FUNDAMENTAL CONCEPTS IN PARTICLE PHYSICS

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Particle physics

- Understand forces (interactions) governing the physics of the most elementary components of Nature.
- Elementary: without structure, within distances accessible to experiments.
- Elementary particles are then “pointlike objects" with some intrinsic properties: mass, spin, electric charge, colour charge, . . .
- Elementary does not mean stable: most elementary particles decay.
- Describe these forces in a coherent (mathematical) formulation: quantitative and predictive theory.
- Find the “fundamental" physical laws of Nature. Reductionist approach.

Great achievement of the last $\sim$50 years: the Standard Model of Glashow, Salam and Weinberg, verified to an impressive precision in very many experiments.

Particle physics / Standard Model

Brief summary:

I: “Our" elementary particles are:

- Quarks, (u, d, s, c, b, t), charged leptons (e, μ, τ) and their neutral lepton partners which are neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$.
- These are often called “matter particles", they have spin 1/2 (hence fermions: Pauli exclusion principle would apply in a multi-particle system), vastly different masses (from ~ 0 (ν's), $m_e \sim .5$ MeV to 171 GeV).
- Photon γ, gluons, $W^\pm$ and Z^0 are “force carriers". They are bosons, with spin 1 ($W^\pm, Z^0$) or helicity ± 1 (γ, gluons).
- The Standard Model, in its minimal version, predicts the existence of a single Higgs boson (should be seen at LHC).

Actually invented by Brout, Englert and by Higgs.

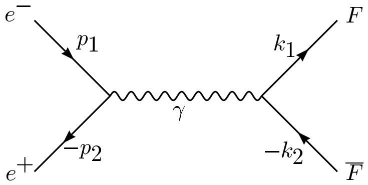
Particle physics / Standard Model

II: The forces, dynamical aspects:

- The Standard Model describes the strong, weak and electromagnetic (quantum) interactions of the elementary particles.
- Electromagnetic interaction (quantum electrodynamics, QED): photons are the quanta of the electromagnetic field (Maxwell, Einstein), massless, long range force.

  Coulomb: $\Phi \sim 1/r$ $|\vec{F}| \sim 1/r^2$

  The electromagnetic field of Maxwell is a superposition of photons.

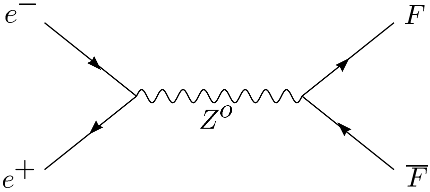


Particle Physics / Standard Model

- Weak force carried by very massive $W^\pm$ ($M \sim$ 80 GeV) and Z^0 ($M \sim$ 91 GeV)

  (discovered at CERN).

  For instance:



- Very short range, $\phi \sim e^{-Mr}/r$ (instead of Coulomb $1/r$).
- At the origin of the very common β decays of elementary particles, neutron, nuclei . . .
- Important role in the formation of the Universe (synthesis of light nuclei).

Particle Physics / Standard Model

- Strong interaction (quantum chromodynamics, QCD): carried by massless gluons.
- If exchanged energy large enough (at least several GeV): relatively weak interaction, scattering of quarks or gluons by quarks or gluons.
  Weaker interaction with larger (exchanged) energy: strong interaction is asymptotically-free.
- At low energies, confining force: no “free" quarks or gluons, only allows colour-neutral bound states, hadrons:

  $q\bar{q}$ (mesons), qqq (baryons), $\overline{qqq}$, glueballs, $\ldots$

  Proton is $u + u + d$, neutron is $u + d + d$, ...
- A long-range force mediated by massless gluons, but a confining force. Hadrons interact then only by a short-range residual strong force, the strong nuclear force, at the origin of the stability of nuclei.

Particle Physics / Standard Model

III: Open problems:

- Find the Higgs (-Brout-Englert) boson.
  If not seen at LHC, reconsider the theory which predicts its existence ...
- Measure Standard Model processes in the energy range of LHC.
- Prove that QCD, the theory of strong interactions, predicts confinement of coloured quarks and gluons.
- Obtain that the hadronic spectrum is correctly predicted by QCD.
- There should also be a graviton (not seen yet), carrier of the gravitational force (massless, helicity ± 2).

Physics is an experimental science:
LHC may also show more than simply the Standard Model, it may display physics beyond the Standard Model.

Concepts in Particle Physics

Subjects, concepts to be discussed in the lectures include:

1. The Standard Model is a theory of quantum fields.
2. Observables relevant to high-energy particle physics: cross sections, decay rates, . . .
3. The Standard Model is mostly defined by its symmetries.
4. It has many parameters: in particular, can we understand fermion masses ?
5. Proton stability, baryon and lepton number conservation / violation . . .
6. Unified theories, can we understand the three particle forces ?
7. The role of gravitation, unification with gravity, extra dimensions, strings.

Points 4 - 7 consider physics beyond the Standard Model, theoretical ideas with increasing level of speculation.

Quantum field theory

Maxwell theory of electromagnetic phenomena is a classical field theory: sources carrying electric charges produce electric and magnetic fields everywhere in space-time:

$$\left.\begin{array}{ll}\text{Charge density} & \rho(\vec{x},t) \\ \text{Current density} & \vec{j}(\vec{x},t)\end{array}\right\} \Longrightarrow \left\{\begin{array}{ll}\vec{E}(\vec{x},t) & \text{Electric field} \\ \vec{B}(\vec{x},t) & \text{Magnetic field}\end{array}\right.$$

Propagation is defined by coupled differential equations:

Maxwell equations (vacuum):

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = -\frac{1}{c}\vec{j} \qquad \vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} = 0$$

Maxwell equations:

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad\qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = -\frac{1}{c}\vec{j} \qquad\qquad \vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} = 0$$

- A physical theory has a set of differential equations describing the evolution in space and time of the state of the system.
- Maxwell: the state is defined by the fields $\vec{E}(\vec{x},t)$, $\vec{B}(\vec{x},t)$, six functions of the space-time point.
- There is a conservation law (electric charge conservation):

  $$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$$

  The sources are “external" (no evolution equation).
- Maxwell equations are relativistic.
  The speed of light is $c \sim 3\cdot 10^8$ m/s for all observers.

Non-relativistic quantum mechanics

The state of the system at time t is a ket $|\psi(t)\rangle$ (Dirac) (in some Hilbert space).

Dynamics of the system, evolution of the state $\sim$ Schrödinger equation:

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \mathcal{H}|\psi(t)\rangle$$

Description of the system defined by the Hamiltonian operator $\mathcal{H}$. Its eigenvalues are allowed energies of the system.

Applies to atomic phenomena:

- Energies $\sim$ electron-volt (eV),
- DIstance $\sim$ Angström 10^{-10} m.

Nuclear, particle physics processes: energies $\gtrsim$ MeV $= 10^6$ eV. Quantum mechanics and special relativity needed.

Special relativity [Einstein 1905]

- Energy of a massive particle is

$$E = \sqrt{m^2c^4 + \vec{p}^{\,2}c^2} \qquad\qquad E \geq mc^2$$

- Relativistic if $|\vec{p}| \gtrsim mc$. If $m = 0$, speed of light c.
- Does not make sense to fix the number of particles: a charged particle radiates photons, the photon is massless and in the limit of zero energy (infinite wavelength, $E \to 0$), photons cannot be counted.
- Similarly, pairs particle – antiparticle may be produced.

Hence: Particle physics needs a relativistic version of quantum mechanics, in which the state of the system can have an arbitrary number of particles.

Quantum field theory is quantum mechanics “plus" special relativity.

We want to consider, for all possible final states $C_1 + C_2 + \ldots$:

- Decay processes $A \longrightarrow C_1 + C_2 + \ldots$
- Scattering processes $A + B \longrightarrow C_1 + C_2 + \ldots$

Intrinsically quantum processes. For instance, A decays with lifetime τ:

$$\frac{d}{dt}N_A = -\frac{1}{\tau}N_A \qquad\qquad N_A(t) = N_A(0)e^{-t/\tau}$$

Decay probability constant, does not depend on “history" of the decaying particles. Predictions probabilistic only.

Lifetime τ should be calculable from the theory describing the dynamics of A.

Quantum field theory

To formulate a quantum field theory, the standard procedure is:

- Choose a set of fields $\Phi_i(x)$ corresponding to the particles to be described.
- Write an action functional

$$S[\Phi_i] = \int d^4x\, \mathcal{L}(\Phi_i, \partial_\mu \Phi_i)$$

The Lagrange function $\mathcal{L}$ should be a relativistic invariant (invariant under Poincaré transformations of special relativity).

- The dynamical field equations follow from the action principle

$$\delta S[\Phi_i] = 0 \qquad \Longrightarrow \qquad \frac{\partial \mathcal{L}}{\partial \Phi_i} - \frac{\partial}{\partial x^\mu} \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi_i} = 0$$

They encode the interactions (nonlinearities in the diff. equations).

Quantum field theory

- Transformations of Φ_i leaving $\mathcal{L}$ unchanged are (internal) symmetries.
- Usually, selected symmetries are imposed and interactions are all possible terms in $\mathcal{L}$ respecting the imposed symmetries:

$$\text{Symmetry} \Longrightarrow \text{Lagrangian } \boldsymbol{\mathcal{L}(\Phi_i)}$$

- Interactions are then understood as consequences of symmetries (which are however not understood, simply “observed").

One should then quantize the fields.
As in quantum mechanics, they should be operators with specific commutation rules, and one should then find, by solving these operators equations, the space of physical states of the interacting theory.

In all physically-relevant theories, a too complicated problem (nonlinear operator equations).

Calculate physical processes in perturbation theory of the quantum field theory of free quarks, gluons, leptons, ...

Back to (nonrelativistic) quantum mechanics

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \mathcal{H}|\psi(t)\rangle$$ Schrödinger equ.

A linear, 1st order equation. State $|\psi(t)\rangle$ at all times depends on $|\psi(t=t_0)\rangle$, time evolution dictated by Hamiltonian $\mathcal{H}(P,Q)$.

To solve, in principle:

- Find in which (Hilbert) space does the Hamiltonian act.
- *i.e.* find a basis of eigenvectors of $\mathcal{H}$

  $\mathcal{H}|u_n\rangle = E_n|u_n\rangle$ to find the allowed energies.
- Use linearity of Schrödinger equation

  $|\psi(t)\rangle = \sum_n c_n(t)|u_n\rangle$ and find the functions $c_n(t)$.

Too hard in many interesting cases (for instance in molecular systems, chemistry), use perturbation theory.

Quantum field theory, perturbation theory

In practice, the aim is to calculate in perturbation theory an S–matrix element:

$$S_{fi} = \langle\, out \mid in \rangle \qquad \begin{cases} \mid in\,\rangle &= \text{initial state at } t \to -\infty \\ \mid out\,\rangle &= \text{final state at } t \to +\infty \end{cases}$$

While $\mid in\,\rangle$ is either a one- (decay) or two-particle state (scattering), $\mid out\,\rangle$ covers all possible final states relevant to the experiment under interest.

- Perturbation theory around free quarks or gluons excludes hadrons. They cannot be studied in perturbation theory.
  QCD confinement is nonperturbative.
- Perturbation theory can be defined (order by order) only for a particular class of quantum field theories, which are then called **renormalizable**.
- The standard perturbative expansion has a nice graphical representation, in terms of **Feynman diagrams**.

QFT, perturbation theory

Split the Lagrangian:

$$\mathcal{L}(\Phi_i, \partial_\mu \Phi_i) = \mathcal{L}_{free} + \mathcal{L}_{int.}(g, g', g_s)$$

$\mathcal{L}_{free}$: free fields, linear field equation, solve and quantize.

$\mathcal{L}_{int.}$: interactions, nonlinear, treat as a perturbation of $\mathcal{L}_{free}$.

- Depends on some coupling constants g, g', g_s (parameters).
- Assume parameters small enough to expand physical quantities in powers of g, g' and g_s.
- Hope that few terms only give an approximation of the full result.

For instance, expand a scattering amplitude $\mathcal{A}(p_i, \ldots, g, g', g_s)$:

$$\mathcal{A}(p_i, \ldots, g, g', g_s) = \sum_{m,n,r \geq 0} \mathcal{A}_{m,n,r}(p_i)\, g^m g'^n g_s{}^r$$

QFT, perturbation theory

For the perturbative expansion to make sense:

- All $\mathcal{A}_{m,n,r}(p_i)$ should be well-defined (finite), for all physical processes: a very non-trivial condition on the structure of $\mathcal{L}_{int.}$:

  The quantum field theory should be renormalisable.

- Renormalisability implies (in particular) that the Lagrangian does not have parameters with negative energy dimension, or, that all operators in $\mathcal{L}$ should have dimension four or less.

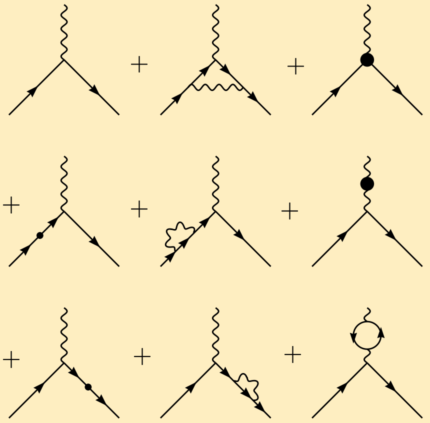
- In the Standard Model, all parameters are dimensionless numbers (coupling constants), except for the *Higgs boson mass* (in $\mathcal{L}_{free}$).

- Note: for a given theory, perturbation theory is not unique, the splitting $\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int.}$ is ambiguous: parts of $\mathcal{L}_{free}$ can be used as interactions to imporove the expansion (counter-terms).

- Renormalisability selects a class of well-defined perturbative expansions. It no one exists, non-renormalisable theory. It applies up to a maximal UV scale, physical processes depend in general on this cutoff scale.

QFT, perturbation theory

Feynman diagrams:



QED: $e^- e^- \gamma$ interaction to order e^3.

For a generic physical process:

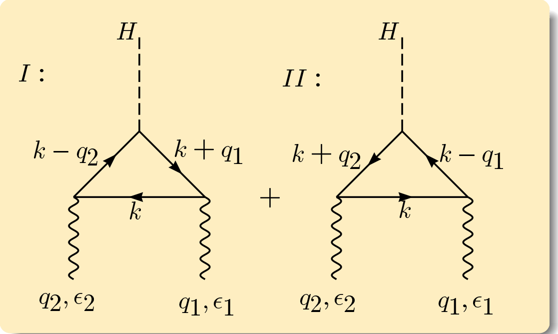
- Each $\mathcal{A}_{m,n,r}(p_i)$ is a sum of various contributions.
- Each contribution has a graphical representation.
- At a given order, the sum of all allowed diagrams gives $\mathcal{A}_{m,n,r}(p_i)$, with specific Feynman rules to calculate the value of each diagram.
- A systematic, intuitive and then very popular method to generate perturbative calculations.

QFT, perturbation theory

Using diagrams, an example: Higgs $\longrightarrow \gamma\gamma$

- The Higgs boson has zero electric charge: no direct interaction with photons.
- Does interact will all massive fermions, with coupling $m_f[\sqrt{2}G_F]^{1/2}$.

The following Feynman diagrams indicate immediately a decay amplitude $\sim m_f G_F \alpha$ (except if they would vanish. . .).

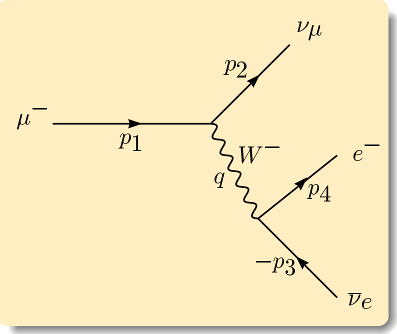


Expect then a decay width

$$\Gamma_{H\to\gamma\gamma} \sim \alpha^2 G_F M_H^3 \mathcal{I}$$

The function $\mathcal{I}(m_f^2/M_H^2)$ vanishes if $m_f/M_H \to 0$. Top quark relevant (if M_H light enough).

QFT, perturbation theory, Feynman diagrams



- Diagrams very useful to visualize and then evaluate perturbative contributions to physical processes.
- However: a single diagram is not a physical process, only the complete amplitudes $\mathcal{A}(p_i, \ldots, g, g', g_s)$ have physical significance.
- Internal lines in a diagram correspond to propagators (Green's functions). They are often called virtual particles (not on their mass-shell).
- Do not think of detecting the W^- boson in the diagram for muon decay. . .
- External lines correspond to particles in states $|\, in\, \rangle$ and $|\, out\, \rangle$.

Observables

Quantum mechanics:

We are interested in the probability that an experiment measuring quantity $\mathcal{A}$ gives result a. Quantum mechanics prescribes:

- To consider the operator A (observable) corresponding to $\mathcal{A}$; a is an eigenvalue of A.
- To look for a basis $\{|u_n\rangle\}$ of the eigenspace of the eigenvalue a.
- For a system in state $|\psi\rangle$, to calculate the amplitudes
$$\mathcal{M}_n = \langle u_n|\psi\rangle.$$

The probability is then

$$P(a) = \sum_n |\mathcal{M}_n|^2$$

if all states are correctly normalized, $\langle\psi|\psi\rangle = 1$, $\langle u_m|u_n\rangle = \delta_{mn}$.
A dimensionless number, $0 \leq P(a) \leq 1$.

The sum is over all states compatible with the result expected from the experiment.

Observables

In quantum field theory (in particle physics):

- The initial state $|in\rangle$ is either one particle (decay) or two well-separated particles (scattering), long before an interaction happens: $t \longrightarrow -\infty$.
- The final state $|out\rangle$ is a set of well-separated particles long after the interaction: $t \longrightarrow \infty$.
- Both states are described by free fields ($\mathcal{L}_{free}$).

From quantum mechanics to QFT, the correspondence is:

$$|\psi\rangle \quad \Longleftrightarrow \quad |in\rangle \qquad\qquad |u_n\rangle \quad \Longleftrightarrow \quad |out\rangle$$

$$\mathcal{M}_n = \langle u_n|\psi\rangle \quad \Longleftrightarrow \quad S_{fi} = \langle out|in\rangle$$

$\sum_n \quad \Longleftrightarrow \quad$ Sum over all states seen by the detector

QFT: the sum over final states may include polarisation (spin) sums, an integral over all configurations allowed by energy-momentum conservation and a sum over states with very different particle content.

Observables

The result is a transition rate per volume unit $\mathcal{T}_{fi}$ (length^{-3} time^{-1}).

To obtain normalized quantities, one defines:

- Scattering: cross section:

$$\sigma = \frac{\mathcal{T}_{fi} \quad (\text{length}^{-3}\ \text{time}^{-1})}{\text{incoming flux } (\text{length}^{-2}\ \text{time}^{-1}) \times \text{target density } (\text{length}^{-3})}$$

The cross section is a surface, measured in multiples of 1 barn = 10^2 fm^2.

- Decay: width:

$$\Gamma = \frac{\mathcal{T}_{fi} \quad (\text{length}^{-3}\ \text{time}^{-1})}{\text{density of decaying particles } (\text{length}^{-3})}$$

The lifetime is $\tau = 1/\Gamma$.

Symmetries

From classical field theory, three facts:

1. Noether theorem: to each continous symmetry of an action corresponds a conserved current, and a conserved charge:

$$\partial_\mu J^\mu_{Noether} = 0 \qquad Q_{Noether} = \int d^3x \, J^0_{Noether} \qquad \frac{d}{dt} Q_{Noether} = 0$$

2. Maxwell electromagnetism has gauge invariance: it eliminates the third, longitudinal, polarisation of the electromagnetic potential $A_\mu(x)$.

3. Fields transforming under symmetries can be characterized by invariant (Casimir) numbers which are intrinsic properties.
   An example is: a sphere has a radius R (rotation invariance).

All three survive in quantum field theory.
Since Noether currents are now operators, current conservation is equivalent to an infinite set of relations between matrix elements: Ward identities.

Symmetries

Relativistic field theories used in particle physics have four kinds of symmetries:

1. Continuous, global space-time symmetries: relativistic invariance, Poincaré symmetry. Implies that every field has a mass m, if $m^2 > 0$, the field has a spin (0, 1/2, 1, ...), if $m = 0$ it has a helicity:

   All particles have mass and spin/helicity.

   Conservation laws: total energy, momentum, angular momentum

   (Or supersymmetry . . .)

2. Global (continous) internal symmetries acting only on fields. Examples are baryon or lepton numbers.

3. Local or gauge internal symmetries.
   Standard Model: $SU(3)_c \times SU(2)_L \times U(1)_Y$.
   Conservation laws: colour and electric charge

4. Discrete symmetries. CPT is a symmetry of all quantum field theories.

Gauge invariance, classical electromagnetism

Maxwell equations (vacuum):

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad\qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = -\frac{1}{c}\vec{j} \qquad\qquad \vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} = 0$$

Solve the 2nd column:

$$\vec{B} = \vec{\nabla}\times\vec{A} \qquad\qquad \vec{E} = -\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{A}$$

Invariant under the gauge transformation with parameter $\Lambda(x)$

$$\vec{A} \longrightarrow \vec{A} + \vec{\nabla}\Lambda \qquad\qquad \Phi \longrightarrow \Phi - \frac{1}{c}\frac{\partial}{\partial t}\Lambda$$

We are left with two 2nd order Maxwell equations for $\vec{A}$ and Φ.

Take then $c = 1$, define two four-vectors

$$A_\mu = (\Phi, \vec{A}) \qquad j_\mu = (\rho, \vec{j}) \qquad \partial^\mu j_\mu = 0$$

A relativistic Lagrangian for Maxwell equations is then

$$\mathcal{L}_{Maxwell} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A^\mu j_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The action is invariant under the gauge transformation

$$A_\mu \longrightarrow A_\mu - \partial_\mu \Lambda$$

since the electromagnetic current is conserved, $\partial^\mu j_\mu = 0$.

Add a Dirac spinor $\psi(x)$, define the current $J^\mu = q\,\overline{\psi}\gamma^\mu\psi$ and consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A^\mu J_\mu + \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

Dirac equation:

$$i\gamma^\mu \partial_\mu \psi = m\psi + qA_\mu \gamma^\mu \psi \qquad -i\partial_\mu \overline{\psi}\gamma^\mu = m\overline{\psi} + qA_\mu \overline{\psi}\gamma^\mu$$

Gauge invariance, QED

As a result of Dirac equation, the electromagnetic current J_μ is conserved: $\partial^\mu J_\mu = 0$.

The result is QED Lagrangian for a spin 1/2 field ψ with electric charge q:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^\mu D_\mu - m)\psi \qquad D_\mu = \partial_\mu + iqA_\mu$$

It is invariant under gauge transformations

$$\begin{aligned} A_\mu &\longrightarrow A'_\mu = A_\mu - \partial_\mu \Lambda(x) \\ \psi &\longrightarrow \psi' = e^{iq\Lambda(x)}\psi \end{aligned}$$

Notice that

$$D_\mu\psi \longrightarrow e^{iq\Lambda(x)}D_\mu\psi$$

$D_\mu\psi$ is the covariant derivative of ψ (it transforms like ψ).

Gauge invariance, QED

We have derived a gauge theory with gauge group $U(1)$ (phase rotation of a complex field) for a Dirac spinor with charge q and the $U(1)$ gauge field (or connection) A_μ.

The argument can be reversed:

1. Postulate the gauge group: $U(1)$.
2. Postulate that the theory includes a massive fermion with $U(1)$ charge q, *i.e.* postulate the $U(1)$ transformation of the spinor field.
3. Write the most general $U(1)$-invariant (relativistic) Lagrangian compatible with the constraints from quantum field theory (renormalizability).

The result is again the QED Lagrangian.

The form of the fermion–fermion–photon interaction follows from the symmetry postulates.

These three steps : the gauge principle. Powerful and mysterious. . .

Gauge invariance, QCD

1: Gauge group: $SU(3)$ (colour gauge group).

2: Quarks are massive fermions transforming in representation $\mathbf{3}$ of the gauge group $SU(3)$:

$$Q_a \longrightarrow Q'_a = e^{i\alpha^A(x)\lambda^A} Q_a \qquad a\text{: flavour index}$$

λ^A: 3×3 matrices, the eight generators of $SU(3)$.

$$[\lambda^A, \lambda^B] = i \sum_C f^{ABC} \lambda^C \qquad f^{ABC}\text{ : structure constants}$$

α^a: the eight parameters of the $SU(3)$ transformation.

3: Since the gauge group has dimension eight, we need eight gauge bosons: gluon fields $A_\mu^A(x)$.

4: The gauge group is non-abelian. Gauge variations of gluon fields depend on the structure constants:

$$A_\mu^A \longrightarrow A_\mu^A + \partial_\mu \alpha^A + \sum_{B,C} f^{ABC} A_\mu^B \alpha^C$$

Gauge invariance, QCD

5: Covariant derivatives of quark fields are then

$$D_\mu Q_a = \partial_\mu Q_a - i A_\mu^A \lambda^A Q_a$$

Rescale $A_\mu^A \to g_s A_\mu^A$ to introduce the strong coupling constant g_s.

6: Write the most general renormalizable $SU(3)$–invariant Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4}\sum_A F_{\mu\nu}^A F^{A\,\mu\nu} + i\sum_a \overline{Q}_a \gamma^\mu D_\mu Q_a - \sum_a m_a \overline{Q}_a Q_a$$

$$F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + g_s \sum_{B,C} f^{ABC} A_\mu^B A_\nu^C$$

- All interactions follow from the gauge principle.
- A single interaction parameter, the strong coupling constant g_s.
- Since the group is non-abelian, gluons have octet (adjoint) colour charge, and self-interactions.

Symmetries of the Standard Model

Gauge symmetries:

- Gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y$.
- All gauge boson self-interactions follow from the choice of the gauge group.
- $SU(3)_c$ acts on quarks and gluons, the eight gauge bosons of $SU(3)$ (QCD).
- $SU(2)_L \times U(1)_Y$ includes the $U(1)_{em}$ of QED, and weak interactions. $SU(2)_L$ only acts on left-handed fermions: parity violation of weak interactions.
- $SU(2)_L \times U(1)_Y$ is spontaneously broken to $U(1)_{em}$ by the Higgs mechanism, which also generates masses for quarks and leptons.
- Strong and electromagnetic interactions are parity-conserving.

Symmetries of the Standard Model

Fermions: quarks and leptons:

Three generations with identical transformation properties ($\mathbf{3} \times \mathbf{16}$ fermions):

Fermions	$SU(3)$	$SU(2)_L$	$U(1)_Y$	B	L
Q_L	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$1/3$	0
U_R	$\mathbf{3}$	$\mathbf{1}$	$2/3$	$1/3$	0
D_R	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$1/3$	0
L_L	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0	1
E_R	$\mathbf{1}$	$\mathbf{1}$	-1	0	1
N_R	$\mathbf{1}$	$\mathbf{1}$	0	0	1

B: baryon number

($B = 1$ for baryons, $B = -1$ for antibaryons, $B = 0$ for mesons).

L: lepton number.

This table allows to write the Lagrangian for all fermion-gauge boson interactions.

$SU(2)$ doublet $\mathbf{2}$: generators are Pauli matrices.

LEP experiments (Z^0 invisible width, ...) and cosmology indicate that there exists only 3 light neutrinos.

Symmetries of the Standard Model

Global (continuous) symmetries:

B and L. Proton exactly stable.

Discrete symmetries:

Parity P, charge conjugation C, time reversal T, CP are broken by weak interaction or in the Yukawa sector (interactions Higgs – fermions).

CPT is an exact symmetry, as in all quantum field theories.

Gauge principle: SYMMETRIES $\Longrightarrow$ DYNAMICS

The symmetry content (gauge group and the transformations of fermion and scalar fields) and renormalizability suffice to write the Lagrangian of the theory and then all dynamical equations

Perhaps the most important concept in theoretical particle physics.
Powerful and mysterious. . .

“Beyond the Standard Model"

The Standard Model gives a coherent and accurate description of elementary particles physics, as far as strong and electroweak interactions at the level of quarks and leptons are concerned.

However:

- The gauge group and the scalar and fermion gauge transformations are postulated, not understood.
- It still has many parameters (18 + neutrino masses and mixings + θ vacuum angle). Their values are not understood.
- In particular, the fermion mass spectrum is a mystery,
$$m_{e^-}/m_{top} \sim 10^{-5} \text{ !}$$
- The value of the weak scale, $M_W \sim 100$ GeV, in relation to the other scale of fundamental forces, the Planck scale $M_{Planck} \sim 10^{18}$ GeV controlling gravitation, is an enigma.
- This huge ratio of scales $M_W/M_{Pl.} \sim 10^{-16}$ poses a severe technical problem, the so-called hierarchy problem.

Beyond the Standard Model

“New physics beyond the Standard Model" could bring more information and constraints on the theory, leading to a deeper synthesis.

Typically, theoretical approach is speculative model-building, experiments search for violations of Standard Model physics (rare processes) or direct discovery of new physics (high-energy, LHC, exotic searches, . . .).

Theoretical directions include (non exhaustive):

- Construct unified theories of the three interactions (Grand Unified Theories, GUTs). Study larger symmetries broken at a scale $\gg M_W$.

  Understand the gauge group, values of the gauge couplings, fermion representations and mass relations.

- Theories with milder hierarchy problem, supersymmetric models of particle interactions.

  Understand fundamental scales. Space-time symmetry beyond the Poincaré group of special relativity.

Beyond the Standard Model

- Develop theories where mass generation ($W^{\pm}$, Z^0, fermion masses) in the Standard Model is not related to a fundamental Higgs boson (LHC will tell . . .).
- Unification with gravitation. The structure of space-time at very short distances. Includes string theories and models with extra dimensions.

  Complete unification of fundamental forces, quantum gravity, understand scales, the cosmological constant, early cosmology, primordial singularity.
- Low-energy (*i. e.* Standard Model energies) signatures of physics beyond the Standard Model.

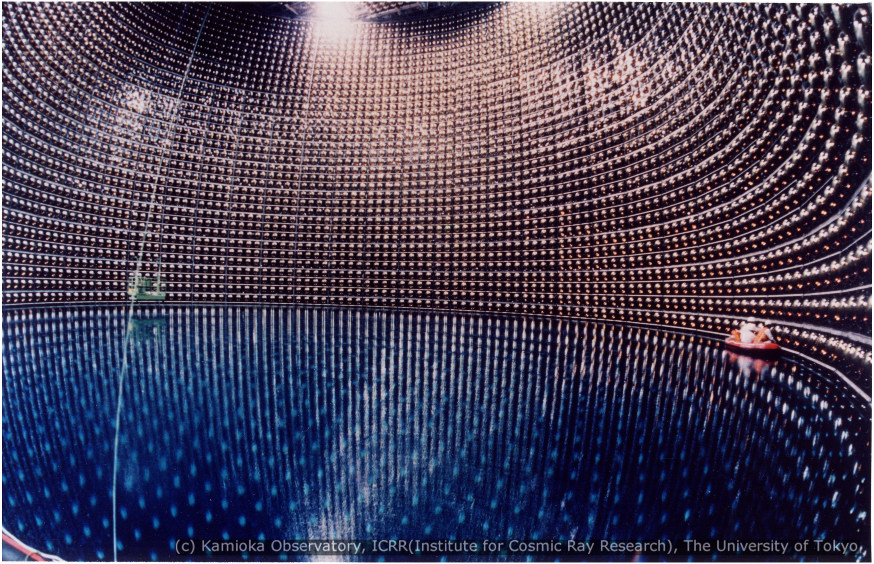
  Processes forbidden in the Standard Model like proton decay. Deviations from Standard Model.

Begin with the last point.

Beyond the Standard Model

- Extensions of the Standard Model usually have a scale Λ much larger than the weak scale M_W, at which some spontaneous symmetry breaking “freezes" most of the new physics: gauge bosons or other states acquire masses $\mathcal{O}(\Lambda)$.
- One cannot directly access these states (accelerator with energy $\sim \Lambda$ needed). One then searches for effective deviations from Standard Model predictions, due to residual effects in processes involving light states of the Standard Model.
- These are often described using non-renormalizable operators made with the light fields of the Standard Model.
- The best-known example is proton decay predicted by several GUTs, which gives access to physics at $\Lambda \sim 10^{16}$ GeV.
- Actually, the simplest GUTs are experimentally excluded: they violate the limit $\tau_p > 10^{31}$ to 10^{33} years.

Beyond the Standard Model

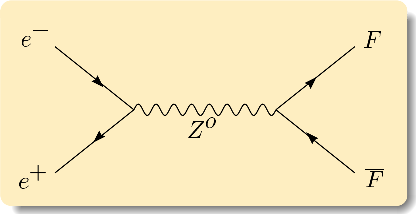


Super-Kamiokande, water filling, 1996.

Effective four-fermion interactions

Effective four-fermion interactions: a tool to study low-energy effects of physics characterized by a very large scale Λ.

For instance, the Standard Model has fermion – gauge (or Higgs) boson interactions which leads to a four-fermion interaction $e^-e^+F\overline{F}$ mediated, by Z^0 (or by any boson coupling to e^- and F).



Z^0 line contributes $g^2/(E^2_{e^+e^-} - M^2_Z)$

If $E_{e^+e^-} \ll M_Z$, it contributes a constant, $-g^2/M^2_Z$

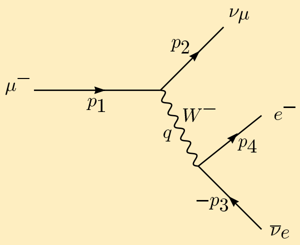
$\Longrightarrow$ An effective four fermion interaction.

Symmetries require: an $SU(3) \times SU(2)_L \times U(1)_Y$ singlet and a Lorentz scalar.

$\Longrightarrow$ The old Fermi theory of weak interactions.

Effective, four-fermion interactions

Fermi theory describes weak interactions when (transferred) energies are much smaller than M_W. It it not renormalizable, it has four-fermion interactions.



Muon decay:
Since $q^2 \ll M_W^2$, the W propagator in the Standard Model diagram reduces to

$$\frac{g^2}{|q^2 - M_W^2|} \quad \longrightarrow \quad \frac{g^2}{M_W^2} = 4\sqrt{2}G_F$$

For $q^2 \ll M_W^2$, generate the decay by an effective four-fermion interaction:

$$\mathcal{L}_{Fermi} \simeq G_F[\overline{\psi}_{(\nu_\mu)L}\gamma^\mu\psi_{(\mu)L}][\overline{\psi}_{(e)L}\gamma^\mu\psi_{(\nu_e)L}] + \ldots$$

Parity violation: involves only left-handed fermion fields.
Dots indicate similar four-fermion terms for other processes.

Effective four-fermion interactions

A similar situation would arise if there exists a scale $\Lambda \gg M_W$ corresponding to “new physics".

This new physics would induce effective four-fermion operators for the light (with respect to Λ) fermions of the Standard Model.

And these operators may violate some symmetries of the Standard model. Finding such violations would show the existence of the “new physics".

Consider then four-fermion operators

$$\mathcal{O}_\Lambda = \frac{1}{\Lambda^2}[\overline{\psi}_1\,\psi_2][\overline{\psi}_3\,\psi_4]$$

where $\psi_{1,\ldots,4}$ are quarks or leptons. Since $SU(3)_c \times SU(2)_L \times U(1)_Y$ is an exact symmetry at energies $\ll \Lambda$, these operators must be invariant under the gauge symmetry of the Standard Model and can then easily be classified.

Effective four-fermion interactions

Operators violating baryon number conservation

In principle, the quantum numbers of quarks and leptons do not exclude proton decay.

- A proton is (uud). $SU(3)_c \times U(1)_{em}$ invariance allows processes like

$$\begin{aligned} u + u &\rightarrow \overline{d} + \ell^+ & \qquad p &\rightarrow \pi^0 + \ell^+ \\ u + d &\rightarrow \overline{u} + \ell^+ \,,\; \overline{d} + \nu & \qquad p &\rightarrow \pi^0 + \ell^+ \,,\; \pi^+ + \nu \end{aligned}$$

  ($\ell = e, \mu$, ν could be $\overline{\nu}$, K modes also exist).

- They violate B and L conservations.
- Since the Standard Model has only B- and L-conserving interactions, it forbids these processes to all orders in perturbation theory: proton stable.

Effective four-fermion interactions

The Standard Model has various types of four-fermion operators invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$ violating B and L.

They could be effective contributions from new physics at some scale Λ and detecting proton decay would then bring evidence for these new interactions.

The relevant operators are (symbolically):

$$\frac{1}{\Lambda^2} Q_L Q_L Q_L L_L \quad \frac{1}{\Lambda^2} D_L^c D_L^c U_L^c N_L^c \quad \frac{1}{\Lambda^2} \overline{E_L^c U_L^c} Q_L Q_L \quad \frac{1}{\Lambda^2} \overline{D_L^c U_L^c} Q_L L_L$$

- The first two could result from the exchange of a scalar boson with quantum numbers $(3, 1, -1/3)$, $(3, 1, 1/3)$ or $(3, 1, -2/3)$.
- The last two could arise from the exchange of a vector boson with quantum numbers $(3, 2, -5/6)$ or $(3, 2, 1/6)$.
- They leave $B - L$ invariant.

Effective four-fermion interactions

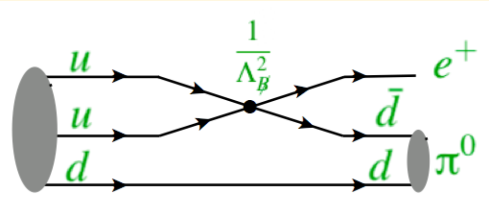
These operators induce proton decay with lifetime

$$\tau_{proton} \sim \Lambda^4 \,/\, m_{proton}^5$$

For the decay mode $p \rightarrow \pi^0 e^+$:

Experimental limit:
$\tau_{proton} > 1.6 \cdot 10^{33}$ years
(super-Kamiokande).

Then: $\Lambda > 10^{15} - 10^{16}$ GeV.



$\Longrightarrow$ Experiment probes extremely high scales

Proton decay in a Grand Unified Theory

The simplest Grand Unified Theory (GUT) is $SU(5)$.
It is defined by:

1. Gauge group: $SU(5)$.
2. Fermions fields in representation $\mathbf{3(10 + \overline{5} + 1)}$,
   spin zero scalar fields in representation $\mathbf{24 + 5 + \overline{5}}$.
3. Write the most general renormalizable Lagrangian.

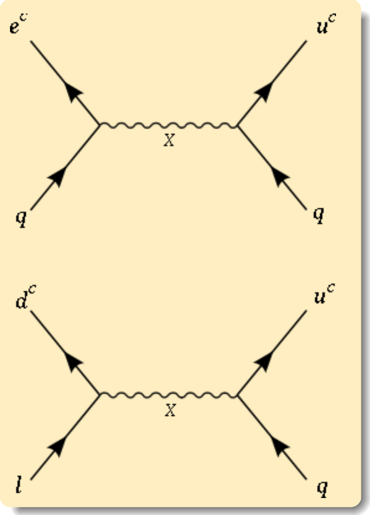
Two successive spontaneous symmetry breakings:

1. $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ induced by scalars in $\mathbf{24}$, scale is M_X.
2. $SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{e.m.}$, induced by scalars in $\mathbf{5 + \overline{5}}$, scale is $v = 246$ GeV.

At scale M_X: $SU(5)$ gauge bosons in representation $(\mathbf{3}, \mathbf{2}, -5/6)$ get a mass $\sim M_X$.

They induce proton decay.

Proton decay in Grand Unified Theories



Proton decay induced by gauge bosons $(\mathbf{3}, \mathbf{2}, -5/6)$.

Limit on τ_{proton} indicates:
$M_X \sim \Lambda > 10^{15}$ GeV.

Effective four-fermion operators:

$$\frac{1}{M_X^2}\overline{e_L^c u_L^c} q_L q_L \qquad \frac{1}{M_X^2}\overline{d_L^c}\,\overline{u_L^c} q_L \ell_L$$

However:

- $SU(5)$ has a single gauge coupling constant.
- $SU(3)_c \times SU(2)_L \times U(1)_Y$ has three couplings $g_s,\ g,\ g'$.
- $SU(5)$ unification leads then to predictions on Standard Model gauge couplings.

Plot then the energy-dependent (measured) gauge running coupling constants in the Standard Model. (see later)

Conclusions are:

- The three gauge couplings do not meet (an any energy scale), except if the Standard Model and the GUT are made supersymmetric.
- Moreover: the most plausible unification scale M_X is too small. It predicts a too fast proton decay.
- (Non supersymmetric) $SU(5)$ is excluded.
- Supersymmetric unification favoured (in support to the Supersymmetric Standard Model).

Running coupling constants

What is the strength of an interaction mesured using a physical process with characteristic energy E ?

[Electromagnetism, fine structure constant: $\alpha(m_{e^-}) \simeq 1/137$, $\alpha(M_Z) \simeq 1/128$].

Obtained from the corresponding QFT probability amplitudes $\mathcal{A}(p_i, \ldots, g)$ with external momenta verifying $E_i = p_i^0 = \mathcal{O}(E)$.

In quantum field theory (QFT) perturbation theory:

1st order:
The amplitude $\mathcal{A}(p_i, \ldots, g)$ is actually a function of the 1st order (one-loop) coupling constant

$$g(E) \equiv g_{corr.} = g - \frac{b_0}{16\pi^2} g^3 \ln(E/m)$$

m: a particle mass,
b_0: a number depending on the field content of the theory.

Running coupling constants

Compare two different reference energies E and E':

$$g(E) = g - \frac{b_0}{16\pi^2}\, g^3 \, \ln(E/m) \qquad\qquad g(E') = g - \frac{b_0}{16\pi^2}\, g^3 \, \ln(E'/m)$$

$$g(E') = g(E) - \frac{b_0}{16\pi^2}\, g^3 \, \ln(E'/E)$$

Equivalent to the **renormalisation-group equation**

$$E\,\frac{d}{dE}\,g = \beta(g) = -\frac{b_0}{16\pi^2}\, g^3 + \mathcal{O}(g^5)$$

For a generic gauge theory with gauge group G, Weyl fermion representation R_f and real scalar representation R_s,

$$b_0 = \frac{22}{3}C(G) - \frac{2}{3}T(R_f) - \frac{1}{6}T(R_s)$$

Running coupling constants

In the Standard Model with 3 generations and 1 Higgs boson (SMS, Minimal Standard Model):

Strong coupling, $SU(3)_c$:	$b_0 = 18 > 0$,	asymptotically-free
Weak coupling, $SU(2)_L$:	$b_0 = 21/2 > 0$,	asymptotically-free
Weak hypercharge $U(1)_Y$:	$b_0 = -41/6 < 0$,	infrared-free
Electric charge $U(1)_{e.m.}$:	$b_0 = -11 < 0$,	infrared-free

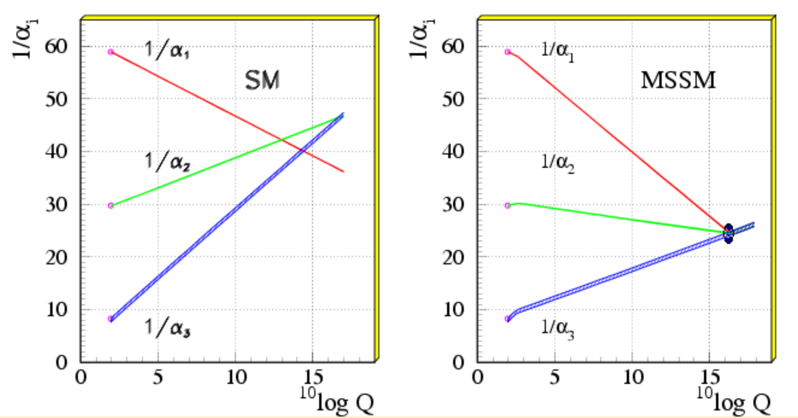
Then:

- Define $\alpha_3 = g_s^2/4\pi$ (strong), $\alpha_2 = g^2/4\pi$ (weak) and $\alpha_1 = {g'}^2/4\pi$ (hypercharge Y).
- Use the values measured at LEP energies: $\alpha_{3,2,1}(M_Z)$.
- Running coupling constants to extrapolate to E:

$$\frac{1}{\alpha(E)} = \frac{1}{\alpha(M_Z)} + \frac{b_0}{2\pi} \ln(E/M_Z)$$

- They measure the strength of each interaction probed at energy E.

Energy-dependent (measured) gauge couplings in the Standard Model



SM = Standard Model, MSSM = Minimal Supersymmetric Standard Model

$\alpha_1 = \frac{5}{3}g'^2/4\pi$: $U(1)_Y$ (weak hypercharge) coupling,

$\alpha_2 = g^2/4\pi$: weak coupling, $\alpha_3 = g_s^2/4\pi$: strong coupling

The problem of scale hierarchies

Known in literature as the "hierarchy problem".

Scales in particle physics:

- The Standard Model has a single scale which can be taken as $M_W \sim 80$ GeV or as $v = 246$ GeV (the expectation value of the Higgs field).

  The Higgs mass is then $M_H^2 = \lambda v^2$ (λ: Higgs self-interaction).

- The typical scale of a GUT unifying strong and electroweak interactions would be $10^{15} - 10^{16}$ GeV.

- Gravitation has its own scale: the Planck scale $M_{Pl.} \sim 10^{18}$ GeV.

  Up to this scale, quantum mechanics does not affect the gravitational field: gravitational waves instead of gravitons. Gravitation can be considered as a background classical force to the Standard Model.

- *Theories "beyond the Standard Model" have in general vastly different fundamental scales.*

The problem of scale hierarchies

The technical problem:

Consider a field theory with two scales Λ_1, Λ_2. We want $\Lambda_1 \ll \Lambda_2$:

- A free choice of parameters in the Lagrangian $\mathcal{L}(\Lambda_1, \Lambda_2, g, \ldots)$.
  (Lowest order of perturbation theory).
- Corrected in perturbation theory.

Suppose that a spin zero particle has mass $M^2 = g\Lambda_1^2$ at lowest order (g: a dimensionless coupling constant).
Think of the Higgs boson in the Standard Model.
Perturbative corrections lead to:

$$
\begin{aligned}
g_{corr.} &= g + A\, g^3 \ln(\Lambda_2/\Lambda_1) + \ldots \\
M^2_{corr.} &= g\Lambda_1^2 + B\, g^3 \Lambda_2^2 + C\, g^3 \Lambda_1^2 \ln(\Lambda_2/\Lambda_1) + \ldots
\end{aligned}
$$

A, B, C: calculable coefficients (of the same order of magnitude).

The problem of scale hierarchies

$$g_{corr.} = g + A\, g^3 \ln(\Lambda_2/\Lambda_1) + \dots$$

$$M^2_{corr.} = g\Lambda_1^2 + B\, g^3 \Lambda_2^2 + C\, g^3 \Lambda_1^2 \ln(\Lambda_2/\Lambda_1) + \dots$$

- We naturally expect that $(A \text{ or } C)\, g^3 \ln(\Lambda_2/\Lambda_1) \lesssim g$ (g small).
- Then: g and $g_{corr.}$: same order of magnitude.
- But: $Bg^3\Lambda_2^2 \gg g\Lambda_1^2$ except if B happens to (almost exactly) vanish.
- We may still decide to finely adjust parameters g, Λ_1 and Λ_2 to keep $M^2_{corr.} \ll \Lambda_2^2$ at order g^3 of perturbation theory.
- This tuning is destroyed at order g^5.
- Tune again, destroyed at order g^7, tune again, and so on ...
- A strikingly inelegant procedure (called a fine-tuning).

$\Longrightarrow$ *Field theory rejects large scale hierarchies.*

$\Longrightarrow$ Only for spin zero fields: either eliminate them, or remove quadratic corrections ($B = 0$), as in supersymmetric theories.

The problem of scale hierarchies

To cancel the coefficient B: the theory should have a symmetry

bosons $\Longleftrightarrow$ fermions

Boson–fermion partners have same interactions, spins differing by 1/2 unit. They can have different masses [since cancelling quadratic divergences with $B = 0$ is primarily an ultraviolet ($E \to \infty$) problem]. Mass differences should be $\mathcal{O}(\Lambda_1)$, the small scale to protect against the hierarchy problem.

We then have <u>softly broken supersymmetry</u>.

Standard Model: double the spectrum with supersymmetric partners

Gauge bosons (helicity ± 1) $\Longrightarrow$ Gauginos, spin 1/2
Quarks and leptons (spin 1/2) $\Longrightarrow$ Scalar quarks and scalar leptons, spin 0
Higgs scalars (spin 0) $\Longrightarrow$ Higgsinos, spin 1/2

PLUS: A second Higgs doublet required: the Minimal Supersymmetric Standard Model (MSSM) has FIVE physical Higgs bosons
[2 neutral scalars, 1 neutral pseudoscalar, 1 charged scalar (+ antiparticle)].

The problem of scale hierarchies

Hence, if physics beyond the Standard Model involves one or several scales $\gg M_W$ (the Planck scale of gravitation could be one of them), options are:

1. The theory has a supersymmetry, softly broken at the lowest scale, and the hierarchy problem does not arise.
2. The theory does not have scalar fields: the dynamical mechanism generating scales is not the Higgs mechanism (new strong forces, fermion condensates, . . .). Speculative ideas however, coherent quantitative theory missing, more convincing concrete and quantitative theoretical work strongly wanted.
3. Or accept extreme fine-tunings of parameters. Inelegant, unintuitive, technically problematic.

Since the first option requires the doubling of the spectrum at the weak interaction scale, it will be tested by LHC experiments.
No observation means exclusion, confirmation harder (a sizeable part of the spectrum should be seen).

MSSM physics is also LHC physics !

Supersymmetry

- Relativistic quantum field theory has global Poincaré symmetry (Lorentz transformations $SO(1,3)$ and space-time translations).
- Can we extend this symmetry (In $D = 4$ Minkowski space-time) ?
- A unique possibility with massive particles: supersymmetry (Haag, Lopuszanski, Sohnius theorem)
- New supersymmetry transformations do not commute with Lorentz transformations, they have spin 1/2.
- They relate bosons and fermions.
- They commute with gauge group transformations. Any gauge theory can be supersymmetrized.
- *The Standard Model can then be supersymmetrized.*
  MSSM and extensions.

Supersymmetry

Supersymmetry algebra

In the simplest case, $N = 1$ SUSY, relevant to MSSM.

Q_α: Majorana spinor, generates supersymmetry transformations
P_μ: generates space-time translations (4-momentum operator)
$M_{\mu\nu}$: generates Lorentz transformations ($SO(1,3)$)

$N = 1$ SUSY

$$\begin{aligned} \{Q_\alpha, Q_\beta\} &= Q_\alpha Q_\beta + Q_\beta Q_\beta \\ &= -2(\gamma^\mu C)_{\alpha\beta} P_\mu \\ [M_{\mu\nu}, Q_\alpha] &= -\frac{i}{2}(\gamma_{\mu\nu} Q)_\alpha \end{aligned}$$

1st relation: "Q_α is the square root of a translation".

2nd relation: Q_α has spin 1/2
SUSY relates then bosons and fermions.

On fields:

$$\delta_Q(\mathbf{BOSON}) = \mathbf{FERMION} \qquad \delta_Q(\mathbf{FERMION}) = \mathbf{BOSON}$$

Supersymmetry

In supersymmetric gauge theories, two supermultiplets:

- Vector or gauge supermultiplet: (A_μ^A, λ^A),
  gauge field (helicities ± 1) and gaugino (helicities $\pm 1/2$).
- Chiral supermultiplet: (ψ^i, z^i),
  Weyl fermion (helicities $\pm 1/2$) and complex scalar (helicities $0, 0$)

Chiral supermultiplet

$$\begin{aligned} \delta z &= \sqrt{2}\,\epsilon\psi \\ \delta\psi_\alpha &= -\sqrt{2}\,f\epsilon_\alpha - \sqrt{2}i(\sigma\bar{\epsilon})_\alpha\partial_\mu z \\ \delta f &= -\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\epsilon} \end{aligned}$$

- Free massive theory: auxiliary field $f = mz$
- Same number of bosons and fermions states.
- All states of a supermultiplet have same masses.

- Possible QFT interactions restricted by supersymmetry.
- Supersymmetry preserved by quantization: supersymmetric QFT.

Supersymmetry

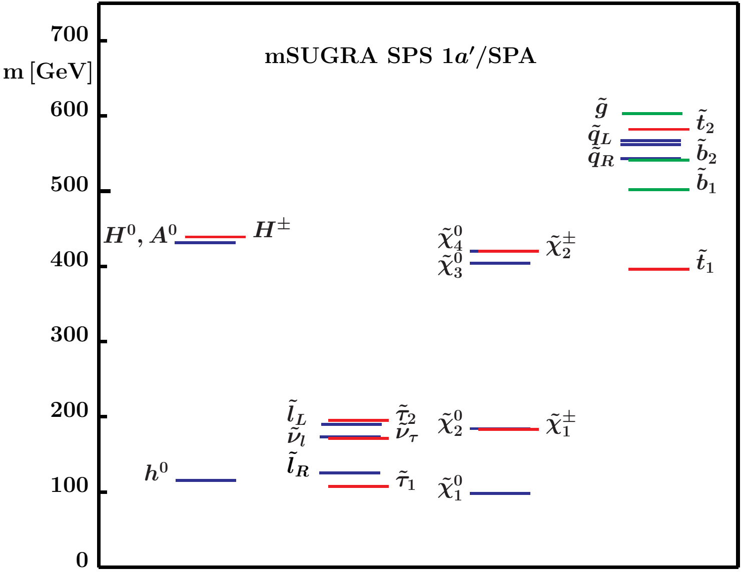
- Not an exact symmetry of Nature: supersymmetric partners with same masses do not exist.
- Maybe an approximate or spontaneously broken symmetry: softly broken supersymmetry: mass differences between SUSY partners of order $\boldsymbol{\Delta}$, the order parameter of supersymmetry breaking.
- Soft breaking: a QFT without potential hierarchy problem.
- MSSM: to protect the weak scale, $\boldsymbol{\Delta} \sim \mathbf{10^2}$ GeV

$$m_{gauginos} - m_{gauge\ bosons} \leq \mathcal{O}(\boldsymbol{\Delta})$$

$$m_{scalar\ quarks\ or\ leptons} - m_{quarks\ or\ leptons} \leq \mathcal{O}(\boldsymbol{\Delta})$$

$$m_{Higgsinos} - m_{Higgs\ bosons} \leq \mathcal{O}(\boldsymbol{\Delta}),$$

Supersymmetry: MSSM, a typical spectrum



Particle Data Group 2009

Supersymmetry

Arguments in support of supersymmetry:

- Values of low-energy coupling constants favour a supersymmetric Standard Model if unification is seeked.
- To protect the weak scale against a much higher unification or Planck scale, associate supersymmetry breaking and the weak scale.
- The supersymmetric extension of the Standard Model is a coherent scheme which can be extensively tested with present experiments.
- *An elegant space-time symmetry, linking bosons and fermions.*

However:

- What is the source of supersymmetry breaking ?
- A realistic spontaneous breaking actually requires LOCAL supersymmetry, this is supergravity which is only a classical theory. Get gravity, lose quantum mechanics . . .
- Need much more, superstrings maybe, to recover quantum mechanics.

Supersymmetry

Adding supersymmetry to the Standard Model calls for
new physics at much higher scales and
for unification with (quantum) gravity

We wait for data