Fluctuation Dynamo in Collisionless and Weakly Collisional Magnetized Plasmas

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Random velocity shears stretch and twist a seed magnetic field:

\[
\frac{d \ln B}{dt} = \hat{b} \hat{b} : \nabla u - \nabla \cdot u,
\]

arranging the magnetic fields into long, thin folds.

\( B \) anti-correlated with field-line curvature \( \hat{b} \cdot \nabla \hat{b} \)

The $\text{Pm} \gg 1$ MHD fluctuation dynamo

Four phases:
1. Diffusion-free
2. Kinematic
   ▶ Kazantsev $k^{3/2}$ spectrum.
3. Nonlinear
   ▶ $\rho u \cdot \nabla u \sim B \cdot \nabla B / 4\pi$
   ▶ smallest-scale stretching suppressed
   ▶ Secular growth of $\langle B^2 \rangle$
4. Saturation
   ▶ minimization of $\hat{b} \hat{b} : \nabla u$
   ▶ $v_A \sim u_{\text{rms}}$ (not scale-by-scale!)
Figure: Simulation results of the $P_m \gg 1$ turbulent MHD dynamo.

(See also Haugen et al. 2004, Beresnyak 2012, Beresnyak & Lazarian 2014)
Theoretical ingredients of the plasma dynamo

ICM only requires $B \sim 10^{-18} \, \text{G}$ to be magnetized (i.e. $\rho_i \sim \lambda_{\text{mfp}}$). Conservation of magnetic moment $\mu = w^2_\perp / B \rightarrow d_t(p_\perp/nB) = 0$. Thus

1. As $B$ increases, $p_\perp$ increases $\rightarrow p_\perp \neq p_\parallel$ (Bad for dynamo! – Helander et al. 2016)

2. Estimate size of $\Delta p = p_\perp - p_\parallel$ in weakly collisional plasmas using CGL equations and collisions:

$$\frac{d}{dt} \frac{p_\perp - p_\parallel}{p} \approx \frac{3}{\nu_i} \frac{d \ln B}{dt} - \nu_i \frac{p_\perp - p_\parallel}{p}$$

(Recall: $d_t \ln B = \hat{b} \hat{b} : \nabla u - \nabla \cdot u$). So

$$\nabla \cdot (P - Ip) \approx -\frac{3p}{\nu_i} \nabla \cdot \left[ \hat{b} \hat{b} \left( \hat{b} \hat{b} : \nabla u \right) \right]$$

Results in $\sim 1\%$ deviations from local thermodynamic equilibrium.
Mirror and firehose instabilities

These instabilities arise in high-$\beta$ (\( \equiv 8\pi p/B^2 \)) plasmas.

Firehose (\( \Delta \equiv p_\perp/p_\parallel - 1 < -2/\beta \)):

Mirror (\( \Delta > 1/\beta \)):

Saturation at \( \nu_{\text{eff}} \sim |\hat{b}\delta b : \nabla u|/\beta \) (Kunz+ 2014; Melville+ 2016).
Mirror and firehose instabilities

These instabilities set thresholds on the pressure anisotropy in the solar wind:

\[
\Delta = \left| \frac{p_\perp}{p_\parallel} - 1 \right| \lesssim \frac{1}{\beta} \quad (\text{see Chen et al. 2016})
\]

Anisotropy can be tightly regulated if \( \nu_{\text{eff}} \sim |\hat{b}\hat{b} : \nabla \mathbf{u}|\beta \). (see also: MRI turbulence, Kunz, Stone & Quataert, 2016 PRL)
Three regimes for plasma dynamo

This physics suggests three dynamo regimes:

1. Unmagnetized regime ($\Omega_i \ll \nu_i$, see Rincon et al. 2016)
2. Magnetized ‘kinetic’ regime ($\Omega_i \ll |\hat{b}\hat{b}:\nabla u|\beta$)
3. Magnetized ‘fluid’ regime ($\Omega_i \gg |\hat{b}\hat{b}:\nabla u|\beta$)

I now present results from:

1. Hybrid-kinetic simulations (St-Onge & Kunz 2017)
   ▶ How does the dynamo operate in a collisionless plasma?
   ▶ Ab initio measurement of $\nu_{\text{eff}}$ motivates...

2. Braginskii-MHD simulations (St-Onge+, JPP (in review)).
   ▶ Given a prescribed viscosity, how does the plasma self-organize itself to amplify the magnetic field?

3. Analytic Modeling
   ▶ Predicting the dynamo in certain asymptotic regimes.
Hybrid-Kinetic simulations

- Full-\( f \) Hybrid Kinetics
  - kinetic ions,
    \[
    \frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla f_i + \left[ \frac{e}{m_i} \left( \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) + \frac{\mathbf{F}}{m_i} \right] \cdot \nabla \mathbf{v} f_i = 0.
    \]
  - isothermal fluid electrons,
    \[
    \mathbf{E} + \frac{1}{c} \mathbf{u} \times \mathbf{B} - \frac{\eta}{c} \nabla \times \mathbf{B} + \frac{\eta^{\text{hyper}}}{c} \nabla \times \nabla^2 \mathbf{B} = -\frac{T_e \nabla n}{en} + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi en}.
    \]
- Non-helical, incompressible, time-correlated (\( \sim L/\mathbf{u}_{\text{rms}} \)) forcing
- Focus on two specific runs:

  1. \( L = 16 \rho_{i0} \quad \beta_{i0} = 10^6 \quad N = 504^3 \quad \text{PPC} = 216 \)
  2. \( L = 10 \rho_{i0} \quad \beta_{i0} = 10^4 \quad N = 252^3 \quad \text{PPC} = 216 \)
$|\mathbf{B}|/B_{\text{rms}}$
$\frac{|B|}{B_{\text{rms}}}$
The Punchline\textsuperscript{1}…

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\end{figure}

Mirror and firehose instabilities redux

- Kinetic instabilities generate magnetic energy above $\rho_i$.
- Evidence of firehose visually and in curvature PDFs.
- Plasma becomes Braginskii-like ($3\hat{b}\hat{b}: \nabla u \sim \nu_{\text{eff}} \Delta_i$):

```
\begin{align*}
\langle B B : \nabla u \rangle / \langle B^2 \rangle & \quad \text{vs} \quad \langle \Delta_i \rangle
\end{align*}
```
Regulation of the pressure anisotropy is *imperfect*:

![Graph](image)

**Figure:** Solar wind measurements – *strong regulation*

**Figure:** Plasma dynamo simulation – *weak regulation*

This suggests ‘hard-wall’ limiters may not be the ideal closure for kinetic microphysics.
Braginskii-MHD simulations\textsuperscript{2}

- Dilute magnetized plasma 
  \( \Omega_i \gg \nu_i \gg \omega \)
- Incompressible Braginskii MHD equations
  \[
  d_t u = B \cdot \nabla B - \nabla p + \nabla \cdot (\hat{b}\hat{b}\Delta p) + \mu \nabla^2 u,
  \]
  \[
  d_t B = B \cdot \nabla u + \eta \nabla^2 B.
  \]
- Nonhelical, incompressible, time-correlated forcing
- Pressure anisotropy
  \[
  \Delta p = 3\mu_B \hat{b}\hat{b} : \nabla u, \text{ both:}
  \]
  - unlimited (parameter scan on \( \mu_B \))
  - hard-wall limited:

  \[
  \Delta p = \begin{cases} 
  \min \left( B^2/2, 3\mu_B \hat{b}\hat{b} : \nabla u \right), & \Delta p > 0 \\
  \max \left( -B^2, 3\mu_B \hat{b}\hat{b} : \nabla u \right), & \Delta p < 0 
  \end{cases}
  \]

Figure: \(|B|/B_{\text{rms}}\) of the unlimited Braginskii-MHD dynamo.

\textsuperscript{2}Submitted to JPP
Hard-walled Braginskii looks like $P_m \gtrsim 1$ MHD

(in box-averaged evolution)
Hard-walled Braginskii looks like $P_m \gtrsim 1$ MHD

(in spectra)

Figure: Kinetic and magnetic energy spectra
Unlimited Braginskii dynamo mimics saturated MHD

- Unlimited regime relevant to early stages of plasma dynamo
- Mimics saturated MHD in:
  - statistics of $\nabla u$ and alignment with respect to $\hat{b}$
  - magnetic spectrum
  - fold geometry (including PDF of $\hat{b} \cdot \nabla \hat{b}$)
  - spectral anisotropy of turbulent velocity

Why is this? Compare

$$B \cdot \nabla B = \nabla \cdot (\hat{b} \hat{b} B^2)$$

to

$$\nabla \cdot (\hat{b} \hat{b} \Delta p) \propto \nabla \cdot (\hat{b} \hat{b} \, dt \ln B).$$

Pressure anisotropy plays the role of magnetic-field strength in tension force.
A modified Kazantsev model for \( \text{Re}_\parallel/\text{Re}_\perp \ll 1 \)

Consider a velocity field with prescribed statistics

\[
\begin{align*}
\overline{u^i(t, x)} &= 0, \\
\overline{u^i(t, x) u^j(t', x')} &= \delta(t - t') \kappa^{ij}(x - x'),
\end{align*}
\]

which are anisotropic with respect to \( \hat{b} \):

\[
\kappa^{ij}(k) = \kappa^{(i)}(k, |\xi|) (\delta^{ij} - \hat{k}_i \hat{k}_j) + \kappa^{(a)}(k, |\xi|) (\hat{b}^i \hat{b}^j + \xi^2 \hat{k}_i \hat{k}_j - \xi \hat{b}^i \hat{k}_j - \xi \hat{k}_i \hat{b}^j),
\]

where \( \xi \equiv \hat{k} \cdot \hat{b} \). We derive an equation for the joint PDF of \( B, k \) and \( \hat{b} \):

\[
\mathcal{P}(B, k, \hat{b}) = \delta(|\hat{b}|^2 - 1) \delta(\hat{b} \cdot k)(4\pi^2 k)^{-1} P(B, k).
\]

This model was originally developed for the saturated state in MHD by Schekochihin (2004)
A modified Kazantsev model for $\text{Re}_\parallel/\text{Re}_\perp \ll 1$

We then derive an equation for the magnetic energy spectrum $M(k) = (1/2) \int_0^\infty dB \, B^2 P(B, k)$:

\[
\frac{\partial M}{\partial t} = \frac{\gamma_\perp}{8} \frac{\partial}{\partial k} \left[ (1 + 2\sigma_\parallel) k^2 \frac{\partial M}{\partial k} - (1 + 4\sigma_\perp + 10\sigma_\parallel) k M \right]
\]

**drift–diffusion in $k$-space**

\[
+ 2(\sigma_\perp + \sigma_\parallel) \gamma_\perp M - 2\eta k^2 M,
\]

**growth**

**decay**

where

\[
\gamma_\perp = \int \frac{d^3k}{(2\pi)^3} \, k^2_\perp \kappa_\perp(k),
\]

**mixing**

\[
\sigma_\perp = \frac{1}{\gamma_\perp} \int \frac{d^3k}{(2\pi)^3} \, k^2_\parallel \kappa_\perp(k),
\]

**shearing**

\[
\sigma_\parallel = \frac{1}{\gamma_\perp} \int \frac{d^3k}{(2\pi)^3} \, k^2_\parallel \kappa_\parallel(k),
\]

**stretching**
A modified Kazantsev model for $\text{Re}_\parallel / \text{Re}_\perp \ll 1$

Figure: Comparison of predicted versus simulation magnetic energy spectra.
A modified Kazantsev model for $\Re_{\parallel}/\Re_{\perp} \ll 1$

In the limit $Rm \to \infty$, the dynamo growth rate $\gamma$ is given by

$$\frac{\gamma}{\gamma_{\perp}} = \frac{1}{8(1 + 2\sigma_{\parallel})} \left[ 16(\sigma_{\perp} + \sigma_{\parallel})(1 + 2\sigma_{\parallel}) - (1 + 2\sigma_{\perp} + 6\sigma_{\parallel})^2 \right].$$

Sufficiently large $\gamma_{\perp}$ (mixing) or $\mu_B$ (parallel viscosity) kills the dynamo!
A new “Prandtl” number

Unlimited Braginskii MHD has two important dimensionless numbers:

\[
\frac{\mu_\parallel}{\eta}, ~ \frac{\mu_\parallel}{\mu_\perp}.
\]

\text{MHD Pm} \quad \text{NEW!}

Ratio of stretching and mixing in the dynamo matters, and is controlled by \(\mu_\parallel/\mu_\perp\).
Predictions of the model for $\text{Re}_\perp \gg 1$, $\text{Re}_\parallel \sim 1$

Figure: Evolution of magnetic energy (left) and $\sigma_\perp$, $\sigma_\parallel$ as a function of $\mu^{-1}$ (right).
Examine the opposite limit: $Re_\parallel \ll 1$ Stokes flow

In the Stokes flow regime, viscosity is so large that the velocity is determined by a balance between dissipation and driving alone:

$$-\mu \nabla^2 u = \tilde{f}$$

As $\nu \to \infty$, flow becomes $\delta$-correlated in time.
Unlimited Braginskii Dynamo for $\text{Re}_\parallel \ll 1$ Stokes flow.

Figure: Evolution of the magnetic energy for MHD and unlimited Braginskii-MHD in the Stokes flow regime for fixed $u_{\text{rms}}$. 

$\eta_{-1}^{-1} = 1.8 \times 10^7$
The take-away points

To summarize,

▶ **Dynamo exists in a collisionless magnetized plasma.** (See also Rincon *et al.* 2016 for unmagnetized regime)

▶ Larmor-scale instabilities play a crucial role.

▶ Many features appear MHD-like ($P_m \gtrsim 1$), despite collisionless plasma.

▶ Saturation at $u \sim v_A$.

For weakly collisional plasmas,

▶ Too anisotropic a viscosity is deleterious for the dynamo (*controls ratio of mixing to stretching*)

▶ Perfect pressure-anisotropy regulation $\rightarrow P_m \sim 1$ MHD

▶ Weak pressure-anisotropy regulation $\rightarrow$ saturated MHD
Future research directions

- Exact determination of $\nu_{\text{eff}}$ in the magnetized kinetic regime
- Other components of the Braginskii viscosity (i.e. gyro-viscosity)
- Kinetic electron effects
  - Dynamo relies on magnetized electrons (flux-freezing)!
  - Resistive scale (i.e. fold separation) set by electron physics
- Interplay between mean-field and fluctuation dynamos:
  - Historical anxiety about mean-field dynamo in the face of fluctuation dynamo
  - Fluctuation dynamo can lead to catastrophic $\alpha$ quenching!
  - Could kinetic effects alleviate these concerns?
Questions?