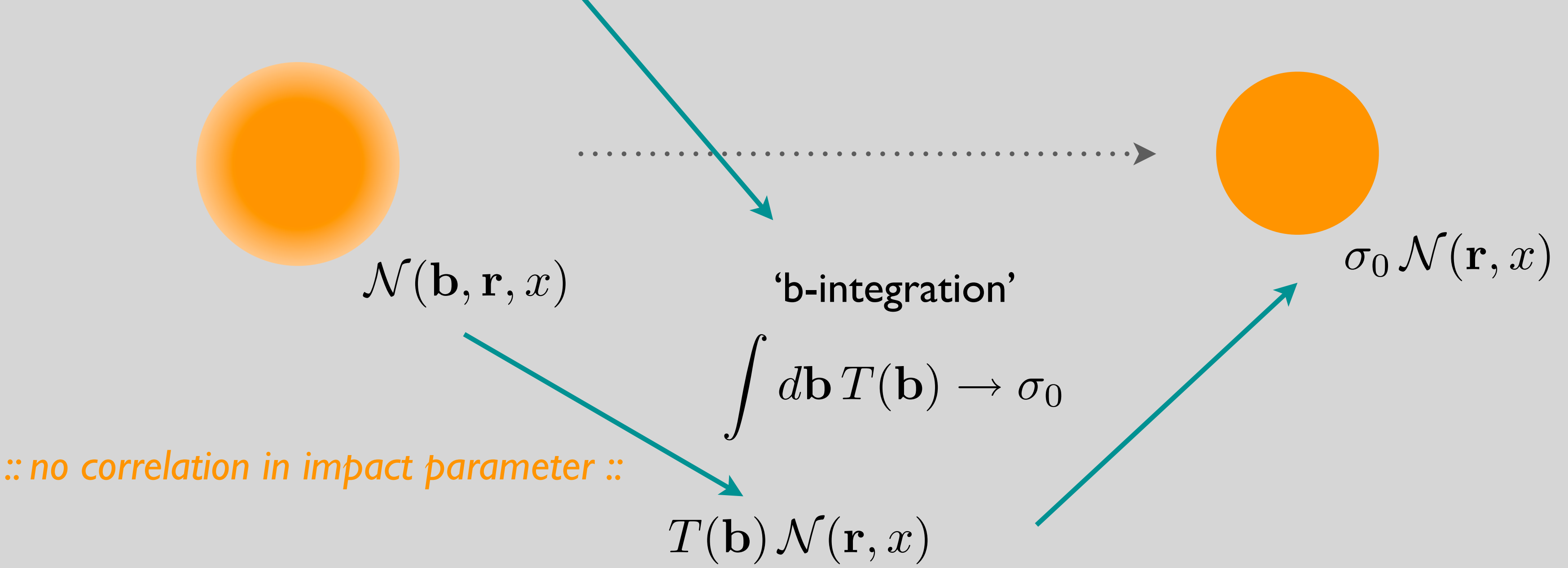
$$\sigma_r(x,y,Q^2) = \frac{Q^2}{4\,\pi^2\alpha_{em}}\left(\sigma_T + \frac{2(1-y)}{1+(1-y)^2}\sigma_L\right)$$

$$\sigma_{T,L}(x,Q^2) = 2\sum_f \int_0^1 dz \int d\mathbf{b}\, d\mathbf{r}\, |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}(\mathbf{b},\mathbf{r},x)$$

- b-dependence governed by long-distance non-perturbative physics [extra model input]
- AAMQ$_s$ resorts to translational invariance approximation
  - proton homogeneous in transverse plane

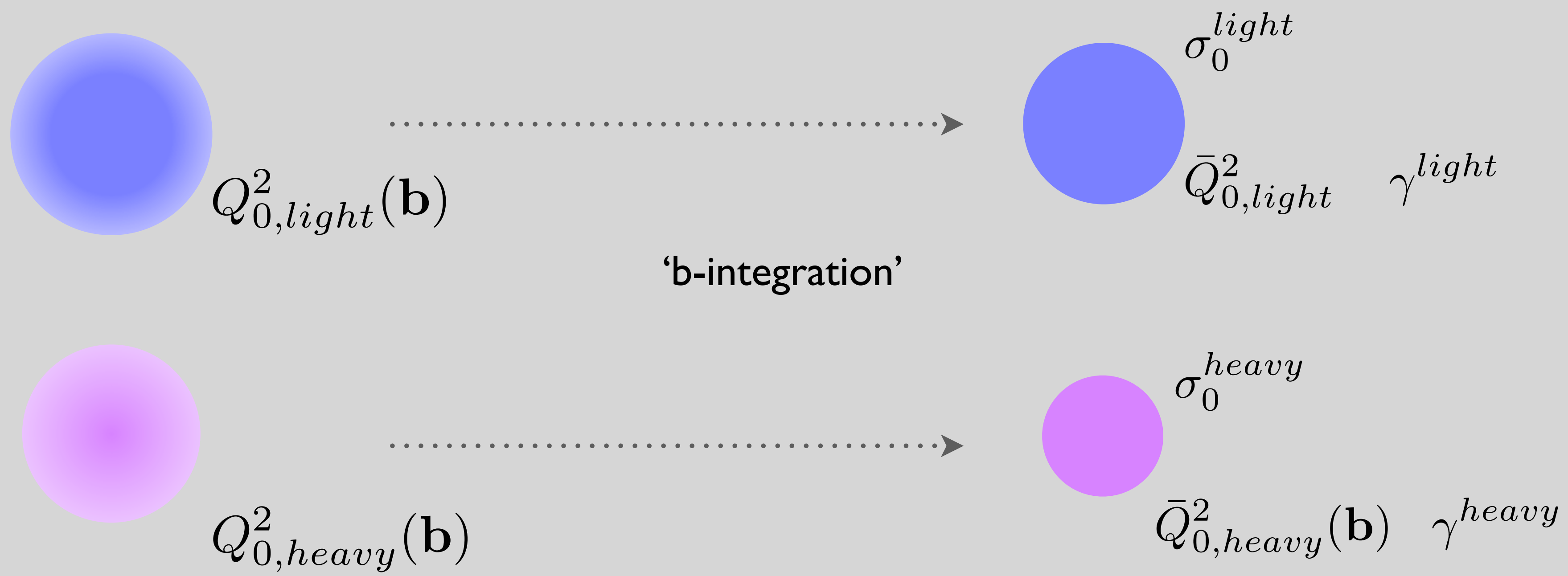
$$\sigma_{T,L}(x,Q^2) = \sigma_0 \int_0^1 dz \int d\mathbf{r}\, \left|\Psi_{T,L}(z,Q^2,\mathbf{r})\right|^2 \mathcal{N}(r,x)$$

only LO
[QED calculation]
NLO now available
[Balitsky & Chirilli]



◌ AAMQ$_s$ setup :: including heavy quarks

- allow for independent light and heavy i.c.



- should follow from 'better' treatment of b-dependence
  - heavy quark dipoles couple differently to target ...

- additional fit parameters ...

$$\sigma_{T,L}(x,Q^2) = \sigma_0 \sum_{f=u,d,s} \int_0^1 dz\, d\mathbf{r}\, |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2\, \mathcal{N}^{light}(\mathbf{r}, x)$$
$$+\, \sigma_0^{heavy} \sum_{f=c,b} \int_0^1 dz\, d\mathbf{r}\, |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2\, \mathcal{N}^{heavy}(\mathbf{r}, x)$$

AAMQs setup :: initial conditions

- 2 families of initial conditions
  - generalized GBW and MV forms

$$\mathcal{N}^{GBW}(r, x\!=\!x_0) = 1 - \exp\left[-\frac{\left(r^2\, Q_{s\,0}^2\right)^{\gamma}}{4}\right]$$

:: differ in UV behaviour ::

$$\mathcal{N}^{MV}(r, x\!=\!x_0) = 1 - \exp\left[-\frac{\left(r^2\, Q_{s\,0}^2\right)^{\gamma}}{4}\ln\left(\frac{1}{r\,\Lambda} + e\right)\right]$$

  ↪ fit parameters
  - initial saturation scale $Q_{s\,0}^2$
  - anomalous dimension γ [sharpness of edge fall-off]

  - scaling ic

  ↪ evolve rcBK to very high [asymptotic] rapidity and re-scale back to initial rapidity
  - Weigert et al. report viability of this approach
  - AAMQs unable to fit the data with scaling i.c.

AAMQ$_s$ setup :: running coupling

- running coupling in rcBK is 1-loop in *coordinate* space

$$\alpha_{s,n_f}(r^2) = \frac{4\pi}{\beta_{0,n_f} \ln\left(\frac{4C^2}{r^2 \Lambda_{n_f}}\right)} \qquad \beta_{0,n_f} = 11 - \frac{2}{3} n_f$$

- for light only :: fixed flavour number $N_f = 3$
- for light+heavy :: variable flavour scheme, coupling matched at quark masses

- C accounts for uncertainty in Fourier transform from momentum to coordinate space [fit parameter]

- Λs fixed by reference to experimentally measured value of α_s [either Z^0 mass or τ mass]

- IR regulated by setting $r > r_{fr}, \;\; \alpha_s(r_{fr}^2) \equiv \alpha_{fr} = 0.7$, or any other suitable value.

AAMQ$_s$ setup :: data set

- combined H1/ZEUS data for reduced cross-section
  - cuts :: $x \leq 10^{-2},\ Q^2 \leq 50\,\mathrm{GeV}^2$
  - very small errors
  - sigma reduced better [no extraction uncertainty]

- old non-HERA data [E665, NMC]

- including charm [all available F_2^{charm} within cuts]
  - normalization uncertainties not considered

    [solved by recent H1/ZEUS combined]

- kinematical redefinition of Bjorken x

$$\tilde{x} = x\left(1 + \frac{4m_f^2}{Q^2}\right)$$

  - only use data where full evolution can be computed from i.c.

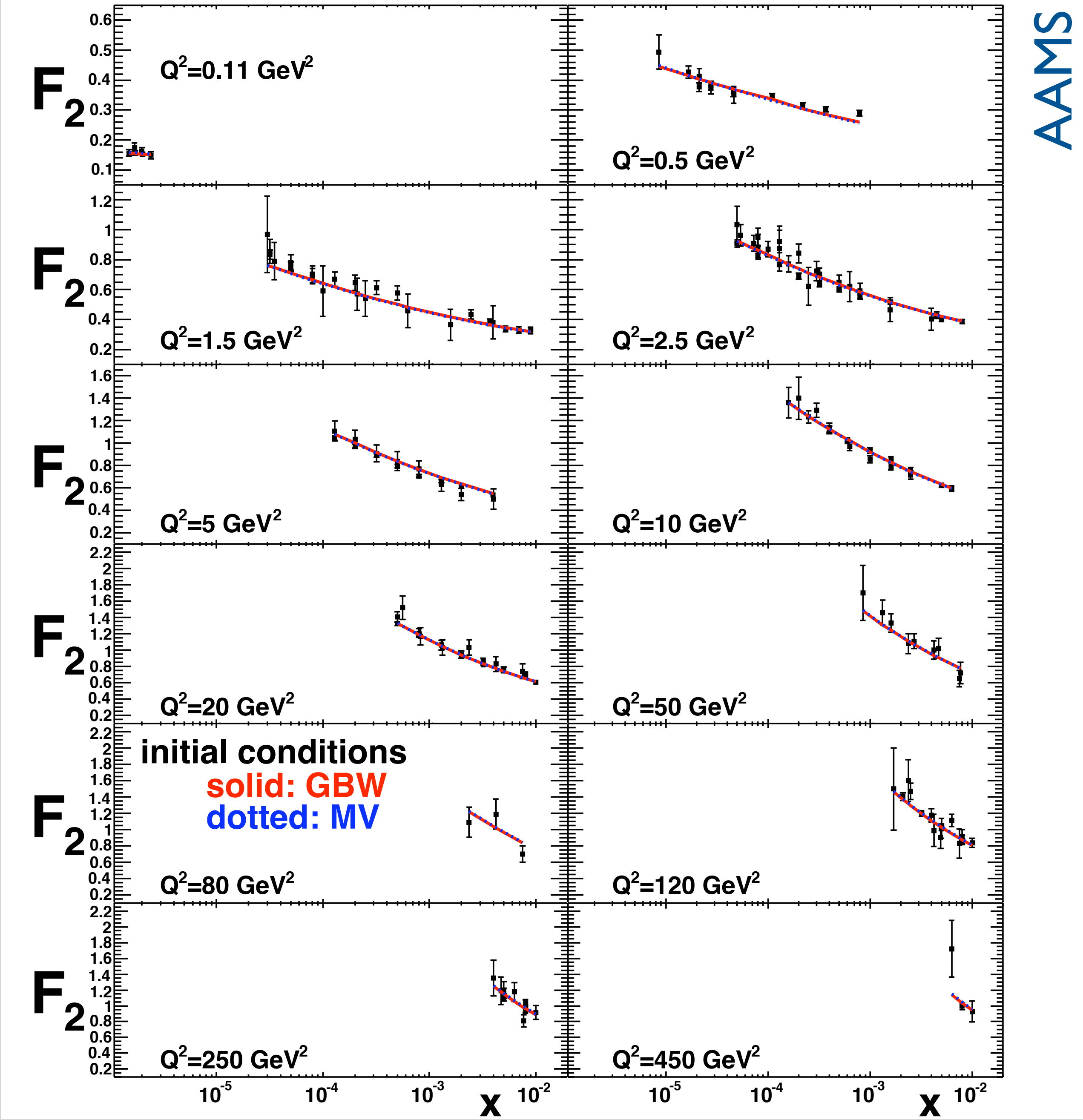
⥀ AAMQ$_s$ setup :: summary

- rcBK + virtual photon-proton cross section
- LO photon impact factor
- translational invariance [no b dependence]

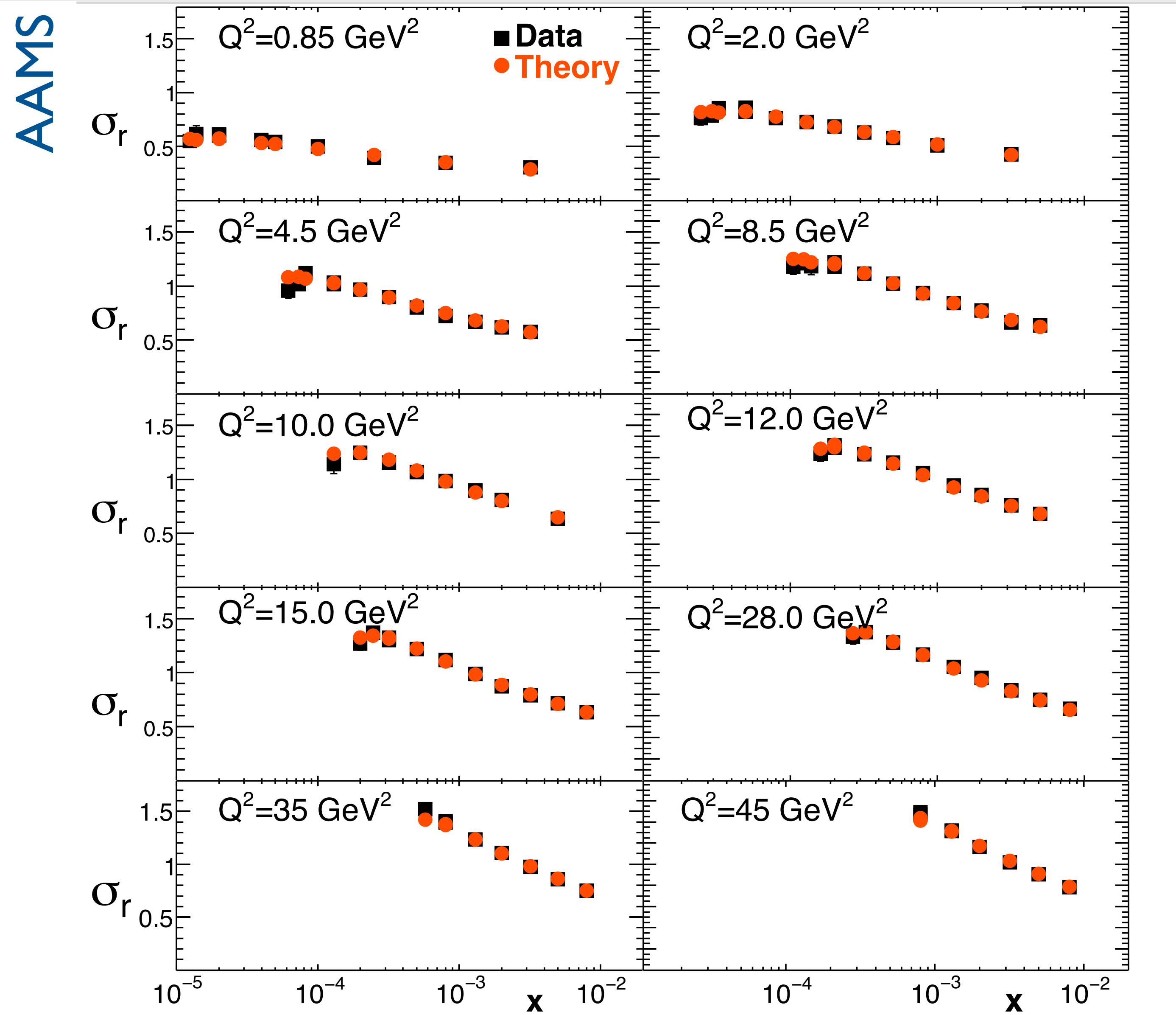
- fit parameters
  - total normalization of cross section [effective area with uniform initial condition: from b-integration]
  - IR uncertainty in running coupling [from FT]
  - initial saturation scale [in ic]
  - anomalous dimension [in ic]

  - 'light only' :: 4 parameters [#data=345]
  - with heavy :: 7 parameters [8 parameters if free light mass] [#data=349]
    - ↪ [allowing for different distribution of heavy in proton]

old vs. new

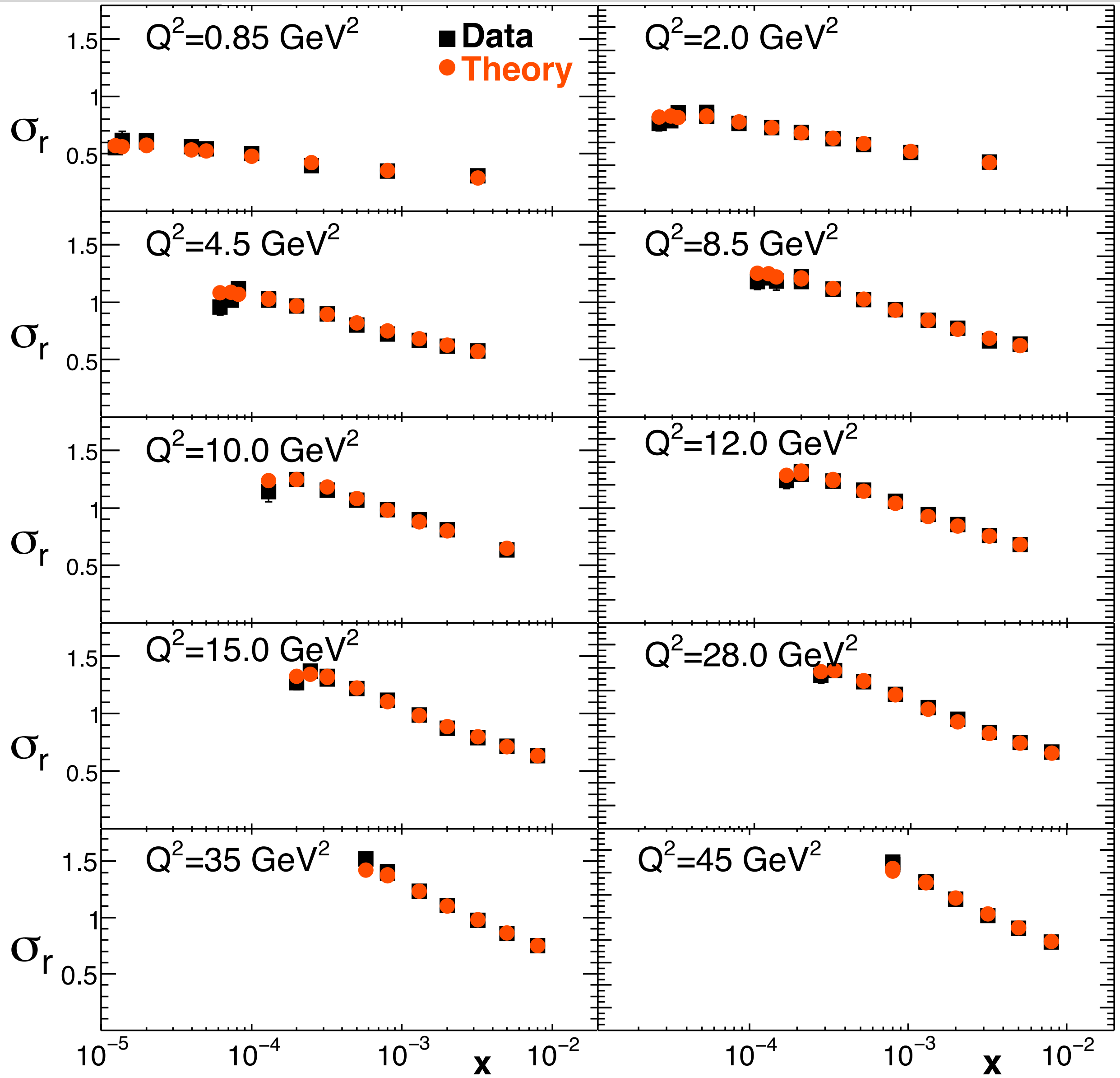


F_2
Q^2=0.11 GeV^2
Q^2=0.5 GeV^2
Q^2=1.5 GeV^2
Q^2=2.5 GeV^2
Q^2=5 GeV^2
Q^2=10 GeV^2
Q^2=20 GeV^2
Q^2=50 GeV^2
initial conditions
solid: GBW
dotted: MV
Q^2=80 GeV^2
Q^2=120 GeV^2
Q^2=250 GeV^2
Q^2=450 GeV^2
x



AAMS
σ_r
Q^2=0.85 GeV^2
Data
Theory
Q^2=2.0 GeV^2
Q^2=4.5 GeV^2
Q^2=8.5 GeV^2
Q^2=10.0 GeV^2
Q^2=12.0 GeV^2
Q^2=15.0 GeV^2
Q^2=28.0 GeV^2
Q^2=35 GeV^2
Q^2=45 GeV^2
x

results [light only]



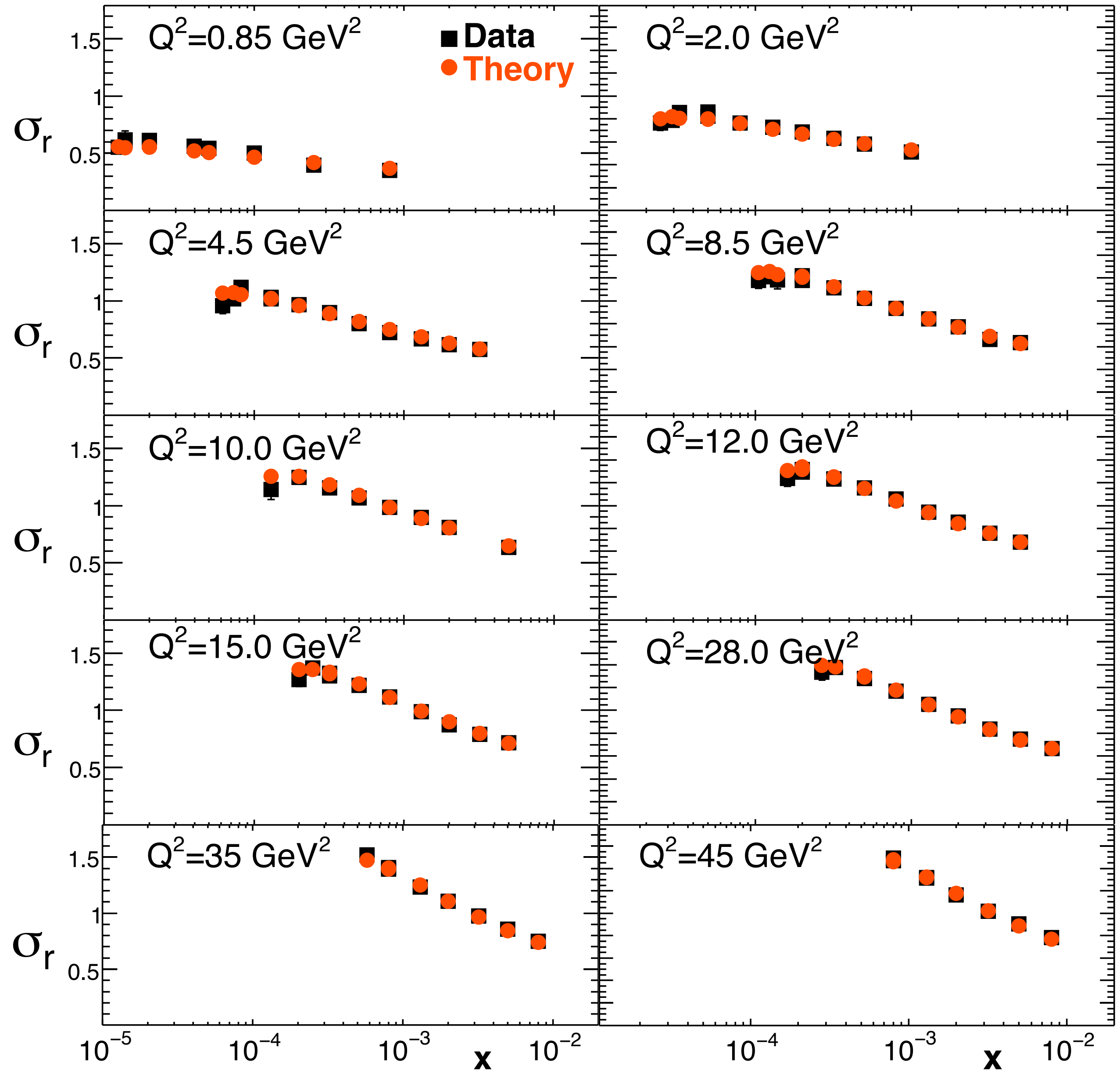
- fully consistent with results obtained with ‘old’ HERA data [AAMS]
  - very mild change of parameters
  - tension with high Q^2 data [and this is good] :: not shown
- fitted initial conditions are numerically ‘essentially identical’
  - physically meaningful

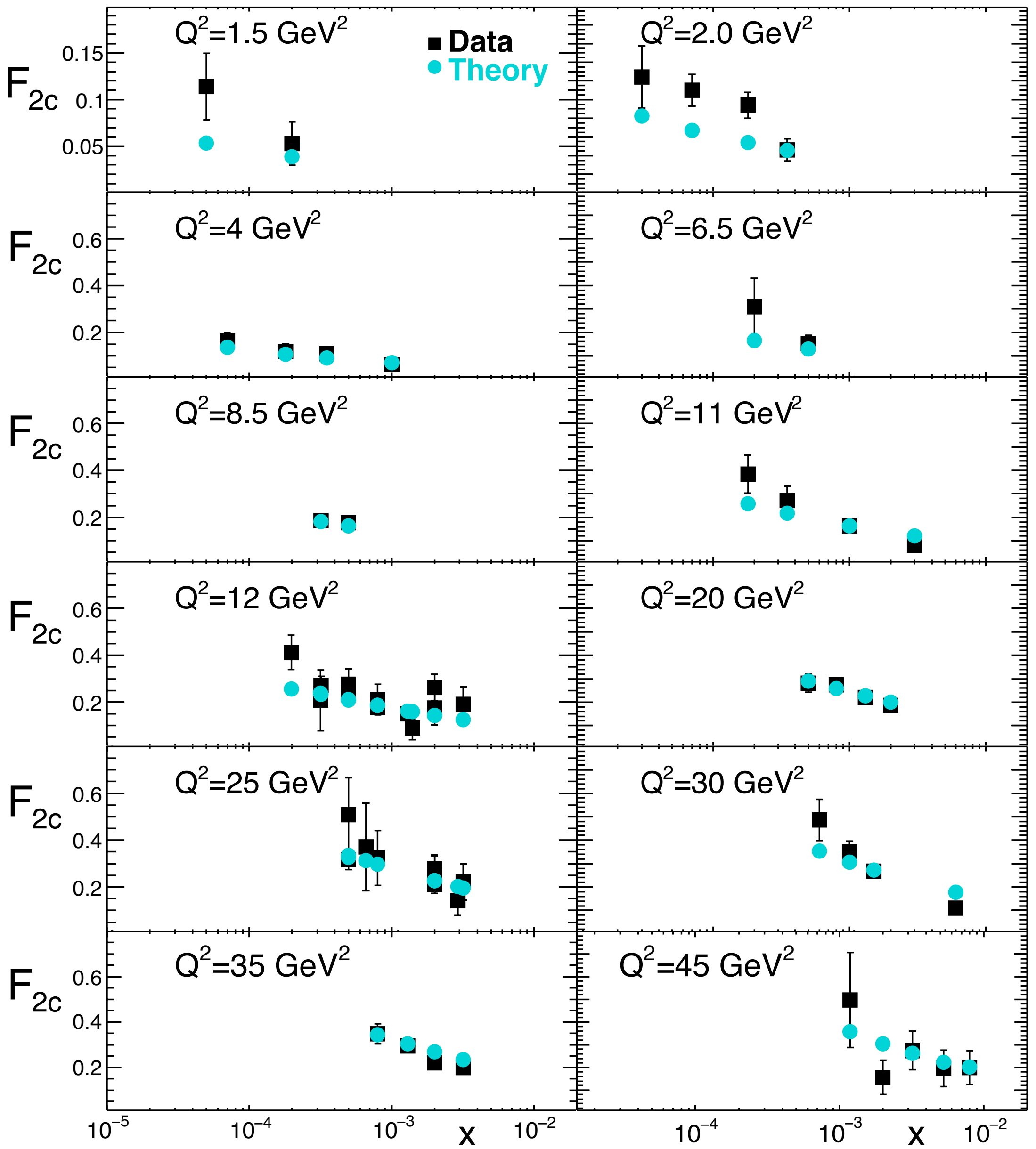
	fit	$\chi^2_{d.o.f}$	$Q^2_{S,0}$	σ_0	γ	C	m_l^2
	GBW						
a	$\alpha_f = 0.7$	1.226	0.241	32.357	0.971	2.46	fixed
a’	$\alpha_f = 0.7\ (\Lambda_{m_\tau})$	1.235	0.240	32.569	0.959	2.507	fixed
b	$\alpha_f = 0.7$	1.264	0.2633	30.325	0.968	2.246	1.74E-2
c	$\alpha_f = 1$	1.279	0.254	31.906	0.981	2.378	fixed
c’	$\alpha_f = 1\ (\Lambda_{m_\tau})$	1.244	0.2329	33.608	0.9612	2.451	fixed
d	$\alpha_f = 1$	1.248	0.239	33.761	0.980	2.656	2.212E-2
	MV						
e	$\alpha_f = 0.7$	1.171	0.165	32.895	1.135	2.52	fixed
f	$\alpha_f = 0.7$	1.161	0.164	32.324	1.123	2.48	1.823E-2
g	$\alpha_f = 1$	1.140	0.1557	33.696	1.113	2.56	fixed
h	$\alpha_f = 1$	1.117	0.1597	33.105	1.118	2.47	1.845E-2
h’	$\alpha_f = 1\ (\Lambda_{m_\tau})$	1.104	0.168	30.265	1.119	1.715	1.463E-2

AAMS

Initial condition	σ_0 (mb)	Q^2_{s0} (GeV^2)	C^2	γ	χ^2/d.o.f.
GBW	31.59	0.24	5.3	1 (fixed)	916.3/844=1.086
MV	32.77	0.15	6.5	1.13	906.0/843=1.075

◌ results [light + heavy]





- excellent global σ_{red} / F_2 + F_2^c description χ^2/#dof ~ 1.3

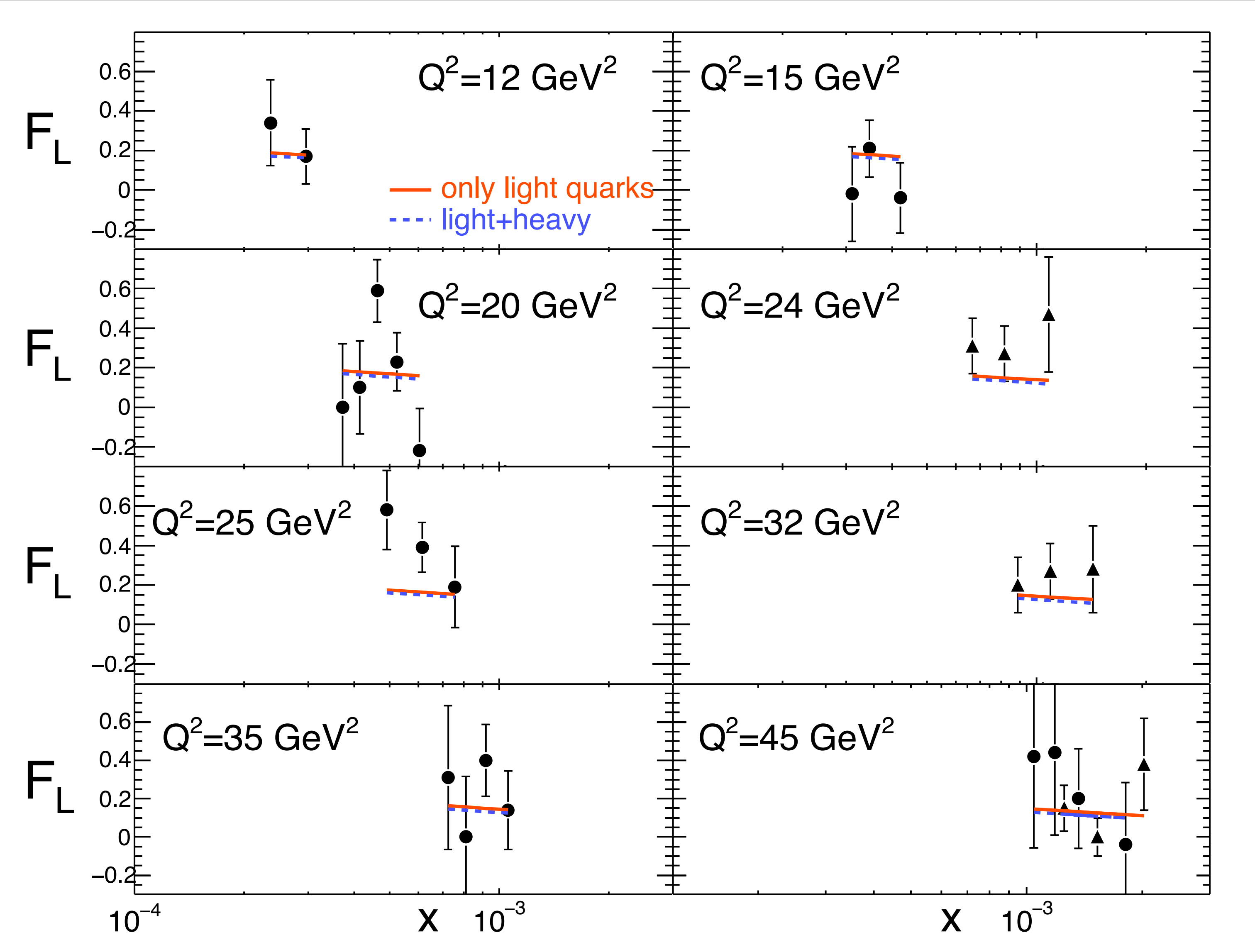
results [light + heavy]

	fit	$\frac{\chi^2}{d.o.f}$	$Q^2_{S,0}$	σ_0	γ	$Q^2_{S,0,c}$	$\sigma_{0,c}$	γ_c	C	m_l^2
	GBW									
a	$\alpha_f = 0.7$	1.269	0.2294	36.953	1.259	0.2289	18.962	0.881	4.363	fixed
a’	$\alpha_f = 0.7\ (\Lambda_{m_\tau})$	1.302	0.2341	36.362	1.241	0.2249	20.380	0.919	7.858	fixed
b	$\alpha_f = 0.7$	1.231	0.2386	35.465	1.263	0.2329	18.430	0.883	3.902	1.458E-2
c	$\alpha_f = 1$	1.356	0.2373	35.861	1.270	0.2360	13.717	0.789	2.442	fixed
d	$\alpha_f = 1$	1.221	0.2295	35.037	1.195	0.2274	20.262	0.924	3.725	1.351E-2
	MV									
e	$\alpha_f = 0.7$	1.395	0.1673	36.032	1.355	0.1650	18.740	1.099	3.813	fixed
f	$\alpha_f = 0.7$	1.244	0.1687	35.449	1.369	0.1417	19.066	1.035	4.079	1.445E-2
g	$\alpha_f = 1$	1.325	0.1481	40.216	1.362	0.1378	13.577	0.914	4.850	fixed
h	$\alpha_f = 1$	1.298	0.156	37.003	1.319	0.147	19.774	1.074	4.355	1.692E-2

- $\sigma_0^{charm} < \sigma_0^{light}$
  - also charm has a gentler fall-off in i.c [$\gamma^{charm} < \gamma^{light}$]

- F_2^c constrained

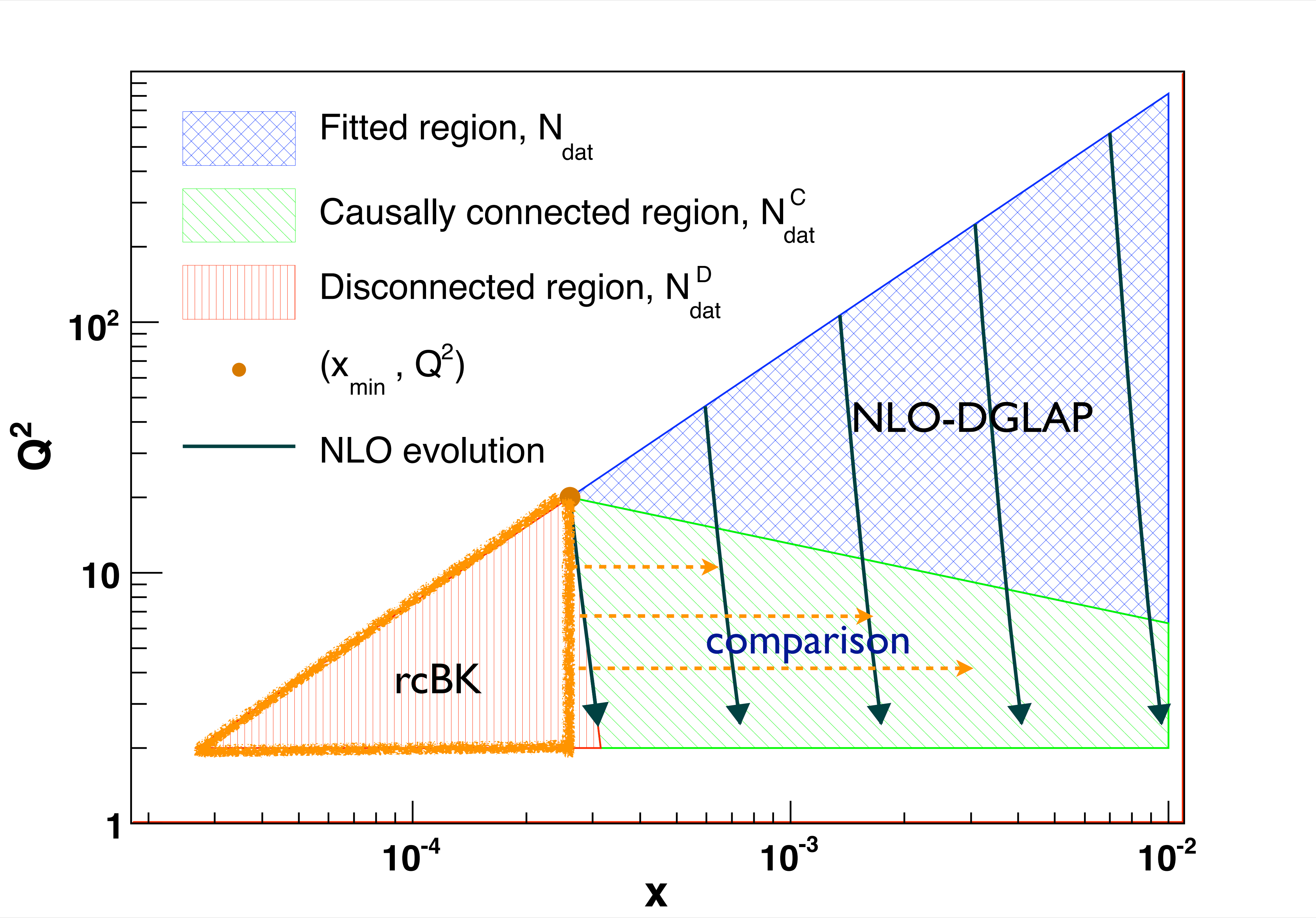
efele



- H1 and ZEUS direct measurements
  - not included in the fit [independent test]

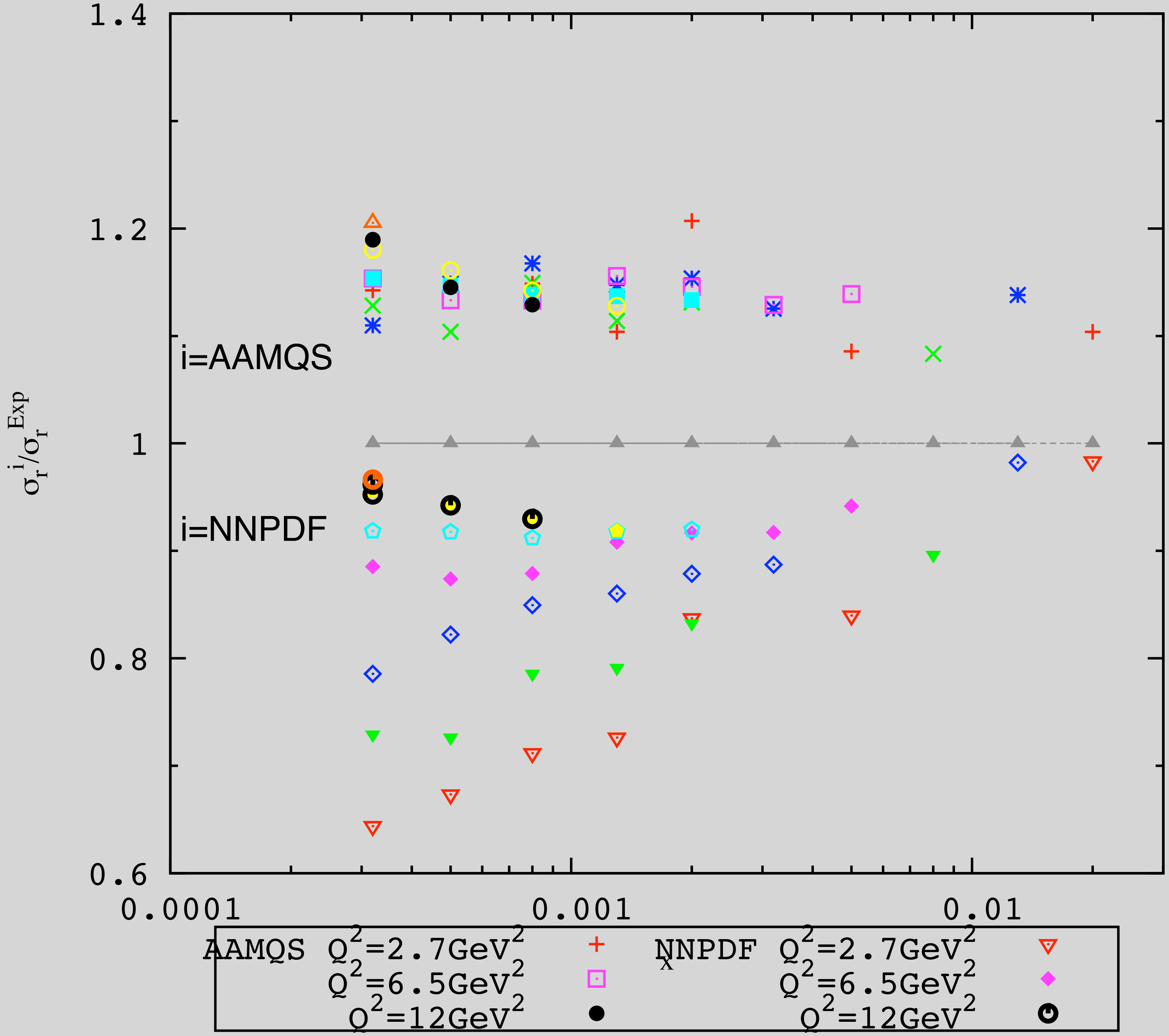
DGLAP comparison

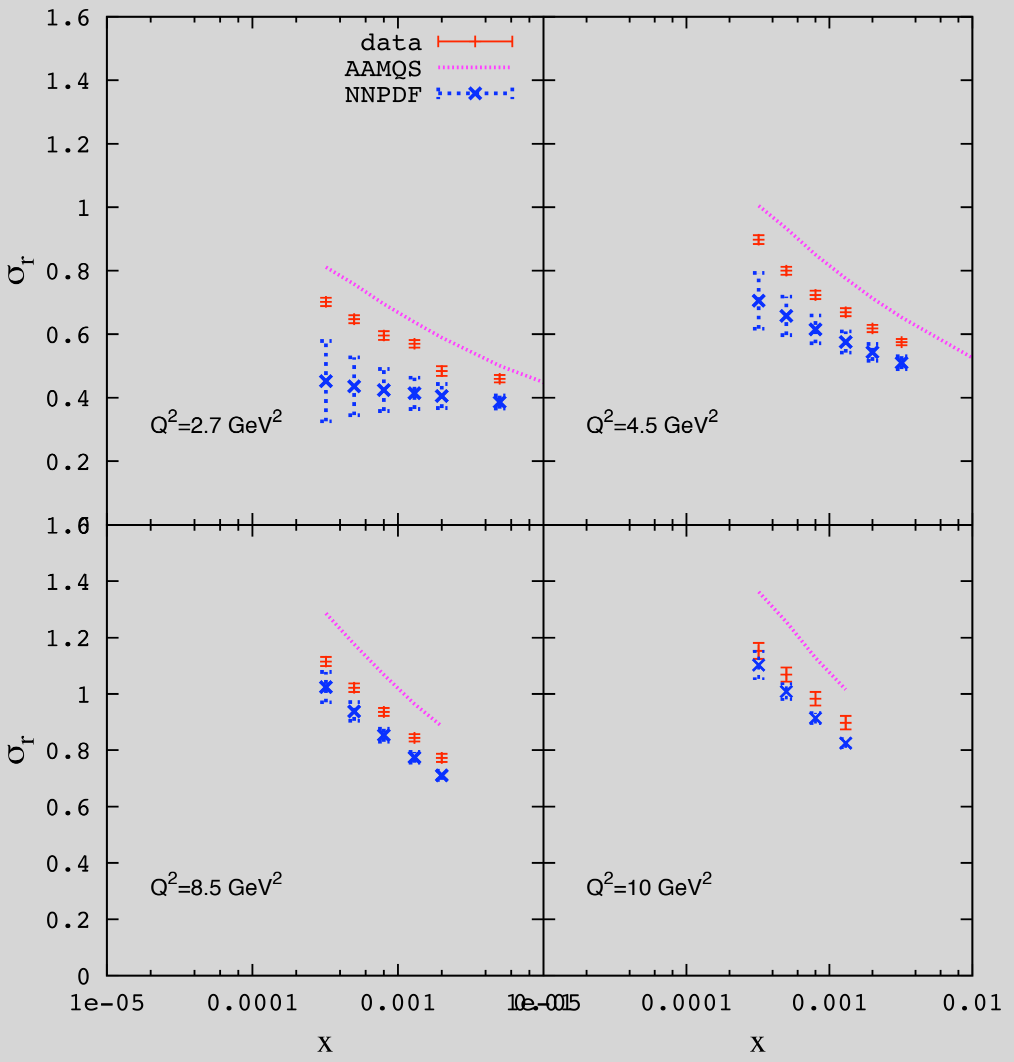
- comparison with NNPDF underway [JGM, J. Rojo, P. Quiroga]



test the evolution NOT the choice of initial conditions

unfitted region [preliminary]





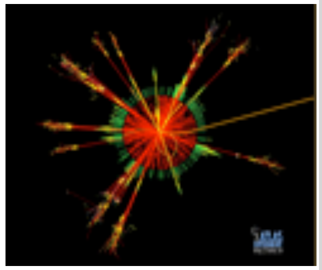
⥀ as a matter of principle

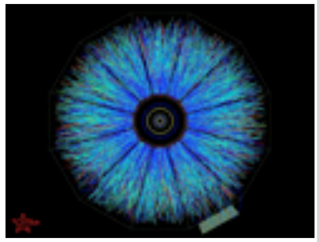
Conclusions are up to you

AAMQs 1.0

http://www-fp.usc.es/phenom/aamqs/aamqs.html

Dipole-proton cross section





The imaginary part of the dipole-proton scattering amplitude is available as a FORTRAN routine for public use. This quantity has been fitted to lepton-proton data using the Balitsky-Kovchegov evolution equations with running coupling. More details can be found at

J. L. Albacete, N. Armesto, J. G. Milhano, P. Quiroga Arias and C. A. Salgado, arXiv:1012.4408

Please refer to this publication when using the routine.

In order to compute the dipole cross section, simply multiply the output from the routine by the corresponding values in Table 1 of arXiv:1012.4408 (the actual values depend on the chosen set of parameters). These values are

For the fits with only light flavors (subroutine aamqs10l):
sigma0=32.357 mb for GBW initial conditions, set a
sigma0=32.895 mb for MV initial conditions, set e

For the fits with light+heavy flavors (subroutine aamqs10h):
sigma0=35.465 mb for GBW initial conditions, light, set b
sigma0=18.430 mb for GBW initial conditions, heavy, set b
sigma0=35.449 mb for MV initial conditions, light, set f
sigma0=19.066 mb for MV initial conditions, heavy, set f

Full instructions and explanations can also be found at the headers of the routines.

To download the code, please follow this link

The main novelties on these parametrizations with respect to our older one arXiv:0902.1112 are the use of the new (H1 and ZEUS combined) HERA data with much smaller error bars as well as the inclusion of heavy flavors in the fits.

If you find any problem, please, let us know

backups

↻ linearity

both DGLAP and BFKL are linear approaches

evolution independent of ensemble

:: underlying assumption ::

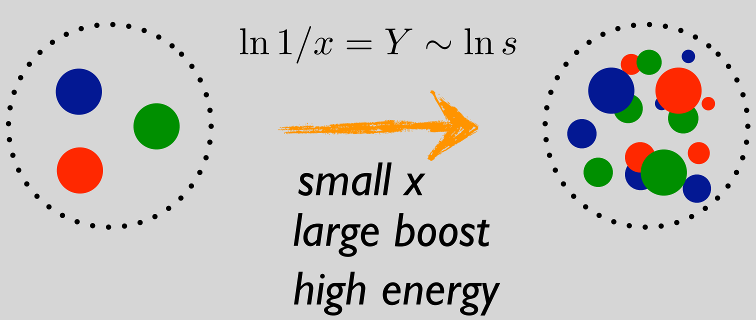
- ensemble is dilute and remains so throughout evolution
- no collective behaviour in parton splitting

DGLAP

- evolution towards larger Q^2 increases dilution
- assumption of linearity is self-consistent

evolution [DGLAP] in Q^2 is intrinsically linear

BFKL



- evolution towards smaller x increases density
  - assumption of perpetual linearity violated
  - evolution in x should account for the ensemble

:: parton overlap, parton recombination, phase space reduction ::

evolution in x becomes naturally non-linear

neither approach is sufficient :: need non-linear generalization of BFKL

gluon dominance

- the infrared sensitivity of parton splitting [bremsstrahlung] favours the emission of soft [small-x] gluons :: at small-x the ensemble is gluon dominated
- both BFKL and DGLAP (DLA) predict a very steep rise of the gluon distribution
  - observed at HERA
  - if perpetuated leads to unitarity [Froissart bound] violation

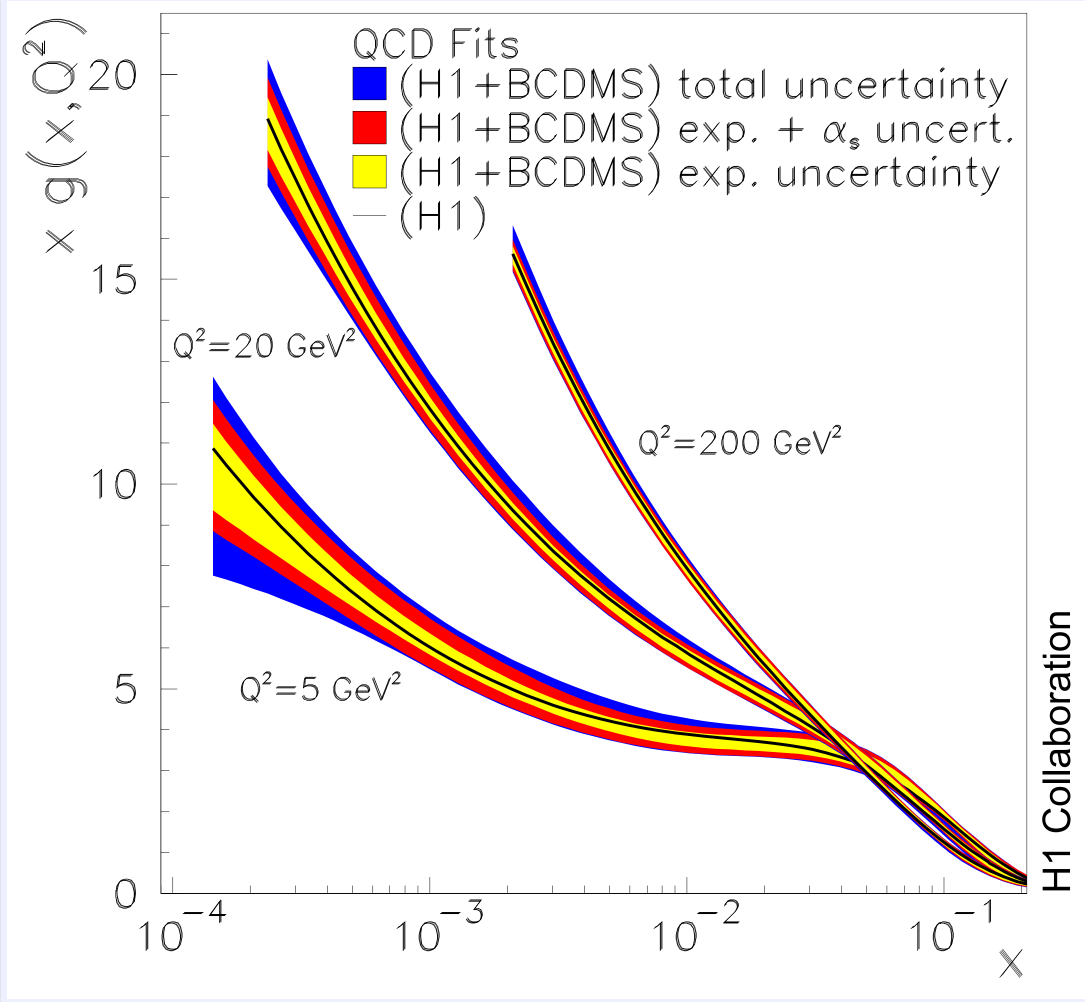
$$xg(x,Q^2) \sim x^{-12\ln 2 \frac{\alpha_s}{\pi}} \sim x^{-0.5}$$

BFKL

$$xg(x,Q^2) \sim \exp\left\{\left(\frac{48}{11-\frac{2}{3}N_f}\ln\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}\ln 1/x\right)^{1/2}\right\}$$

DLA-DGLAP

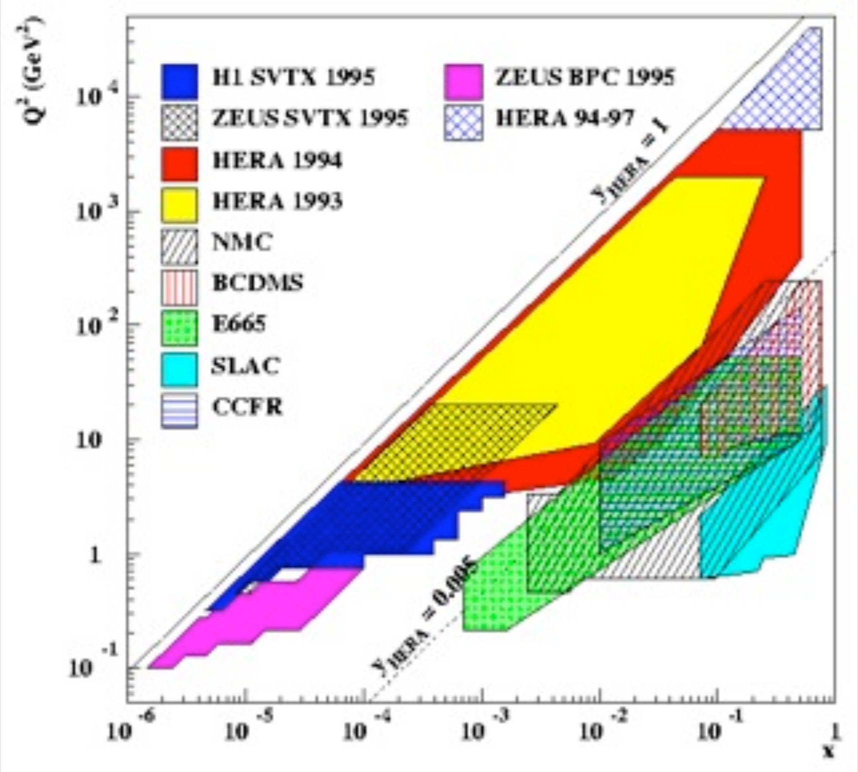
the growth of partonic density should be tempered by non-linearities when the density becomes large



proton vs. nuclear pdfs

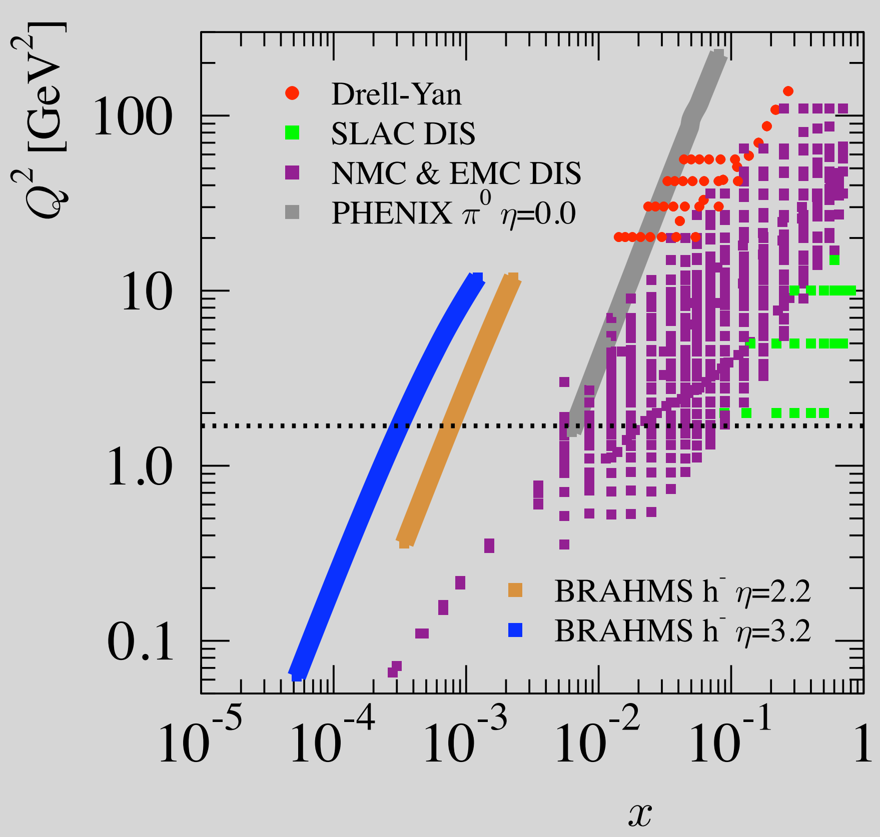
- *proton case*
  - collinear factorization theorems proven for some processes
  - wealth of data (DIS, DY, jets)
    - ↪ very reliable pdfs in 'data covered' kinematical range
  - large number of parameters in i.c.
    - ↪ very 'accommodating'
    - ↪ large uncertainty where data not available [small-x for moderate Q^2]

- but [see later] small x effects beyond collinear approach



proton vs. nuclear pdfs

- *nuclear case*
  - collinear factorizability is a working assumption
    - encoding of all nuclear effects in npdfs is a huge leap of faith
    - could be reliably tested in pA LHC collisions [will discuss later]
  - relatively scarce data
  - standardly encoded as nuclear modification of proton pdfs [inherits proton pdf uncertainties]



$$f_{i/A}(x,Q^2) = R_i^A(x,Q^2)\, f_{i/p}(x,Q^2)$$

pdf of parton in nucleon inside nucleus

nuclear modification

pdf of parton in free nucleon

Balitsky-Kovchegov (BK) equation

$$\frac{\partial N(r,Y)}{\partial Y} = \int \frac{d^2z}{2\pi} K(\vec{r},\vec{r}_1,\vec{r}_2)\Big[N(r_1,Y)+N(r_2,Y)-N(r,Y)-N(r_1,Y)N(r_2,Y)\Big]$$

- closed evolution for scattering probability
- is unitary [scattering probability cannot grow above 1]
- large Nc limit (mean field) of infinite hierarchy [equivalent to target evolution]

$$N(x,y) = \langle n(x,y)\rangle_{\text{target}} = 1/N_c\langle 1 - V(x)V^\dagger(y)\rangle_{\text{target}}$$

$$\frac{\partial}{\partial Y}\langle n(x,y)\rangle = \cdots \langle n(x,y)n(y,z)\rangle$$

$$\frac{\partial}{\partial Y}\langle n(x,y)n(y,z)\rangle = \cdots$$

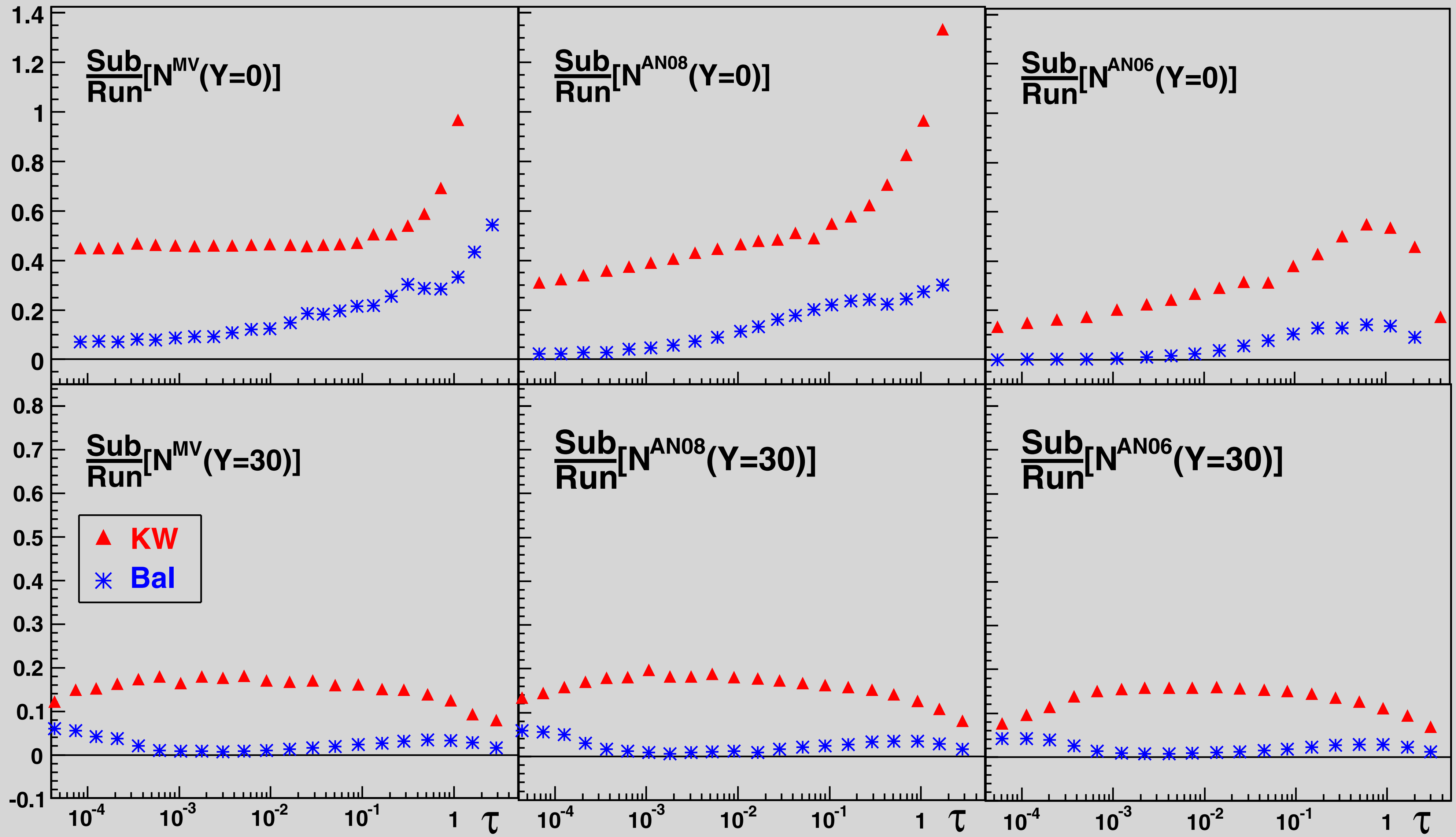
$$\ldots$$

large N_c limit

$$\langle n(x,y)n(y,z)\rangle = \langle n(x,y)\rangle\langle n(y,z)\rangle + o(1/N_c^2) = N(x,y)N(y,z) + o(1/N_c^2)$$

- no other consistent truncation possible
- numerical results for full hierarchy deviate 10% (0.1%) at most from full B-JIMWLK

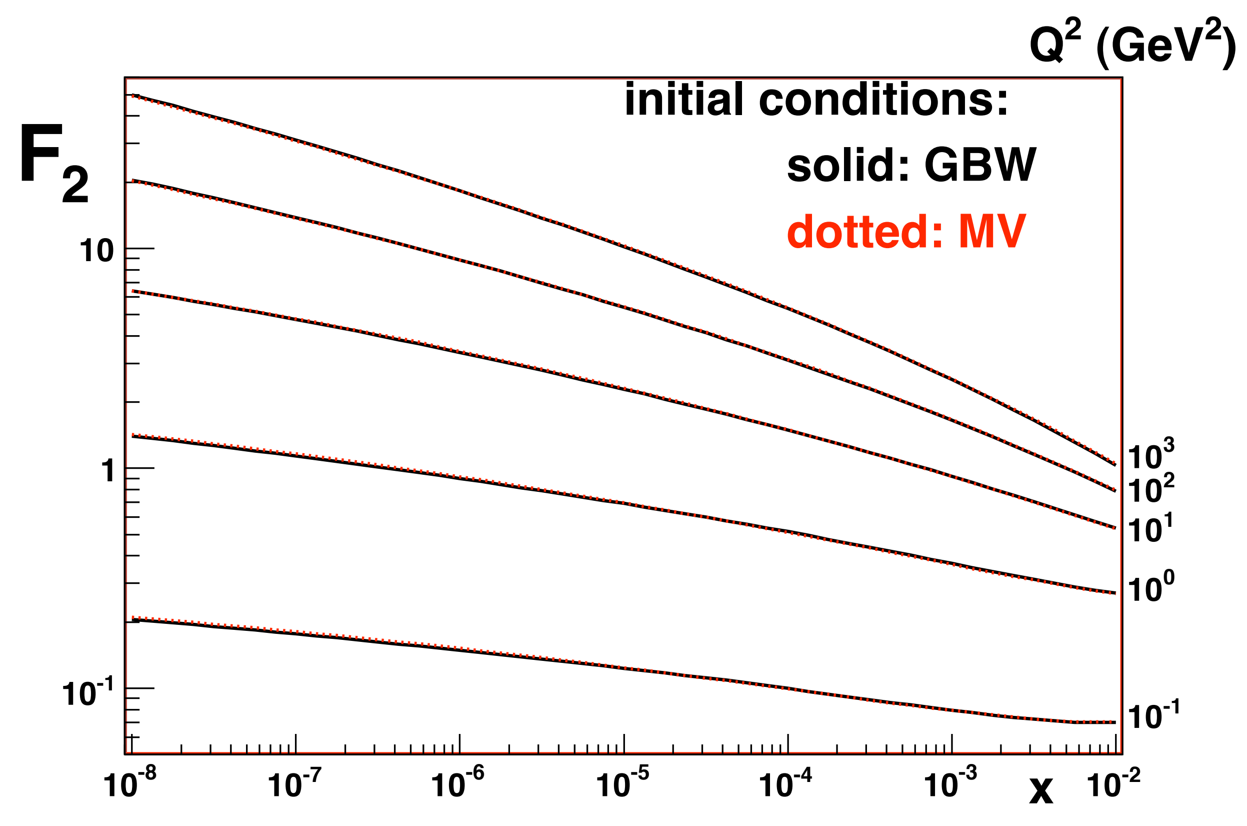
⟳ on why B is B'

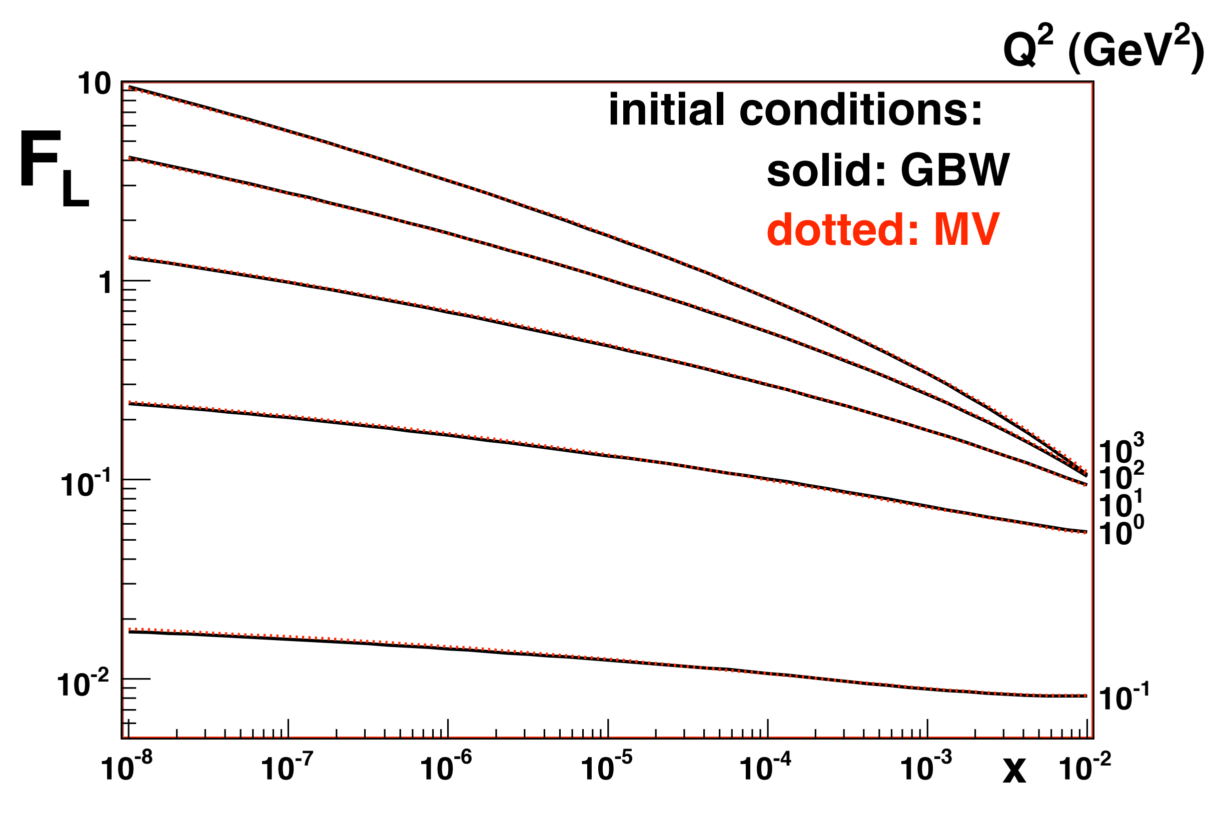


[Albacete, Kovchegov]

$$\frac{\partial \mathcal{N}(r,Y)}{\partial Y} = \mathcal{R}[\mathcal{N}] - \mathcal{S}[\mathcal{N}]$$

predictions

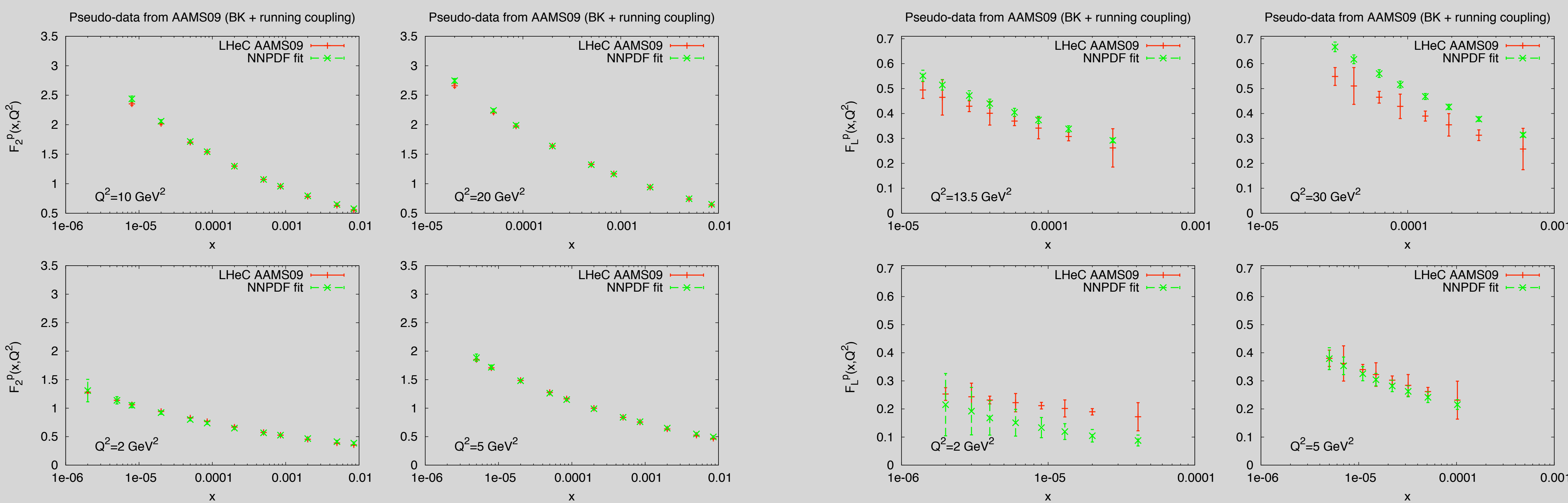




- F_2 and F_L extrapolated to LHeC and UHECR kinematical conditions
  - near independence on [tested] initial conditions
  - first principle approach allows for credible extrapolation
    - ↪ 'all' relevant physics included

⟳ vs. DGLAP

- AAMS 1.0 F_2 and F_L cannot be fitted by NLO-DGLAP [Rojo, LHeC working group]

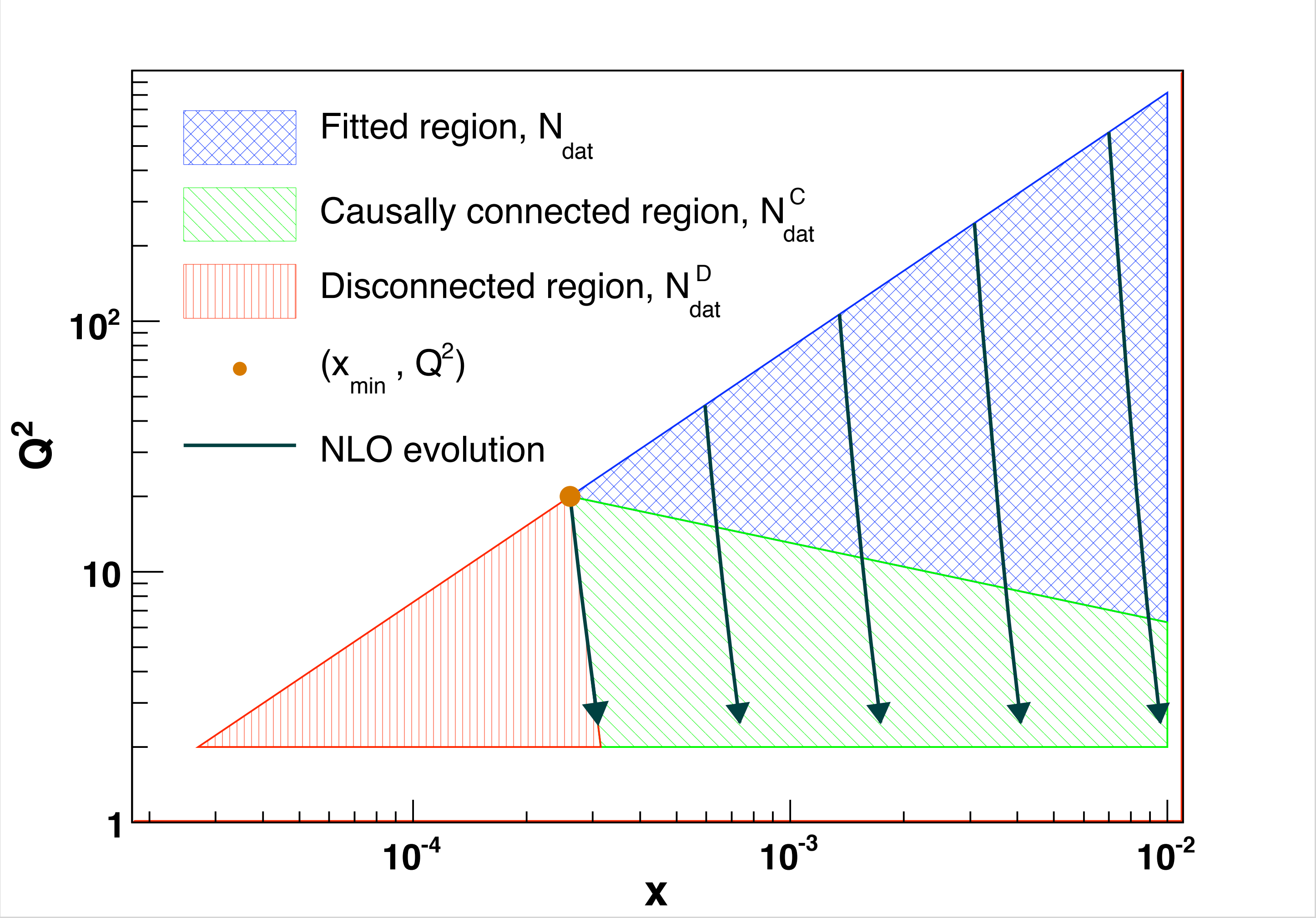


- i.e., pseudo-data (for LHeC) generated from AAMS is inconsistent with NLO-DGLAP
- differences cannot be absorbed into initial condition [in which there are ~200 parameters]

DGLAP i.c. independent statements (i)

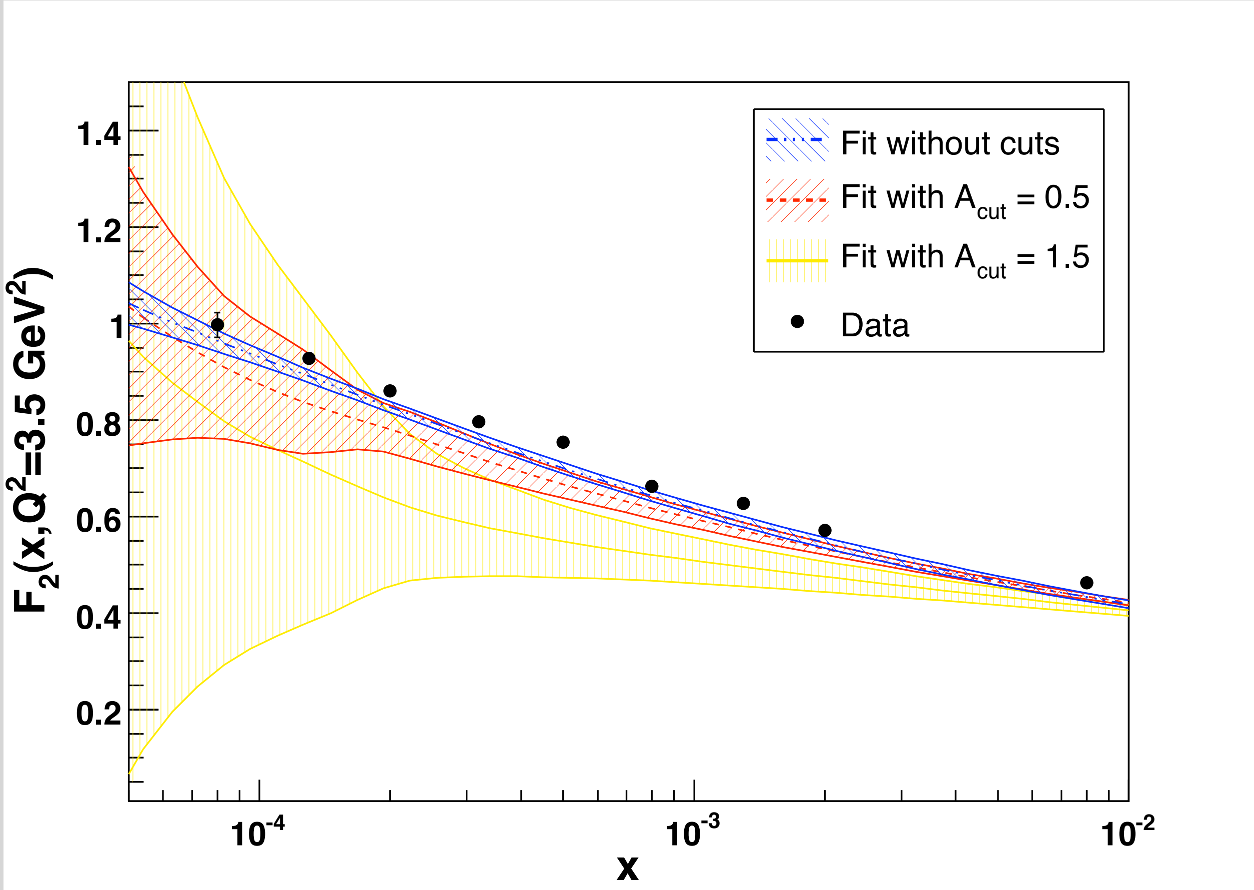
Caola, Forte, Rojo

- explore entire functional space for i.c. [NNPDF]
- perform NLO fit to subset of data
- extrapolate only to causally connected region where data exists and compare



DGLAP i.c. independent statements (ii)

Caola, Forte, Rojo



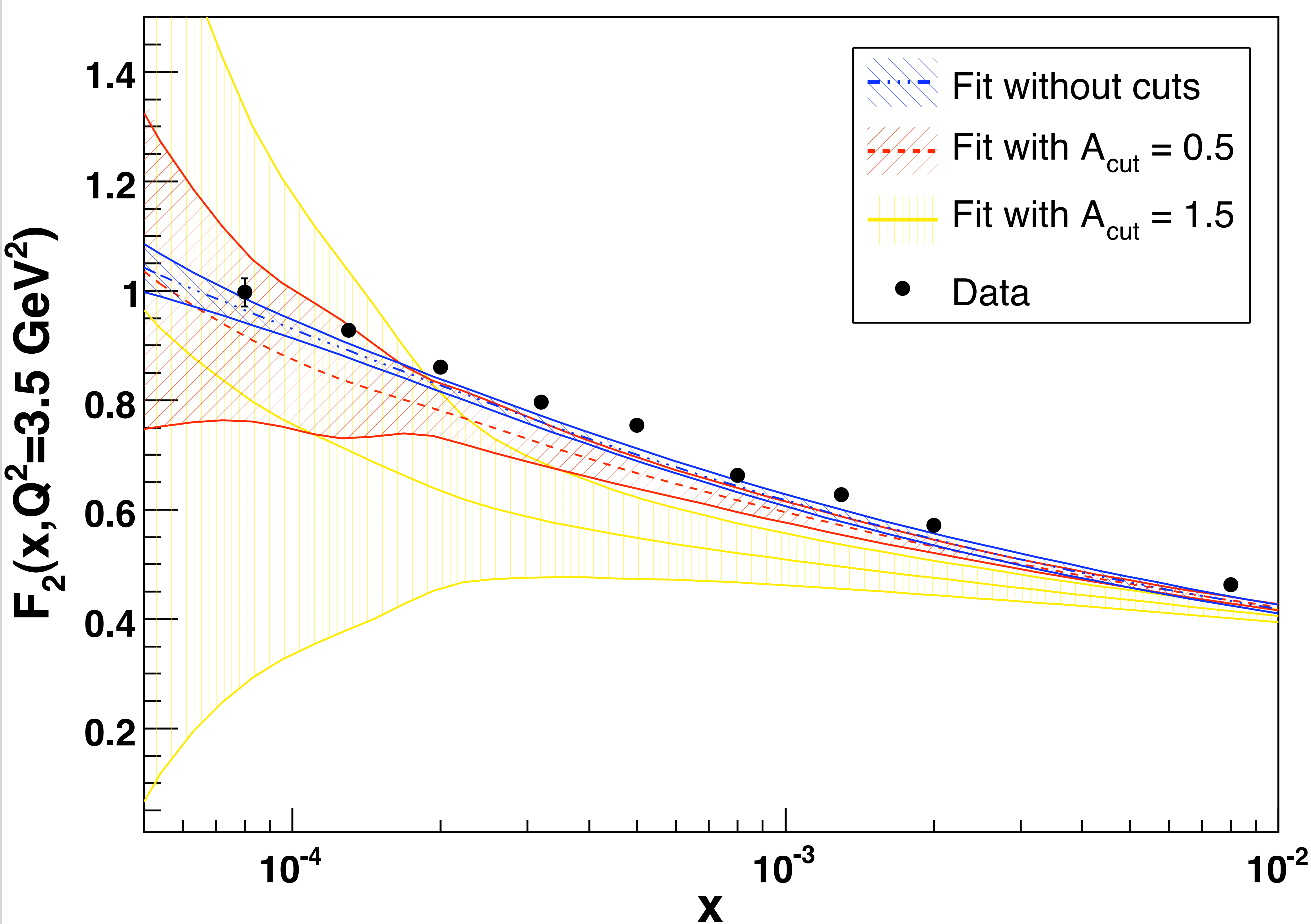
$$Q^2 \geq A_{\rm cut} \cdot x^{\lambda}$$

$$d_{\rm stat}(x, Q^2) \equiv \frac{F_{\rm data} - F_{\rm fit}}{\sqrt{\sigma^2_{\rm data} + \sigma^2_{\rm fit}}},$$

- small, but systematic, deviations from data found for F_2 [of all places...] HERA data
  - not NNLO [goes in the wrong direction]
  - not a mass effect [too small] :: should be settled by NNPDF 2.1
  - solvable by BFKL resummation in DGLAP kernel [interim fix...]

- predictive power rapidly degrades with 1/x

DGLAP dev



Fit without cuts
Fit with A_{cut} = 0.5
Fit with A_{cut} = 1.5
Data
$F_2(x,Q^2=3.5\ GeV^2)$
1.4
1.2
1
0.8
0.6
0.4
0.2
10^{-4}
10^{-3}
10^{-2}
x

nuclear AAMQs

- nuclei
  - direct fit of nuclear DIS data with proton parameters [2 nuclear params]

$$Q_{s,A}^2 = c\,A^\delta\,Q_{s,p}^2$$

  - data not sufficient for independent fit

- impact parameter dependence essential for access to larger range of observables
  - requires non-perturbative input [therefore modelling...]

- strong reliance on 'k_t-factorized' formulae [but see Horowitz & Kovchegov]
  - include further [non-DIS] observables in fit
    - ↪ risk of exhausting prediction ground by fitting it ...

AAMQs collaboration

installation [Spring 2009]

- 4 institutes
- 5 scientists
- ~200 cpu cores
- installation and commissioning in 2009
- online from early 2010

http://www-fp.usc.es/phenom/aamqs/aamqs.html