Introduction to Accelerator Physics
Beam Dynamics for „Summer Students"

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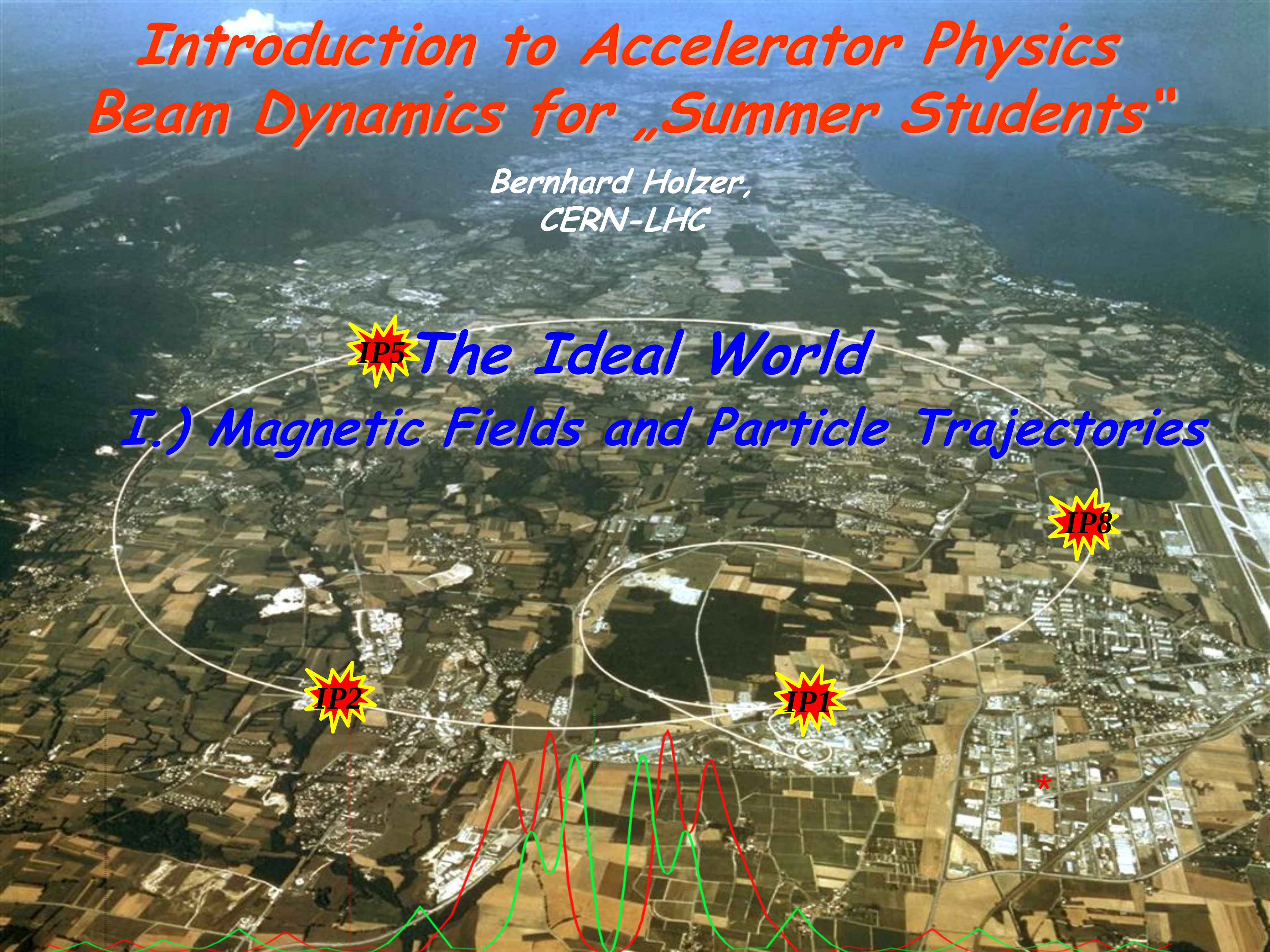


The Ideal World

I.) Magnetic Fields and Particle Trajectories

Luminosity Run of a typical storage ring:

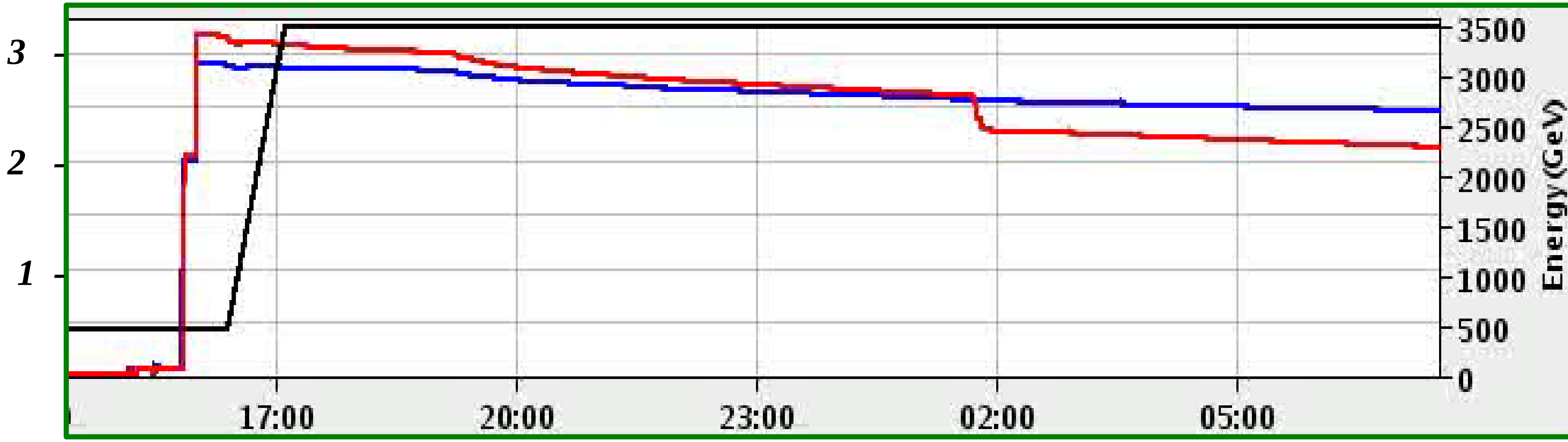
LHC Storage Ring:* *Protons* accelerated and *stored for 12 hours

distance of particles travelling at about $v \approx c$

$L = 10^{10}$-10^{11} km

... several times Sun - Pluto and back

intensity (10^{11})



→ ***guide the particles* on a well defined orbit *(„design orbit“)***

→ ***focus the particles* to keep each single particle trajectory within the vacuum chamber of the storage ring, i.e. close to the design orbit.**

1.) Introduction and Basic Ideas

„ … in the end and after all it should be a kind of circular machine"

→ need transverse deflecting force

Lorentz force

$$\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines:

$$v \approx c \approx 3 * 10^8 \, m/s$$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q * 300 \, \frac{MV}{m}$$

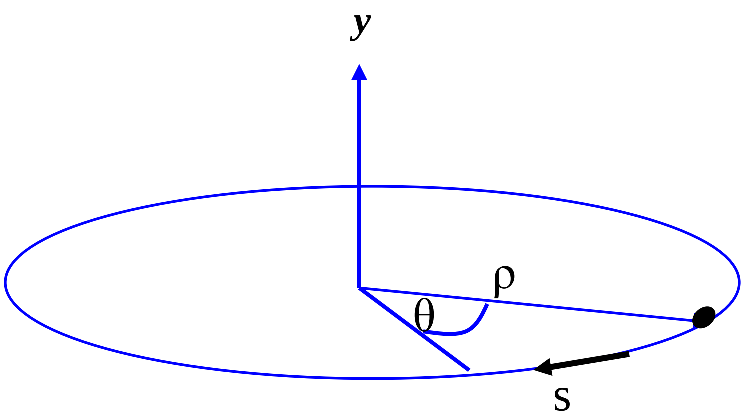
equivalent el. field … E

technical limit for el. field:

$$E \leq 1 \, \frac{MV}{m}$$

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $F_L = e\,v\,B$

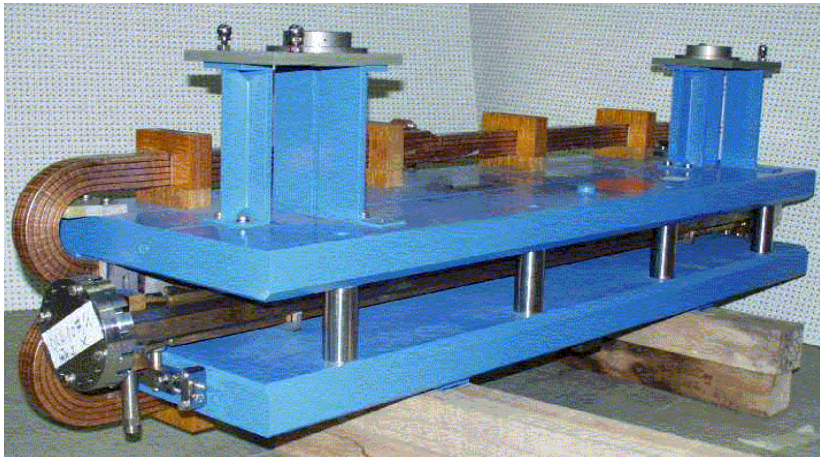
centrifugal force $F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$

$$\frac{\gamma\, m_0\, v^2}{\rho} = e\,v\,B$$

$$\frac{p}{e} = B\,\rho$$

$B\rho$ = "beam rigidity"

2.) The Magnetic Guide Field



Dipole Magnets:

define the ideal orbit
homogeneous field created
by two flat pole shoes

$$B = \frac{\mu_0 \, n \, I}{h}$$

Normalise magnetic field to momentum:

$$\frac{p}{e} = B \rho \quad \longrightarrow \quad \frac{1}{\rho} = \frac{e B}{p}$$

convenient units:

$$B = [T] = \left[\frac{Vs}{m^2}\right] \qquad p = \left[\frac{GeV}{c}\right]$$

Example LHC:

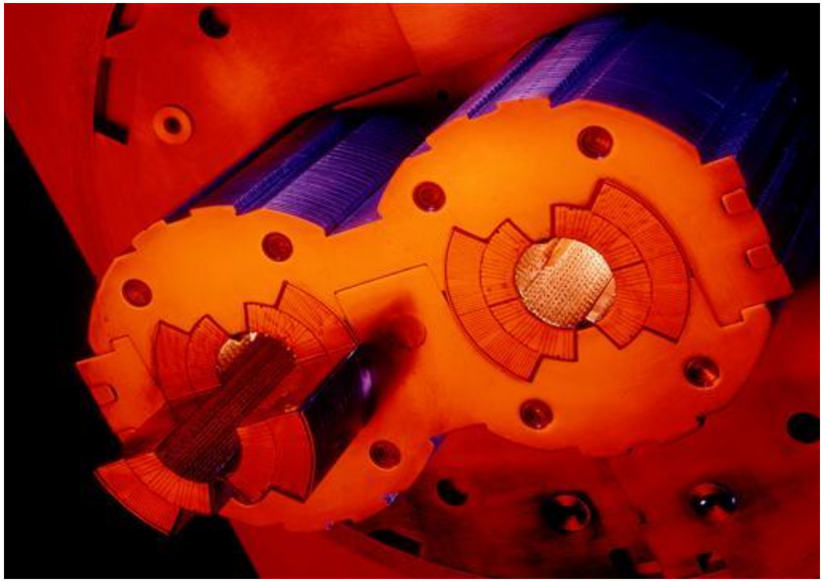
$$B = 8.3\, T$$

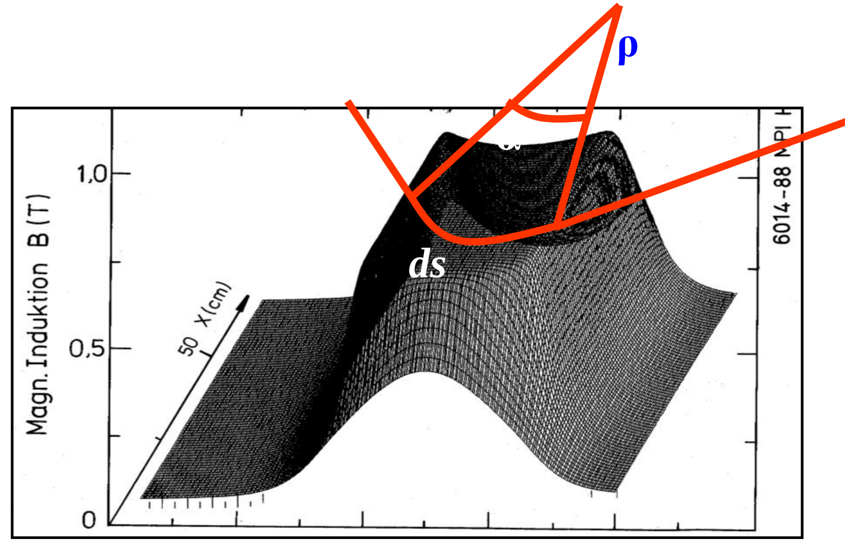
$$p = 7000 \, \frac{GeV}{c}$$

$$\frac{1}{\rho} = e \, \frac{8.3 \, Vs / m^2}{7000 * 10^9 \, eV / c} = \frac{8.3 \, s * 3 * 10^8 \, m / s}{7000 * 10^9 \, m^2}$$

$$\frac{1}{\rho} = 0.333 \, \frac{8.3}{7000} \, 1/m$$

The Magnetic Guide Field





field map of a storage ring dipole magnet

$\rho = 2.53\ km$ → $2\pi\rho = 17.6\ km$

$\approx 66\%$

$B \approx 1 \dots 8\ T$

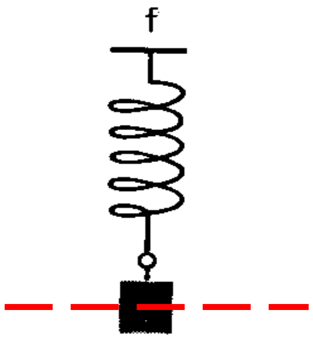
rule of thumb:

$$\frac{1}{\rho} \approx 0.3 \frac{B[T]}{p[GeV/c]}$$

„normalised bending strength“

2.) Focusing Properties – Transverse Beam Optics

classical mechanics: pendulum



there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2 x}{dt^2} = -c * x$$

general solution: free harmonic oszillation

$$x(t) = A * \cos(\omega t + \varphi)$$

***Storage Ring:** we need a Lorentz force that rises as a function of the distance to .... ?*

.... the design orbit

$$F(x) = q * v * B(x)$$

Quadrupole Magnets:

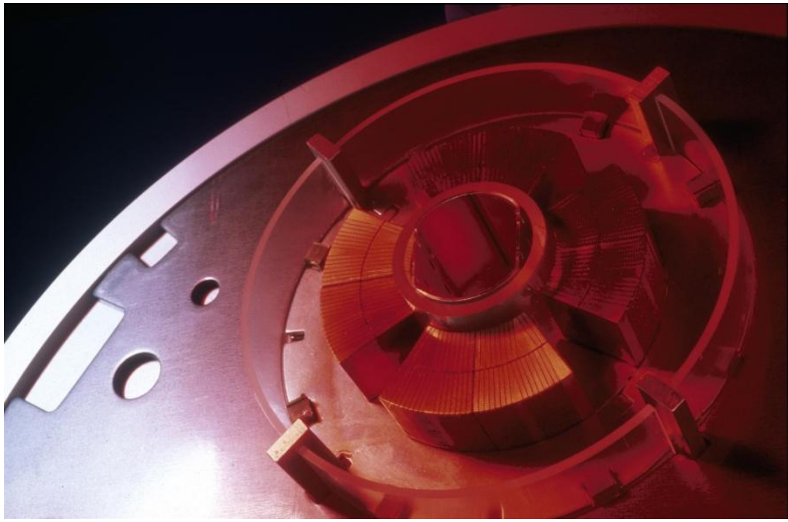
required: *focusing forces* *to keep trajectories in vicinity of the ideal orbit*

linear increasing Lorentz force

linear increasing magnetic field $B_y = g\,x \qquad B_x = g\,y$

normalised quadrupole field:

$$\longrightarrow \qquad k = \frac{g}{p/e}$$



LHC main quadrupole magnet

$$g \approx 25 \ldots 220 \quad T/m$$

simple rule:

$$k = 0.3 \frac{g(T/m)}{p(GeV/c)}$$

what about the vertical plane:
... Maxwell

$$\vec{\nabla} \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t} = 0 \qquad \Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = g$$

(The terms $\vec{j}$ and $\frac{\partial \vec{E}}{\partial t}$ are crossed out.)

Focusing forces and particle trajectories:

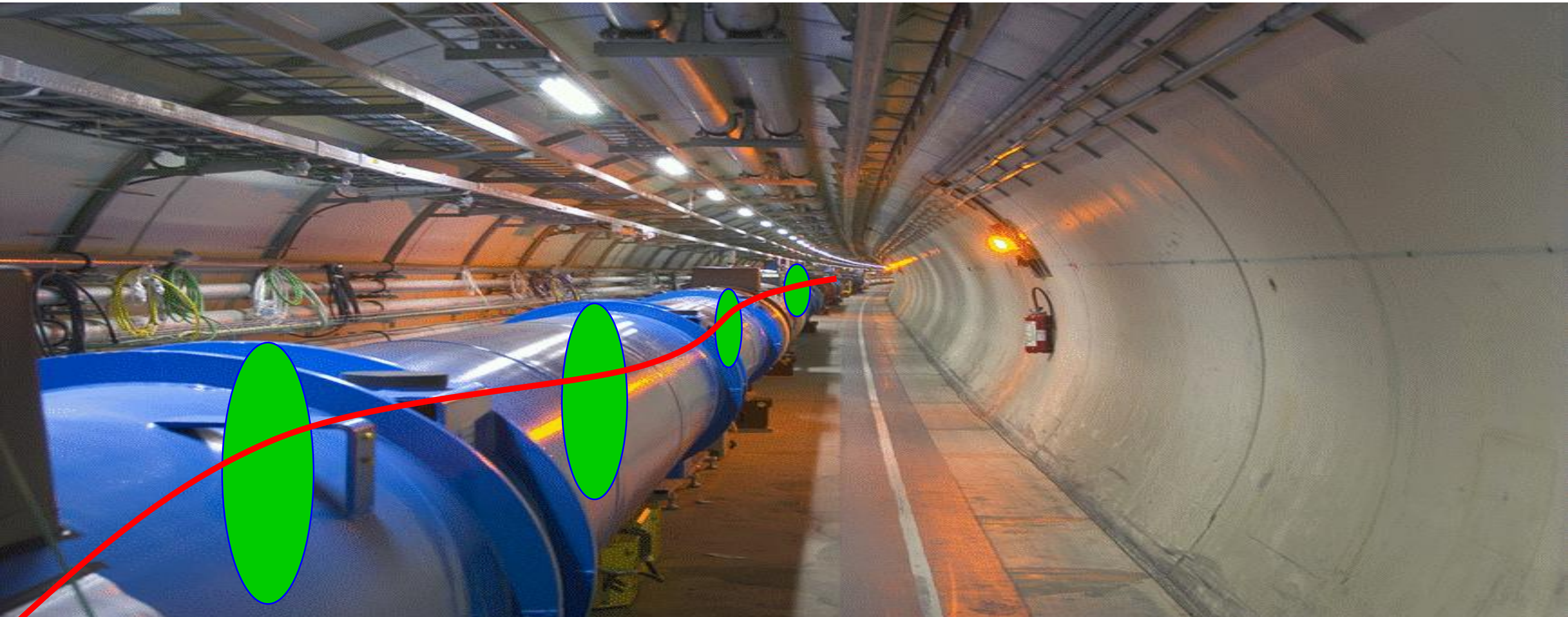
normalise magnet fields to momentum
(remember: $\boldsymbol{B*\rho = p / q}$ *)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

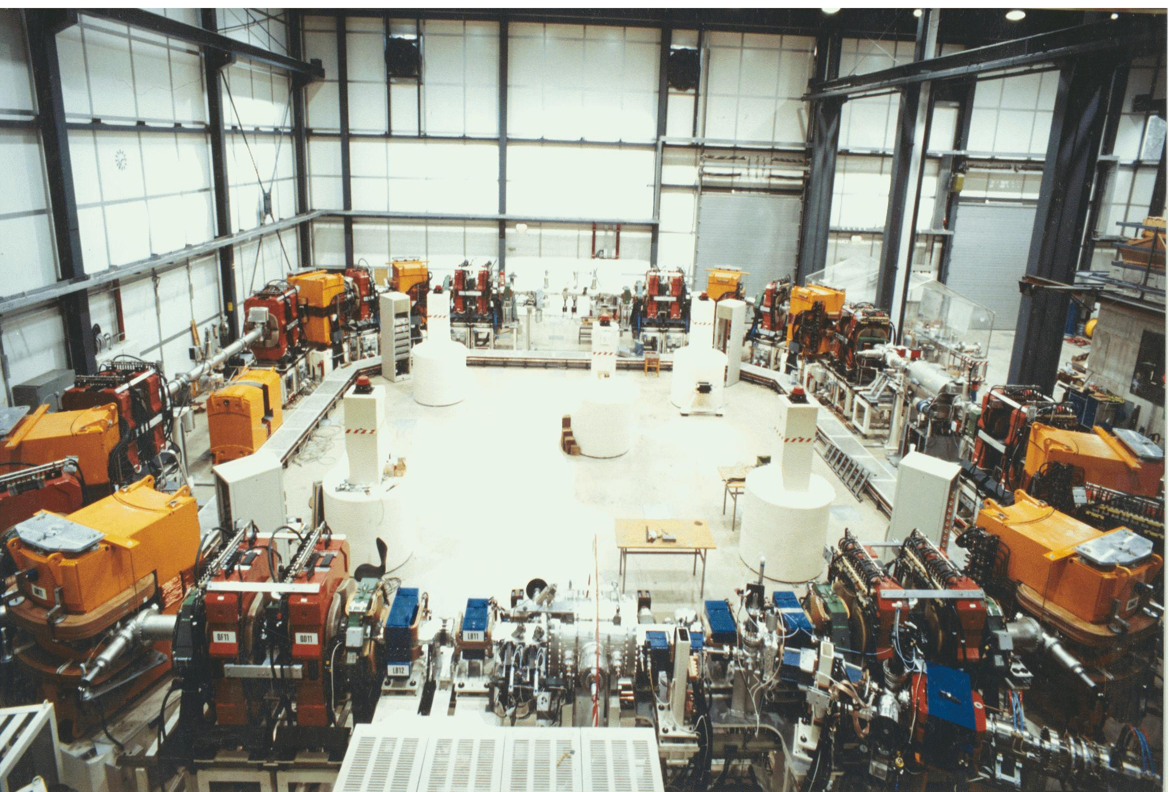
$$k := \frac{g}{p/q}$$



3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}\,m\,x^2} + \cancel{\frac{1}{3!}\,n\,x^3} + \ldots$$

only terms linear in x, y taken into account *dipole fields*
quadrupole fields



Separate Function Machines:

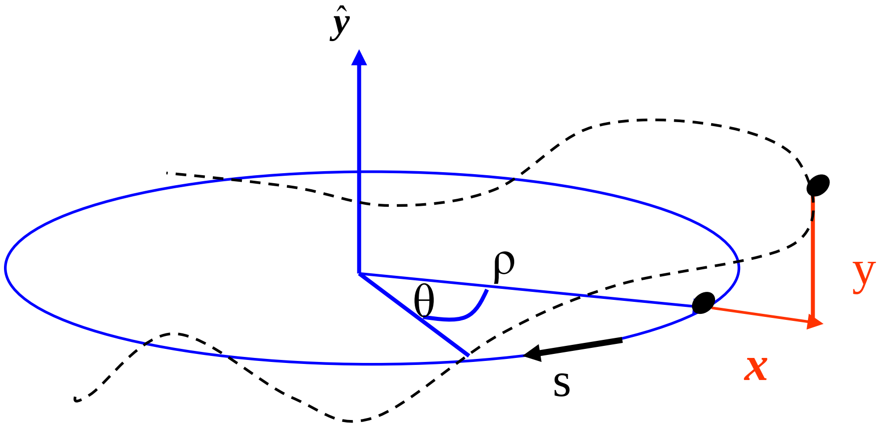
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

*** man sieht nur dipole und quads → linear**

The Equation of Motion:



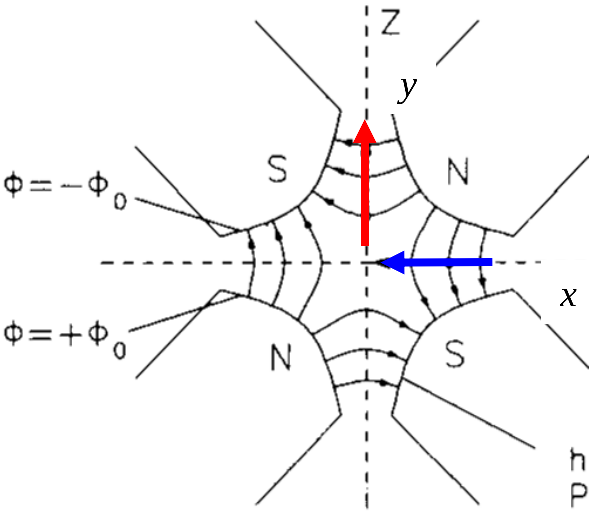
* *Equation for the* ***horizontal motion:***

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = 0$$

* *Equation for the* ***vertical motion:***

$\frac{1}{\rho^2} = 0$ *no dipoles … in general …*

$k \leftrightarrow -k$ *quadrupole field changes sign*



$$y'' + k\,y = 0$$

4.) Solution of Trajectory Equations

Define … hor. plane: $K = 1/\rho^2 - k$

… vert. Plane: $K = k$

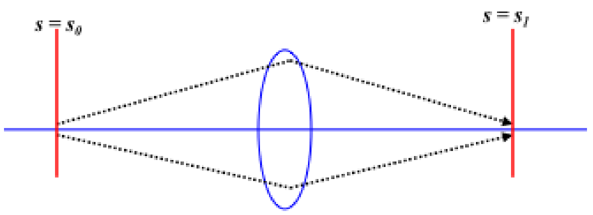
$$x'' + K\,x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole K > 0:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$

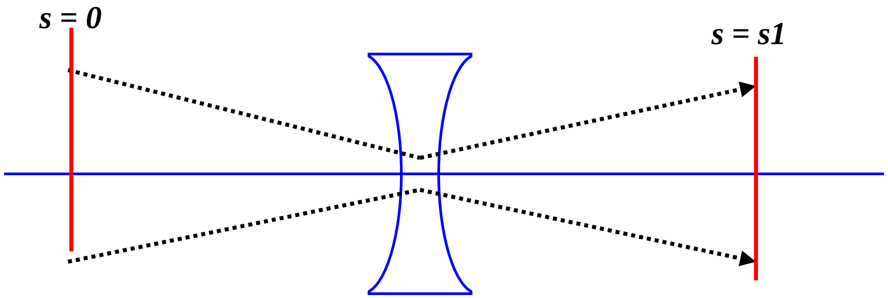


For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}\,l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}\,l\right) \\ -\sqrt{|K|}\sin\left(\sqrt{|K|}\,l\right) & \cos\left(\sqrt{|K|}\,l\right) \end{pmatrix}$$

hor. defocusing quadrupole:



$$x'' - K\,x = 0$$

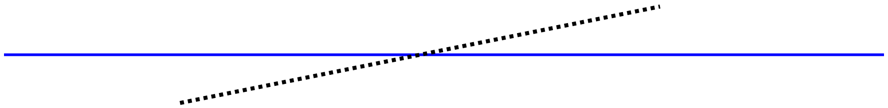
Ansatz: Remember from school

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\,l \\ \sqrt{|K|}\sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}$$

drift space:

K = 0



$$x(s) = x'_0 * s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

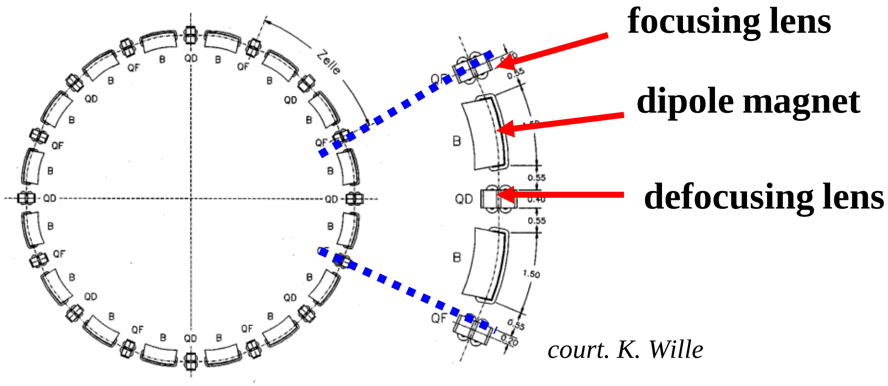
! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

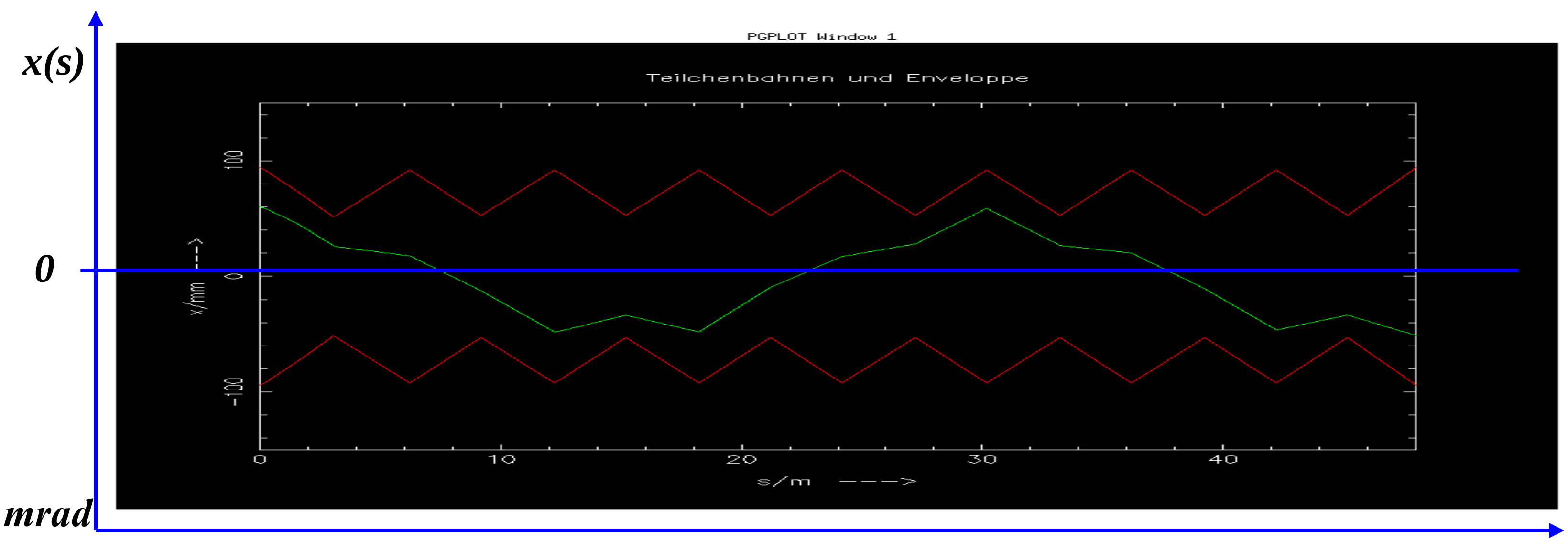
$$M_{total} = M_{QF} * M_{D} * M_{QD} * M_{Bend} * M_{D*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



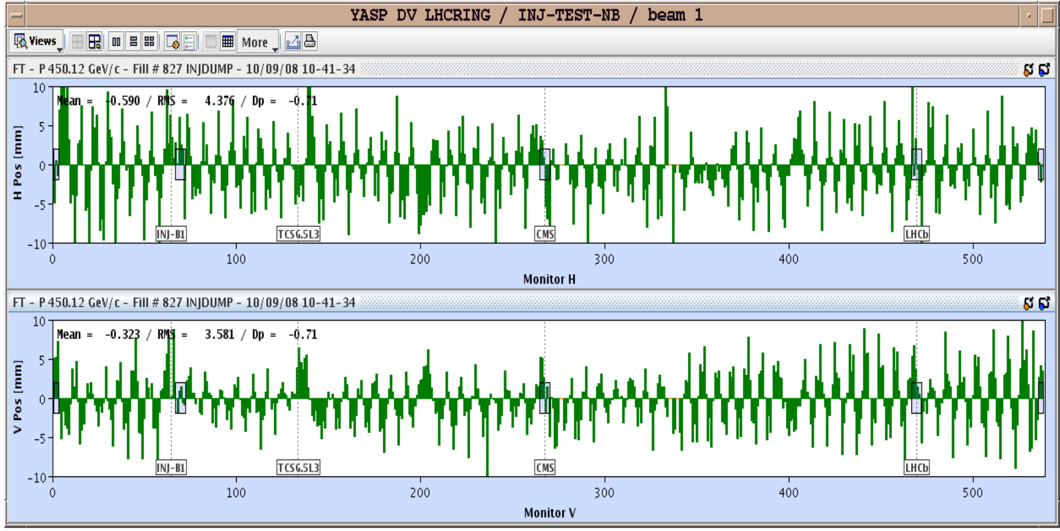
court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



typical values in a strong foc. machine:
x ≈ mm, x´ ≤ mrad

5.) Orbit & Tune:



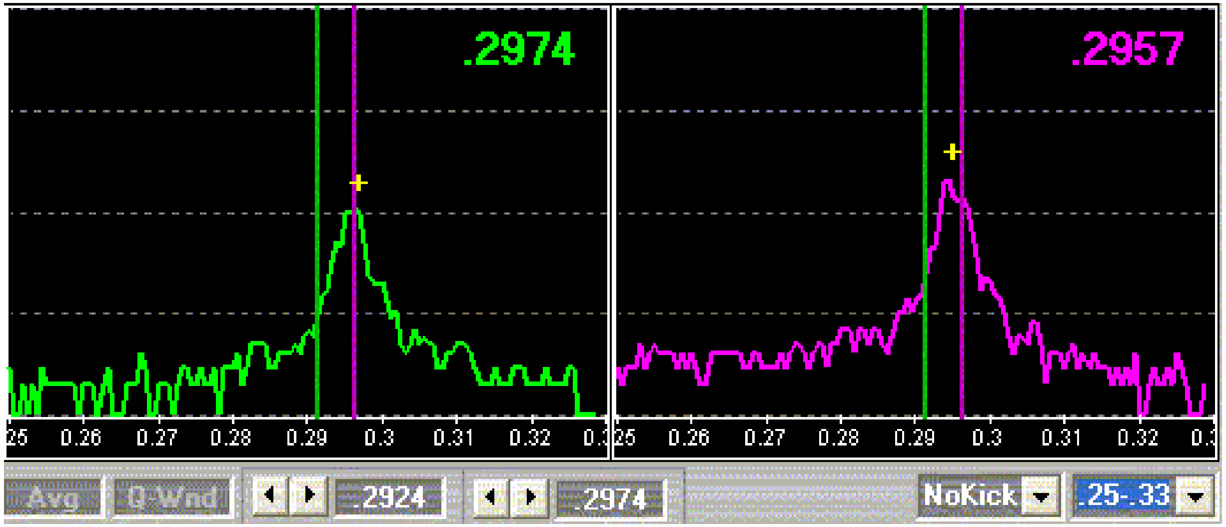
Tune: number of oscillations per turn

64.31
59.32

Relevant for beam stability:
non integer part

LHC revolution frequency: 11.3 kHz

$0.31 * 11.3 = 3.5 kHz$

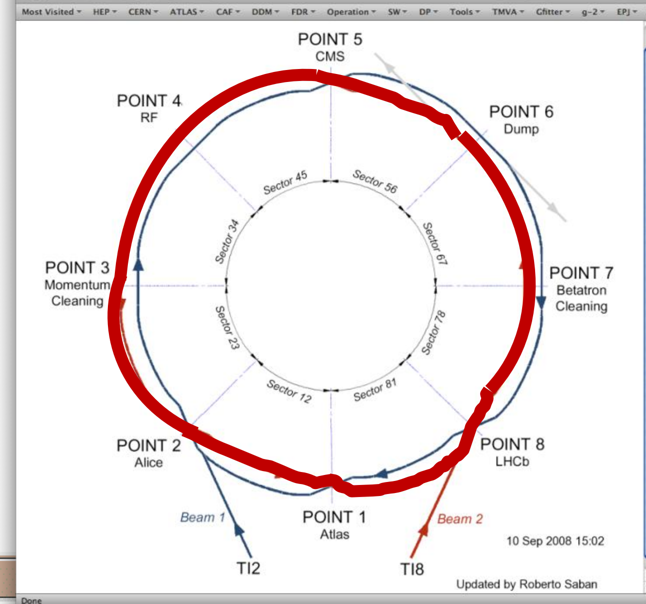


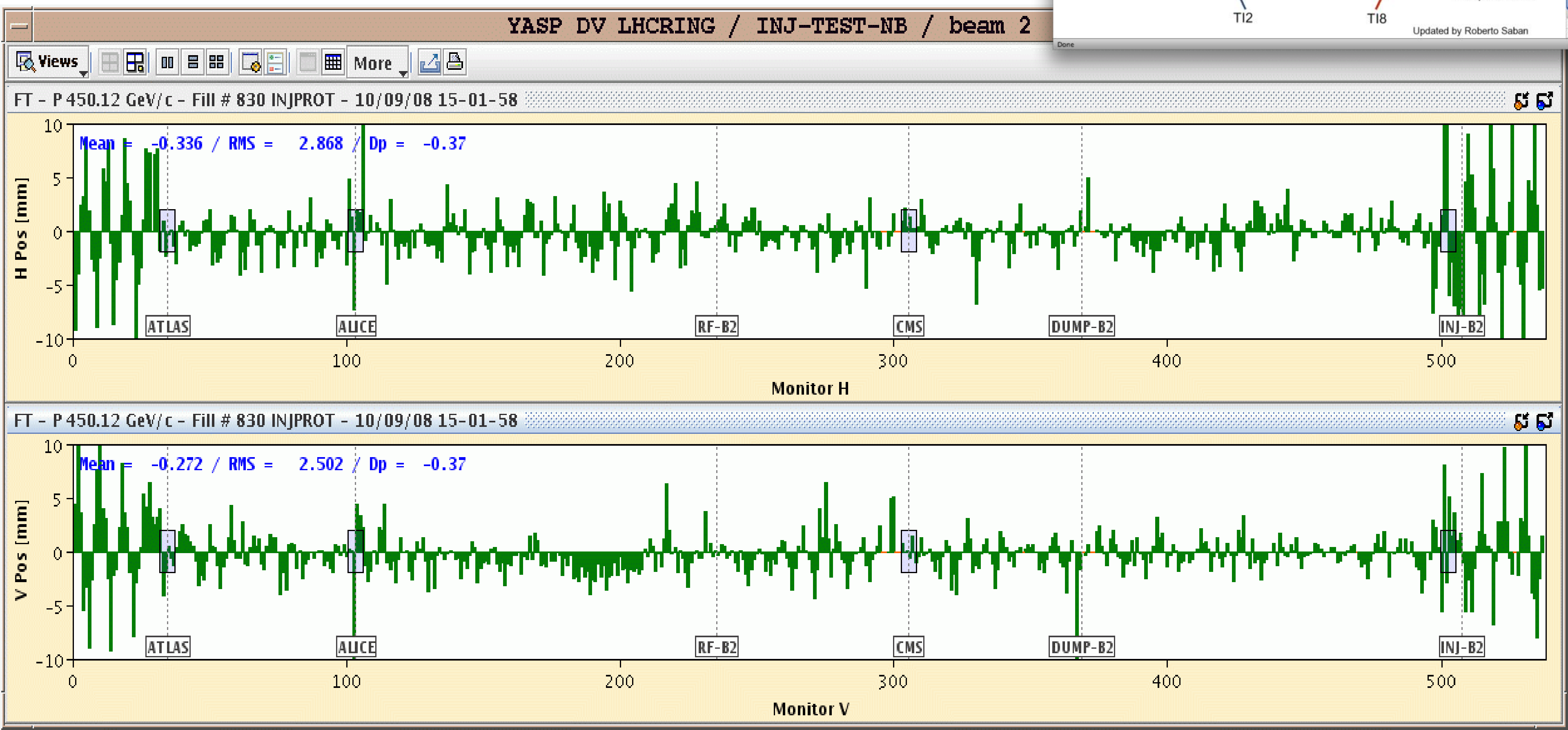
LHC Operation: Beam Commissioning

First turn steering "by sector:"

- ❑One beam at the time
- ❑Beam through 1 sector (1/8 ring),

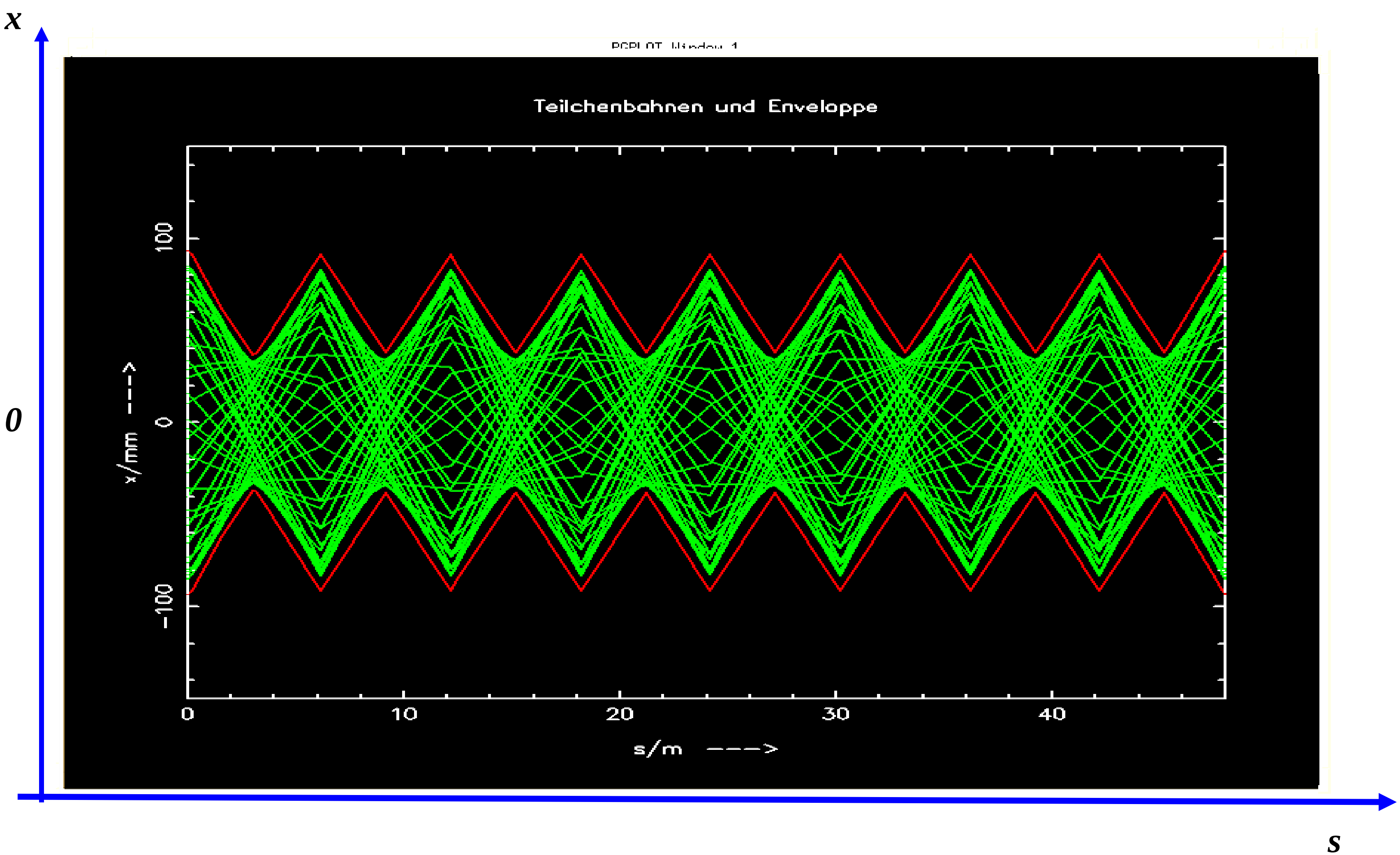
correct trajectory, open collimator and move on.





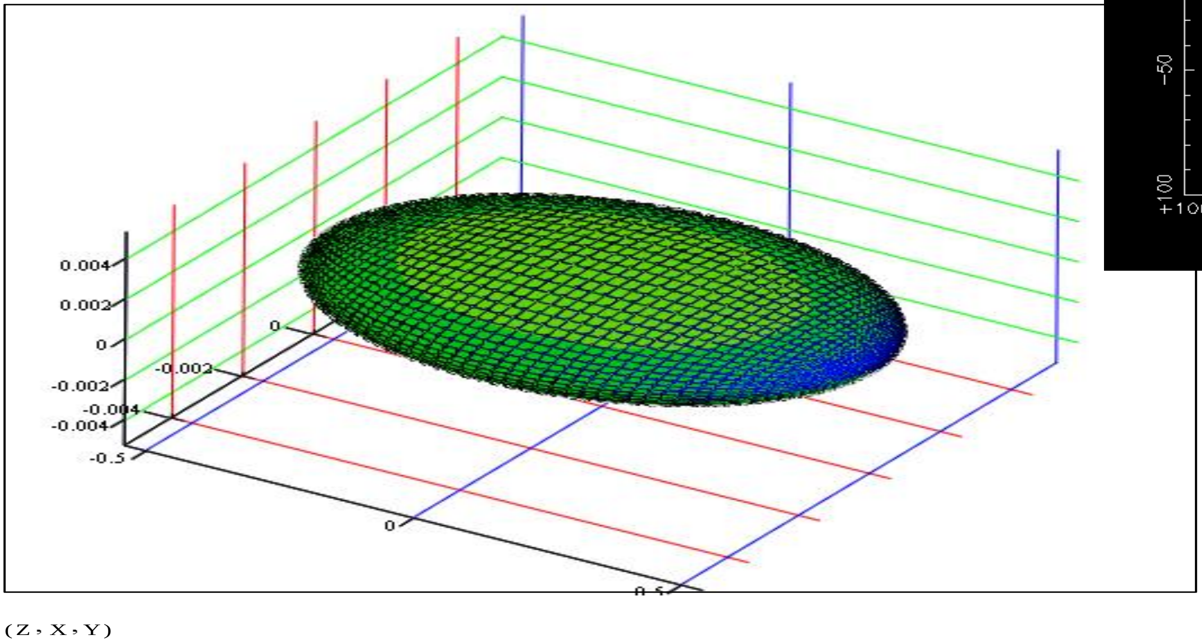
Question: ***what will happen, if the particle performs a second turn ?***

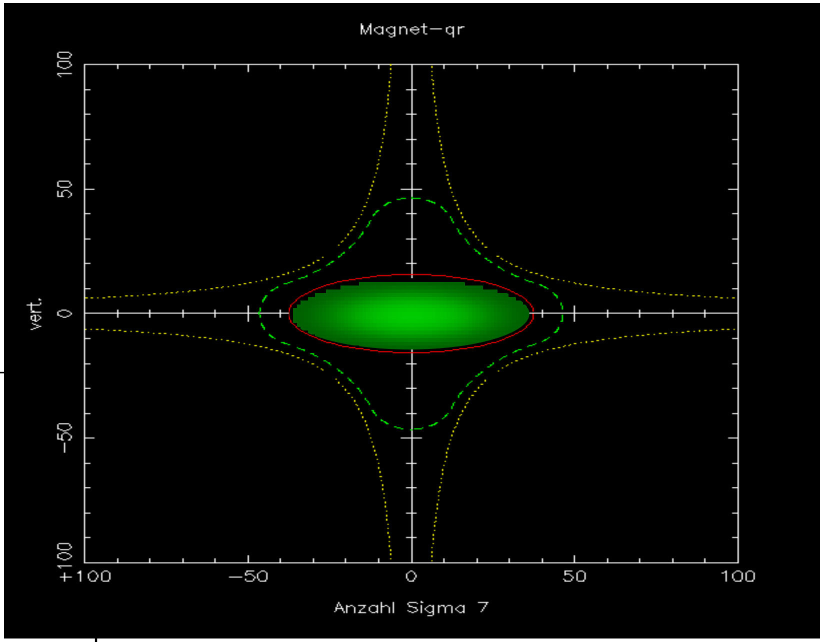
... or a third one or ... 10^{10} turns



II.) The Ideal World:

Particle Trajectories, Beams & Bunches





Bunch in a Storage Ring

Astronomer Hill:

differential equation for motions with periodic focusing properties
„Hill's equation"

Example: particle motion with periodic coefficient

equation of motion: $x''(s) - k(s)x(s) = 0$

restoring force ≠ const,
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

6.) The Beta Function

„it is convenient to see“

... after some beer ... general solution of Mr Hill can be written in the form:

Ansatz:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

ε, Φ = integration constants determined by initial conditions

β(s) periodic function given by focusing properties of the lattice ↔ quadrupoles

$$\beta(s + L) = \beta(s)$$

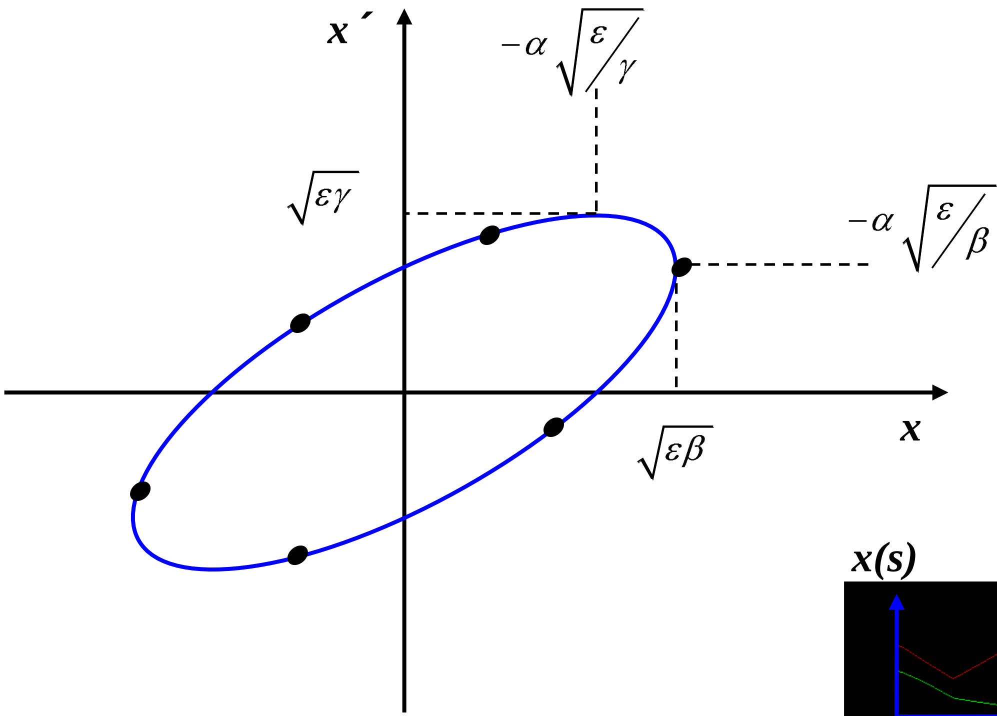
ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
scientifiquely spoken: area covered in transverse x, x´phase space … and it is
constant !!!

Ψ(s) = „phase advance“ of the oscillation between point „0“ and „s“ in the lattice.
For one complete revolution: number of oscillations per turn „Tune“

$$Q_y = \frac{1}{2\pi} \cdot \int \frac{ds}{\beta(s)}$$

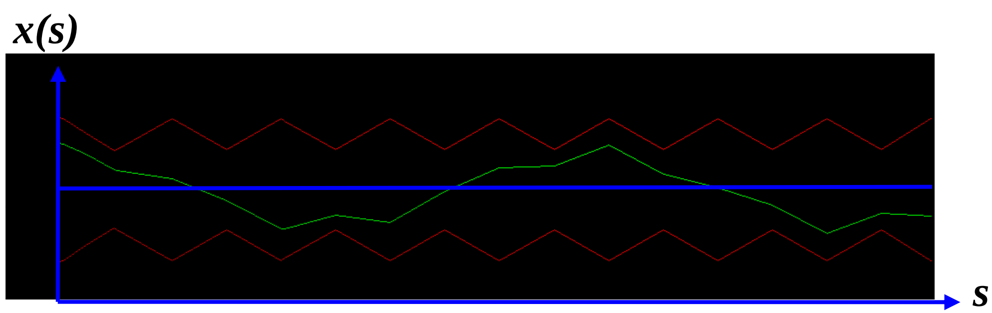
7.) Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$



***Liouville:* in reasonable storage rings**
area in phase space is constant.

A = π*ε=const

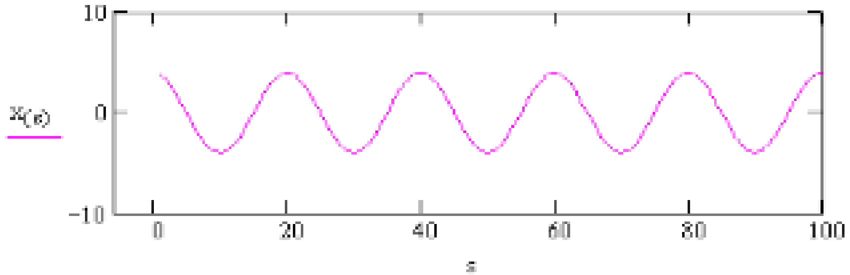


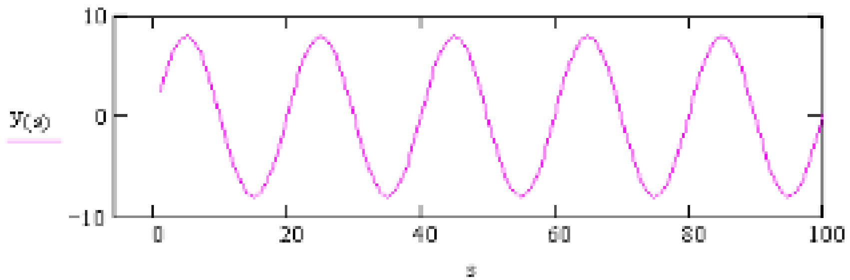
ε beam emittance* = *woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
Scientifiquely spoken: area covered in transverse x, x´phase space … and it is constant !!!

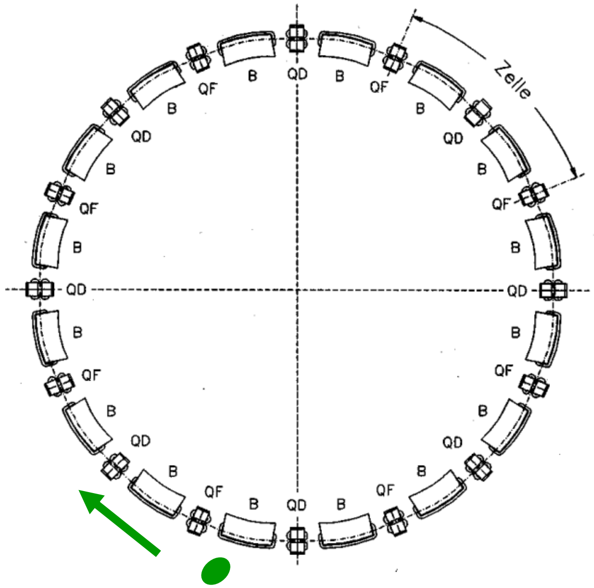
Particle Tracking in a Storage Ring

Calculate x, x´ for each linear accelerator element according to matrix formalism

plot x, x´as a function of „s"

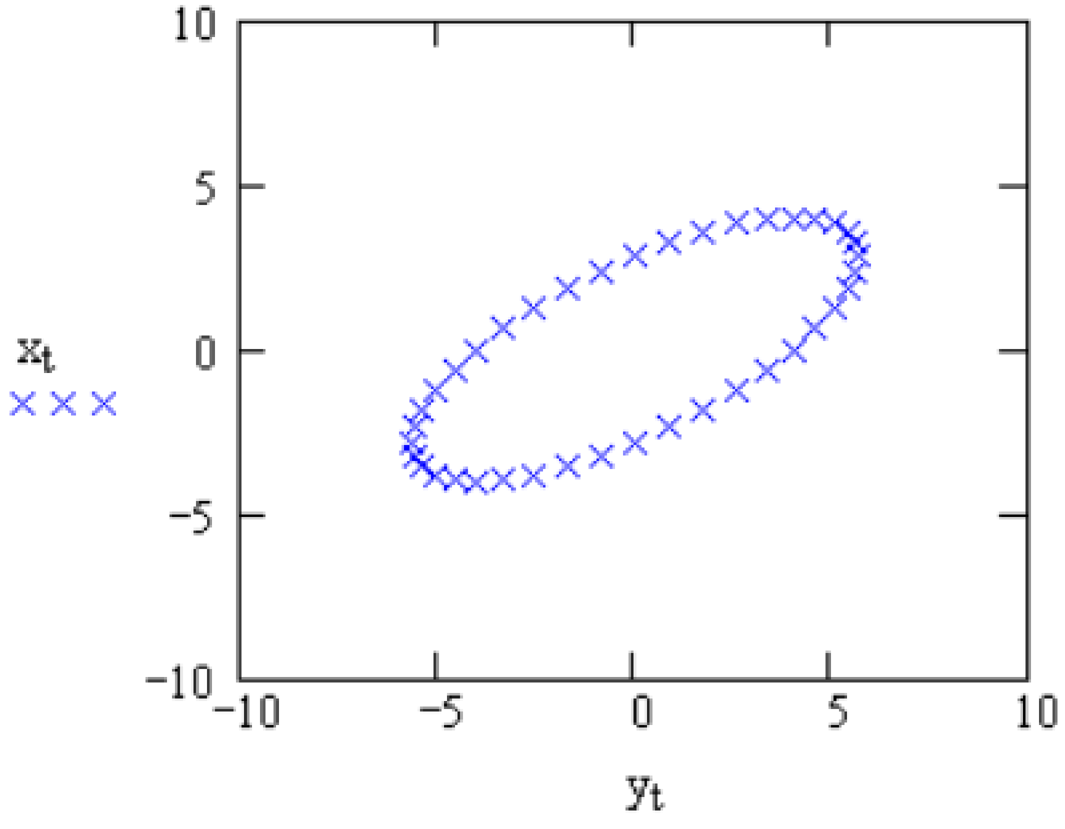




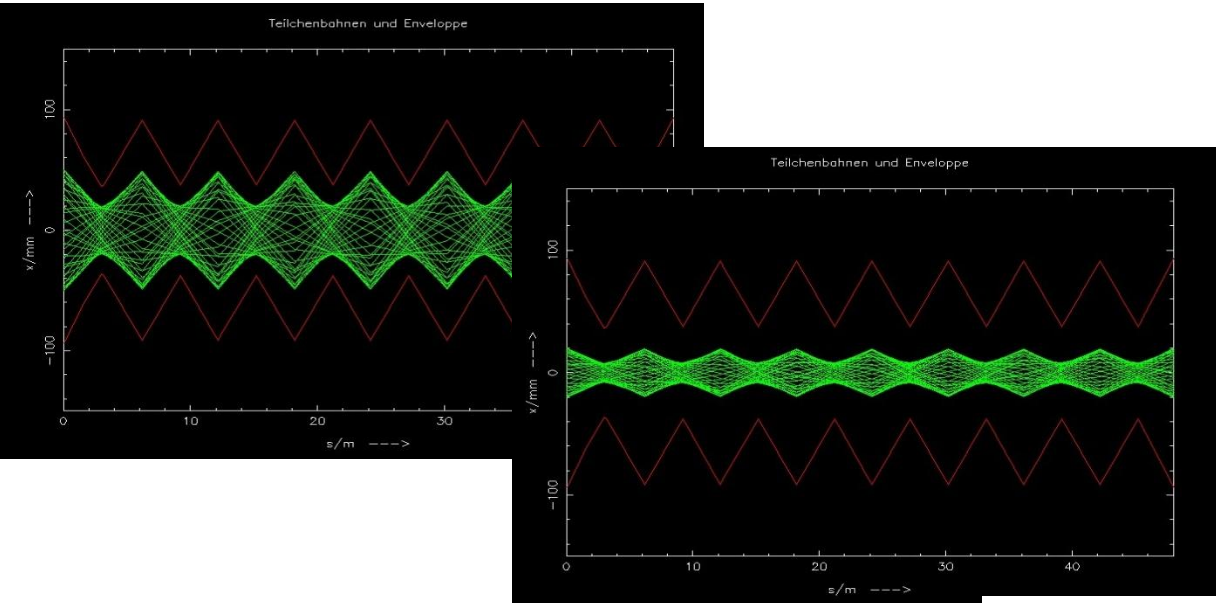


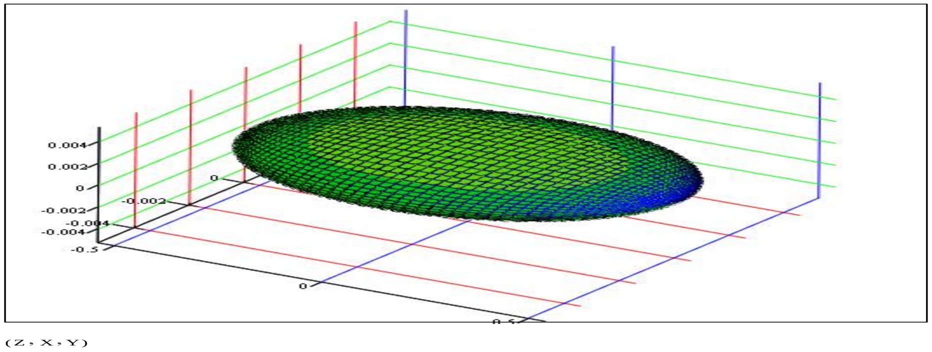
… and now the ellipse:

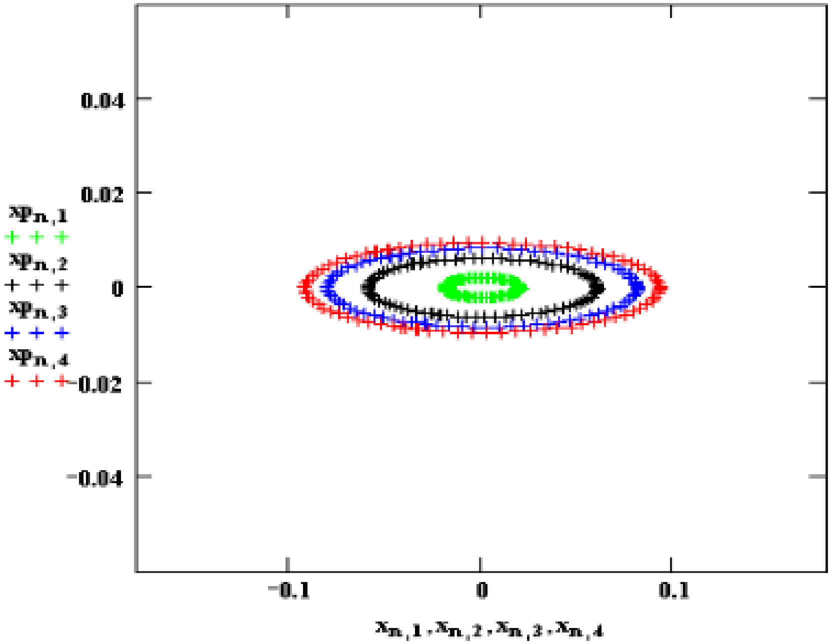
note for each turn x, x´at a given position „s_1" and plot in the phase space diagram



Emittance of the Particle Ensemble:



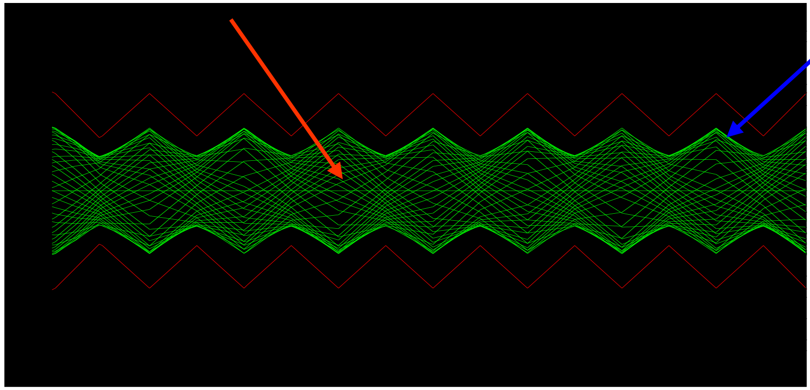




Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi)$$

$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



single particle trajectories, N ≈ 10 [11] per bunch

Gauß Particle Distribution:

$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\,\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

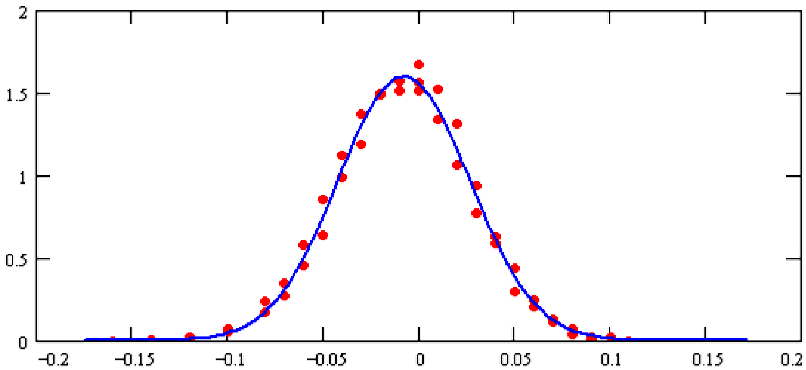
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

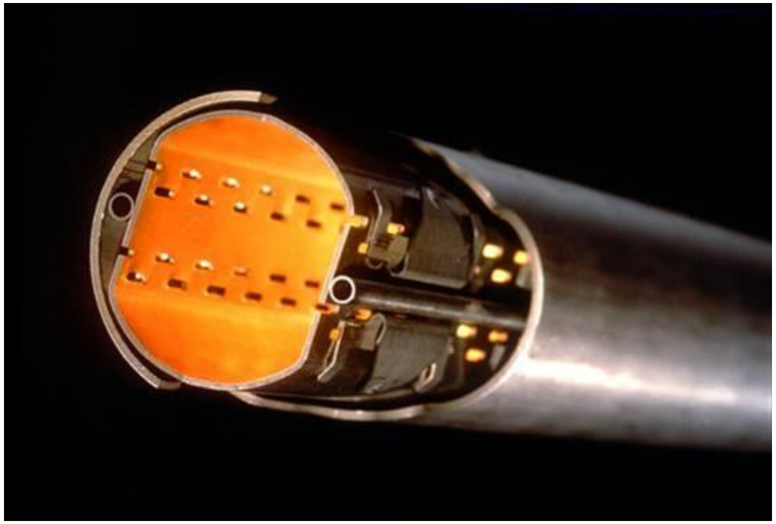
LHC:

$$\beta = 180\ m$$

$$\varepsilon = 5 * 10^{-10}\ m\ rad$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5 * 10^{-10}\ m * 180\ m} = 0.3\ mm$$

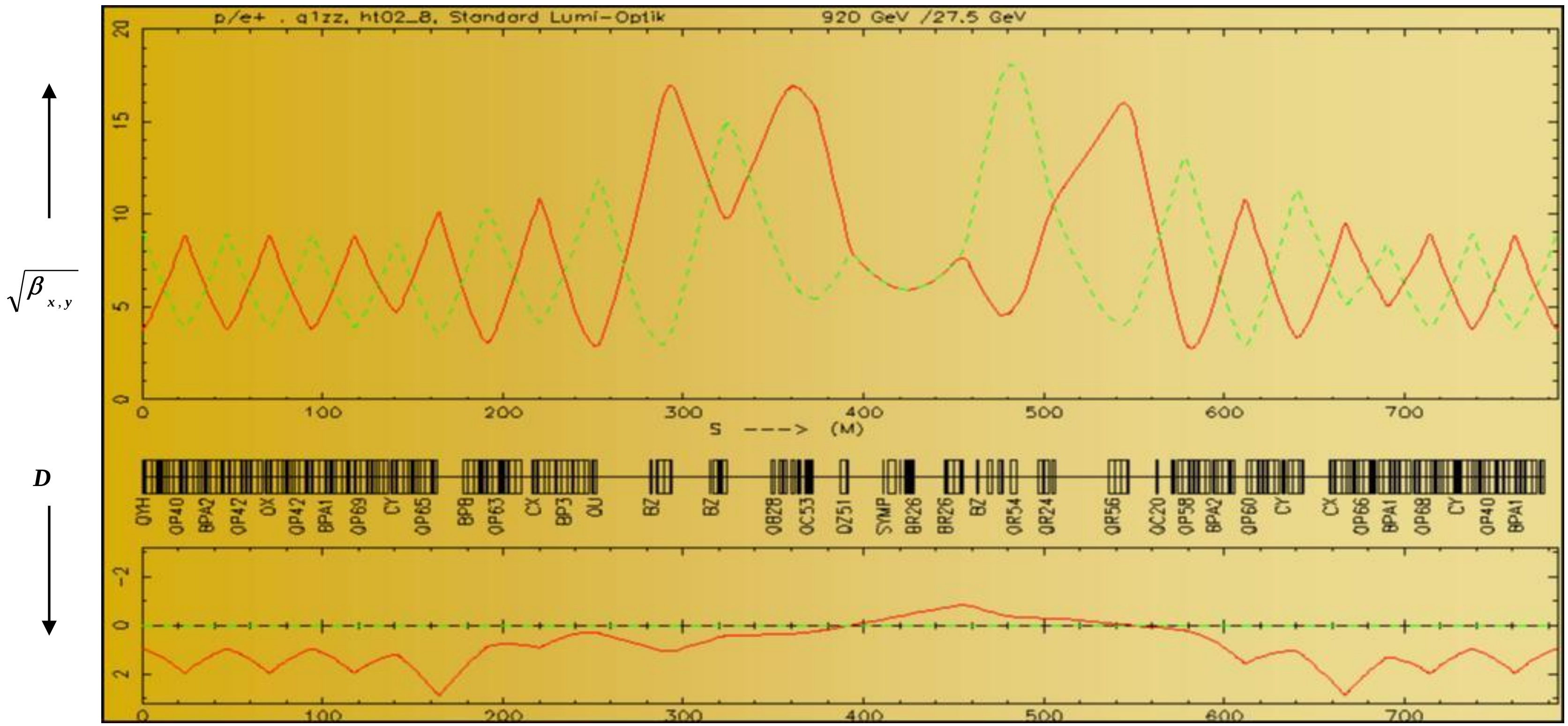




aperture requirements: $r_0 = 12 * \sigma$

III.) The „not so ideal“ World

Lattice Design in Particle Accelerators



1952: Courant, Livingston, Snyder:

Theory of strong focusing in particle beams

Recapitulation: ...the story with the matrices !!!

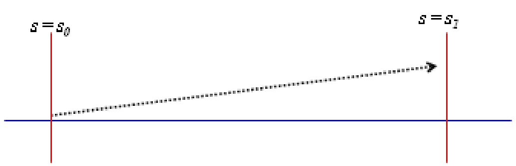
Equation of Motion:

$$x'' + K\,x = 0 \qquad K = 1/\rho^2 - k \quad \dots \textit{ hor. plane:}$$

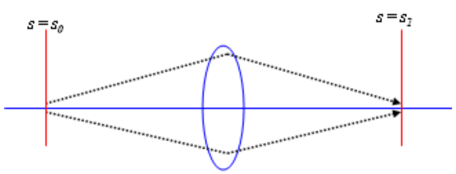
$$K = k \quad \dots \textit{ vert. Plane:}$$

Solution of Trajectory Equations

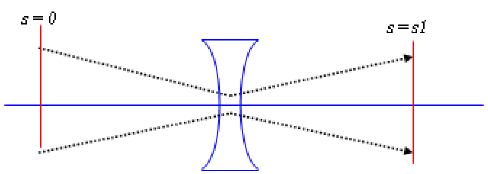
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$



$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}\,l) & \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}\,l) \\ -\sqrt{|K|}\sin(\sqrt{|K|}\,l) & \cos(\sqrt{|K|}\,l) \end{pmatrix}$$



$$M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|}\,l) & \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}\,l) \\ \sqrt{|K|}\sinh(\sqrt{|K|}\,l) & \cosh(\sqrt{|K|}\,l) \end{pmatrix}$$

$$M_{total} = M_{QF} * M_D * M_B * M_D * M_{QD} * M_D * \quad \dots$$

8.) Lattice Design: *„… how to build a storage ring"*

Geometry of the ring:

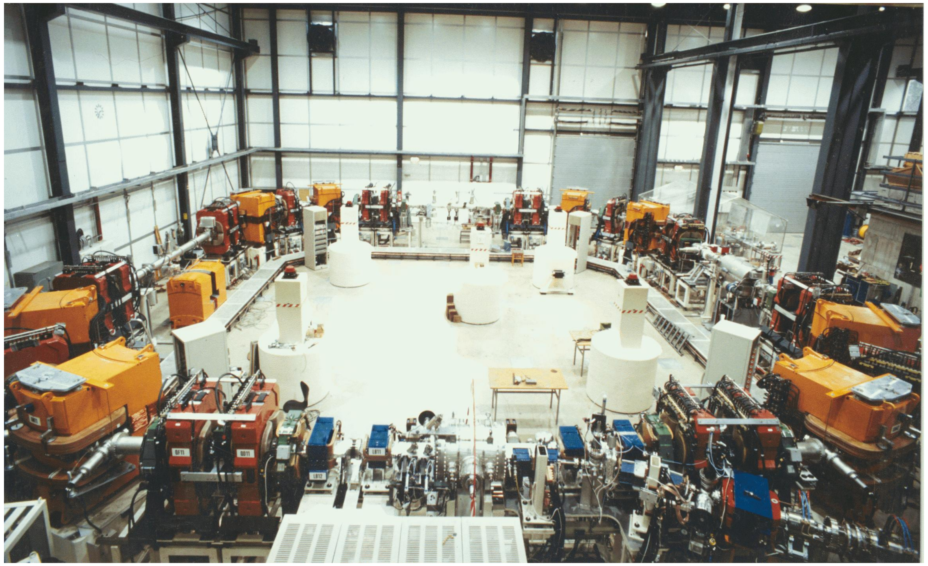
$$B * \rho = p / e$$

p = momentum of the particle,
ρ = curvature radius

Bρ= beam rigidity

Circular Orbit: bending angle of one dipole

$$\alpha = \frac{ds}{\rho} \approx \frac{dl}{\rho} = \frac{Bdl}{B\rho}$$



The angle run out in one revolution must be 2π, so for a full circle

$$\alpha = \frac{\int Bdl}{B\rho} = 2\pi$$

$$\int Bdl = 2\pi \frac{p}{q}$$

… defines the integrated dipole field around the machine.

Example LHC:

7000 GeV Proton storage ring
dipole magnets N = 1232
l = 15 m
q = +1 e

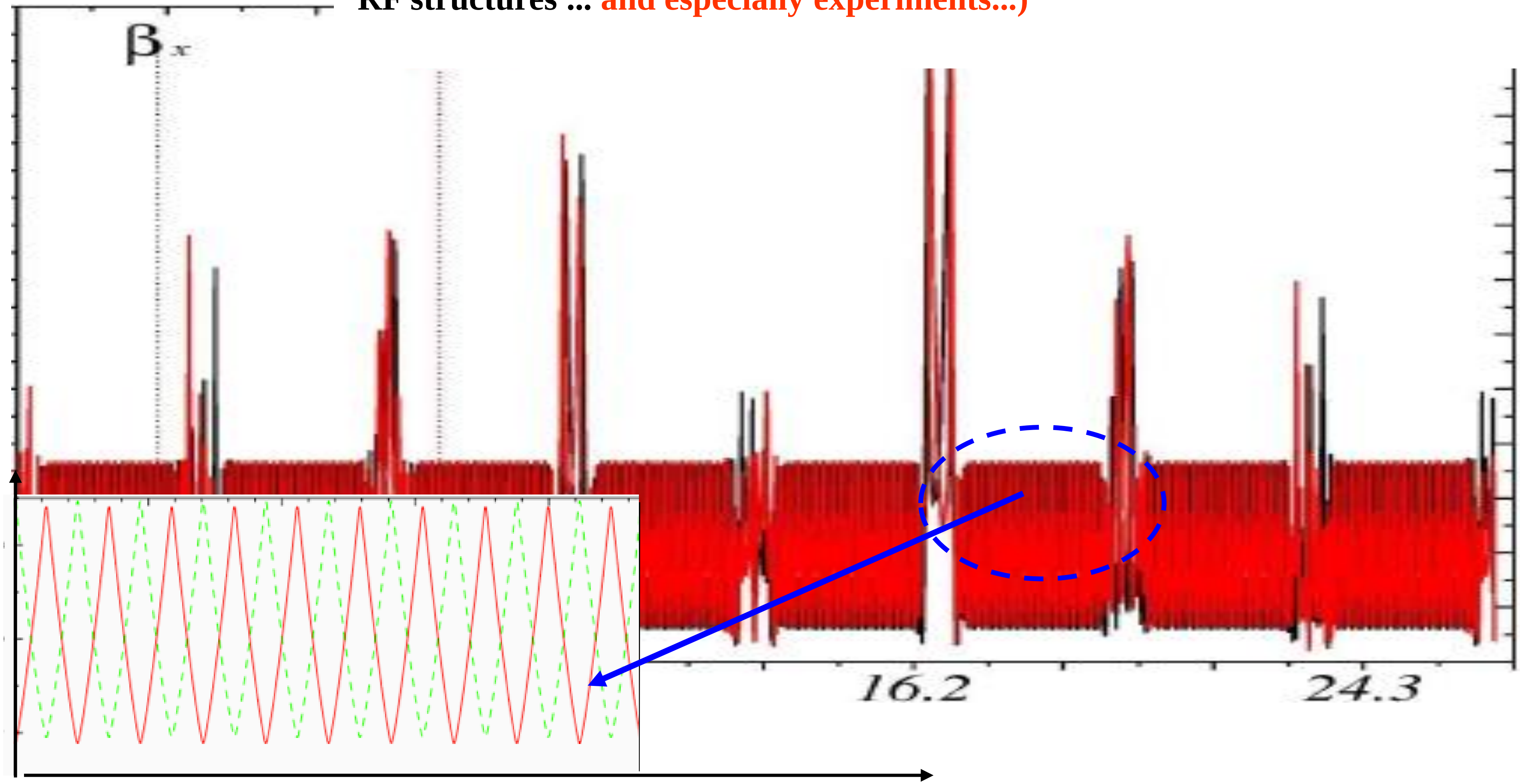
$$\int B\, dl \approx N\, l\, B = 2\pi\, p / e$$

$$B \approx \frac{2\pi\ 7000\ 10^{9}\, eV}{1232\ \ 15\, m\ \ 3\ 10^{8}\, \frac{m}{s}\ \ e} = 8.3\ Tesla$$

FoDo-Lattice

A magnet structure consisting of focusing and defocusing quadrupole lenses in alternating order with nothing in .

(Nothing = elements that can be neglected on first sight: drift, bending magnets, RF structures ... and especially experiments...)

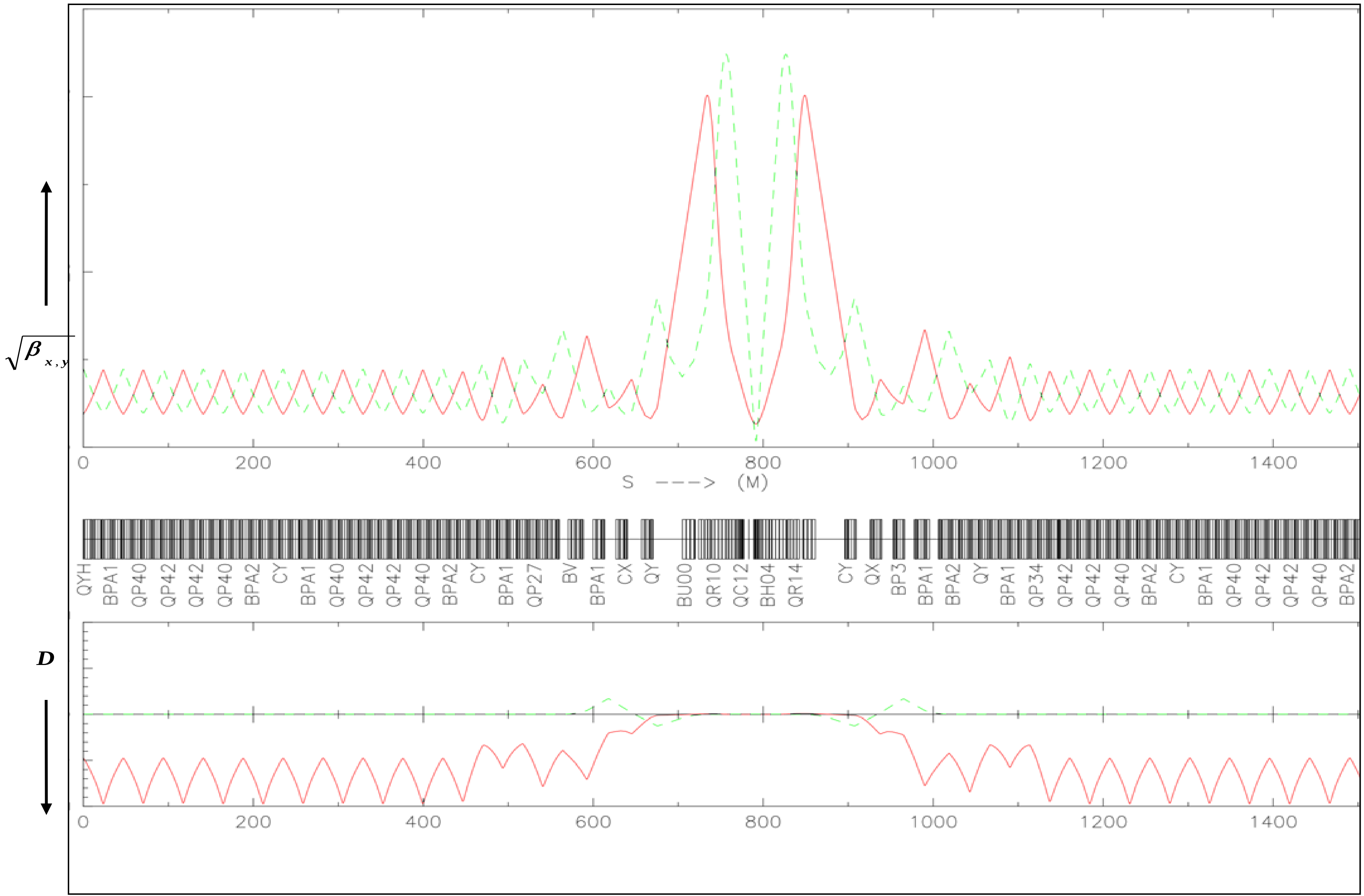


Starting point for the calculation: in the middle of a focusing quadrupole

Phase advance per cell $\mu = 45°$,

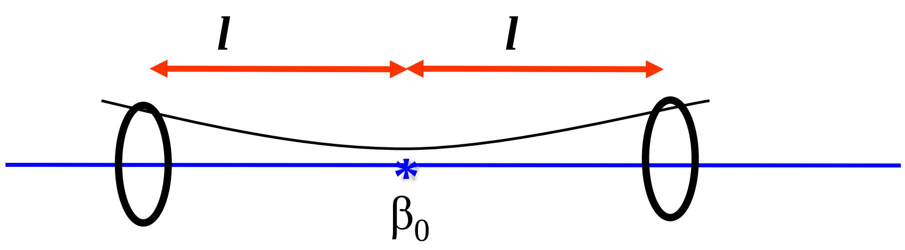
→ calculate the twiss parameters for a periodic solution

9.) Insertions



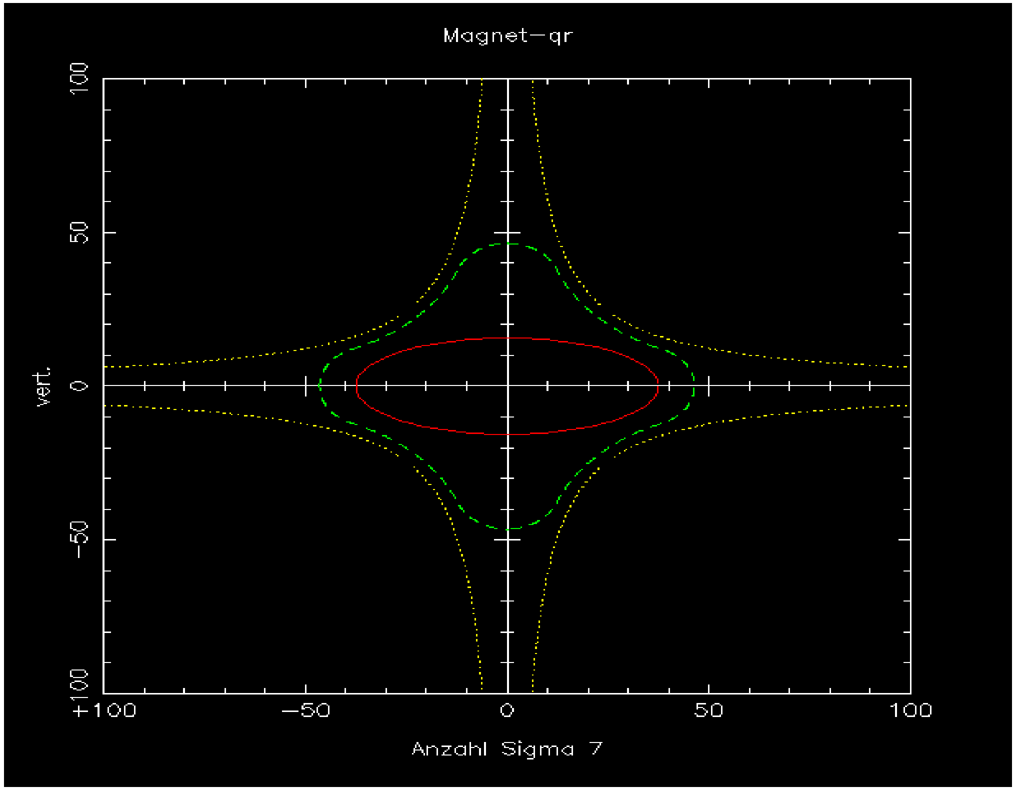
β-Function in a Drift:

$$\beta(\ell) = \beta_0 + \frac{\ell^2}{\beta_0}$$



At the end of a long symmetric drift space the beta function reaches its maximum value in the complete lattice.
-> here we get the largest beam dimension.

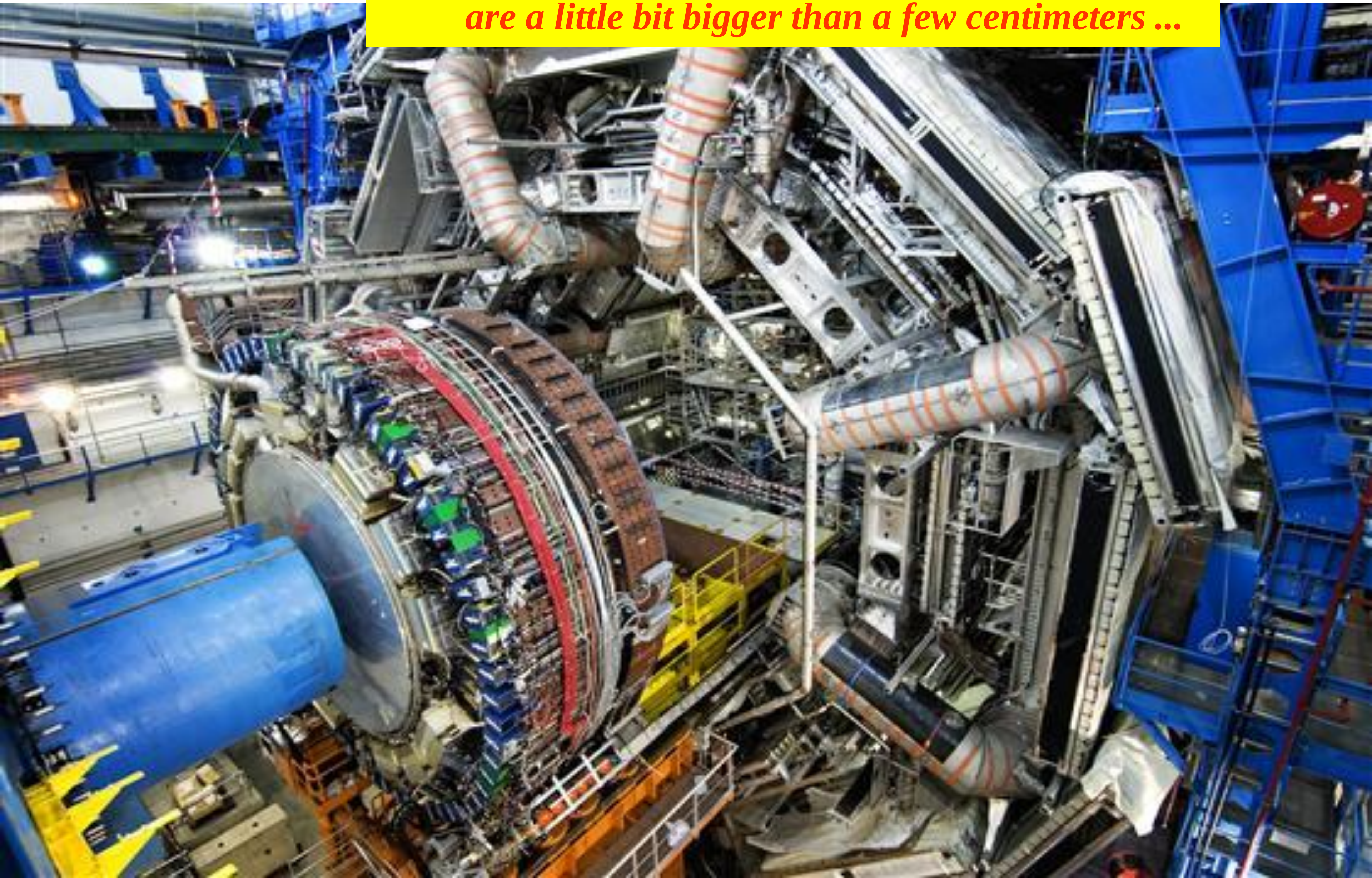
-> keep l as small as possible



7 sima beam size iside a mini beta quadrupole

... clearly there is an

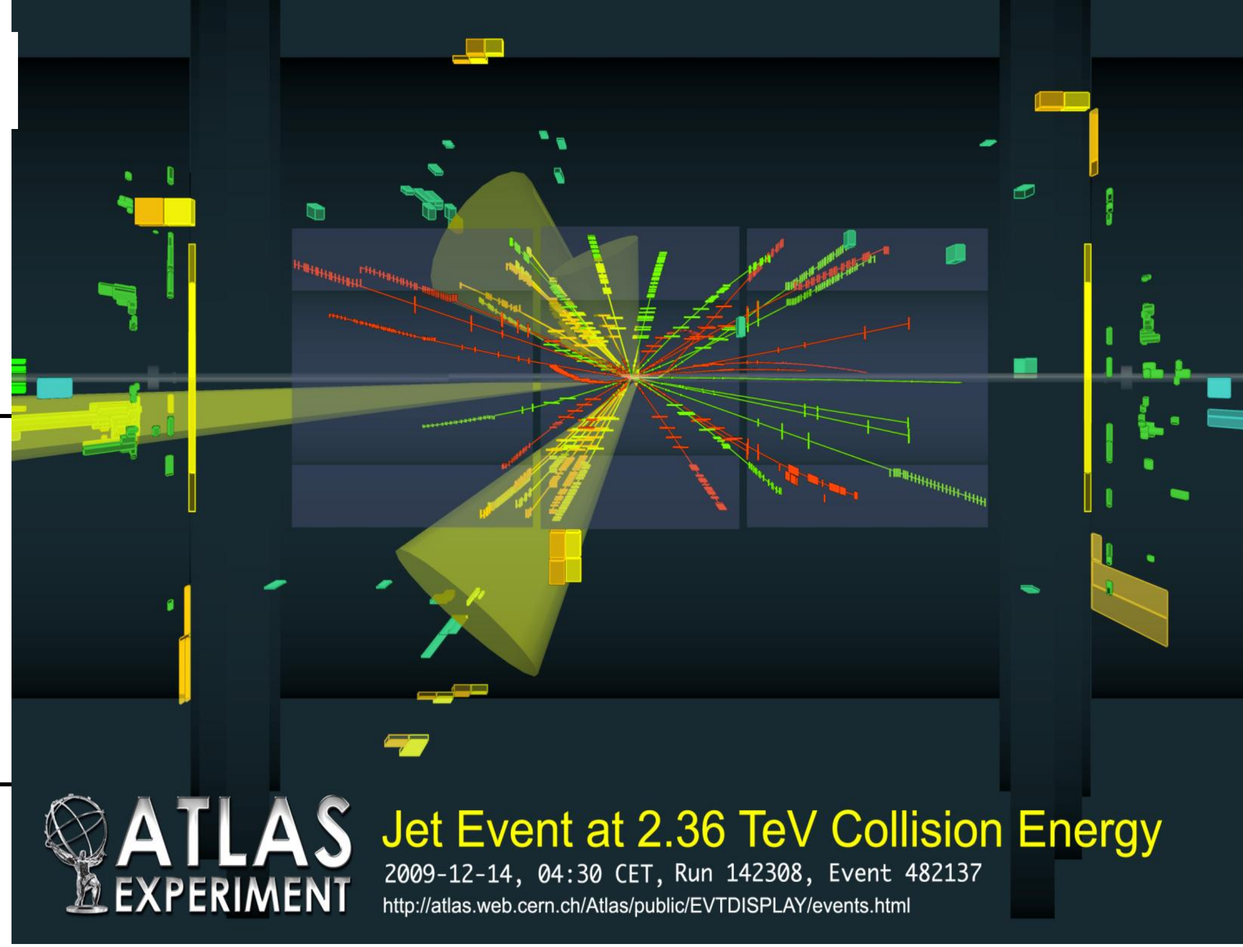
... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...



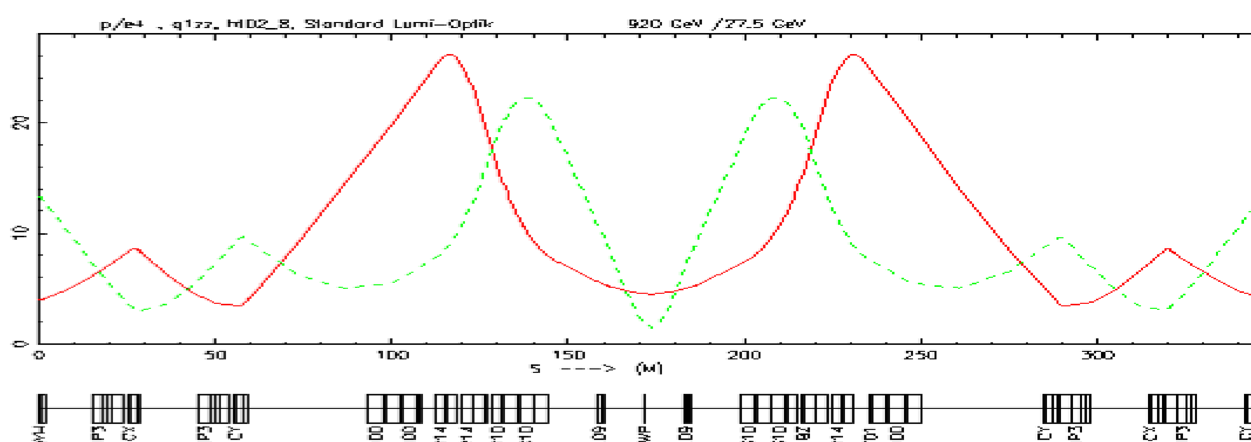
The Mini-β Insertion:

$$R = L * \Sigma_{react}$$

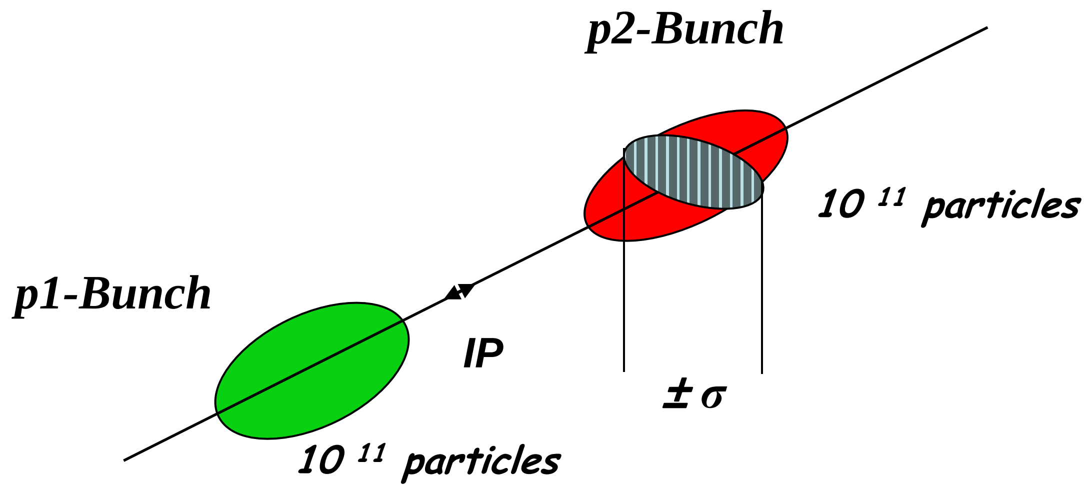
production rate of events
is determined by the
cross section Σ_{react}
and a parameter L that is given
by the design of the accelerator:
… the luminosity



$$L = \frac{1}{4\pi e^2 f_0 b} * \frac{I_1 * I_2}{\sigma^*_x * \sigma^*_y}$$



10.) Luminosity



Example: Luminosity run at LHC

$\beta_{x,y} = 0.55\ m$

$f_0 = 11.245\ kHz$

$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ \ m$

$n_b = 2808$

$\sigma_{x,y} = 17\ \mu m$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

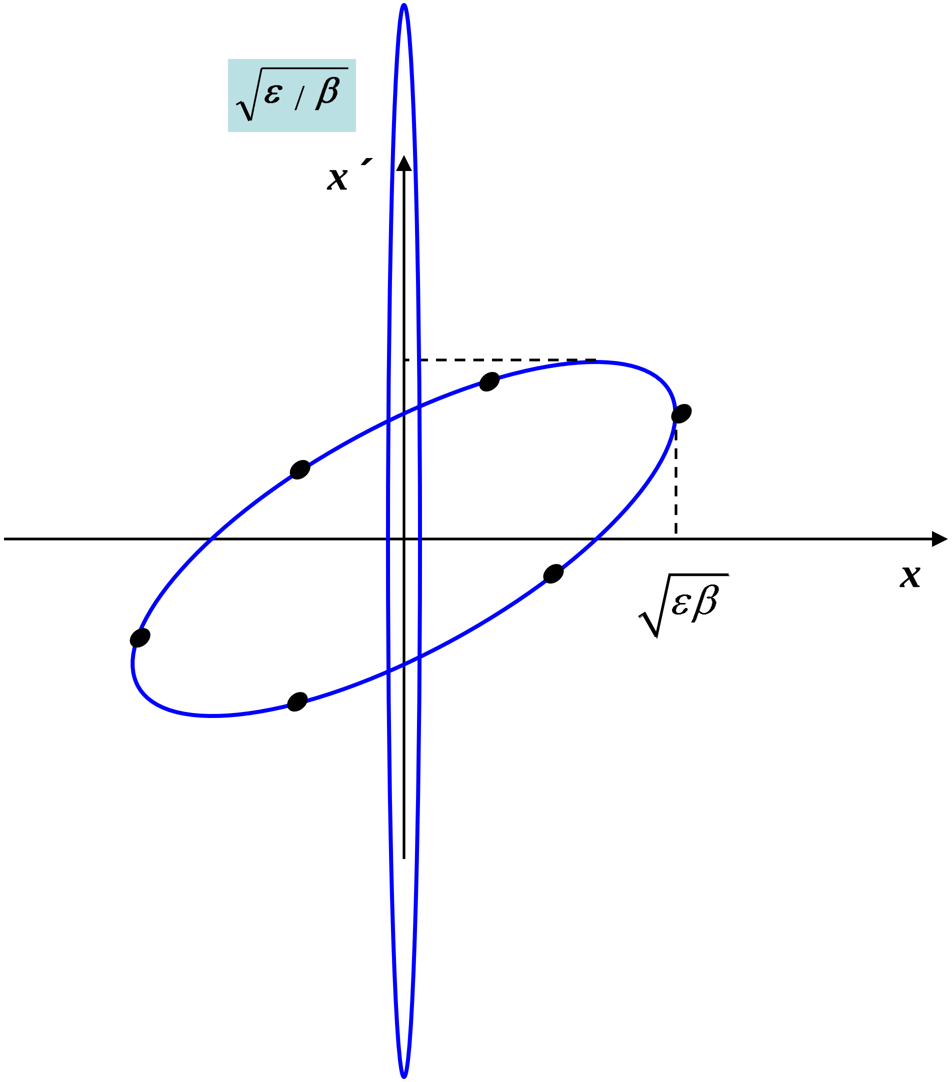
$I_p = 584\ mA$

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

Mini-β Insertions: Betafunctions

A mini-β insertion is always a kind of special symmetric drift space.
→greetings from Liouville

the smaller the beam size
the larger the bam divergence



Mini-β Insertions: some guide lines

* calculate the periodic solution in the arc

* introduce the drift space needed for the insertion device (detector ...)

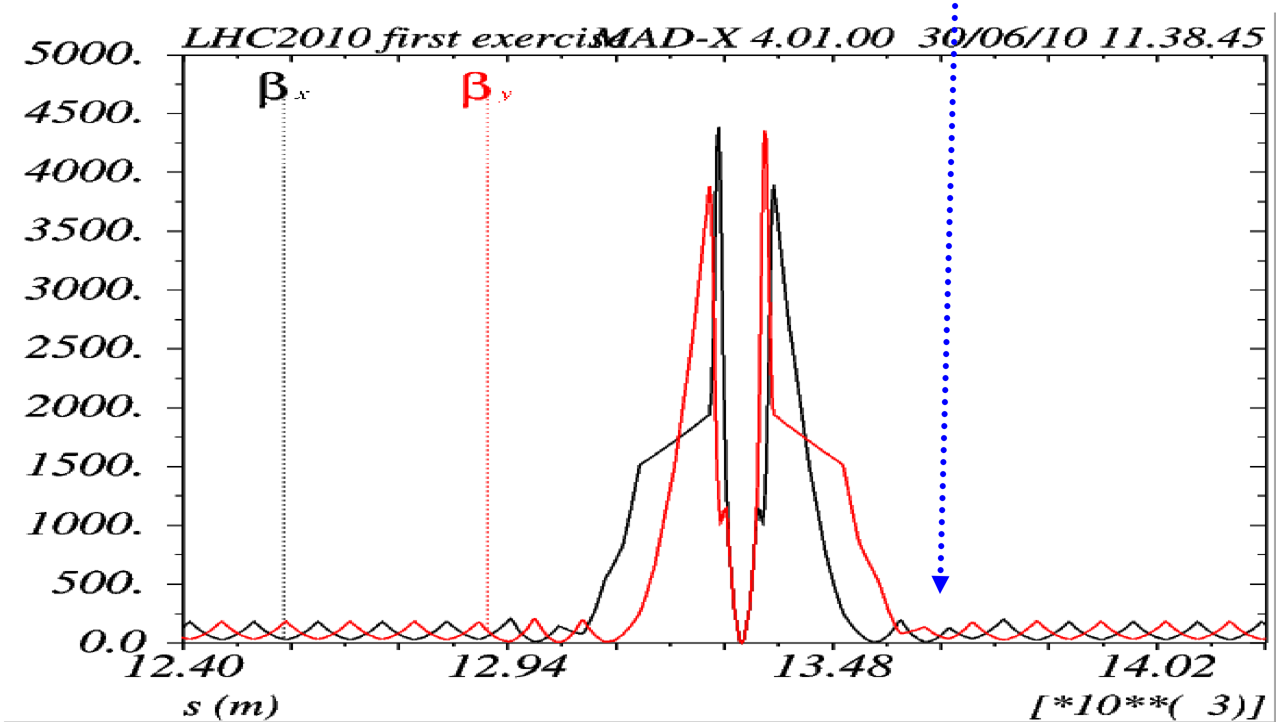
* put a quadrupole doublet (triplet ?) as close as possible

* introduce additional quadrupole lenses to match the beam parameters to the values at the beginning of the arc structure

parameters to be optimised & matched to the periodic solution:

$$\alpha_x, \ \beta_x \qquad D_x, \ D_x'$$
$$\alpha_y, \ \beta_y \qquad Q_x, \ Q_y$$

8 individually powered quad magnets are needed to match the insertion
(... at least)

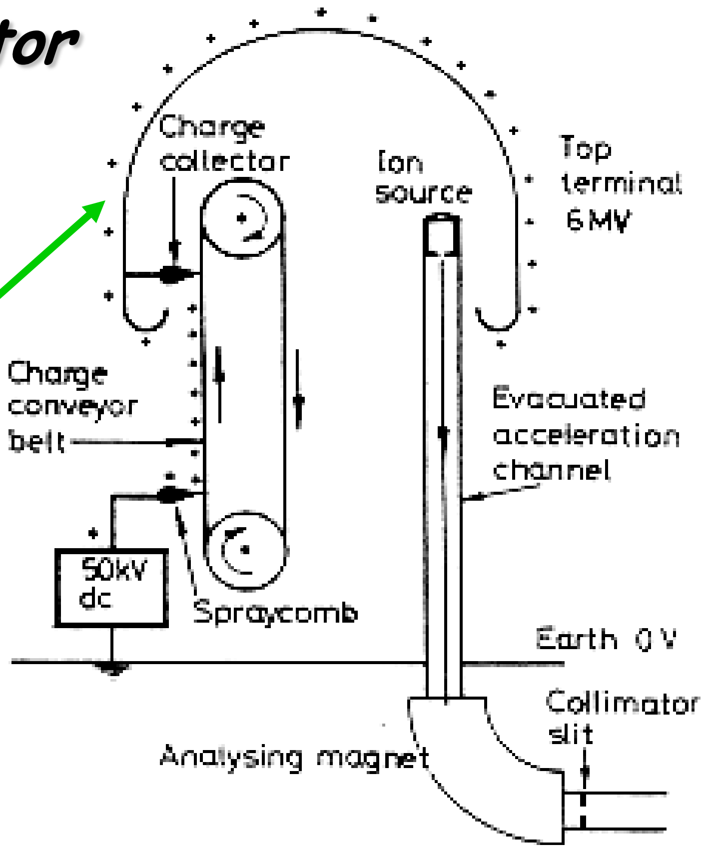


IV) … let´s talk about acceleration

Electrostatic Machines

(Tandem –) van de Graaff Accelerator

creating high voltages by mechanical transport of charges



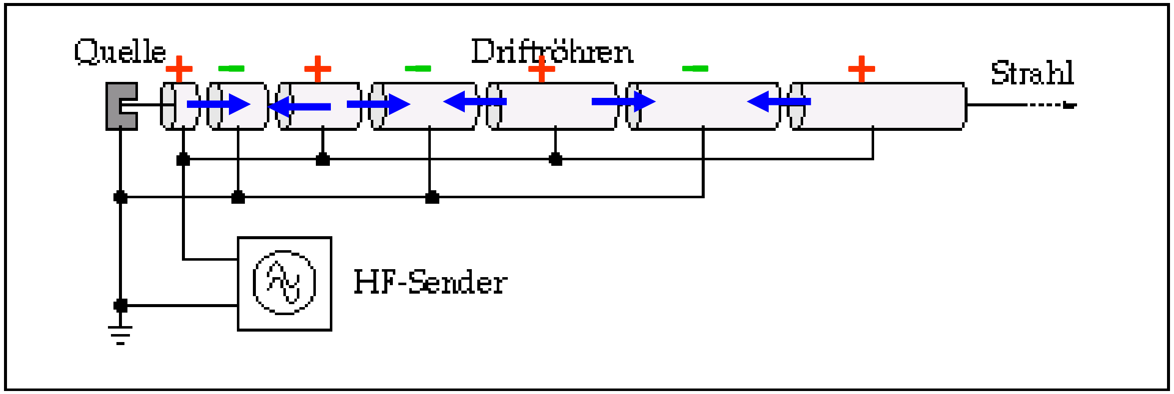
** Terminal Potential: U ≈ 12 ...28 MV*
using high pressure gas to suppress discharge (SF_6)

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

* *The „Tandem principle“:* *Apply the accelerating voltage twice ...*
... by working with negative ions (e.g. H^-) and
stripping the electrons in the centre of the structure

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

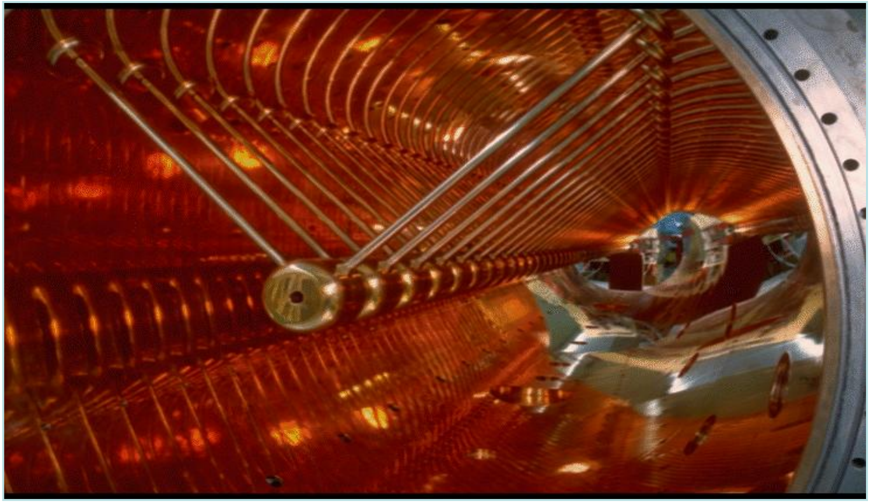
12.) Linear Accelerator 1928, Wideroe



Energy Gain per „Gap":

$$W = q\, U_0 \sin \omega_{RF}\, t$$

drift tube structure at a proton linac (GSI Unilac)



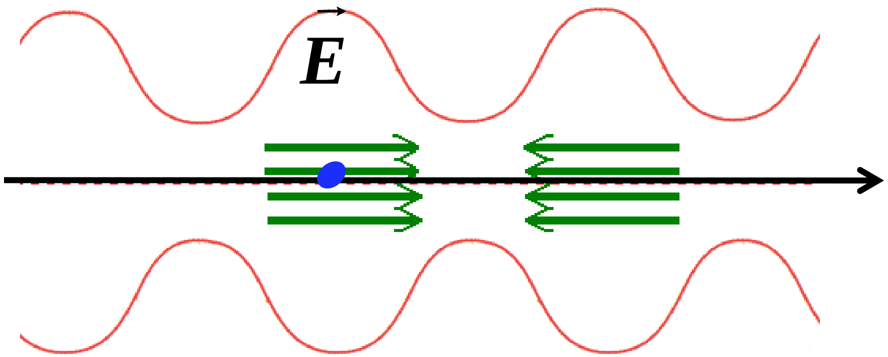
500 MHz cavities in an electron storage ring

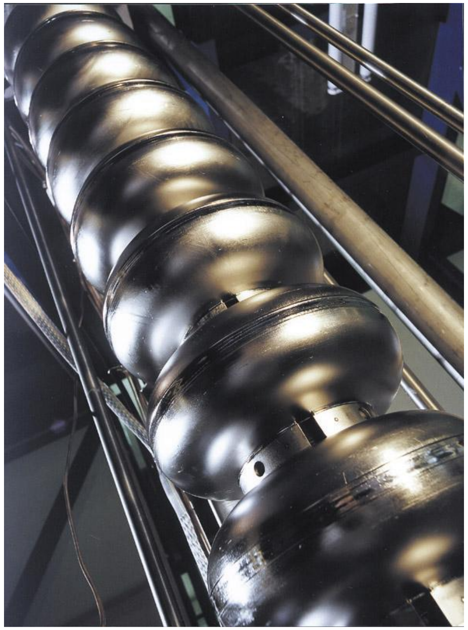
* *RF Acceleration: multiple application of the same acceleration voltage; brillant idea to gain higher energies*

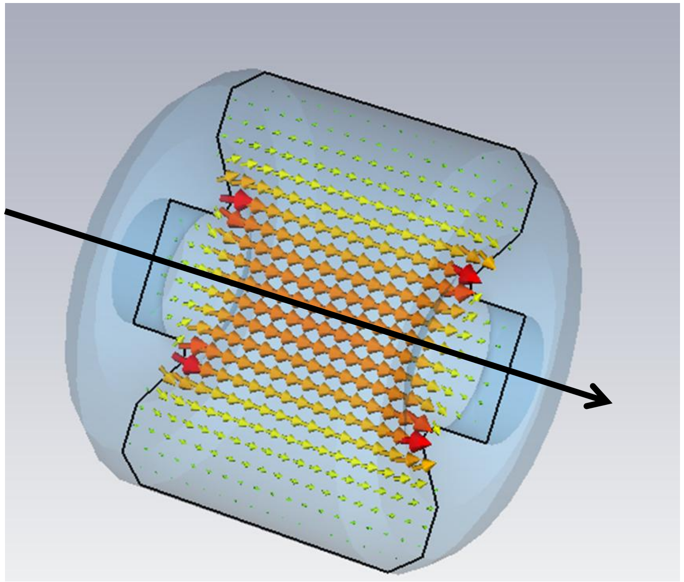
13.) The Acceleration

Where is the acceleration?

Install an RF accelerating structure in the ring:







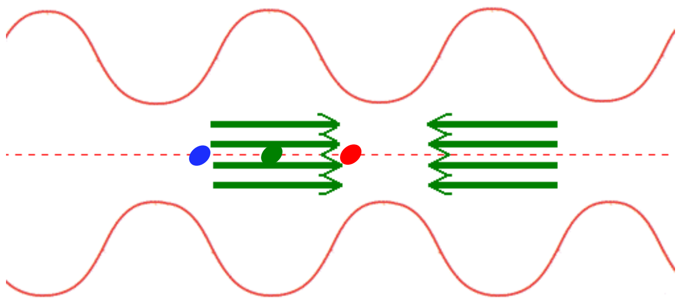
B. Salvant
N. Biancacci

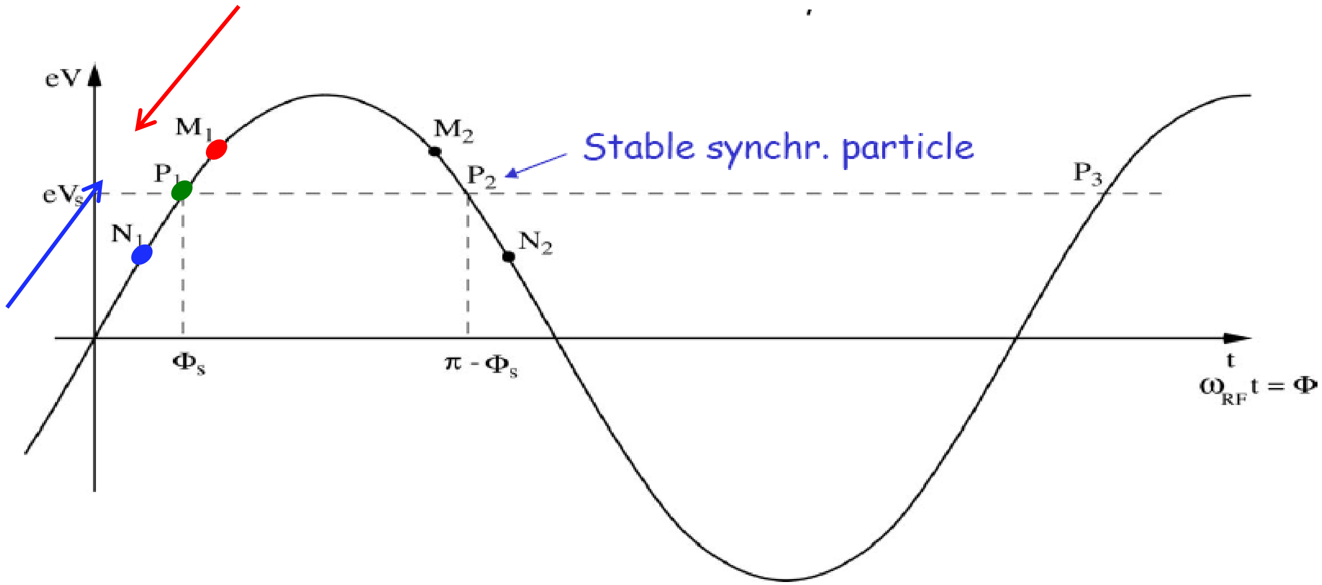
14.) The Acceleration for $\Delta p/p \neq 0$
"Phase Focusing" below transition

ideal particle •

particle with $\Delta p/p > 0$ • *faster*

particle with $\Delta p/p < 0$ • *slower*



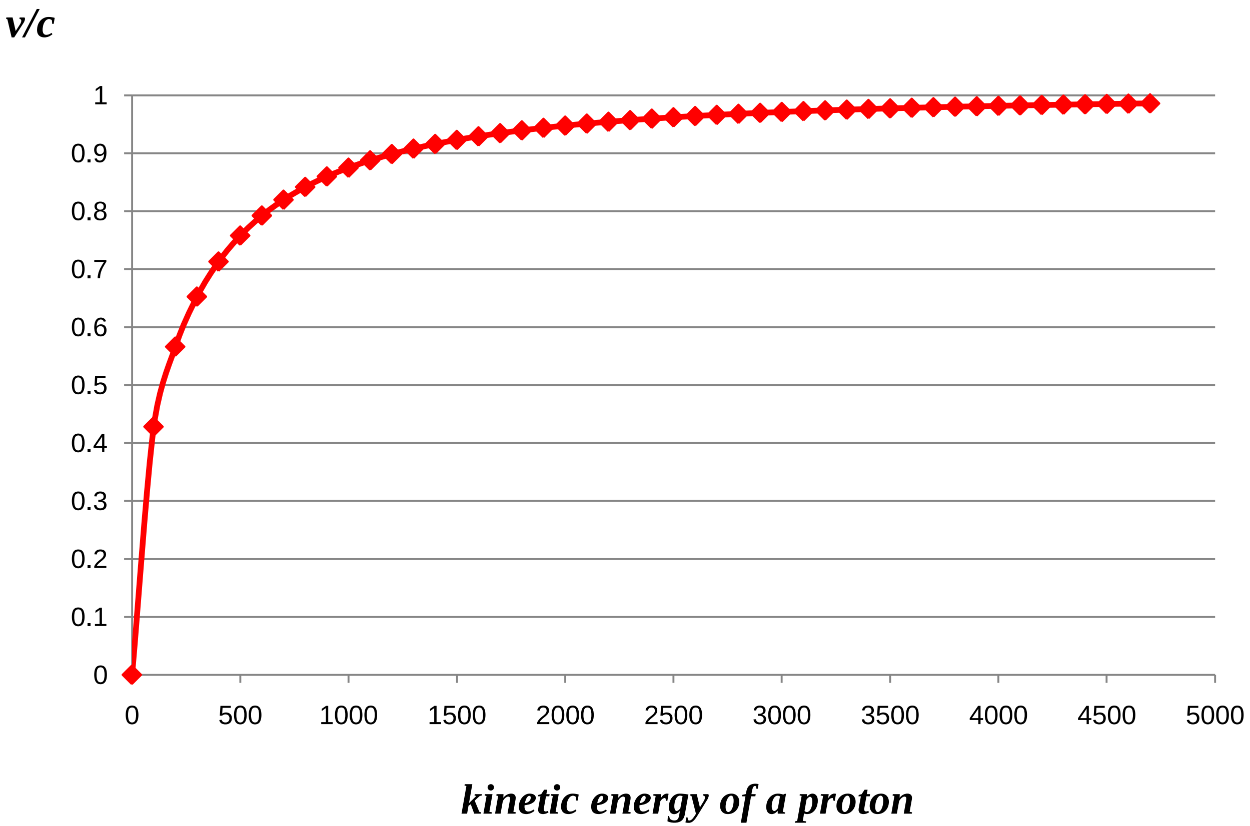


Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

oscillation frequency: $$f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0\cos\phi_s}{E_s}}$$ **≈ some Hz**

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1 - \frac{mc^2}{E^2}}$$



... some when the particles do not get faster anymore

.... but heavier !

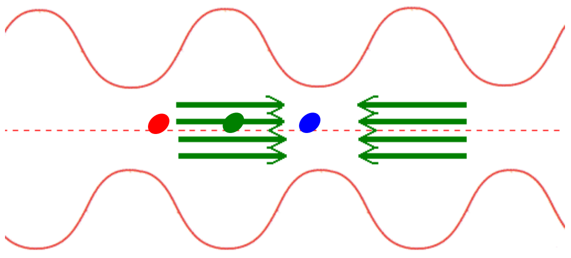
15.) The Acceleration for Δp/p≠0

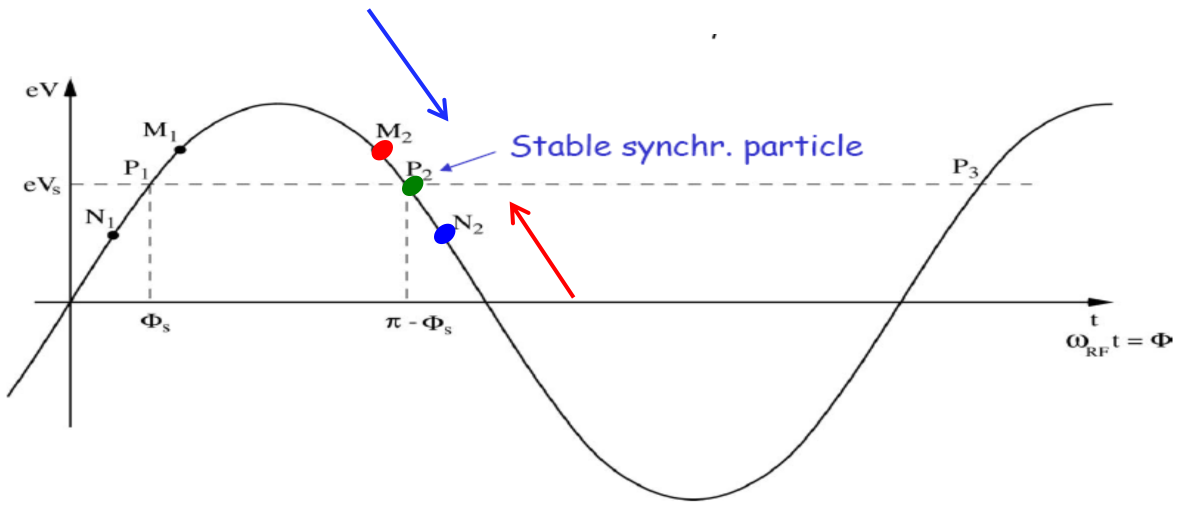
"Phase Focusing" ***above transition***

ideal particle •

particle with Δp/p > 0 • ***heavier***

particle with Δp/p < 0 • ***lighter***





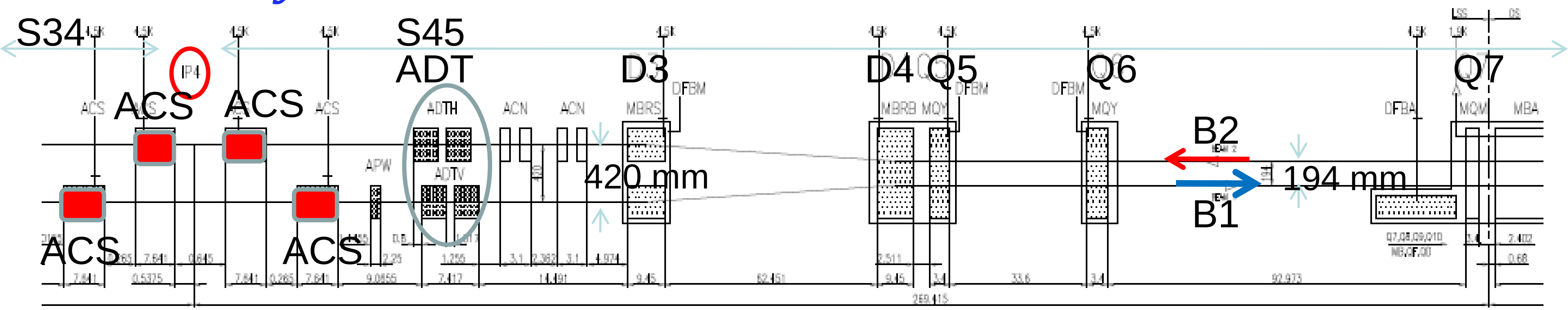
Focussing effect in the longitudinal direction

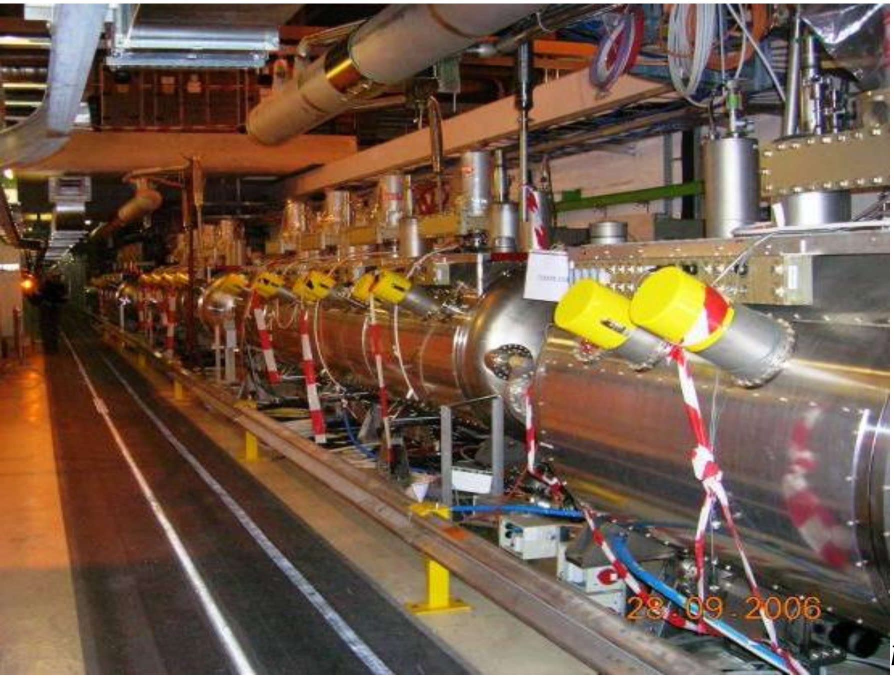
keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4



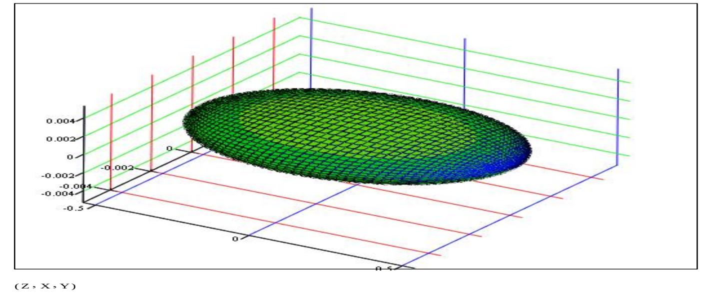


m

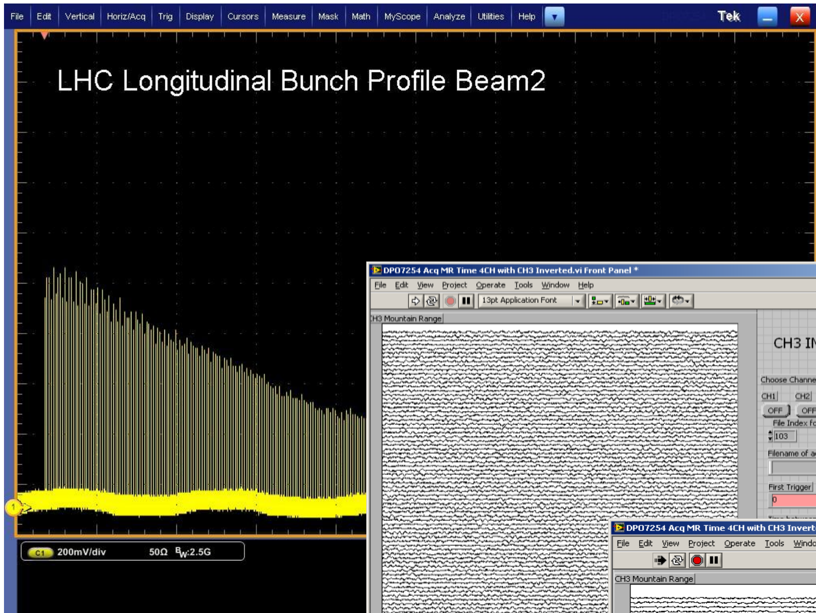
Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

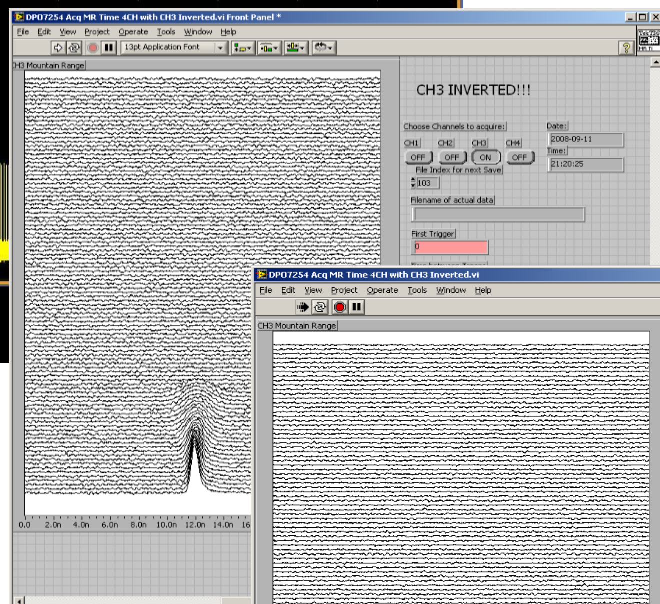
Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	10^{-3}	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*M Hz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

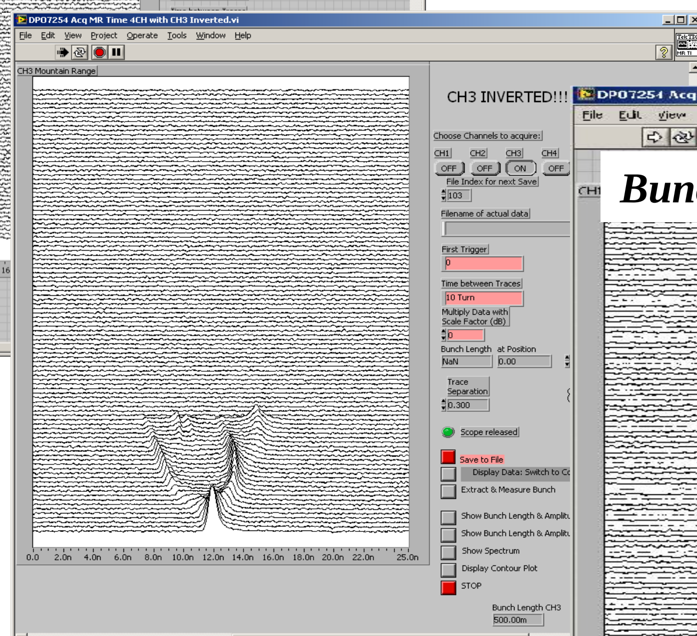
LHC Commissioning: RF

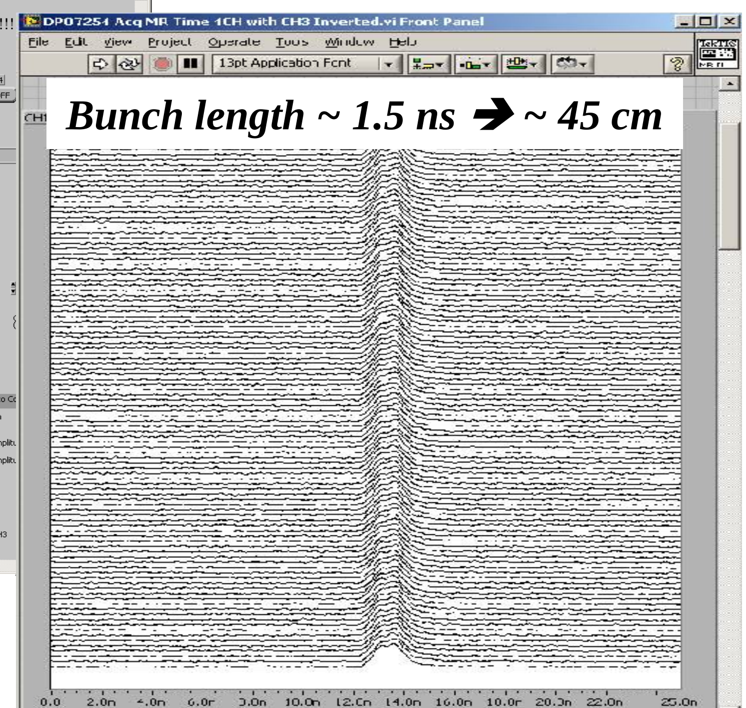


a proton bunch: focused longitudinal by the RF field









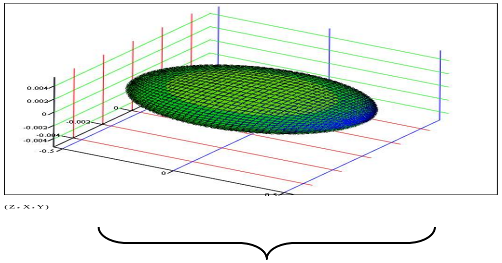
RF off

RF on, phase optimisation

RF on, phase adjusted, beam captured

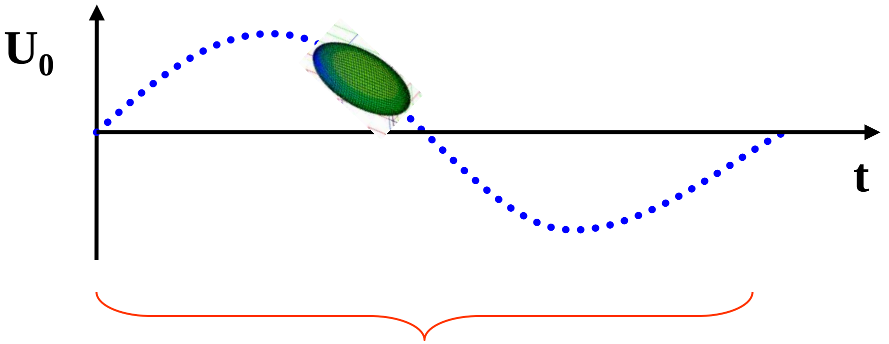
Problem: panta rhei !!!

(Heraklit: 540-480 v. Chr.)



Bunch length of Electrons ≈ 1cm

just a stupid (and nearly wrong) example)



$\lambda = 60\ cm$

$$\left.\begin{array}{l}\nu = 500\ MHz \\ c = \lambda\ \nu\end{array}\right\}\ \lambda = 60\ cm$$

$\sin(90^o) = 1$

$\sin(84^o) = 0.994$

$$\frac{\Delta U}{U} = 6.0\ 10^{-3}$$

typical momentum spread of an electron bunch:

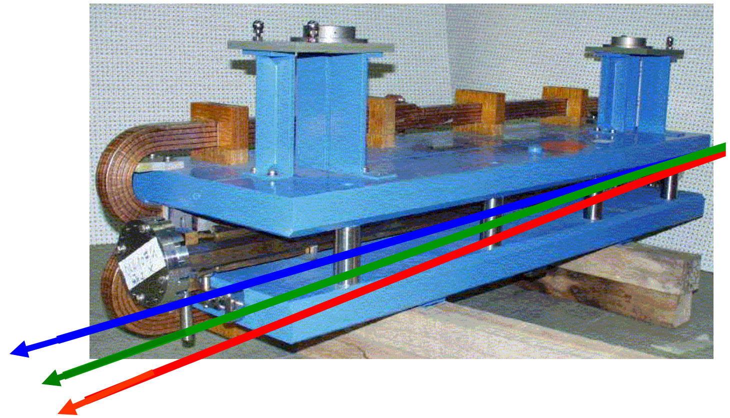
$$\frac{\Delta p}{p} \approx 1.0\ 10^{-3}$$

17.) Dispersion and Chromaticity:
Magnet Errors for $\Delta p/p \neq 0$

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

dipole magnet

$$\alpha = \frac{\int B\, dl}{p/e}$$



$$x_D(s) = D(s)\,\frac{\Delta p}{p}$$

focusing lens

$$k = \frac{g}{p/e}$$

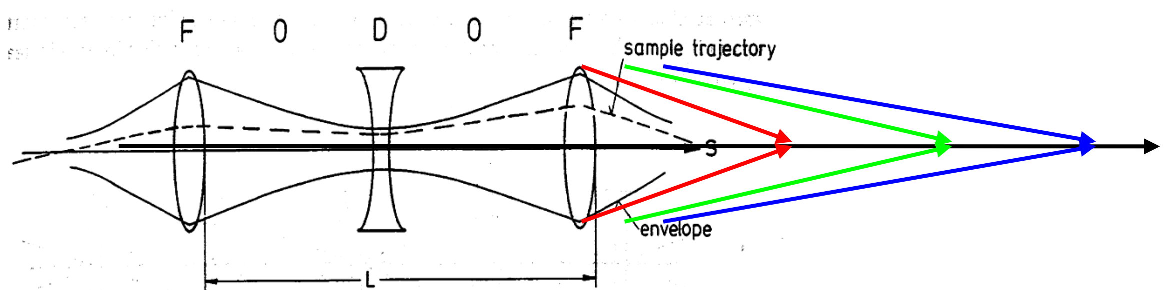
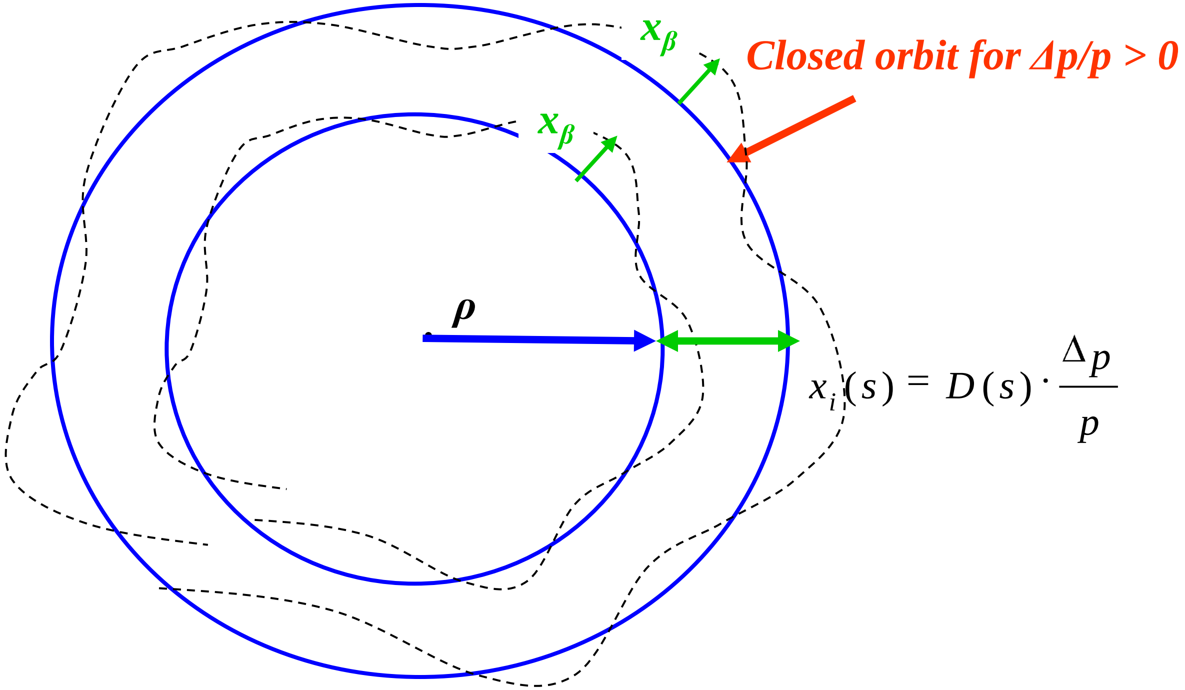


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

Dispersion

Example: homogeneous dipole field



$$x_i(s) = D(s) \cdot \frac{\Delta p}{p}$$

Matrix formalism:

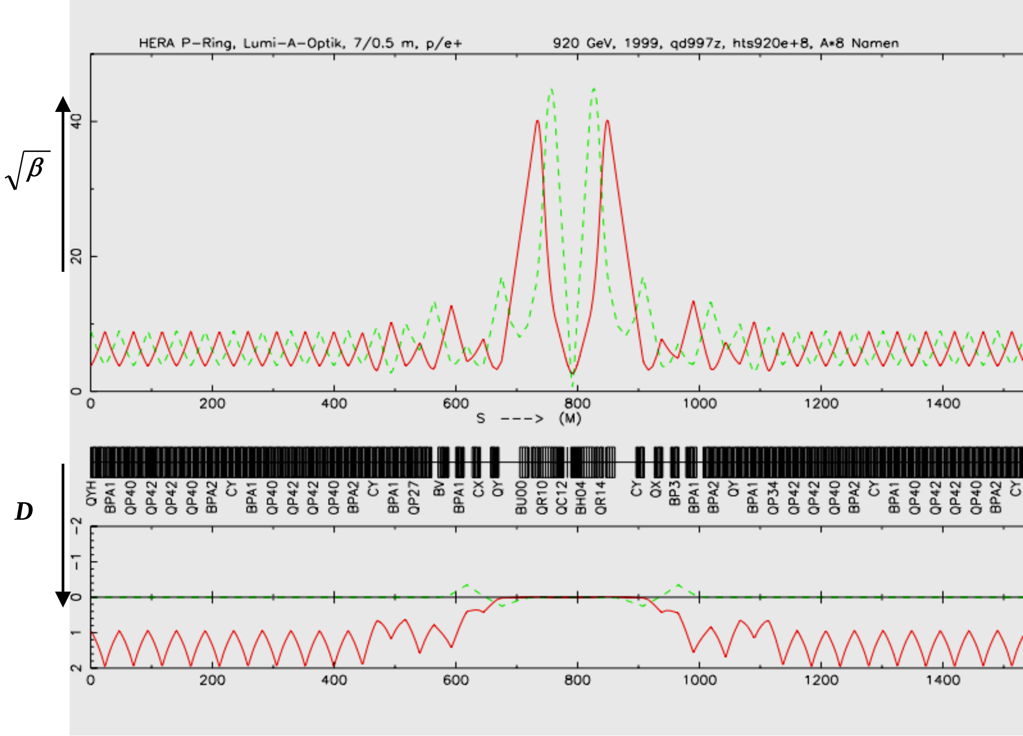
$$x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}$$

$$x(s) = C(s) \cdot x_0 + S(s) \cdot x_0' + D(s) \cdot \frac{\Delta p}{p}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0 + \frac{\Delta p}{p} \begin{pmatrix} D \\ D' \end{pmatrix}_0$$

or expressed as 3x3 matrix

$$\begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_s = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_0$$



Example

$$x_\beta = 1 \dots 2\ mm$$

$$D(s) \approx 1 \dots 2\ m$$

$$\Delta p / p \approx 1 \cdot 10^{-3}$$

Amplitude of Orbit oscillation contribution due to Dispersion ≈ beam size

→ Dispersion must vanish at the collision point

!

Calculate D, D′: ... takes a couple of sunny Sunday evenings !

V.) Are there Any Problems ???

sure there are

Some Golden Rules to Avoid Trouble

I.) Golden Rule number one: do not focus the beam !

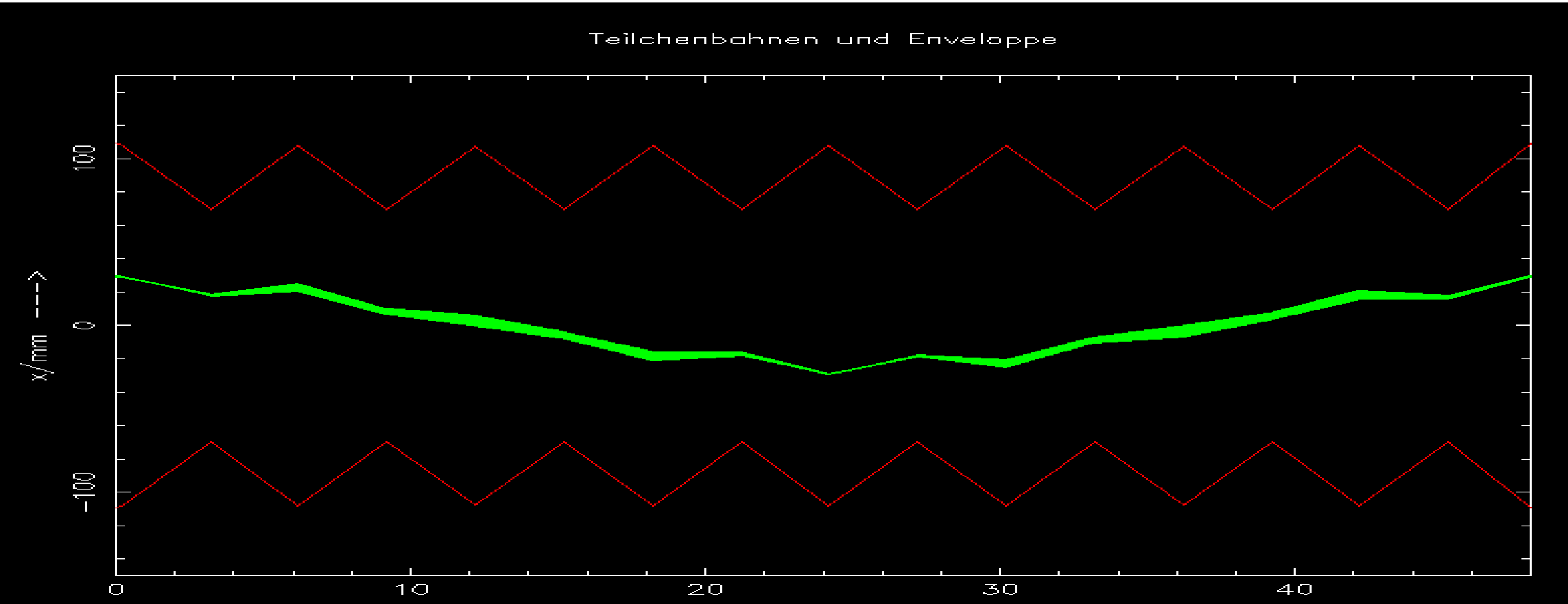
Problem: Resonances

$$x_{co}(s) = \frac{\sqrt{\beta(s)} * \int \frac{1}{\rho_{s1}} \sqrt{\beta_{s1}} * \cos(\psi_{s1} - \psi_s - \pi Q)\, ds}{2 \sin \pi Q}$$

Assume: Tune = integer $Q = 1 \rightarrow 0$

Qualitatively spoken:

Integer tunes lead to a resonant increase of the closed orbit amplitude in presence of the smallest dipole field error.

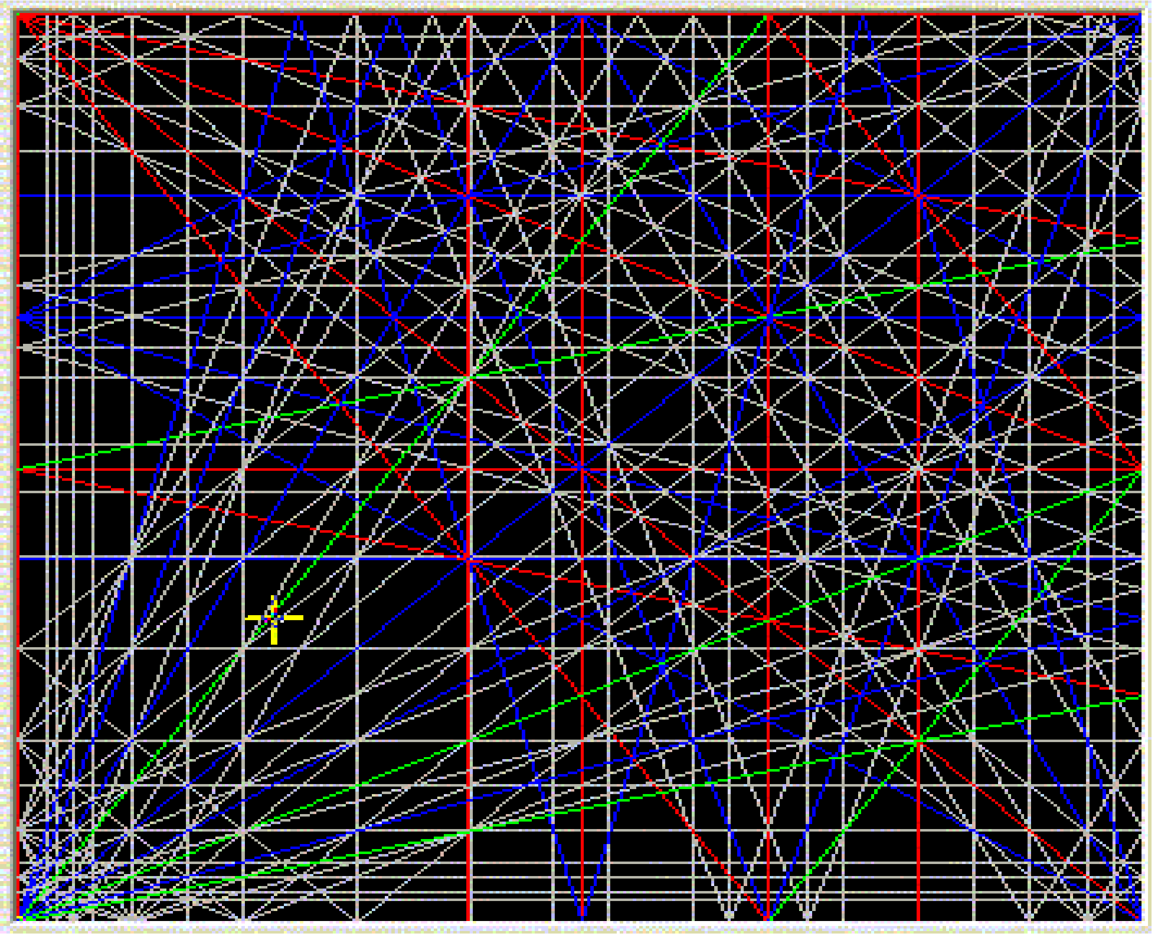


Tune and Resonances

$$m*Q_x+n*Q_y+l*Q_s = integer$$

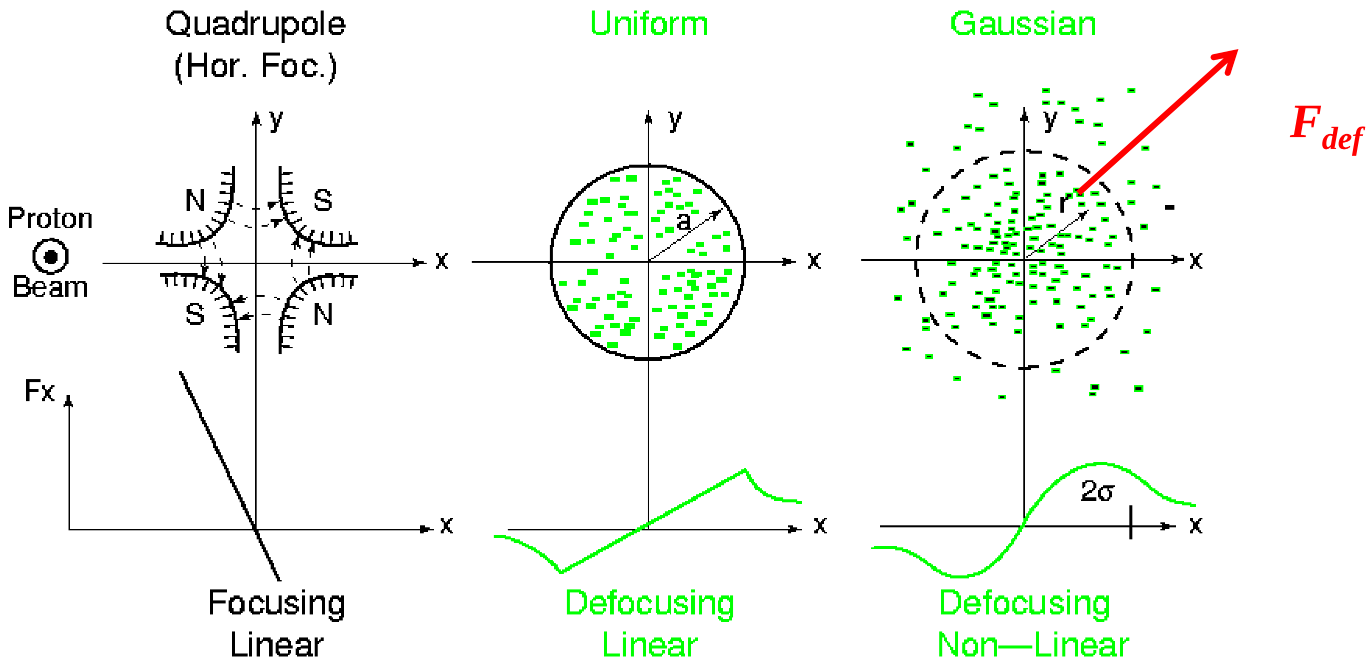
Tune diagram up to 3rd order

… and up to 7th order



Homework for the operateurs: find a nice place for the tune where against all probability the beam will survive

II.) Golden Rule number two: Never accelerate charged particles !



Transport line with quadrupoles

$$x'' + K(s)x = 0$$

Transport line with quadrupoles and space charge

$$x'' + (K(s) + K_{SC}(s))x = 0$$

$$x'' + \left(K(s) - \frac{2 r_0 I}{e a^2 \beta^3 \gamma^3 c} \right) x = 0$$

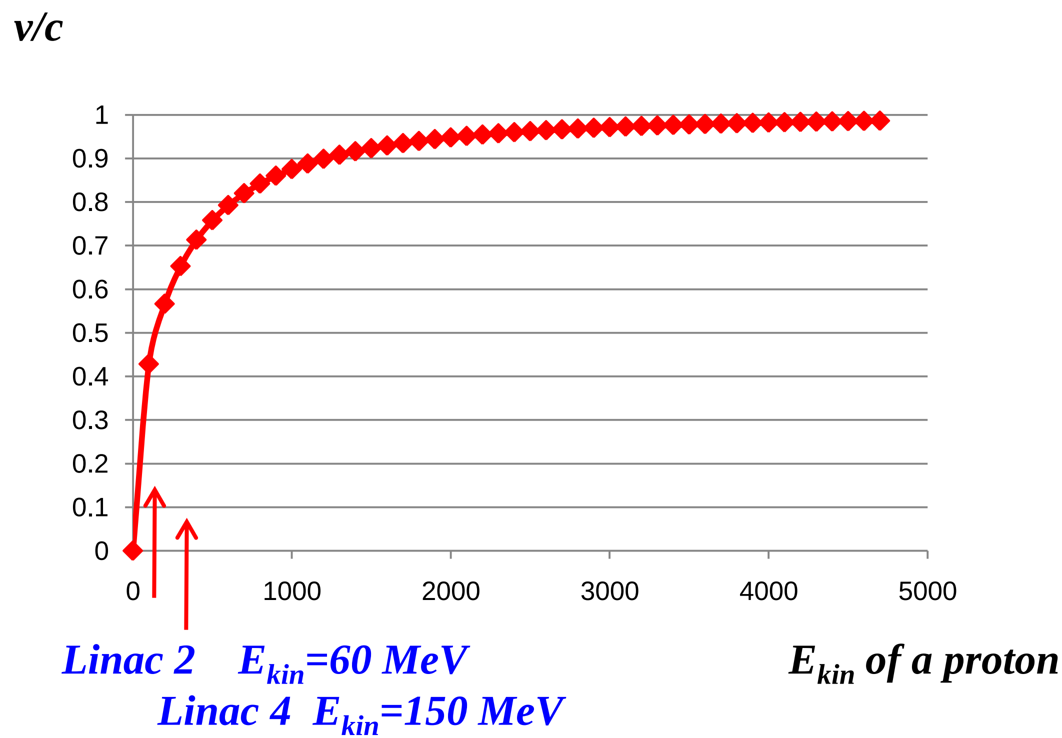
K_{SC}

Golden Rule number two: ***Never accelerate charged particles !***

Tune Shift due to Space Charge Effect

Problem at low energies

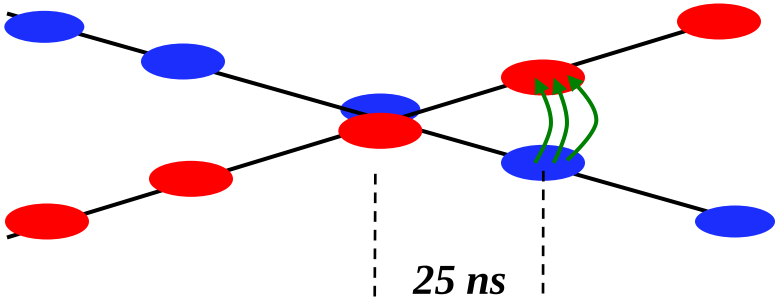
$$\Delta Q_{x,y} = -\frac{r_0 N}{2\pi \varepsilon_{x,y} \beta \gamma^2}$$



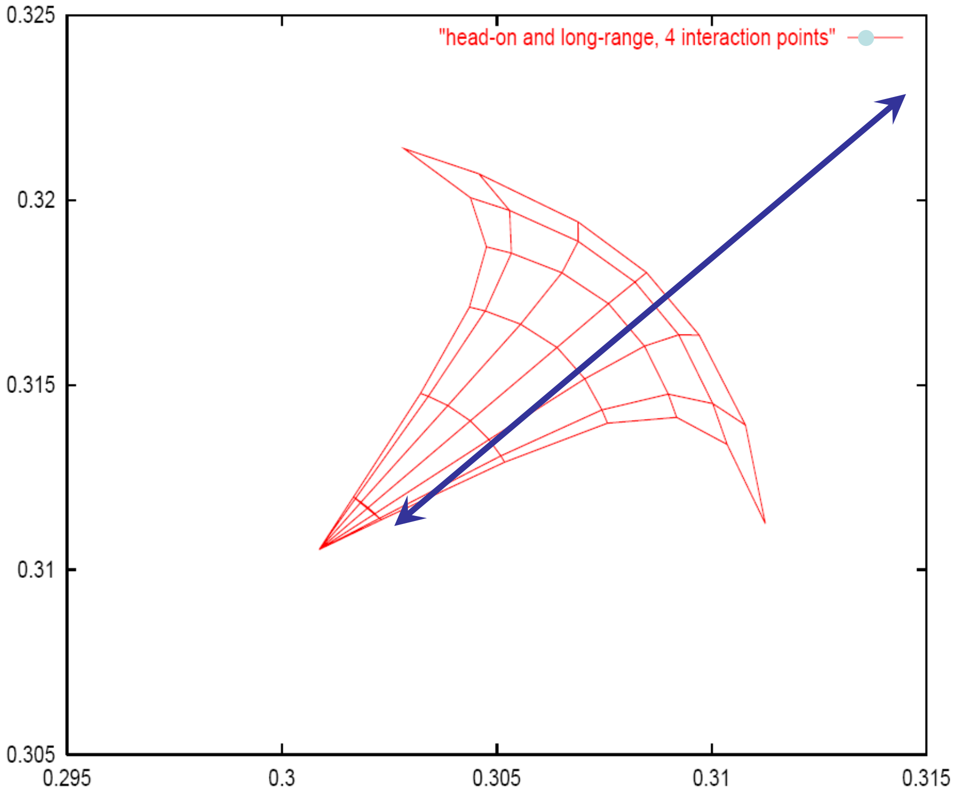
... at low speed the particles repel each other

III.) Golden Rule number three:

Never Collide the Beams !



the colliding bunches influence each other
→ change the focusing properties of the ring !!

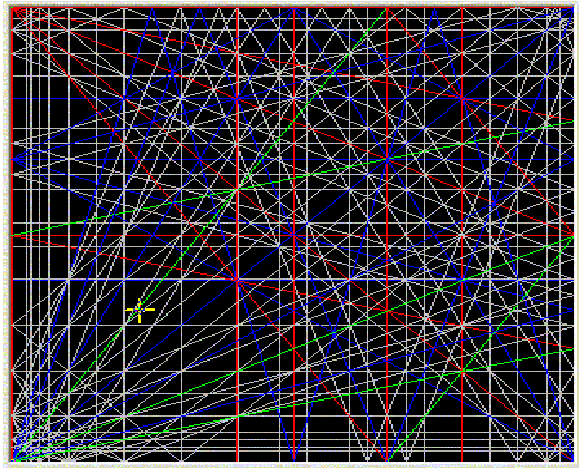


Courtesy W. Herr

most simple case:
linear beam beam tune shift

$$\Delta Q_x = \frac{\beta_x^* * r_p * N_p}{2\pi \gamma_p (\sigma_x + \sigma_y) * \sigma_x}$$

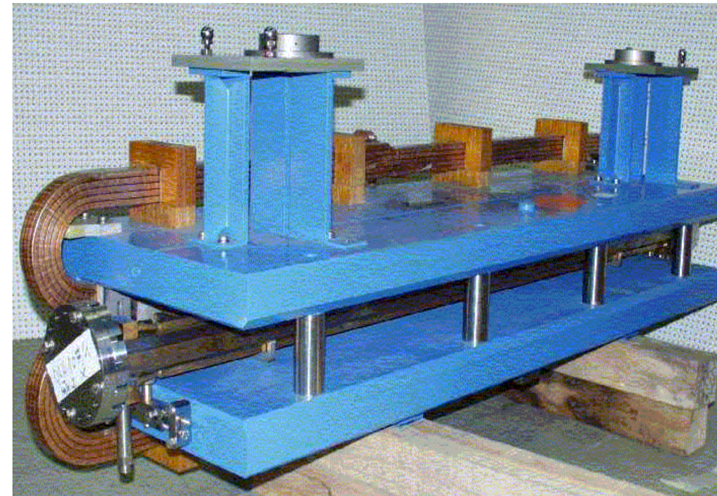
and again the resonances !!!



IV.) Golden Rule Number four: Never use Magnets

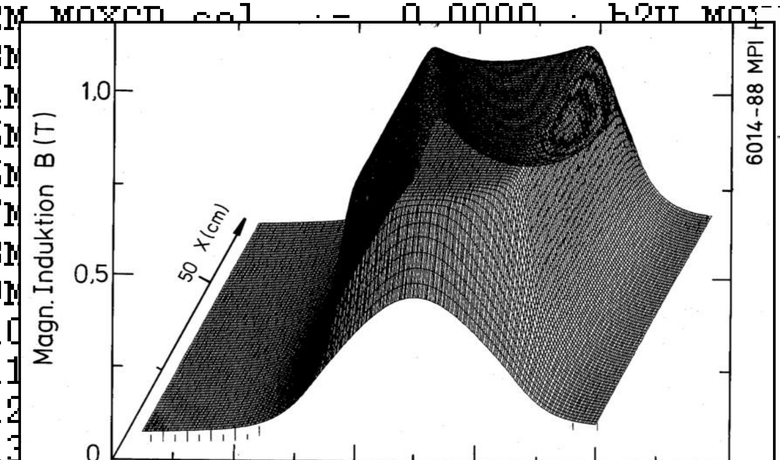
```
! ------------------------------------------------------------
! ***********
! -----------
!                                 bn at injection
b1M_MQXCD_inj   :=   0.0000 ; b1U_MQXCD_inj   :=   0000
b2M_MQXCD_inj   :=   0.0000 ; b2U_MQXCD_inj   :=   0000
b3M                                            :=   8900
b4M                                            :=   6400
b5M                                            :=   4600
b6M                                            :=   2800
b7M                                            :=   2100
b8M_MQXCD_inj   :=   0.0000 ; b8U_MQXCD_inj   :=   1600
b9M_MQXCD_inj   :=   0.0000 ; b9U_MQXCD_inj   :=   0800
b10M_MQXCD_inj  :=   0.5000 ; b10U_MQXCD_inj  :=   0600
b11M_MQXCD_inj  :=   0.0000 ; b11U_MQXCD_inj  :=   0300
b12M                           12U_MQXCD_inj  :=   0200
b13M                           13U_MQXCD_inj  :=   0100
b14M_MQXCD_inj  :=  -0.2700 ; b14U_MQXCD_inj  :=  0.0300 ; b14R_MQXCD_inj  :=  0.0100
b15M_MQXCD_                   b15U_MQXCD_inj  :=  0.0000 ; b15R_MQXCD_inj  :=  0.0000
!                                 bn in collision
b1M_MQXCD_c                   b1U_MQXCD_col   :=  0.0000 ; b1R_MQXCD_col   :=  0.0000
b2M_MQXCD_col   :=   0.0000 ; b2U_MQ                        :=  0.0000
b3M
b4M
b5M
b6M
b7M
b8M
b9M
b10
b11
b12
b13
b14M_MQXCD_col  :=   0.0000 ; b14U_M
b15M_MQXCD_col  :=   0.0000 ; b15U_MQXCD_col  :=  0.0000 ; b15R_MQXCD_col  :=  0.0000
```

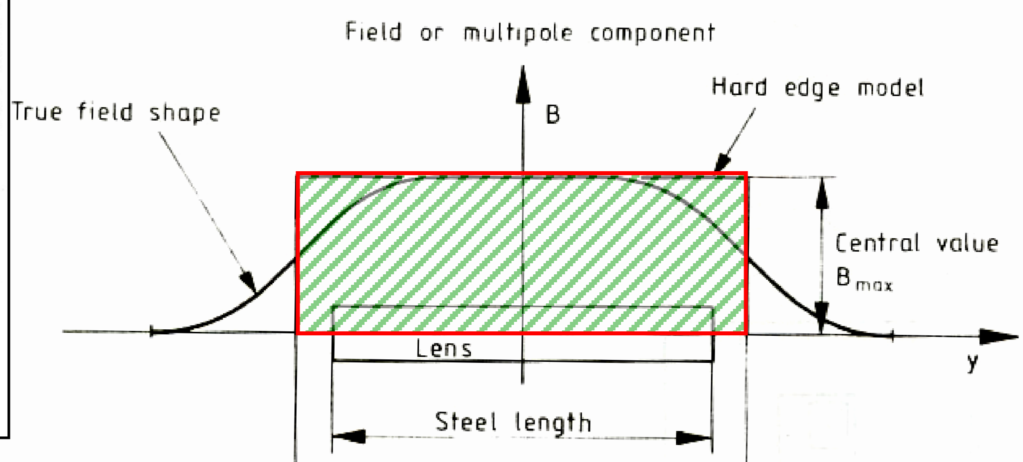
$$B_y + iB_x = B_{ref} * \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{r_0} \right)^{n-1}$$



"effective magnetic length"

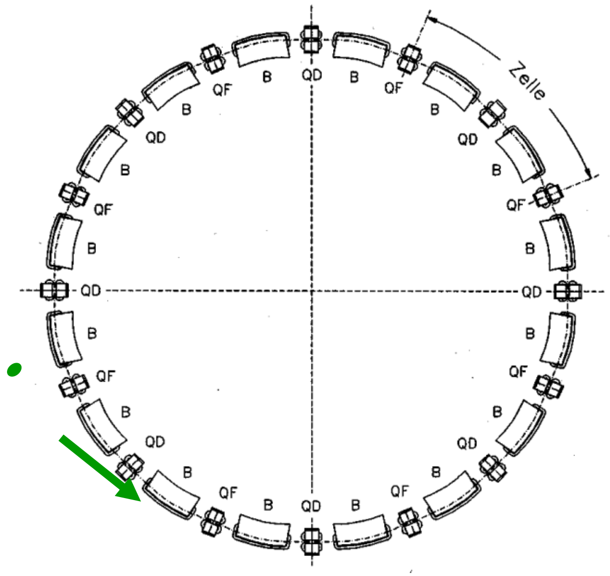
$$B * l_{eff} = \int_0^{l_{mag}} B ds$$





Clearly there is another problem ...

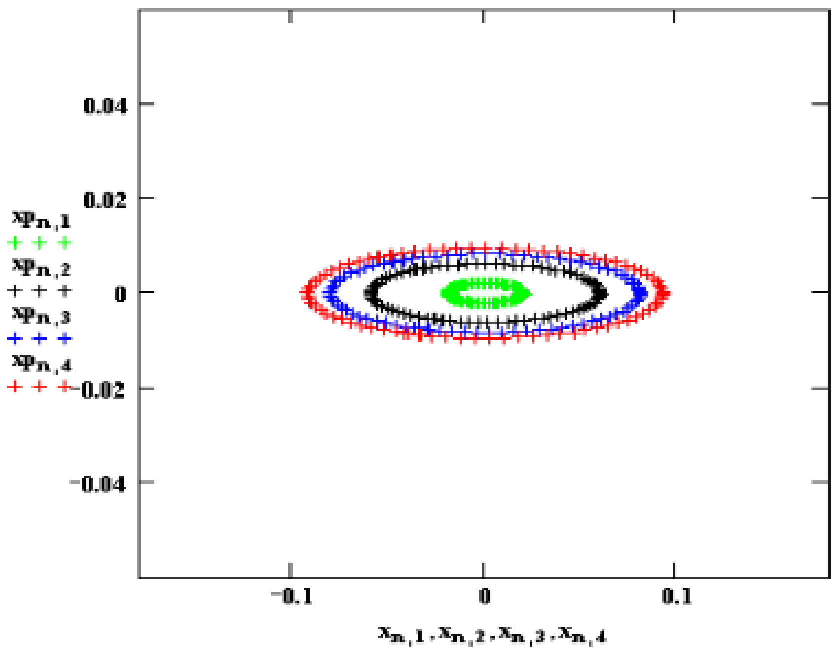
... if it were easy everybody could do it



Again: the phase space ellipse

for each turn write down – at a given position „s" in the ring – the single partilce amplitude x and the angle x´... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



A beam of 4 particles

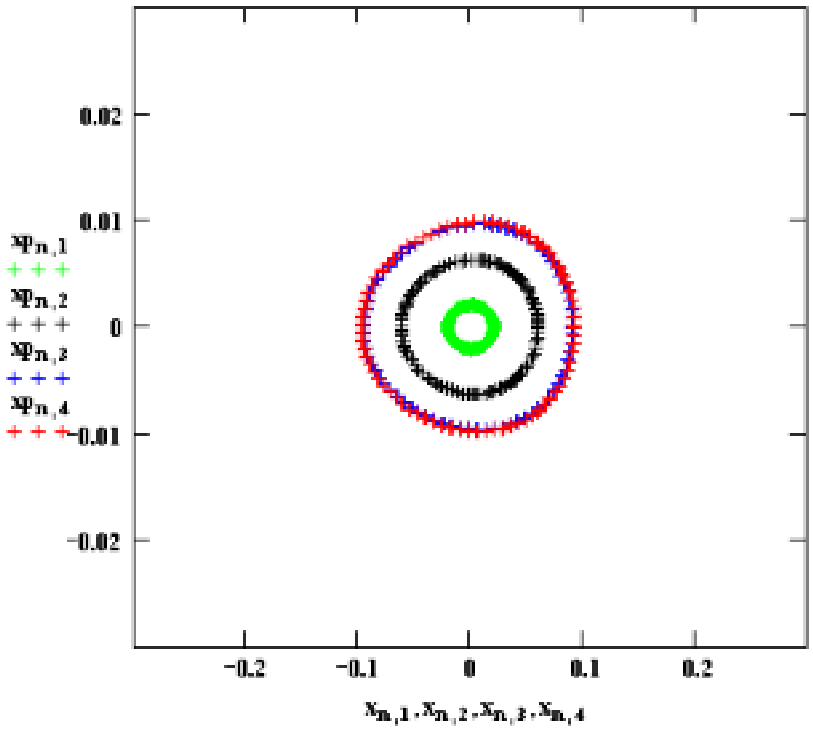
– each having a slightly different emittance:

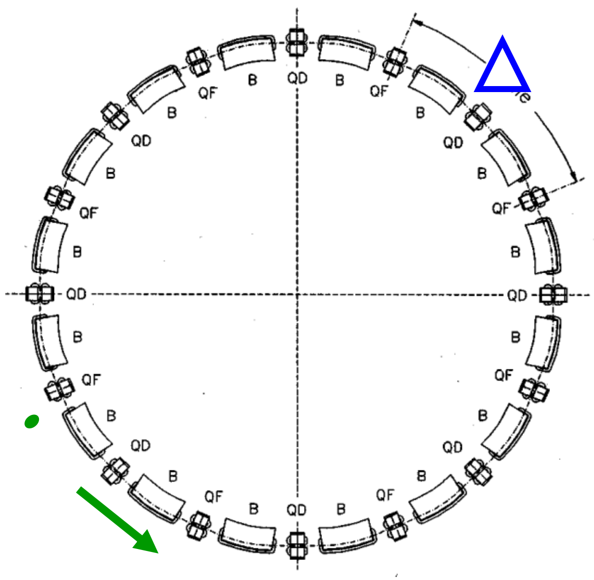
Installation of a weak (!!!) sextupole magnet

The good news: sextupole fields in accelerators cannot be treated analytically anymore.

→ no equatiuons; instead: Computer simulation

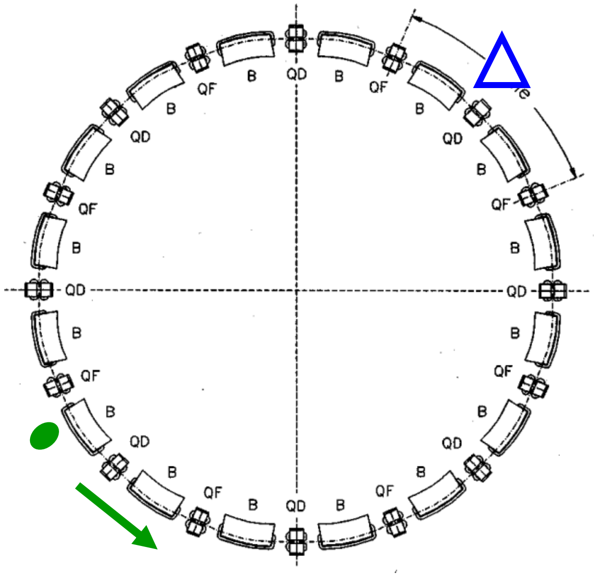
„ particle tracking "

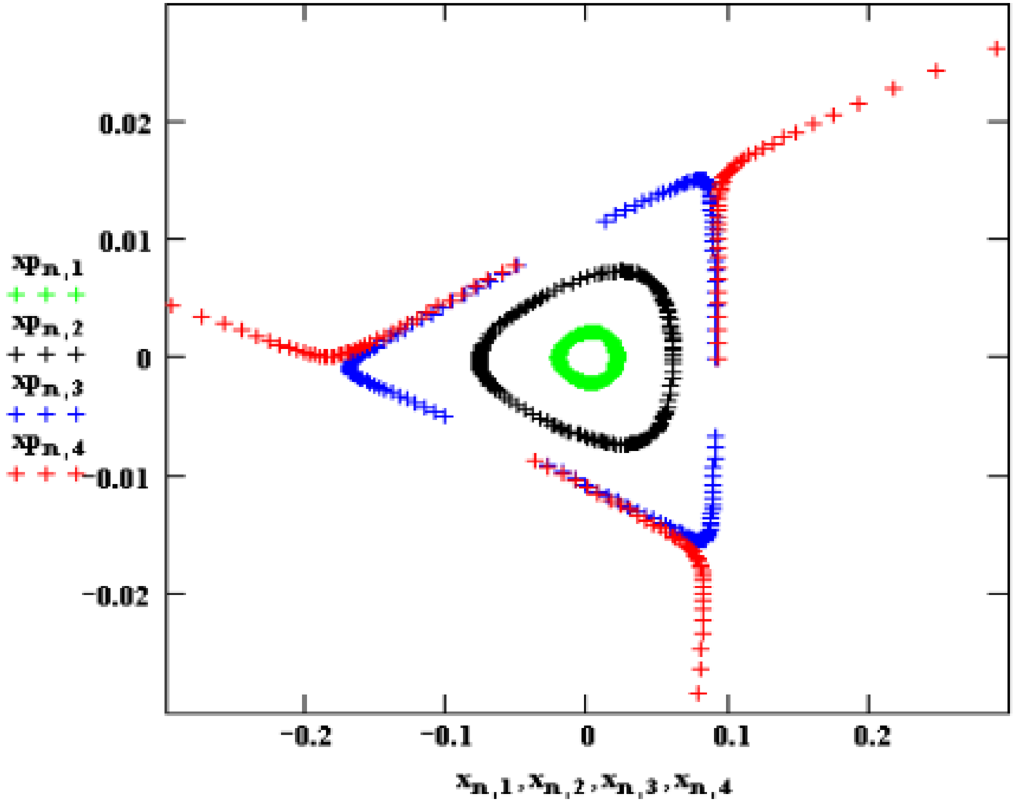




Effect of a strong (!!!) Sextupole …

→ *Catastrophy !*





„dynamic aperture“

Golden Rule XXL: COURAGE

and with a lot of effort from Bachelor / Master / Diploma / PhD and Summer-Students the machine is running !!!

thank'x for your help and have a lot of fun