

# Particle Detectors

Summer Student Lectures 2010

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- ◆ **History of Instrumentation ↔ History of Particle Physics**
- ◆ **The 'Real' World of Particles**
- ◆ **Interaction of Particles with Matter**
- ◆ **Tracking Detectors, Calorimeters, Particle Identification**
- ◆ **Detector Systems**

# Detector Physics

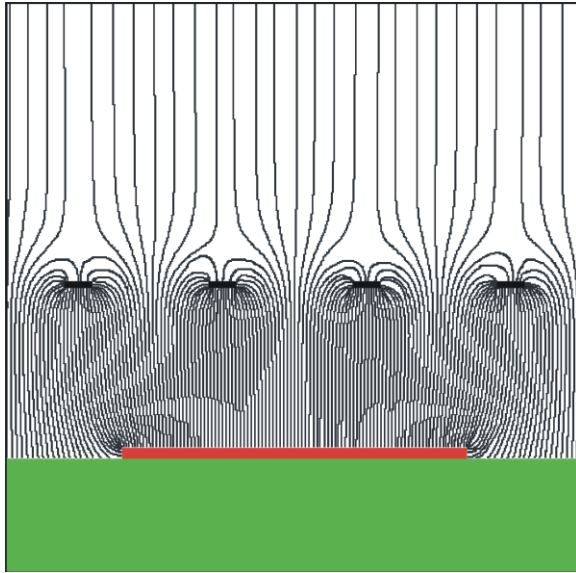
Precise knowledge of the processes leading to signals in particle detectors is necessary.

The detectors are nowadays working close to the limits of theoretically achievable measurement accuracy – even in large systems.

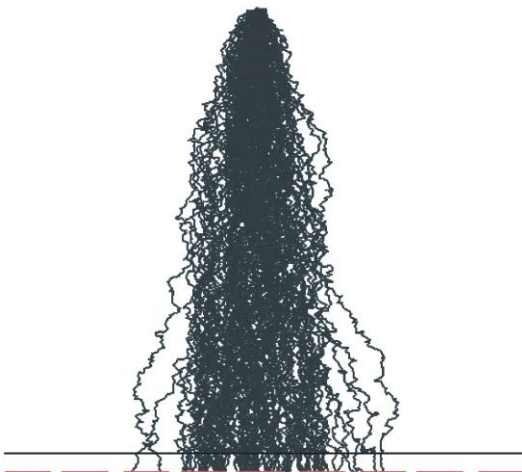
Due to available computing power, detectors can be simulated to within 5-10% of reality, based on the fundamental microphysics processes (atomic and nuclear crosssections).

# Particle Detector Simulation

Electric Fields in a Micromega Detector



Electrons avalanche multiplication

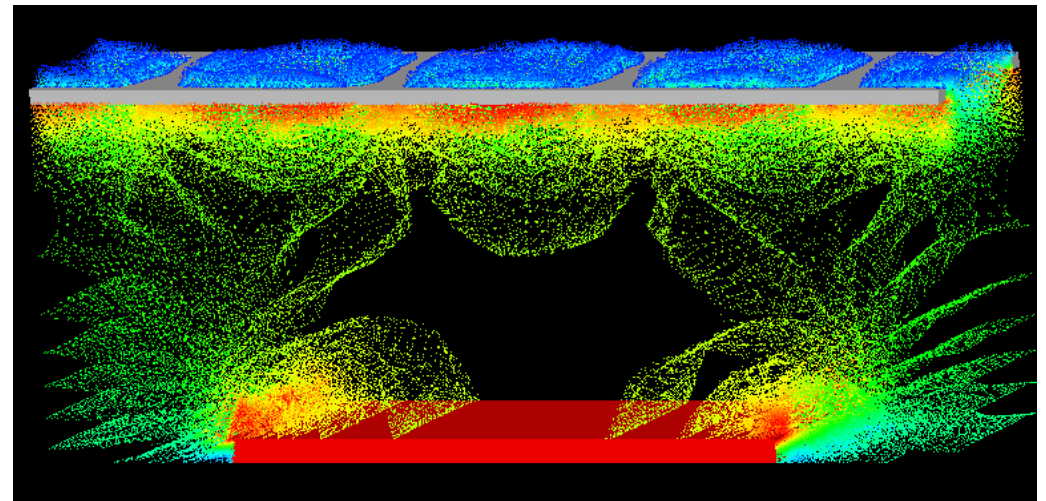
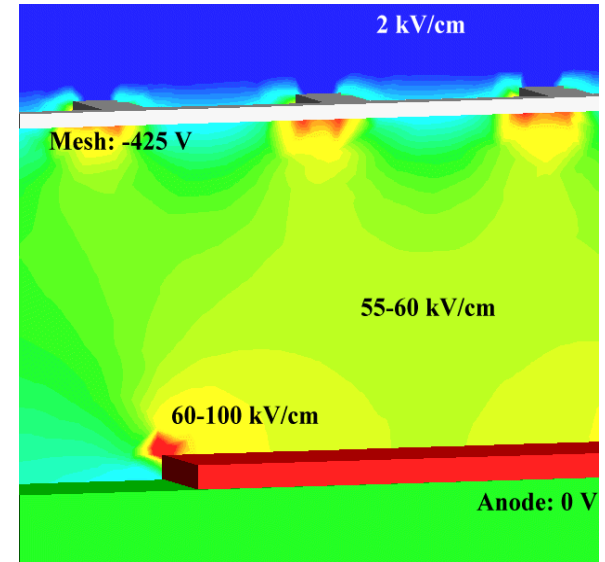


Very accurate simulations of particle detectors are possible due to availability of Finite Element simulation programs and computing power.

Follow every single electron by applying first principle laws of physics.

For Gaseous Detectors:  
GARFIELD by R. Veenhof

Electric Fields in a Micromega Detector

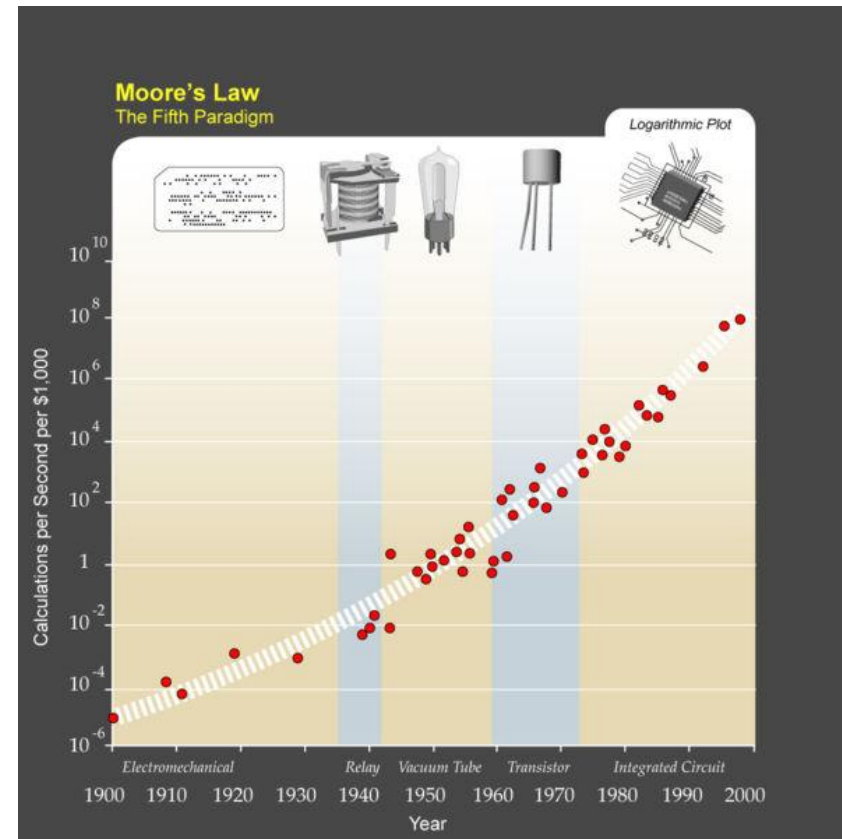


# Particle Detector Simulation

I) C. Moore's Law:  
Computing power doubles 18 months.

II) W. Riegler's Law:  
The use of brain for solving a problem  
is inversely proportional to the available  
computing power.

→ I) + II) = ...



Knowing the basics of particle detectors is essential ...

# Interaction of Particles with Matter

Any device that is to detect a particle must interact with it in some way → almost ...

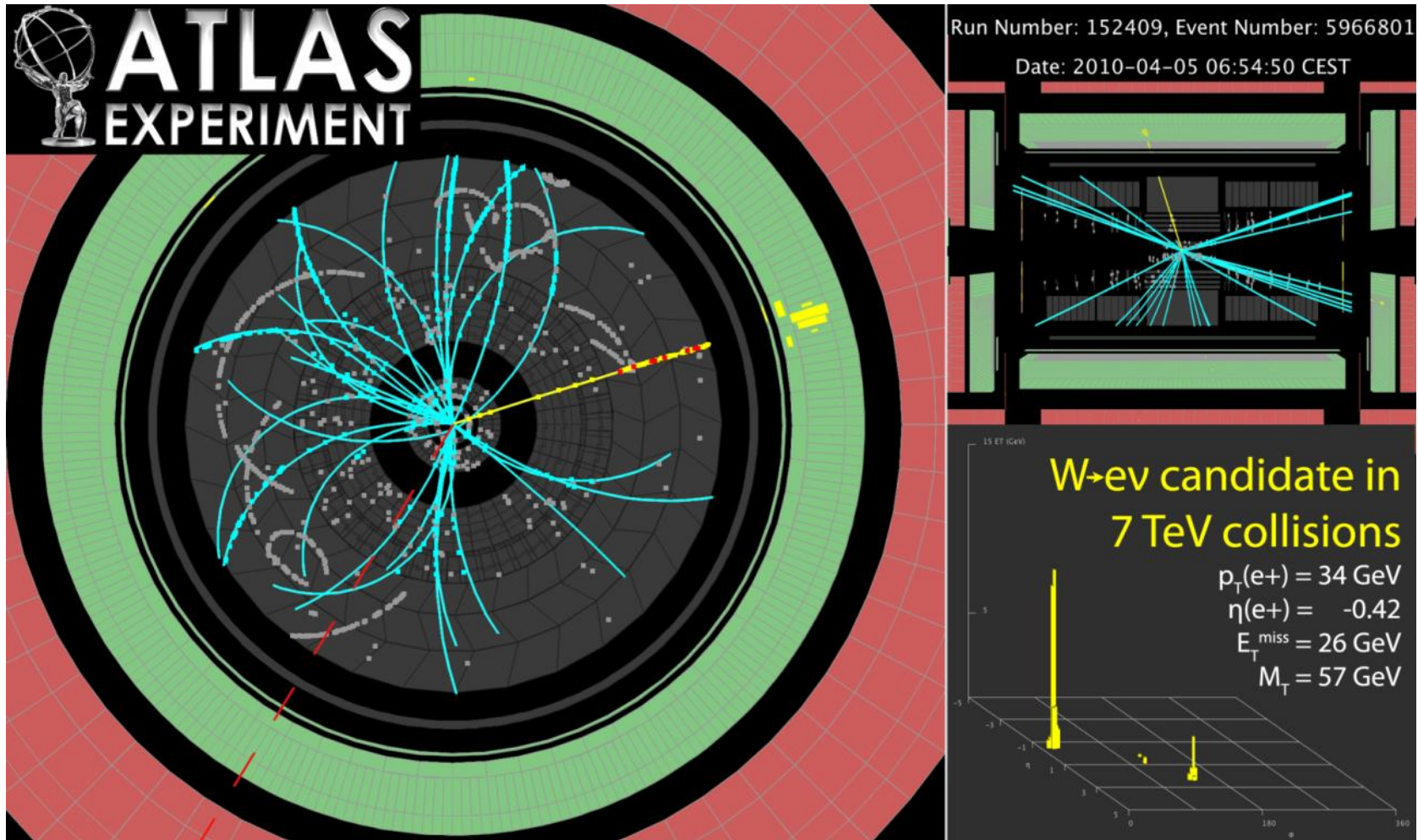
In many experiments neutrinos are measured by missing transverse momentum.

E.g.  $e^+e^-$  collider.  $P_{\text{tot}}=0$ ,  
If the  $\Sigma p_i$  of all collision products is  $\neq 0$  → neutrino escaped.

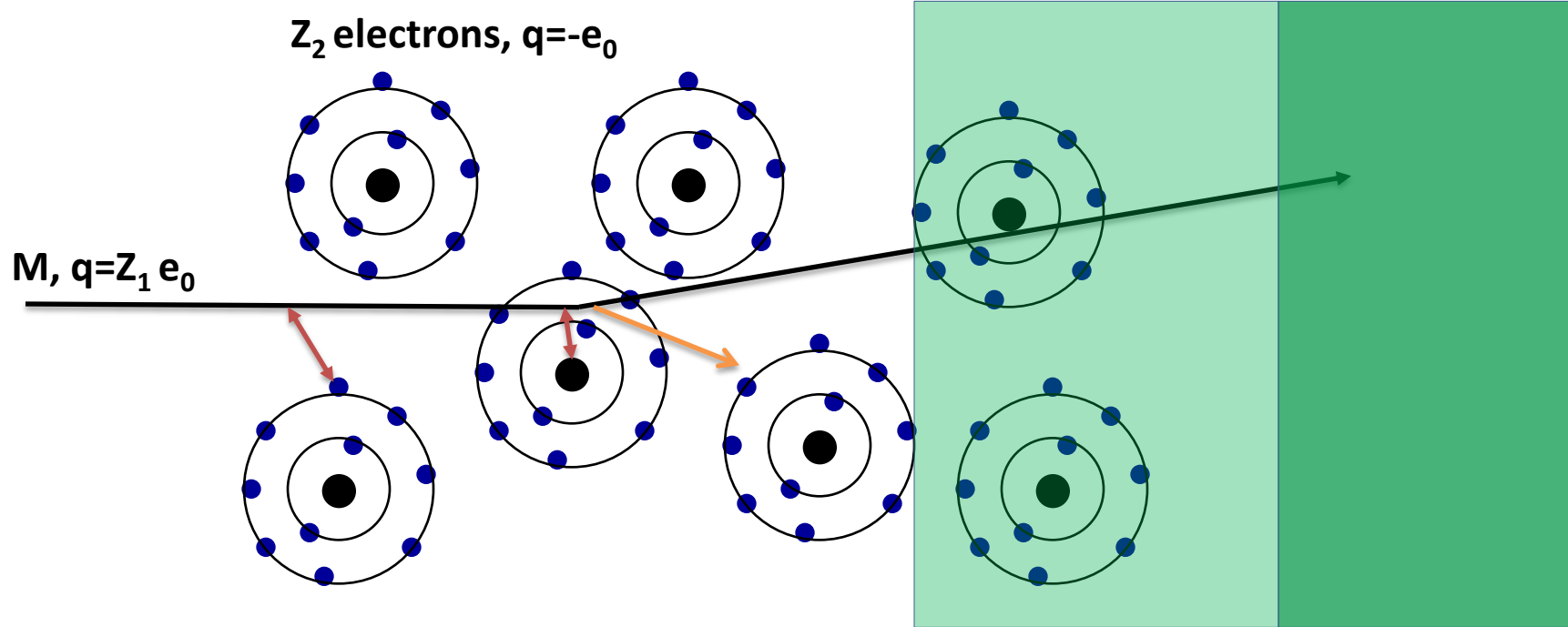


“Did you see it?”  
“No nothing.”  
“Then it was a neutrino!”

# Interaction of Particles with Matter



# Electromagnetic Interaction of Particles with Matter

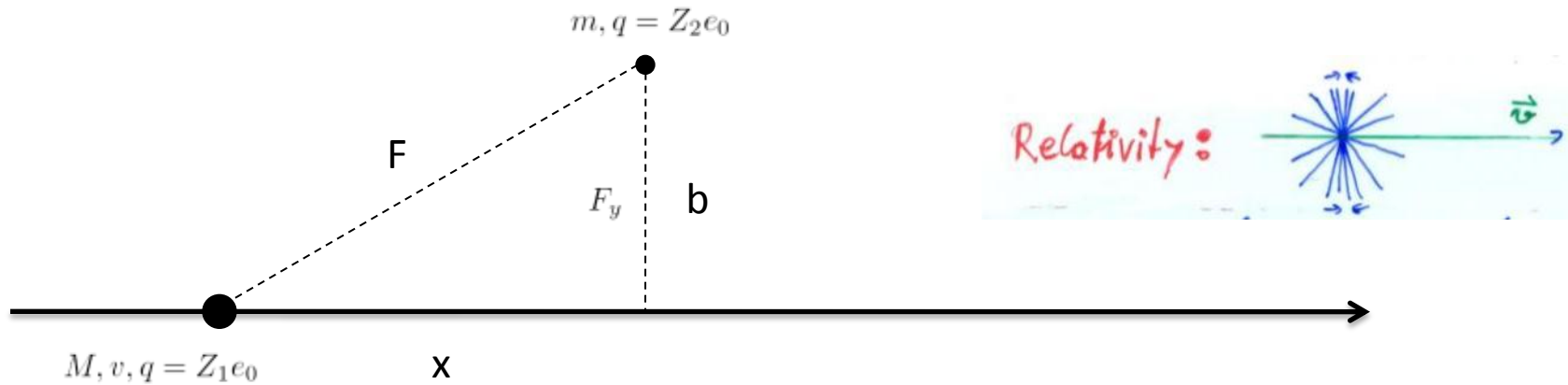


Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produced and X ray photon, called Transition radiation.

# Ionization and Excitation



**While the charged particle is passing another charged particle, the Coulomb Force is acting, resulting in momentum transfer**

$$F_y = \frac{Z_1 Z_2 e_0^2}{4\pi\epsilon_0 (b^2 + v^2 t^2)} \frac{b}{\sqrt{b^2 + v^2 t^2}} \quad \Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2Z_1 Z_2 e_0^2}{4\pi\epsilon_0 v b}$$

**The relativistic form of the transverse electric field doesn't change the momentum transfer. The transverse field is stronger, but the time of action is shorter**

$$F_y = \frac{\gamma Z_1 Z_2 e_0^2 b}{4\pi\epsilon_0 (b^2 + \gamma^2 v^2 t^2)^{3/2}} \quad \Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2Z_1 Z_2 e_0^2}{4\pi\epsilon_0 v b}$$

**The transferred energy is then**

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{Z_2^2}{m} \frac{2Z_1^2 e_0^4}{(4\pi\epsilon_0)^2 v^2 b^2}$$

$$\Delta E(\text{electrons}) = Z_2 \frac{1}{m_e} \frac{2Z_1^2 e_0^4}{(4\pi\epsilon_0)^2 v^2 b^2} \quad \Delta E(\text{nucleus}) = \frac{Z_2^2}{2Z_2 m_p} \frac{2Z_1^2 e_0^4}{(4\pi\epsilon_0)^2 v^2 b^2} \quad \frac{\Delta E(\text{electrons})}{\Delta E(\text{nucleus})} = \frac{2m_p}{m_e} \approx 4000$$

**→ The incoming particle transfer energy only (mostly) to the atomic electrons !**



# Ionization and Excitation

Target material: mass A, Z<sub>2</sub>, density ρ [g/cm<sup>3</sup>], Avogadro number N<sub>A</sub>

A gramm → N<sub>A</sub> Atoms:

Number of atoms/cm<sup>3</sup>

n<sub>a</sub> = N<sub>A</sub> ρ / A [1/cm<sup>3</sup>]

Number of electrons/cm<sup>3</sup>

n<sub>e</sub> = N<sub>A</sub> ρ Z<sub>2</sub> / A [1/cm<sup>3</sup>]

$$\Delta E(\text{electrons}) = \frac{2Z_2 Z_1^2 m_e c^2}{\beta^2 b^2} \frac{e_0^4}{(4\pi\epsilon_0 m_e c^2)^2} = \frac{2Z_2 Z_1^2 m_e c^2}{\beta^2 b^2} r_e^2$$



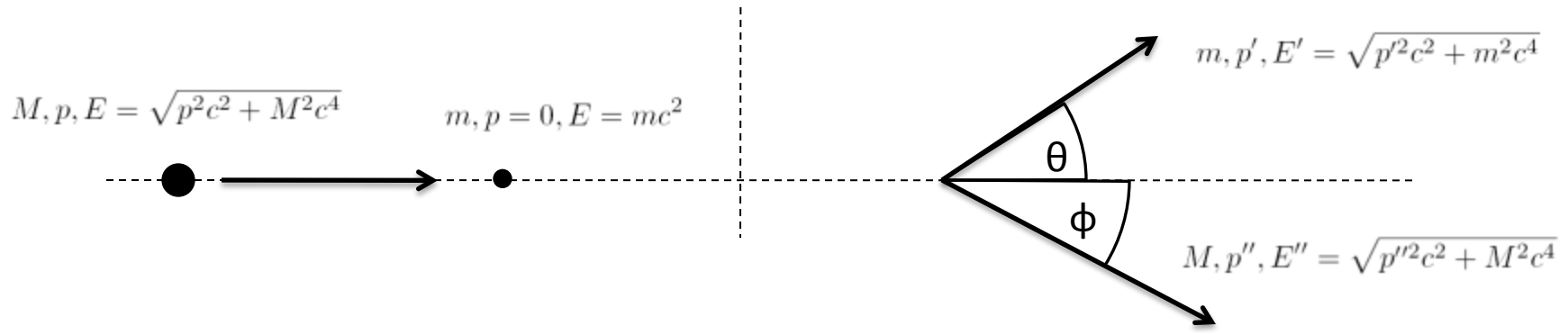
$$dE = - \int_{b_{min}}^{b_{max}} n_e \Delta E dx 2b\pi db = - \frac{4\pi Z_2 Z_1^2 m_e c^2 r_e^2}{\beta^2} \frac{N_A \rho}{A} \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

With  $\Delta E(b) \rightarrow db/b = -1/2 dE/E \rightarrow E_{max} = \Delta E(b_{min}) \quad E_{min} = \Delta E(b_{max})$

$$\frac{dE}{dx} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \int_{E_{min}}^{E_{max}} \frac{dE}{E} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \ln \frac{E_{max}}{E_{min}}$$

**E<sub>min</sub> ≈ I (Ionization Energy)**

# Relativistic Collision Kinematics, $E_{\max}$



$$1) \quad \sqrt{p^2 c^2 + M^2 c^4} + mc^2 = \sqrt{p'^2 c^2 + m^2 c^4} + \sqrt{p''^2 c^2 + M^2 c^4}$$

$$2) \quad p = p' \cos \theta + p'' \cos \phi \quad p'^2 = p^2 + p''^2 - 2pp'' \cos \theta$$

$$0 = p' \sin \theta + p'' \sin \phi$$

$$1+2) \quad E^{k'} = \sqrt{p'^2 c^2 + m^2 c^4} - mc^2 = \frac{2mc^2 p^2 c^2 \cos^2 \theta}{\left[ mc^2 + \sqrt{p^2 c^2 + M^2 c^4} \right]^2 - p^2 c^2 \cos^2 \theta}$$

$$E_{\max}^{k'} = \frac{2mc^2 p^2 c^2}{(m^2 + M^2)c^4 + 2m\sqrt{p^2 c^2 + M^2 c^4}} = 2mc^2 \beta^2 \gamma^2 F \quad F = \left( 1 + \frac{2m}{M} \sqrt{1 + \beta^2 \gamma^2} + \frac{m^2}{M^2} \right)^{-1}$$

# Classical Scattering on Free Electrons

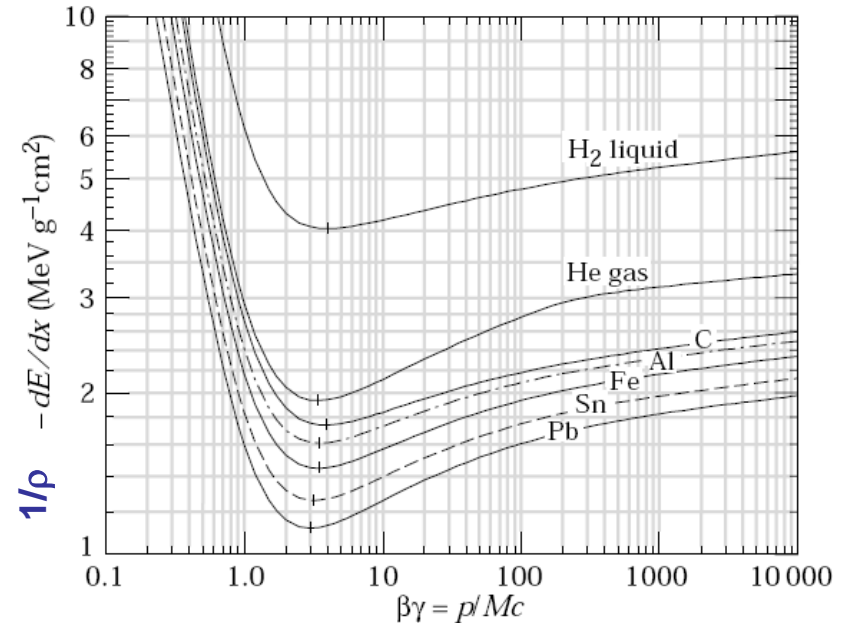
$$\frac{1}{\rho} \frac{dE}{dx} = -2\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2} N_A \frac{Z}{A} \ln \frac{2m_e c^2 \beta^2 \gamma^2 F}{I}$$

This formula is up to a factor 2 and the density effect identical to the precise QM derivation →

## Bethe Bloch Formula

$$\frac{1}{\rho} \frac{dE}{dx} = \underline{-4\pi r_e^2 m_e c^2} \frac{Z_1^2}{\beta^2} N_A \frac{Z}{A} \left[ \ln \frac{2m_e c^2 \beta^2 \gamma^2 F}{I} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

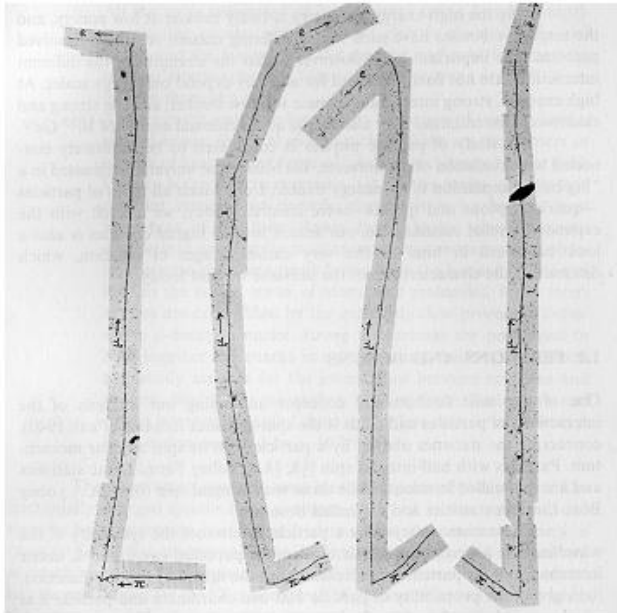
Electron Spin



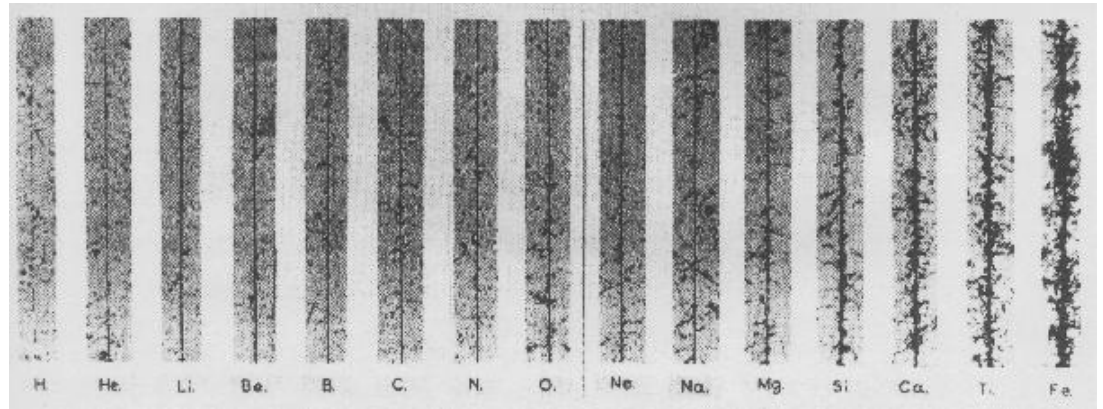
$$\delta(\beta\gamma) = \ln h\omega_p/I + \ln \beta\gamma - \frac{1}{2}$$

Density effect. Medium is polarized  
Which reduces the log. rise.

Small energy loss  
→ Fast Particle

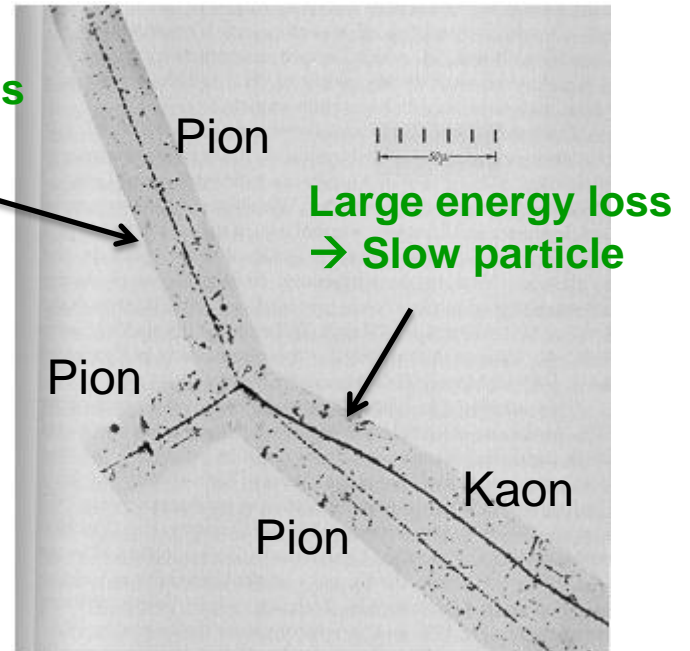


Discovery of muon and pion



Cosmic rays:  $dE/dx \propto Z^2$

Small energy loss  
→ Fast particle



Large energy loss  
→ Slow particle

# Bethe Bloch Formula

$$\frac{1}{\rho} \frac{dE}{dx} = -4\pi r_e^2 m_e c^2 \frac{Z_1^2}{\beta^2} N_A \frac{Z}{A} \left[ \ln \frac{2m_e c^2 \beta^2 \gamma^2 F}{I} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Für  $Z > 1$ ,  $I \approx 16Z^{0.9} \text{ eV}$

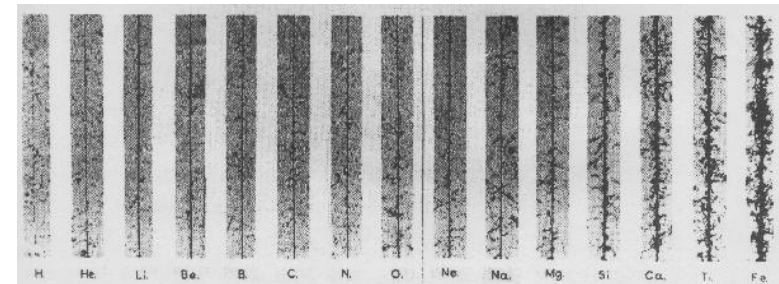
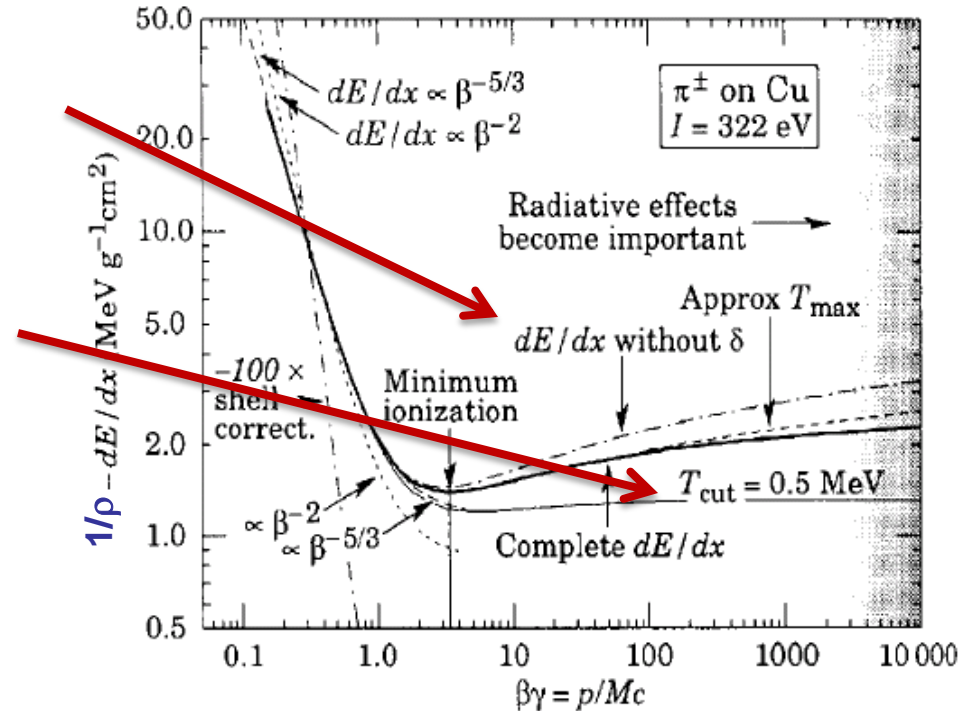
For Large  $\beta\gamma$  the medium is being polarized by the strong transverse fields, which reduces the rise of the energy loss  $\rightarrow$  density effect

At large Energy Transfers (delta electrons) the liberated electrons can leave the material. In reality,  $E_{\text{max}}$  must be replaced by  $E_{\text{cut}}$  and the energy loss reaches a plateau (Fermi plateau).

Characteristics of the energy loss as a function of the particle velocity ( $\beta\gamma$ )

The specific Energy Loss  $1/\rho \text{ dE/dx}$

- first decreases as  $1/\beta^2$
- increases with  $\ln \gamma$  for  $\beta = 1$
- is  $\approx$  independent of  $M$  ( $M \gg m_e$ )
- is proportional to  $Z_1^2$  of the incoming particle.
- is  $\approx$  independent of the material ( $Z/A \approx \text{const}$ )
- shows a plateau at large  $\beta\gamma$  ( $\gg 100$ )
- $dE/dx \approx 1-2 \times \rho \text{ [g/cm}^3\text{] MeV/cm}$



# Bethe Bloch Formula

Bethe Bloch Formula, a few Numbers:

For  $Z \approx 0.5 A$

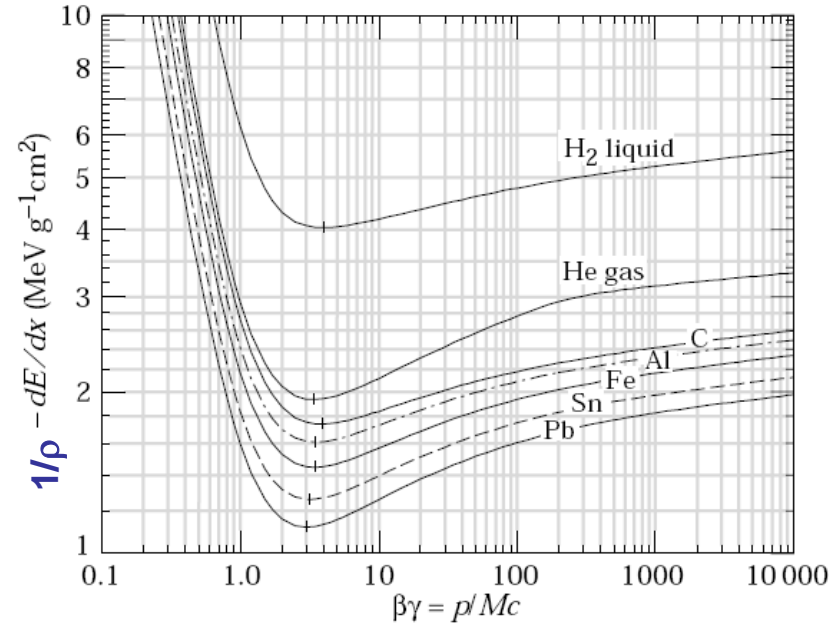
$1/\rho \text{ dE}/\text{dx} \approx 1.4 \text{ MeV cm}^2/\text{g}$  for  $\beta\gamma \approx 3$

**Example :**

Iron: Thickness = 100 cm;  $\rho = 7.87 \text{ g/cm}^3$

$\text{dE} \approx 1.4 * 100 * 7.87 = 1102 \text{ MeV}$

→ A 1 GeV Muon can traverse 1m of Iron



This number must be multiplied with  $\rho$  [ $\text{g/cm}^3$ ] of the Material →  $\text{dE}/\text{dx}$  [ $\text{MeV}/\text{cm}$ ]

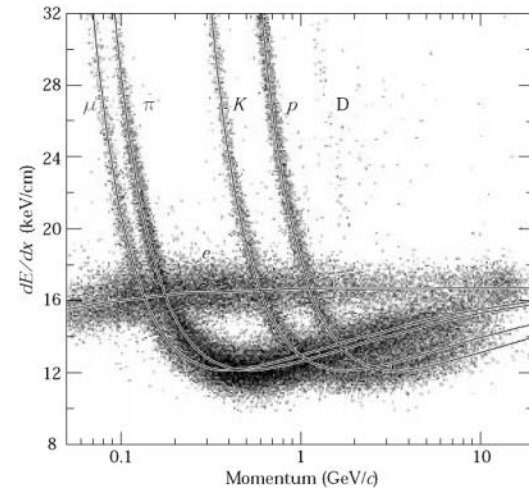
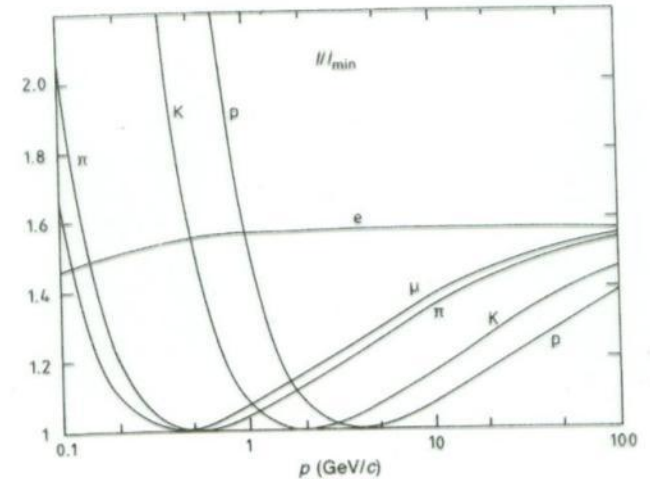
# Energy Loss as a Function of the Momentum

Energy loss depends on the particle velocity and is  $\approx$  independent of the particle's mass  $M$ .

The energy loss as a function of particle Momentum  $P = Mc\beta\gamma$  IS however depending on the particle's mass

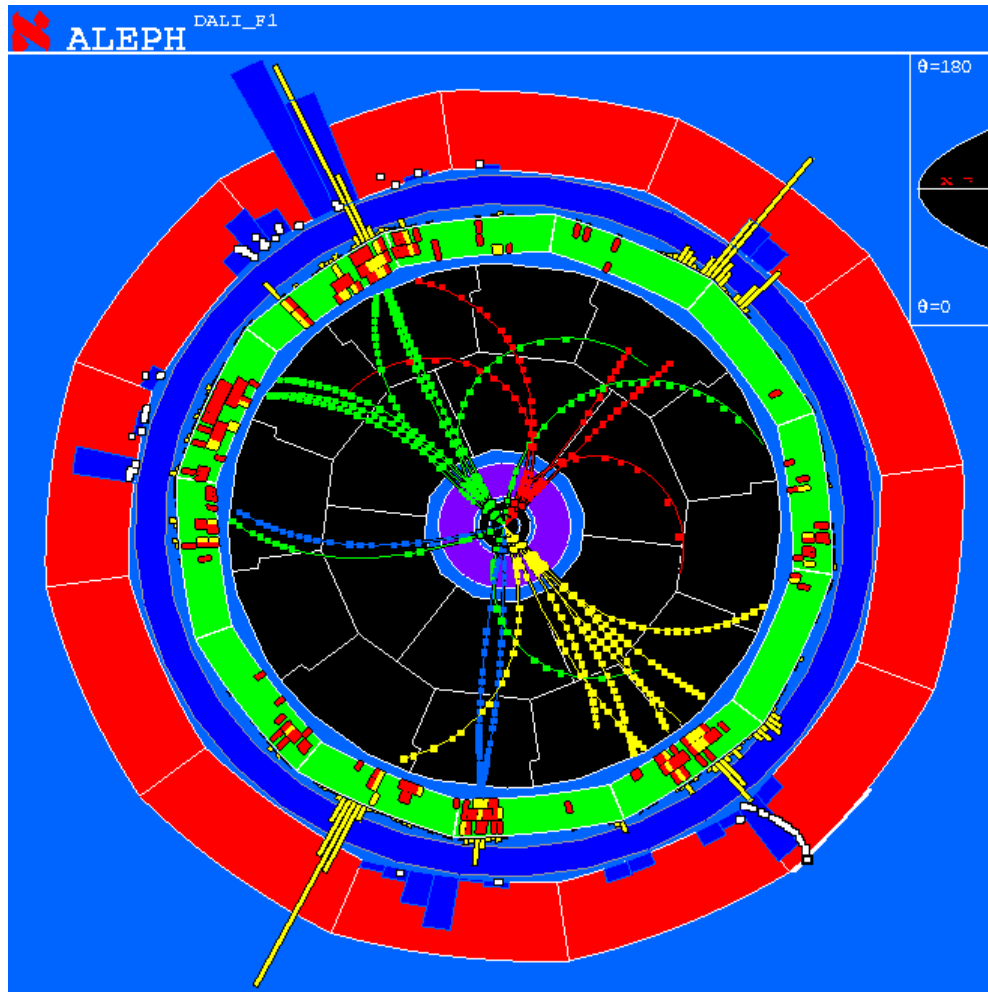
By measuring the particle momentum (deflection in the magnetic field) and measurement of the energy loss one can measure the particle mass

→ Particle Identification !



$$\frac{1}{\rho} \frac{dE}{dx} = -4\pi r_e^2 m_e c^2 Z_1^2 \frac{p^2 + M^2 c^2}{p^2} N_A \frac{Z}{A} \left[ \ln \frac{2m_e c^2 F}{I} \frac{p^2}{M^2 c^2} - \frac{p^2}{p^2 + M^2 c^2} \right]$$

# Energy Loss as a Function of the Momentum



Measure momentum by curvature of the particle track.

Find  $dE/dx$  by measuring the deposited charge along the track.

→ Particle ID



# Range of Particles in Matter

Particle of mass  $M$  and kinetic Energy  $E_0$  enters matter and loses energy until it comes to rest at distance  $R$ .

$$R(E_0) = \int_{E_0}^0 \frac{-1}{dE/dx} dE$$

$$R(\beta_0 \gamma_0) = \frac{Mc^2}{\rho} \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0 \gamma_0)$$

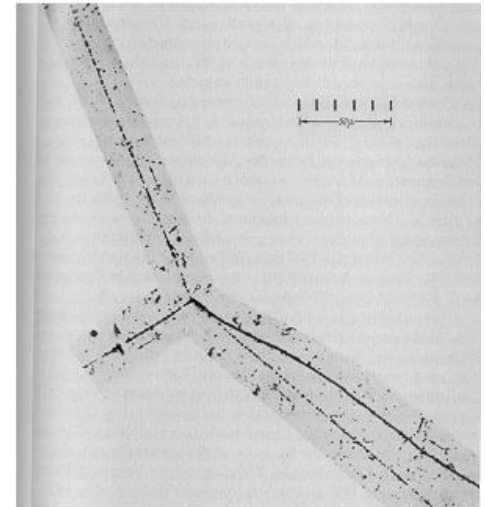
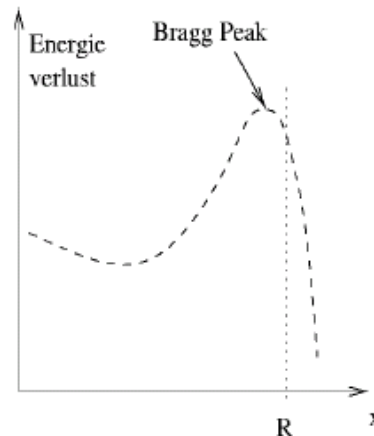
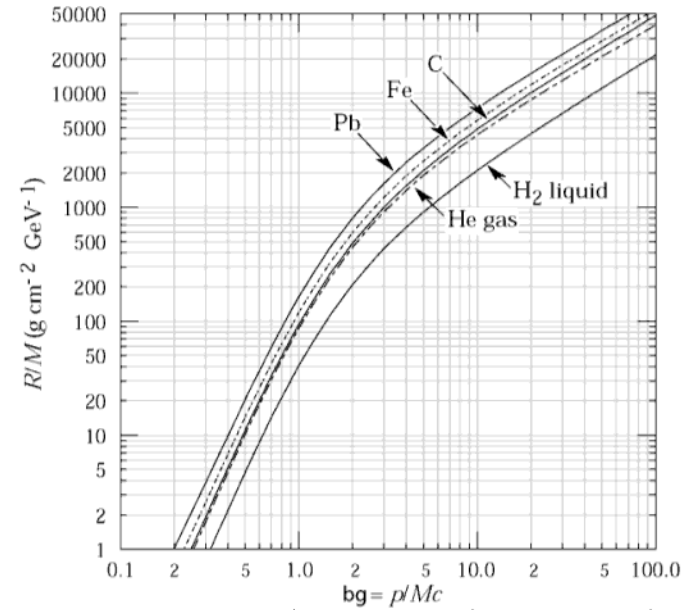
$$\frac{\rho}{Mc^2} R(\beta_0 \gamma_0) = \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0 \gamma_0) \approx \text{Independent of the material}$$

## Bragg Peak:

For  $\beta\gamma > 3$  the energy loss is  $\approx$  constant (Fermi Plateau)

If the energy of the particle falls below  $\beta\gamma=3$  the energy loss rises as  $1/\beta^2$

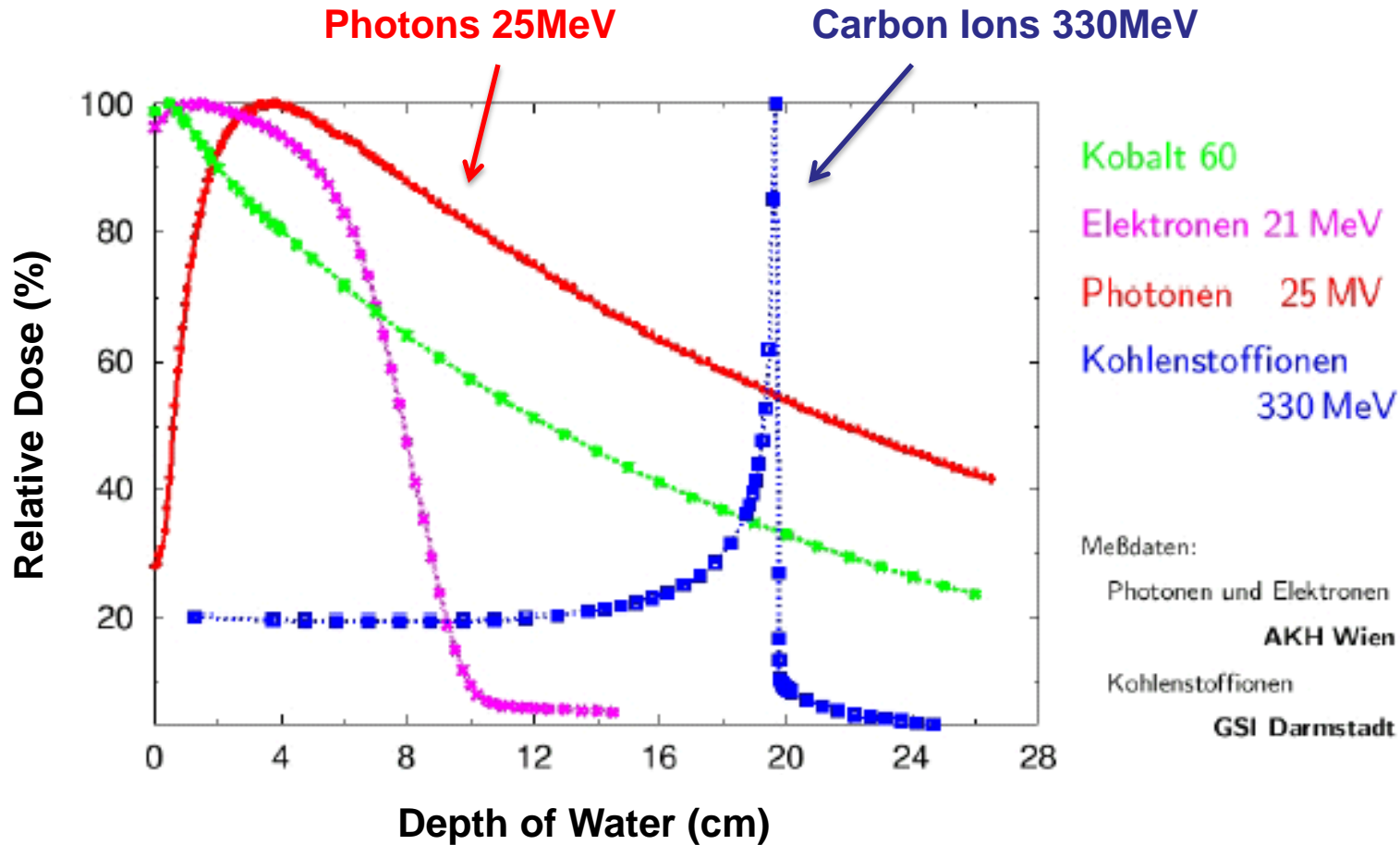
Towards the end of the track the energy loss is largest  $\rightarrow$  Cancer Therapy.



# Range of Particles in Matter

## Average Range:

Towards the end of the track the energy loss is largest → Bragg Peak → Cancer Therapy



# Search for Hidden Chambers in the Pyramids

The structure of the Second Pyramid of Giza is determined by cosmic-ray absorption.

Luis W. Alvarez, Jared A. Anderson, F. El Bedwei, James Burkhard, Ahmed Fakhry, Adib Girgis, Amr Goneid, Fikhray Hassan, Dennis Iverson, Gerald Lynch, Zenab Miligy, Ali Hilmy Moussa, Mohammed-Sharkawi, Lauren Yazolino

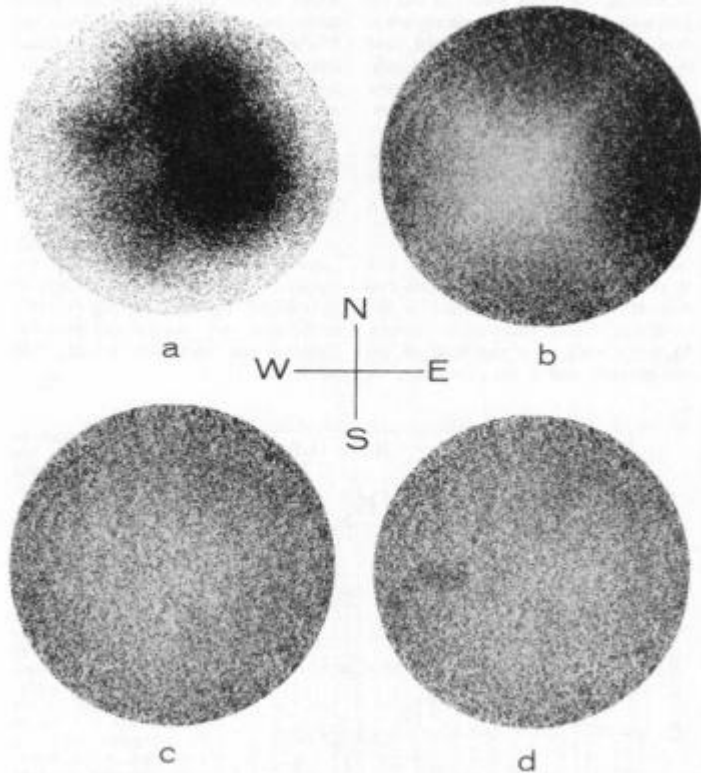


Fig. 13. Scatter plots showing the three stages in the combined analytic and visual analysis of the data and a plot with a simulated chamber, (a) Simulated "x-ray photograph" of uncorrected data. (b) Data corrected for the geometrical acceptance of the apparatus. (c) Data corrected for pyramid structure as well as geometrical acceptance. (d) Same as (c) but with simulated chamber, as in Fig. 12.

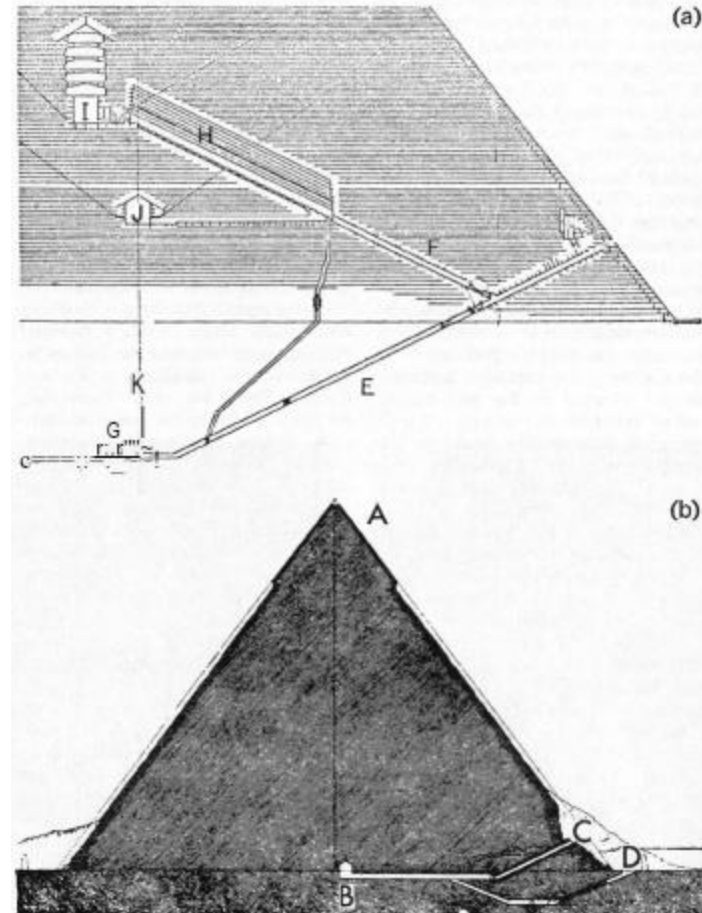
Fig. 2 (bottom right). Cross sections of (a) the Great Pyramid of Cheops and (b) the Pyramid of Chephren, showing the known chambers: (A) Smooth limestone cap, (B) the Belzoni Chamber, (C) Belzoni's entrance, (D) Howard-Vyse's entrance, III descending passageway, (F) ascending passageway, (G) underground chamber, (-1) Grand Gallery, (I) King's Chamber, (J) Queen's Chamber, (K) center line of the pyramid.

6 FEBRUARY 1970



Luis Alvarez used the attenuation of muons to look for chambers in the Second Giza Pyramid → Muon Tomography

He proved that there are no chambers present.



# Intermezzo: Crosssection

Crosssection  $\sigma$ : Material with Atomic Mass  $A$  and density  $\rho$  contains  $n$  Atoms/cm<sup>3</sup>

$$n[\text{cm}^{-3}] = \frac{N_A[\text{mol}^{-1}] \rho[\text{g}/\text{cm}^3]}{A[\text{g}/\text{mol}]} \quad N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

E.g. Atom (Sphere) with Radius  $R$ : Atomic Crosssection  $\sigma = R^2\pi$

A volume with surface  $F$  and thickness  $dx$  contains  $N=nFdx$  Atoms.

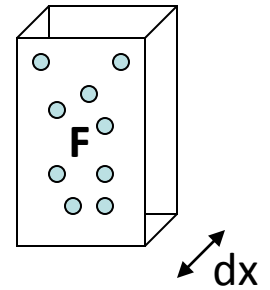
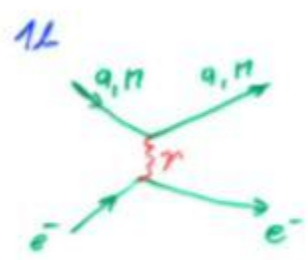
The total 'surface' of atoms in this volume is  $N \sigma$ .

The relative area is  $p = N \sigma / F = N_A \rho \sigma / A dx =$

Probability that an incoming particle hits an atom in  $dx$ .

What is the probability  $P$  that a particle hits an atom between distance  $x$  and  $x+dx$  ?

$P$  = probability that the particle does NOT hit an atom in the  $m=x/dx$  material layers and that the particle DOES hit an atom in the  $m^{\text{th}}$  layer



$$P(x)dx = (1-p)^m p \approx e^{-m} p = \exp\left(-\frac{N_A \rho \sigma}{A} x\right) \frac{N_A \rho \sigma}{A} dx = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right) dx \quad \lambda = \frac{A}{N_A \rho \sigma}$$

Mean free path  $= \int_0^{\infty} x P(x) dx = \int_0^{\infty} \frac{x}{\lambda} e^{-\frac{x}{\lambda}} dx = \lambda$

Average number of collisions/cm  $= \frac{1}{\lambda} = \frac{N_A \rho \sigma}{A}$

# Intermezzo: Differential Crosssection



**Differential Crosssection:**  $\frac{d\sigma(E, E')}{dE'}$

→ Crosssection for an incoming particle of energy  $E$  to lose an energy between  $E'$  and  $E'+dE'$

**Total Crosssection:**  $\sigma(E) = \int \frac{d\sigma(E, E')}{dE'} dE'$

**Probability  $P(E)$  that an incoming particle of Energy  $E$  loses an energy between  $E'$  and  $E'+dE'$  in a collision:**

$$P(E, E')dE' = \frac{1}{\sigma(E)} \frac{d\sigma(E, E')}{dE'} dE'$$

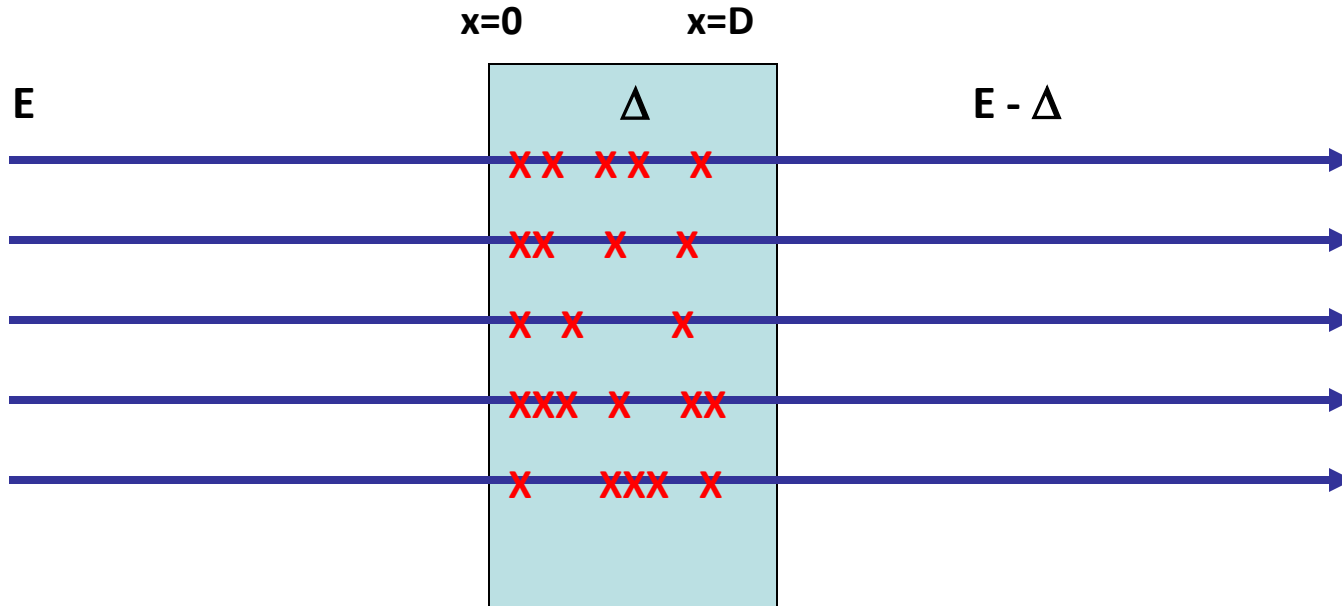
**Average number of collisions/cm causing an energy loss between  $E'$  and  $E'+dE'$**   $= \frac{N_A \rho}{A} \frac{d\sigma(E, E')}{dE'}$

**Average energy loss/cm:**  $\frac{dE}{dx} = -\frac{N_A \rho}{A} \int E' \frac{d\sigma(E, E')}{dE'} dE'$

7/15/2010

# Fluctuation of Energy Loss

Up to now we have calculated the average energy loss. The energy loss is however a statistical process and will therefore fluctuate from event to event.



**$P(\Delta) = ?$  Probability that a particle loses an energy  $\Delta$  when traversing a material of thickness  $D$**

We have seen earlier that the probability of an interaction occurring between distance  $x$  and  $x+dx$  is exponentially distributed

$$P(x)dx = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right) dx \quad \lambda = \frac{A}{N_A \rho \sigma}$$

# Probability for n Interactions in D

We first calculate the probability to find n interactions in D, knowing that the probability to find a distance x between two interactions is  $P(x)dx = 1/\lambda \exp(-x/\lambda) dx$  with  $\lambda = A/N_A\rho\sigma$

Probability to have no interaction between 0 und D:

$$P(x > D) = \int_D^\infty P(x_1)dx_1 = e^{-\frac{D}{\lambda}}$$

Probability to have one interaction at  $x_1$  and no other interaction:

$$P(x_1, x_2 > D) = \int_D^\infty P(x_1)P(x_2 - x_1)dx_2 = \frac{1}{\lambda} e^{-\frac{D}{\lambda}}$$

Probability to have one interaction independently of  $x_1$ :

$$\int_0^D P(x_1, x_2 > D) = \frac{D}{\lambda} e^{-\frac{D}{\lambda}}$$

Probability to have the first interaction at  $x_1$ , the second at  $x_2$  .... the  $n^{th}$  at  $x_n$  and no other interaction:

$$P(x_1, x_2 \dots x_n > D) = \int_D^\infty P(x_1)P(x_2 - x_1) \dots P(x_n - x_{n-1})dx_n = \frac{1}{\lambda^n} e^{-\frac{D}{\lambda}}$$

Probability for n interactions independently of  $x_1, x_2 \dots x_n$

$$\int_0^D \int_0^{x_{n-1}} \int_0^{x_{n-1}} \dots \int_0^{x_1} P(x_1, x_2, \dots, x_n > D) dx_1 \dots dx_{n-1} = \frac{1}{n!} \left(\frac{D}{\lambda}\right)^n e^{-\frac{D}{\lambda}}$$

# Probability for n Interactions in D

For an interaction with a mean free path of  $\lambda$ , the probability for n interactions on a distance D is given by

$$P(n) = \frac{1}{n!} \left( \frac{D}{\lambda} \right)^n e^{-\frac{D}{\lambda}} = \frac{\bar{n}^n}{n!} e^{-\bar{n}} \quad \bar{n} = \frac{D}{\lambda} \quad \lambda = \frac{A}{N_A \rho \sigma}$$

→ Poisson Distribution !

If the distance between interactions is exponentially distributed with an mean free path of  $\lambda$  → the number of interactions on a distance D is Poisson distributed with an average of  $\bar{n}=D/\lambda$ .

How do we find the energy loss distribution ?

If  $f(E)$  is the probability to lose the energy  $E'$  in an interaction, the probability  $p(E)$  to lose an energy E over the distance D ?

$$f(E) = \frac{1}{\sigma} \frac{d\sigma}{dE}$$

$$p(E) = P(1)f(E) + P(2) \int_0^E f(E-E')f(E')dE' + P(3) \int_0^E \int_0^{E'} f(E-E'-E'')f(E'')f(E')dE''dE' + \dots$$

$$F(s) = \mathcal{L}[f(E)] = \int_0^\infty f(E)e^{-sE}dE$$

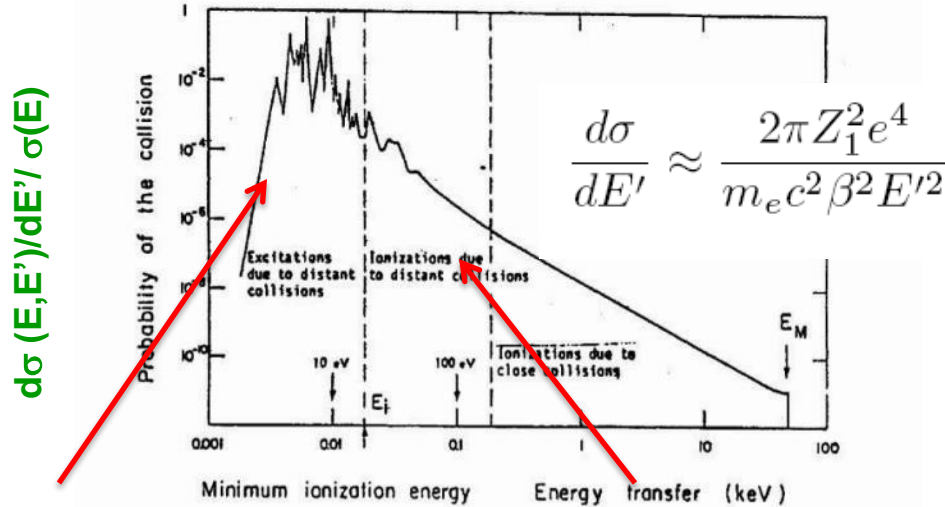
$$\mathcal{L}[p(E)] = P(1)F(s) + P(2)F(s)^2 + P(3)F(s)^3 + \dots = \sum_{n=1}^{\infty} P(n)F(s)^n = \sum_{n=1}^{\infty} \frac{\bar{n}^n F^n}{n!} e^{-\bar{n}} = e^{\bar{n}(F(s)-1)} - 1 \approx e^{\bar{n}(F(s)-1)}$$

$$p(E) = \mathcal{L}^{-1} \left[ e^{\bar{n}(F(s)-1)} \right] = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{\bar{n}(F(s)-1)+sE} ds$$



# Fluctuations of the Energy Loss

Probability  $f(E)$  for losing energy between  $E'$  and  $E'+dE'$  in a single interaction is given by the differential cross section  $d\sigma(E, E')/dE'$  which is given by the Rutherford cross section at large energy transfers



Excitation and ionization

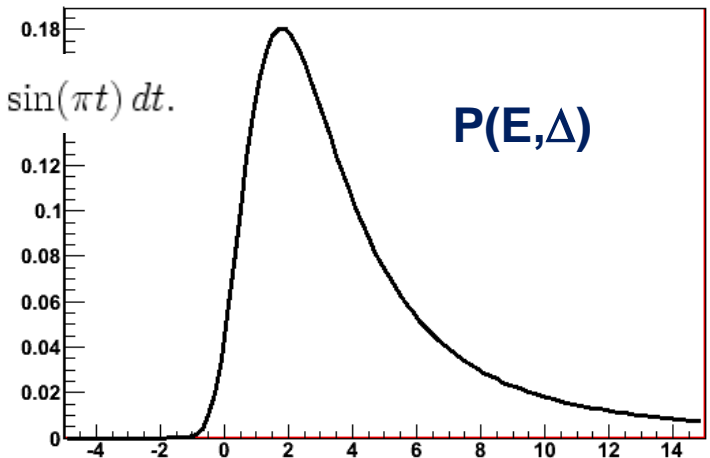
Scattering on free electrons

$$p(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \exp(s \log s + xs) ds = \frac{1}{\pi} \int_0^{\infty} \exp(-t \log t - xt) \sin(\pi t) dt.$$

$$x = \frac{E}{\bar{n}\epsilon} + C_\gamma - 1 - \ln \bar{n}$$

$$\bar{n} = \frac{N_A \rho Z_2 k D}{A \epsilon}$$

$$\ln \epsilon = \ln \frac{I^2}{E_{max}} + 2\beta^2$$



# Landau Distribution

## Landau Distribution

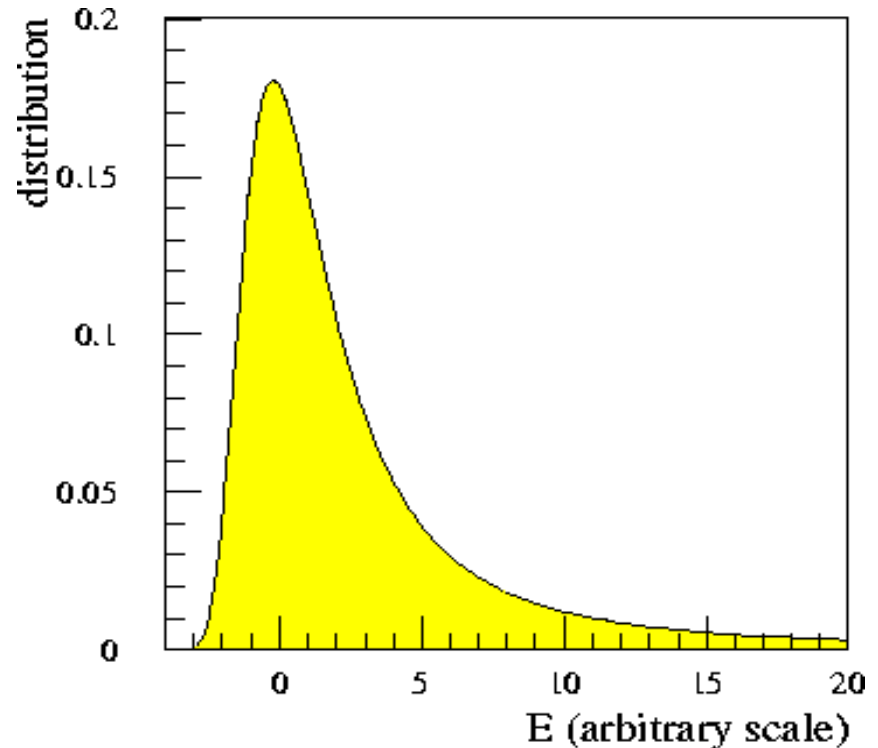
$P(\Delta)$ : Probability for energy loss  $\Delta$   
in matter of thickness  $D$ .

Landau distribution is very  
asymmetric.

Average and most probable  
energy loss must be  
distinguished !

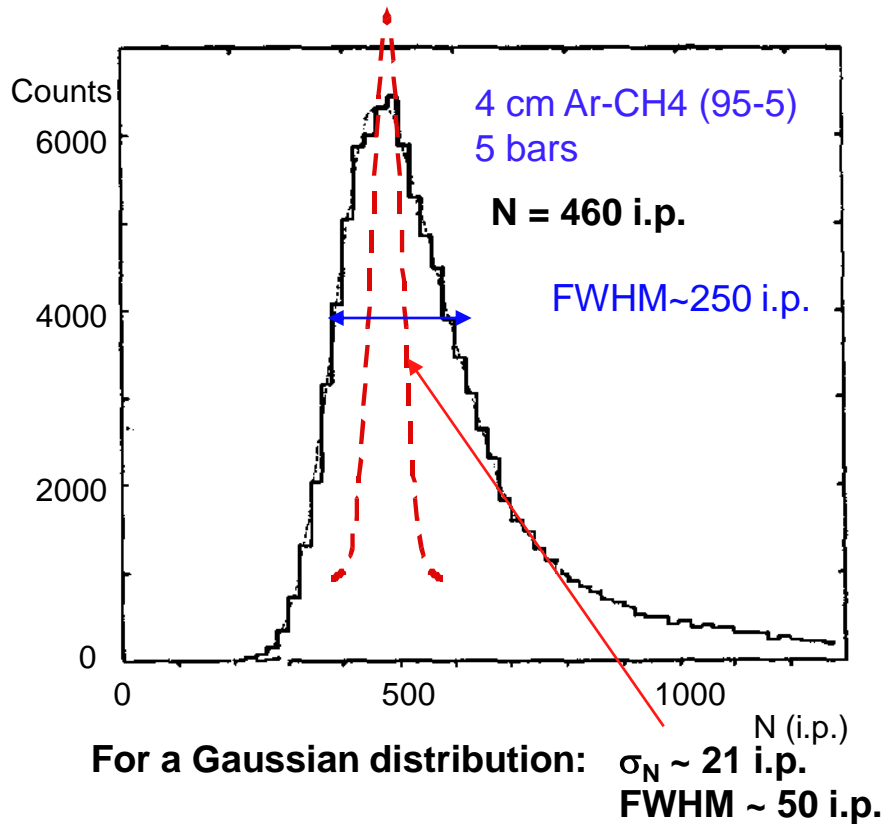
Measured Energy Loss is usually  
smaller than the real energy loss:

3 GeV Pion:  $E'_{\max} = 450\text{MeV} \rightarrow$  A  
450 MeV Electron usually leaves  
the detector.



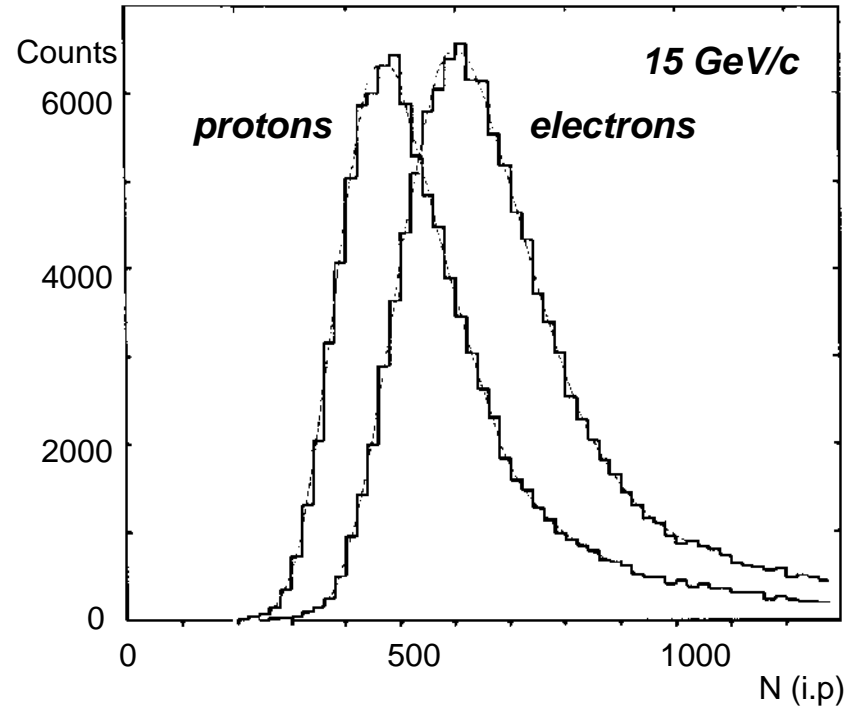
# Landau Distribution

## LANDAU DISTRIBUTION OF ENERGY LOSS:



## PARTICLE IDENTIFICATION

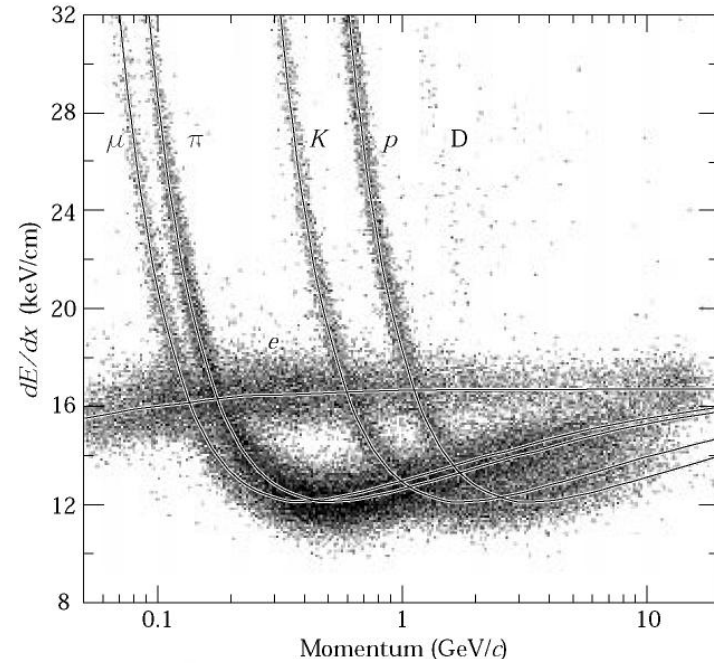
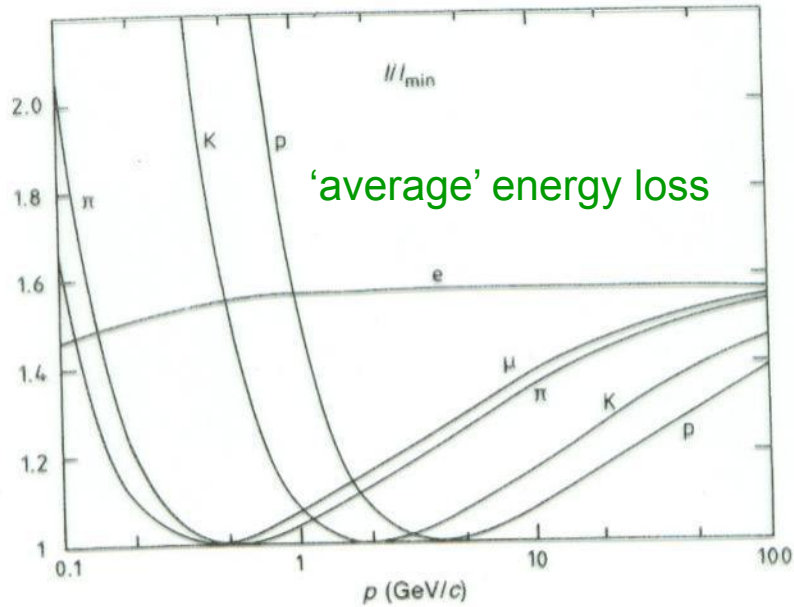
Requires statistical analysis of hundreds of samples



*I. Lehraus et al, Phys. Scripta 23(1981)727*

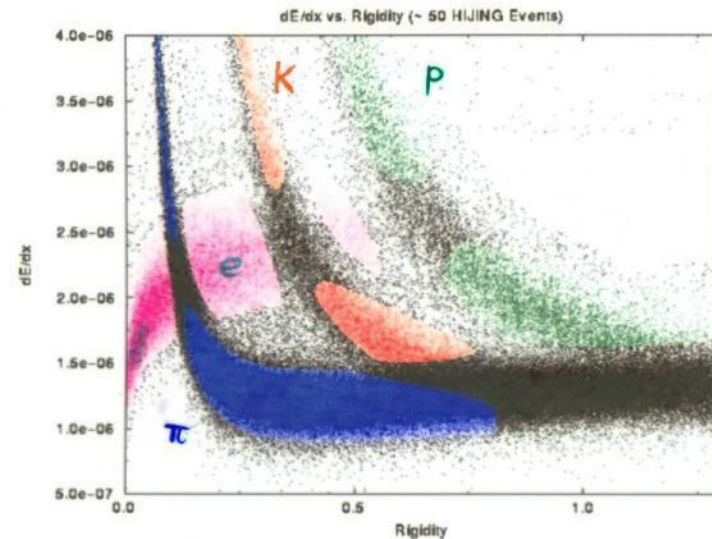
# Particle Identification

## Measured energy loss



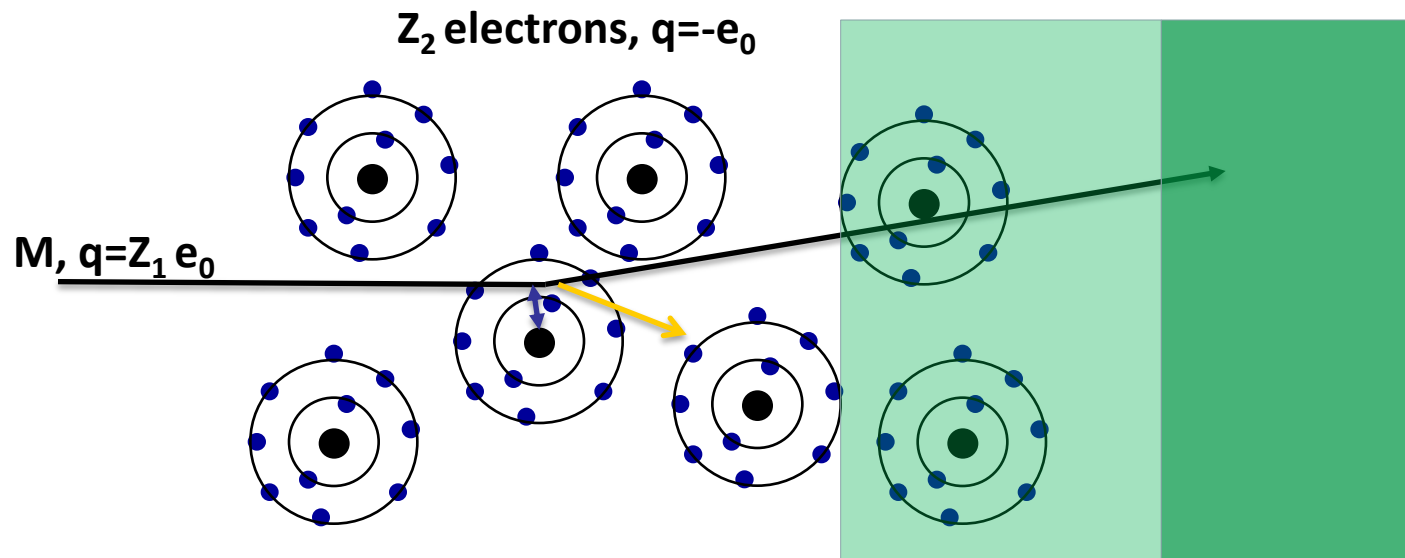
In certain momentum ranges, particles can be identified by measuring the energy loss.

STAR  
TPC



# Bremsstrahlung

A charged particle of mass  $M$  and charge  $q=Z_1e$  is deflected by a nucleus of charge  $Ze$  which is partially 'shielded' by the electrons. During this deflection the charge is 'accelerated' and it therefore radiated  $\rightarrow$  Bremsstrahlung.



7/15/2010

# Bremsstrahlung, Classical



$$\frac{d\sigma'}{d\Omega} = \left( \frac{2Z_1Z_2e^2}{4\pi\epsilon_0 p \cdot v} \right)^2 \frac{1}{(2\sin(\frac{\theta}{2}))^4} \quad p = Mv$$

"Rutherford Scattering"

Written in Terms of Momentum Transfer  $Q^2 = 2p^2(1 - \cos\theta)$

$$\frac{d\sigma'}{dQ} = 8\pi \left( \frac{Z_1Z_2e^2}{4\pi\epsilon_0 \beta c} \right)^2 \cdot \frac{1}{Q^3}$$



$$Q = |\vec{p} - \vec{p}'|$$

$\lim_{\omega \rightarrow 0} \frac{dI}{d\omega} \sim \frac{2}{3\pi} \frac{Z_1^2 e^2}{M^2 c^3} \frac{1}{4\pi\epsilon_0} Q^2$  *From Maxwell's eq (Jackson)*  
*Radiated Energy between  $\omega, \omega + d\omega$*

$$\frac{dE}{dx} = \frac{N_A g}{A} \cdot \int_0^{Q_{max}} \int_{Q_{min}} dQ \frac{dI}{d\omega} \cdot \frac{d\sigma'}{dQ} \quad , \quad Q_{max} = \frac{E}{\hbar}$$

$$\frac{dE}{dx} = \frac{N_A g}{A} \cdot \frac{16}{3} d \cdot Z^2 \cdot \left( \frac{Z_1^2 e^2}{4\pi\epsilon_0 M c^2} \right)^2 \cdot E \cdot \ln \frac{Q_{max}}{Q_{min}}$$

$$d = \frac{e^2}{4\pi\epsilon_0 \hbar c} \sim \frac{1}{137}$$

A charged particle of mass M and charge  $q=Z_1e$  is deflected by a nucleus of Charge Ze.

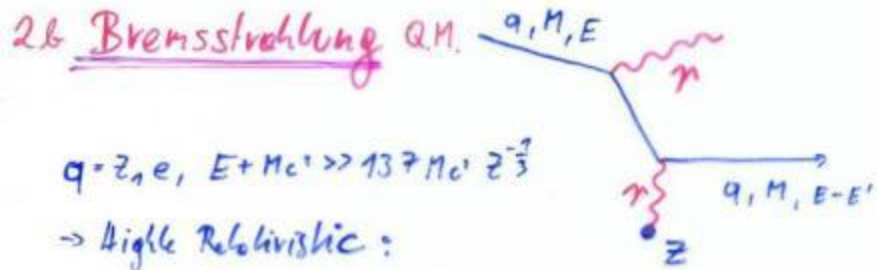
Because of the acceleration the particle radiated EM waves  $\rightarrow$  energy loss.

Coulomb-Scattering (Rutherford Scattering) describes the deflection of the particle.

Maxwell's Equations describe the radiated energy for a given momentum transfer.

$\rightarrow dE/dx$

# Bremsstrahlung, QM



$$\frac{d\sigma(E, E')}{dE'} = 4Z^2 Z_1^4 \left( \frac{1}{4\pi\epsilon_0} \frac{e^2}{Mc^2} \right)^2 \left( \frac{1}{E'} \right) F(E, E')$$

$$F(E, E') = \left[ 1 + \left( 1 - \frac{E'}{E + Mc^2} \right)^2 - \frac{2}{3} \left( 1 - \frac{E'}{E + Mc^2} \right) \right] \ln 183 Z^{-\frac{2}{3}} + \frac{1}{9} \left( 1 - \frac{E'}{E + Mc^2} \right)$$

$$\frac{dE}{dx} = - \frac{N_A \rho}{A} \int_0^E E' \frac{d\sigma}{dE'} dE' \approx 4Z^2 Z_1^4 \left( \frac{1}{4\pi\epsilon_0} \frac{e^2}{Mc^2} \right)^2 E \left[ \ln 183 Z^{-\frac{2}{3}} + \frac{1}{18} \right]$$

$$\underline{\underline{\frac{dE}{dx} = - \frac{N_A \rho}{A} 4Z^2 Z_1^4 \left( \frac{1}{4\pi\epsilon_0} \frac{e^2}{Mc^2} \right)^2 E \ln(183 Z^{-\frac{2}{3}})}}$$

$$E(x) = E_0 e^{-\frac{x}{X_0}} \quad X_0 = \frac{A}{4Z^2 N_A \rho Z_1^4 \left( \frac{1}{4\pi\epsilon_0} \frac{e^2}{Mc^2} \right)^2 \ln 183 Z^{-\frac{2}{3}}}$$

$X_0$  ... Radiation length

Proportional to  $Z^2/A$  of the Material.

Proportional to  $Z_1^4$  of the incoming particle.

Proportional to  $\rho$  of the material.

Proportional  $1/M^2$  of the incoming particle.

Proportional to the Energy of the Incoming particle →

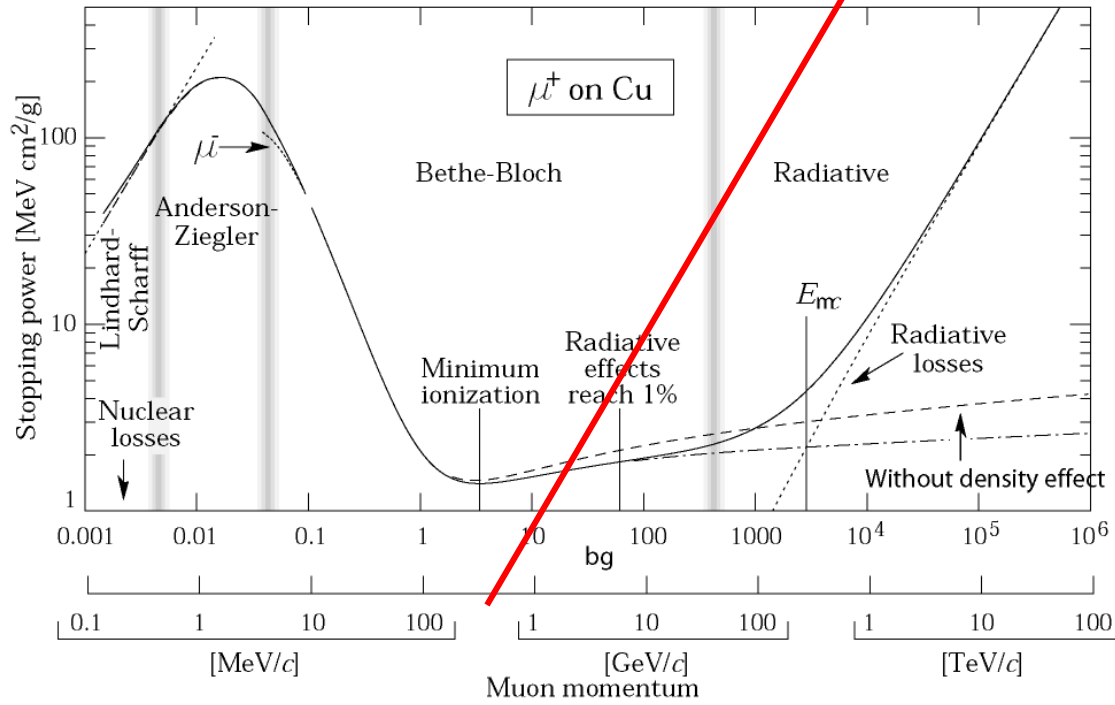
$E(x) = \text{Exp}(-x/X_0)$  – ‘Radiation Length’

$$X_0 \propto M^2 A / (\rho Z_1^4 Z^2)$$

$X_0$ : Distance where the Energy  $E_0$  of the incoming particle decreases  $E_0 \text{Exp}(-1) = 0.37 E_0$ .

# Critical Energy

such as copper to about 1% accuracy for energies between about 6 MeV and 6 GeV



Electron Momentum      5      50      500      MeV/c

For the muon, the second lightest particle after the electron, the critical energy is at 400GeV.

The EM Bremsstrahlung is therefore only relevant for electrons at energies of past and present detectors.

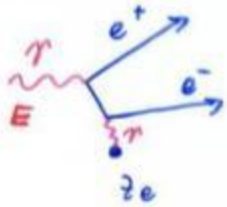
**Critical Energy: If  $dE/dx$  (Ionization) =  $dE/dx$  (Bremsstrahlung)**

**Myon in Copper:       $p \approx 400\text{GeV}$**

**Electron in Copper:    $p \approx 20\text{MeV}$**

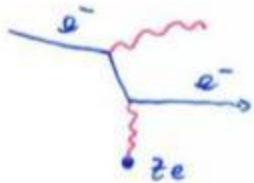


# Pair Production, QM



$$\gamma + \text{Nucl.} \rightarrow e^+ + e^- + \text{Nucl.}$$

The Diagram is very similar to Bremsstrahlung



$e^- + \text{Nucl.} \rightarrow \gamma + e^- + \text{Nucl.}$   
 Crossing Symmetry: bring particle to the other side and make it the antiparticle  $\rightarrow$  'same' correction ...

$$\frac{d\sigma(E, E')}{dE'} = 4\alpha Z^2 v_0^2 \frac{1}{E} G(E, E') \quad E \gg 137 m_e c^2 Z^{-1/3}$$

$$G(E, E') = \left[ \left( \frac{E'+m_e c^2}{E} \right)^2 \left( 1 - \frac{E'+m_e c^2}{E} \right)^2 + \frac{2}{3} \frac{E'+m_e c^2}{E} \left( 1 - \frac{E'+m_e c^2}{E} \right) \ln \frac{E}{E'} \right. \\ \left. - \frac{1}{3} \frac{E'+m_e c^2}{E} \left( 1 - \frac{E'+m_e c^2}{E} \right) \right]$$

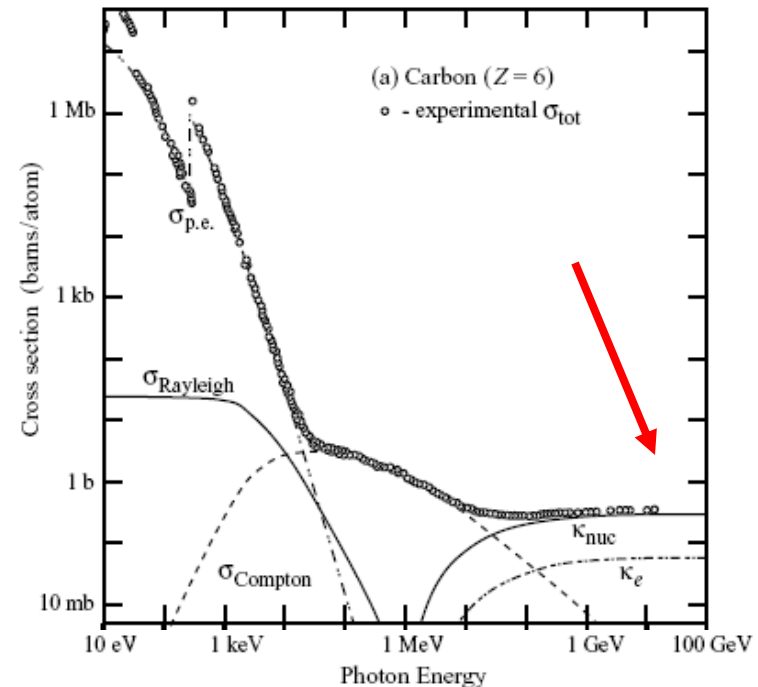
$$\sigma = \int_0^{E-2m_e c^2} \frac{d\sigma}{dE'} dE' = 4\alpha Z^2 v_0^2 \cdot \frac{7}{3} \ln 183 Z^{-1/3}$$

$$P(x) = \frac{1}{2} e^{-\frac{x}{\lambda}} \quad \lambda = \frac{A}{9 N_A \sigma} = \frac{9}{7} X_0$$

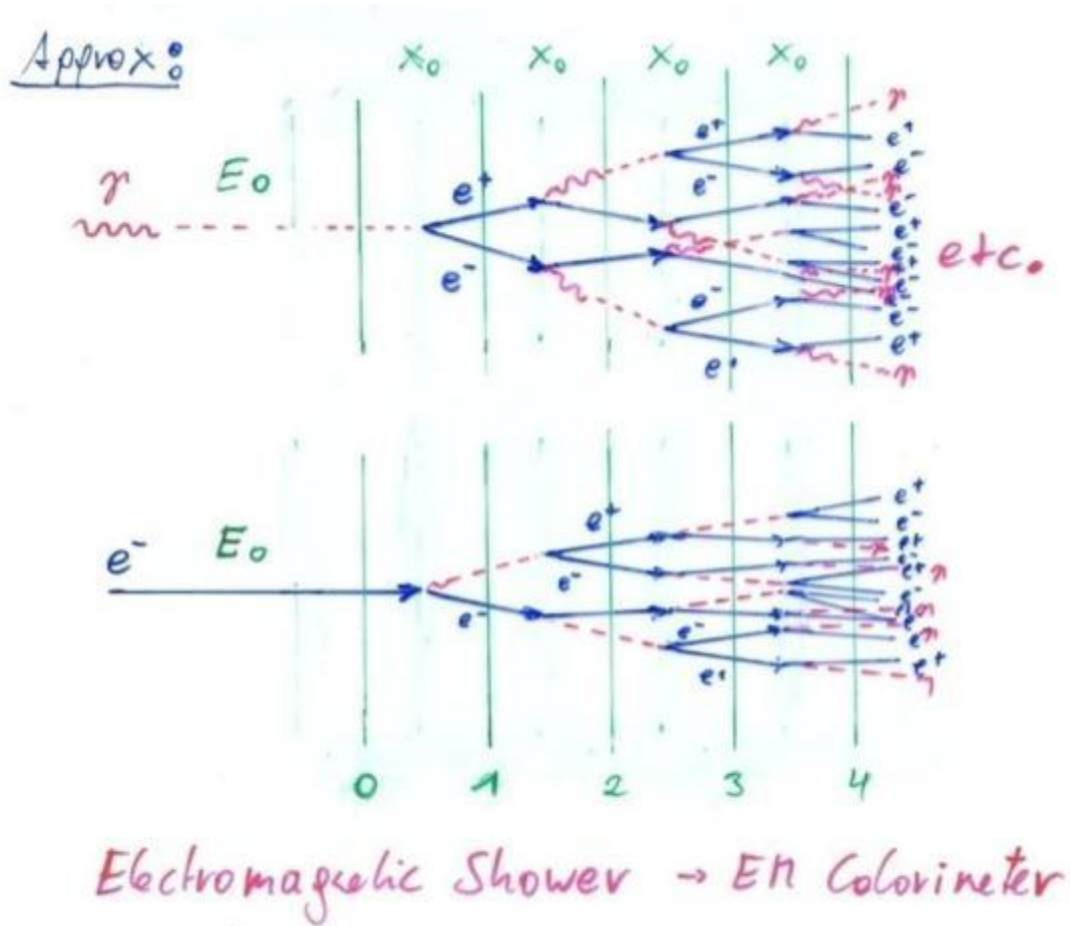
$\hookrightarrow$  Probability that Photon converts to  $e^+ e^-$  after a distance  $x$ .

For  $E_\gamma \gg m_e c^2 = 0.5 \text{ MeV}$  :  $\lambda = 9/7 X_0$

Average distance a high energy photon has to travel before it converts into an  $e^+ e^-$  pair is equal to 9/7 of the distance that a high energy electron has to travel before reducing it's energy from  $E_0$  to  $E_0 \cdot \text{Exp}(-1)$  by photon radiation.



# Bremsstrahlung + Pair Production $\rightarrow$ EM Shower



# Multiple Scattering

**Statistical (quite complex) analysis of multiple collisions gives:**

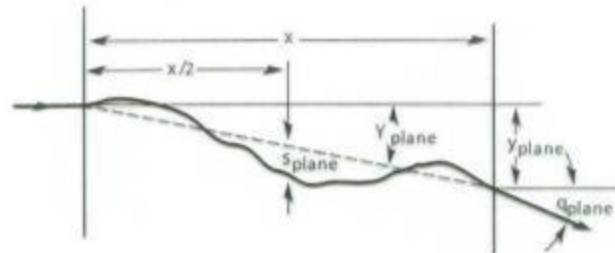
**Probability that a particle is deflected by an angle  $\theta$  after travelling a distance  $x$  in the material is given by a Gaussian distribution with sigma of:**

$$\Theta_0 = \frac{0.0136}{\beta c p [\text{GeV}/c]} Z_1 \sqrt{\frac{x}{X_0}}$$

**$X_0$  ... Radiation length of the material**

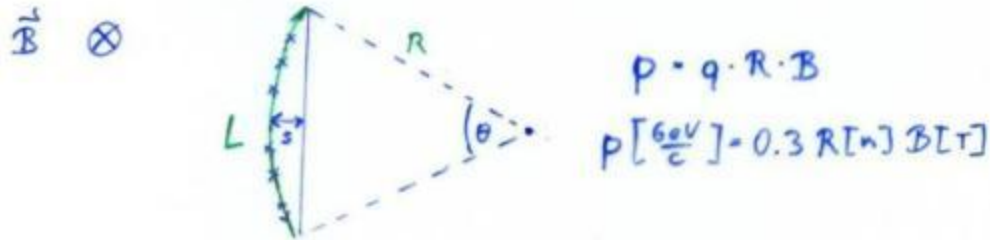
**$Z_1$  ... Charge of the particle**

**$p$  ... Momentum of the particle**



# Multiple Scattering

Magnetic Spectrometer: A charged particle describes a circle in a magnetic field:



$$L = R \cdot \theta$$

$$S = R \left( 1 - \cos \frac{\theta}{2} \right) \sim R \frac{\theta^2}{8} = \frac{L^2}{8R} \rightarrow R = \frac{L^2}{8S}$$

$$\Delta p = 0.3 B \Delta R = 0.3 B \frac{L^2}{8S^2} \Delta S$$

$$\Delta S = \frac{\sigma_x}{\sqrt{N}} \quad \sigma_x \dots \text{point resolution, } N \dots \text{Measurement Points}$$

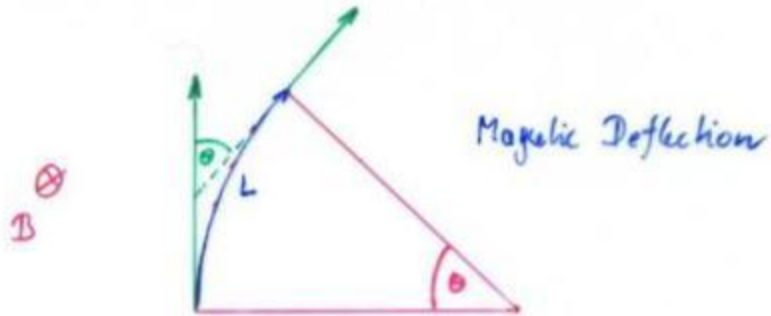
$$\frac{\Delta p}{p} = \frac{\Delta S}{S} = \frac{\sigma_x [\text{m}]}{\sqrt{N}} \cdot \frac{3.3 \cdot 8 p \left[ \frac{\text{GeV}}{c} \right]}{B [\text{T}] \cdot L^2 [\text{m}^2]}$$

E.g:  $p = 10 \frac{\text{GeV}}{c}$ ,  $B = 1 \text{T}$ ,  $L = 1 \text{m}$ ,  $\sigma_x = 200 \mu\text{m}$ ,  $N = 25$

$$\frac{\Delta p}{p} = 0.01 \rightarrow 1\%$$

Limit  $\rightarrow$  Multiple Scattering

# Multiple Scattering



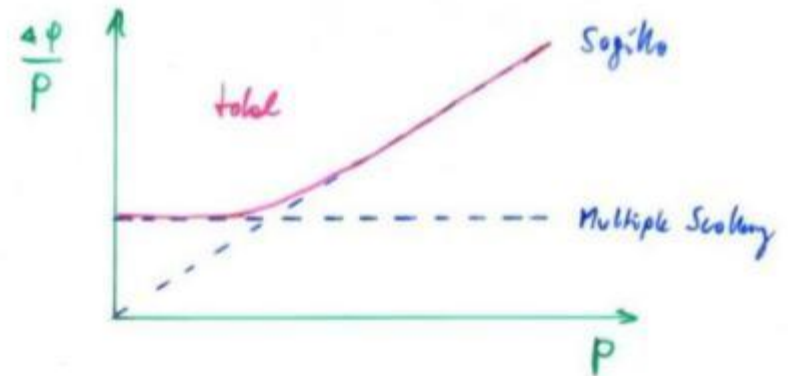
$$\rho \left[ \frac{c \cdot v}{c} \right] = 0.3 R [m] B [T]$$

$$\theta = \frac{L}{R} = \frac{L}{\rho} \cdot 0.3 B$$

$$\frac{\Delta p}{p} = \frac{\Delta \theta}{\theta} = \frac{\theta_0}{\theta} \sim \frac{0.05}{3 B [T] L [m]} \sqrt{\frac{L}{x_0}}$$

→ Independent of  $p$

$$\frac{\Delta p}{p} \Big|_{tot} = \sqrt{\left( \frac{\Delta p}{p} \Big|_{Sog} \right)^2 + \left( \frac{\Delta p}{p} \Big|_{ms} \right)^2}$$



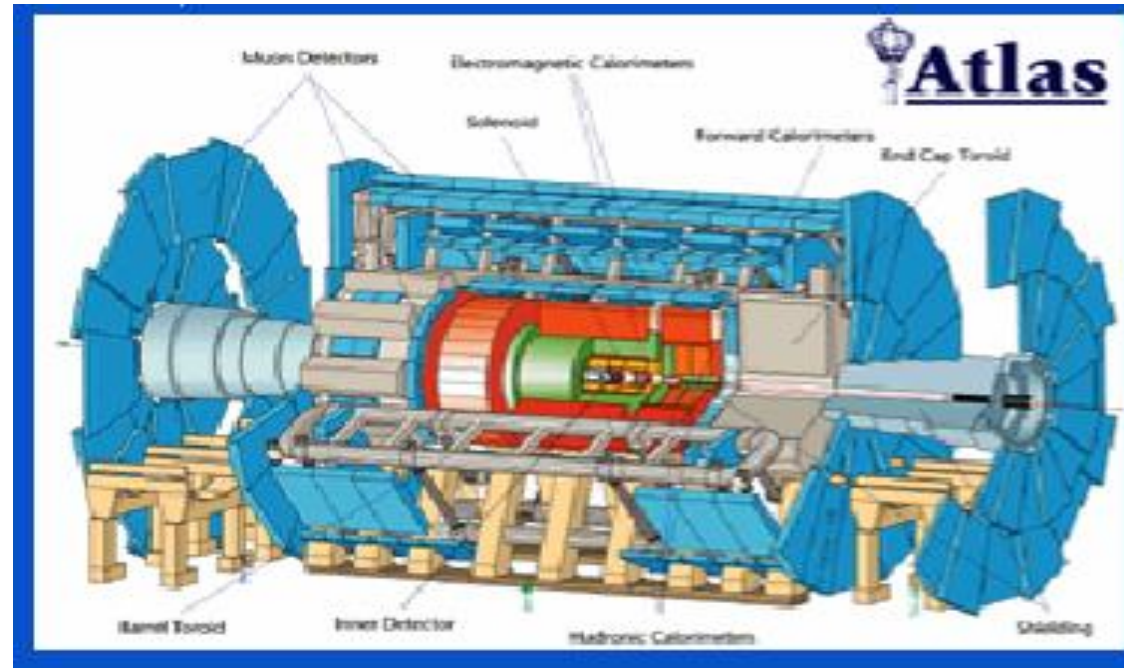
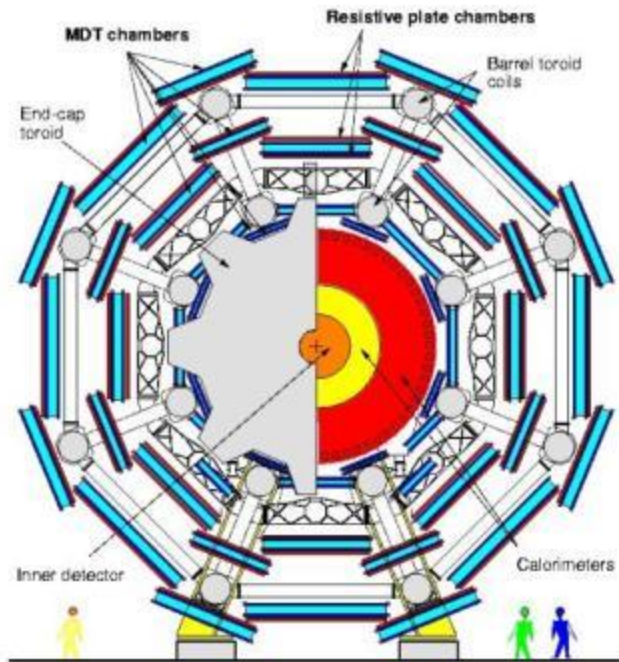
# Multiple Scattering

ATLAS Muon Spectrometer:

$N=3$ ,  $\sigma=50\mu\text{m}$ ,  $P=1\text{TeV}$ ,

$L=5\text{m}$ ,  $B=0.4\text{T}$

$\Delta p/p \sim 8\%$  for the most energetic muons at LHC



# Cherenkov Radiation

If we describe the passage of a charged particle through material of dielectric permittivity  $\epsilon_1$  (using Maxwell's equations) the differential energy crosssection is  $>0$  if the velocity of the particle is larger than the velocity of light in the medium is

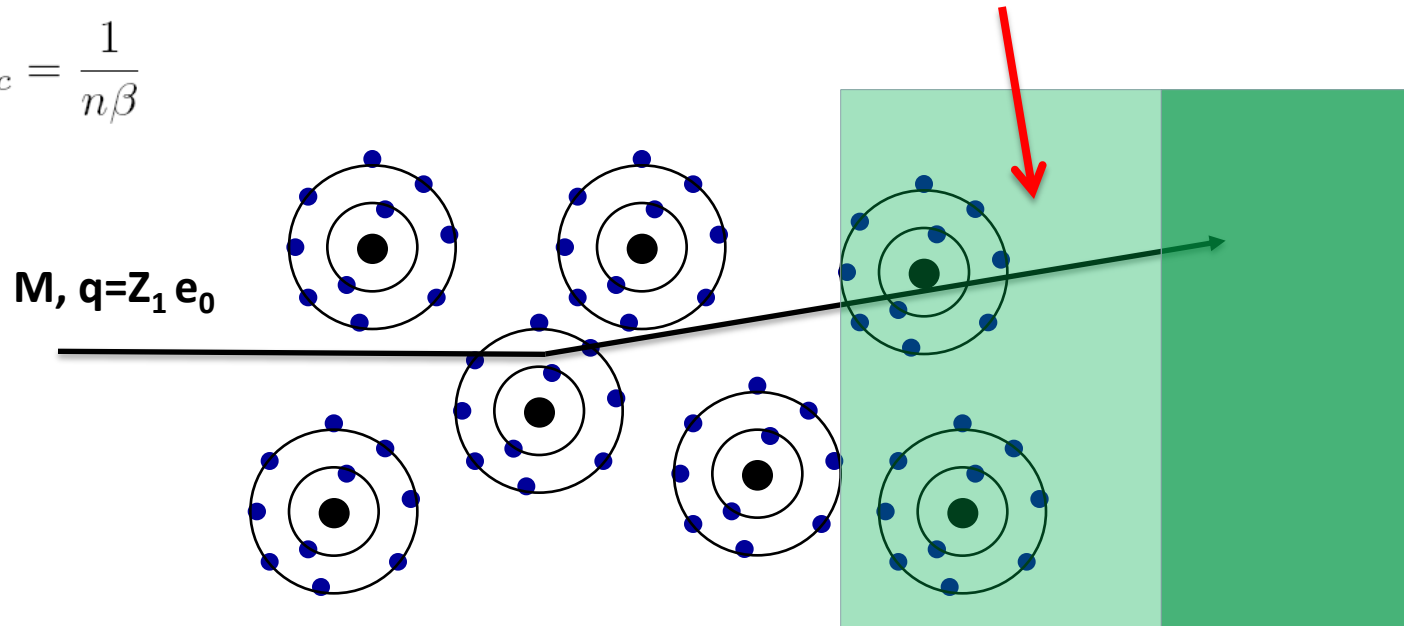
$$\frac{d\sigma}{dE} = \frac{\alpha}{\beta^2 \pi} \frac{A}{N_A \rho Z_2 \hbar c} \left( \beta^2 - \frac{1}{\epsilon_1} \right) \rightarrow \frac{N_A \rho Z_2}{A} \frac{d\sigma}{d\omega} \frac{d\omega}{dE} = \frac{\alpha}{c} \left( 1 - \frac{1}{\beta^2 n^2} \right) \quad n = \sqrt{\epsilon_1} \quad E = \hbar \omega$$

$$\frac{dE}{dx d\omega} \frac{1}{\hbar} = \frac{\alpha}{c} \left( 1 - \frac{1}{\beta^2 n^2} \right) \rightarrow \frac{dN}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2} \left( 1 - \frac{1}{\beta^2 n^2} \right) \quad \omega = \frac{2\pi c}{\lambda}$$

**N** is the number of Cherenkov Photons emitted per cm of material. The expression is in addition proportional to  $Z_1^2$  of the incoming particle.

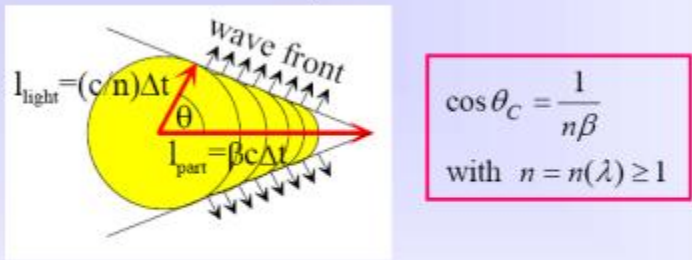
The radiation is emitted at the characteristic angle  $\Theta_c$ , that is related to the refractive index  $n$  and the particle velocity by

$$\cos \Theta_c = \frac{1}{n\beta}$$



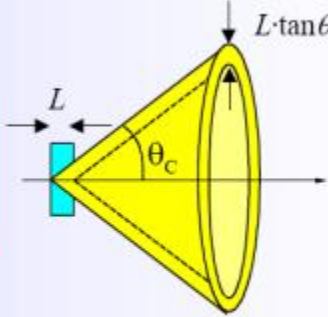
# Cherenkov Radiation

with velocity  $\beta \geq \beta_{thr} = \frac{1}{n}$   $n$ : refractive index



$I_{light} = (c/n)\Delta t$   
 $I_{part} = \beta c \Delta t$   
 $\cos \theta_c = \frac{1}{n\beta}$   
 with  $n = n(\lambda) \geq 1$

■  $\beta_{thr} = \frac{1}{n} \rightarrow \theta_c \approx 0$  Cherenkov threshold



■  $\theta_{max} = \arccos \frac{1}{n}$  'saturated' angle ( $\beta=1$ )

If the velocity of a charged particle is larger than the velocity of light in the medium  $v > \frac{c}{n}$  ( $n$ ... Refractive Index of Material) it emits 'Cherenkov' radiation at a characteristic angle of  $\cos \theta_c = \frac{1}{n\beta}$  ( $\beta = \frac{v}{c}$ )

$$\frac{dN}{dx} \sim 2\pi d z_1^2 \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{\lambda_2 - \lambda_1}{\lambda_2 \cdot \lambda_1}$$

= Number of emitted photons/length with  $\lambda$  between  $\lambda_1$  and  $\lambda_2$

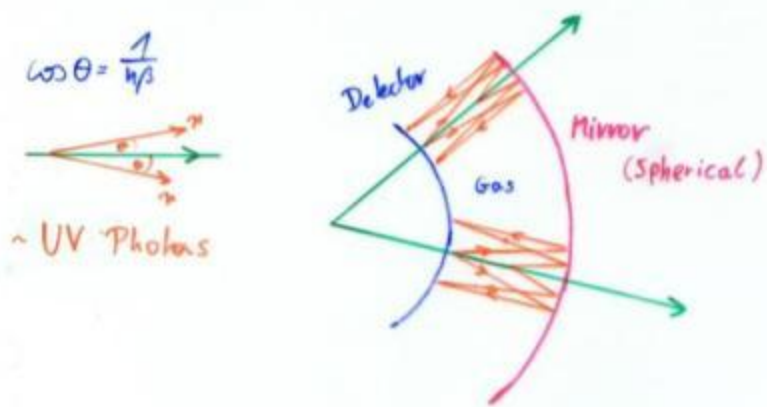
With  $\lambda_1 = 400\text{nm}$   $\lambda_2 = 700\text{nm}$

$$\frac{dN}{dx} = 490 \left(1 - \frac{1}{\beta^2 n^2}\right) \left[\frac{1}{\text{cm}}\right]$$

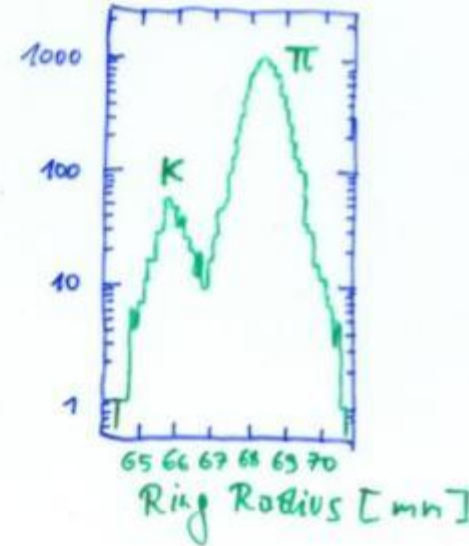
Material	$n-1$	$\beta$ threshold	$\gamma$ threshold
solid Sodium	3.22	0.24	1.029
lead glass	0.67	0.60	1.25
water	0.33	0.75	1.52
Silica aerogel	0.025-0.075	0.93-0.976	2.7 - 4.6
air	$2.93 \cdot 10^{-4}$	0.9997	41.2
He	$3.3 \cdot 10^{-5}$	0.99997	123



# Ring Imaging Cherenkov Detector (RICH)



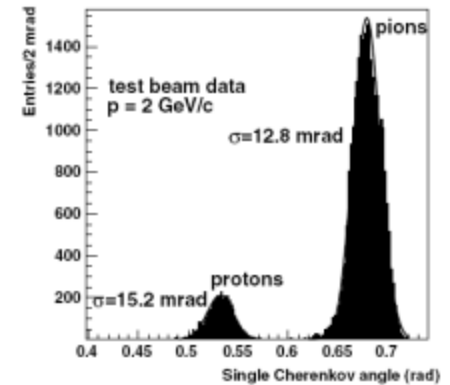
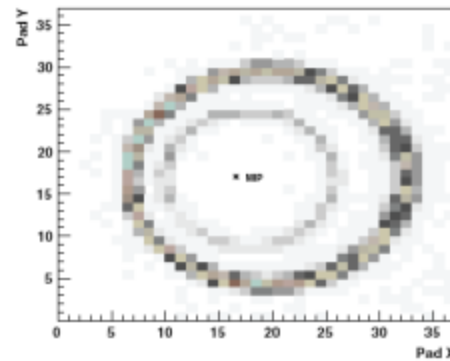
200 GeV/c  $K, \pi$



Resolution  $\frac{\Delta n}{n} = \gamma^2 \beta^3 n \Delta \theta \frac{1}{\sqrt{N_{ph} L}}$  ( $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ )

Angle Measurement Accuracy

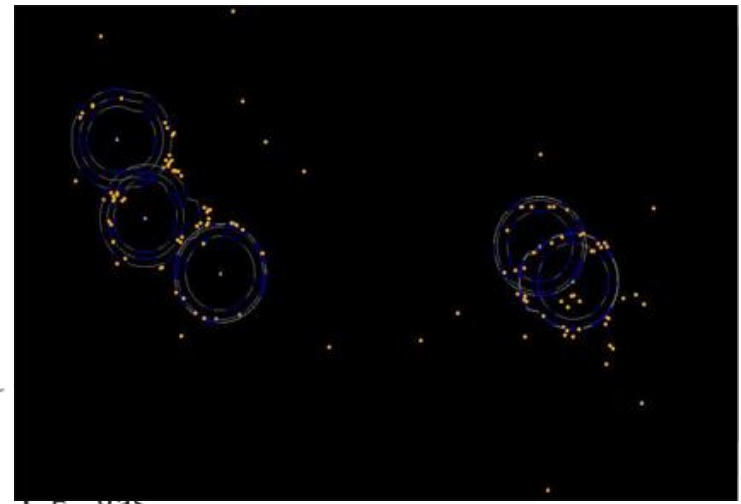
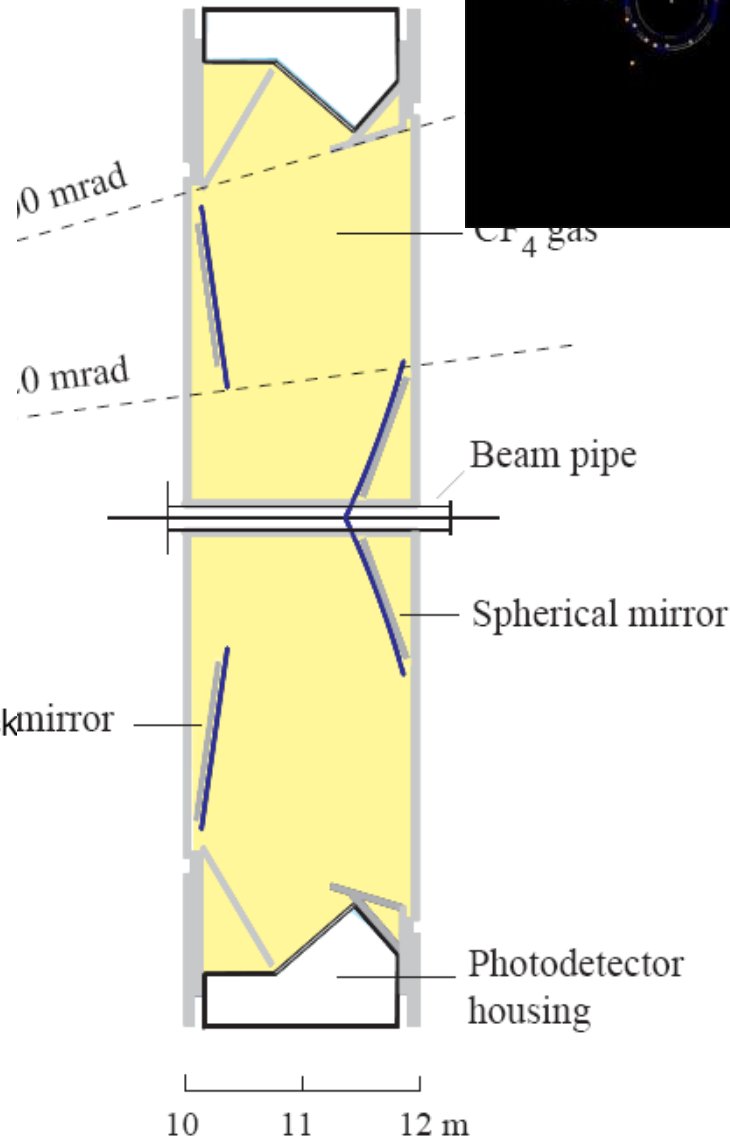
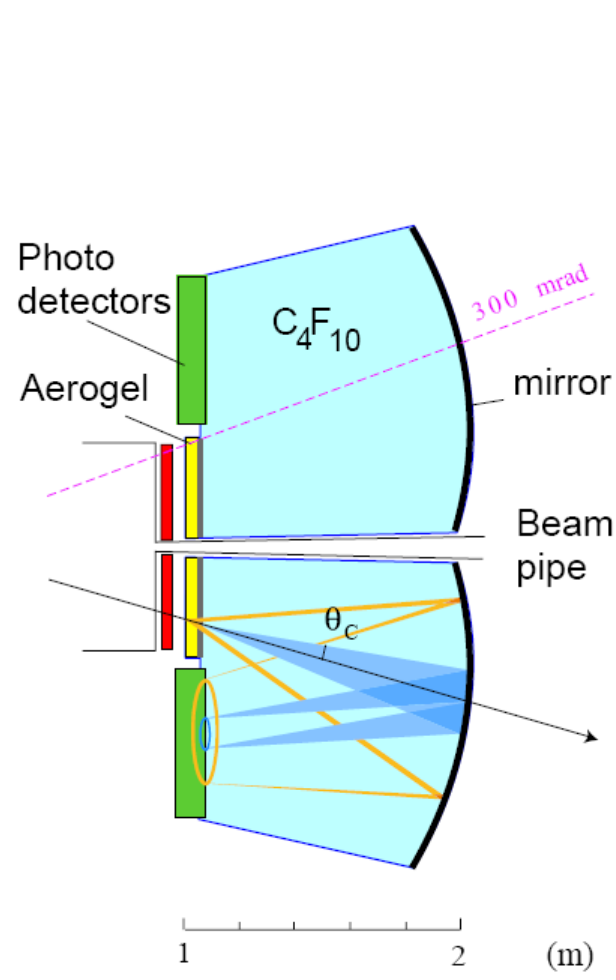
Photon Statistics



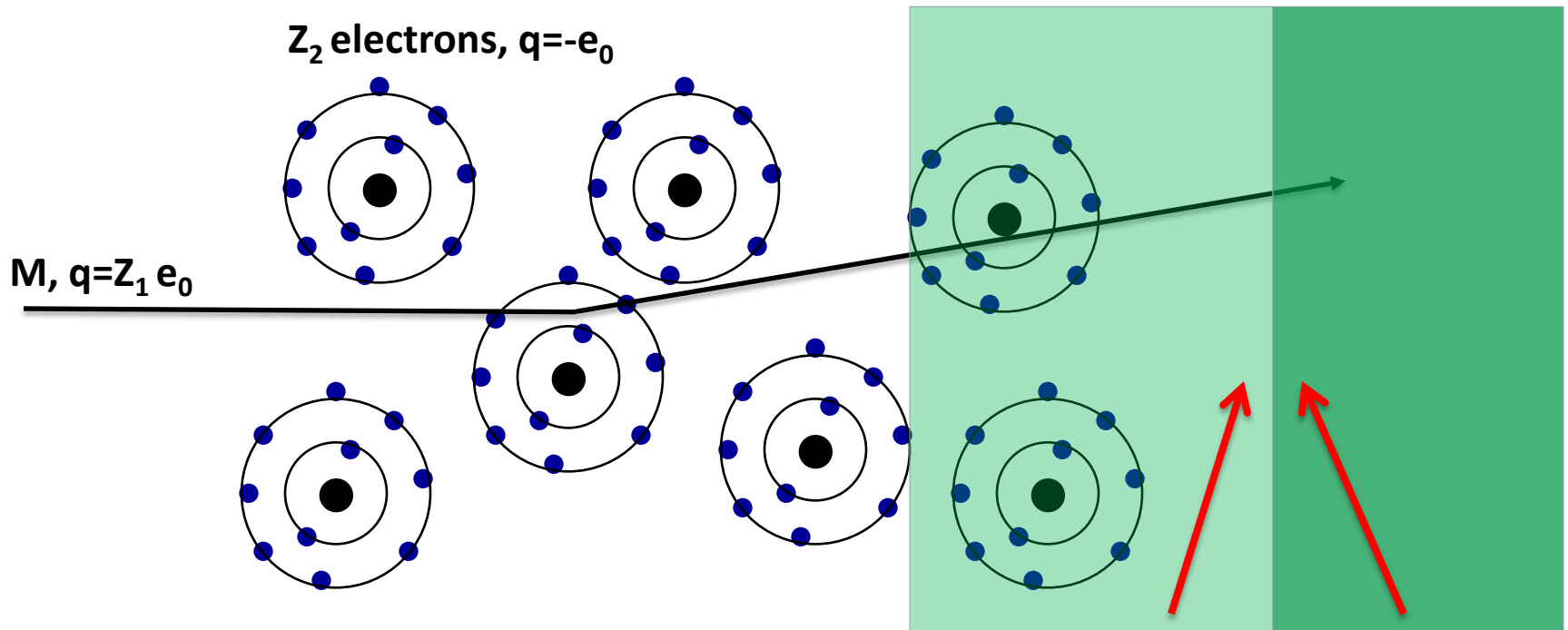
medium	n	$\theta_{\max}$ (deg.)	$N_{\text{ph}}$ ( $\text{eV}^{-1} \text{cm}^{-1}$ )
air*	1.000283	1.36	0.208
isobutane*	1.00127	2.89	0.941
water	1.33	41.2	160.8
quartz	1.46	46.7	196.4

There are only 'a few' photons per event  $\rightarrow$  one needs highly sensitive photon detectors to measure the rings !

# LHCb RICH



# Transition Radiation



When the particle crosses the boundary between two media, there is a probability of the order of 1% to produced and X ray photon, called Transition radiation.

# Transition Radiation

Radiation ( $\sim$  keV) emitted by ultra-relativistic particles when they traverse the border of 2 materials of different dielectric permittivity ( $\epsilon_1, \epsilon_2$ )



$$q = Z_1 e$$

$$I = \frac{1}{3} d Z_1^2 (\hbar \omega_p) \gamma \dots \text{Radiated Energy per Transition}$$

$\hbar \omega_p \dots$  plasma frequency of the medium  
 $\dots \sim 20$  eV for Styrene

About half the Energy is radiated between

$$0.1 \hbar \omega_p \gamma < \hbar \omega < \hbar \omega_p \gamma$$

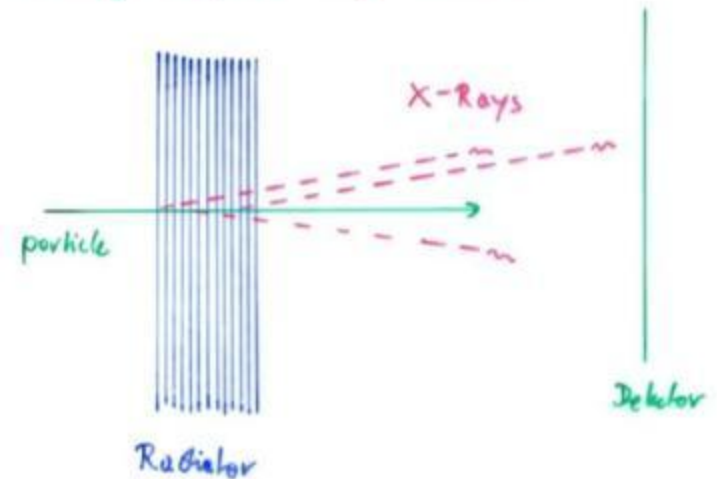
E.g.  $\gamma = 1000$  2-20 keV X-Rays

$$N_\gamma \sim \frac{2}{3} d Z_1^2 \sim 5 \cdot 10^{-3} \cdot Z_1^2$$

$\gamma$ -dependence from hardening rather than  $N_\gamma$

$$\text{Emission Angle} \sim \frac{1}{\gamma}$$

The Number of Photons can be increased by placing many foils of material.



# Electromagnetic Interaction of Particles with Matter

## Ionization and Excitation:

Charged particles traversing material are exciting and ionizing the atoms.

The average energy loss of the incoming particle by this process is to a good approximation described by the Bethe Bloch formula.

The energy loss fluctuation is well approximated by the Landau distribution.

## Multiple Scattering and Bremsstrahlung:

The incoming particles are scattering off the atomic nuclei which are partially shielded by the atomic electrons.

Measuring the particle momentum by deflection of the particle trajectory in the magnetic field, this scattering imposes a lower limit on the momentum resolution of the spectrometer.

The deflection of the particle on the nucleus results in an acceleration that causes emission of Bremsstrahlungs-Photons. These photons in turn produced  $e^+e^-$  pairs in the vicinity of the nucleus, which causes an EM cascade. This effect depends on the 2<sup>nd</sup> power of the particle mass, so it is only relevant for electrons.

# Electromagnetic Interaction of Particles with Matter

## Cherenkov Radiation:

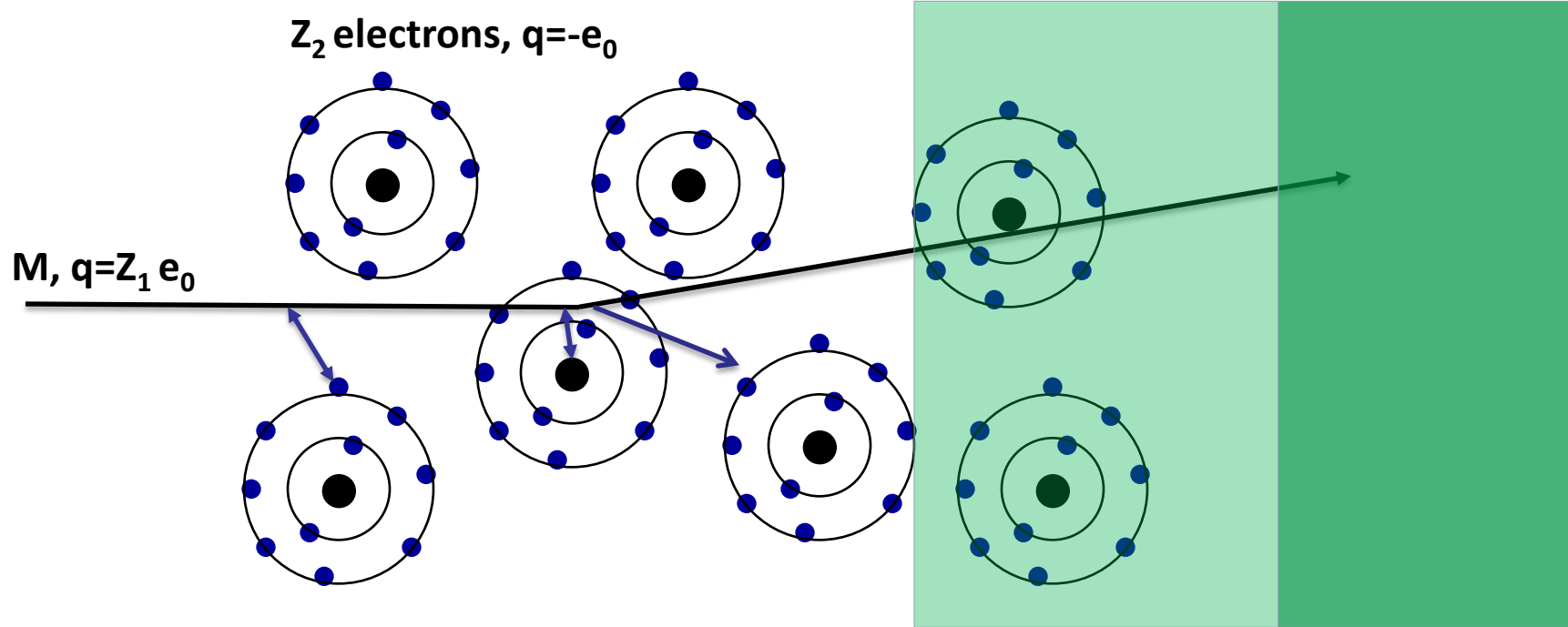
If a particle propagates in a material with a velocity larger than the speed of light in this material, Cherenkov radiation is emitted at a characteristic angle that depends on the particle velocity and the refractive index of the material.

## Transition Radiation:

If a charged particle is crossing the boundary between two materials of different dielectric permittivity, there is a certain probability for emission of an X-ray photon.

→ The strong interaction of an incoming particle with matter is a process which is important for Hadron calorimetry and will be discussed later.

# Electromagnetic Interaction of Particles with Matter



**Now that we know all the Interactions we can talk about Detectors !**

Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produce and X ray photon, called Transition radiation.

# Now that we know all the Interactions we can talk about Detectors !

