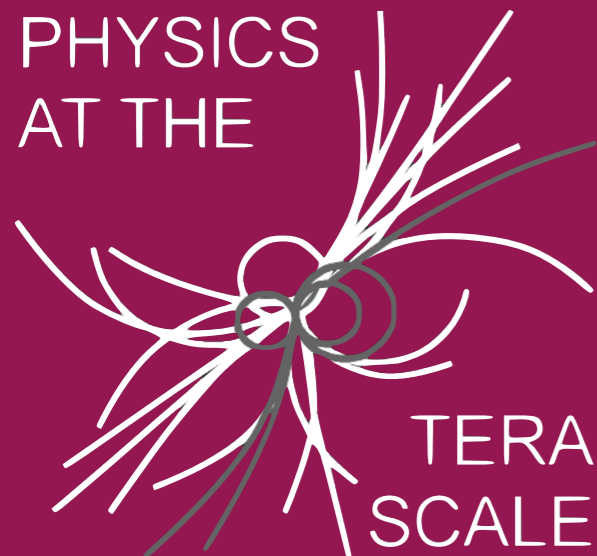


PHYSICS  
AT THE



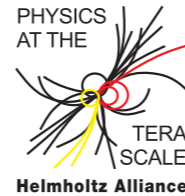
TERA  
SCALE

**Helmholtz Alliance**

# MULTI PARTON INTERACTION: SOME THEORETICAL CONSIDERATIONS

ZOLTÁN NAGY  
*DESY*

# MPI10 @ DESY



Helmholtz Alliance

## PHYSICS AT THE TERASCALE

Deutsches Elektronen-Synchrotron DESY +++ Karlsruher Institut für Technologie - Großforschungsbereich +++ Max-Planck-Institut für Physik +++ Rheinisch-Westfälische Technische Hochschule Aachen +++ Humboldt-Universität zu Berlin +++ Rheinische Friedrich-Wilhelms-Universität Bonn +++ Technische Universität Dortmund +++ Technische Universität Dresden +++ Albert-Ludwigs-Universität Freiburg +++ Justus-Liebig-Universität Gießen +++ Georg-August-Universität Göttingen +++ Universität Hamburg +++ Ruprecht-Karls-Universität Heidelberg +++ Karlsruher Institut für Technologie - Universitätsbereich +++ Johannes Gutenberg-Universität Mainz +++ Ludwig-Maximilians-Universität München +++ Universität Regensburg +++ Universität Rostock +++ Universität Siegen +++ Julius-Maximilians-Universität Würzburg +++ Bergische Universität Wuppertal +++



- ✦ The workshop was supported by Helmholtz Alliance “Physics at the Terascale”
- ✦ 36 registered participants and 17 talks.
- ✦ Focus on theoretical issues.
- ✦ All the talks were recorded and they are available from our website.

#### Topics:

- improving theoretical understanding of multiple parton interactions (MPI)
- implementation in general-purpose Monte Carlo (MC) programs
- current development of double parton distribution functions and their use in MC programs
- MPI contributions to signal and background processes
- recent experimental results from the LHC and the Tevatron
- plans for future measurements of MPI at the LHC
- continued development of better MC event generators

#### Advisory Committee:

J. Bartels (Univ. of Hamburg)  
M. Diehl (DESY)  
H. Jung (DESY)  
D. Soper (Univ. of Oregon)  
J. Stirling (Univ. of Cambridge)  
G. Weiglein (DESY)

Registration deadline: 5 September 2010. Please register via the workshop webpage.

Organising Committee: Judith Katzy (DESY), Albert Knutsson (DESY), Anna Kulesza (RWTH Aachen), Zoltan Nagy (DESY)

<http://www.terascale.de/mpl10>

# MPI10 @ DESY

## Experimentalists' reviews

- ATLAS review (Bill Murray, Deepak Kar)
- CMS review (Livio Fano)
- LHCb review (Michael Schmelling)

## Double parton PDF

- GS09 (Jonathan Gaunt)
- PDF review (James Stirling)

## Core Theory

- MPI:Theoretical Considerations (Markus Diehl)
- MPI, Diffraction, BFKL pomeron (Gösta Gustafson)
- Recombination within multi-chain contributions in pp scattering (Jochen Bartels)

## Phenomenology

- W pair production at LHC (Steve Kom)
- MPI in W+jets production (Ezio Maina)
- Dynamical characteristics of DPS (Ed Berger)
- 4-jet correlations at Tevatron and LHC (Boris Blok)

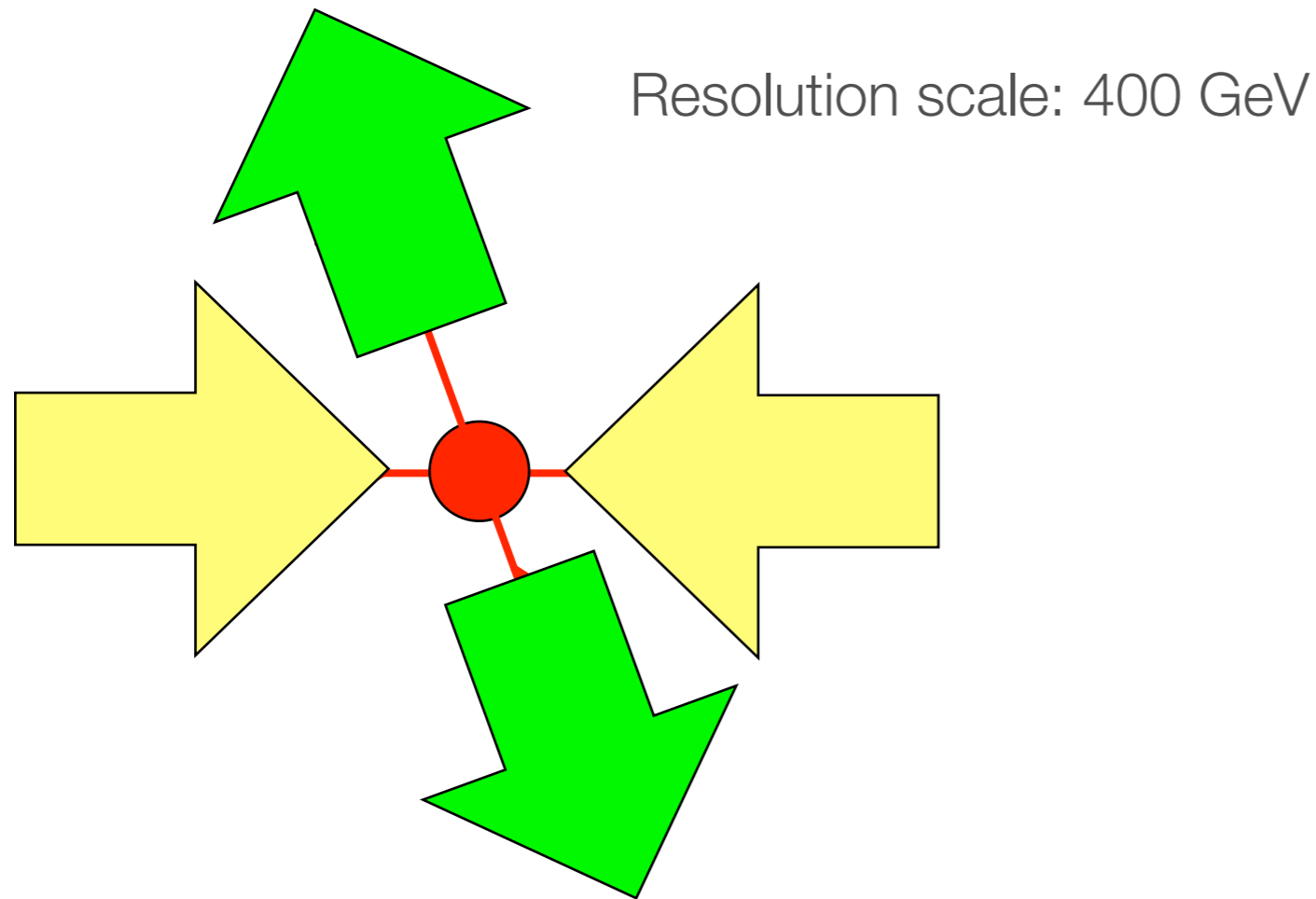
## MC Developement

- PYTHIA review (Richard Corke)
- Herwig review (Stefan Gieseke)
- SHERPA review (Korinna Zapp)
- DIPSY (Gösta Gustafson)

<http://mpi10.desy.de>

# Hadron-Hadron Collision

In hadron-hadron collision the picture is more complicated.

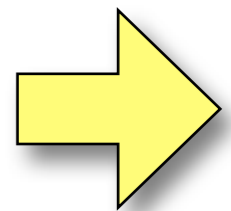


Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

**Important observation:** The total cross section *is independent of* the resolution of the measurement (or detector).

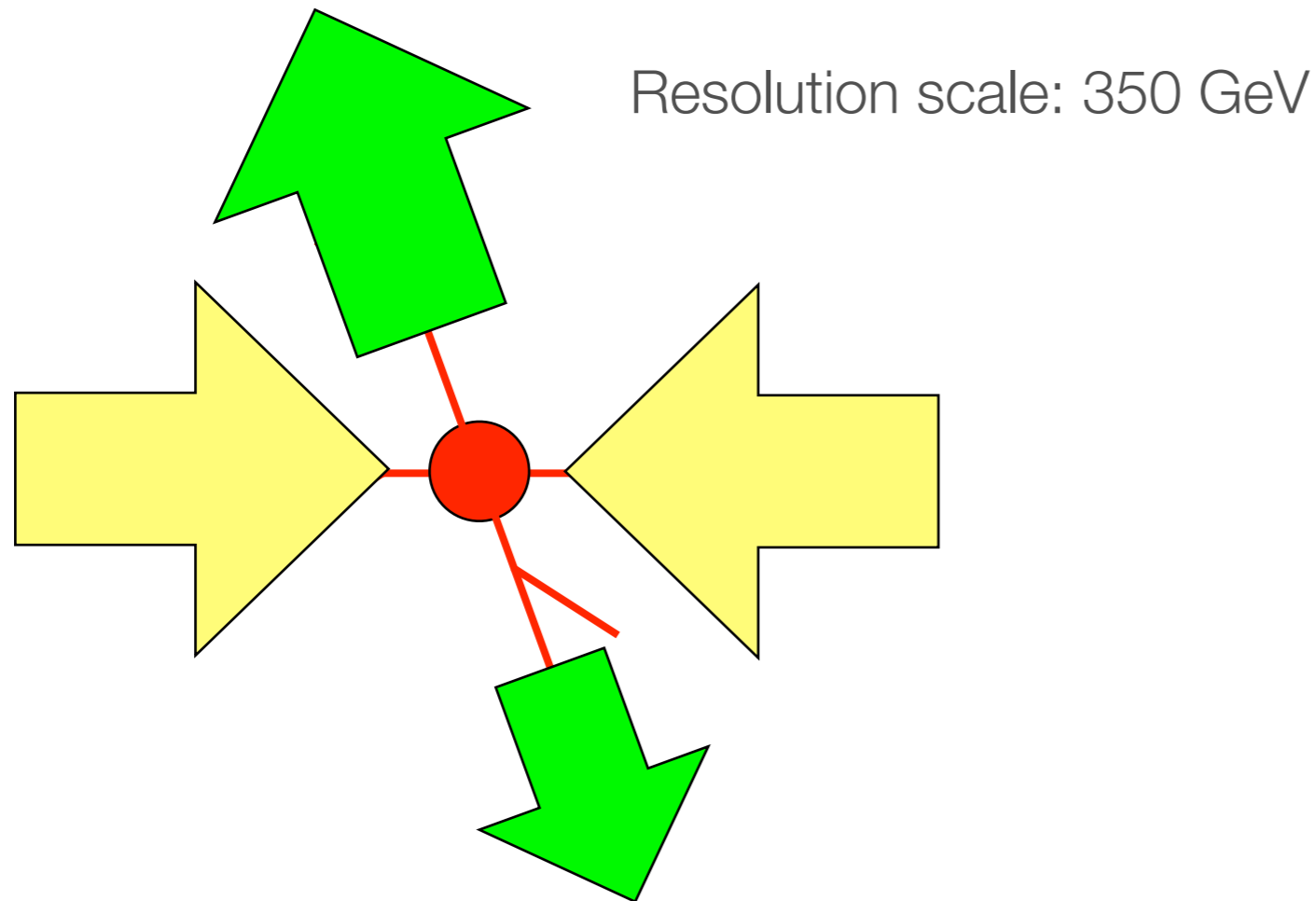
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*Does perturbative QCD support this nice intuitive picture?*



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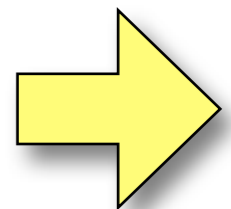


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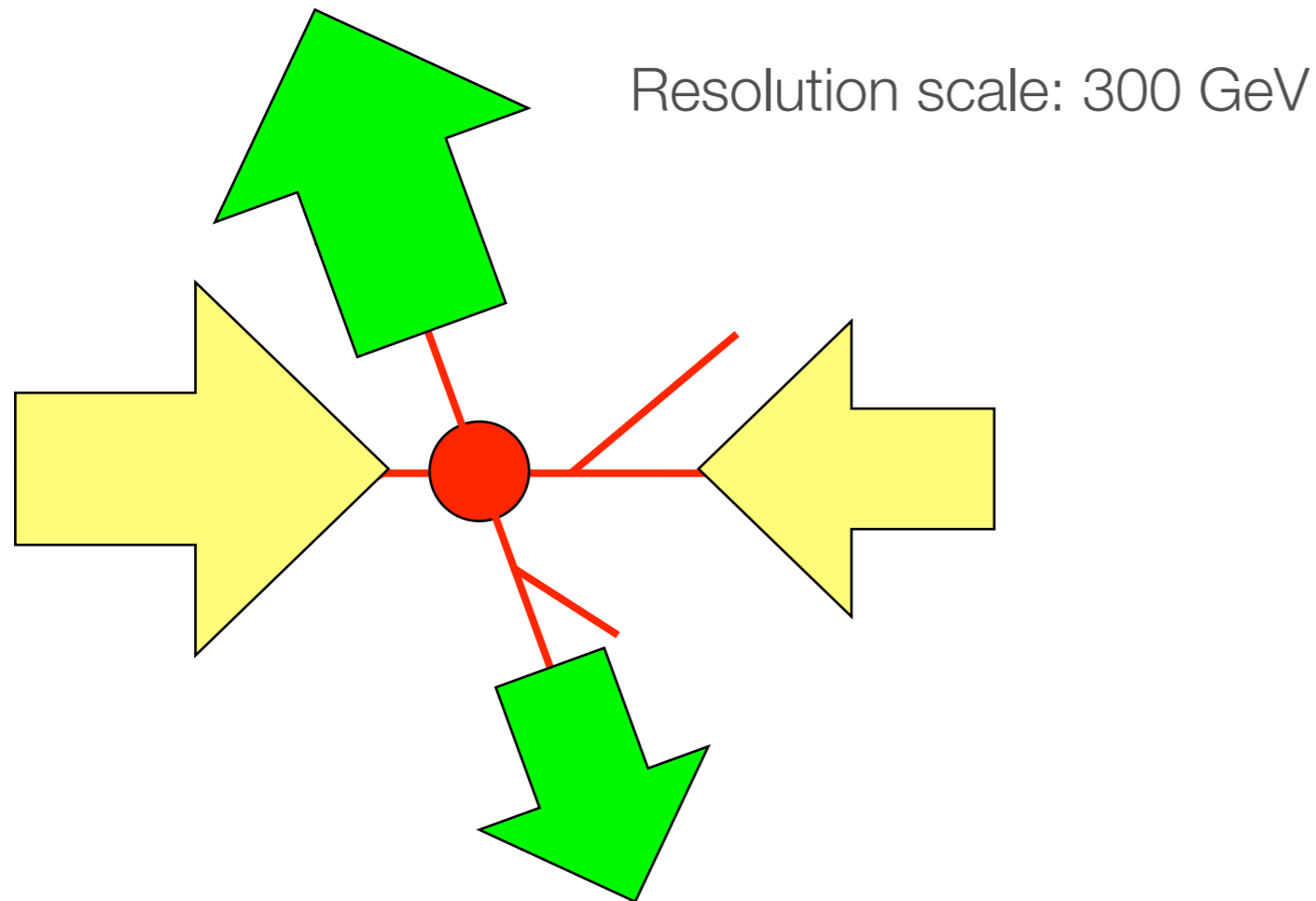
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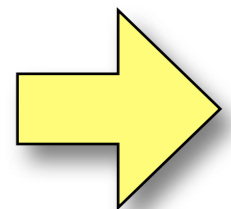


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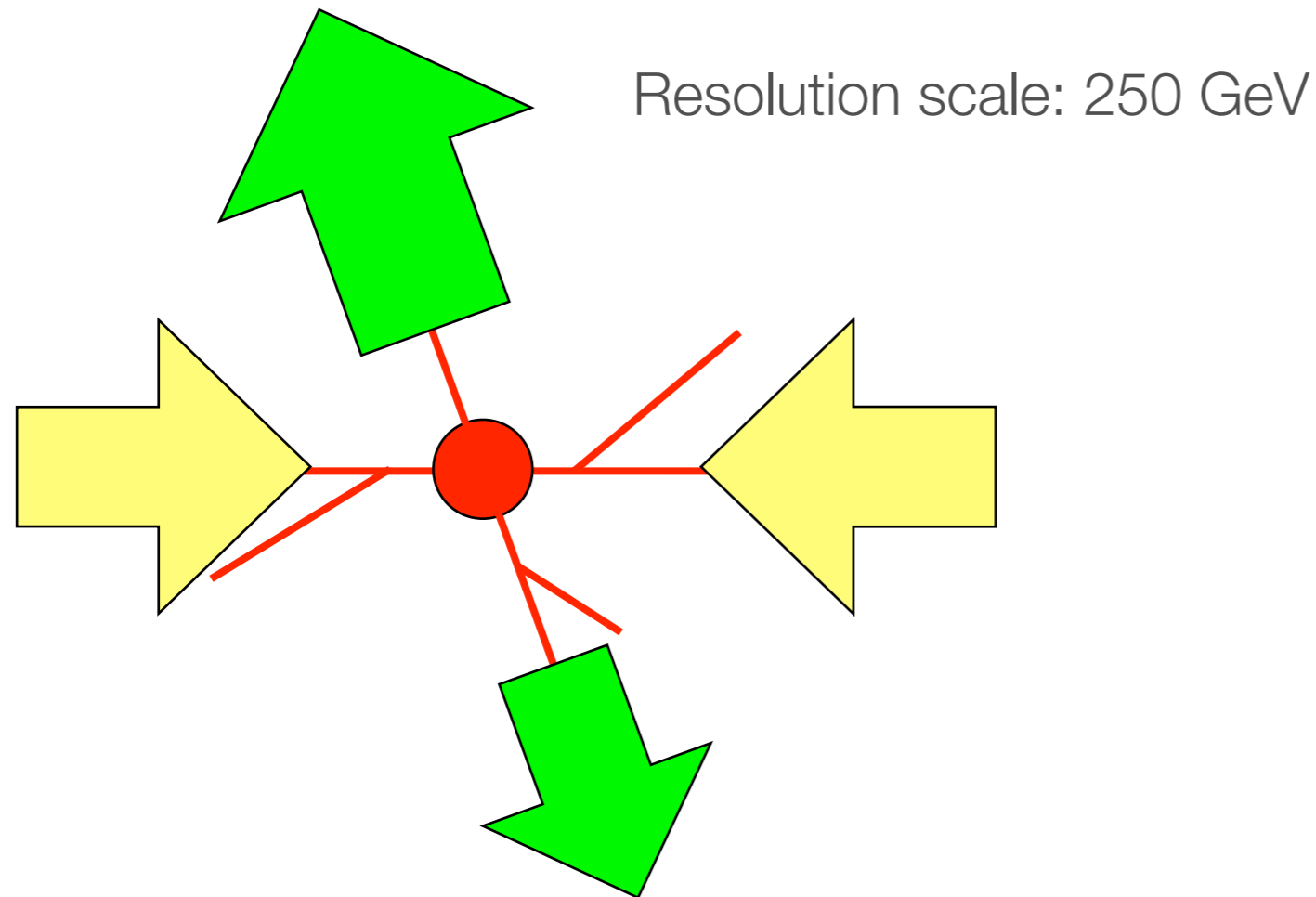
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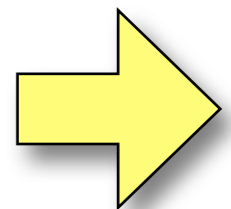


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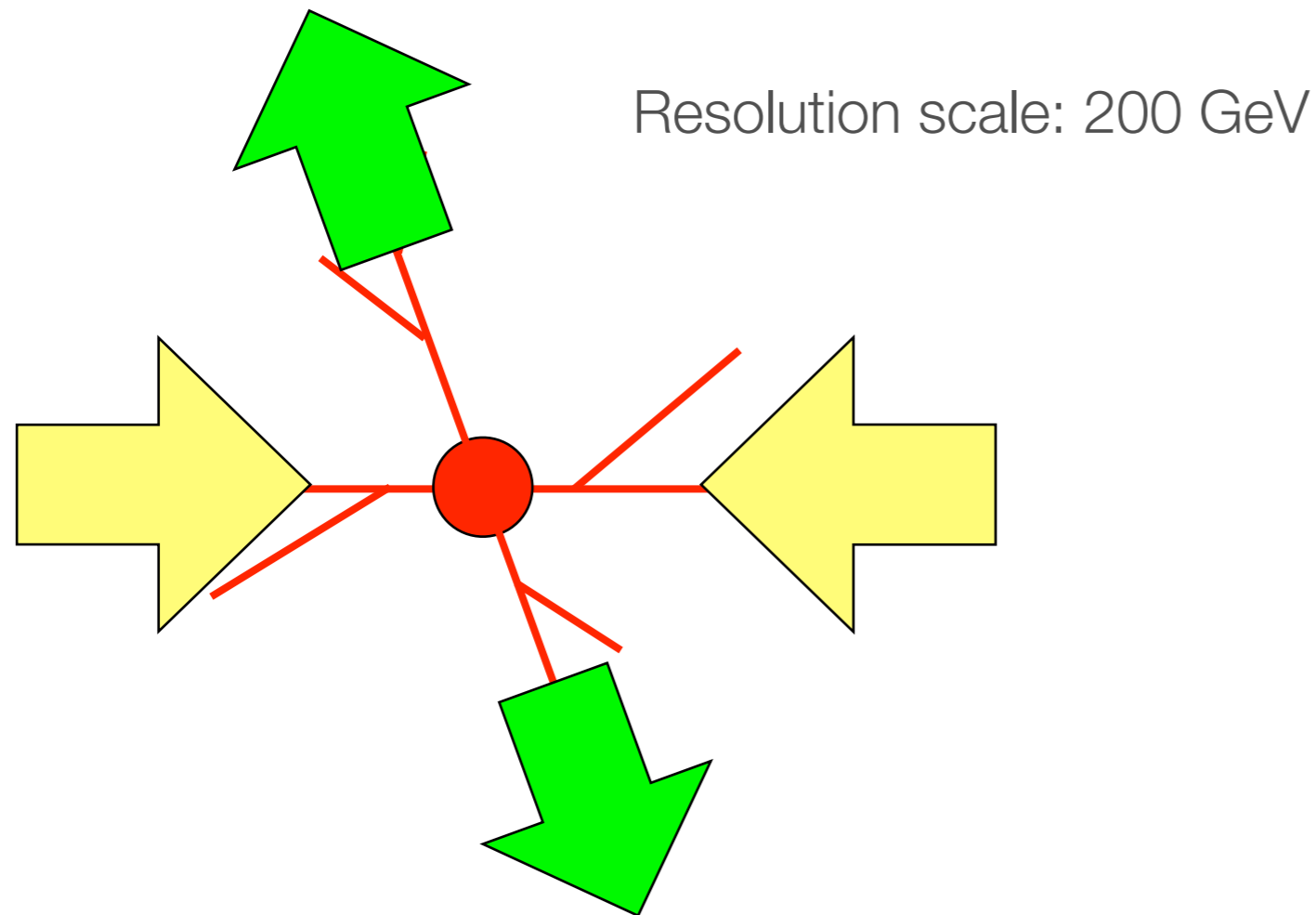
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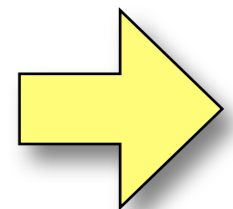


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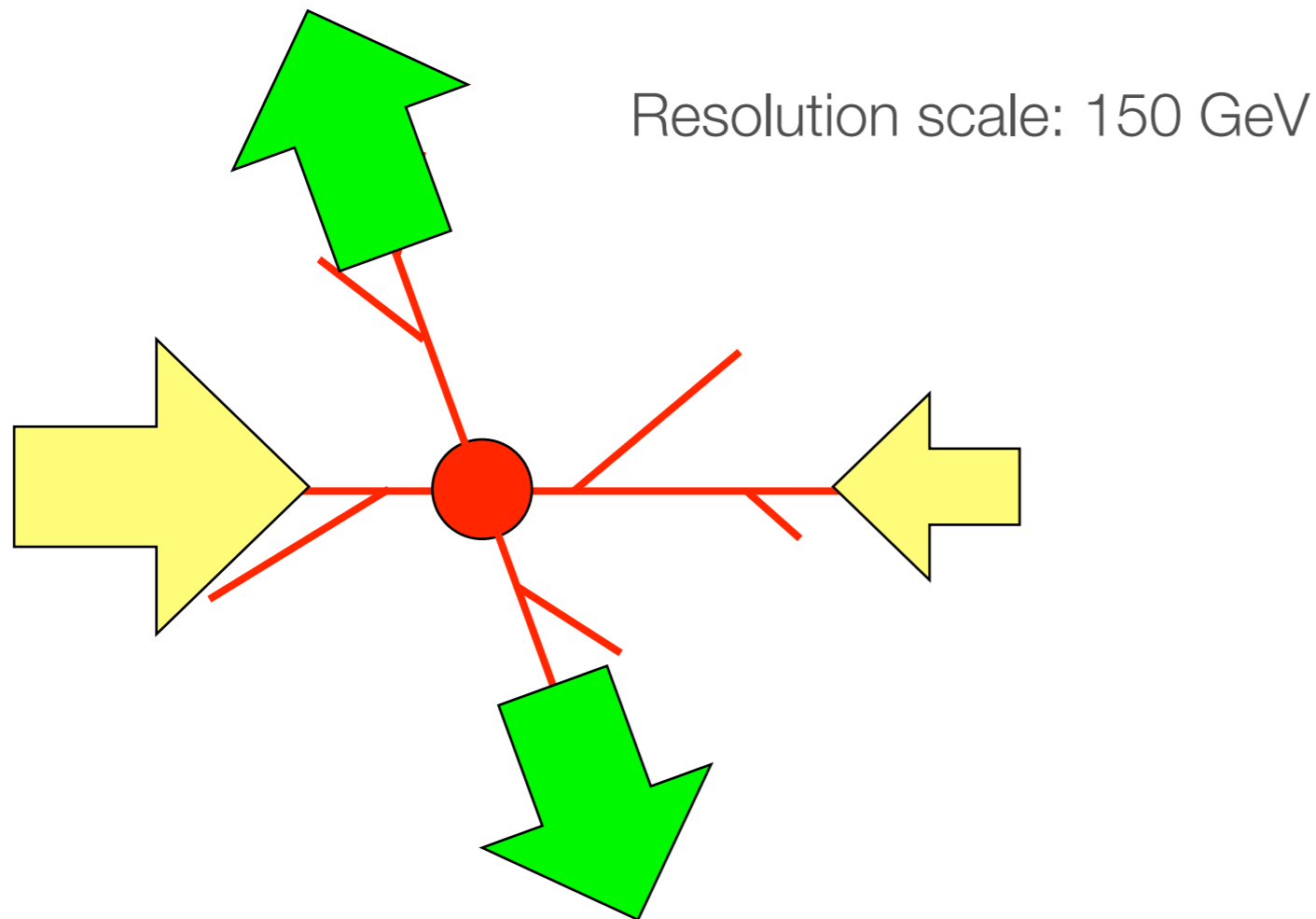
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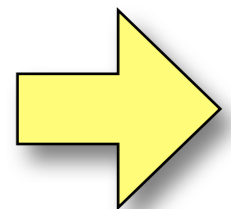


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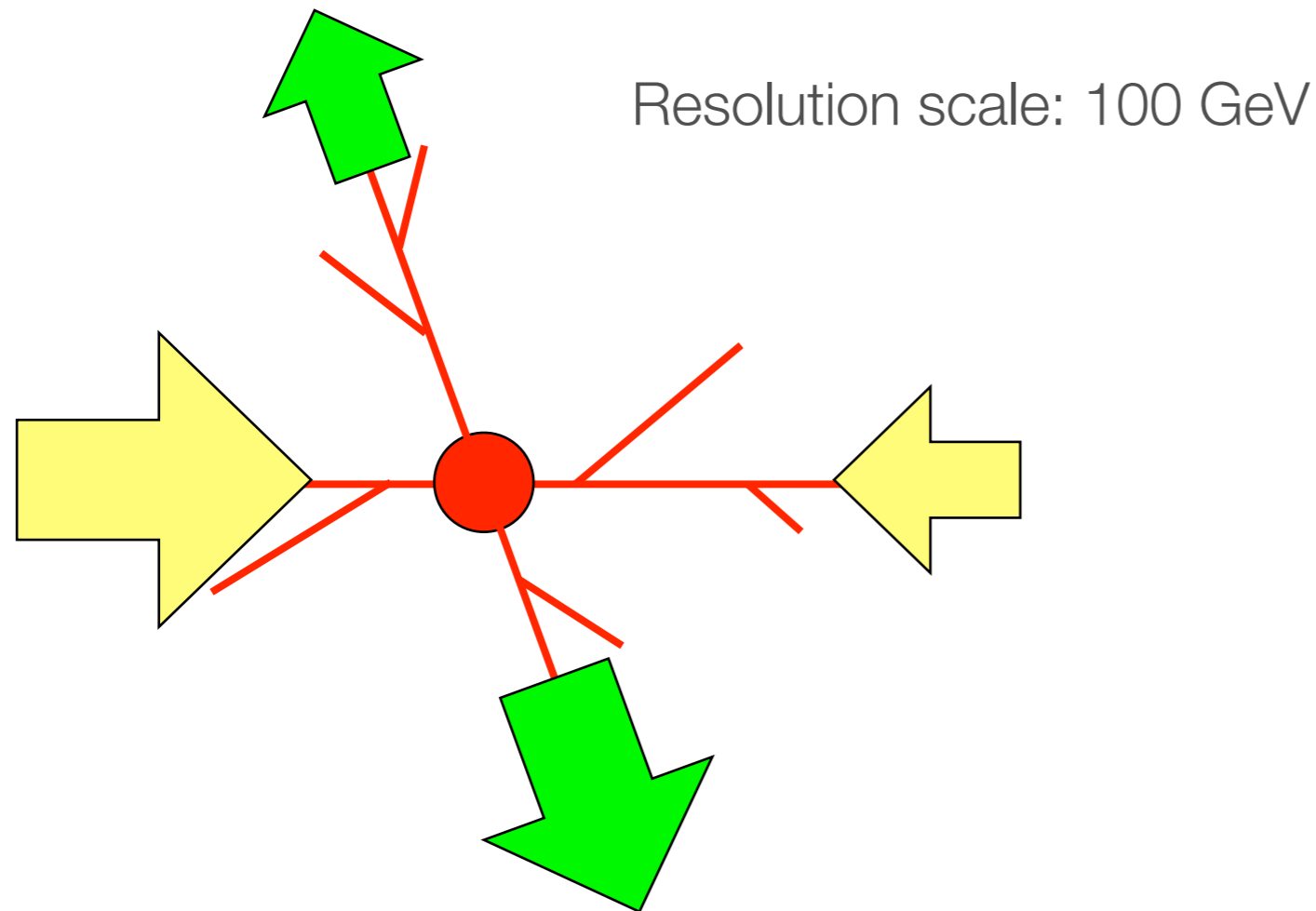
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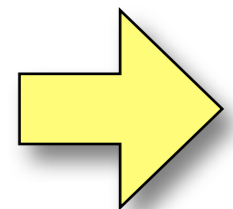


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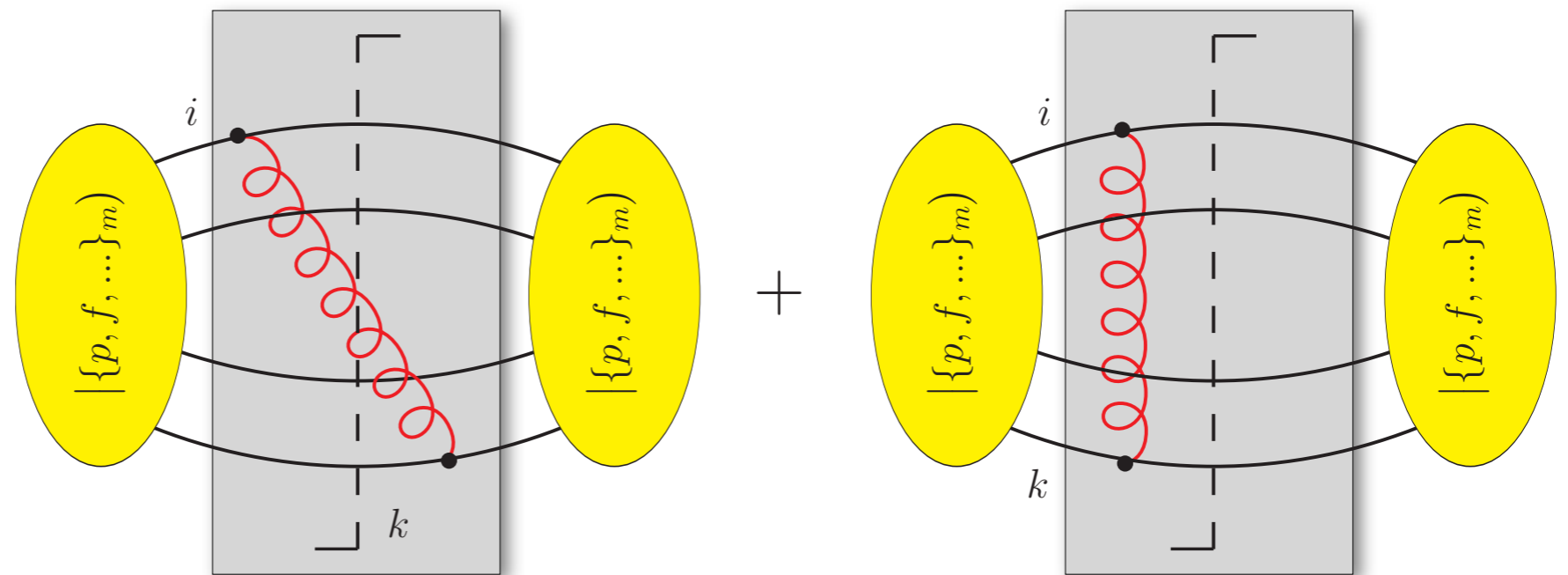
# Parton Shower

The parton shower is an fully exclusive evolution of the partonic final state.

$$\mathcal{U}(t, t') | \rho(t') \rangle = \mathbb{T} \exp \left\{ \int_t^{t'} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) \right] \right\} | \rho(t') \rangle$$

*Resolvable radiations*

*Unresolvable radiations*

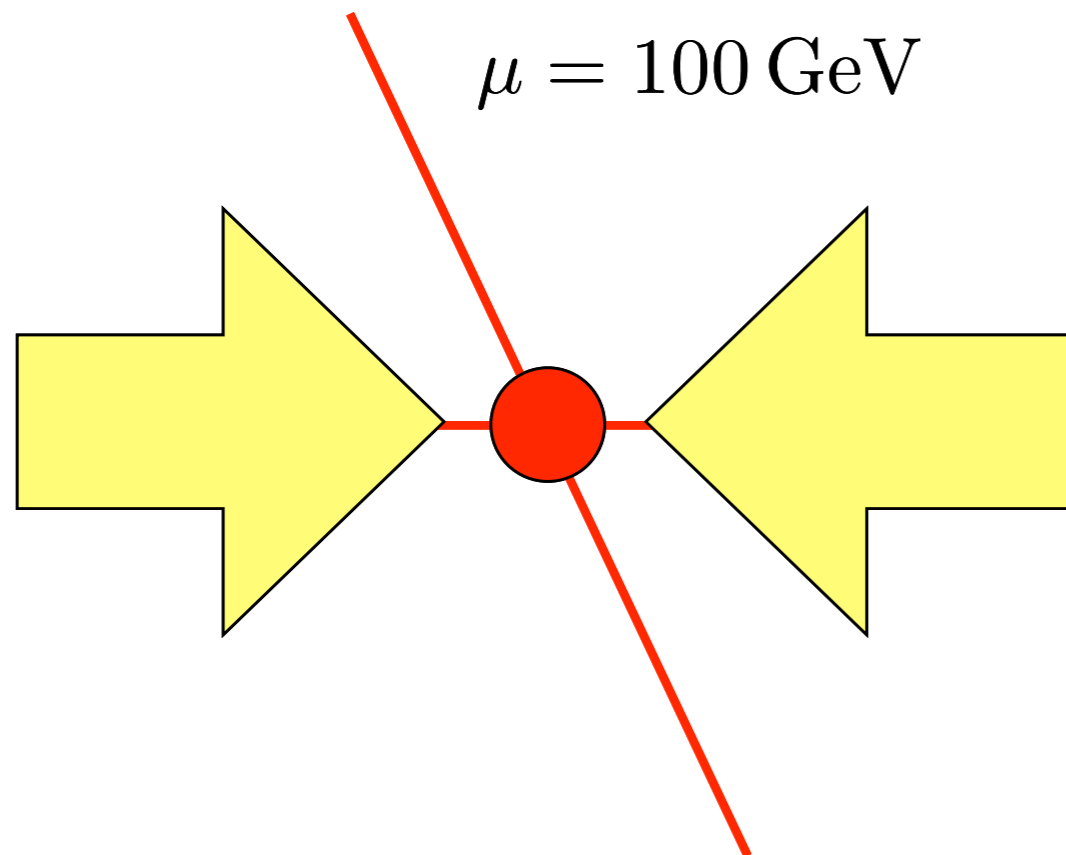


*Note, in parton shower basically we don't have tunable parameter.*

*Is it possible to use this framework for MPI events? Does QCD support it?*

# Multi Parton Interaction

Let us see how it looks at hadron collider



In hadron-hadron collision the parton distribution function also absorbs the contribution of the **multiple interactions** and **rescattering**.

**Our strategy:**

- Identify **factorizable** singular contributions systematically.
- Sum up the **strongly ordered** radiations.
- Minimize the number of the **ad-hoc assumptions** and **tuning parameters**.

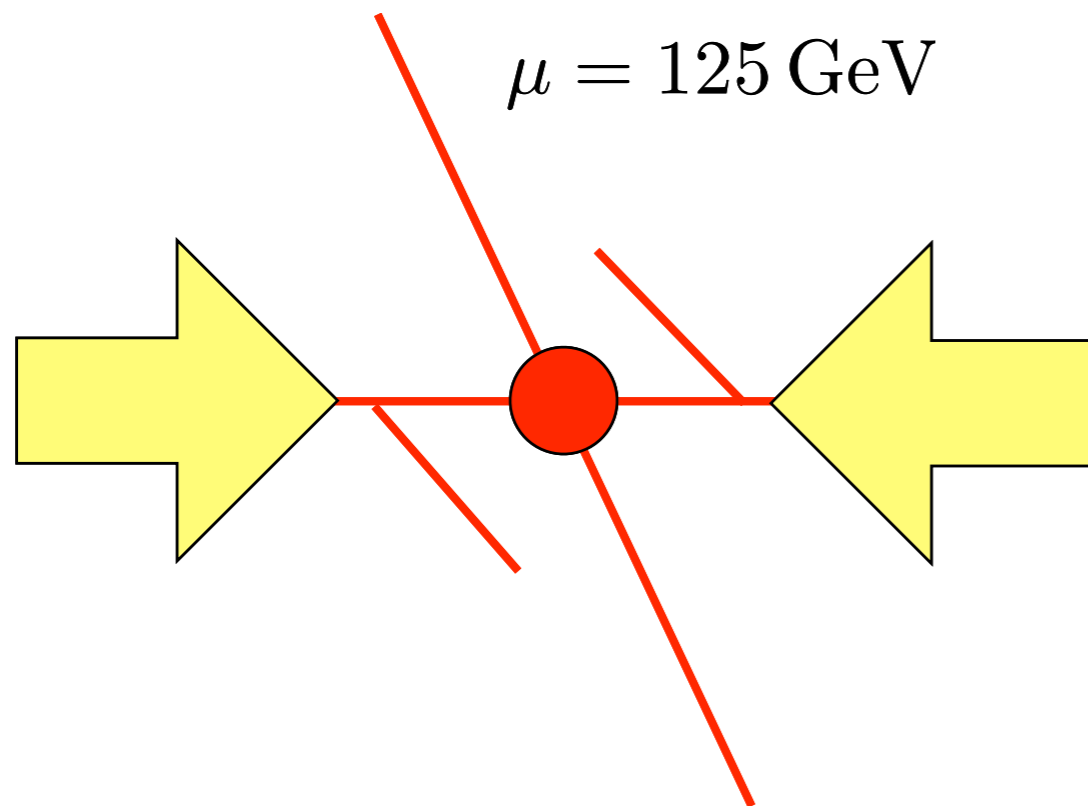
Now, one can try to define the evolution operator in the following form

$$\mathcal{U}(t, t') = \mathbb{T} \exp \left\{ \int_t^{t'} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}_I(\tau) + \sum_{\beta=\text{MI, RS}} \{ \mathcal{H}_\beta(\tau) - \mathcal{V}_\beta(\tau) \} \right] \right\}$$

Single radiations  Everything else

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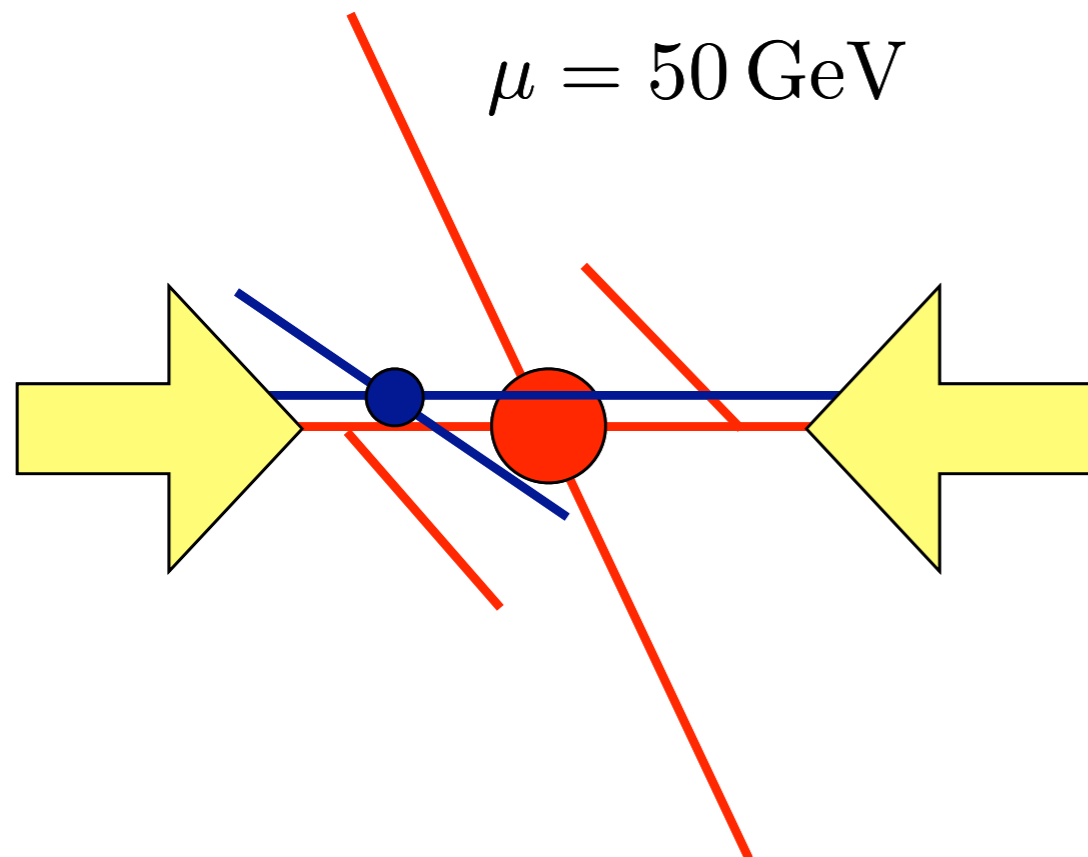
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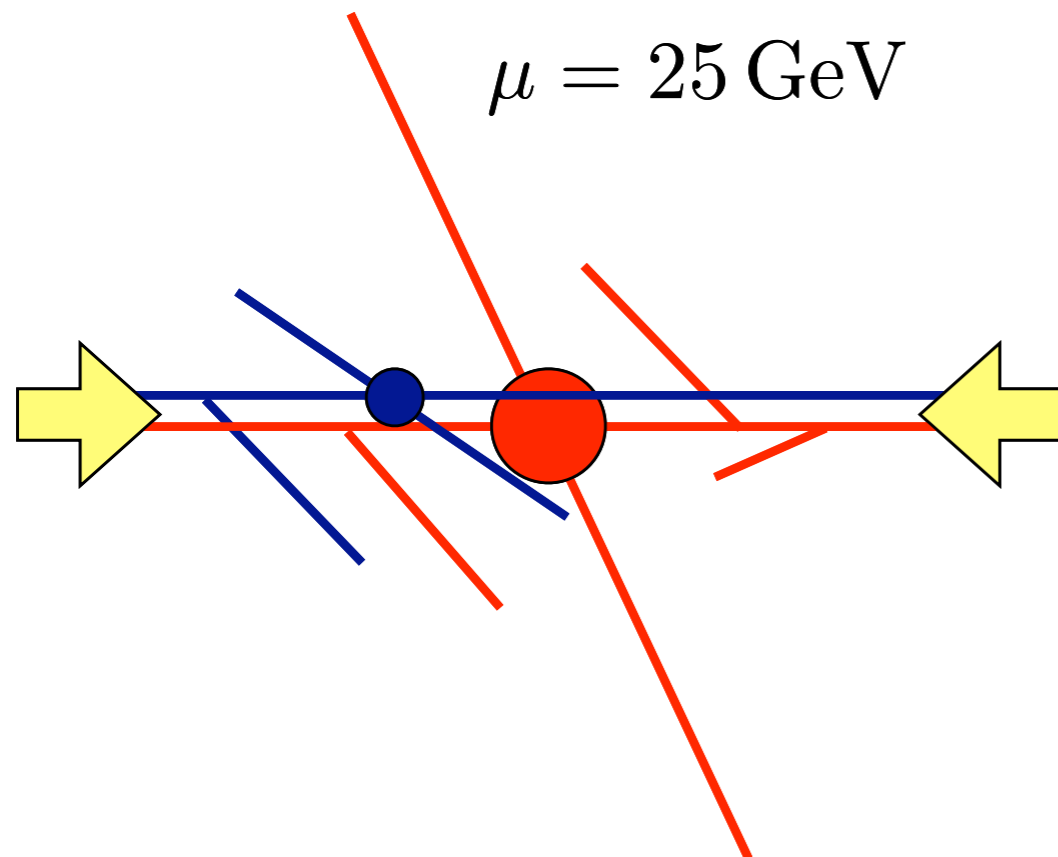
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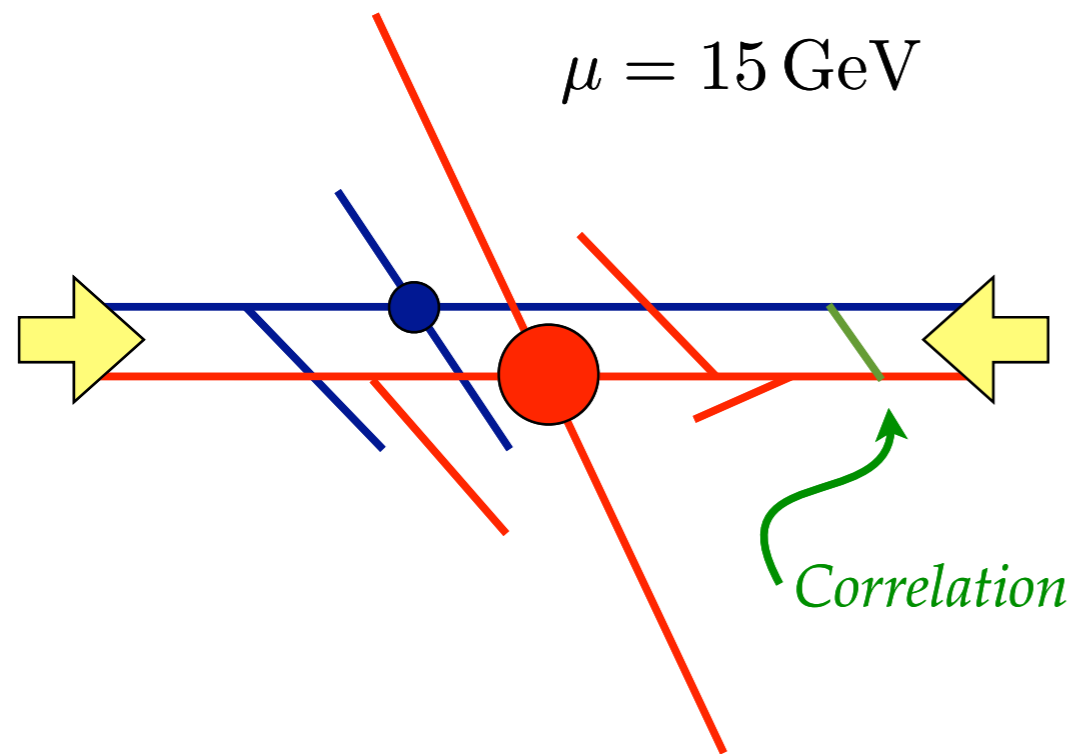
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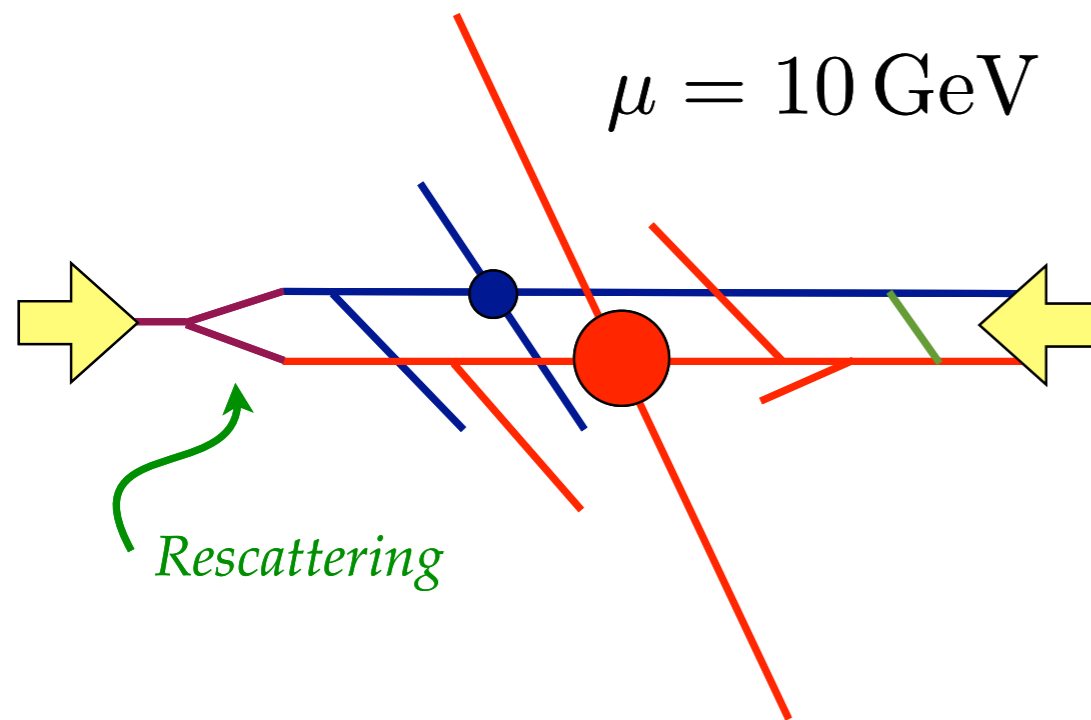
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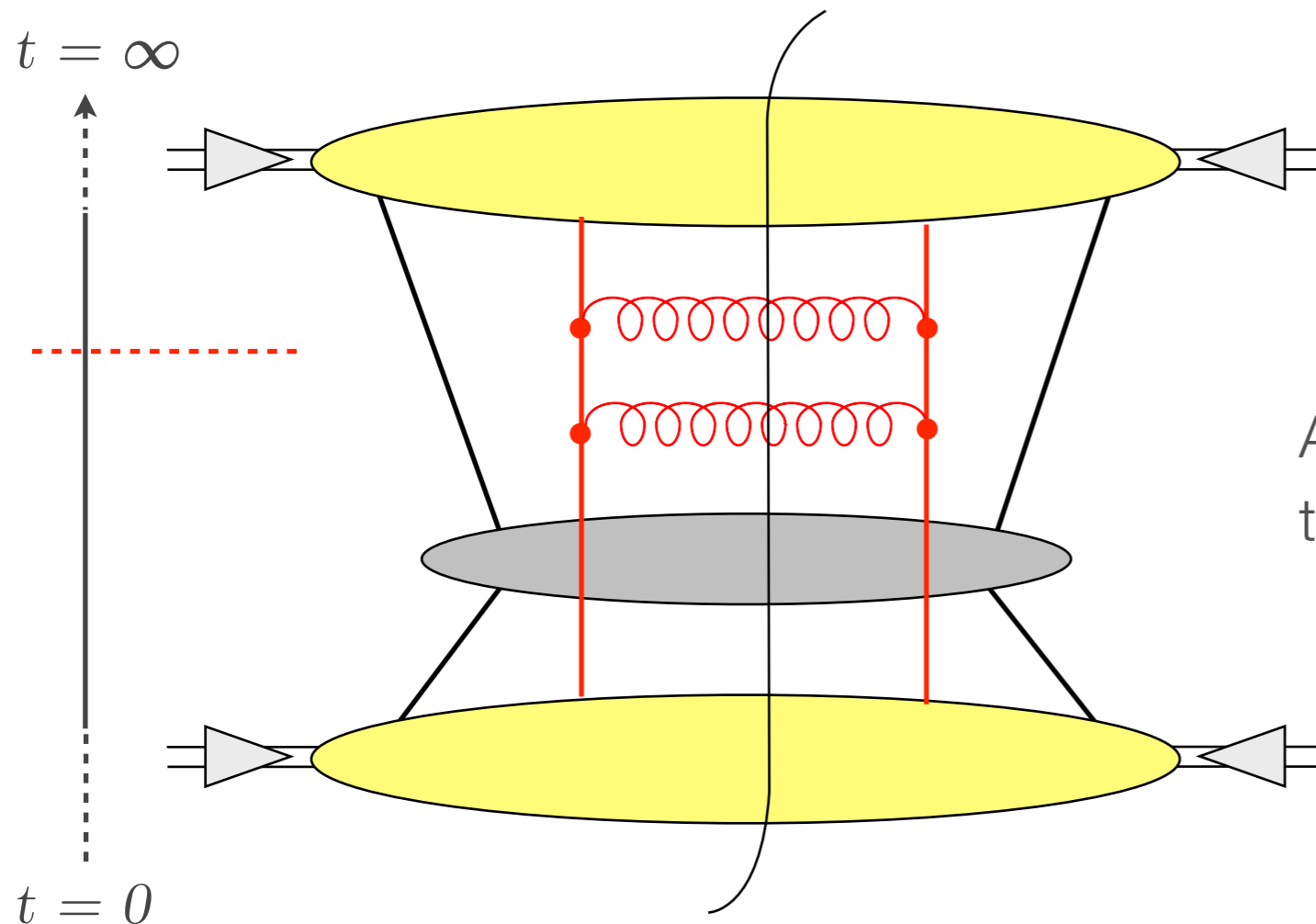
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Single radiations
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# Multi Parton Interaction



We need this at least at leading color level.

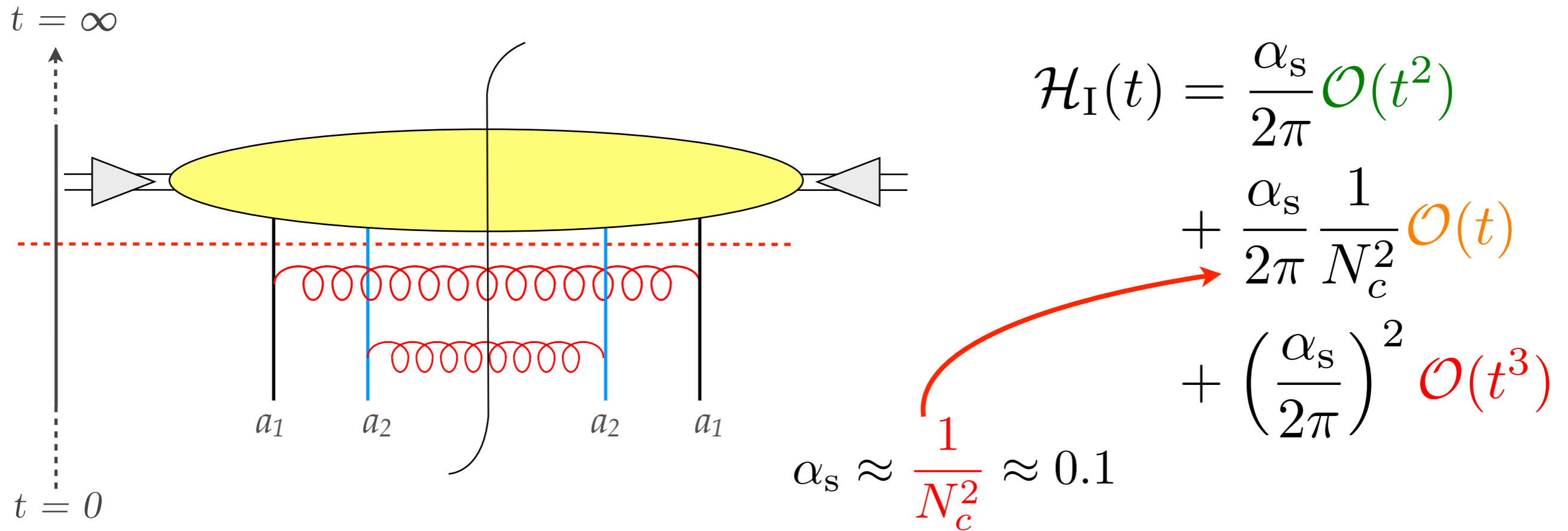
$$\mathcal{H}_{\text{MI}}(t) = \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{O}(e^t)$$

Actually the real scaling is weaker due to the *power suppression*:

$$e^{t-t_0} \sim \frac{\Lambda_{\text{QCD}}^2}{p_{\perp}^2}$$

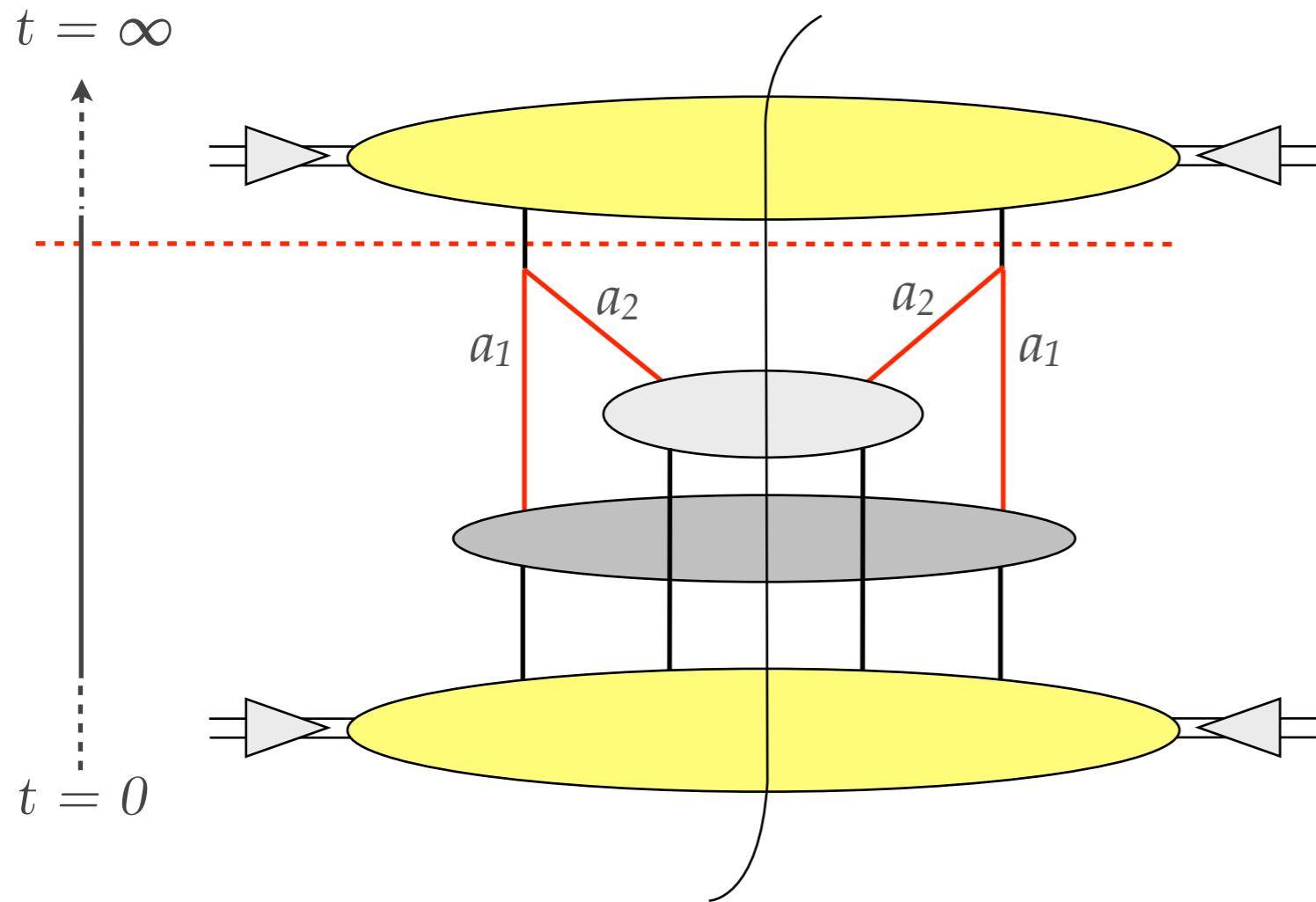
- This is important in the very small  $p_T$  regions and negligible in the large  $p_T$  regions but it is hard to tell how important in the intermediate region. **The cumulative effect could be sizable.**
- Important to note that this is an **NLO contributions**. Thus, compared to the standard shower this is also suppressed by an extra power of  $\alpha_s$ .
- Requires **multi parton PDF (mPDF)**.
- Implemented in **HERWIG & PYTHIA**. (No “proper” mPDF implemented.)

# Standard IRS



- This is the standard shower evolution. Adds LL and NLL contributions. Not power suppressed.
- Since the MPI kernel is NLO contribution we should consider the standard shower at NLO level as well. (Just to be systematic.)
- If we consider NLO terms then we need subleading color contributions, too.
- Adds correction to the primary interaction as well as to the MPI contributions.
- It is implemented **only at LO level** in **HERWIG & PYTHIA**.

# Rescattering



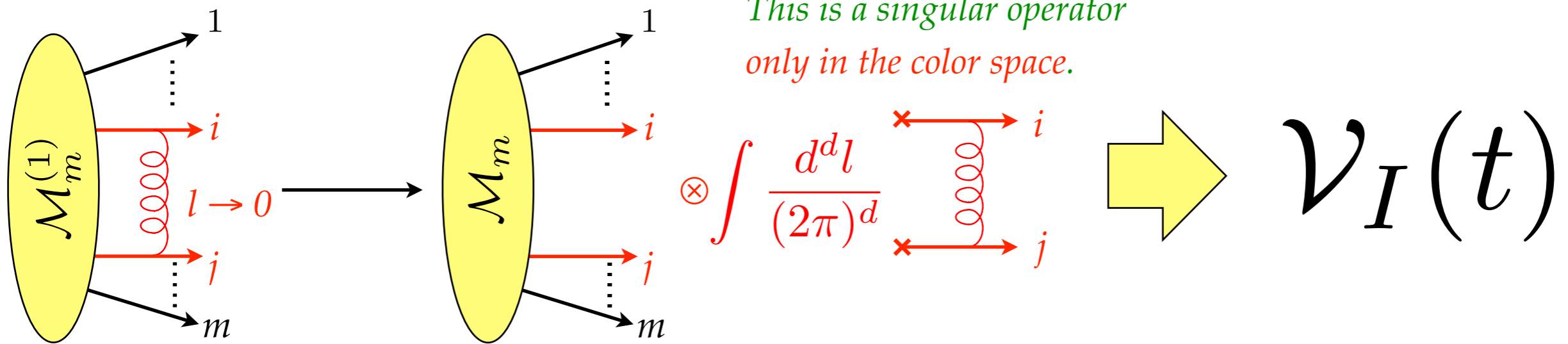
$$\mathcal{H}_{\text{RS}}(t) = \frac{\alpha_s}{2\pi} \mathcal{O}(t) + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{O}(t^2)$$

*This is the most problematic contribution*

$$\mathcal{V}_{\text{RS}}(t) = 0$$

- This operator can be applied on states with at least two chains. (They are **already power suppressed**.)
- No corresponding factorizable virtual contribution. ➡ **No associated Sudakov factor.**
- Only NLL contribution to the MPI terms.
- Some level it is implemented in **PYTHIA**.

# Virtual Contributions



In standard parton shower this operator is obtained from the **unitarity** condition

$$(1 | \mathcal{V}_I(t) = (1 | \mathcal{H}_I(t) \quad \Rightarrow \quad \text{Always real}$$

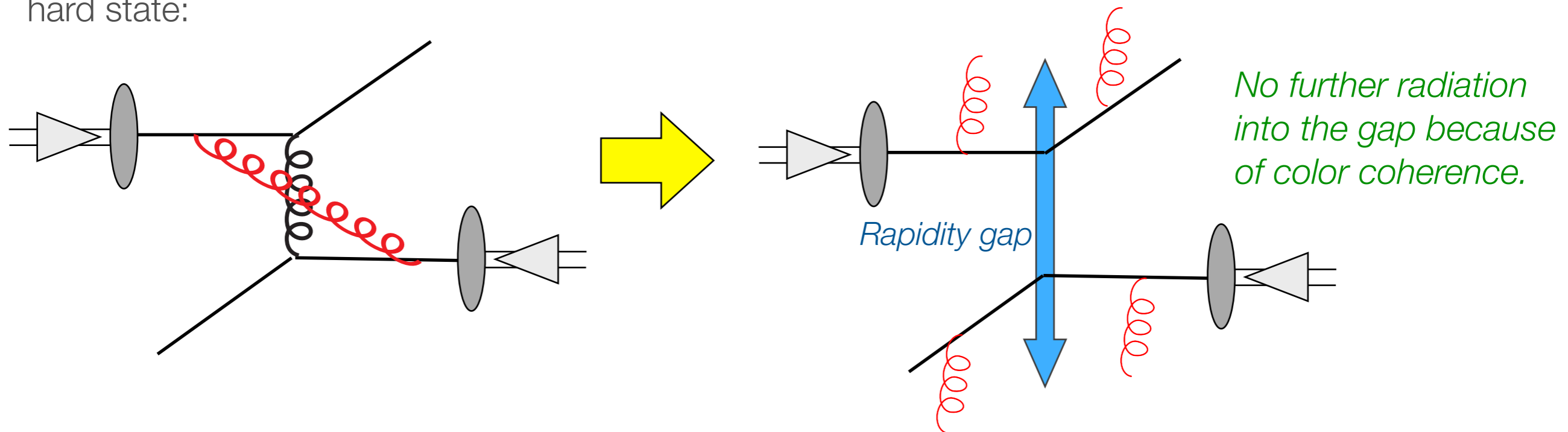
But it turns out that we have imaginary contribution from the virtual graphs

$$\int \frac{d^d l}{(2\pi)^d} \text{ [Loop Diagram] } \propto \mathcal{V}_I(t) + \underbrace{i\pi \tilde{\mathcal{V}}(t)}_{\text{Coulomb gluon}} \quad \text{and} \quad (1 | \tilde{\mathcal{V}}(t) = 0$$

*What can Coulomb gluon do?*

# Coulomb Gluon

1. Coulomb gluon changes the color configuration and the color flow. It is pure virtual contribution, thus it is unresolvable. *It does the same thing what color reconnection does.*
2. It always make color correlation between the two incoming partons. Let's consider a color octet hard state:



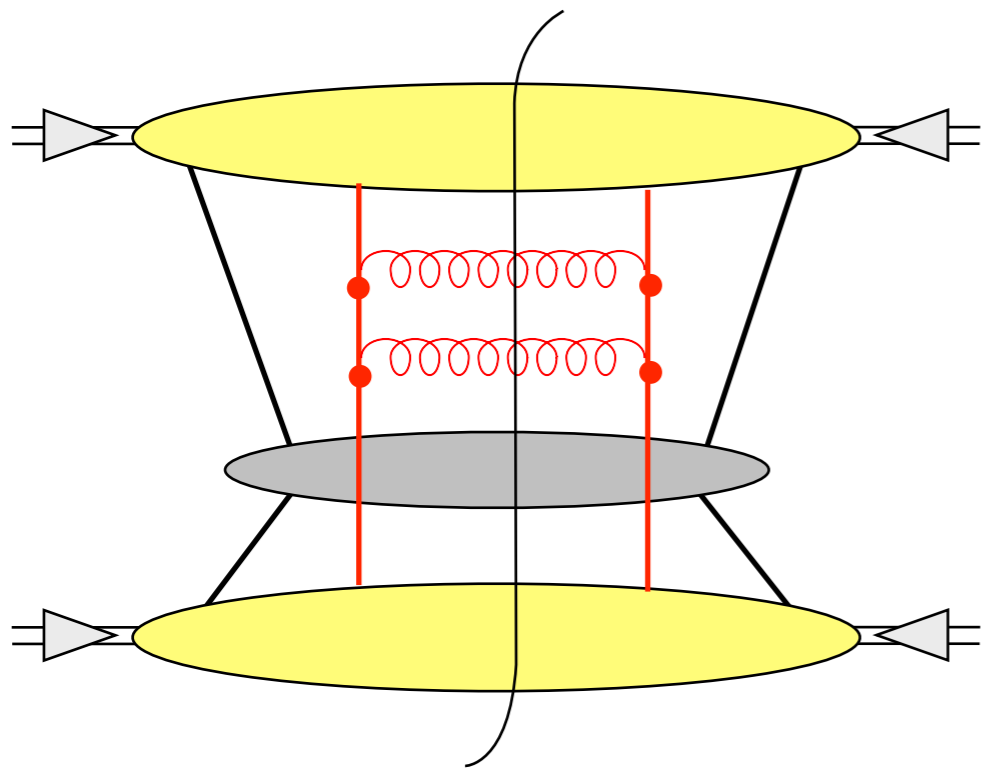
*This is a contribution to the diffractive events.*

3. Leads to "*Super Leading Logs*" in the case of some non-global observables.

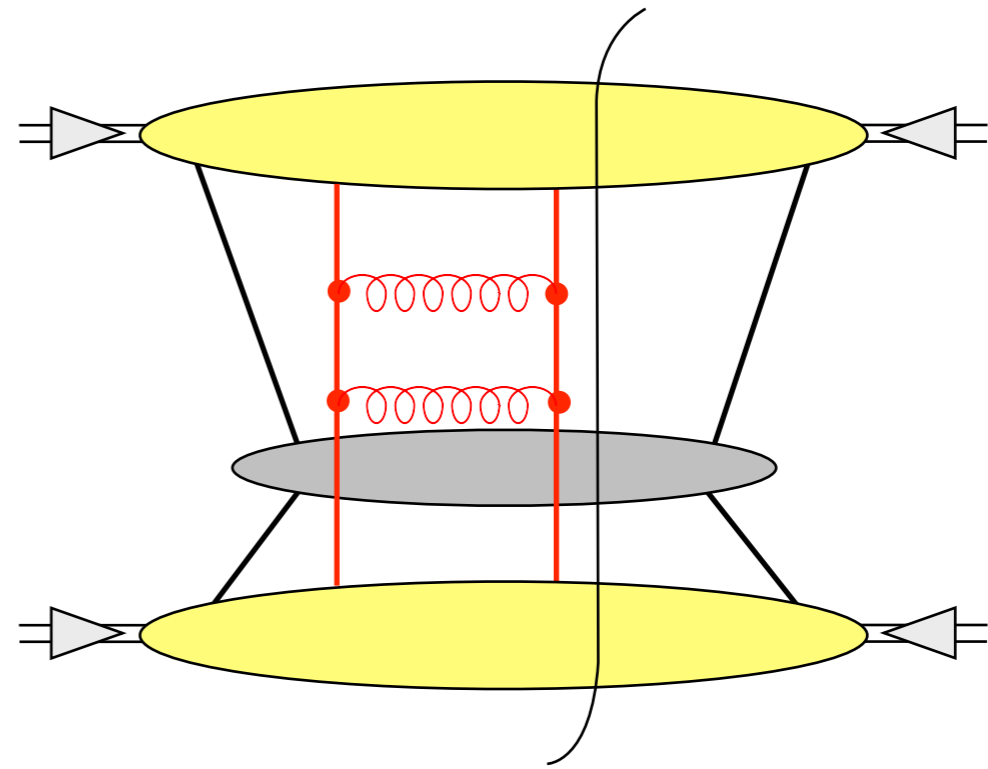
*Do we have Coulomb like contribution in the MPI virtual graphs?*

# MPI: Coulomb Gluon

In the MPI part the “resolvable” radiation comes from extra  $2 \rightarrow 2$  scattering. This is very singular in the low  $p_T$  region. This singularity must be cancelled by the corresponding virtual graphs.



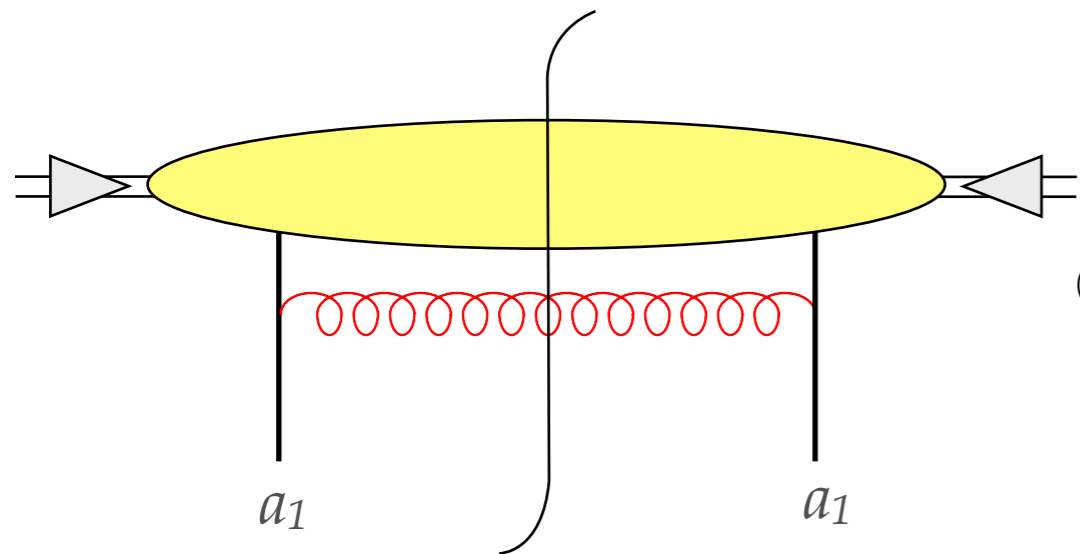
*Real  $2 \rightarrow 2$  scattering adds two extra jets*



Corresponding virtual graph.  
This is a forward elastic scattering contribution.  
It can produce Coulomb gluon term  $\Rightarrow$  *Color reconnection effect*

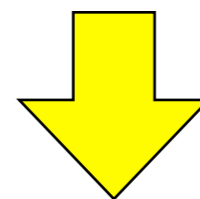
# Single Parton PDF

The PDF has a operator product definition



$$\propto 2p^+ \int \frac{dz^-}{2\pi} e^{ixz^- p^+} \langle p | \bar{q}(0) q(z) | p \rangle$$

*This expression is UV divergent and needed to be renormalized.*



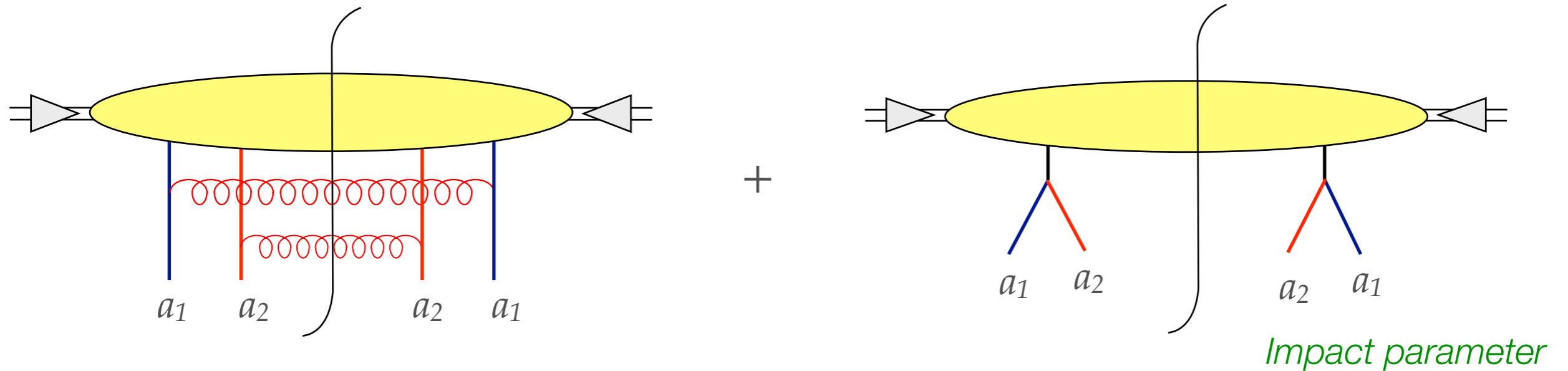
$$\mu \frac{d}{d\mu} f_{a/H}(x, \mu) = \sum_b [P_{a,b} \otimes f_{b/H}](x, \mu)$$

The UV singularity in the PDF corresponds to the IR singularity in hard part of the cross section. Everything is consistent.

*How does it work in the mPDF case?*



# Multi Parton PDF



$$\propto 2p^+ \int \frac{dz_2^-}{2\pi} e^{ix_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi} e^{ix_1 z_1^- p^+} \langle p | \bar{q}(-\frac{1}{2}z_2) \Gamma_2 q(\frac{1}{2}z_2) \bar{q}(y - \frac{1}{2}z_1) \Gamma_1 q(y + \frac{1}{2}z_1) | p \rangle$$

This operator is also UV divergent and needed to be renormalized. RGE provides the generalized DGLAP equation.

For  $y \neq 0$  we have a homogeneous DGLAP equation, there is no contribution from  $2 \rightarrow 4$  transitions

$$\frac{d}{dt} F(x_i, y) = P \otimes_{x_1} F + P \otimes_{x_2} F$$

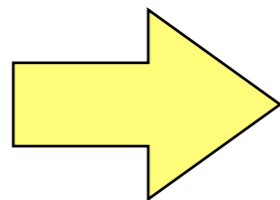
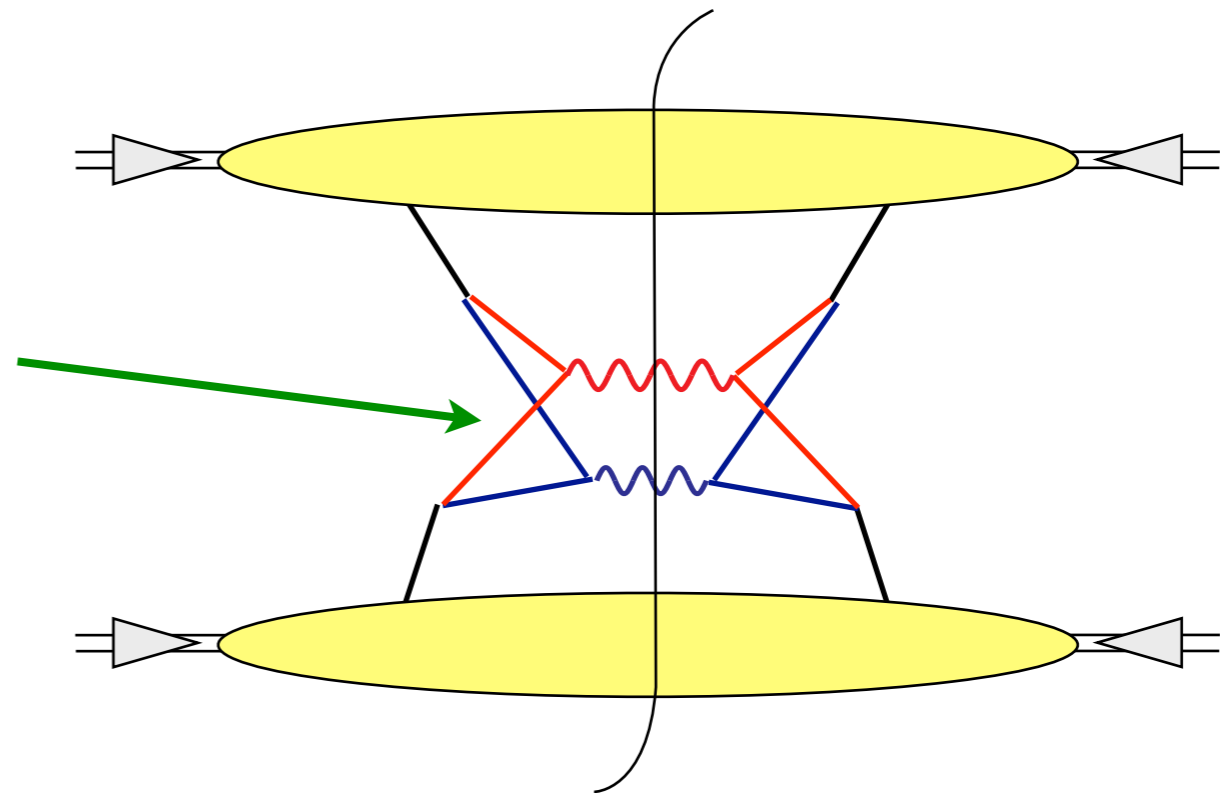
For  $\int dy F(x, y)$  we have contribution from  $2 \rightarrow 4$  transitions and leads to the well known inhomogeneous evolution equation.

# Multi Parton PDF

Let us study the  $2 \rightarrow 4$  transitions in the hard matrix elements. In this example we have double Z boson production

There is a 1-loop graph in this process. This loop integral is perfectly finite, there is *NO IR singularities*.

This tells we should **NOT** consider the  $2 \rightarrow 4$  transitions.



*Look like there is some inconsistency between the two approaches...*

# Conclusion

- Multiple Interaction is very complicated from theory point of view.
- There are MC tool available mostly based on some tunable models (Color reconnection, simple mPDF assumption,...)
- Some perturbative effects are not included in our MC (Coulomb gluon,...)
- Lack of factorization theorems

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## Wizard of **ID**

