Diffraction at hadron colliders: a theoretical review

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Workshop on Multiple-Partonic Interactions at the LHC

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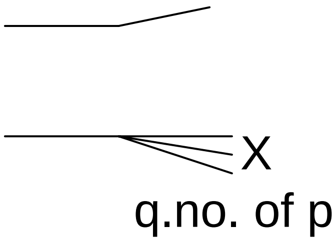
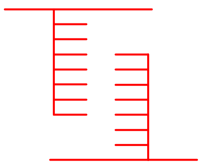
No unique definition of diffraction

1. Diffraction is elastic (or quasi-elastic) scattering caused, via s-channel unitarity, by the absorption of components of the wave functions of the incoming particles
   e.g. pp→pp,
   pp→pX (single proton dissociation, SD),
   pp→XX (both protons dissociate, DD)



Good for quasi-elastic proc.
– but not high-mass dissoc^{n}



2. A diffractive process is characterized by a large rapidity gap (LRG), which is caused by t-channel Pomeron exch.
   (or, to be more precise, by the exchange corresponding to the rightmost singularity in the complex angular momentum plane with vacuum quantum numbers).

Only good for very LRG events – otherwise Reggeon/fluctuation contaminations

Why study diffraction ?

Intrinsic interest. The LHC should reach, for the first time, sufficiently HE to distinguish between the different theoretical asymptotic scenarios for HE interactions.

In HE pp collisions about 40% of σ_{tot} comes from diffractive processes, like elastic scatt., SD, DD. Need to study diffraction to understand the structure of σ_{tot} and the nature of the underlying events which accompany the sought-after rare hard subprocesses. (Note the LHC detectors do not have 4π geometry and do not cover the whole rapidity interval. So minimum-bias events account for only part of total $\sigma_{inelastic}$.)

Study needed to estimate the survival probabilities of LRG to soft rescattering.
Recall "hard" exclusive diffractive processes (e.g., pp $\rightarrow$ p + Higgs + p) are an excellent means of suppressing the background for New Physics signals

Needed so as to understand the structure of HE cosmic ray phenomena (e.g. Auger experiment).

Finally, the hope is that a study of diffraction may allow the construction of a MC which merges "soft" and "hard" HE hadron interactions in a reliable and consistent way.

s-channel unitarity

diagonal in b $\sim \ell/p$

$SS^\dagger = I$ with $S = I + iT$ $\rightarrow$ $T - T^\dagger = iT^\dagger T$

elastic unitarity →

$$2\,\mathrm{Im}\,T_{el}(s,b) = |T_{el}(s,b)|^2 + G_{inel}(s,b)$$

$$\begin{cases} \dfrac{d^2\sigma_{tot}}{d^2b} = 2\,\mathrm{Im}\,T_{el} = 2\left(1 - e^{-\Omega/2}\right) \\ \dfrac{d\sigma_{el}}{d^2b} = |T_{el}|^2 = \left(1 - e^{-\Omega/2}\right)^2 \\ \dfrac{d\sigma_{inel}}{d^2b} = 2\,\mathrm{Im}\,T_{el} - |T_{el}|^2 = 1 - e^{-\Omega} \end{cases}$$

Prob. of no inel inter^n

Opacity / Eikonal $\Omega(s,b) \geq 0$

e.g. black disc

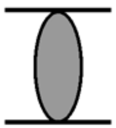
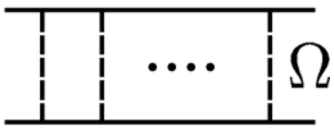
$\mathrm{Im}\,T_{el} = 1, \quad b < R$

$$\left.\begin{array}{l} \sigma_{tot} = 2\pi R^2 \\ \sigma_{el} = \sigma_{inel} = \pi R^2 \end{array}\right.$$

total absorption gives elastic scatt

bare Pomeron-exchange amp. $\Omega/2$ = 

Elastic amp. $T_{el}(s,b)$

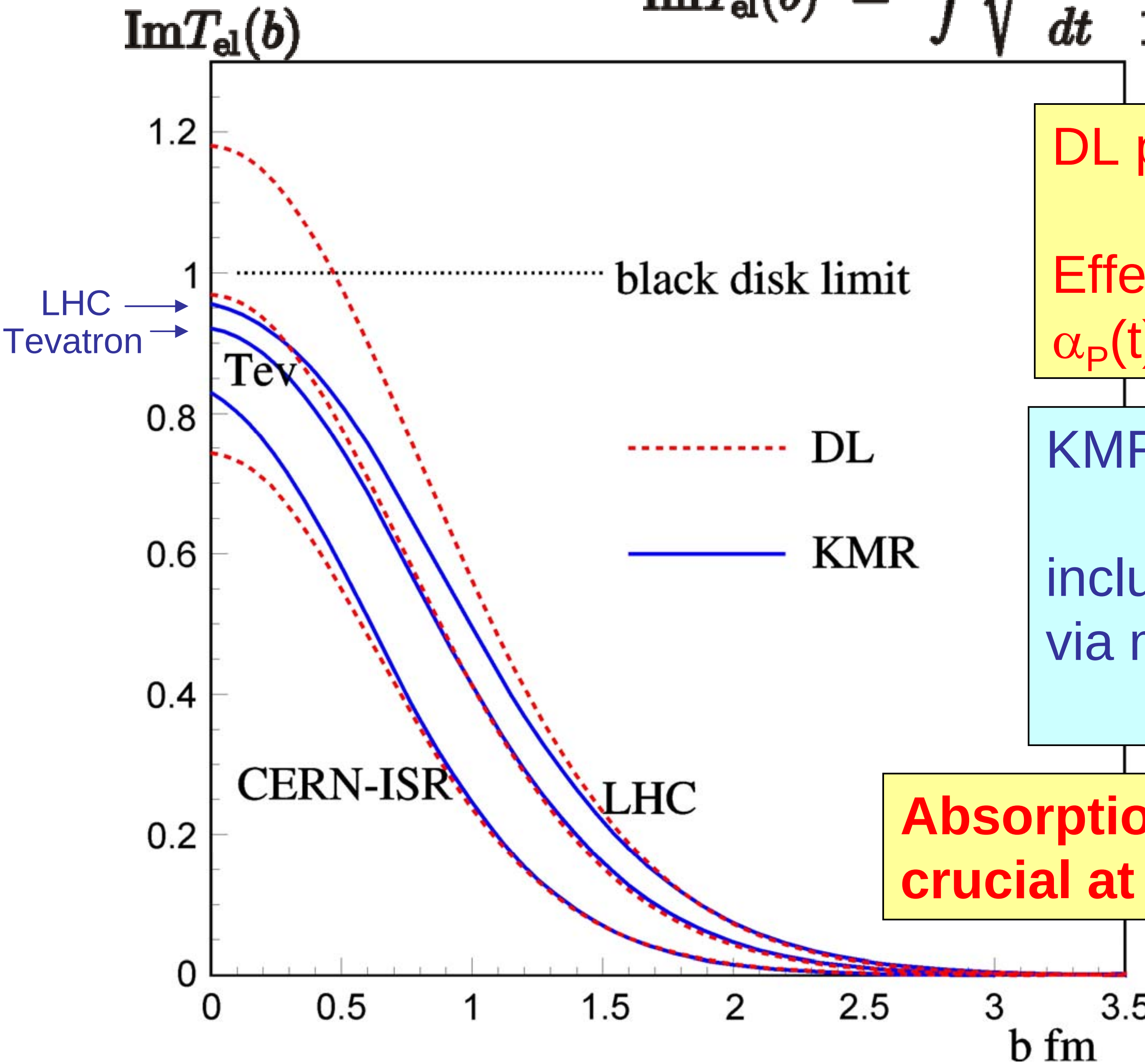
Im $T_{\rm el}$ =  $= 1 - e^{-\Omega/2} = \sum_{n=1}^{\infty}$  $\Omega/2$

(s-ch unitarity)

Can get impact parameter profile of $\mathrm{Im}T_{el}(s,b)$ direct from

$$d\sigma_{el}/dt \sim |\,\mathrm{Im}T_{el}(s,t)\,|^2 \qquad (\rho \ll 1)$$

$$\mathrm{Im}T_{\mathrm{el}}(b) = \int \sqrt{\frac{d\sigma_{\mathrm{el}}}{dt}\,\frac{16\pi}{1+\rho^2}}\; J_0(qb)\,\frac{qdq}{2\pi}$$



DL parametrization:

Effective Pom. pole
$\alpha_P(t) = 1.08+0.25t$

KMR parametrization

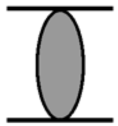
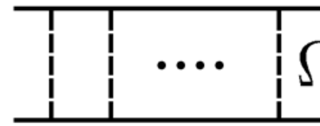
includes absorption
via multi-Pomeron
effects

Absorption/ s-ch unitarity crucial at small b at LHC

proton dissociation ?

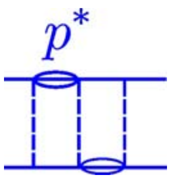
bare amp. $\Omega/2$ = 

Elastic amp. $T_{el}(s,b)$

$\text{Im } T_{\text{el}} =$  $= 1 - e^{-\Omega/2} = \sum_{n=1}^{\infty}$  $\Omega/2$

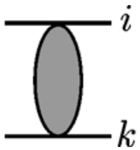
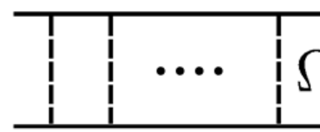
(s-ch unitarity)

Low-mass diffractive dissociation



→multichannel eikonal

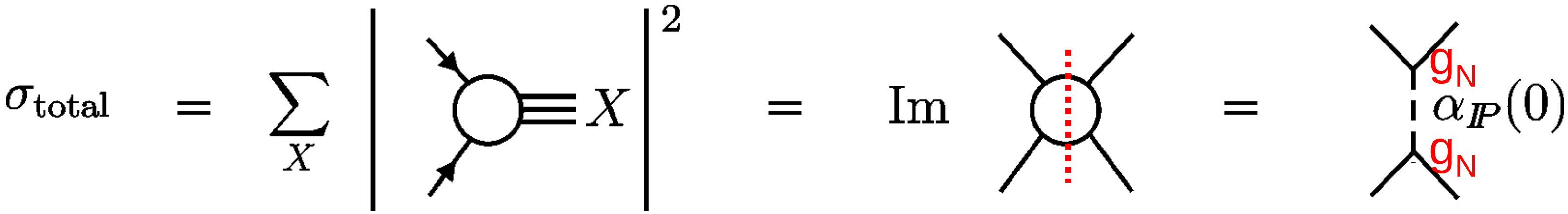
introduce diff^{ve} estates ϕ_i, ϕ_k (comb^{ns} of p,p*,..) which only undergo "elastic" scattering (Good-Walker)

$\text{Im } T_{ik} =$  $= 1 - e^{-\Omega_{ik}/2} = \sum$  $\Omega_{ik}/2$

what about high-mass diffraction ?

Optical theorems

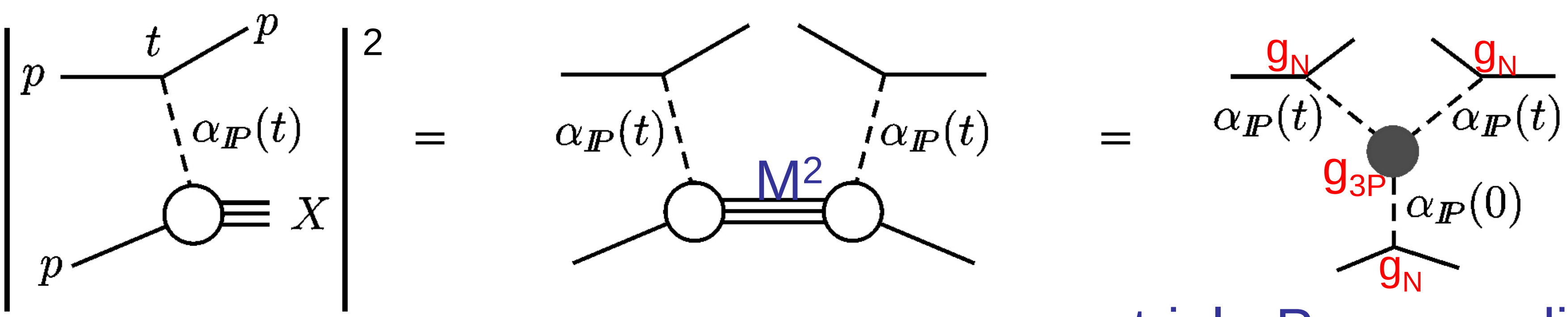
at high energy
use Regge



but screening/s-ch unitarity important so σ_{total} suppressed

$g_N^2 \left(\frac{s}{s_0}\right)^{\alpha_{I\!P}(0)-1}$

High-mass diffractive dissociation



triple-Pomeron diag

but screening important

$g_N^3 g_{3P} \left(\frac{M^2}{s_0}\right)^{\alpha_{I\!P}(0)-1} \left(\frac{s}{M^2}\right)^{2\alpha_{I\!P}(t)-2}$

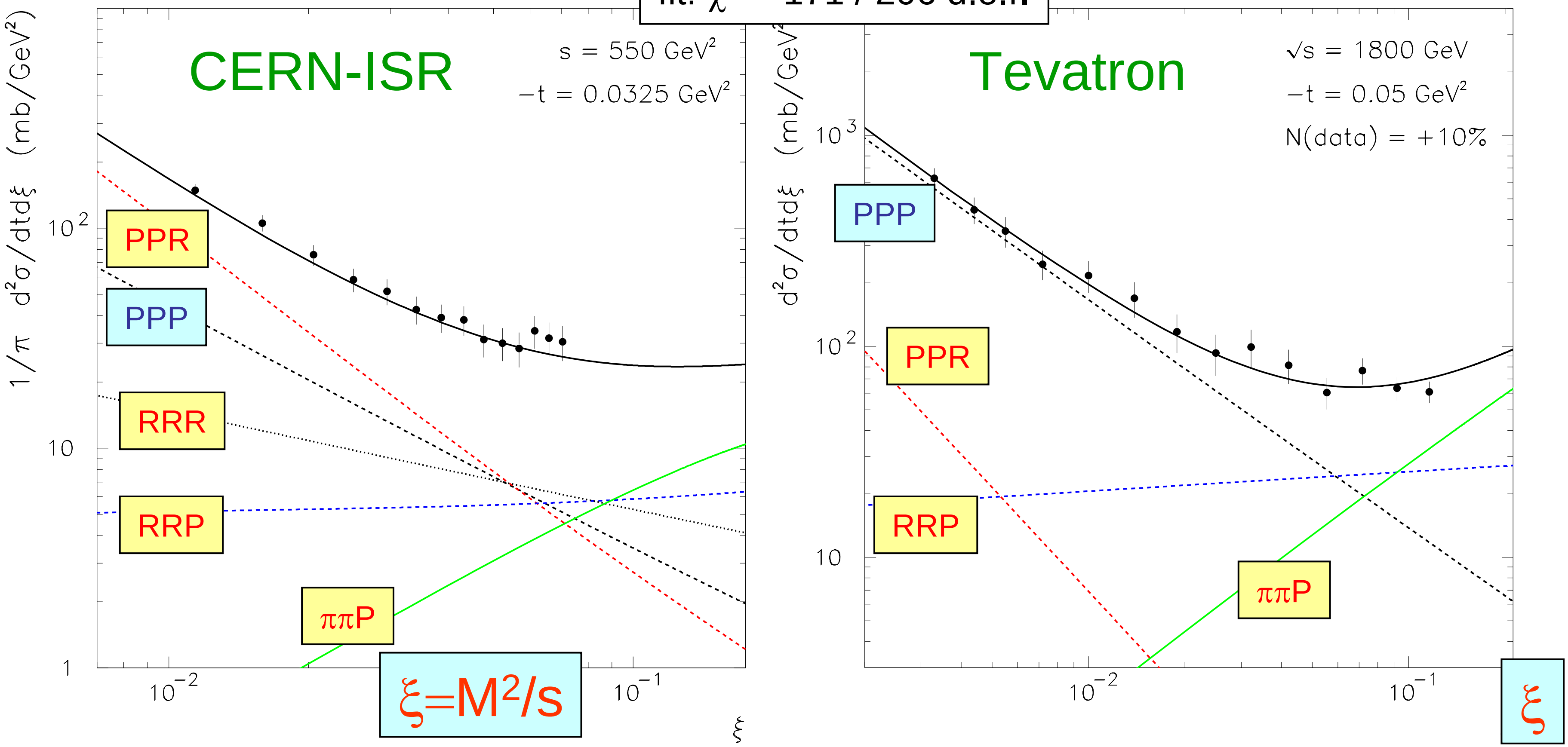
$(g_{3P})_{\text{effective}} = S^2 (g_{3P})_{\text{bare}}$ so $(g_{3P})_{\text{bare}}$ increased

g_{3P}?

triple-Regge analysis of $d\sigma/dtd\xi$, including screening

(includes compilation of SD data by Goulianos and Montanha)

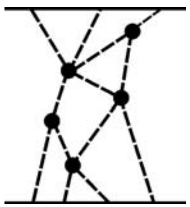
fit: χ^2 = 171 / 206 d.o.f.



Luna+KMR
(Oliveira+MR)

$g_{3P}=\lambda g_N \qquad \lambda \sim 0.2$

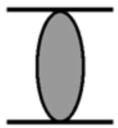
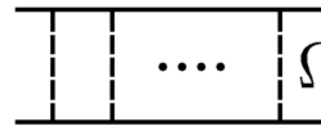
g_{3P} large, need to include multi-Pomeron effects



Elastic amp. $T_{el}(s,b)$

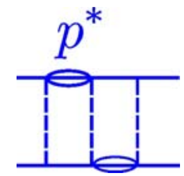
bare amp. $\Omega/2$ = 

(-20%)

Im T_{el} =  $= 1 - e^{-\Omega/2} = \sum_{n=1}^{\infty}$  $\Omega/2$

(s-ch unitarity)

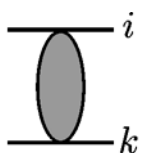
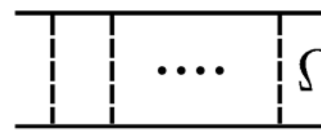
Low-mass diffractive dissociation



→multichannel eikonal

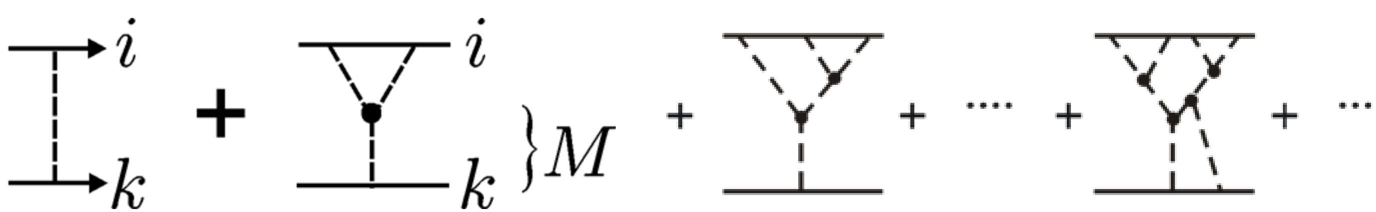
introduce diff^ve estates ϕ_i, ϕ_k (comb^ns of p,p*,..) which only undergo “elastic” scattering (Good-Walker)

(-40%)

Im T_{ik} =  $= 1 - e^{-\Omega_{ik}/2} = \sum$  $\Omega_{ik}/2$

include high-mass diffractive dissociation

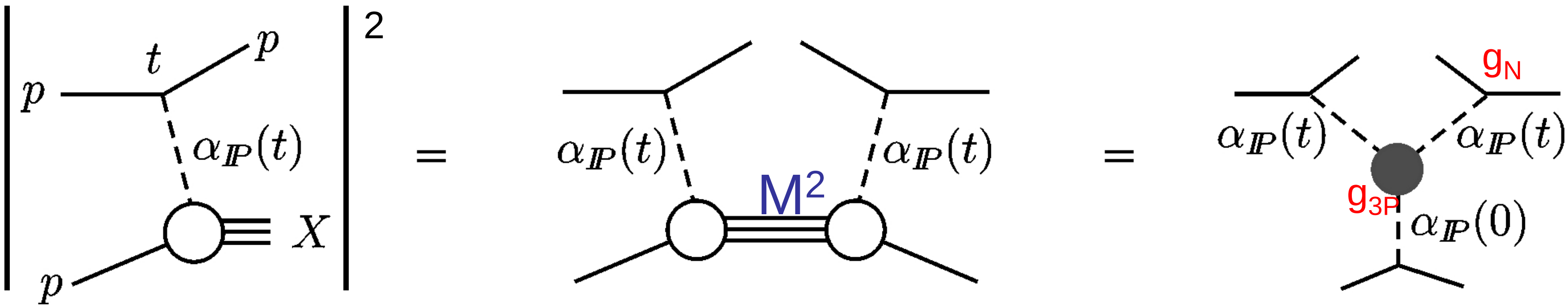
(SD -80%)

Ω_{ik} = 

$g_{3P}=\lambda\ g_N \quad \lambda \sim 0.2$

← why do we say λ is large, and multi-Pom diagrams important at HE ?

naïve argument without absorptive effects:



$\boxed{M^2 d\sigma_{SD}/dM^2 \sim g_N^3\ g_{3P} \sim \lambda \sigma_{el}}$ $\boxed{\ln s/M_0^2}$ $(\sigma_{el} \sim g_N^4)$

$$\sigma_{\rm SD} = \int \frac{M^2 d\sigma_{\rm SD}}{dM^2} \frac{dM^2}{M^2} \sim \lambda \ln(s/M_0^2)\ \sigma_{el}$$

so at HE collider energies $\sigma_{SD} \sim \sigma_{el}$

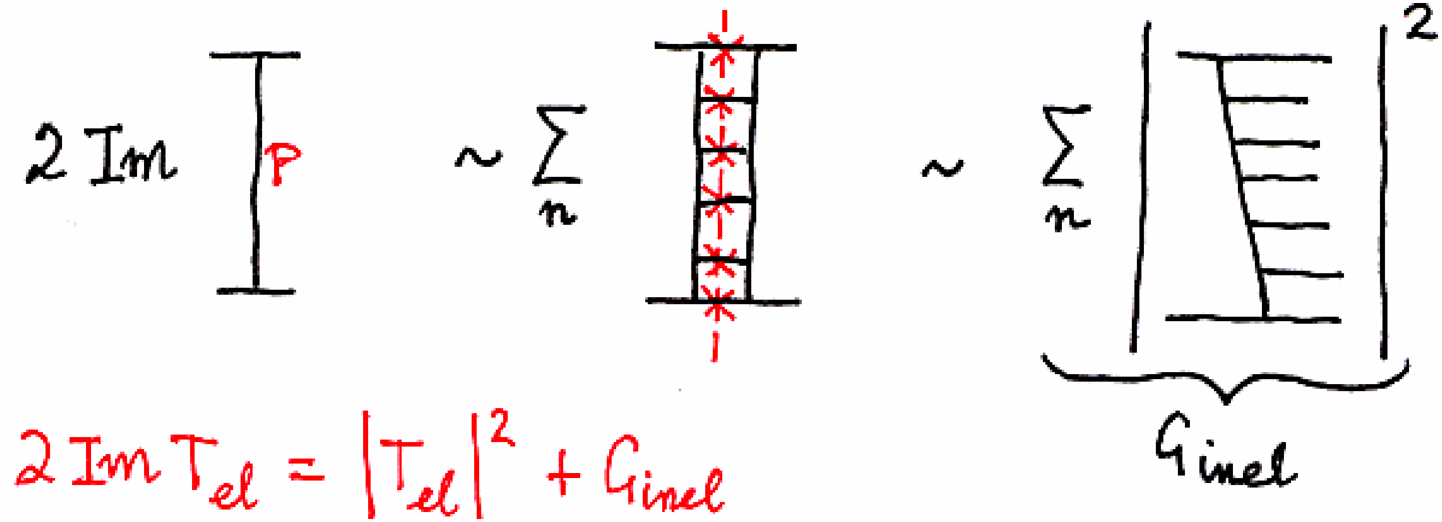
SD is “enhanced” by larger phase space available at HE.

s-channel unitarity and Pomeron exchange

Exch. of one Pomeron

Unitarity relates the Im part of ladder diagrams (disc T = 2 Im T) to cross sections for multiparticle production

Pomeron is sum of ladder graphs formed from colourless two-gluon exchange



one P exch. (1 cut)	two P exch. (4 cuts)
P	$P \otimes P$

$$\frac{d^2\sigma_{tot}}{d^2b} = 2\,\mathrm{Im}\,T_{el} = 2\left(1-e^{-\Omega/2}\right) = \Omega - \frac{\Omega^2}{4} + \dots$$

all cuts

$$\frac{d^2\sigma_{el}}{d^2b} = |T_{el}|^2 = \left(1-e^{-\Omega/2}\right)^2 = +\frac{\Omega^2}{4} + \dots$$

$$\frac{d^2\sigma_{ind}}{d^2b} = tot - el = \left(1-e^{-\Omega}\right) = \Omega - \Omega^2 + \dots$$

cut 1 Pom

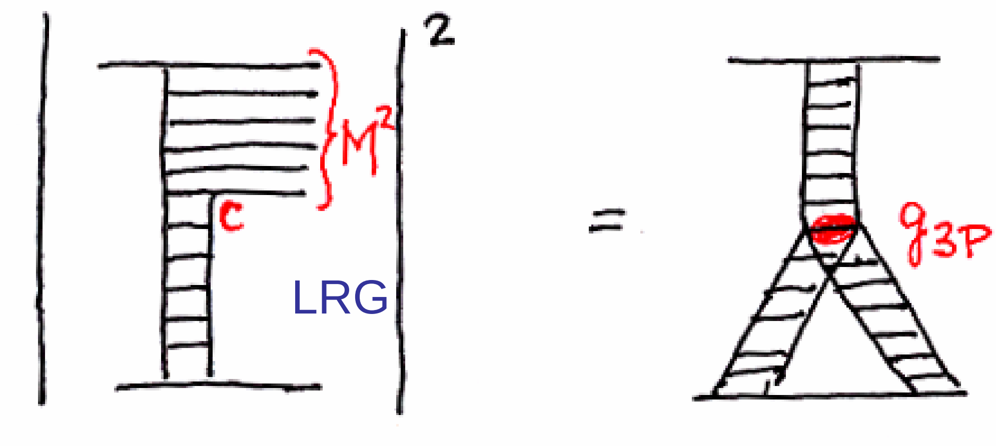
$$+\frac{\Omega^2}{2} + \dots$$

cut 2 Pom

A multi-Pomeron diagram describes several different processes

“enhanced” multi-Pomeron exchange diagrams

So much for elastic “eikonal” unitarity.
However an intermediate parton c may be scattered elastically.



The prob. of splitting within unit rapidity, g_{3P}, is relatively small. However each parton in ladder can generate a splitting so the effect is enhanced by parton multiplicity, that is by the size of LRG. In fact σ_{SD} (and σ_{DD}) would exceed the G_{inel} contribution arising from single Pomeron exch. at HE.
Unitarity is restored by small prob. S^2 that LRG survives the soft rescatt. between protons (and also intermediate partons)

Calculation of S^2

average over diff. estates i,k

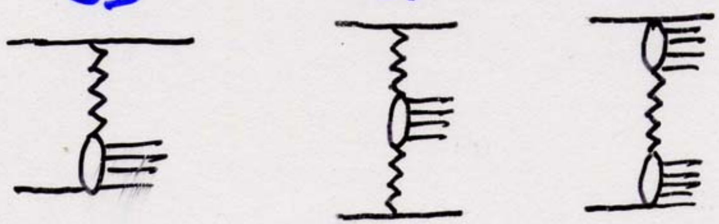
over b

prob. of proton to be in diffractive estate i

hard m.e. $i\,k \rightarrow H$

$$\overline{S^2} = \frac{\sum_{i,k} \int d^2b\, |a_{pi}|^2\ |a_{p'k}|^2\ |\mathcal{M}_{ik}|^2\ \exp(-\Omega_{ik}(s,b))}{\sum_{i,k} \int d^2b\, |a_{pi}|^2\ |a_{p'k}|^2\ |\mathcal{M}_{ik}|^2}$$

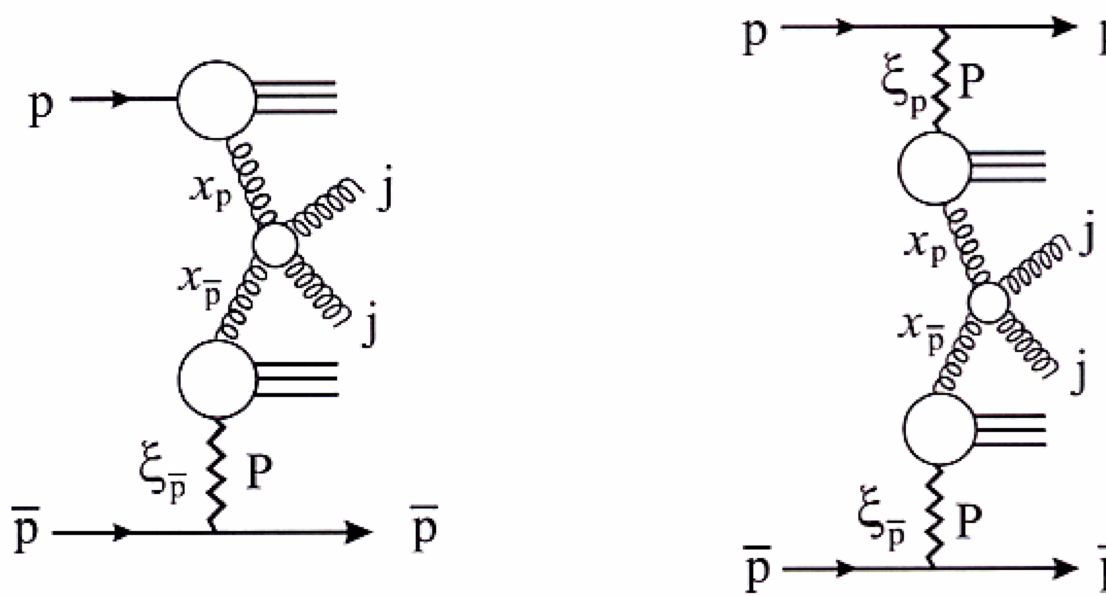
survival factor w.r.t. soft i-k interaction. Recall that $e^{-\Omega}$ is the prob. of no inelastic scatt. (which would otherwise fill the gap)



Values of S^2	SD	CD	DD
Tevatron	0·10	0·05	0·15
LHC	0·06	0·02	0·10

Example 1: Dijet production at the Tevatron

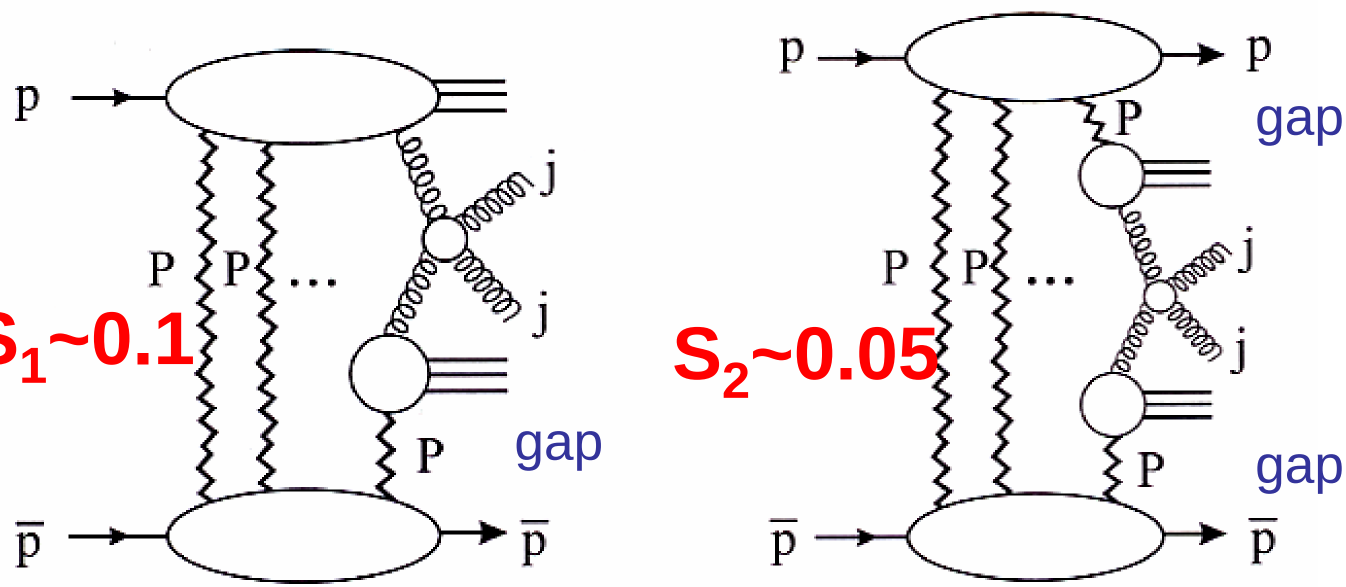
A.B. Kaidalov et al. / Physics Letters B 559 (2003) 235–238



SD

DPE

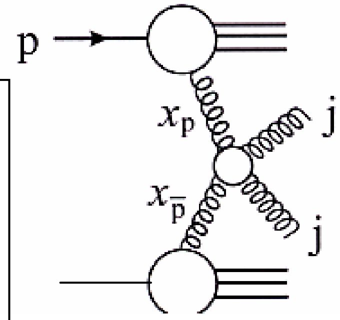
Survival prob. of gaps:



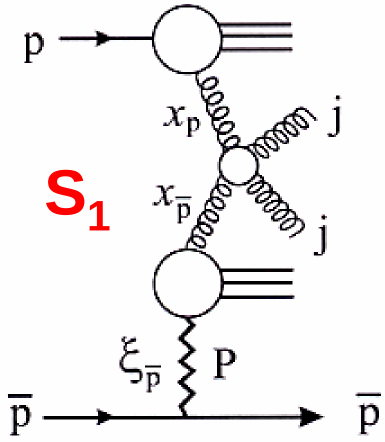
S_1~0.1

S_2~0.05

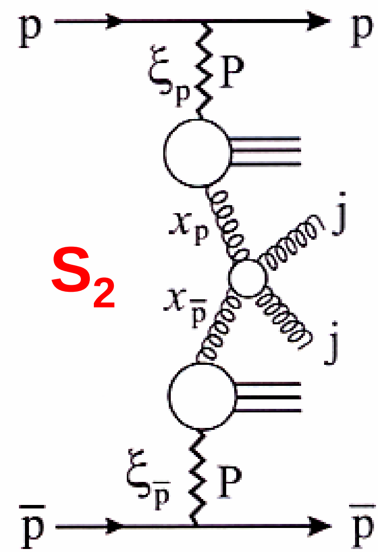
F_P is Pomeron "flux factor"
ξ is fraction of incoming mom. carried by Pom.
$x = \beta\xi$
f are the effective PDFs



ND



SD



DP

$$R_{\mathrm{ND}}^{\mathrm{SD}} \equiv \frac{\sigma_{jj}^{\mathrm{SD}}}{\sigma_{jj}^{\mathrm{ND}}} = \frac{F_P(\xi_{\bar{p}})\, f_P(\beta)\,\beta}{f_{\bar{p}}(x_{\bar{p}})\, x_{\bar{p}}}\, S_1$$

$$R_{\mathrm{SD}}^{\mathrm{DP}} \equiv \frac{\sigma_{jj}^{\mathrm{DP}}}{\sigma_{jj}^{\mathrm{SD}}} = \frac{F_P(\xi_p)\, f_P(\beta_1)\,\beta_1}{f_p(x_p)\, x_p}\, \frac{S_2}{S_1}$$

Need same kinematics.
Uncertainities cancel.
Could study $S(\beta)$

$$D = \frac{R_{\mathrm{ND}}^{\mathrm{SD}}}{R_{\mathrm{SD}}^{\mathrm{DP}}} = \frac{F_P(\xi_{\bar{p}})\, f_P(\beta)\,\beta}{F_P(\xi_p)\, f_P(\beta_1)\,\beta_1}\, \frac{f_p(x_p)\, x_p}{f_{\bar{p}}(x_{\bar{p}})\, x_{\bar{p}}}\, \frac{S_1^2}{S_2}$$

$= S_1^2/S_2$ (if $\beta=\beta_1$, same ξ)

$\sim 0.1^2/0.05 = 0.2$

CDF data D = 0.19 +/- 0.07

Example 2: exclusive Higgs production

Advantages of $pp \rightarrow p + H + p$ with $H \rightarrow bb_{bar}$

- If outgoing protons are tagged far from IP then $\sigma(M) \sim 1$ GeV (mass also from H decay products)

- Unique chance to study $H \rightarrow bb_{bar}$:

  QCD bb_{bar} bkgd suppressed by $J_z=0$ selection rule

  S/B~1 for SM Higgs $M < 140$ GeV

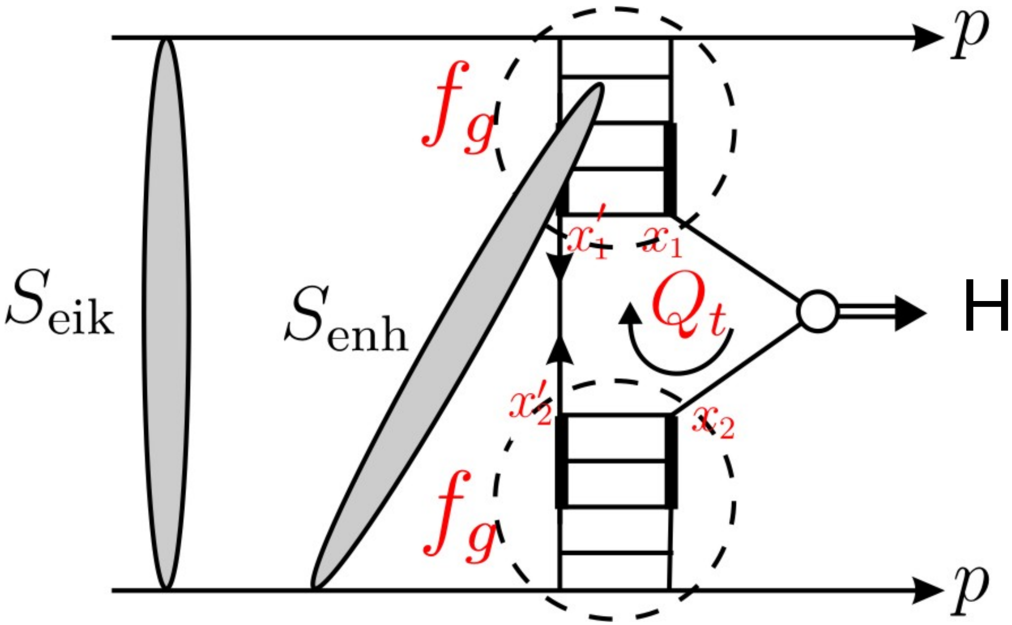
- Very clean environment, even with pile-up---10 ps timing

- SUSY Higgs: parameter regions with larger signal S/B~10, even regions where conv. signal is challenging and diffractive signal enhanced----h, H both observable

- Azimuth angular distribution of tagged p's→ spin-parity 0^{++}

…but what price do we pay for the gaps?

… "soft" scatt. can easily destroy the gaps



gap

gap

soft-hard factoriz^n

eikonal rescatt: between protons ← conserved

enhanced rescatt: involving intermediate partons ← broken

→ need a model for soft physics at HE, see later..

Recall the 2nd definition of diffraction

Diffraction is any process caused by Pomeron exchange.

(Old convention was any event with LRG of size $\delta\eta>3$, since Pomeron exchange gives the major contribution)

However LRG in the distribution of secondaries can also arise from

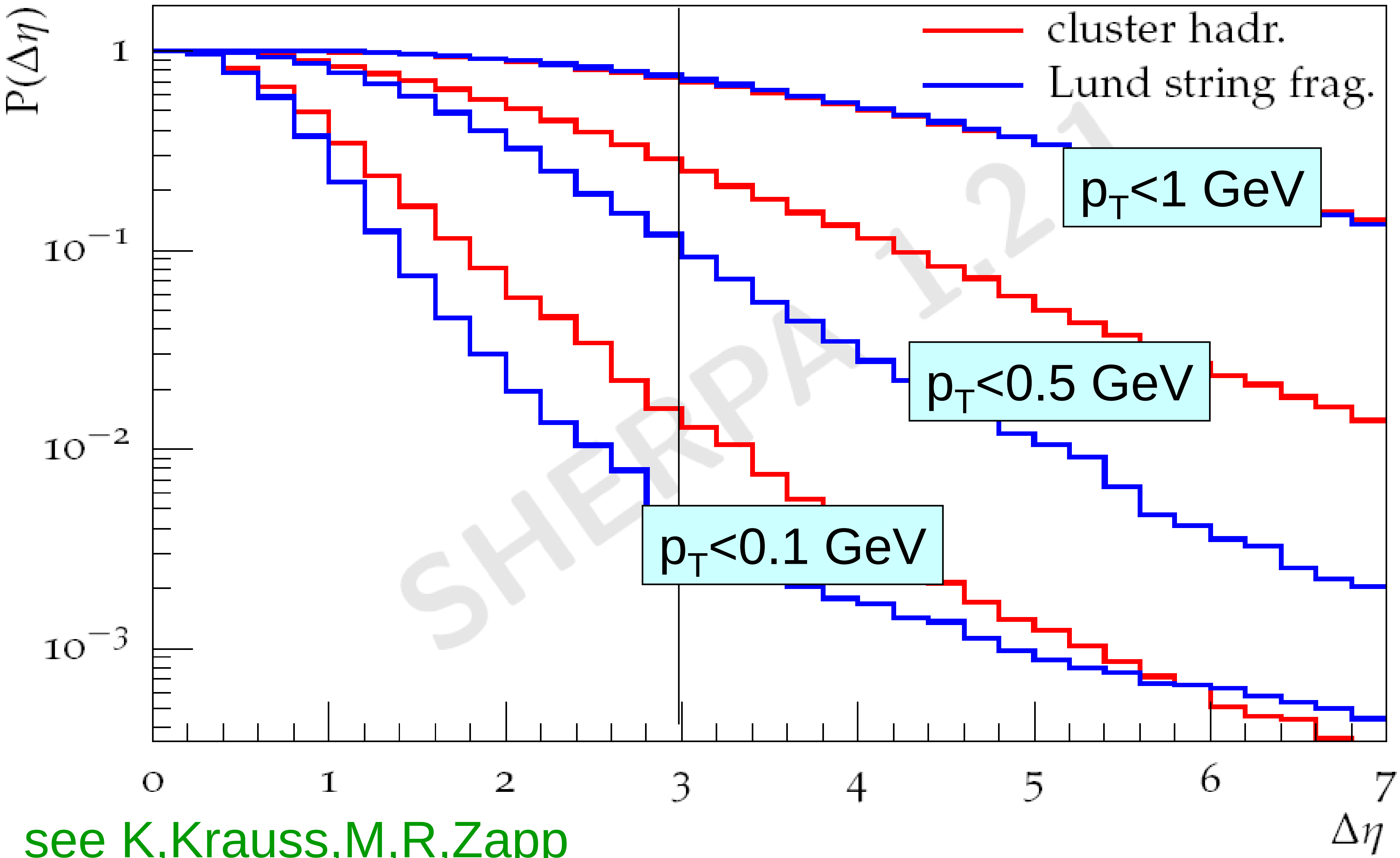
(a) Reggeon exchange

(b) fluctuations during the hadronization process

Indeed, at LHC energies LRG of size $\delta\eta>3$ do not unambiguously select diffractive events.

Prob. of finding gap larger than Δη in inclusive event at 7 TeV due to fluctuations in hadronization

gap anywhere in -5 <η <5, different threshold p_T



see K,Krauss,M,R,Zapp

So to study pure Pomeron exchange we have

either to select much larger gaps

or to study the Δy dependence of the data, fitting so as to subtract the part caused by Reggeon and/or fluctuations.

At the LHC: can study fluctuations due to hadronization

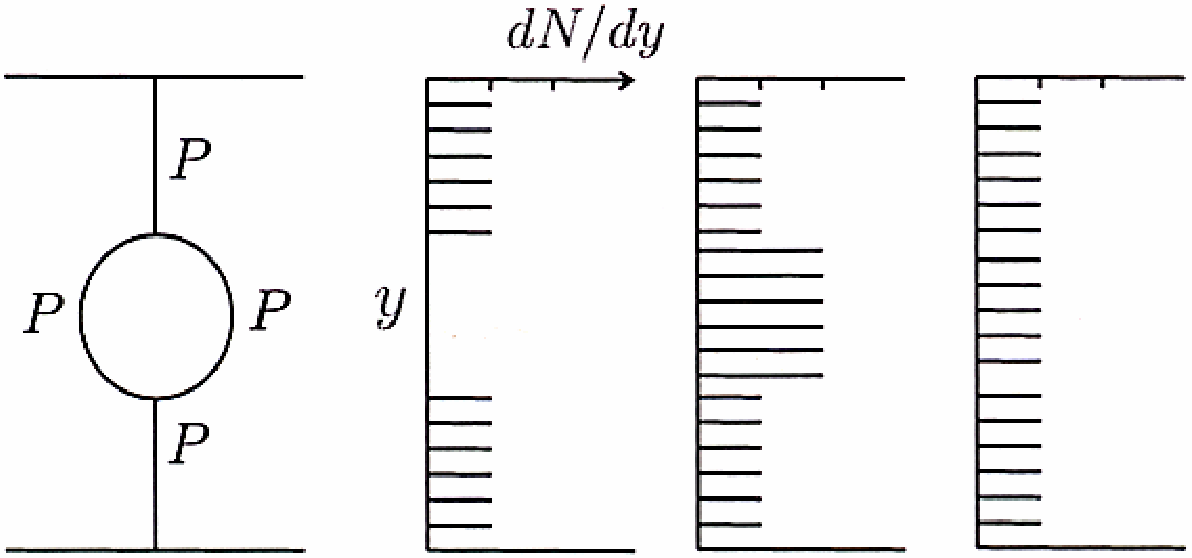
Accumulate events with LRG using "veto" trigger with no particles seen with $p_T > p_{Tcut}$ in rapidity intervals $\delta y > \Delta y$ in either calorimeter or tracker.

The dependence of σ on Δy and p_{Tcut} will constrain the model for hadronization and hence allow the selection of true diffractive events

Long-range rapidity correlations in multiplicity

Multi-Pomeron diagrams describe not only LRG processes, but also events with larger density of secondaries.

Example:



If n Pomerons are cut in some dy interval then the multiplicity is n times cutting just one Pomeron suppose in some rapidity interval, including y_a and y_b, n Pomerons are cut

$$\sigma_{inel} = \sum_n \sigma_n$$

one P cut

1-particle inclusive $$\frac{d\sigma}{dy} = \sum_n n\,\sigma_n \frac{dN^{(1)}}{dy}$$

2-particle inclusive $$\frac{d^2\sigma}{dy_a\,dy_b} = \sum_n n^2 \sigma_n \left(\frac{dN^{(1)}}{dy}\right)^2$$

$$R_2 \equiv \frac{\sigma_{inel}\, d^2\sigma/dy_a\,dy_b}{(d\sigma/dy_a)(d\sigma/dy_b)} - 1 = \frac{\langle n^2\rangle}{\langle n\rangle^2} - 1 > 0$$

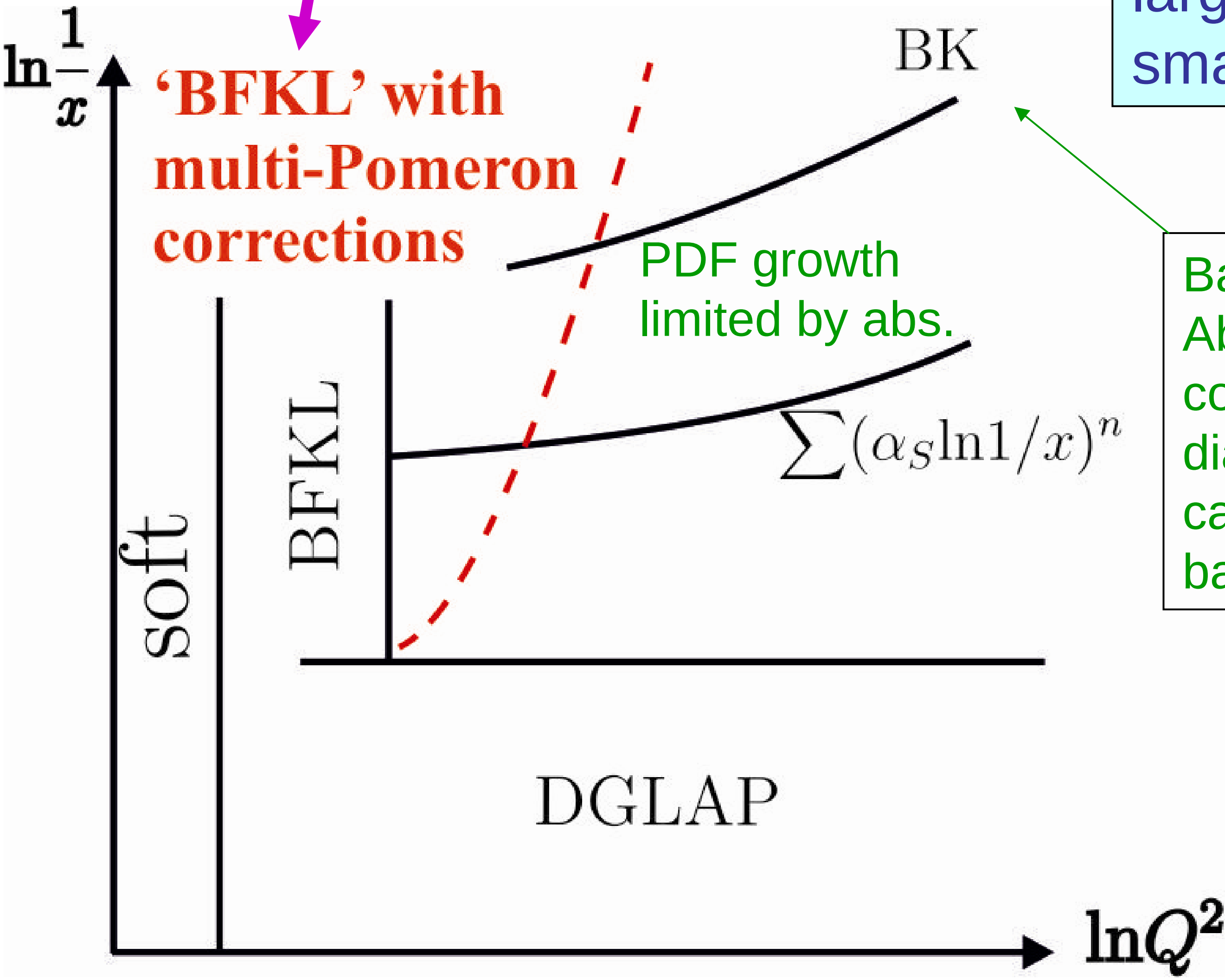
observation of similar rapidity distributions in both LRG events and in two-particle correlation R_2, would indicate an effect due to Pomeron exchange (that is diffraction) and not due to fluctuations

Model to merge 'soft' and 'hard' HE hadron interactions

- Up to now no complete model (Monte Carlo) including all facets
  --- elastic scattering, diffractive events, hard jets, etc. ---
  on the same footing. Important for the LHC
- We seek a model that not only describes pure soft HE low k_t data, (via Pomeron exchange and Reggeon FT), but which also extends into the large k_t pQCD domain
- To do this we need to introduce the partonic structure of the Pomeron:
  "soft" $\leftarrow\rightarrow$ "hard" Pomeron

domain populated by LHC data

DGLAP-based MCs tend to have too large <p_T> and too small $d\sigma/dy$ at LHC



Balitsky-Kovchegov eq.
Above this line more complicated multi-Pom diagrams enter and so cannot justify BK eq. based on "fan" diag^s

"Soft" and "Hard" Pomerons ?

A vacuum-exchange object drives soft HE interactions. Not a simple pole, but an enigmatic non-local object. Rising σ_{tot} means multi-Pom diags (with Regge cuts) are necessary to restore unitarity. σ_{tot}, $d\sigma_{el}/dt$ data, described, in a limited energy range, by eff. pole $\alpha_P^{eff} = 1.08 + 0.25t$

Sum of ladders of Reggeized gluons with, in LLx BFKL, a singularity which is a cut and not a pole (or with running α_s a series of poles). When HO are included the intercept of the BFKL/hard Pomeron is $\Delta = \alpha_P(0)-1 \sim 0.3$

We argue that there exists only one Pomeron, which makes a smooth transition between the hard and soft regimes.

Evidence that the soft Pomeron in soft domain has qualitatively similar structure to the hard Pomeron

No irregularity in HERA data in the transition region $Q^2 \sim 0.3 – 2$ GeV^2. Data are smooth thro'out this region

Small slope $\alpha' < 0.05$ GeV^{-2} of bare Pomeron trajectory is found in global analyses of "soft" data after accounting for absorptive corr^{ns} and secondary Reggeons. So typical values of k_T inside Pomeron amp. are relatively large ($\alpha' \sim 1/k_T^2$)

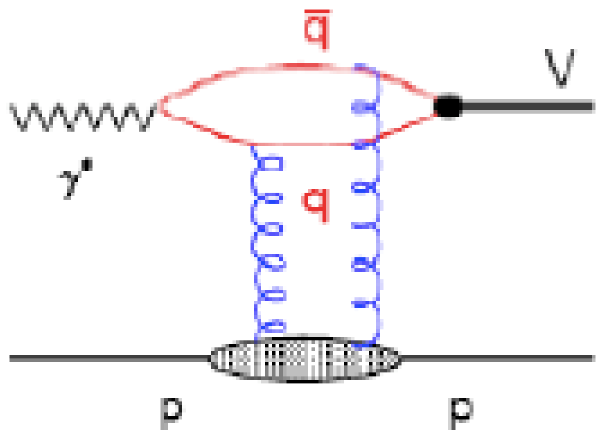
Such global analyses of "soft" data find bare Pomeron intercept $\Delta = \alpha_P(0) - 1 \sim 0.3$ close to the intercept of the hard/pQCD Pomeron after NLL corr^{ns} are resummed

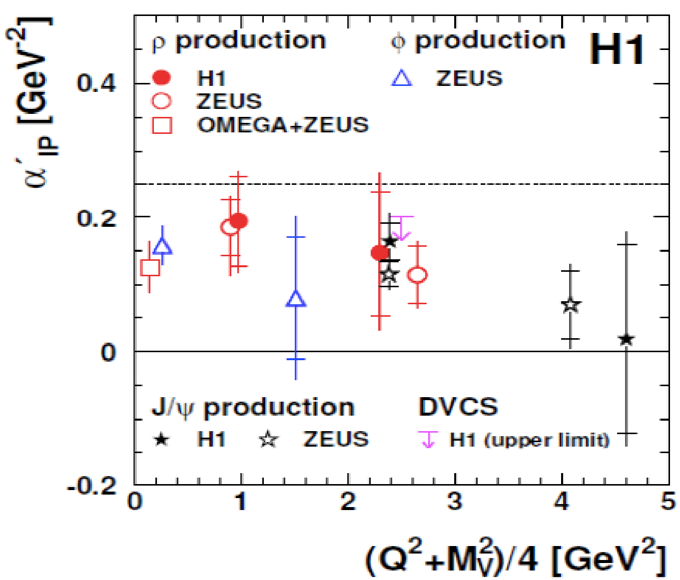
The data on vector meson electro-production at HERA imply a power-like behaviour which smoothly interpolates between the "effective" soft value ~ 1.1 at $Q^2 \sim 0$, and a hard value ~ 1.3 at large Q^2

In summary, the bare pQCD Pomeron amplitude, with trajectory $\alpha_P \sim 1.3 + 0\,t$, is subject to increasing absorptive effects as we go to smaller k_T which allow it to smoothly match on to the attributes of the soft Pomeron.

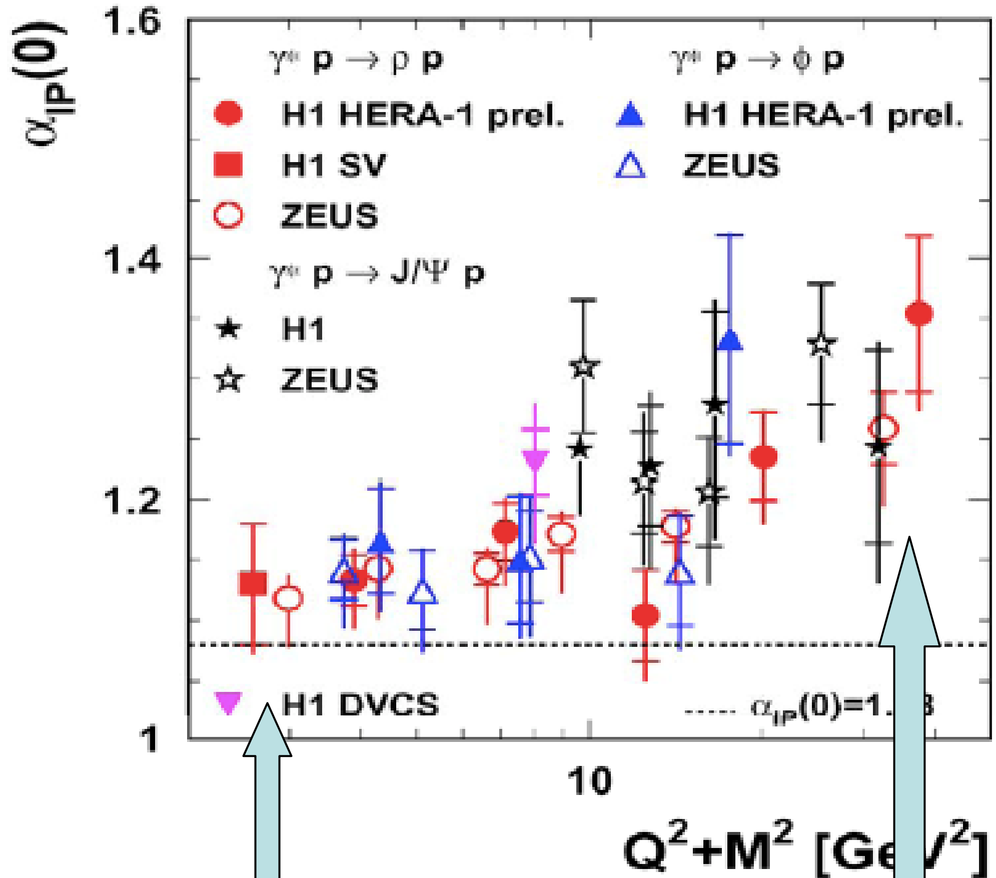
In the limited energy interval up to the Tevatron energy, some of these attributes (specifically those related to the elastic amplitude) can be mimicked by an effective Pomeron pole with trajectory $\alpha_P^{eff} = 1.08 + 0.25\,t$.

Vector meson prod^n at HERA





hard energy dependences



soft
$\alpha_P(0)$~1.1

hard
$\alpha_P(0)$~1.3

A partonic approach to soft interactions

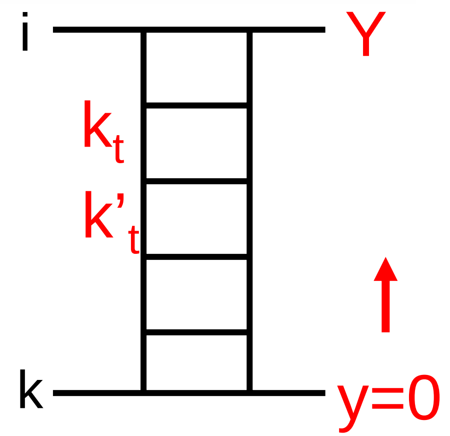
We have seen that it is reasonable to assume that in the soft domain the soft Pomeron has the general properties expected from the hard/QCD Pomeron; at least there is a smooth transition from the soft to hard Pomeron

This opens the way to extend the description of HE "soft" interactions to the perturbative very low x, k_T ~few GeV domain, a region relevant to the LHC

We start from the partonic ladder structure of the Pomeron, generated by the BFKL-like evolution in rapidity

Partonic structure of "bare" Pomeron

$$\Omega = \Omega_{ik}(y, k_t, b)$$

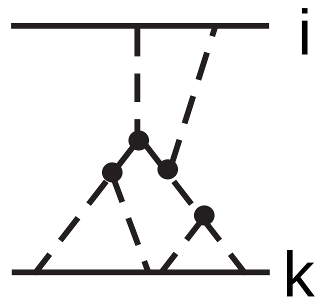


BFKL evol^n in rapidity generates ladder

$$\frac{\partial \Omega(y, k_t)}{\partial y} = \bar{\alpha}_s \int d^2k'_t \, K(k_t, k'_t) \, \Omega(y, k'_t)$$

- At each step k_t and b of parton can be be changed – so, in principle, we have 3-variable integro-diff. eq. to solve
- **Inclusion of k_t crucial to match soft and hard domains. Moreover, embodies less screening for larger k_t comp^{ts}.**
- We use a simplified form of the kernel K with the main features of BFKL – diffusion in log k_t^2, $\Delta = \alpha_P(0) - 1 \sim 0.3$
- b dependence during the evolution is prop' to the Pomeron slope α', which is v.small ($\alpha' < 0.05$ GeV^{-2}) -- so ignore. Only b dependence comes from the starting evol^n distrib^n
- Evolution gives $\Longrightarrow$ $\Omega = \Omega_{ik}(y, k_t, b)$

Multi-Pomeron contributions



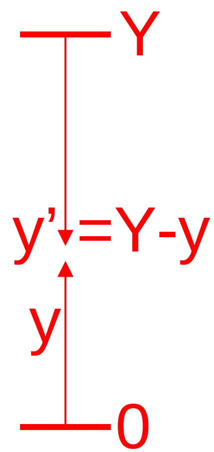
Now include rescatt of intermediate partons with the "beam" i and "target" k

evolve up from y=0

$$\frac{\partial \Omega_k(y)}{\partial y} = \bar{\alpha}_s \int d^2k_t' \; \exp(-\lambda(\Omega_k(y) + \Omega_i(y'))/2) \; K(k_t, k_t') \; \Omega_k(y)$$

evolve down from y'=Y-y=0

$$\frac{\partial \Omega_i(y')}{\partial y'} = \bar{\alpha}_s \int d^2k_t' \; \exp(-\lambda(\Omega_i(y') + \Omega_k(y))/2) \; K(k_t, k_t') \; \Omega_i(y')$$



where $\lambda\Omega_{i,k}$ reflects the different opacity of protons felt by intermediate parton, rather the proton-proton opacity $\Omega_{i,k}$ $\lambda \sim 0.2$

solve iteratively for $\Omega_{ik}(y,k_t,b)$ **inclusion of k_t crucial**

Note: data prefer $\exp(-\lambda\Omega)$ → $[1 - \exp(-\lambda\Omega)] / \lambda\Omega$

Form is consistent with generalisation of AGK cutting rules

In principle, knowledge of $\Omega_{ik}(y,k_t,b)$ allows the description of all soft, semi-hard pp high-energy data:

σ_{tot}, $d\sigma_{el}/dt$, $d\sigma_{SD}/dtdM^2$, DD, DPE…

LRG survival factors S^2 (to both eikonal, enhanced rescatt)

PDFs and diffractive PDFs at low x and low scales

Indeed, such a model can describe the main features of all the data, in a semi-quantitative way, with just a few physically motivated parameters.

Some preliminary results of this Khoze-Martin-Ryskin model:

Cross sections (in mb) versus collider energy (in TeV)

energy	σ_{tot}	σ_{el}	$\sigma_{SD}^{\mathrm{low}M}$	$\sigma_{SD}^{\mathrm{high}M}$	$\sigma_{SD}^{\mathrm{tot}}$
1.8	72.8/72.5	16.3/16.8	4.4/5.2	8.3/11.1	12.7/16.3
14	98.3/94.6	25.1/24.2	6.1/7.5	14.0/15.9	20.1/23.4
100	127.1/117.4	35.2/31.8	8.0/9.9	20.6/20.0	28.6/29.9

$d\sigma/dy \sim s^{0.2}$ like the LHC data for 0.9 to 7 TeV

$S^2 = 0.010\text{-}0.016$ for gaps in $pp \rightarrow p + H + p$ (120GeV SM Higgs at 14TeV)

Conclusions

- s-ch unitarity is important for quasi-elastic scatt or LRG events

- Multi-Pomeron exchange diagrams restore unitarity:
  (i) eikonal pp rescatt. (ii) enhanced with intermediate partons

- Altho' $g_{3P} \sim 0.2 g_N$, high-mass p dissoc^n is enhanced at the LHC

- Unitarity is restored for LRG by small survival prob. S^2 of gaps
  e.g. $S^2 \sim 0.015$ for $pp \rightarrow p + H + p$ (M_H=120 GeV at 14 TeV)

- LRG also from fluct^{ns} in had^n: study different p_T cuts and Δy
  also study long-range rapidity correlations at the LHC

- QCD/BFKL Pom. → Pomeron describing soft physics

- Partonic struct. of Pom, with multi-Pom contrib^{ns} can describe
  all soft ($\sigma_{tot,el,SD}$..) and semihard (PDFs, minijets..) physics-KMR

- Forms the basis of "all purpose" MC - Krauss, Hoeth, Zapp