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Multiparton distribution functions in perturbative QCD

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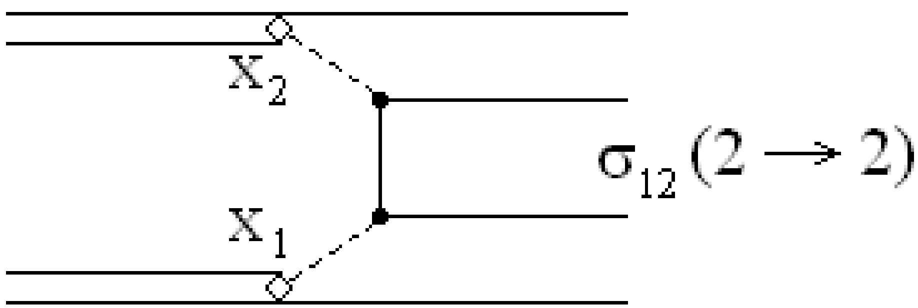
Phys. Lett. 113 B, 325 (1982) (with V.P. Shelest, G.M. Zinovjev);
Phys. Rev. D 68, 114012 (2003);
Phys. Lett. B 594, 171 (2004) (with V.L. Korotkikh);
Phys. Rev. D 81, 065014 (2010);
arXiv:1010.4874 [hep-ph]

This talk brings attention to what is knowable from perturbative QCD theory on multiparton distribution functions in the light of CDF and D0 measurements of the inclusive cross section for double parton scattering.

In a parton model (which was established in the quantum field theory in the leading logarithm approximation) the differential cross section for the two-jet process, for instance, is given by

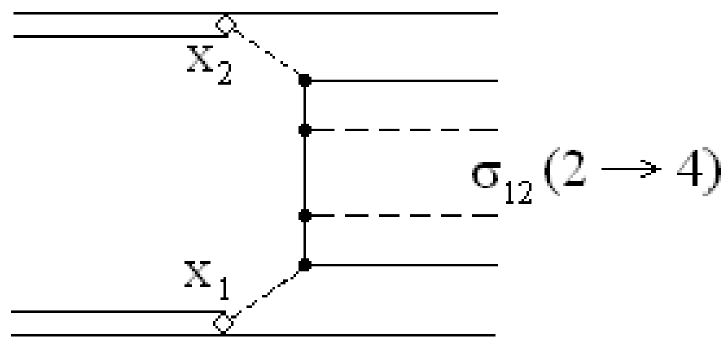
$$d\sigma = \sum_{q/g} d\sigma_{12}\ D_p(x_1, Q^2)\ D_{\bar{p}}(x_2, Q^2)$$



where $D_p(x_i, Q^2)$ **are the single quark/gluon momentum distributions at the scale** Q^2 **(determined by a hard process).**

And the differential cross section for the four-jet process is given by

$$d\sigma = \sum_{q/g} d\sigma_{12}(2 \to 4)\ D_p(x_1, Q^2)\ D_{\bar{p}}(x_2, Q^2)$$



But there is the possibility of the simultaneous interaction of two parton pairs which has been proposed since long

Landshoff, Polkinghorne, Goebel, Halzen, Scott,... (∼ 1980)

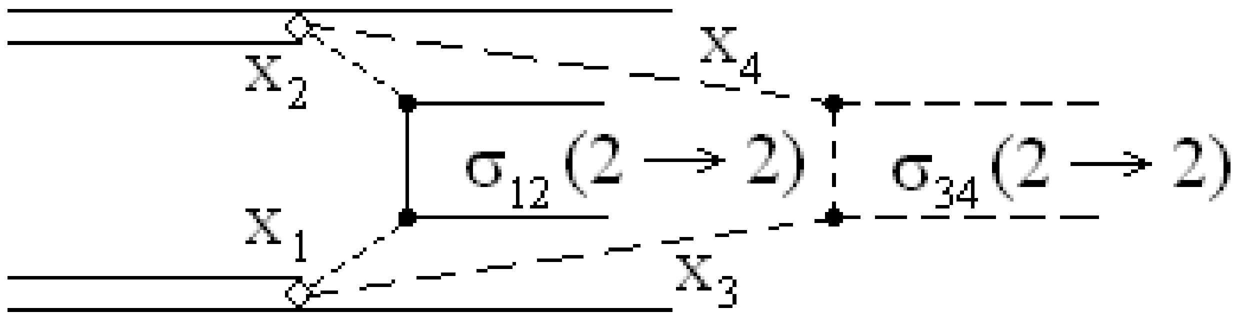
and from that has also developed in a number of works.

Takagi, Paver, Treleani, Humpert, Odorico, Sjostrand, van Zijl, Calucci, Del Fabbro, Stirling,...

CDF and D0 collaborations have MEASURED a large number of double parton scattering

The differential cross section for the four-jet process (due to the simultaneous interaction of two parton pairs) is given by

$$d\sigma = \sum_{q/g} \frac{d\sigma_{12}\ d\sigma_{34}}{2\sigma_{eff}}\ D_p(x_1, x_3)\ D_{\bar{p}}(x_2, x_4), \qquad (1)$$



where $d\sigma_{ij}$ stands for the two-jet cross section. The dimensional factor σ_{eff} in the denominator represents the total inelastic cross section which is an estimate of the size of the hadron.

With the effective cross section measured by CDF and DO

$$(\sigma_{eff})_{CDF} \simeq (\sigma_{eff})_{D0} \simeq 15 \textbf{ mb,}$$

one can estimate the transverse size r_p**, which is too small in comparison with the proton radius** R_p **extracted from** ep **elastic scattering experiments. The relatively small value of** $(\sigma_{eff})_{CDF}$ **with respect to the naive expectation was, in fact, considered as evidence of nontrivial correlation effects in transverse space (***Treleani ....***).**

But, apart from these correlations, the longitudinal momentum correlations can also exist and they are under consideration. The factorization ansatz is just applied to the two-parton distributions incoming in eq. (1):

$$\boldsymbol{D_p(x_i, x_j) \; = \; D_p(x_i, Q^2) \; D_p(x_j, Q^2) \; (1 - x_i - x_j),} \qquad \textbf{(2)}$$

where $D_p(x_i, Q^2)$ **are the single quark/gluon momentum distributions at the scale** Q^2 **(determined by a hard process).**

However many parton distributions satisfy the generalized DGLAP evolution equations (derived by *Kirschner; Shelest, Snigirev, Zinovjev***) as well as single parton distributions.**

Under certain initial conditions these generalized equations lead to solutions, which are identical with the jet calculus rules proposed originally for multiparton fragmentation functions by *Konishi-Ukawa-Veneziano* and are in some contradiction with the factorization hypothesis (2). Here one should note that at the parton level this is the strict assertion within the leading logarithm approximation.

After introducing the natural dimensionless variable

$$t = \frac{1}{2\pi b} \ln\left[1 + \frac{g^2(\mu^2)}{4\pi} b \ln\left(\frac{Q^2}{\mu^2}\right)\right] = \frac{1}{2\pi b} \ln\left[\frac{\ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}{\ln\left(\frac{\mu^2}{\Lambda^2_{QCD}}\right)}\right], \quad b = \frac{33 - 2n_f}{12\pi},$$

where $g(\mu^2)$ is the running coupling constant at the reference scale μ^2, n_f is the number of active flavours, Λ_{QCD} is the dimensional QCD parameter, the DGLAP equations read

$$\frac{dD_i^j(x,t)}{dt} = \sum_{j'} \int_x^1 \frac{dx'}{x'} D_i^{j'}(x',t) P_{j' \to j}\left(\frac{x}{x'}\right).$$

They describe the scaling violation of the parton distributions $D_i^j(x,t)$ inside a dressed quark or gluon $(i, j = q/g)$.

$$\frac{dD_i^{j_1 j_2}(x_1, x_2, t)}{dt} = \sum_{j_1{}'} \int_{x_1}^{1-x_2} \frac{dx_1{}'}{x_1{}'} D_i^{j_1{}' j_2}(x_1{}', x_2, t) P_{j_1{}' \to j_1}\left(\frac{x_1}{x_1{}'}\right)$$

$$+ \sum_{j_2{}'} \int_{x_2}^{1-x_1} \frac{dx_2{}'}{x_2{}'} D_i^{j_1 j_2{}'}(x_1, x_2{}', t) P_{j_2{}' \to j_2}\left(\frac{x_2}{x_2{}'}\right)$$

$$+ \sum_{j'} D_i^{j'}(x_1 + x_2, t) \frac{1}{x_1 + x_2} P_{j' \to j_1 j_2}\left(\frac{x_1}{x_1 + x_2}\right)$$

We will not write the kernels P explicitly and derive the generalized equations for two-parton distributions $D_i^{j_1 j_2}(x_1, x_2, t)$, representing the probability that in a dressed constituent i one finds two bare partons of types j_1 and j_2 with the given longitudinal momentum fractions x_1 and x_2 (referring to our papers for details), we give only their solutions via the convolution of single distributions

$$D_i^{j_1 j_2}(x_1, x_2, t) =$$

$$\sum_{j' j_1' j_2'} \int_0^t dt' \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \; D_i^{j'}(z_1 + z_2, t')$$

$$\frac{1}{z_1 + z_2} P_{j' \to j_1' j_2'}(\frac{z_1}{z_1 + z_2}) D_{j_1'}^{j_1}(\frac{x_1}{z_1}, t - t') D_{j_2'}^{j_2}(\frac{x_2}{z_2}, t - t').$$

This convolution coincides with the jet calculus rules as mentioned above and is the generalization of the well-known *Gribov-Lipatov* relation installed for single functions (the distribution of bare partons inside a dressed constituent is identical to the distribution of dressed constituents in the fragmentation of a bare parton in the leading logarithm approximation). The solution shows that the distribution of partons is *correlated* in the leading logarithm approximation.

Of course, it is interesting to find out the phenomenological issue of this parton level consideration. This can be done within the well-known factorization of soft and hard stages (physics of short and long distances). As a result the DGLAP equations describe the evolution of parton distributions in a hadron with t (Q^2), if one replaces the index i by index h only. However, the initial conditions for new equations at $t = 0$ $(Q^2 = \mu^2)$ are unknown a priori and must be introduced phenomenologically or must be extracted from experiments or some models dealing with physics of long distances [at the parton level: $D_i^j(x, t = 0) = \delta_{ij}\delta(x - 1)$; $D_i^{j_1 j_2}(x_1, x_2, t = 0) = 0$].

Nevertheless the solution of the generalized DGLAP evolution equations with the given initial condition may be written as before via the convolution of single distributions

$$D_h^{j_1 j_2}(x_1, x_2, t) = D_{h(QCD)}^{j_1 j_2}(x_1, x_2, t) +$$

$$\sum_{j_1' j_2'} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D_h^{j_1' j_2'}(z_1, z_2, 0) D_{j_1'}^{j_1}(\frac{x_1}{z_1}, t) D_{j_2'}^{j_2}(\frac{x_2}{z_2}, t) \ ,$$

where

$$D_{h(QCD)}^{j_1 j_2}(x_1, x_2, t) =$$

$$\sum_{j' j_1' j_2'} \int_0^t dt' \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \, D_h^{j'}(z_1 + z_2, t')$$

$$\frac{1}{z_1 + z_2} P_{j' \to j_1' j_2'}\left(\frac{z_1}{z_1 + z_2}\right) D_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, t - t'\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2}, t - t'\right) \quad (3)$$

are the dynamically correlated distributions given by perturbative QCD.

The reckoning for the unsolved confinement problem (physics of long distances) is the unknown nonperturbative two-parton correlation function $D_h^{j_1' j_2'}(z_1, z_2, 0)$ at some scale μ^2. One can suppose that this function is the product of two single-parton distributions times a momentum conserving factor at this scale μ^2:

$$D_h^{j_1 j_2}(z_1, z_2, 0) \; = \; D_h^{j_1}(z_1, 0) D_h^{j_2}(z_2, 0) \theta(1 - z_1 - z_2).$$

Then

$$D_h^{j_1 j_2}(x_1, x_2, t) = D_{h(QCD)}^{j_1 j_2}(x_1, x_2, t) + \theta(1 - x_1 - x_2)(D_h^{j_1}(x_1, t) D_h^{j_2}(x_2, t) +$$

$$\sum_{j_1' j_2'} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} D_h^{j_1'}(z_1, 0) D_h^{j_2'}(z_2, 0) D_{j_1'}^{j_1}(\frac{x_1}{z_1}, t) D_{j_2'}^{j_2}(\frac{x_2}{z_2}, t)[\theta(1 - z_1 - z_2) - 1]),$$

where

$$D_h^j(x, t) = \sum_{j'} \int_{x}^{1} \frac{dz}{z}\ D_h^{j'}(z, 0)\ D_{j'}^{j}(\frac{x}{z}, t)$$

is the solution of DGLAP eq. with the given initial condition $D_h^j(x, 0)$ for parton distributions inside a hadron expressed via distributions at the parton level.

This MAIN result shows that if the two-parton distributions are factorized at some scale μ^2, then the evolution violates this factorization *inevitably* at any different scale ($Q^2 \neq \mu^2$), apart from the violation due to the kinematic correlations induced by the momentum conservation (given by θ functions)

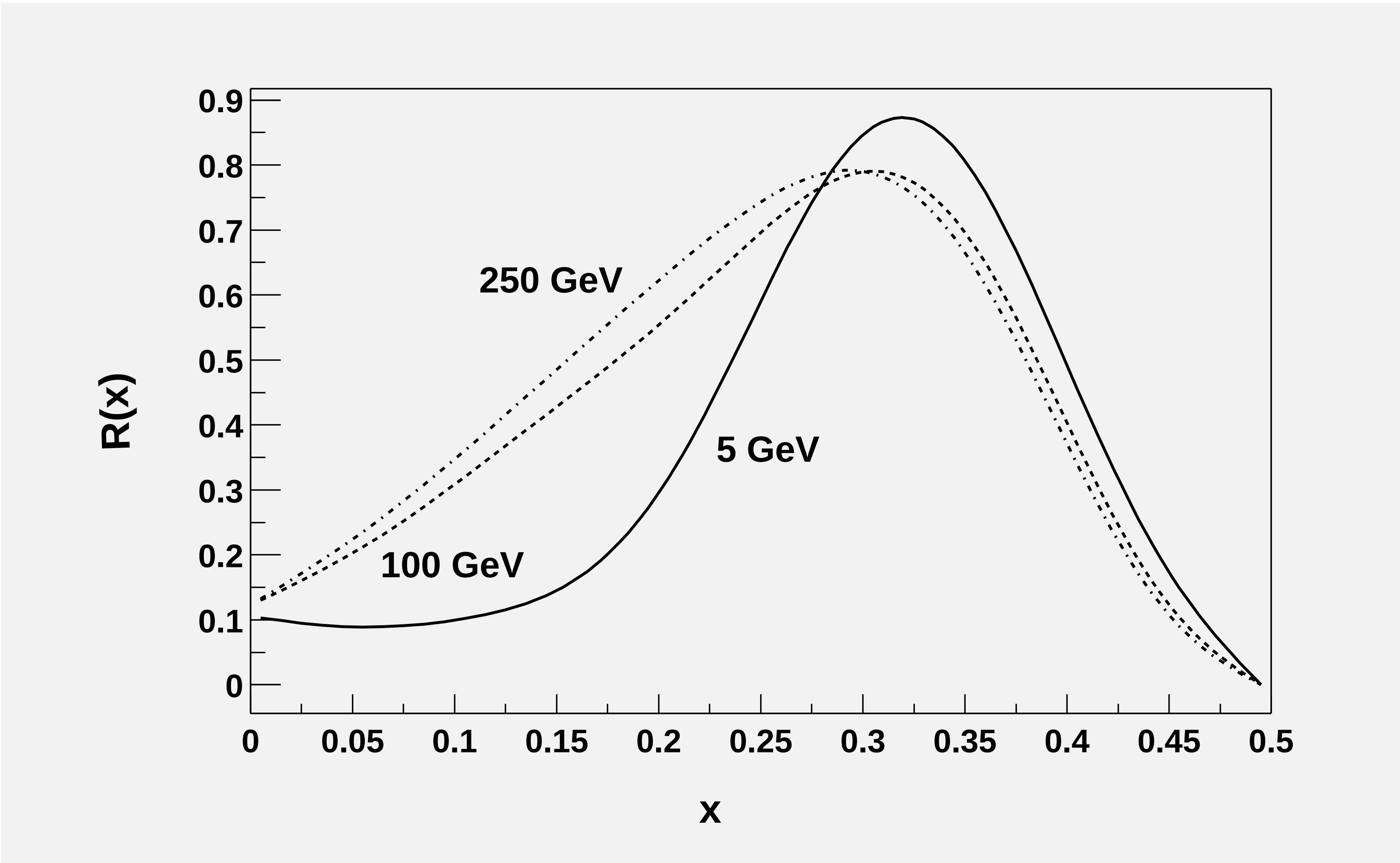
For a practical employment it is interesting to know the degree of this violation. We do it using the CTEQ fit for single distributions as an input in eq. (3). The nonperturbative initial conditions $D_h^j(x,0)$ **are specified in a parametrized form at a fixed low-energy scale** $Q_0 = \mu = 1.3$ **GeV. The particular function forms and the value of** Q_0 **are not crucial for the CTEQ global analysis at the flexible enough parametrization, which reads**

$$\boldsymbol{xD_p^j(x,0) \ = \ A_0^j x^{A_1^j}(1-x)^{A_2^j} e^{A_3^j x}(1+e^{A_4^j} x)^{A_5^j}.}$$

The independent parameters $A_0^j,\ A_1^j,\ A_2^j,\ A_3^j,\ A_4^j,\ A_5^j$ **for parton flavour combinations** $u_v \equiv u - \bar{u}$**,** $d_v \equiv d - \bar{d}$**,** g **and** $\bar{u} + \bar{d}$ **are given in Appendix A of work:** *J.Pamplin, et al., JHEP 0207 (2002) 012.*

The results of numerical calculations are presented on fig. for the ratio:

$$\boldsymbol{R(x,t) \ = \ (D_{p(QCD)}^{gg}(x_1,x_2,t)/D_p^g(x_1,t)D_p^g(x_2,t)(1-x_1-x_2)^2)|_{x_1=x_2=x}.}$$



R(x)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
250 GeV
5 GeV
100 GeV
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
x

One should note that the momentum conserving phase space factor $(1-x_1-x_2)^2$ is introduced instead of $(1-x_1-x_2)$ usually used. The reason is simple: this factor was introduced in eq. (2), generally speaking, “by hand” in order to “save” the momentum conservation law, i.e. in order to make the product of two single distributions is equal to zero smoothly at $x_1+x_2=1$. However the generalized QCD evolution equations demand higher power of $(1-x_1-x_2)$ at $x_1+x_2 \to 1$: only the phase space integrals give

$$\int_{x_1}^{1-x_2} dz_1 \int_{x_2}^{1-z_1} dz_2 \; = \; (1-x_1-x_2)^2/2.$$

In fact this power must depend on t increasing with its growth as this takes place for single distributions at $x \to 1$ Our numerical calculations support this assertion also: the power of $(1-x_1-x_2)$ for the perturbative QCD gluon-gluon correlations is higher than 2 and increases with $t(Q)$ as one can see from fig. However the introduced factor $(1-x_1-x_2)^2$ has not an influence practically on the ratio under consideration in the region of small x_1, x_2. And namely this region, in which multiple interactions can contribute to the cross section visibly, is interesting from experimental point of view.

Fig. shows that at the scale of CDF hard process ($\sim$ 5 GeV) the ratio $R(x)$ is nearly 10% and increases right up to 30% at the LHC scale ($\sim$ 100 GeV) for the longitudinal momentum fractions $x \leq 0.1$ accessible to these measurements.

For the finite longitudinal momentum fractions $x \sim \mathbf{0.2} \div \mathbf{0.4}$ the correlations are large right up to 90% . They become important in more and more x region with the growth of t in accordance with the predicted QCD asymptotic behaviour.

A non minor role of the QCD evolution of multiparton distribution functions has been demonstrated: (*E. Cattaruzza, A. Del Fabbro, D. Treleani, Phys. Rev. D 72 (2005) 034022*)

In the case of multiple production of W bosons with equal sign, the terms with correlations may represent a correction of the order of 40% of the cross sections, for pp collisions at 1 TeV c.m. energy, and a correction of the order of 20% at 14 TeV.

In the case of $b\bar{b}$ pairs the correction terms are of the order of 10-15% at 1 TeV and of the order of 5% at 14 TeV.

J.R. Gaunt, W.J. Stirling
JHEP 1003 (2010) 005:

DGLAP equations have been numerically integrated and a set of publicly available grids covering the ranges:

$$10^{-6} < x_1 < 1, \qquad 10^{-6} < x_2 < 1, \qquad 1 < Q^2 < 10^9 GeV$$

is given.

Possible manifestation of QCD evolution at the LHC:

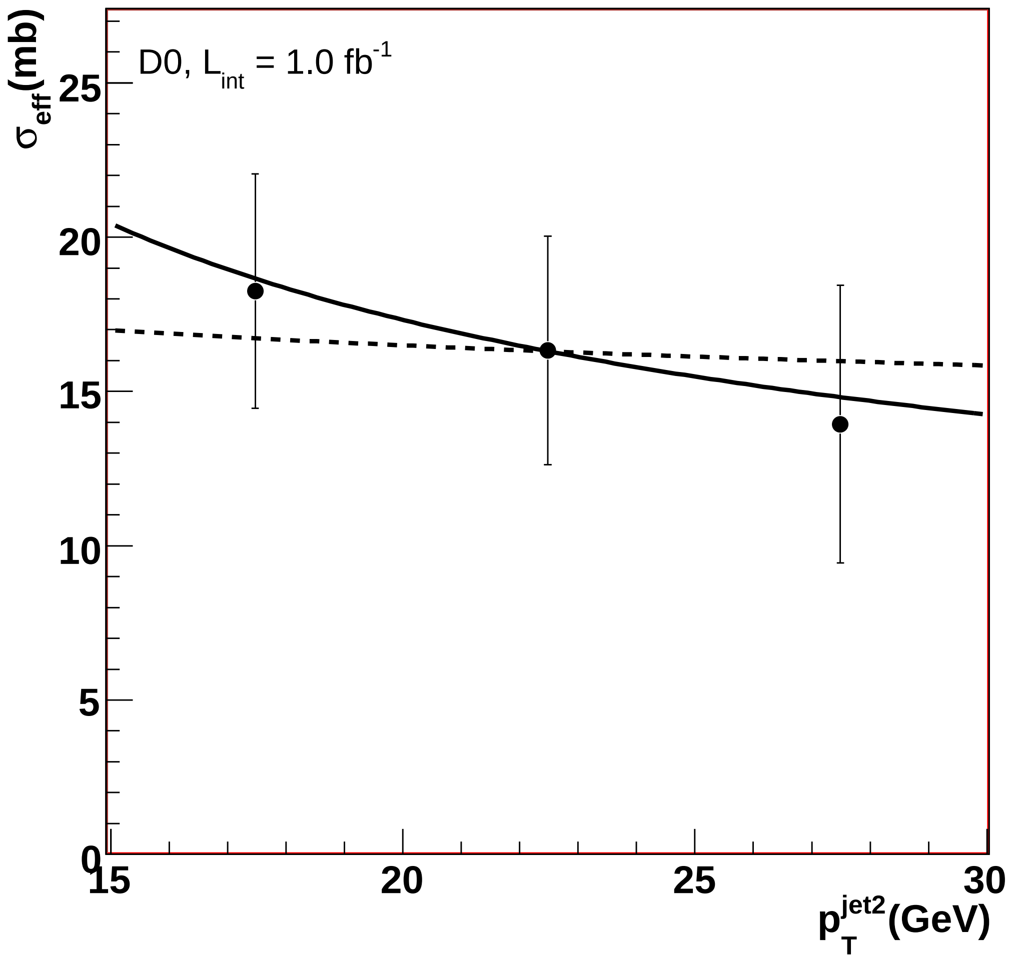
J.R. Gaunt, C.-H. Kom, A. Kulesza, W.J. Stirling
EPJ C 69 (2010) 53.

E. Maina
arXiv:1010.5674 [hep-ph].

DO Collaboration has measured σ_{eff} **at 3 different resolution scales**
Phys. Rev. D 81, 052012 (2010)

These results can be interpreted as a first inderect observation of the QCD evolution of double parton distributions

!? !?



Experimental extraction:

$$\frac{\sigma_{DPS}^{\gamma+3j}}{\sigma^{\gamma j}\sigma^{jj}} = [\sigma_{\rm eff}^{\rm exp}]^{-1}$$

Theoretical "prediction":

$$\sigma_{\rm eff}^{\rm exp} = \sigma_{\rm eff}^{0}[1 + k\ln(p_T^{\rm jet2}/p_{T0}^{\rm jet2}]^{-1}$$

inspired by the explicit expression for the correlation term and the evolution variable t ($k = 0.1$ (dashed line) and $k = 0.5$ (solid line))

Asymptotic behavior of double parton distribution functions:
arXiv:1010.4874 [hep-ph]

At x close to 1 these functions include the factors

$$(1 - x_1 - x_2) \quad (1 - x_1) \quad (1 - x_2)$$

with the exponents depending on parton types. These exponents are known at the parton level and can be calculated in principle at the hadron level fixing the asymptotic form of initial conditions near this kinematical boundary.

The two-parton distribution functions become practically uncorrelated in the kinematical range of relatively small longitudinal momentum fractions. The additional "factorization" contribution induced by evolution being suppressed by the initial gluon and quark multiplicities in comparison with the "genuine" factorization component (the solution of homogeneous equation) in the case of one slow ($x_1 \sim 0$) and one fast ($x_2 =$ finite) parton:

$$\frac{D_{h0}^{gj_2}(x_1, x_2, t)}{D_{h\text{fact}}^{gj_2}(x_1, x_2, t)}\Big|_{x_1 \to 0} \sim \frac{1}{M_h^g(0,0) + \frac{C_F}{N_c} \sum_q M_h^q(0,0)}.$$

OUTLOOK

It is interesting to study the double parton distribution functions beyond the leading logarithm approximation over Q^2:

- **two different scales (done,** *Gaunt, Stirling***)**
- **BFKL regime** $Q^2 = \mathrm{const}$ **,** $\ln(1/x) \to \infty$
- **colour glass condensate approach**
- **fixing relative transverse momentum or invariant mass of partons (DDT-formfactor ?** *Kirschner, Preprint TH 2823-CERN,1980* **(parton level only))**

m**-parton distributions:**

$$\frac{dD_i^{j_1\ldots j_m}(x_1,\ldots,x_m,t)}{dt} = \sum_{l=1}^{m}\sum_{j'}\int\limits_{x_l}^{1-x_1-\ldots-x_{l-1}-x_{l+1}-\ldots-x_m}\frac{dx'}{x'}\times$$

$$\times D_i^{j_1\ldots j_{l-1}j'j_{l+1}\ldots j_m}(x_1,\ldots,x_{l-1},x',x_{l+1},\ldots,x_m,t)P_{j'\to j_l}(\frac{x_l}{x'})$$

$$+\sum_{l=1}^{m}\sum_{p=l+1}^{m}\sum_{j'}\frac{1}{x_l+x_p}P_{j'\to j_l j_p}(\frac{x_l}{x_l+x_p})\times$$

$$\times D_i^{j_1\ldots j_{l-1}j'j_{l+1}\ldots j_{p-1}j_{p+1}\ldots j_m}(x_1,\ldots,x_{l-1},x_l+x_p,x_{l+1},\ldots,x_{p-1},x_{p+1},\ldots,x_m,t)$$

V.P. Shelest, A.M. Snigirev, and G.M. Zinovjev, Preprint ITP-83-46E, Kiev, 1983