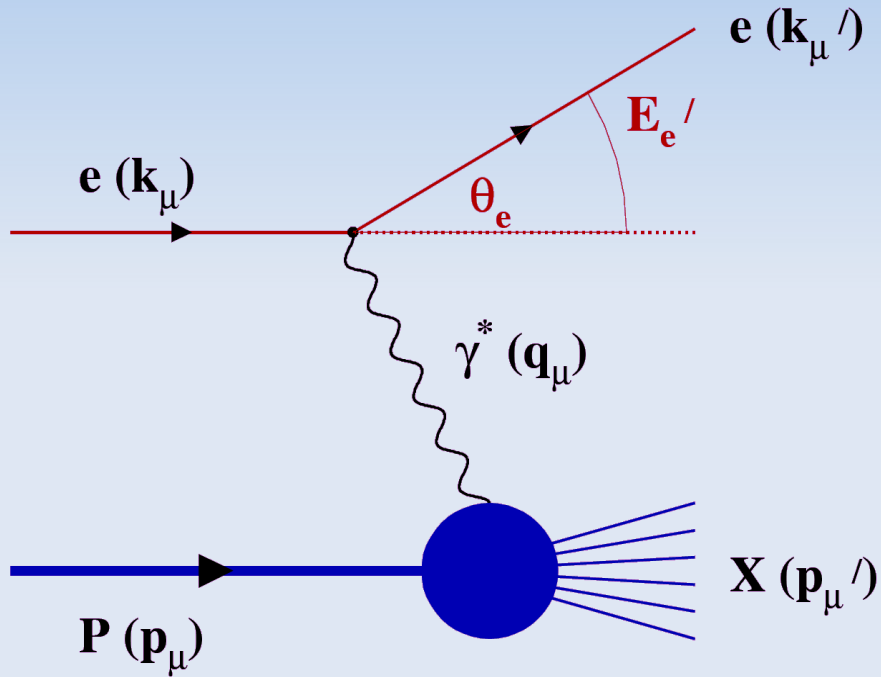


2-particle correlations in pp and AA collisions

Jamal Jalilian-Marian
Baruch College, New York NY

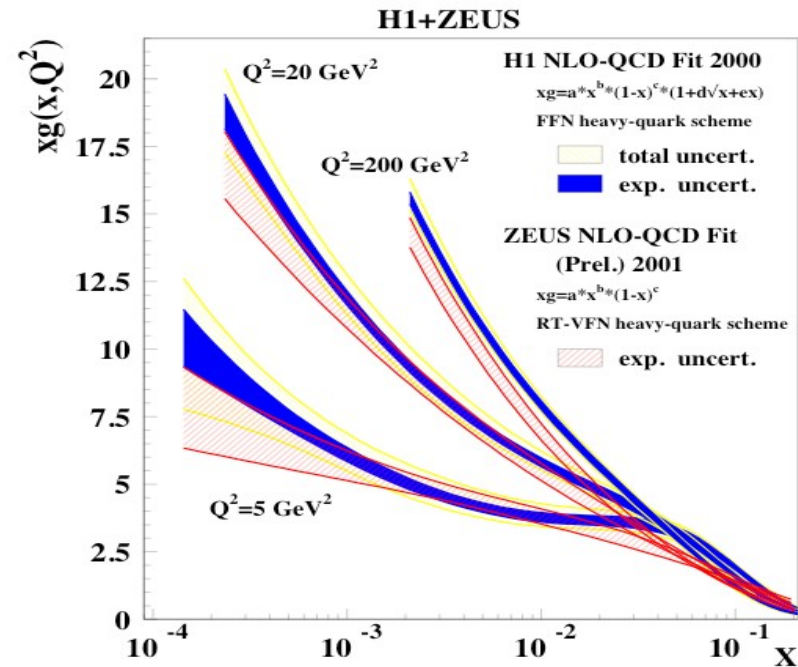
A proton at small x

DIS: $e p \rightarrow e X$



$$x = \frac{p^+}{P^+}$$

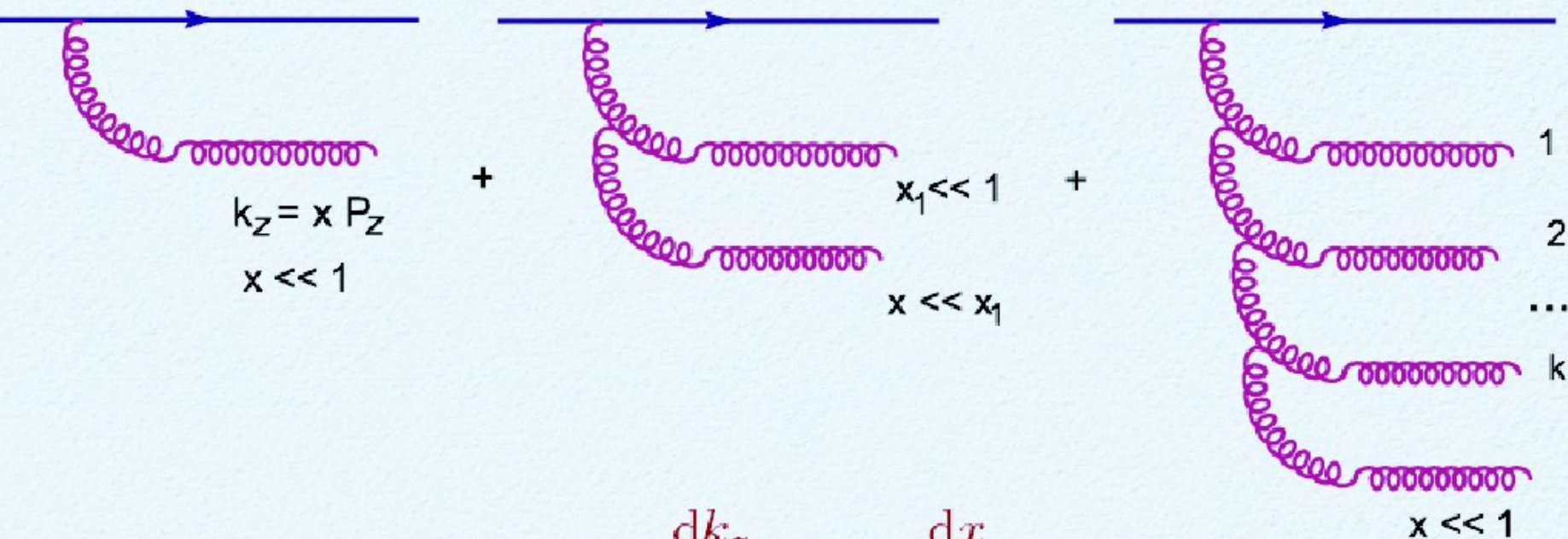
is the fraction of hadron energy carried by a parton



there are a lot of gluons at small x

gluon radiation at small x : pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small- x) gluons

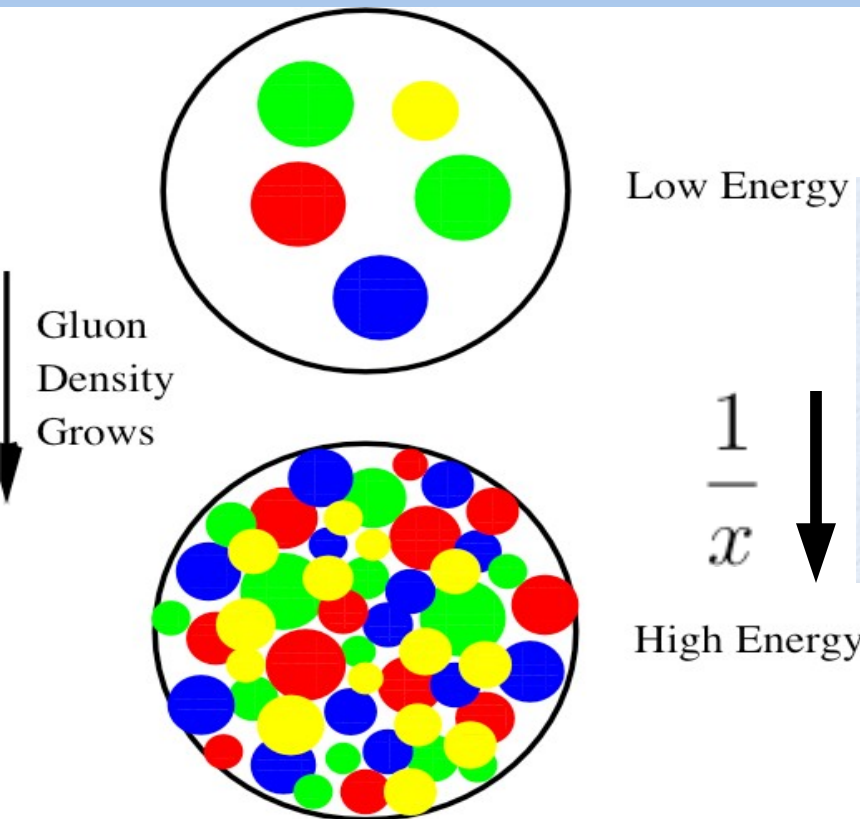


$$d\mathcal{P} \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x}$$

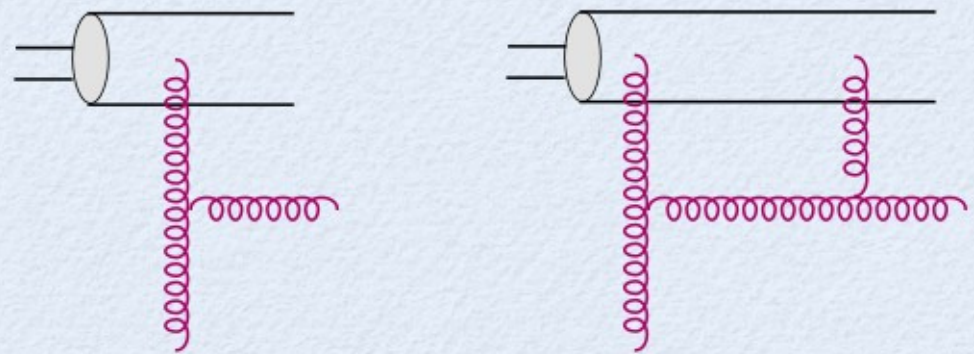
The 'price' of an additional gluon:

$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x} \quad \text{number of gluons grows fast} \quad n \sim e^{\alpha_s \ln 1/x}$$

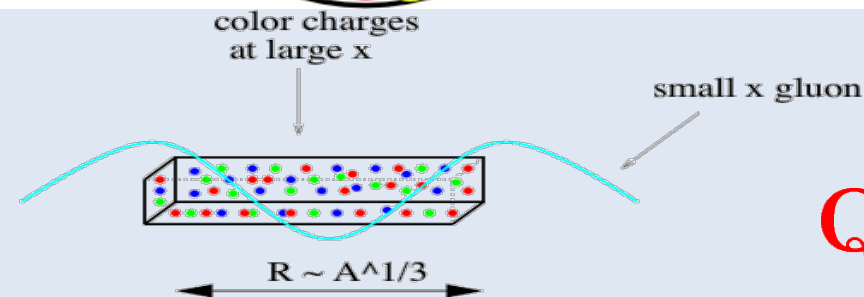
CGC and gluon saturation



“attractive” bremsstrahlung vs. “repulsive” recombination



$$\frac{\alpha_s x G(x, b_t, Q^2)}{S_{\perp} Q^2} \sim 1$$



$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

Effective Action + RGE

$$S[\mathbf{A}, \rho] = -\frac{1}{4} \int d^4x F_{\mu\nu}^2 + \frac{i}{N_c} \int d^2x_t dx^- \delta(x^-) \text{Tr}[\rho(x_t) \mathbf{U}(\mathbf{A}^-)]$$

Large x : color source ρ small x : gluon field \mathbf{A}^μ

$$\mathbf{U}(\mathbf{A}^-) = \hat{\mathbf{P}} \text{Exp} \left[ig \int dx^+ \mathbf{A}_a^- \mathbf{T}_a \right]$$

$$\mathbf{Z}[\mathbf{j}] = \int [\mathbf{D}\rho] \mathbf{W}_{\Lambda^+}[\rho] \left[\frac{\int^{\Lambda^+} [\mathbf{D}\mathbf{A}] \delta(\mathbf{A}^+) e^{iS[\mathbf{A}, \rho] - \int \mathbf{j} \cdot \mathbf{A}}}{\int^{\Lambda^+} [\mathbf{D}\mathbf{A}] \delta(\mathbf{A}^+) e^{iS[\mathbf{A}, \rho]}} \right]$$

weight functional:

$\mathbf{W}_{\Lambda^+}[\rho]$ probability distribution of color source ρ
at longitudinal scale Λ^+

invariance under change of $\Lambda^+ \longrightarrow$ RGE for $\mathbf{W}_{\Lambda^+}[\rho]$

QCD at High Energy: Wilsonian RG

resum $\alpha_s \log \frac{1}{x}$

Fields A^μ

Sources ρ

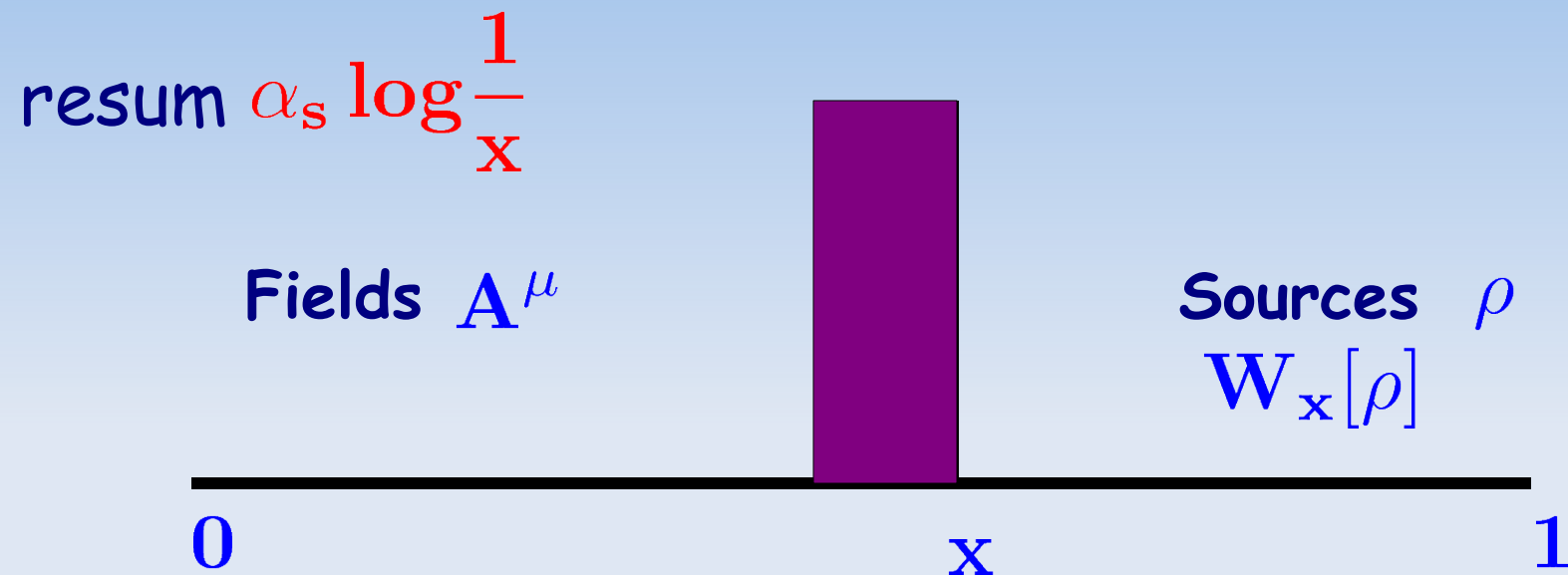
$W_x[\rho]$

0

x

1

QCD at High Energy: Wilsonian RG

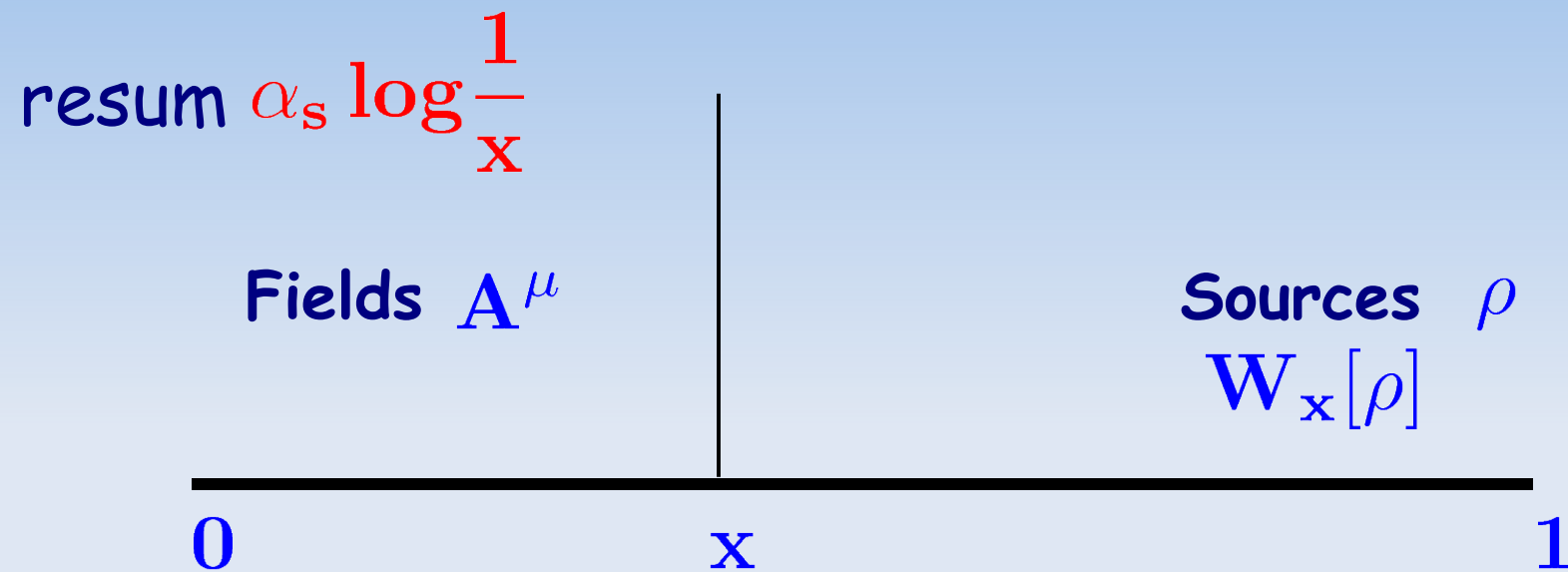


$$A^\mu = A_{\text{class}}^\mu + \delta A^\mu$$

integrate out field fluctuations quadratically

$$\rho \rightarrow \rho' = \rho + \delta \rho$$

QCD at High Energy: Wilsonian RG



$$\frac{\partial W[\rho]}{\partial \ln 1/x} = \frac{1}{2} \int_{\mathbf{x}_t, \mathbf{y}_t} \frac{\delta}{\delta \rho^a(\mathbf{x}_t)} \chi^{ab}(\mathbf{x}_t, \mathbf{y}_t)[\rho] \frac{\delta}{\delta \rho^a(\mathbf{y}_t)} W[\rho]$$

JIMWLK eq. describes x evolution of observables

CGC: QCD at high gluon density

multiple scatterings \longrightarrow p_t broadening

“Cronin” effect

evolution with $\ln(1/x)$ \longrightarrow suppression

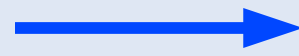
“Leading twist” nuclear shadowing

effective degrees of freedom:

Wilson line $V(x_t)$ re-sums multiple scatterings

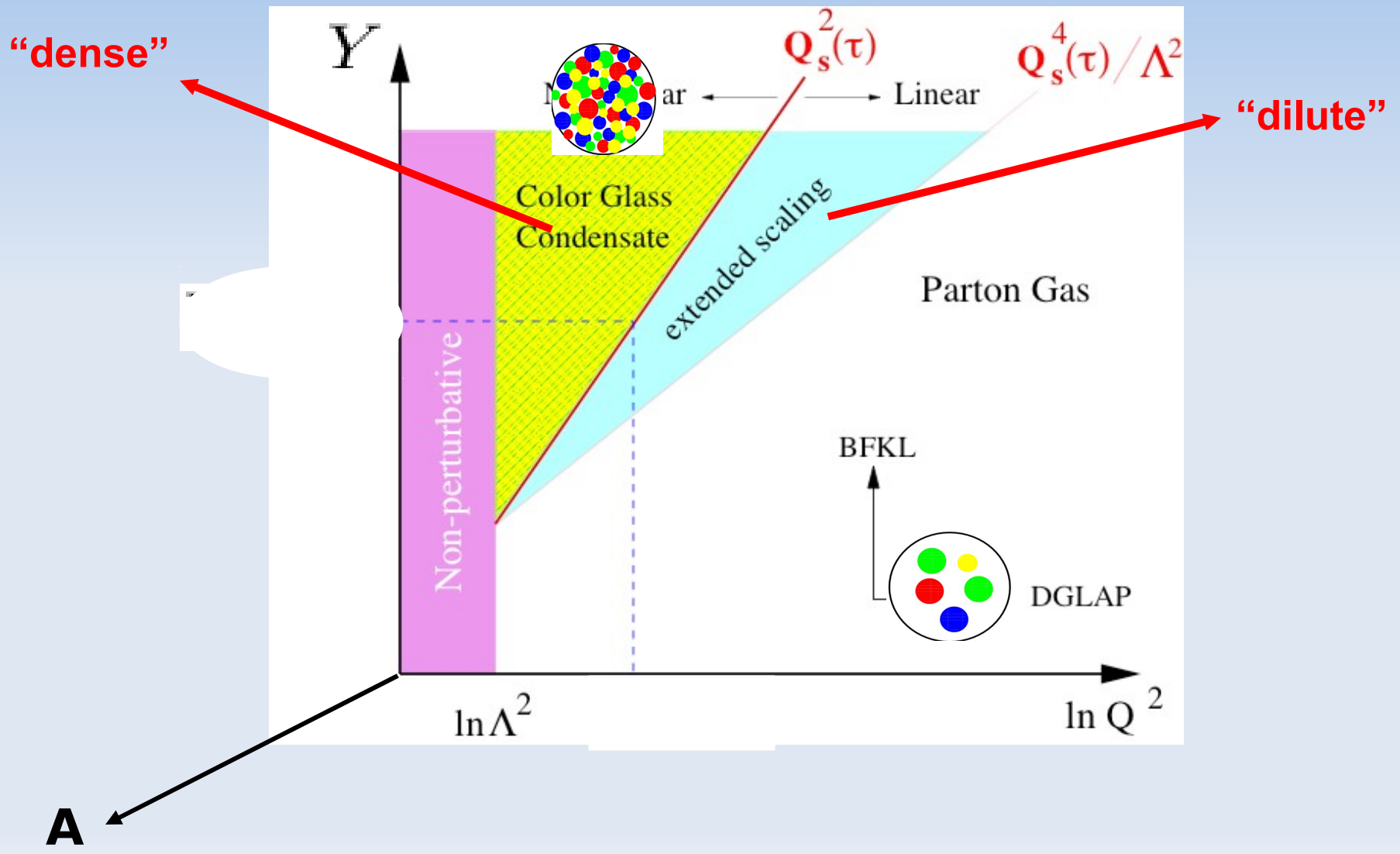
CGC observables are expressed in terms of

$$\langle \text{Tr} V \dots V^\dagger \rangle$$



*satisfy the
JIMWLK equation:
Re-sums $\ln 1/x$*

Road Map of QCD Phase Space



The Classical Field

saddle point of effective action \rightarrow Yang-Mills equations

$$\mathbf{D}_\mu \mathbf{F}_a^{\mu\nu} = \delta^\nu + \delta(\mathbf{x}^-) \rho_a(\mathbf{x}_t)$$

solutions are non-Abelian
Weizsäcker-Williams fields

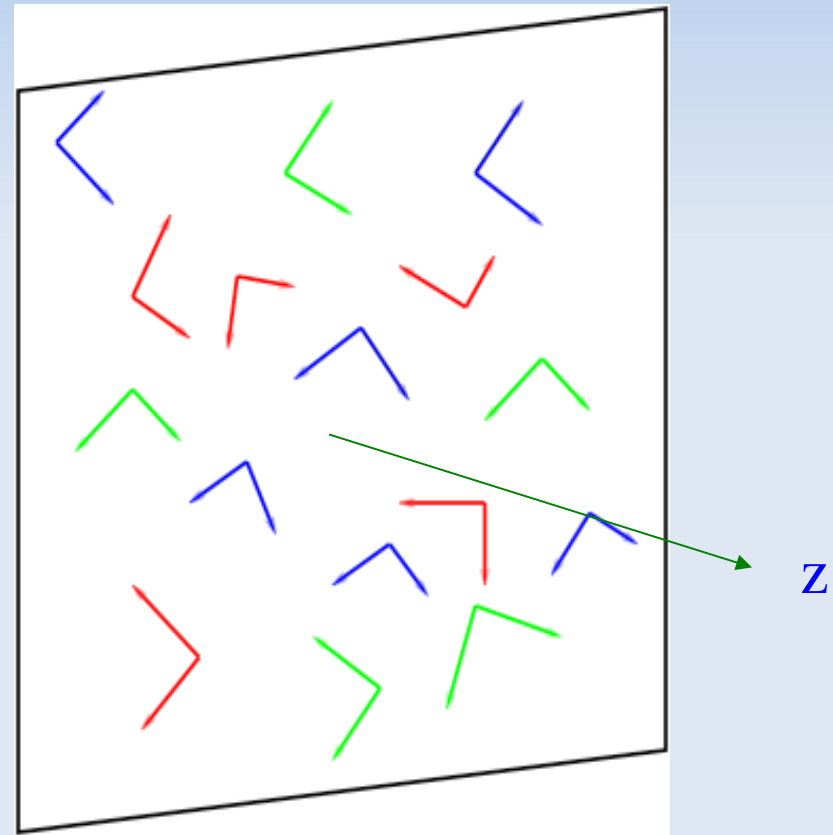
$$\mathbf{A}^+ = \mathbf{0}$$

$$\mathbf{A}^- = \mathbf{0}$$

$$\mathbf{A}_a^i = \theta(\mathbf{x}^-) \alpha_a^i(\mathbf{x}_t)$$

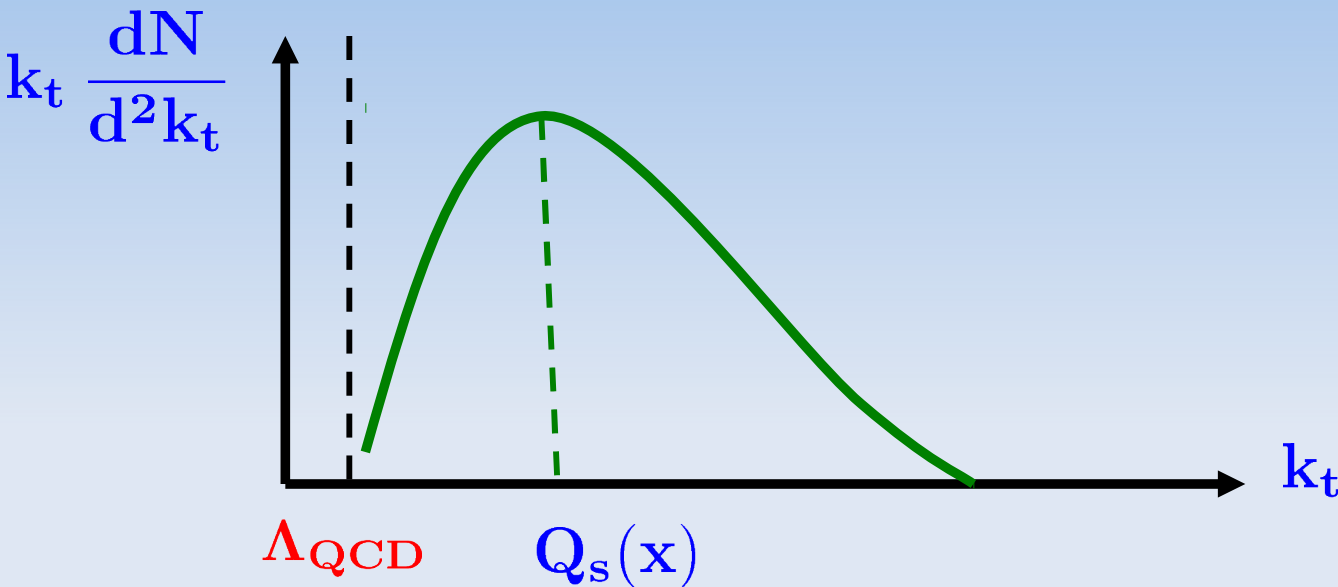
$$\partial^i \alpha_a^i = g \rho_a$$

\swarrow
pure (2d) gauge



color $\mathbf{E}_\perp, \mathbf{B}_\perp$ fields

Intrinsic (un-integrated) gluon distribution at small x



a state with large (gluon) occupation number $O\left[\frac{1}{\alpha_s}\right]$
very different time scales between large and small x modes

$Q_s(x, b_t, A)$ can provide a hard infrared cutoff

Observables

DIS:

*structure functions (inclusive and diffractive)
single and double particle production*

PA (dilute-dense):

*multiplicities
single and double inclusive spectra*

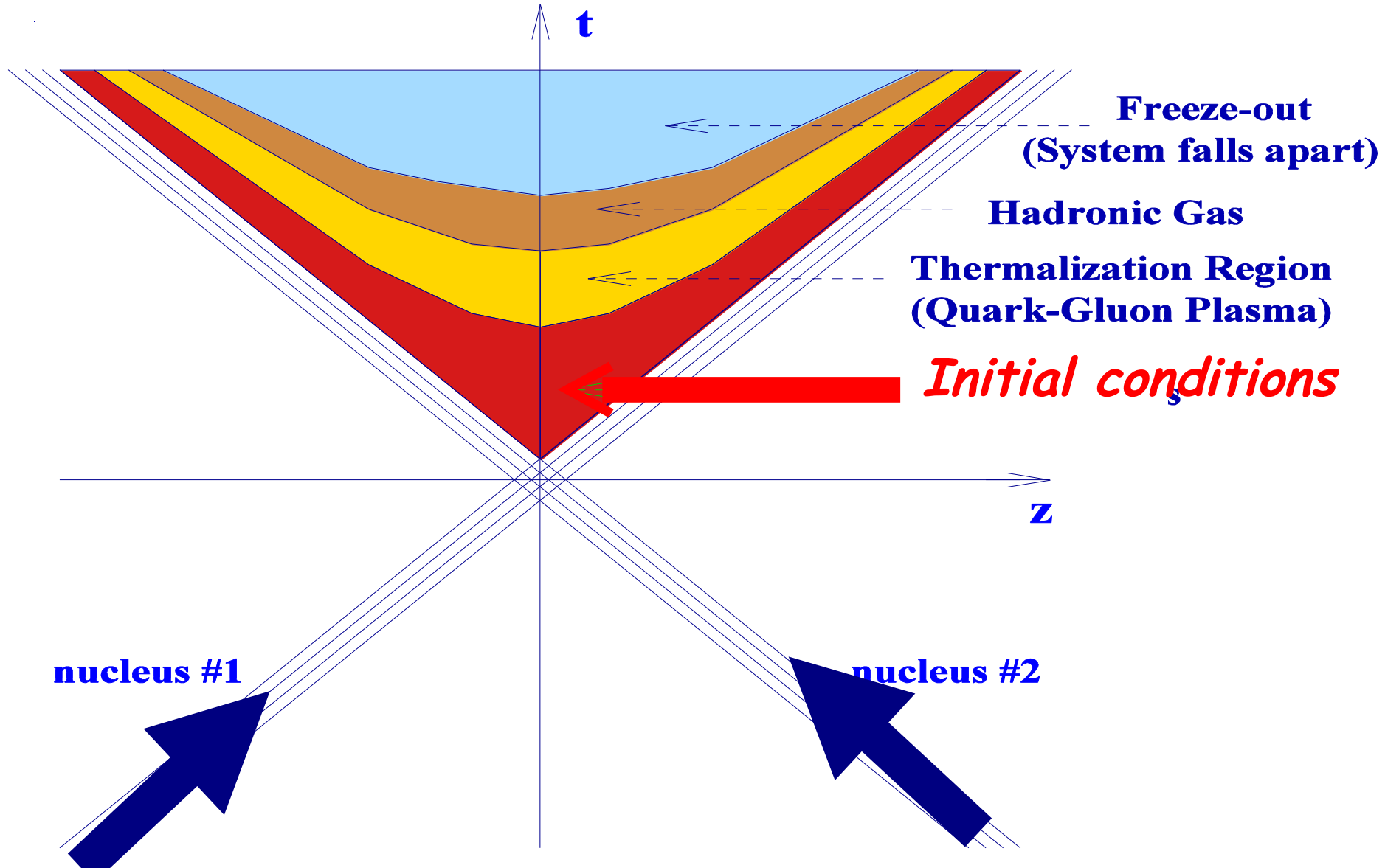
talk by
C. Marquet

AA, pp (dense-dense):

*multiplicities, spectra
near-side long range rapidity correlation*

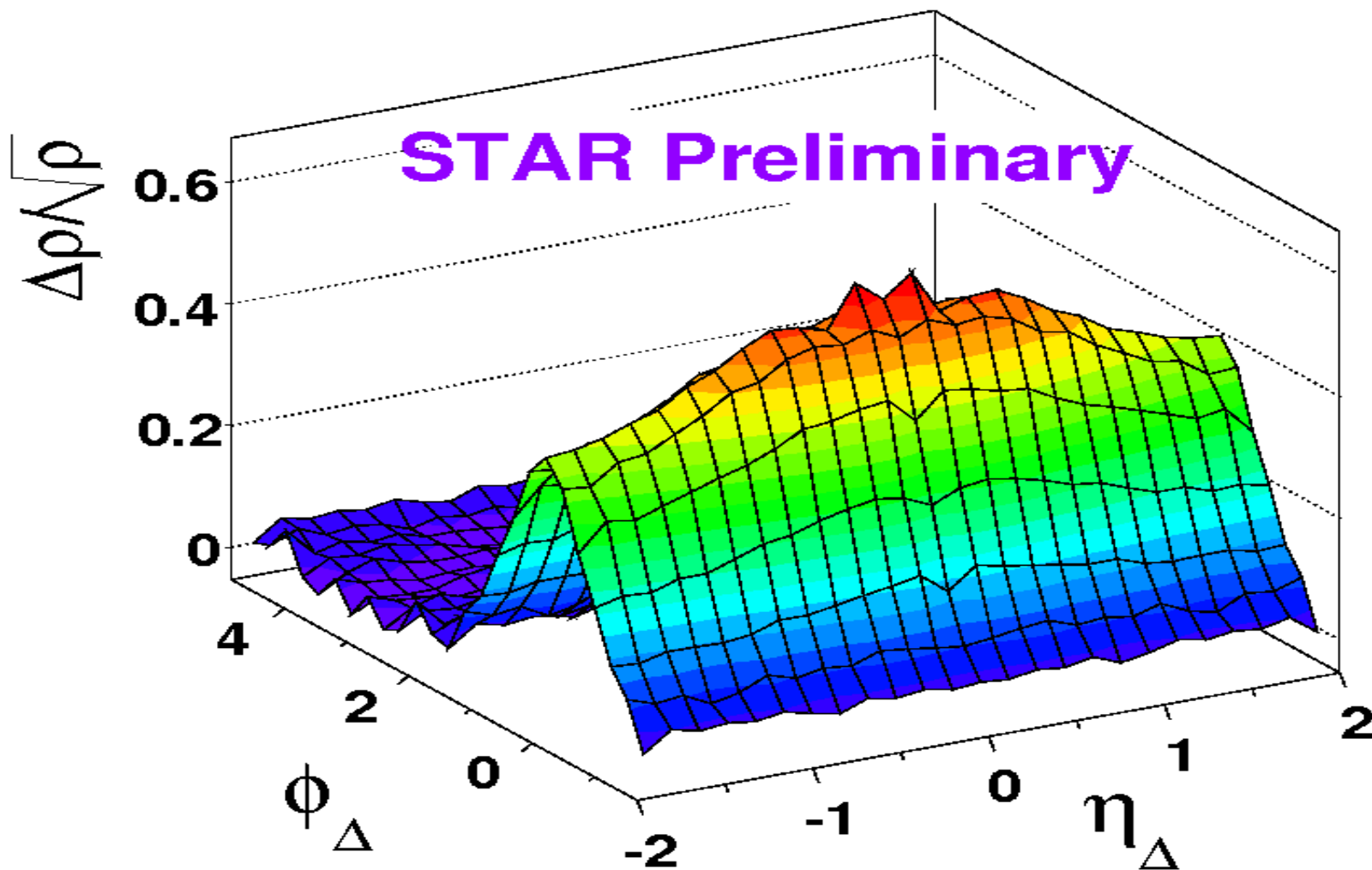
RIDGE

Space-Time History of a Heavy Ion Collision

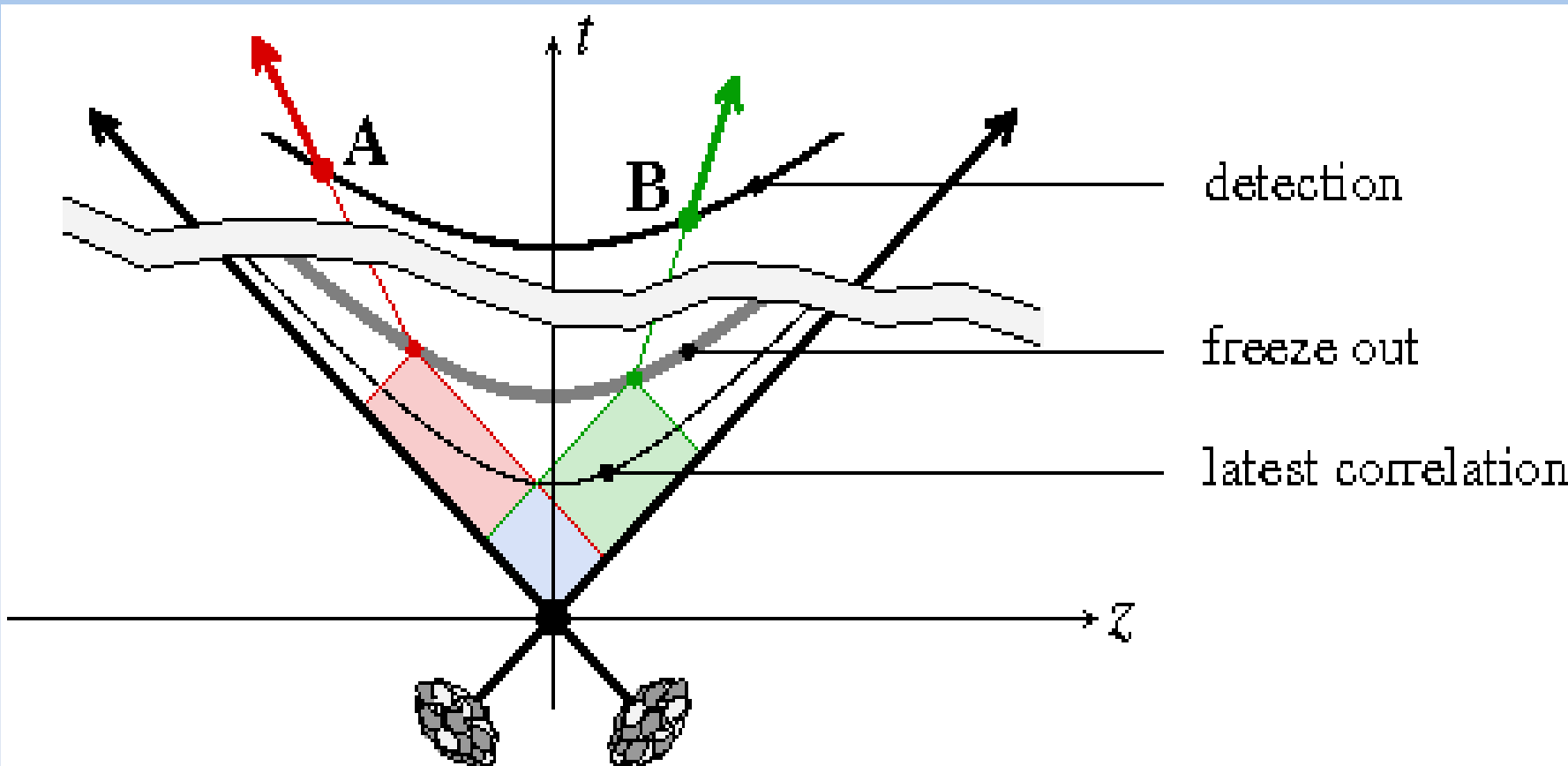


Ridge in AA

(near-side long-range rapidity correlations)



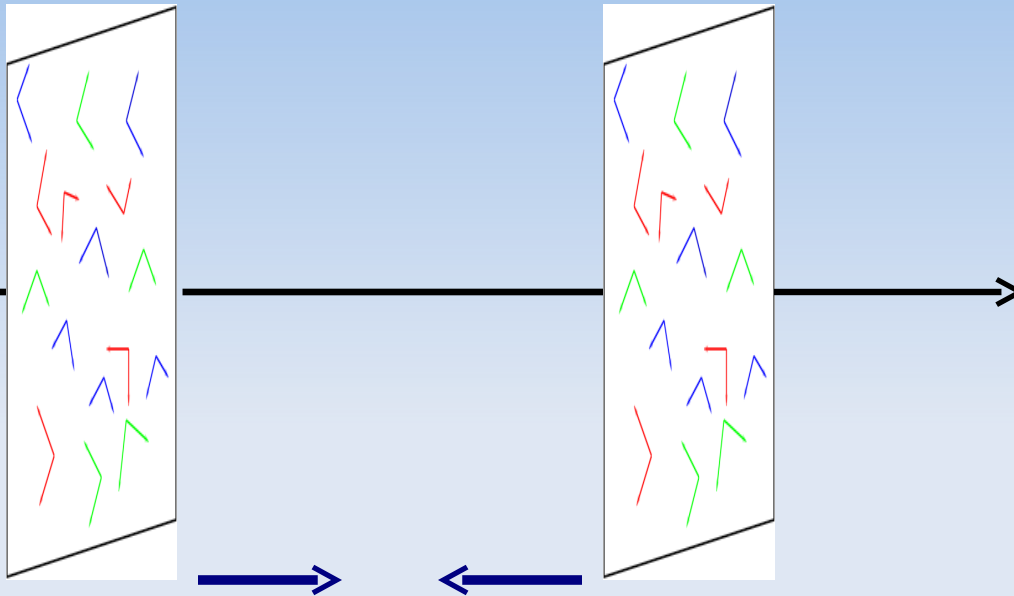
long-range rapidity correlations



$$\tau \leq \tau_{\text{fo}} e^{-\frac{1}{2} |y_A - y_B|}$$

DGMV: NPA810 (2008) 91, DGLV: NPA836 (2010) 159

Colliding Sheets of Color Glass



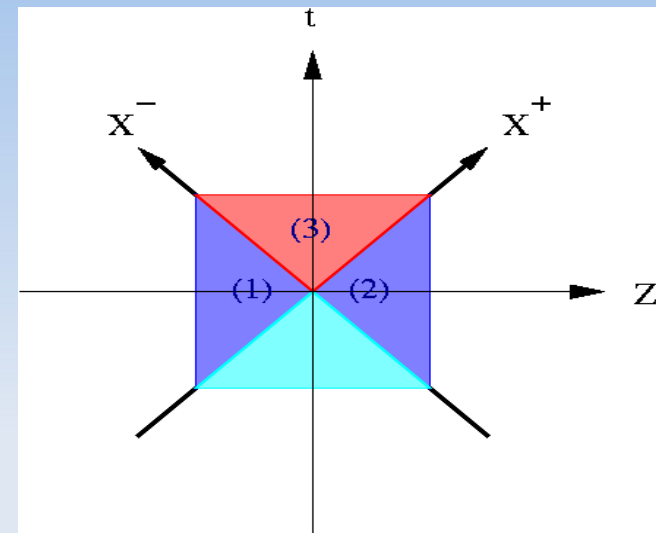
before the collision:

$$\mathbf{A}^+ = \mathbf{A}^- = \mathbf{0}$$

$$\mathbf{A}^i = \mathbf{A}_1^i + \mathbf{A}_2^i$$

$$\mathbf{A}_1^i = \theta(x^-)\theta(-x^+)\alpha_1^i$$

$$\mathbf{A}_2^i = \theta(-x^-)\theta(x^+)\alpha_2^i$$



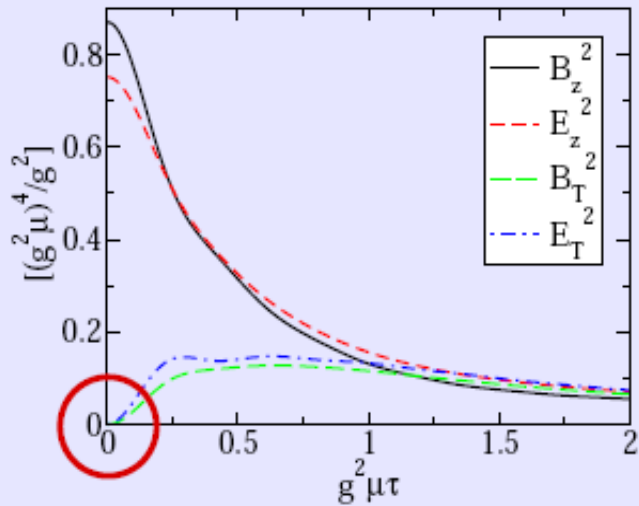
after the collision:

solve for A_μ

in the forward LC

GLASMA:

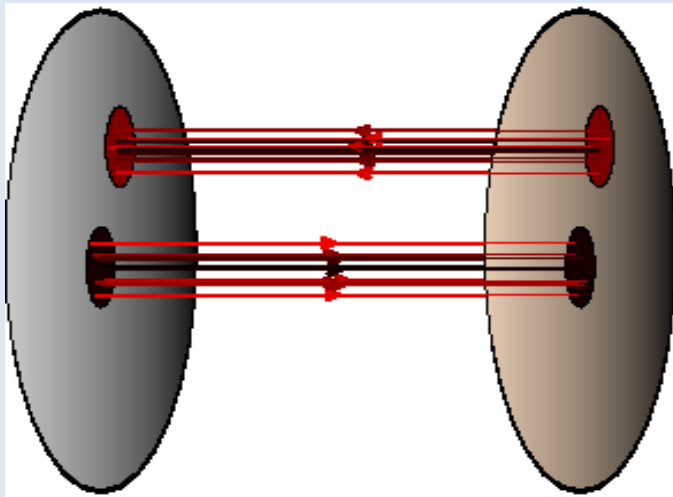
gluon fields produced in collision of two sheets of color glass



Early on glasma fields (E and B) are longitudinal

Classical solutions are boost invariant

Transverse size of these flux tubes is $\sim \frac{1}{Q_s}$



Two-gluon correlation

$$C(p_{\perp}, q_{\perp}) = \frac{g^4}{64(2\pi)^6} (f_{abc} f_{a'\bar{b}\bar{c}} f_{a\hat{b}\hat{c}} f_{a'\tilde{b}\tilde{c}}) \int \prod_{i=1}^4 \frac{d^2 k_{i\perp}}{(2\pi)^2}$$

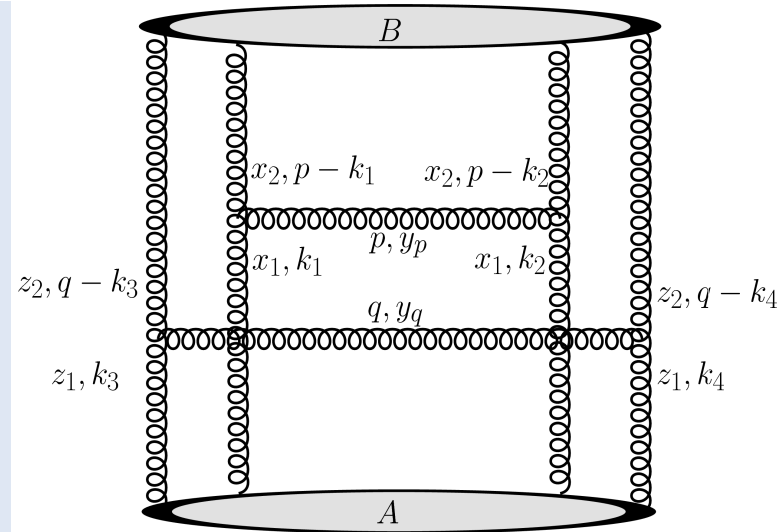
$$L_{\mu}(p_{\perp}, k_{1\perp}) L^{\mu}(p_{\perp}, k_{2\perp}) L_{\nu}(q_{\perp}, k_{3\perp}) L^{\nu}(q_{\perp}, k_{4\perp})$$

$$\left\langle \alpha_1^{*\hat{b}}(k_{2\perp}) \alpha_1^{*\tilde{b}}(k_{4\perp}) \alpha_1^b(k_{1\perp}) \alpha_1^{\bar{b}}(k_{3\perp}) \right\rangle$$

$$\left\langle \alpha_2^{*\hat{c}}(p_{\perp} - k_{2\perp}) \alpha_2^{*\tilde{c}}(q_{\perp} - k_{4\perp}) \alpha_2^c(p_{\perp} - k_{1\perp}) \alpha_2^{\bar{c}}(q_{\perp} - k_{3\perp}) \right\rangle$$

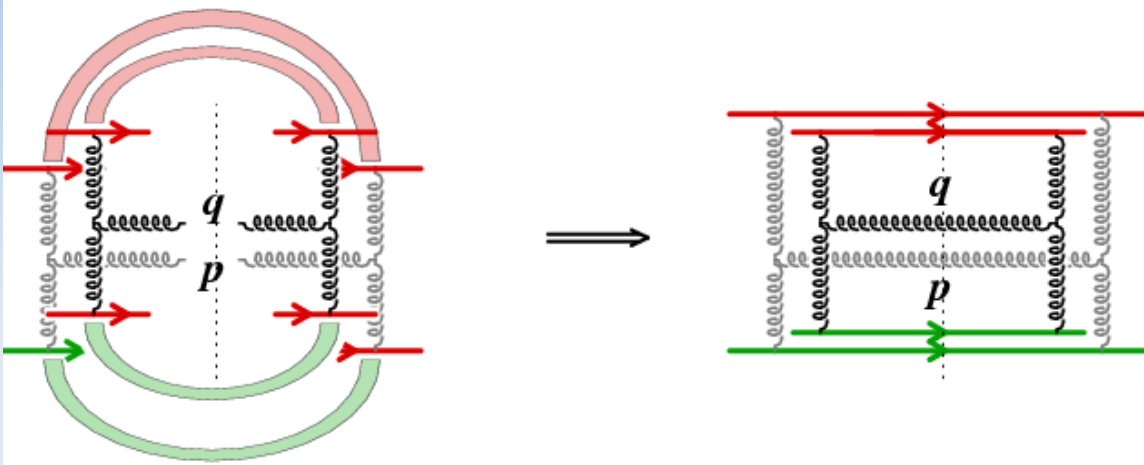
**dilute region: 4-pt function
of gluon fields**

Gaussian averaging: $e^{-\frac{\rho^2}{\mu^2}}$
 uncorrelated: 1 (single inclusive)²
 correlated: 8, suppressed by $\frac{1}{N_c^2 - 1}$



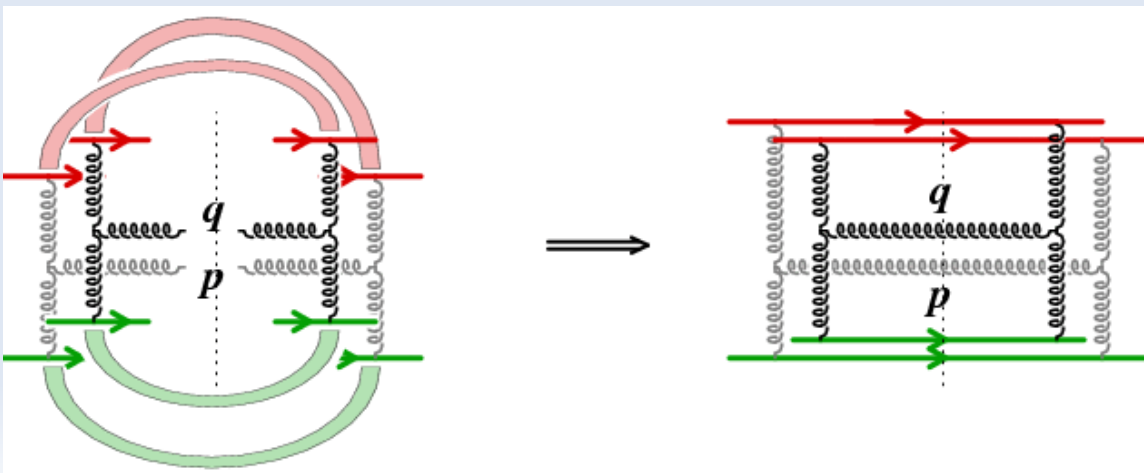
Two-gluon production in AA (pp)

Independent production of two gluons:



PYTHIA:
“independent
multi-parton
interactions”

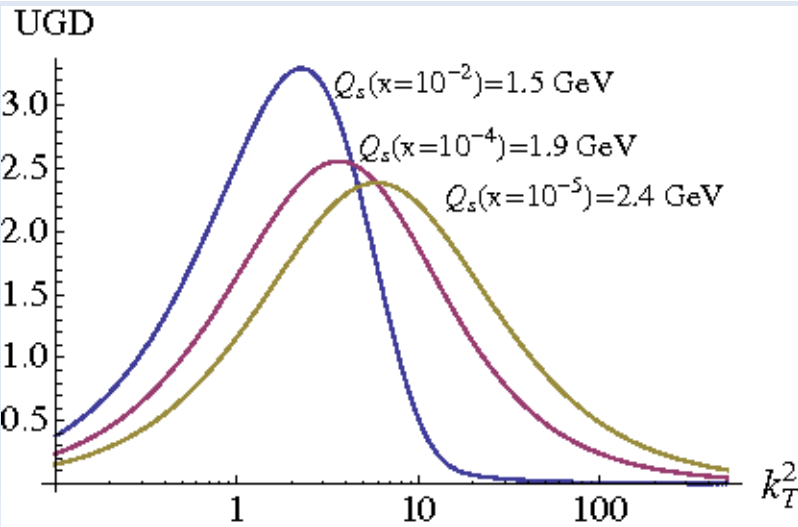
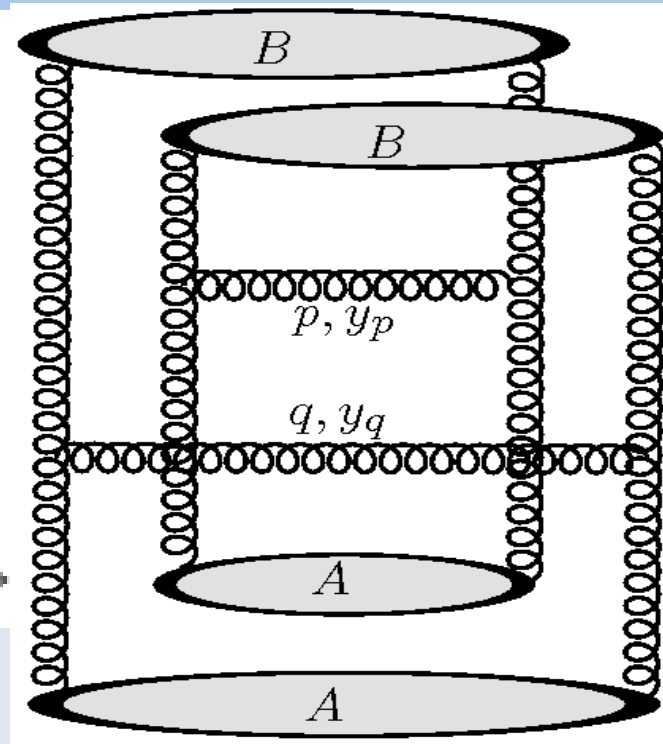
Correlated two-gluon production:



DGMV:
NPA810 (2008) 91

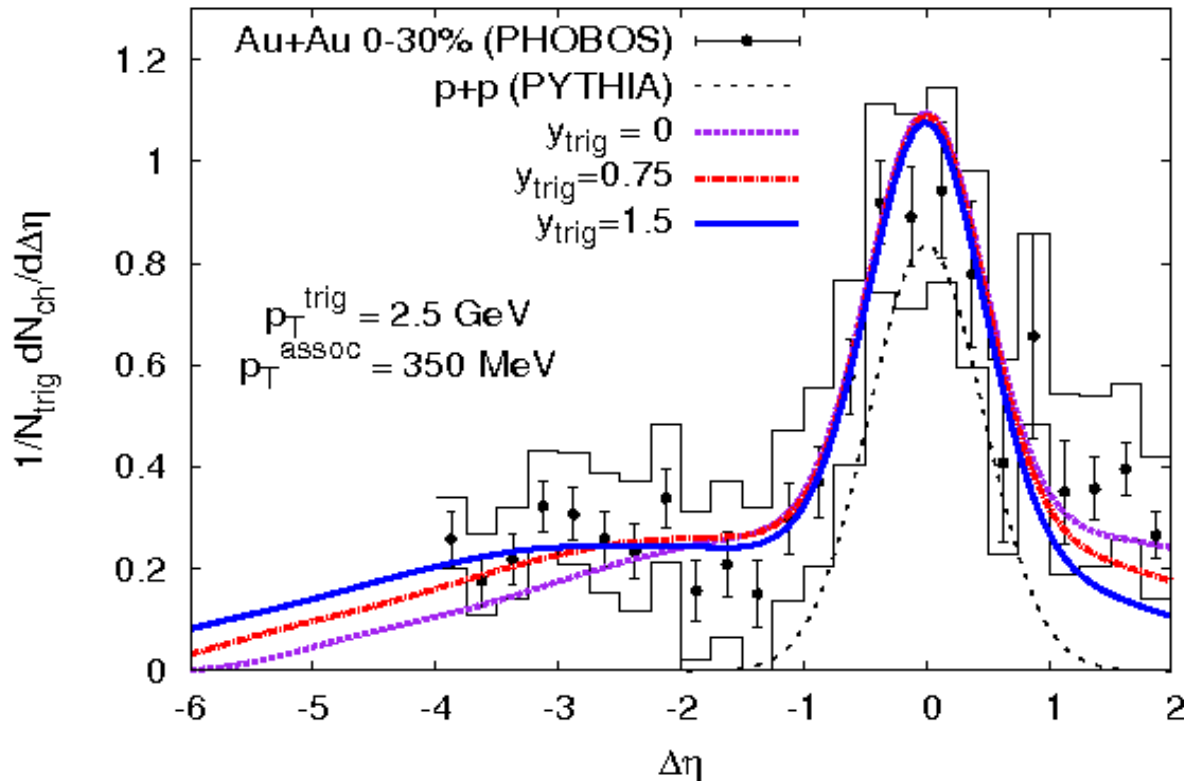
Two-gluon production in AA (pp)

$$\frac{dN_2}{d^2p_\perp dy_p d^2q_\perp dy_q} = \frac{\alpha_s^2}{16\pi^{10}} \frac{N_c^2 S_\perp}{(N_c^2 - 1)^3 p_\perp^2 q_\perp^2} \times \int d^2k_\perp \left\{ \Phi_A^2(y_p, k_\perp) \Phi_B(y_p, p_\perp - k_\perp) \right. \\ \times [\Phi_B(y_q, q_\perp + k_\perp) + \Phi_B(y_q, q_\perp - k_\perp)] \\ + \Phi_B^2(y_q, k_\perp) \Phi_A(y_p, p_\perp - k_\perp) \\ \left. \times [\Phi_A(y_q, q_\perp + k_\perp) + \Phi_A(y_q, q_\perp - k_\perp)] \right\}$$



solutions of rcBK
angular collimation

Ridge in AA

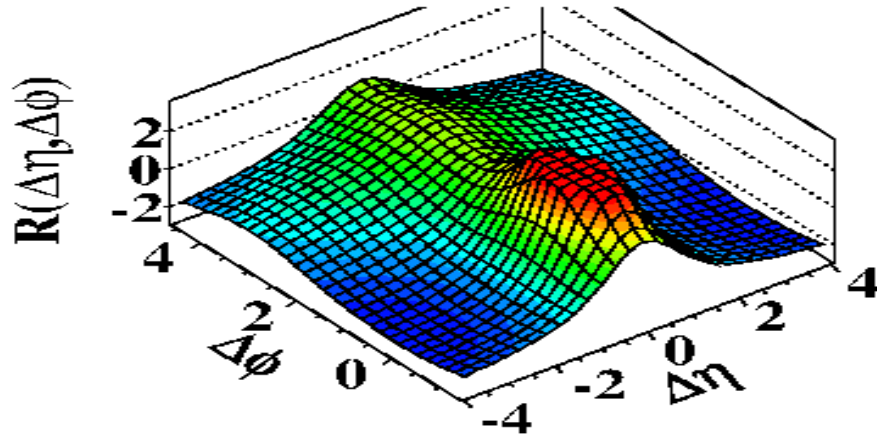


CGC glasma flux tubes
DGMV: NPA810 (2008) 91

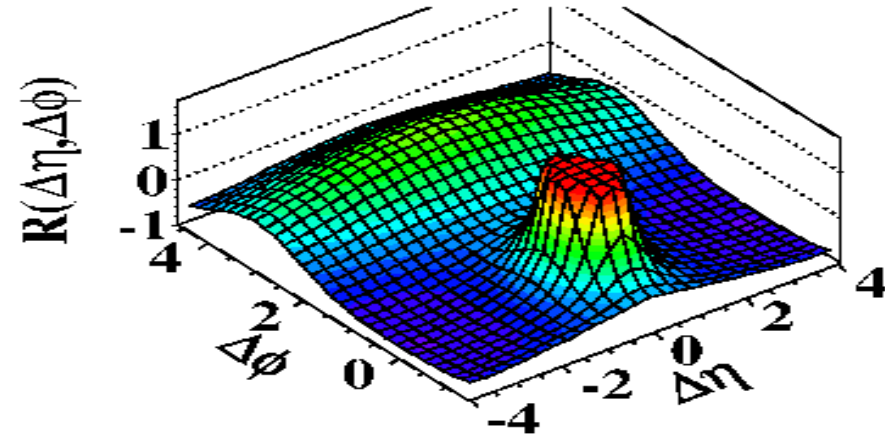
**Azimuthal angle dependence
enhanced by radial flow in QGP**

The CMS ridge at LHC

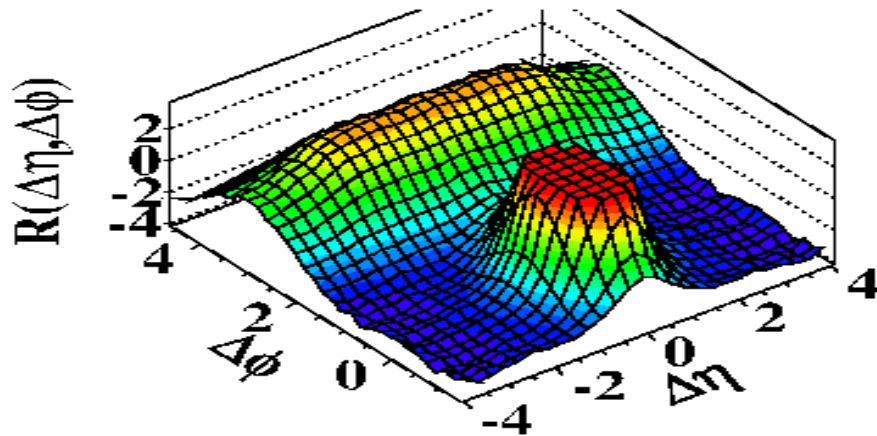
(a) CMS MinBias, $p_T > 0.1 \text{ GeV}/c$



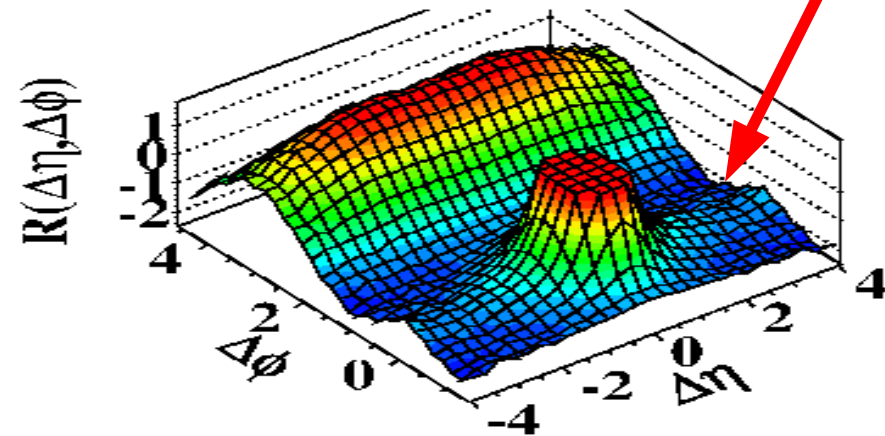
(b) CMS MinBias, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



(c) CMS $N \geq 110$, $p_T > 0.1 \text{ GeV}/c$

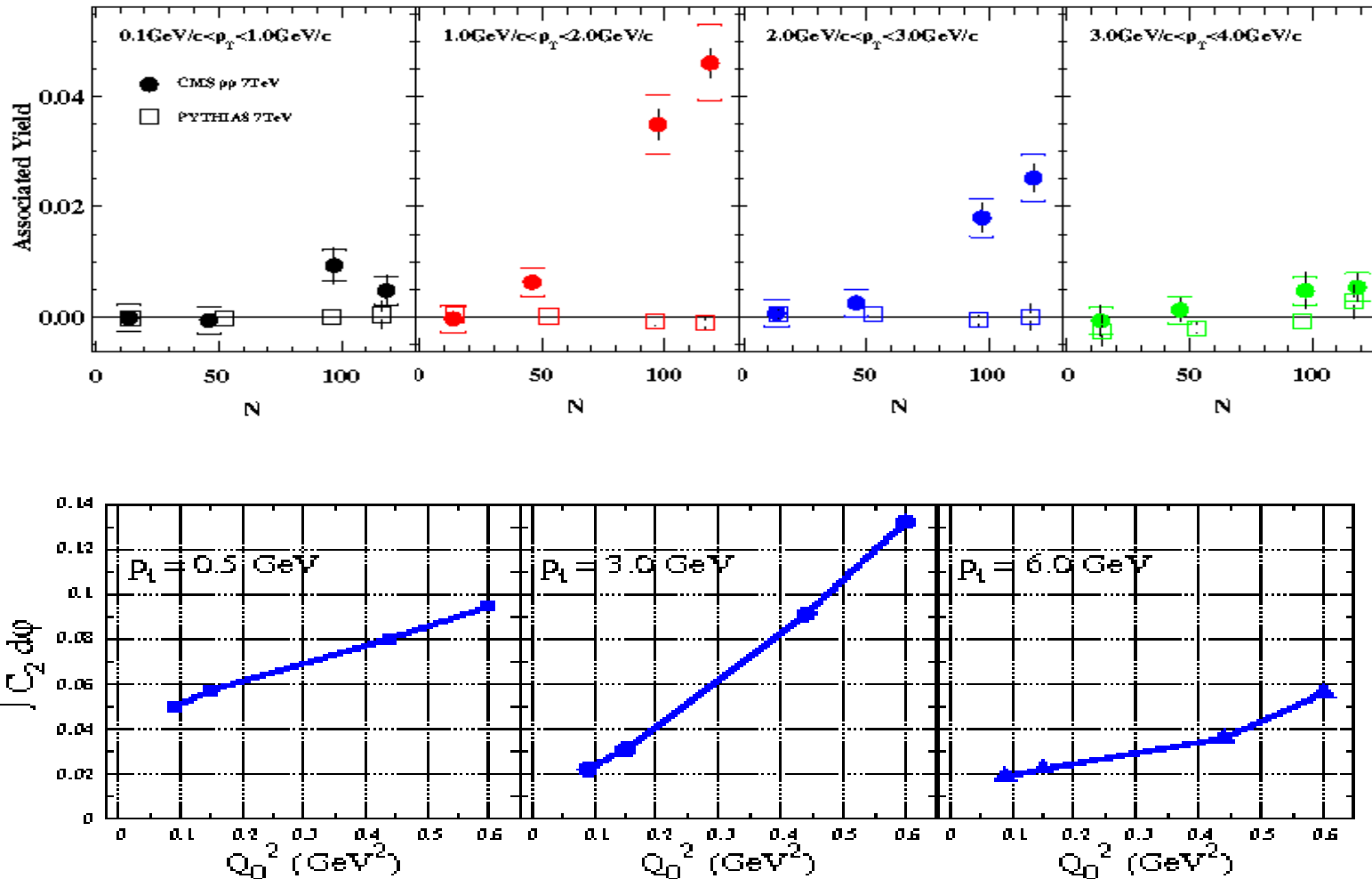


(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



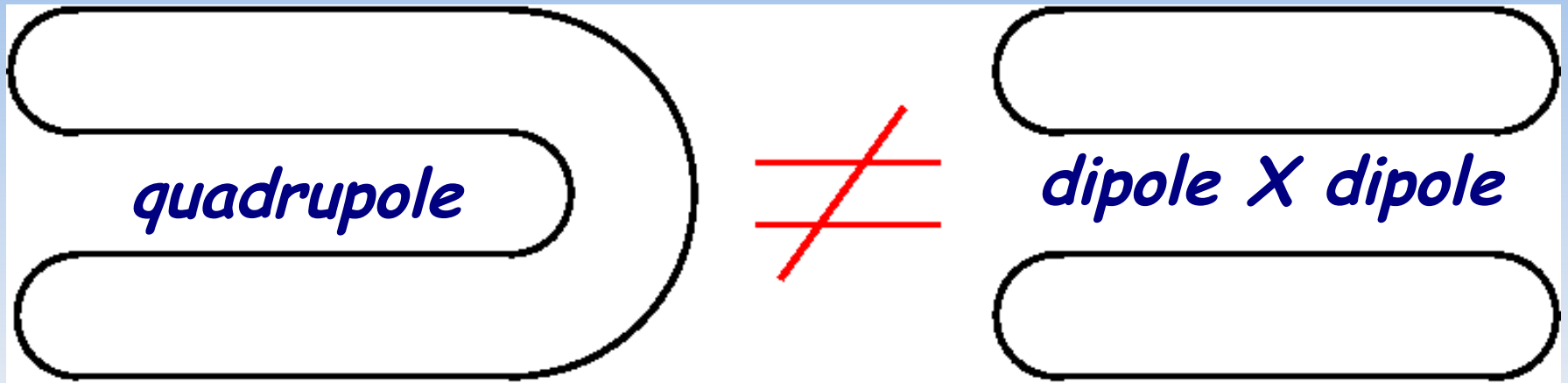
Ridge

The CMS ridge at LHC

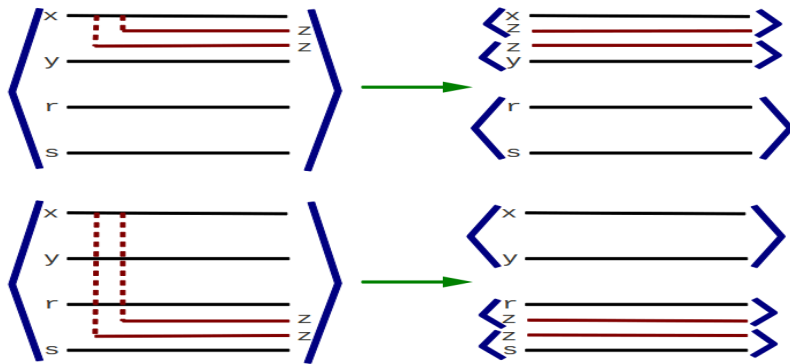


DDGJLV, PLB697 (2011) 21

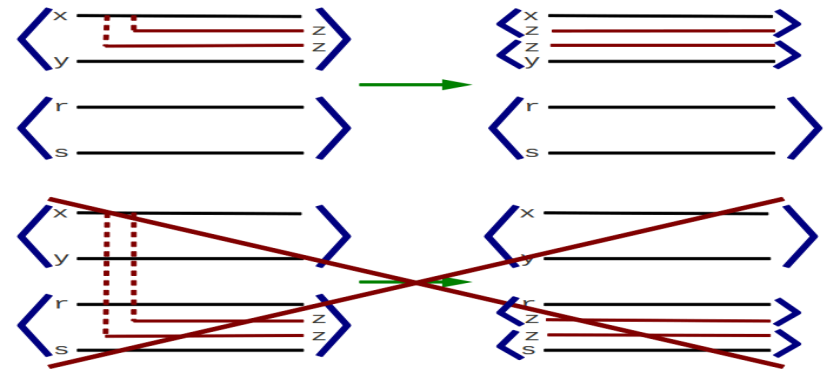
Beyond dipole + large N_c approximation



and they evolve differently even at large N_c



JIMWLK



Dipole approximation

Evolution of gluon 4-pt function

$$\begin{aligned}
 \frac{d}{dY} \langle \alpha_r^a \alpha_{\bar{r}}^b \alpha_s^c \alpha_{\bar{s}}^d \rangle &= \frac{g^2 N_c}{(2\pi)^3} \int d^2 z \\
 &\left\langle \frac{\alpha_z^a \alpha_{\bar{r}}^b \alpha_s^c \alpha_{\bar{s}}^d}{(r-z)^2} + \frac{\alpha_r^a \alpha_z^b \alpha_s^c \alpha_{\bar{s}}^d}{(\bar{r}-z)^2} + \frac{\alpha_r^a \alpha_{\bar{r}}^b \alpha_z^c \alpha_{\bar{s}}^d}{(s-z)^2} + \frac{\alpha_r^a \alpha_{\bar{r}}^b \alpha_s^c \alpha_z^d}{(\bar{s}-z)^2} - 4 \frac{\alpha_r^a \alpha_{\bar{r}}^b \alpha_s^c \alpha_{\bar{s}}^d}{z^2} \right\rangle \\
 &+ \frac{g^2}{\pi} \int \frac{d^2 z}{(2\pi)^2} \\
 &\left\langle f^{\epsilon\kappa a} f^{\kappa b} \frac{(r-z) \cdot (\bar{r}-z)}{(r-z)^2 (\bar{r}-z)^2} \left[\alpha_r^e \alpha_{\bar{r}}^f - \alpha_r^e \alpha_z^f - \alpha_z^e \alpha_{\bar{r}}^f + \alpha_z^e \alpha_z^f \right] \alpha_s^c \alpha_{\bar{s}}^d \right. \\
 &+ f^{\epsilon\kappa a} f^{\kappa c} \frac{(r-z) \cdot (s-z)}{(r-z)^2 (s-z)^2} \left[\alpha_r^e \alpha_s^f - \alpha_r^e \alpha_z^f - \alpha_z^e \alpha_s^f + \alpha_z^e \alpha_z^f \right] \alpha_{\bar{r}}^b \alpha_{\bar{s}}^d \\
 &+ f^{\epsilon\kappa a} f^{\kappa d} \frac{(r-z) \cdot (\bar{s}-z)}{(r-z)^2 (\bar{s}-z)^2} \left[\alpha_r^e \alpha_{\bar{s}}^f - \alpha_r^e \alpha_z^f - \alpha_z^e \alpha_{\bar{s}}^f + \alpha_z^e \alpha_z^f \right] \alpha_{\bar{r}}^b \alpha_s^c \\
 &+ f^{\epsilon\kappa b} f^{\kappa c} \frac{(\bar{r}-z) \cdot (s-z)}{(\bar{r}-z)^2 (s-z)^2} \left[\alpha_{\bar{r}}^e \alpha_s^f - \alpha_{\bar{r}}^e \alpha_z^f - \alpha_z^e \alpha_s^f + \alpha_z^e \alpha_z^f \right] \alpha_r^a \alpha_{\bar{s}}^d \\
 &+ f^{\epsilon\kappa b} f^{\kappa d} \frac{(\bar{r}-z) \cdot (\bar{s}-z)}{(\bar{r}-z)^2 (\bar{s}-z)^2} \left[\alpha_{\bar{r}}^e \alpha_{\bar{s}}^f - \alpha_{\bar{r}}^e \alpha_z^f - \alpha_z^e \alpha_{\bar{s}}^f + \alpha_z^e \alpha_z^f \right] \alpha_r^a \alpha_s^c \\
 &\left. + f^{\epsilon\kappa c} f^{\kappa d} \frac{(s-z) \cdot (\bar{s}-z)}{(s-z)^2 (\bar{s}-z)^2} \left[\alpha_s^e \alpha_{\bar{s}}^f - \alpha_s^e \alpha_z^f - \alpha_z^e \alpha_{\bar{s}}^f + \alpha_z^e \alpha_z^f \right] \alpha_r^a \alpha_{\bar{r}}^b \right\rangle
 \end{aligned}$$

dipole approximation breaks down!
AD-JJM, PRD81:094015 (2010)

JIMWLK: Beyond dipole + large N_c

*2-hadron correlations poses new challenges to CGC
but
every challenge can become an opportunity*

How large are the terms missed by dipole approximation ?

What is their energy dependence ?

What is the role of non-Gaussian (quartic) initial conditions ?

need to solve JIMWLK equation

Two-hadron correlations

qualitative agreement with CGC predictions

A quantitative description of two-hadron correlation requires going beyond dipole approximation in CGC framework