2nd Lecture

When (some) QCD matters

- Flavor symmetries ⇒ Sebastien
- Heavy quark symmetry ⇒ Sebastien
- Operator product expansion for inclusive decays

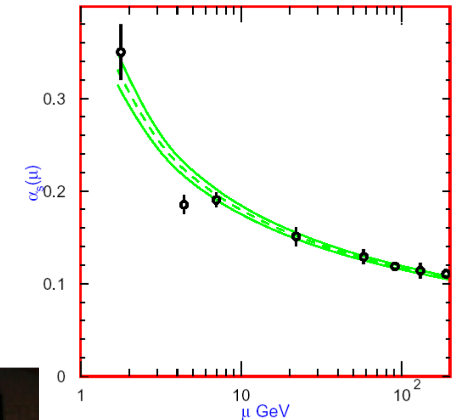
  Semileptonic b decays, $b \to s\gamma$, and friends
- Nonleptonic decays

  B decays to charm, Λ_b decay

  charmless B decays, different approaches

Interplay of electroweak and strong interactions

- How to learn about high energy physics from low energy hadronic processes?
- QCD coupling is scale dependent, $\alpha_s(m_B) \sim 0.2$



$$\alpha_s(\mu) = \frac{\alpha_s(\Lambda)}{1 + \frac{\alpha_s}{2\pi}\,\beta_0 \ln\frac{\mu}{\Lambda}}\,, \qquad \beta_0 = 11 - \frac{2}{3}\,n_f > 0$$

Nobel prize in 2004:

Politzer, Wilczek, Gross

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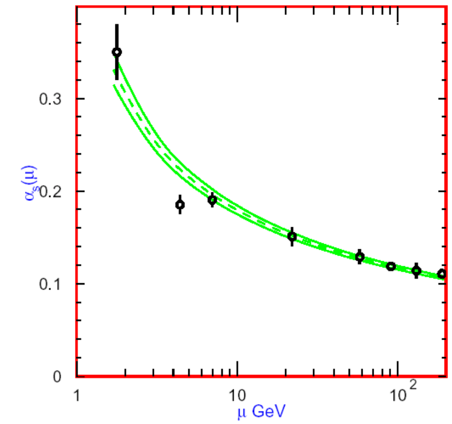


$$\alpha_s(\mu) = \frac{\alpha_s(\Lambda)}{1 + \dfrac{\alpha_s}{2\pi}\,\beta_0 \ln \dfrac{\mu}{\Lambda}}\,, \qquad \beta_0 = 11 - \frac{2}{3}\,n_f > 0$$

High energy (short distance): perturbation theory is useful

Low energy (long distance): QCD becomes nonperturbative $\Rightarrow$ It is usually very hard, if not impossible, to make precise calculations

- Solutions: New symmetries in some limits: effective theories (heavy quark, chiral)
  Certain processes are determined by short-distance physics
  Lattice QCD (bite the bullet — limited cases) $\Rightarrow$ Olivier
- Incalculable nonperturbative hadronic effects are often the limiting factor

Disentangling weak and strong interactions

- Want to learn about electroweak physics, but hadronic physics is nonperturbative

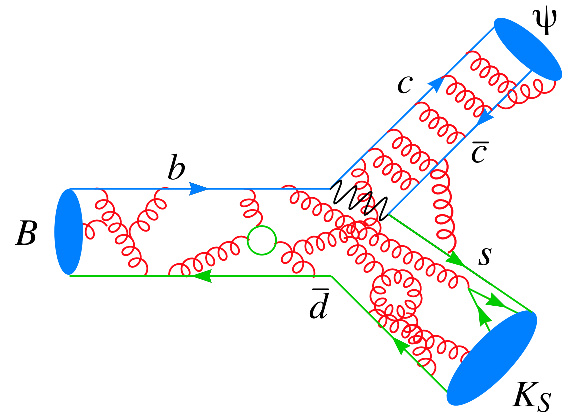
  Model independent continuum approaches:

- (1) Symmetries of QCD (exact or approximate)

  E.g.: $\sin 2\beta$ from $B \to J/\psi K_S$: amplitude not calculable

  Solution: CP symmetry of QCD ($\theta_{\rm QCD}$ can be neglected)

$$\langle \psi K_S | \mathcal{H} | B^0 \rangle = -\langle \psi K_S | \mathcal{H} | \overline{B}^0 \rangle \times [1 + \mathcal{O}(\alpha_s \lambda^2)]$$

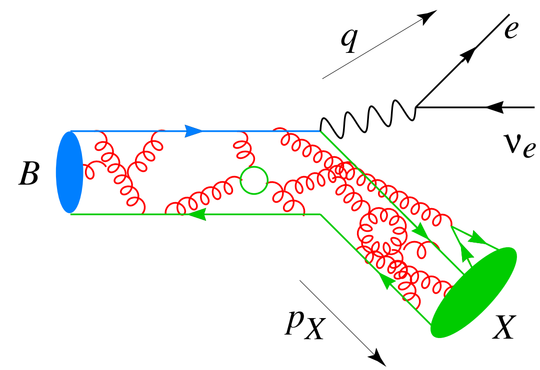


- (2) Effective field theories (separation of scales)

  E.g.: $|V_{cb}|$ and $|V_{ub}|$ from semileptonic B decays

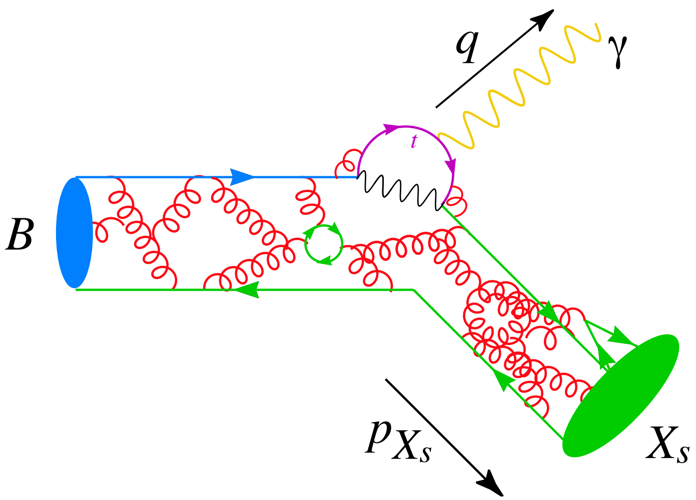
  Solution: Heavy quark expansions

$$\Gamma = |V_{cb}|^2 \times (\text{known factors}) \times [1 + \mathcal{O}(\Lambda_{\rm QCD}^2/m_b^2)]$$



Many relevant scales: $B \to X_s\gamma$

- Separate physics at: $(m_{t,W} \sim 100\,\mathrm{GeV}) \gg (m_b \sim 5\,\mathrm{GeV}) \gg (\Lambda \sim 0.5\,\mathrm{GeV})$



Inclusive decay:
$X_s = K^*,\ K^{(*)}\pi,\ K^{(*)}\pi\pi$, etc.

Diagrams with many gluons are crucial, resumming certain subset of them affects rate at factor-of-two level

Rate calculated at $< 10\%$ level, using several effective theories, renormalization group, operator product expansion... one of the most involved SM analyses

- Solution: Short distance dominated; unknown corrections suppressed by

$$\Gamma(B \to X_s\gamma) = [\text{known}] \times \left\{1 + \mathcal{O}\left(\alpha_s^3 \ln \frac{m_W}{m_b}, \frac{\Lambda_{\mathrm{QCD}}^2}{m_{b,c}^2}, \frac{\alpha_s \Delta m_c}{m_b}\right)\right\}$$

Some caveats

- Lot at stake: theoretical tools for semileptonic and rare decays are the same
  - Measurements of CKM elements
  - Better understanding of hadronic physics improves sensitivity to new physics

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- For today's talk: [strong interaction] model independent

  $\equiv$ theor. uncertainty suppressed by small parameters

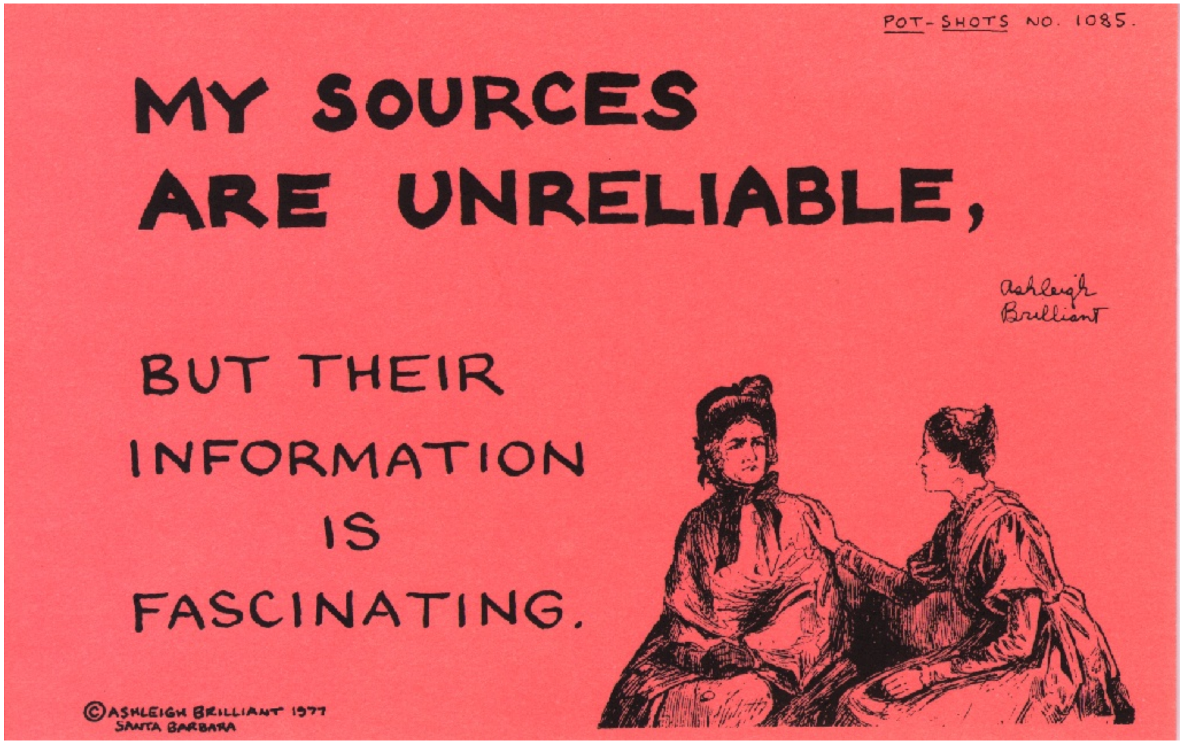
  ... so theorists argue about $\mathcal{O}(1)\times$(small numbers) instead of $\mathcal{O}(1)$ effects

- Most of the progress have come from expanding in powers of $\Lambda/m_Q,\ \alpha_s(m_Q)$

  ... a priori not known whether $\Lambda \sim 200\,\mathrm{MeV}$ or $\sim 2\,\mathrm{GeV}$ $(f_\pi, m_\rho, m_K^2/m_s)$

  ... need experimental guidance to see how well the theory works

The name of the game



The SM shows impressive consistency — even by Stockholm standards

Only robust deviations from model independent theory are likely to be interesting

(2σ: 50 theory papers 3σ: 200 theory papers 5σ: strong sign of effect)

Heavy quark symmetry

$\Rightarrow$ Sebastien

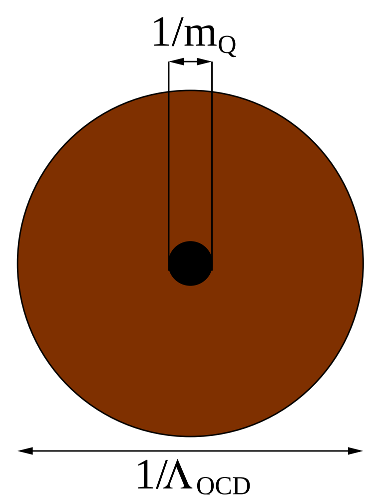
Heavy quark symmetry

- $Q\,\overline{Q}$: positronium-type bound state, perturbative in the $m_Q \gg \Lambda_{\rm QCD}$ limit
- $Q\,\bar{q}$: wave function of the light degrees of freedom
  ("brown muck") insensitive to spin and flavor of Q

B meson is a lot more complicated than just a $b\,\bar{q}$ pair

In the $m_Q \gg \Lambda_{\rm QCD}$ limit, the heavy quark acts as a static color source with fixed four-velocity v^μ

$\Rightarrow SU(2n)$ heavy quark spin-flavor symmetry at fixed v^μ



- Similar to atomic physics: $(m_e \ll m_N)$

1. Flavor symmetry $\sim$ isotopes have similar chemistry [Ψ_e independent of m_N]
2. Spin symmetry $\sim$ hyperfine levels almost degenerate [$\vec{s}_e - \vec{s}_N$ interaction $\to 0$]

Spectroscopy of heavy-light mesons

- In $m_Q \gg \Lambda_{\rm QCD}$ limit, spin of the heavy quark is a good quantum number, and so is the spin of the light d.o.f., since $\vec{J} = \vec{s}_Q + \vec{s}_l$ and

$$\left.\begin{array}{r} \text{angular momentum conservation: } [\vec{J}, \mathcal{H}] = 0 \\ \text{heavy quark symmetry: } [\vec{s}_Q, \mathcal{H}] = 0 \end{array}\right\} \Rightarrow [\vec{s}_l, \mathcal{H}] = 0$$

- For a given s_l, two degenerate states:

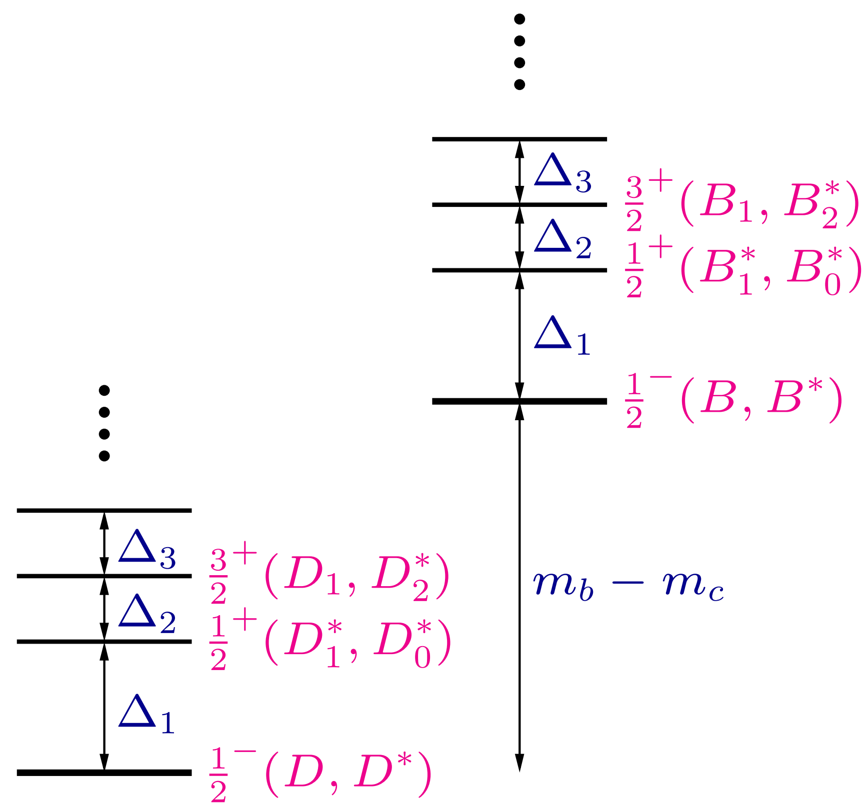
$$J_\pm = s_l \pm \tfrac{1}{2}$$

$\Rightarrow \Delta_i = \mathcal{O}(\Lambda_{\rm QCD})$ — same in B and D sector

Doublets are split by order $\Lambda^2_{\rm QCD}/m_Q$, e.g.:

$m_{D^*} - m_D \simeq 140$ MeV

$m_{B^*} - m_B \simeq 45$ MeV



Aside: a puzzle

- Since vector–pseudoscalar mass splitting $\propto 1/m_Q$, expect: $m_V^2 - m_P^2 =$ const.

  Experimentally:

$$m_{B^*}^2 - m_B^2 = 0.49\,\mathrm{GeV}^2$$

$$m_{B_s^*}^2 - m_{B_s}^2 = 0.50\,\mathrm{GeV}^2$$

$$m_{D^*}^2 - m_D^2 = 0.54\,\mathrm{GeV}^2$$

$$m_{D_s^*}^2 - m_{D_s}^2 = 0.58\,\mathrm{GeV}^2$$

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$$m_{D_s^*}^2 - m_{D_s}^2 = 0.58\,\mathrm{GeV}^2$$

$$m_{K^*}^2 - m_K^2 = 0.55\,\mathrm{GeV}^2$$

$$m_\rho^2 - m_\pi^2 = 0.57\,\mathrm{GeV}^2$$

- The HQS argument relies on $m_Q \gg \Lambda_{\mathrm{QCD}}$, so something more has to go on...

- It's not only important to test how a theory works, but also how it breaks down!

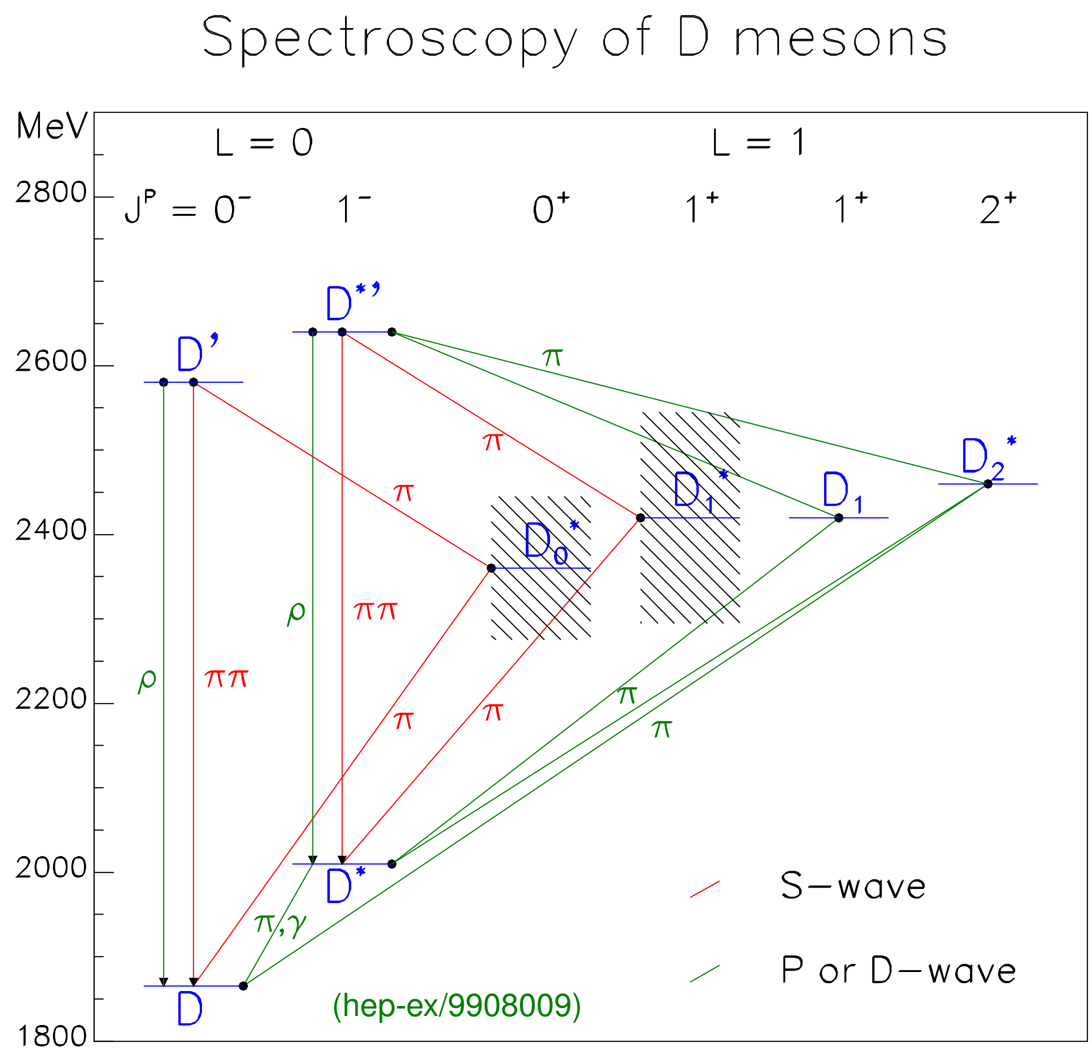
Successes in charm spectrum

- D_1 is narrow: S-wave $D_1 \to D^* \pi$ amplitude allowed by angular momentum conservation, but forbidden in the $m_Q \to \infty$ limit by heavy quark spin symmetry

- Mass splittings of orbitally excited states is small:

$$m_{D_2^*} - m_{D_1} = 37\,\mathrm{MeV} \ll m_{D^*} - m_D$$

vanishes in the quark model, since it arise from $\langle \vec{s}_Q \cdot \vec{s}_{\bar{q}}\, \delta^3(\vec{r}\,) \rangle$



Aside: strong decays of D_1 and D_2^*

- The strong interaction Hamiltonian conserves the spin of the heavy quark and the light degrees of freedom separately

$(D_1, D_2^*) \to (D, D^*)\pi$ — four amplitudes related by heavy quark spin symmetry

$$\Gamma(j \to j'\,\pi) \propto (2s_l+1)(2j'+1) \left| \begin{Bmatrix} L & s_l' & s_l \\ \frac{1}{2} & j & j' \end{Bmatrix} \right|^2$$

Multiplets have opposite parity $\Rightarrow$ π must be in $L=2$ partial wave

$\Gamma(D_1 \to D\pi)$	:	$\Gamma(D_1 \to D^*\pi)$	:	$\Gamma(D_2^* \to D\pi)$	:	$\Gamma(D_2^* \to D^*\pi)$
0	:	1	:	$\frac{2}{5}$	:	$\frac{3}{5}$
0	:	1	:	2.3	:	0.92

- Last line includes large $|p_\pi|^5$ HQS violation from phase space, which changes $\Gamma(D_2^* \to D\pi)/\Gamma(D_2^* \to D^*\pi)$ from $2/3$ to 2.5 (data: 2.3 ± 0.6)

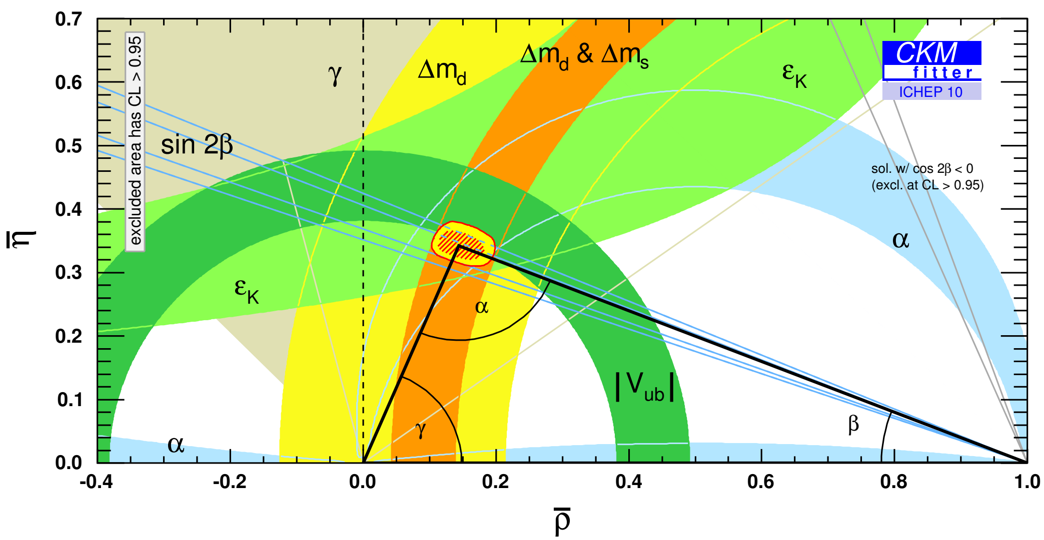
[Note: prediction for ratio of D_1 and D_2^* total widths works less well]

Semileptonic and rare B decays

$|V_{ub}|$ is the dominant uncertainty of the side of the UT opposite to β

$|V_{ub}|$ is crucial for comparing tree-dominated and loop-mediated processes

Error of $|V_{cb}|$ is a large part of the uncertainty in the ϵ_K constraint, and in $K \to \pi\nu\bar{\nu}$ when it's measured



Rare $b \to s\gamma$, $s\,\ell^+\ell^-$, and $s\,\nu\bar{\nu}$ decays are sensitive probes of the Standard Model

Exclusive $B \to D^{(*)}\ell\bar{\nu}$ decay

- In the $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, configuration of brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin

- On a time scale $\ll \Lambda_{\rm QCD}^{-1}$ weak current changes $b \to c$

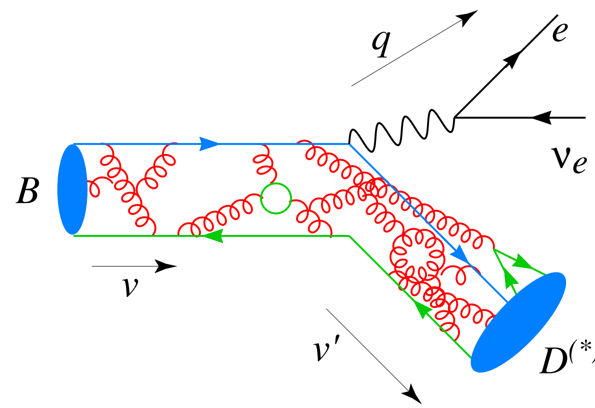
  i.e.: $\vec{p}_b \to \vec{p}_c$ and possibly $\vec{s}_Q$ flips

  In $m_{b,c} \gg \Lambda_{\rm QCD}$ limit brown muck only feels $v_b \to v_c$

  Form factors independent of Dirac structure of weak current $\Rightarrow$ all form factors related to a single function of $w = v \cdot v'$, the Isgur-Wise function, $\xi(w)$

  $\Uparrow$

  Contains all nonperturbative low-energy hadronic physics



- $\xi(1) = 1$, because at "zero recoil" configuration of brown muck not changed at all

$B \to D^{(*)}\ell\bar\nu$ form factors

- Lorentz invariance $\Rightarrow$ 6 form factors

$$\langle D(v')|V_\nu|B(v)\rangle = \sqrt{m_B m_D}\,\left[h_+\,(v+v')_\nu + h_-\,(v-v')_\nu\right]$$

$$\langle D^*(v')|V_\nu|B(v)\rangle = i\sqrt{m_B m_{D^*}}\,h_V\,\epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}v'^\beta v^\gamma$$

$$\langle D(v')|A_\nu|B(v)\rangle = 0$$

$$\langle D^*(v')|A_\nu|B(v)\rangle = \sqrt{m_B m_{D^*}}\,\left[h_{A_1}\,(w+1)\epsilon^*_\nu - h_{A_2}\,(\epsilon^*\cdot v)v_\nu - h_{A_3}\,(\epsilon^*\cdot v)v'_\nu\right]$$

$V_\nu = \bar c\gamma_\nu b,\quad A_\nu = \bar c\gamma_\nu\gamma_5 b,\quad w \equiv v\cdot v' = \dfrac{m_B^2+m_D^2-q^2}{2m_B m_D},$ and $h_i = h_i(w,\mu)$

- In $m_Q \gg \Lambda_{\rm QCD}$ limit, up to corrections suppressed by α_s and $\Lambda_{\rm QCD}/m_{c,b}$

$$h_- = h_{A_2} = 0\,, \qquad h_+ = h_V = h_{A_1} = h_{A_3} = \xi(w)$$

The α_s are corrections calculable

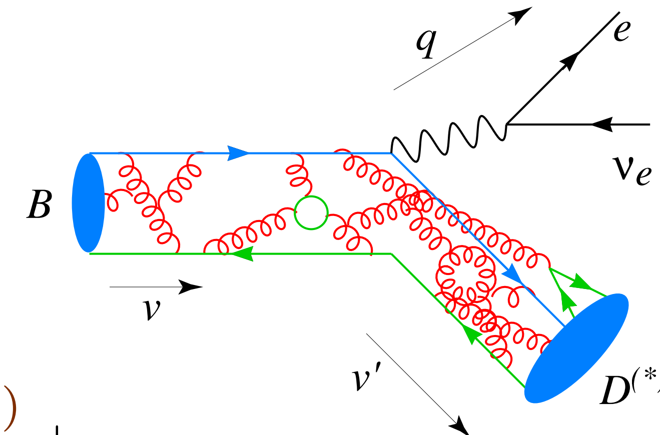
$\Lambda_{\rm QCD}/m_{c,b}$ corrections is where model dependence enters

$|V_{cb}|$ from $B \to D^{(*)}\ell\bar\nu$

- Extract $|V_{cb}|$ from $w \equiv v \cdot v' = (m_B^2 + m_D^2 - q^2)/(2m_B m_D) \to 1$ limit of the rate

$$\frac{\mathrm{d}\Gamma(B \to D^{(*)}\ell\bar\nu)}{\mathrm{d}w} = (\dots)\,(w^2-1)^{3/2(1/2)}\,|V_{cb}|^2\,\mathcal{F}^2_{(*)}(w)$$

$w \equiv v \cdot v'$ Isgur-Wise function $+ \dots$



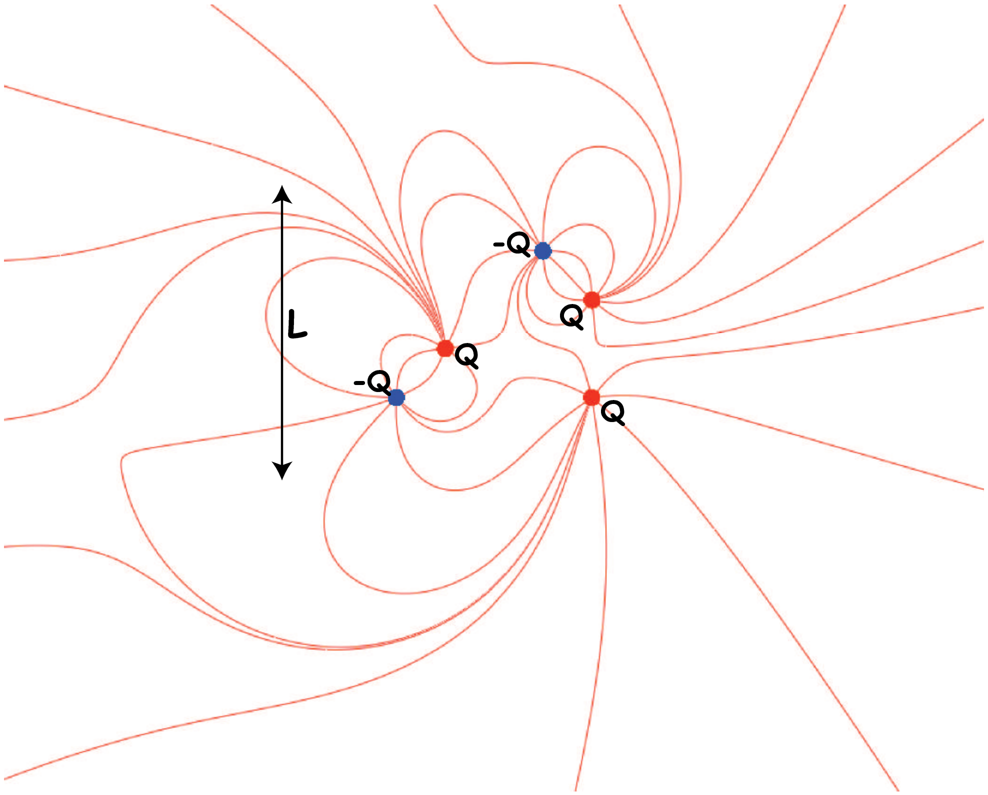
$$\mathcal{F}(1) = 1_{\mathrm{Isgur-Wise}} + 0.02_{\alpha_s,\alpha_s^2} + \frac{\text{(lattice or models)}}{m_{c,b}} + \dots$$

$$\mathcal{F}_*(1) = 1_{\mathrm{Isgur-Wise}} - 0.04_{\alpha_s,\alpha_s^2} + \frac{0_{\mathrm{Luke}}}{m_{c,b}} + \frac{\text{(lattice or models)}}{m_{c,b}^2} + \dots$$

- Lattice QCD: $\mathcal{F}_*(1) = 0.921 \pm 0.024$, $\mathcal{F}(1) = 1.074 \pm 0.024$ [arXiv:0808.2519, hep-lat/0409116]
- Need constraints on shape to fit [Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert]
- Need some understanding of decays to higher mass X_c states (backgrounds)
- Data: $|V_{cb}\,\mathcal{F}_*(1)| = (35.75 \pm 0.42) \times 10^{-3}$, $|V_{cb}\,\mathcal{F}(1)| = (42.3 \pm 1.5) \times 10^{-3}$ [HFAG]
  [note: χ^2/dof $= 39.6/21\ (56.9/21)$, CL $= 0.8\%\ (4\mathrm{E}{-5})$]

Heavy quark expansion

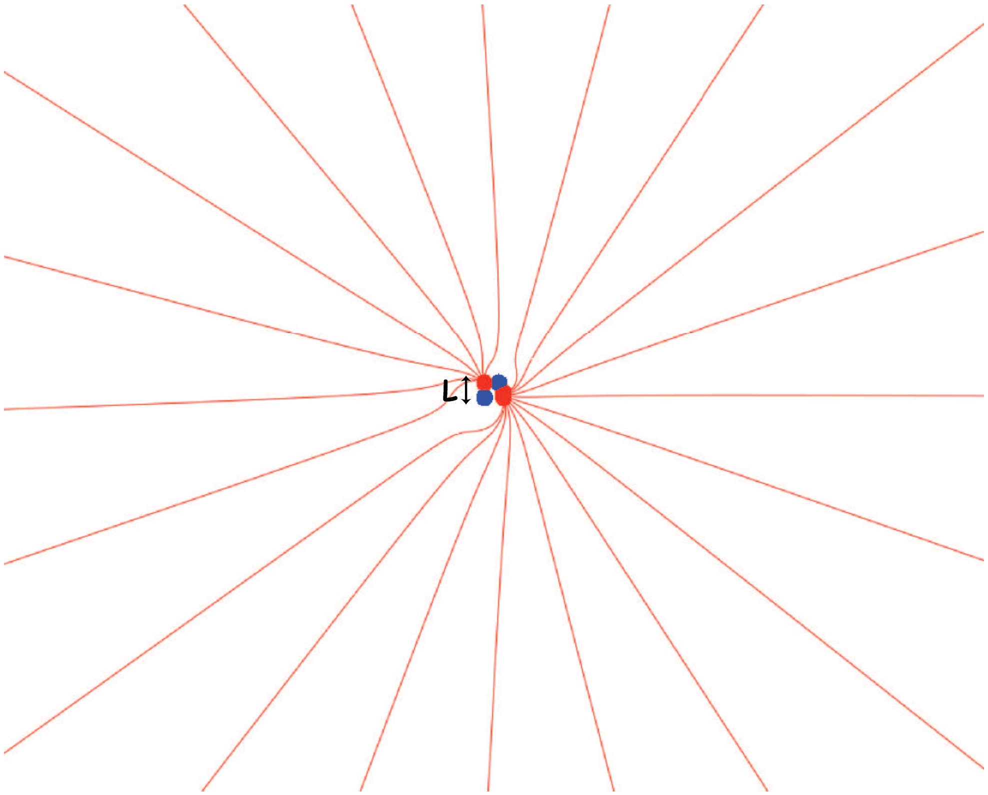
The multipole expansion



Physics at $r \sim L$ is complicated

Depends on the details of the charge distribution

The multipole expansion



Physics at $r \gg L$ is much simpler

Charge distribution characterized by total charge, q

Details suppressed by powers of L/r, and can be parameterized in terms of $p_i,\ Q_{ij},\ \ldots$

Simplifications occur due to separating physics at different distance scales

- Complicated charge distribution can be replaced by a point source with additional interactions (multipoles) — underlying idea of effective theories

The multipole expansion (cont.)

- Potential: $$V(x) = q\,\frac{1}{r} + p_i\,\frac{x_i}{r^3} + \frac{1}{2}\,Q_{ij}\,\frac{x_i x_j}{r^5} + \dots$$

  Short distance quantities: $q = \int \rho(x)\,\mathrm{d}^3x, \quad p_i = \int x_i\,\rho(x)\,\mathrm{d}^3x,$ etc.

  Long distance quantities: $\left\langle \frac{1}{r} \right\rangle, \quad \left\langle \frac{x_i}{r^3} \right\rangle, \quad \left\langle \frac{x_i x_j}{r^5} \right\rangle,$ etc.

- Higher multipoles: new interactions from "integrating out" short distance physics

- Useful tool independent of the fact whether we know the underlying theory or not

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- Any theory at momentum $p \ll M$ can be described by an effective Hamiltonian

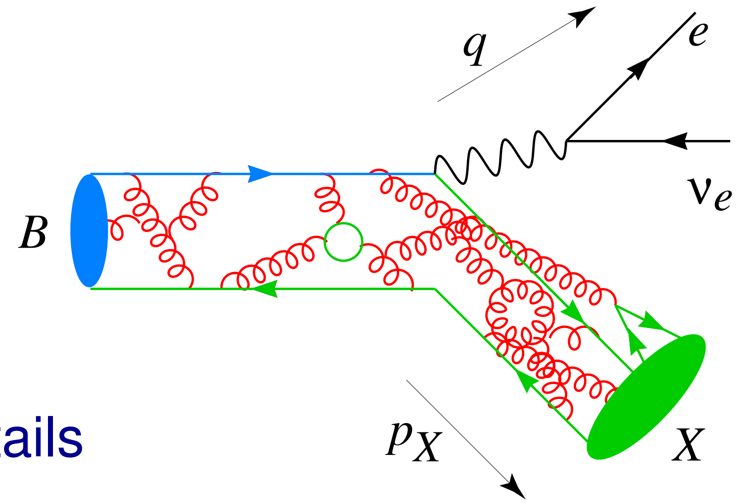
$$H_{\text{eff}} = H_0 + \sum_i \frac{C_i}{M^{n_i}}\,O_i$$

$M \to \infty$ limit + corrections with well-defined power counting
H_0 may have more symmetries than full theory at nonzero p/M
Can work to higher orders in p/M; can sum logs of p/M

NP can modify C_i or give rise to new O_i's — right coefficients? right operators?

Inclusive heavy hadron decays

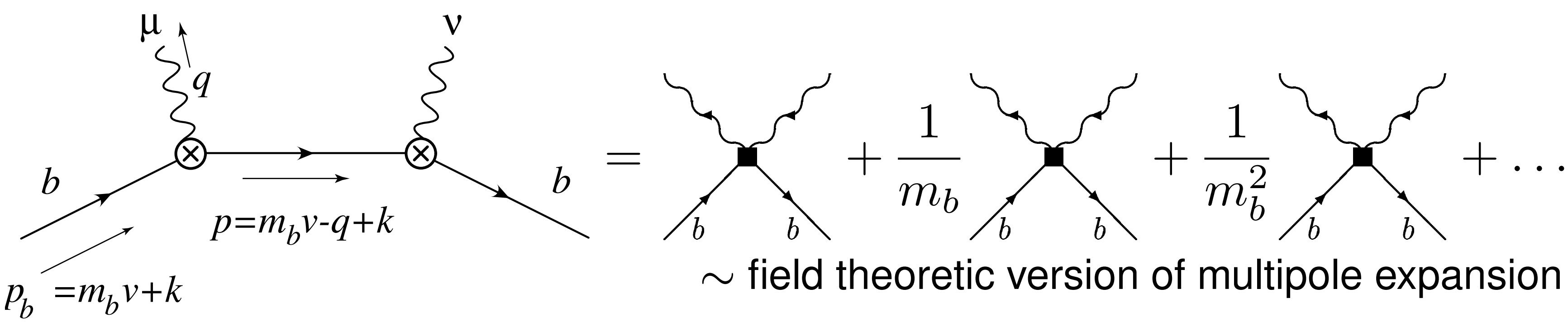
- Sum over hadronic final states, subject to constraints determined by short distance physics



Decay: short distance (calculable)

Hadronization: long distance (nonperturbative), but probability to hadronize is unity; sum over details

- Optical theorem + operator product expansion (OPE) + heavy quark symmetry



$\sim$ field theoretic version of multipole expansion

Can think of the OPE as expansion of forward scattering amplitude in $k \sim \Lambda_{\rm QCD}$

Operator product expansion

- Consider semileptonic $b \to u$ decay: $O_{bu} = -\frac{4G_F}{\sqrt{2}} V_{ub} \underbrace{(\overline{u}\,\gamma^\mu P_L\, b)}_{J^\mu_{bu}} \underbrace{(\bar{\ell}\,\gamma_\mu P_L\,\nu)}_{J_{\ell\nu}}$

  Decay rate: $\Gamma(B \to X_u \ell\bar{\nu}) \sim \sum_{X_c} \int \mathrm{d[PS]} \left|\langle X_u \ell\bar{\nu}|O_{bu}|B\rangle\right|^2$

  Factor to: $B \to X_u W^*$ and $W^* \to \ell\bar{\nu}$, concentrate on hadronic part

$$W^{\mu\nu} \sim \sum_{X_c} \delta^4(p_B - q - p_X) \left|\langle B|J^{\mu\dagger}_{bu}|X_u\rangle\, \langle X_u|J^\nu_{bu}|B\rangle\right|^2 = \mathrm{Im}\, T^{\mu\nu}$$

(optical theorem) $\quad T^{\mu\nu} = i \int \mathrm{d}x\, e^{-iq\cdot x}\, \langle B|\, T\big\{ J^{\mu\dagger}_{bu}(x)\, J^\nu_{bu}(0) \big\}\, |B\rangle$

- Operators: $\bar{b}\, b \to$ free quark decay, $\langle \bar{b} D^2 b\rangle$, $\langle \bar{b}\sigma_{\mu\nu} G^{\mu\nu} b\rangle \sim m^2_{B^*} - m^2_B$, etc.

$$\mathrm{d}\Gamma = \begin{pmatrix} b \text{ quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \alpha_s^2(\ldots) + \ldots \right\}$$

- As for $e^+e^- \to$ hadrons, question is when perturbative calculation can be trusted

Analytic structure for semileptonic decays

- More complicated than $e^+e^- \to$ hadrons

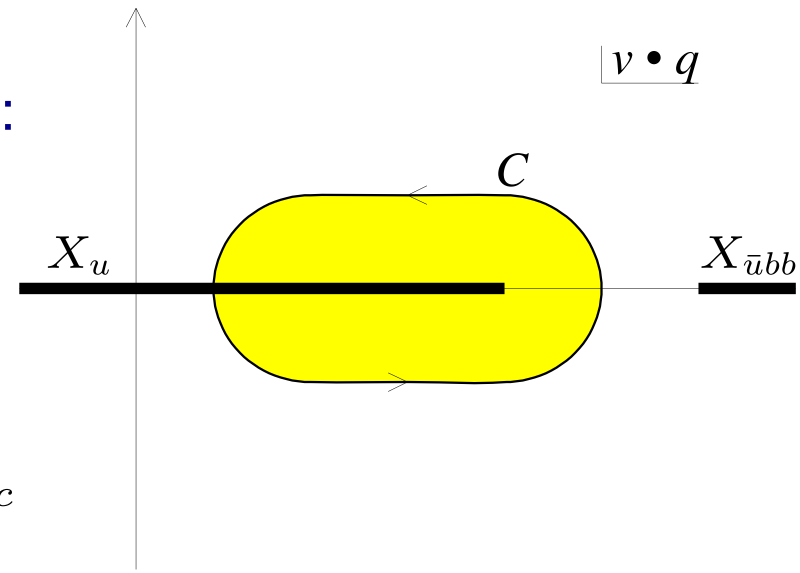
  For fixed q^2, cuts of $T^{\mu\nu}$ in the complex q^0 plane:

  $$q^0 = q \cdot v < (m_B^2 + q^2 - m^2_{X_q^{\min}})/2m_B$$

  $$q^0 = q \cdot v > (m^2_{X_{\bar{q}bb}^{\min}} - m_B^2 - q^2)/2m_B$$

  For $b \to c\ell\bar{\nu}$, two cuts are separated by $> 4m_c$

  For $b \to u\ell\bar{\nu}$ near $q^2_{\max}$ only by $\mathcal{O}(\Lambda_{\rm QCD})$ at)



- To calculate any observable, contour must approach the cut somewhere

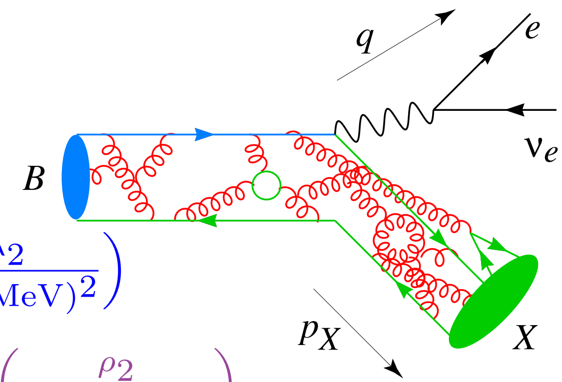
  Integration over neutrino (or kinematic variables) “builds in” some smearing

- Tested in great detail in semileptonic $B \to X_c\ell\bar{\nu}$ decays

- Nonleptonic rates (lifetimes) have to use OPE in the physical region

Classic application: inclusive $|V_{cb}|$

- Want to determine $|V_{cb}|$ from $B \to X_c \ell \bar\nu$:



$$\Gamma(B \to X_c \ell \bar\nu) = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (4.7\,\text{GeV})^5\, (0.534) \times$$

$$\Bigg[1 - 0.22\left(\frac{\Lambda_{1S}}{500\,\text{MeV}}\right) - 0.011\left(\frac{\Lambda_{1S}}{500\,\text{MeV}}\right)^2 - 0.052\left(\frac{\lambda_1}{(500\,\text{MeV})^2}\right) - 0.071\left(\frac{\lambda_2}{(500\,\text{MeV})^2}\right)$$

$$- 0.006\left(\frac{\lambda_1\Lambda_{1S}}{(500\,\text{MeV})^3}\right) + 0.011\left(\frac{\lambda_2\Lambda_{1S}}{(500\,\text{MeV})^3}\right) - 0.006\left(\frac{\rho_1}{(500\,\text{MeV})^3}\right) + 0.008\left(\frac{\rho_2}{(500\,\text{MeV})^3}\right)$$

$$+ 0.011\left(\frac{T_1}{(500\,\text{MeV})^3}\right) + 0.002\left(\frac{T_2}{(500\,\text{MeV})^3}\right) - 0.017\left(\frac{T_3}{(500\,\text{MeV})^3}\right) - 0.008\left(\frac{T_4}{(500\,\text{MeV})^3}\right)$$

$$+ 0.096\epsilon - 0.030\epsilon^2_{\text{BLM}} + 0.015\epsilon\left(\frac{\Lambda_{1S}}{500\,\text{MeV}}\right) + \dots\Bigg]$$

Corrections: $\mathcal{O}(\Lambda/m)$: $\sim 20\%$, $\mathcal{O}(\Lambda^2/m^2)$: $\sim 5\%$, $\mathcal{O}(\Lambda^3/m^3)$: $\sim 1-2\%$, $\mathcal{O}(\alpha_s)$: $\sim 10\%$, Unknown terms: $<$ few %

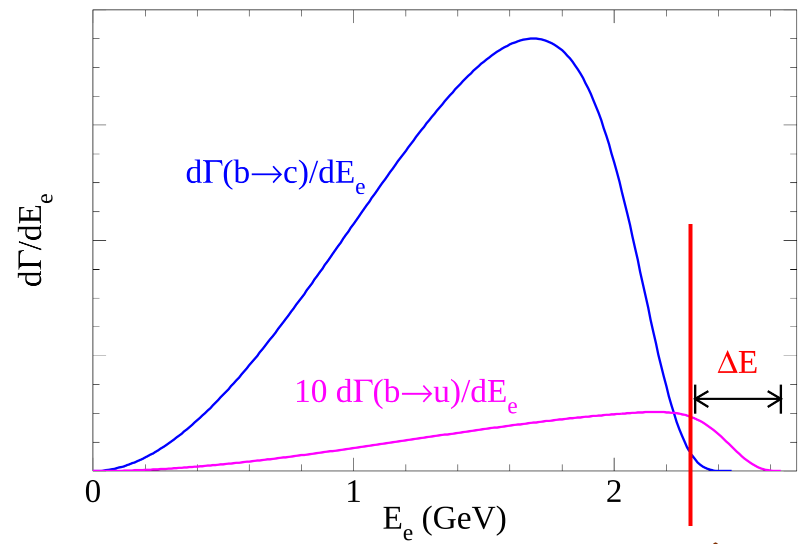
Matrix elements extracted from shape variables — good fit to lots of data

- Error of $|V_{cb}| \sim 2\%$ — a precision field; uncomfortable $\sim 2\sigma$ tension with exclusive

The challenge of inclusive $|V_{ub}|$ measurements

- Total rate predicted with $\sim 4\%$ accuracy, similar to $\mathcal{B}(B \to X_c \ell \bar{\nu})$
- To remove the huge charm background $(|V_{cb}/V_{ub}|^2 \sim 100)$, need phase space cuts

  Can enhance pert. and nonpert. corrections



- Instead of being constants, the hadronic parameters become functions (like PDFs)

  Leading order: universal & related to $B \to X_s \gamma$;
  $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$: several new unknown functions

  Nonperturbative effects shift endpoint $\frac{1}{2} m_b \to \frac{1}{2} m_B$ & determine its shape

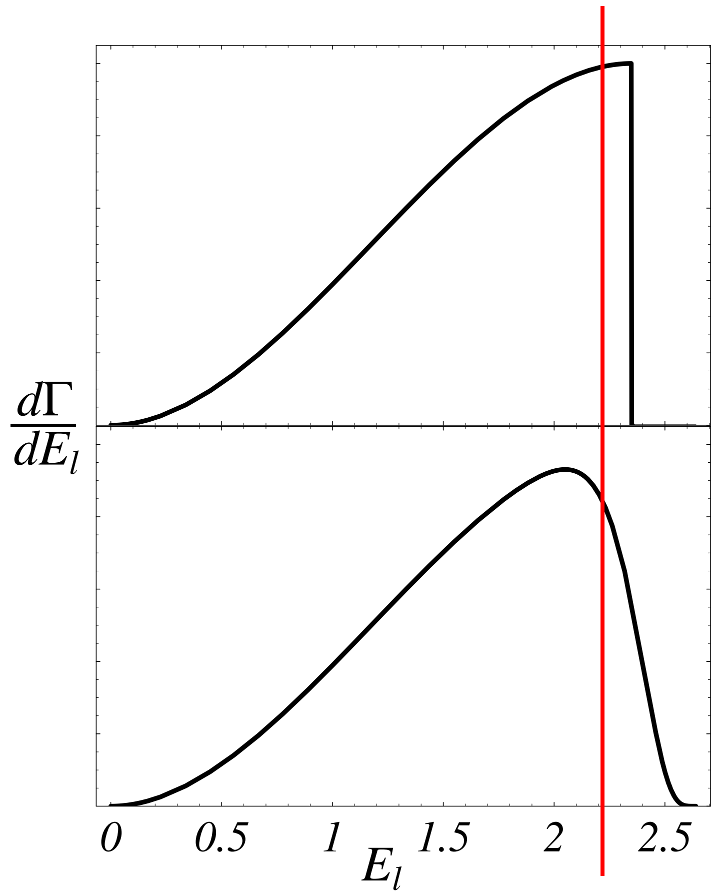
- Shape in the endpoint region is determined by b quark PDF in B — related to the $B \to X_s \gamma$ photon spectrum at lowest order

[Bigi, Shifman, Uraltsev, Vainshtein; Neubert]

Shape function: lepton endpoint vs. $B \to X_s\gamma$

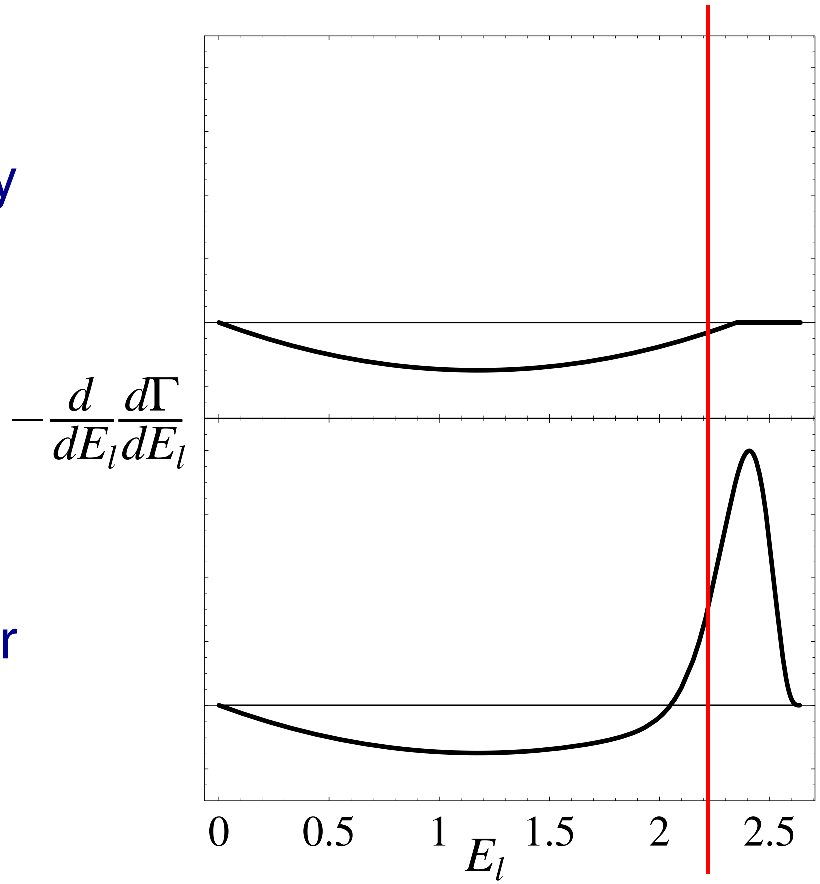
b quark decay spectrum

with a model for b quark PDF



Shape function: lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum



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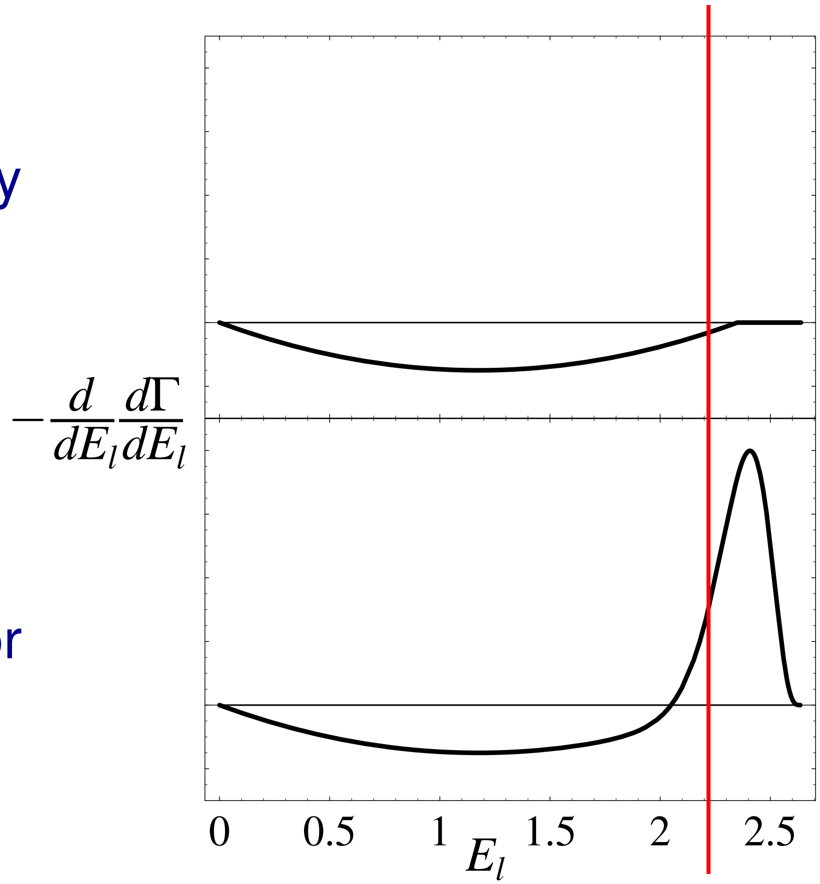
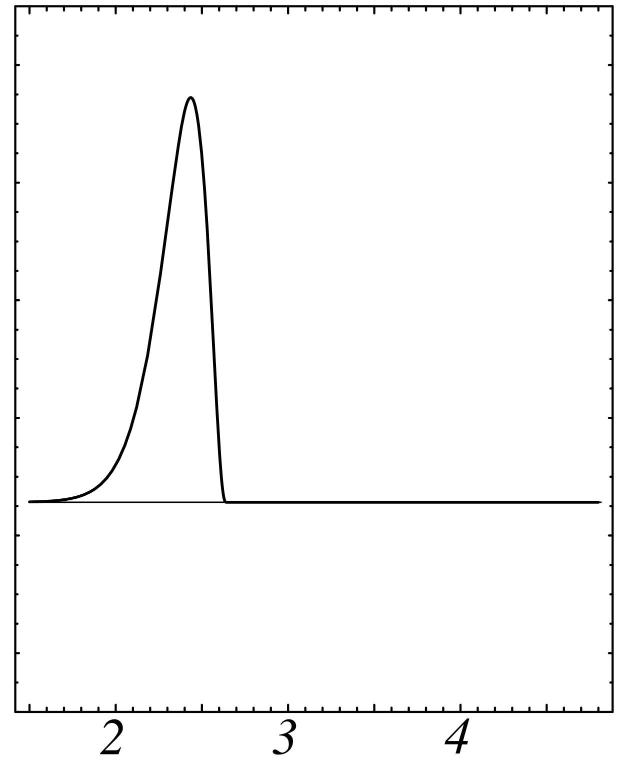


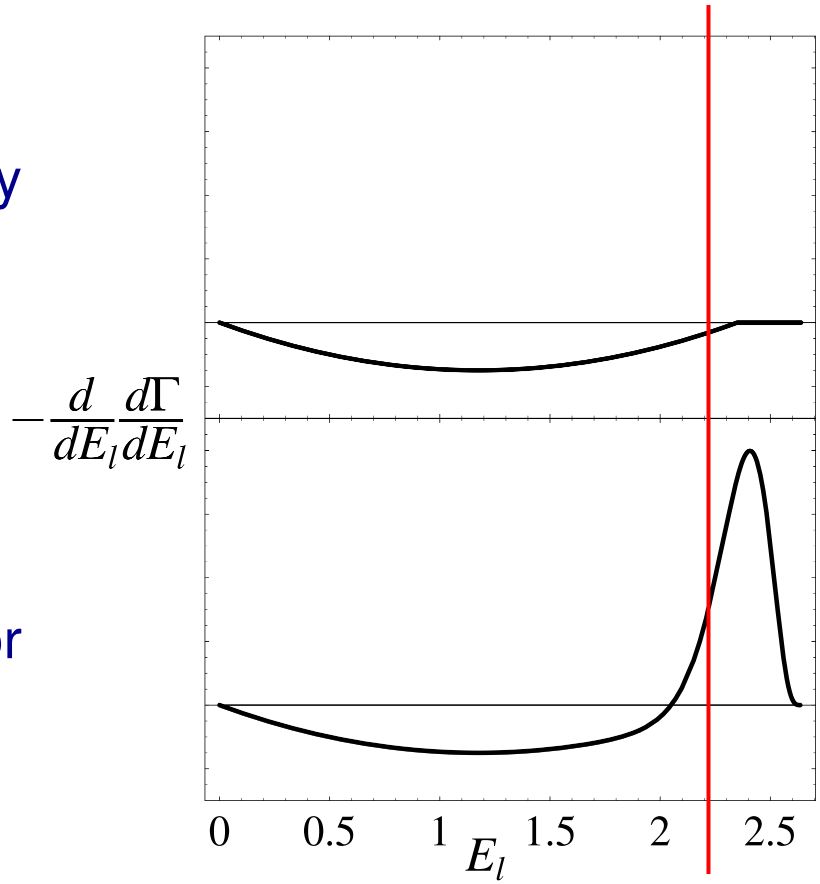
difference:



Shape function: lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum

with a model for b quark PDF



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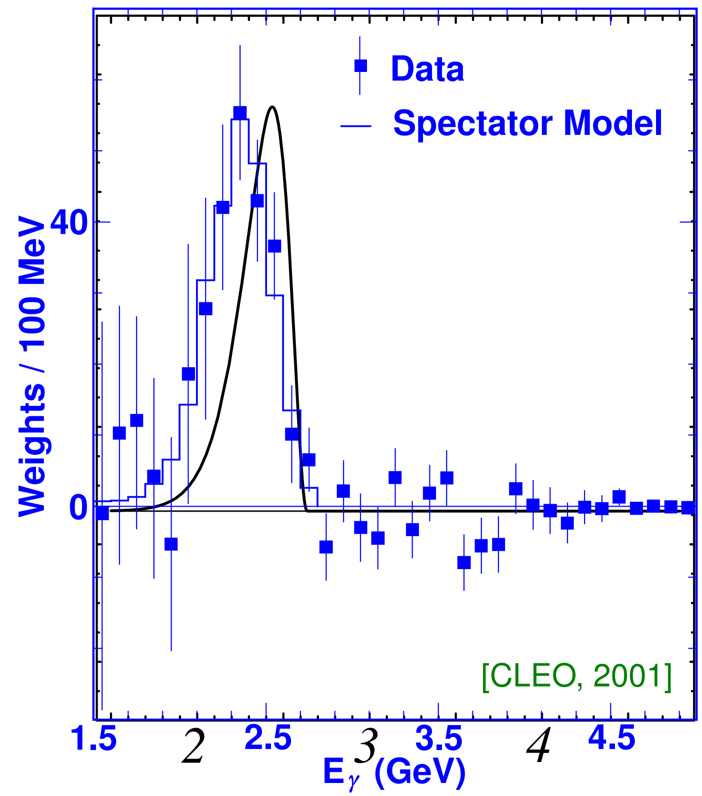
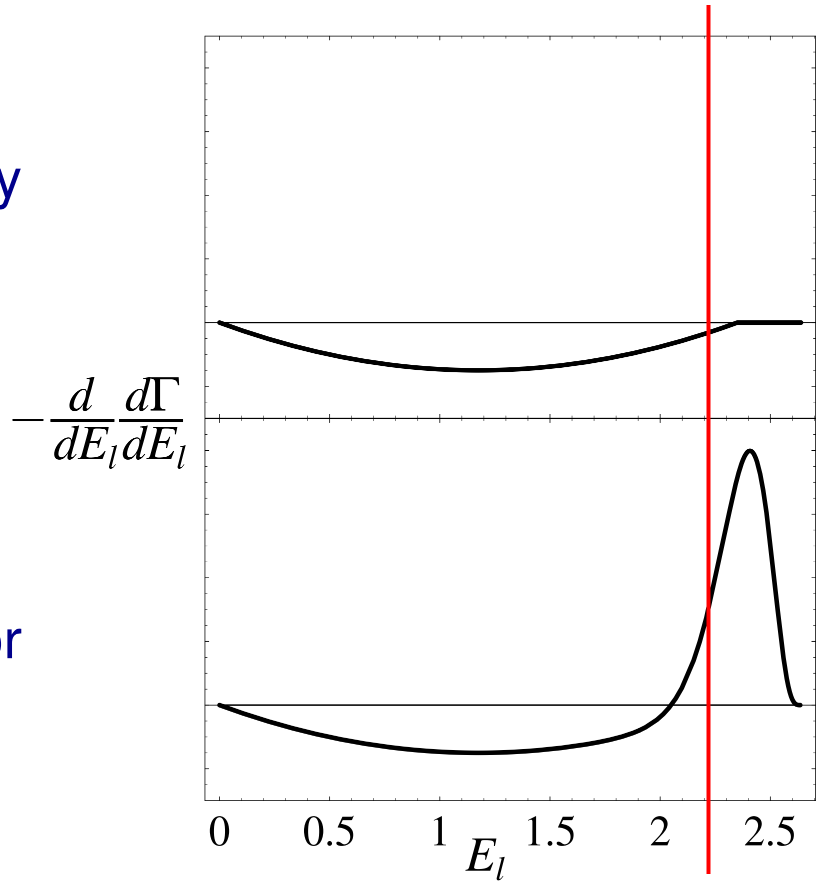


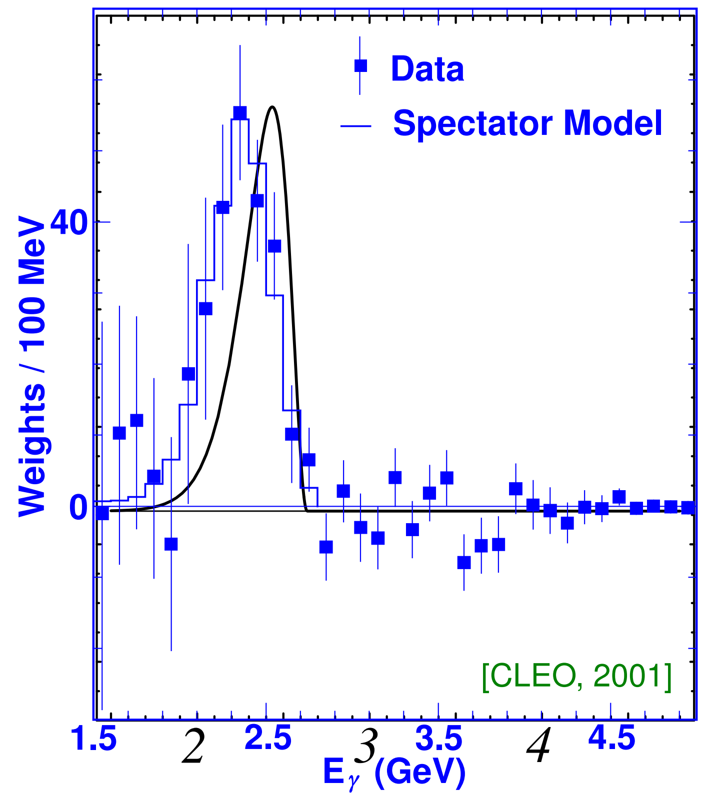
Shape function: lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum

with a model for b quark PDF



difference:



- Both of these spectra determined at lowest order by the b quark PDF in B meson
- Lots of work toward extending beyond leading order; some open issues remain

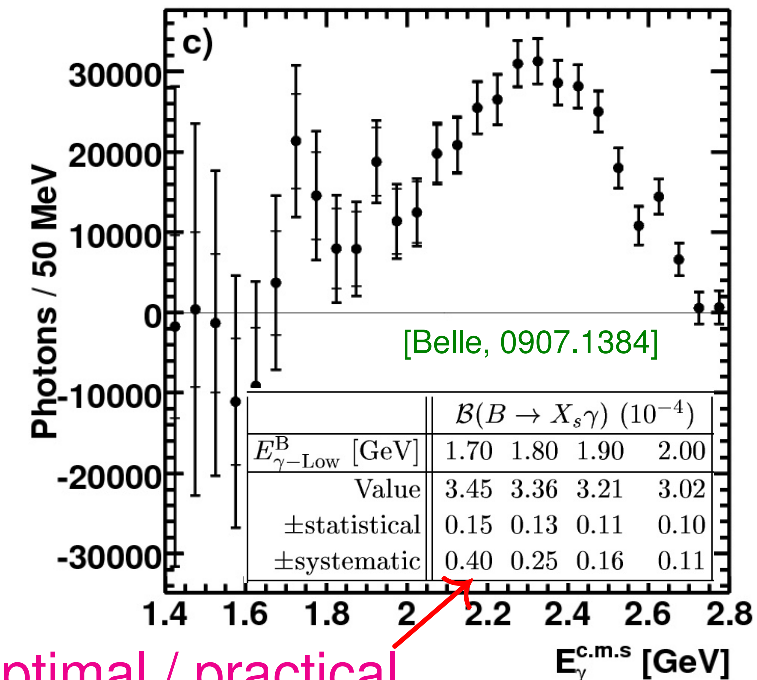
Regions of $B \to X_s\gamma$ phase space

- Important both for $|V_{ub}|$ and constraining NP
- $m_B - 2E_\gamma \lesssim 2\,\mathrm{GeV}$, and $<1\,\mathrm{GeV}$ at the peak

Three cases: 1) $\Lambda_{\mathrm{QCD}} \sim m_B - 2E_\gamma \ll m_B$
2) $\Lambda_{\mathrm{QCD}} \ll m_B - 2E_\gamma \ll m_B$
3) $\Lambda_{\mathrm{QCD}} \ll m_B - 2E_\gamma \sim m_B$

Neither 1) nor 2) is fully appropriate

[Sometimes called: 1) SCET and 2) MSOPE regions]



	$\mathcal{B}(B \to X_s\gamma)$ (10^{-4})			
$E^{\mathrm{B}}_{\gamma-\mathrm{Low}}$ [GeV]	1.70	1.80	1.90	2.00
Value	3.45	3.36	3.21	3.02
±statistical	0.15	0.13	0.11	0.10
±systematic	0.40	0.25	0.16	0.11

- Not clear if reducing E_γ^{cut} to $\sim 1.7\,\mathrm{GeV}$ is indeed optimal / practical

---

- $B \to X_u \ell \bar\nu$ is more complicated: hadronic physics depends not on one (E_γ) but two variables (best choice: $p_X^\pm = E_X \mp |\vec{p}_X|$ — "jettyness" of hadronic final state)
- Existing approaches based on theory in one region, extrapolated / modeled to rest

Approaches to $|V_{ub}|$ — more to come

- BLNP [Bosch *et al.*] — based on SCET region ⇒ Stephane
  - factorization & resummation in shape function region treated correctly
  - crossing into local OPE region not model independent
  - tied to "shape function" scheme
- DGE [Andersen & Gardi] — based on SCET region + perturbative model for the SF
  - SCET region treated correctly; motivated by renormalon resummation
- GGOU [Gambino *et al.*] — based on local OPE region + SF smearing
  - no resummation in SCET region
  - tied to "kinetic" scheme
- BLL [Bauer, ZL, Luke] — based on local OPE at large q^2 (but expansion scale is smaller)
  - combine q^2 and m_X cuts, such that SF effect is kept small
- Shape function independent relations [Leibovich, Low, Rothstein; Hoang, ZL, Luke; Lange, Neubert, Paz; Lange]
  - beautiful at leading order, less so when $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ included

If all else fails: "Grinstein-type double ratios"

- Continuum theory may be competitive using HQS $+$ chiral symmetry suppression
- $\dfrac{f_B}{f_{B_s}} \times \dfrac{f_{D_s}}{f_D}$ — lattice: double ratio $= 1$ within few $\%$ [Grinstein '93]
- $\dfrac{f^{(B\to\rho\ell\bar\nu)}}{f^{(B\to K^*\ell^+\ell^-)}} \times \dfrac{f^{(D\to K^*\ell\bar\nu)}}{f^{(D\to\rho\ell\bar\nu)}}$ or q^2 spectra — accessible soon? [ZL, Wise; Grinstein, Pirjol]

  $D \to \rho\ell\bar\nu$ data still consistent with no $SU(3)$ breaking in form factors

  Could lattice QCD do more to pin down the corrections?

  Worth looking at similar ratio with K, π — role of B^* pole...?
- $\dfrac{\mathcal{B}(B\to\ell\bar\nu)}{\mathcal{B}(B_s\to\ell^+\ell^-)} \times \dfrac{\mathcal{B}(D_s\to\ell\bar\nu)}{\mathcal{B}(D\to\ell\bar\nu)}$ — very clean... after 2015? [Ringberg workshop, '03]
- $\dfrac{\mathcal{B}(B_u\to\ell\bar\nu)}{\mathcal{B}(B_d\to\mu^+\mu^-)}$ — even cleaner... around 2020? [Grinstein, CKM'06]
- For implications for probing SUSY models, ask Nazila [Akeroyd, Mahmoudi, 1007.2757]

$B \to X_s\gamma$ **and** $K^*\gamma$

$\Rightarrow$ Patrick

Inclusive $B \to X_s\gamma$ calculations

- One (if not "the") most elaborate SM calculations

  Constrains many models: 2HDM, SUSY, LRSM, etc.

- NNLO practically completed [Misiak et al., hep-ph/0609232]

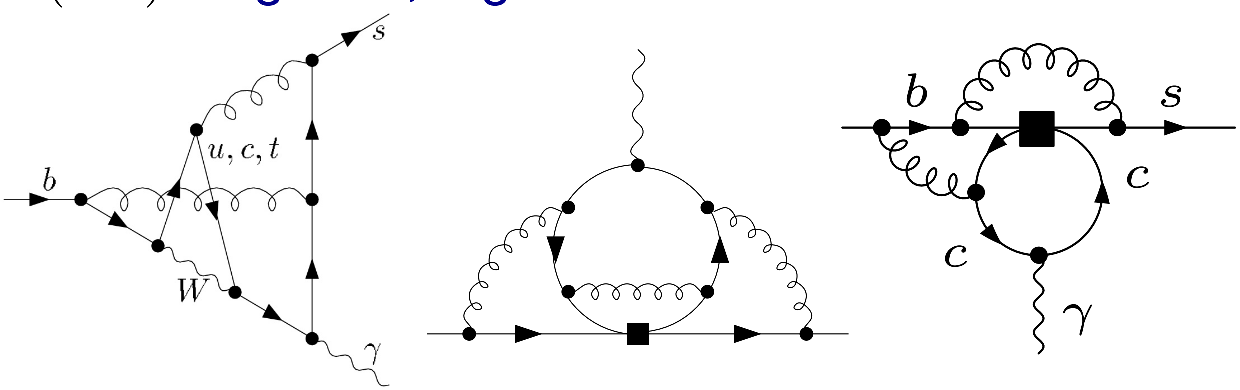
  4-loop running, 3-loop matching and matrix elements

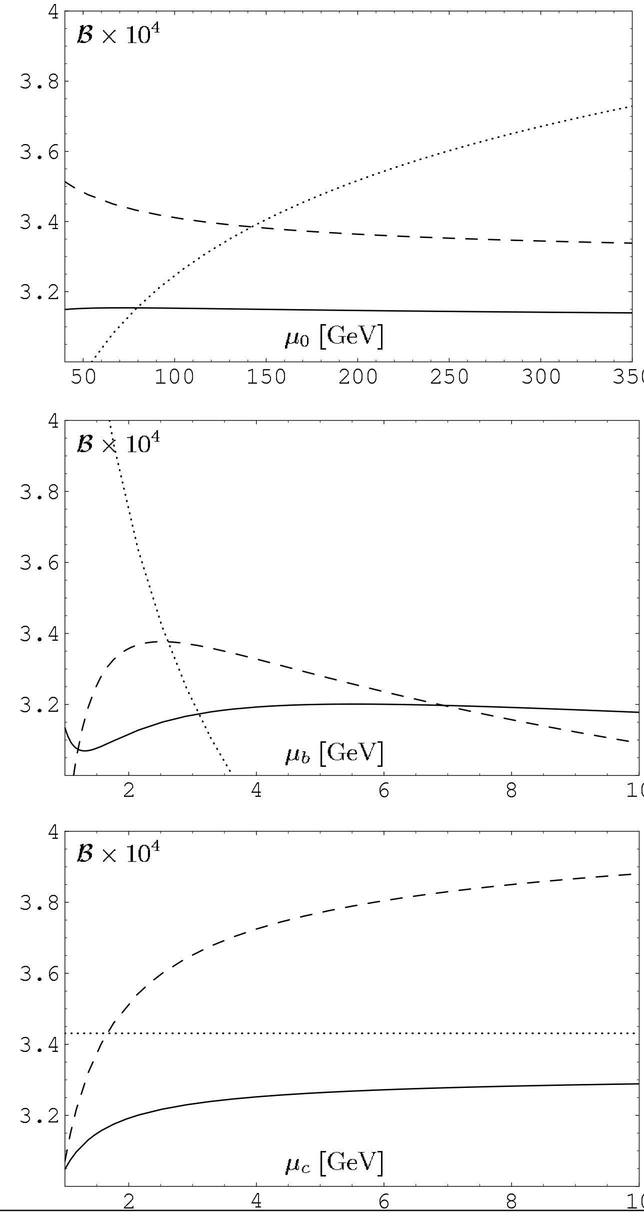
  Scale dependencies significantly reduced $\Rightarrow$

- $\mathcal{B}(B \to X_s\gamma)\big|_{E_\gamma > 1.6\mathrm{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$

  measurement: $(3.52 \pm 0.25) \times 10^{-4}$

- $\mathcal{O}(10^4)$ diagrams, e.g.:





$B \to X_s\gamma$ and neutralino dark matter

- Green: excluded by $B \to X_s\gamma$

  Brown: excluded (charged LSP)

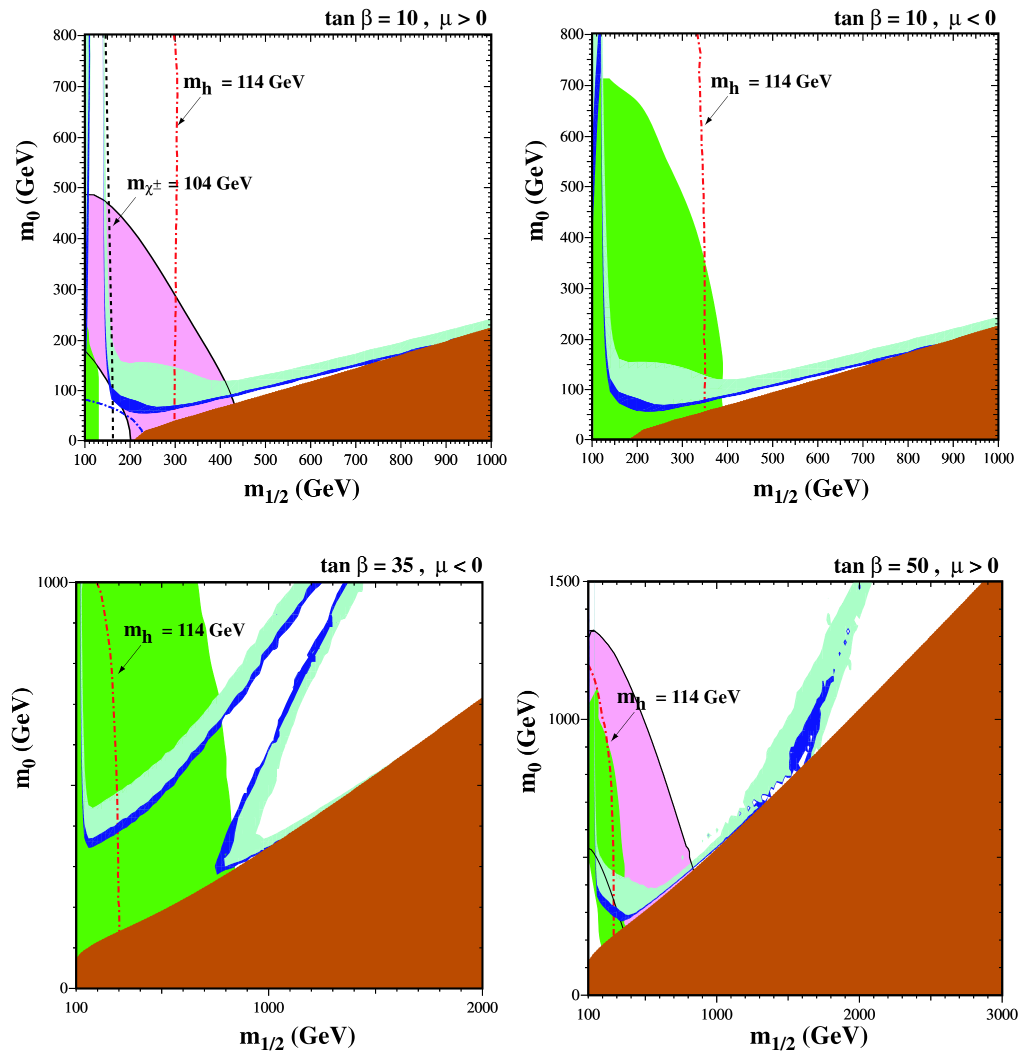
  Magenta: favored by $g_\mu - 2$

  Blue: favored by $\Omega_\chi h^2$ from WMAP

- Analyses assume constrained MSSM

  If either $S_{\eta' K} \neq \sin 2\beta$ or $S_{K^*\gamma} \neq 0$, then has to be redone

  Then $B \to X_s \ell^+\ell^-$ and $B_s \to \mu\mu$ may give complementary constraints



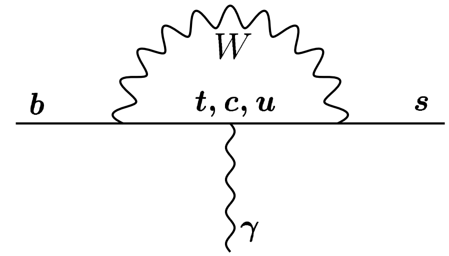
[Ellis, Olive, Santoso, Spanos]

Photon polarization in $B \to X_s\gamma$

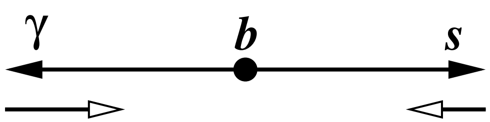
- Is $B \to X_s\gamma$ due to $O_7 \sim \bar{s}\sigma_{\mu\nu}F^{\mu\nu}P_R b$ $(b \to s_L\gamma_L)$ or $O_7' \sim \bar{s}\sigma_{\mu\nu}F^{\mu\nu}P_L b$ $(b \to s_R\gamma_R)$?

  In SM: $C_7'/C_7 = m_s/m_b$, so decays to γ_L dominate

  Left- and right-handed photons do not interfere



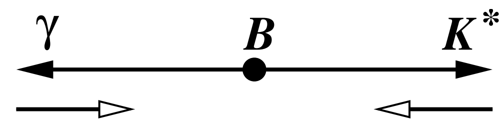
Inclusive $B \to X_s\gamma$



Assumption: 2-body decay

Does not apply for $b \to s\gamma g$

Exclusive $B \to K^*\gamma$



In quark model (s_L implies $J_z^{K^*} = -1$)

Does not apply for higher K^* Fock states

- Had been expected to give $S_{K^*\gamma} = -2\,(m_s/m_b)\sin 2\phi_1$ [Atwood, Gronau, Soni]

$$\frac{\Gamma[\overline{B}^0(t) \to K^*\gamma] - \Gamma[B^0(t) \to K^*\gamma]}{\Gamma[\overline{B}^0(t) \to K^*\gamma] + \Gamma[B^0(t) \to K^*\gamma]} = S_{K^*\gamma}\sin(\Delta m\, t) - C_{K^*\gamma}\cos(\Delta m\, t)$$

- Data: $S_{K^*\gamma} = -0.16 \pm 0.22$ — both the measurement and the theory can progress

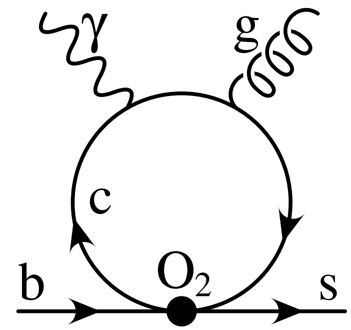
Right-handed photons in the SM

- Dominant source of "wrong-helicity" photons in the SM is O_2 [Grinstein, Grossman, ZL, Pirjol]

  Equal $b \to s\gamma_L,\ s\gamma_R$ rates at $\mathcal{O}(\alpha_s)$; calculated to $\mathcal{O}(\alpha_s^2 \beta_0)$

  Inclusively only rates are calculable: $\Gamma_{22}^{(\mathrm{brem})}/\Gamma_0 \simeq 0.025$

  Suggests: $A(b \to s\gamma_R)/A(b \to s\gamma_L) \sim \sqrt{0.025/2} = 0.11$



- $B \to K^*\gamma$: At leading order in $\Lambda_{\mathrm{QCD}}/m_b$, wrong helicity amplitude vanishes

  Subleading order: no longer vanishes

  Order of magnitude: $$\frac{A(\overline{B}^0 \to \overline{K}^{0*}\gamma_R)}{A(\overline{B}^0 \to \overline{K}^{0*}\gamma_L)} = \mathcal{O}\left(\frac{C_2}{3C_7}\,\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right) \sim 0.1$$

  Some additional suppression expected, but I don't find $\lesssim 0.02$ claims convincing

- Consider pattern in several modes, hope to build a case [Atwood, Gershon, Hazumi, Soni]

Even more observables

- Direct CP asymmetry:

$$A_{B\to X_s\gamma} = -0.012 \pm 0.028$$
$$A_{B\to X_{d+s}\gamma} = -0.011 \pm 0.012$$
$$A_{B\to K^*\gamma} = -0.010 \pm 0.028$$

  SM prediction < 0.01, except for $A_{B\to\rho\gamma}$ which is larger

- Isospin asymmetry: it seems to me that theoretical uncertainties would make it hard to argue for new physics

- If these observables don't show NP, I doubt higher K states could be convincing

Other interesting $b \to s$ decays

- ALEPH $B \to X_c \tau\nu$ search via large $E_{\rm miss}$ also bounded $B \to X_s \nu\bar\nu$ [Grossman, ZL, Nardi]

  ALEPH bound: $\mathcal{B}(B \to X_s \nu\bar\nu) < 6.4 \times 10^{-4}$ still the best to date

  Does only $B \to K\nu\bar\nu$ have a chance at super-B?

- Can also bound $B_{(s)} \to \tau^+\tau^-(X)$, only at few $\%$ level

  Renewed recent interest in connection with DØ anomaly, to enhance $\Delta\Gamma_{B_s}$

  BaBar established: $\mathcal{B}(B \to \tau^+\tau^-) < 4.1 \times 10^{-3}$

- Models with unrelated couplings in each channel, e.g., SUSY without R-parity[1]

  Models with enhanced 3332 generation couplings: $B \to X_s\nu\bar\nu,\ X_s\tau\tau,\ B_s \to \tau\tau$

- Even in 2020, we'll have (exp. bound)$/$(SM prediction) $\gtrsim 10^3$ in some channels

  E.g.: $B_{(s)} \to \tau^+\tau^-(X)$, $B_{(s)} \to e^+e^-$, maybe more...

[1] "Can do everything except make coffee" — Babar Physics Book

Some other rare B decays

- Important probes of new physics (a crude guide, $\ell = e$ or μ) $\Rightarrow$ Patrick

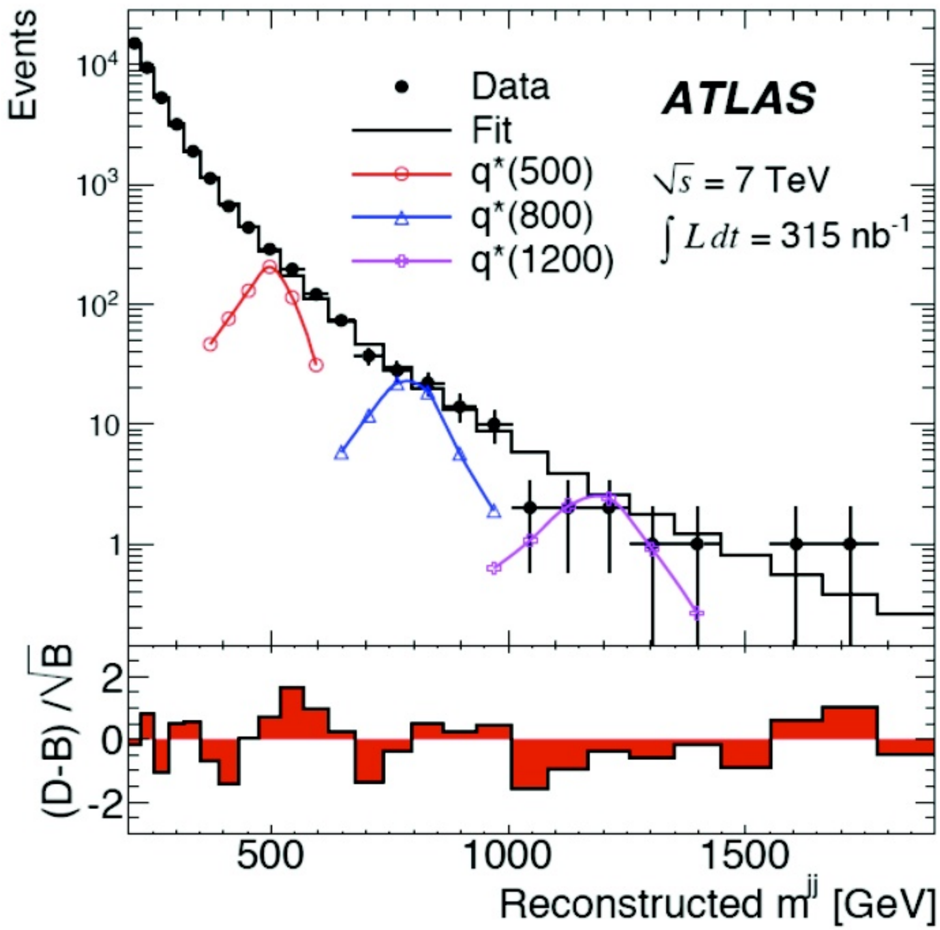
Decay	$\sim$SM rate	present status	expected
$B \to X_s\gamma$	3.2×10^{-4}	$(3.52 \pm 0.25) \times 10^{-4}$	4%
$B \to \tau\nu$	1×10^{-4}	$(1.73 \pm 0.35) \times 10^{-4}$	5%
$B \to X_s\nu\bar{\nu}$	3×10^{-5}	$< 6.4 \times 10^{-4}$	only $K\nu\bar{\nu}$?
$B \to X_s\ell^+\ell^-$	6×10^{-6}	$(4.5 \pm 1.0) \times 10^{-6}$	6%
$B_s \to \tau^+\tau^-$	1×10^{-6}	$<$ few %	$\Upsilon(5S)$ run ?
$B \to X_s\tau^+\tau^-$	5×10^{-7}	$<$ few %	?
$B \to \mu\nu$	4×10^{-7}	$< 1.3 \times 10^{-6}$	6%
$B \to \tau^+\tau^-$	5×10^{-8}	$< 4.1 \times 10^{-3}$	$\mathcal{O}(10^{-4})$
$B_s \to \mu^+\mu^-$	3×10^{-9}	$< 5 \times 10^{-8}$	LHCb
$B \to \mu^+\mu^-$	1×10^{-10}	$< 1.5 \times 10^{-8}$	LHCb

- Many interesting modes will first be seen at super-B (or LHCb)

  Maintain ability for inclusive studies as much as possible (smaller theory errors)

- Some of the theoretically cleanest modes (ν, τ, inclusive) only possible at e^+e^-

Bump hunting: not only for ATLAS & CMS...

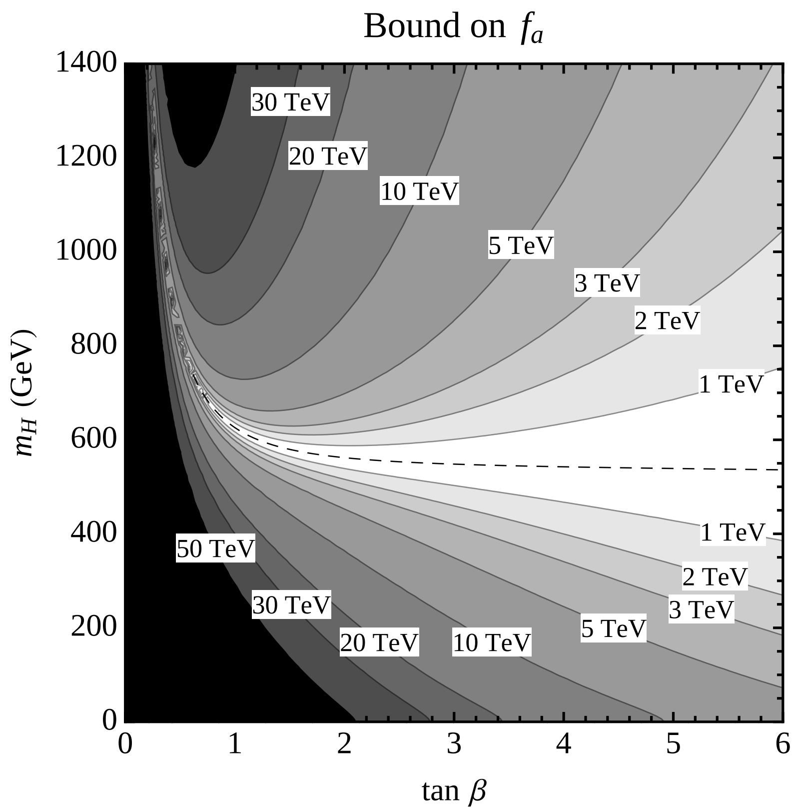


(The first LHC result superseding Tevatron limits)

Bump hunting: dark matter in B decay?

- Recent observations of cosmic ray excesses lead to flurry DM model building

  E.g., "axion portal": light ($\lesssim 1\,\mathrm{GeV}$) scalar particle coupling as $(m_\psi/f_a)\,\bar\psi\gamma_5\psi\,a$

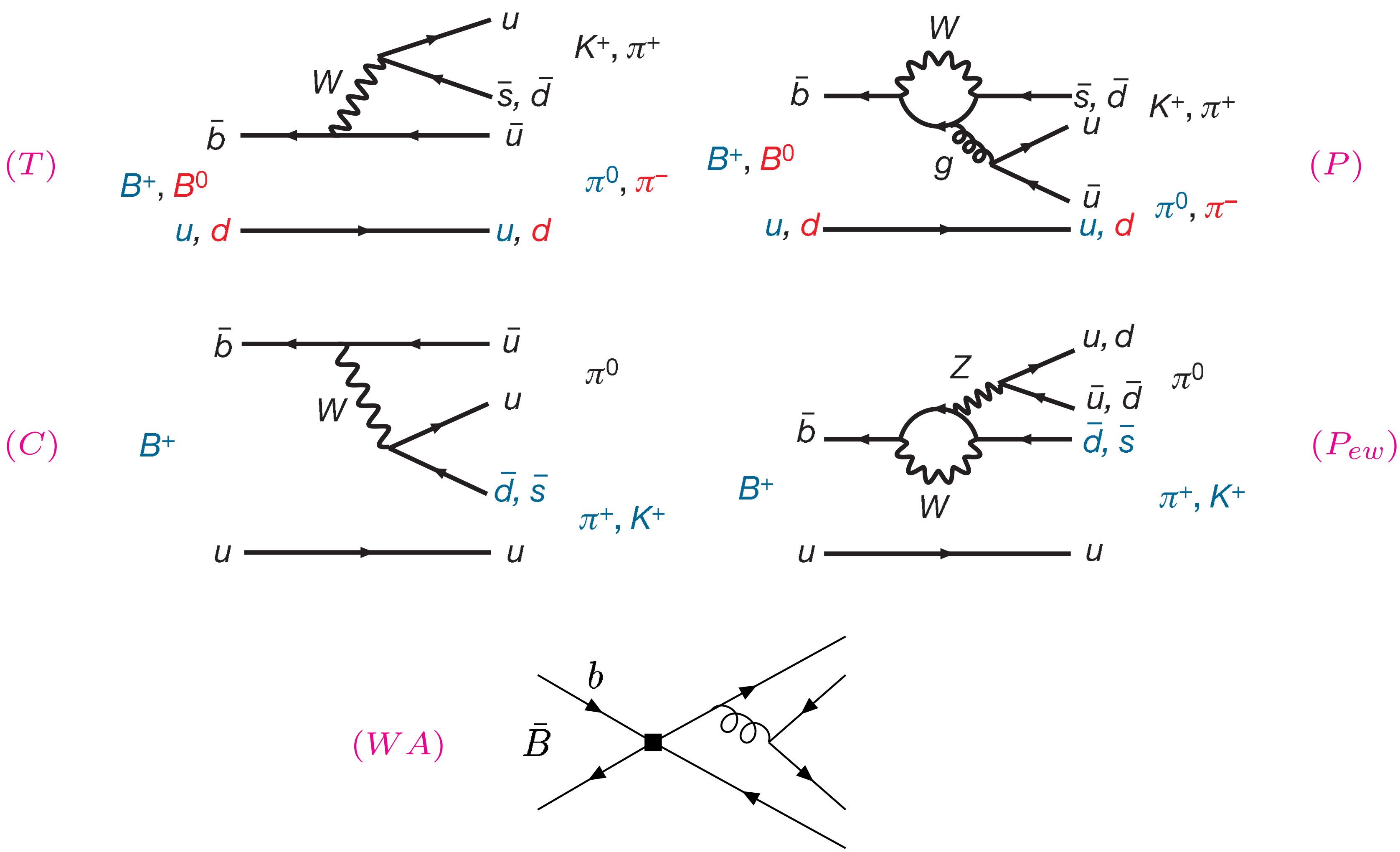


[Freytsis, ZL, Thaler]

- In most of parameter space $B \to K\ell^+\ell^-$ gives best bound, LHCb can improve it

Nonleptonic decays

Terminology



Some motivations

- Two hadrons in the final state are more complicated (also for lattice QCD)

  Lot at stake, even if precision is worse

  Many observables sensitive to NP — can we disentangle from hadronic physics?

  – $B \to \pi\pi, K\pi$ branching ratios and CP asymmetries (related to α, γ in SM)

  – Polarization in charmless $B \to VV$ decays

- First derive correct expansion in $m_b \gg \Lambda_{\rm QCD}$ limit, then worry about predictions

  – Need to test accuracy of expansion (even in $B \to \pi\pi$, $|\vec{p}_q| \sim 1\,\mathrm{GeV}$)

  – Sometimes model dependent additional inputs needed

HQET vs. SCET

- HQET: nonperturbative interactions do not change four-velocity of heavy quark

$p_b^\mu = m_b v^\mu + k^\mu$ — once we fix v, superselection rule; v label, k residual momenta

Project out large component: $h_v^{(b)}(x) = e^{i m_b v\cdot x}\,\dfrac{1+\not{v}}{2}\,b(x)$

- SCET: light-cone momentum of collinear partons change via $\mathcal{O}(1)$ interactions

Collinear quark in n direction: $p^- = \bar{n}\cdot p$ and $p_\perp$ are labels, but not conserved

Define: $n^2 = \bar{n}^2 = 0\,,\ n\cdot\bar{n} = 2\,;$ decompose: $p^\mu = \frac{1}{2}(\bar{n}\cdot p)n^\mu + \frac{1}{2}(n\cdot p)\bar{n}^\mu + p_\perp^\mu$

Collinear partons: $p^\mu = (p^-, p^+, p_\perp) \sim Q\,\big(1, \lambda^2, \lambda\big)$ (Q: large scale, λ: small param.)

Introduce new fields: $\psi(x) = e^{-i\widetilde{p}\cdot x}\,\psi_{n,p}(x) \qquad \xi_{n,p}(x) = \dfrac{\not{n}\not{\bar{n}}}{4}\,\psi_{n,p}(x)$

SCET in a nutshell

- Effective theory for processes involving energetic hadrons, $E \gg \Lambda$

  [Bauer, Fleming, Luke, Pirjol, Stewart, + . . .]

  Introduce distinct fields for relevant degrees of freedom, power counting in λ

modes	fields	$p = (-, +, \perp)$	p^2
collinear	$\xi_{n,p},\, A^\mu_{n,q}$	$E(1, \lambda^2, \lambda)$	$E^2\lambda^2$
soft	$q_q,\, A^\mu_s$	$E(\lambda, \lambda, \lambda)$	$E^2\lambda^2$
usoft	$q_{us},\, A^\mu_{us}$	$E(\lambda^2, \lambda^2, \lambda^2)$	$E^2\lambda^4$

  SCET$_\mathrm{I}$: $\lambda = \sqrt{\Lambda/E}$ — jets $(m{\sim}\Lambda E)$

  SCET$_\mathrm{II}$: $\lambda = \Lambda/E$ — hadrons $(m{\sim}\Lambda)$

  Match QCD $\to$ SCET$_\mathrm{I}$ $\to$ SCET$_\mathrm{II}$

- Can decouple ultrasoft gluons from collinear Lagrangian at leading order in λ

$$\xi_{n,p} = Y_n\, \xi^{(0)}_{n,p} \qquad A_{n,q} = Y_n\, A^{(0)}_{n,q}\, Y_n^\dagger \qquad Y_n = \mathrm{P}\exp\Big[ig \int_{-\infty}^{x} \mathrm{d}s\, n\cdot A_{us}(ns)\Big]$$

  Nonperturbative usoft effects made explicit through factors of Y_n in operators

  New symmetries: collinear / soft gauge invariance

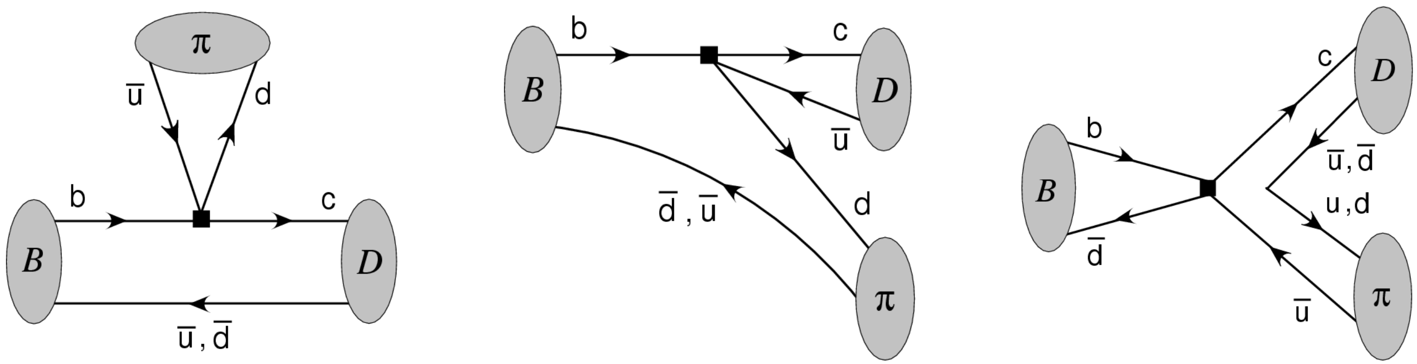
- Simplified / new $(B \to D\pi,\ \pi\ell\bar\nu)$ proofs of factorization theorems [Bauer, Pirjol, Stewart]

- Subleading order untractable before: $B \to D^0\pi^0$, CPV in $B \to K^*\gamma$, etc.

$B \to D^{(*)}\pi$ decays in SCET

- Proven that $A \propto \mathcal{F}^{B\to D} f_\pi$ at leading order [n.b.: $p_\pi = (2.310, 0, 0, 2.306)$ GeV]
  Also holds in large N_c, works at 5–10% level, need precise data to test mechanism

$\overline{B}^0 \to D^+\pi^-$, $B^- \to D^0\pi^-$ | $B^- \to D^0\pi^-$, $\overline{B}^0 \to D^0\pi^0$ | $\overline{B}^0 \to D^+\pi^-$, $\overline{B}^0 \to D^0\pi^0$



SCET: $\mathcal{O}(1)$ $\mathcal{O}(\Lambda_{\rm QCD}/Q)$ $\mathcal{O}(\Lambda_{\rm QCD}/Q)$ $Q = \{E_\pi, m_{b,c}\}$

- Predictions:

$$\frac{\mathcal{B}(B^- \to D^{(*)0}\pi^-)}{\mathcal{B}(\overline{B}^0 \to D^{(*)+}\pi^-)} = 1 + \mathcal{O}(\Lambda_{\rm QCD}/Q)\,,$$

data: $\sim 1.8 \pm 0.2$ (also for ρ)
$\Rightarrow \; \mathcal{O}(30\%)$ power corrections
[Beneke, Buchalla, Neubert, Sachrajda; Bauer, Pirjol, Stewart]

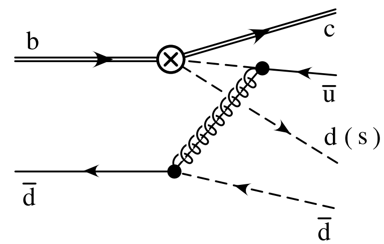
$$\frac{\mathcal{B}(\overline{B}^0 \to D^0\pi^0)}{\mathcal{B}(\overline{B}^0 \to D^{*0}\pi^0)} = 1 + \mathcal{O}(\Lambda_{\rm QCD}/Q)\,,$$

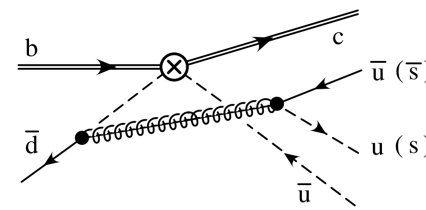
data: $\sim 1.1 \pm 0.25$
Unforeseen before SCET
[Mantry, Pirjol, Stewart]

Color suppressed $B \to D^{(*)0}\pi^0$ decays

- Single class of power suppressed SCET_I operators: $T\{\mathcal{O}^{(0)}, \mathcal{L}^{(1)}_{\xi q}, \mathcal{L}^{(1)}_{\xi q}\}$

[Mantry, Pirjol, Stewart]



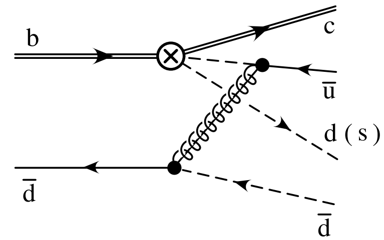


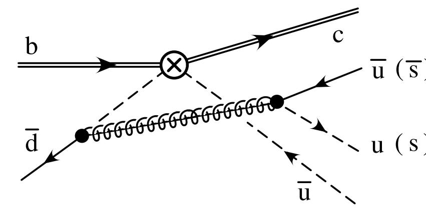
$$A(D^{(*)0}M^0) = N_0^M \int \mathrm{d}z\, \mathrm{d}x\, \mathrm{d}k_1^+ \mathrm{d}k_2^+\, T^{(i)}(z)\, J^{(i)}(z, x, k_1^+, k_2^+)\, \underbrace{S^{(i)}(k_1^+, k_2^+)}_{\text{complex} - \text{nonpert. strong phase}}\, \phi_M(x) + \dots$$

Color suppressed $B \to D^{(*)0}\pi^0$ decays

- Single class of power suppressed $\mathrm{SCET_I}$ operators: $T\{\mathcal{O}^{(0)}, \mathcal{L}^{(1)}_{\xi q}, \mathcal{L}^{(1)}_{\xi q}\}$

[Mantry, Pirjol, Stewart]





$$A(D^{(*)0}M^0) = N_0^M \int \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}k_1^+\mathrm{d}k_2^+\, T^{(i)}(z)\, J^{(i)}(z,x,k_1^+,k_2^+)\, \underbrace{S^{(i)}(k_1^+,k_2^+)}_{\text{complex} - \text{nonpert. strong phase}}\, \phi_M(x) + \dots$$

- Not your garden variety factorization formula... $S^{(i)}(k_1^+,k_2^+)$ know about n

$$S^{(0)}(k_1^+,k_2^+) = \frac{\langle D^0(v')|(\bar h_{v'}^{(c)} S)\, \not{n} P_L (S^\dagger h_v^{(b)})(\bar d S)_{k_1^+}\, \not{n} P_L (S^\dagger u)_{k_2^+} |\bar B^0(v)\rangle}{\sqrt{m_B m_D}}$$

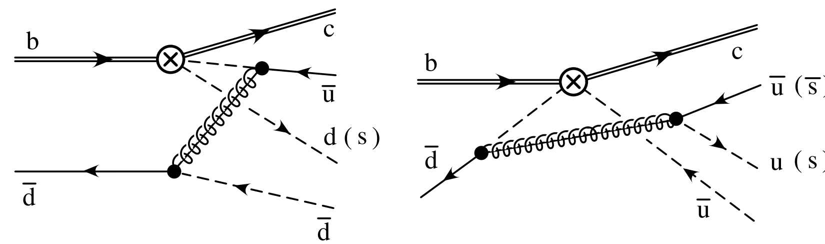
Separates scales, allows to use HQS without $E_\pi/m_c = \mathcal{O}(1)$ corrections

($i = 0, 8$ above)

Color suppressed $B \to D^{(*)0}\pi^0$ decays

- Single class of power suppressed SCET$_\mathrm{I}$ operators: $T\{\mathcal{O}^{(0)}, \mathcal{L}_{\xi q}^{(1)}, \mathcal{L}_{\xi q}^{(1)}\}$

[Mantry, Pirjol, Stewart]



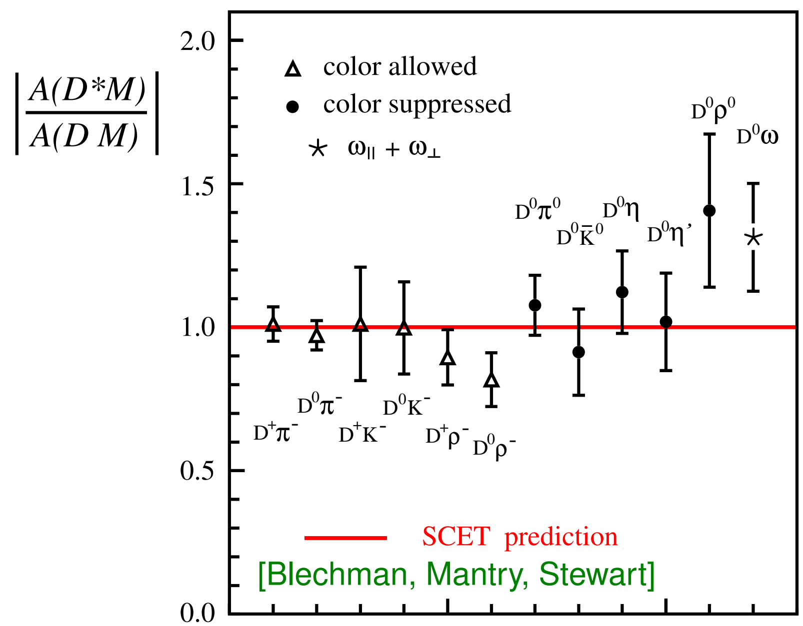
$$A(D^{(*)0}M^0) = N_0^M \int \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}k_1^+\mathrm{d}k_2^+\, T^{(i)}(z)\, J^{(i)}(z,x,k_1^+,k_2^+)\, \underbrace{S^{(i)}(k_1^+,k_2^+)}_{\text{complex}\,-\,\text{nonpert. strong phase}}\, \phi_M(x) + \ldots$$

- Ratios: the $\triangle = 1$ relations follow from naive factorization and heavy quark symmetry

The $\bullet = 1$ relations do not — a prediction of SCET not foreseen by model calculations

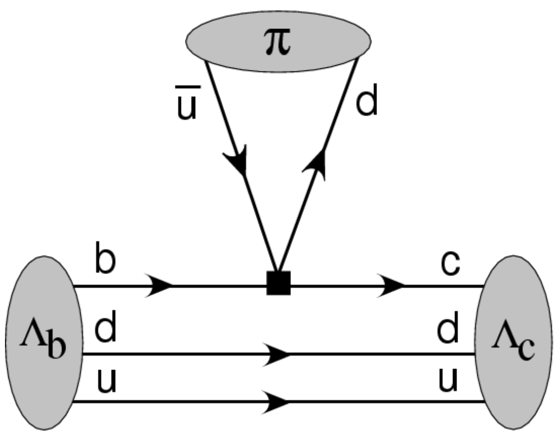
Also predict equal strong phases between $I = 1/2$ and $3/2$ amplitudes in $D\pi$ and $D^*\pi$

Data: $\delta(D\pi) = (28 \pm 3)^\circ,\ \ \delta(D^*\pi) = (32 \pm 5)^\circ$



Λ_b and B_s decays

- CDF measured in 2003: $\Gamma(\Lambda_b \to \Lambda_c^+ \pi^-)/\Gamma(\overline{B}^0 \to D^+\pi^-) \approx 2$



Factorization does not follow from large N_c, but holds at leading order in Λ_{QCD}/Q

$$\frac{\Gamma(\Lambda_b \to \Lambda_c \pi^-)}{\Gamma(\overline{B}^0 \to D^{(*)+}\pi^-)} \simeq 1.8 \left(\frac{\zeta(w_{\max}^{\Lambda})}{\xi(w_{\max}^{D^{(*)}})}\right)^2$$

[Leibovich *et al.*]

Isgur-Wise functions may be expected to be comparable

Lattice could nail this

- $B_s \to D_s \pi$ is pure tree, can help to determine relative size of E vs. C

[CDF '03: $\mathcal{B}(B_s \to D_s^- \pi^+)/\mathcal{B}(B^0 \to D^- \pi^+) \simeq 1.35 \pm 0.43$ (using $f_s/f_d = 0.26 \pm 0.03$)]

Lattice could help: Factorization relates tree amplitudes, need $SU(3)$ breaking in $B_s \to D_s \ell \bar{\nu}$ vs. $B \to D \ell \bar{\nu}$ form factors from exp. or lattice

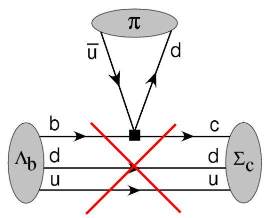
More complicated: $\Lambda_b \to \Sigma_c \pi$

- Recall quantum numbers:

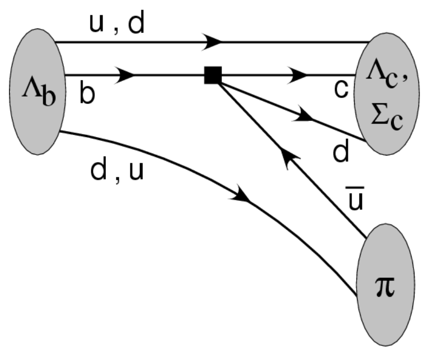
multiplets	s_l	$I(J^P)$
Λ_c	0	$0(\frac{1}{2}^+)$
$\Sigma_c,\ \Sigma_c^*$	1	$1(\frac{1}{2}^+),\ 1(\frac{3}{2}^+)$

$$\Sigma_c = \Sigma_c(2455),\ \Sigma_c^* = \Sigma_c(2520)$$

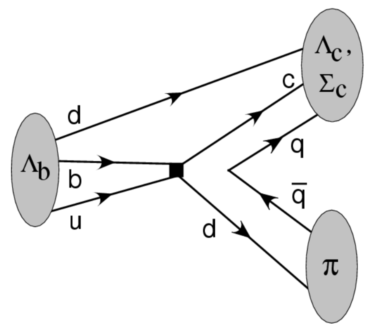
- Can't address in naive factorization, since $\Lambda_b \to \Sigma_c$ form factor vanishes by isospin



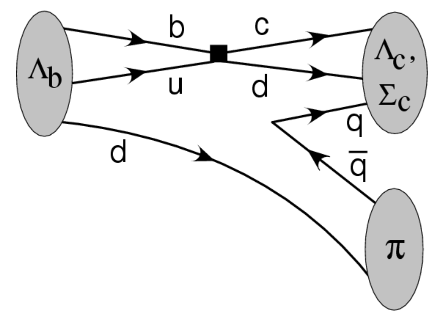
[Leibovich *et al.*]



$C =$ "color commensurate"
$\mathcal{O}(\Lambda_{\rm QCD}/Q)$



$E =$ "exchange"
$\mathcal{O}(\Lambda_{\rm QCD}/Q)$



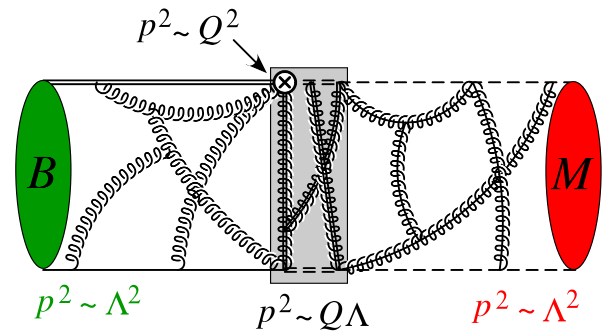
$B =$ "bow-tie"
$\mathcal{O}(\Lambda_{\rm QCD}^2/Q^2)$

- Prediction: $$\frac{\Gamma(\Lambda_b \to \Sigma_c^* \pi)}{\Gamma(\Lambda_b \to \Sigma_c \pi)} = 2 + \mathcal{O}\big[\Lambda_{\rm QCD}/Q\,,\ \alpha_s(Q)\big] = \frac{\Gamma(\Lambda_b \to \Sigma_c^{*0} \rho^0)}{\Gamma(\Lambda_b \to \Sigma_c^0 \rho^0)}$$

Can avoid π^0's from $\Lambda_b \to \Sigma_c^{(*)0}\pi^0 \to \Lambda_c \pi^- \pi^0$ or $\Lambda_b \to \Sigma_c^{(*)+}\pi^- \to \Lambda_c \pi^0 \pi^-$

Semileptonic $B \to \pi, \rho$ form factors

- At leading order in Λ/Q, to all orders in α_s, two contributions at $q^2 \ll m_B^2$: soft form factor & hard scattering
  (Separation scheme dependent; $Q = E, m_b$, omit μ's)
  [Beneke & Feldmann; Bauer, Pirjol, Stewart; Becher, Hill, Lange, Neubert]



$$F(Q) = C_i(Q)\,\zeta_i(Q) + \frac{m_B f_B f_M}{4E^2} \int \mathrm{d}z\mathrm{d}x\mathrm{d}k_+\, T(z,Q)\, J(z,x,k_+,Q)\, \phi_M(x)\phi_B(k_+)$$

- Symmetries $\Rightarrow$ nonfactorizable (1st) term obey form factor relations [Charles *et al.*]

  3 $B \to P$ and 7 $B \to V$ form factors related to 3 universal functions

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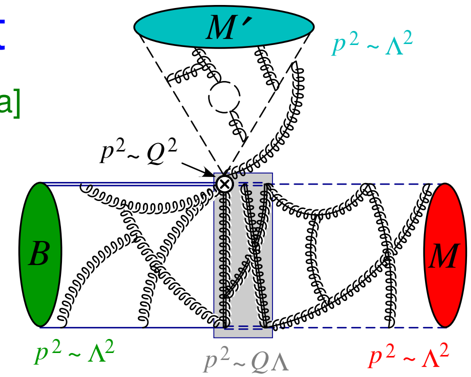
- Relative size? QCDF: 2nd $\sim \alpha_s \times$(1st), PQCD: 1st $\ll$ 2nd, SCET: 1st $\sim$ 2nd
- Whether first term factorizes (involves $\alpha_s(\mu_i)$, as 2nd term does) involves same physics issues as hard scattering, annihilation, etc., contributions to $B \to M_1 M_2$

Charmless $B \to M_1 M_2$ decays

- Limited consensus about implications of the heavy quark limit

[Bauer, Pirjol, Rothstein, Stewart; Chay, Kim; Beneke, Buchalla, Neubert, Sachrajda]



$$A = A_{c\bar{c}} + N\bigg[f_{M_2}\, \zeta^{BM_1} \int \mathrm{d}u\, T_{2\zeta}(u)\, \phi_{M_2}(u) + f_{M_2} \int \mathrm{d}z \mathrm{d}u\, T_{2J}(u,z)\, \zeta_J^{BM_1}(z)\, \phi_{M_2}(u) + (1 \leftrightarrow 2) \bigg]$$

- $\zeta_J^{BM_1} = \int \mathrm{d}x \mathrm{d}k_+\, J(z,x,k_+)\, \phi_{M_1}(x)\, \phi_B(k_+)$ also appears in $B \to M_1$ form factors

  $\Rightarrow$ Relations to semileptonic decays do not require expansion in $\alpha_s(\sqrt{\Lambda Q})$

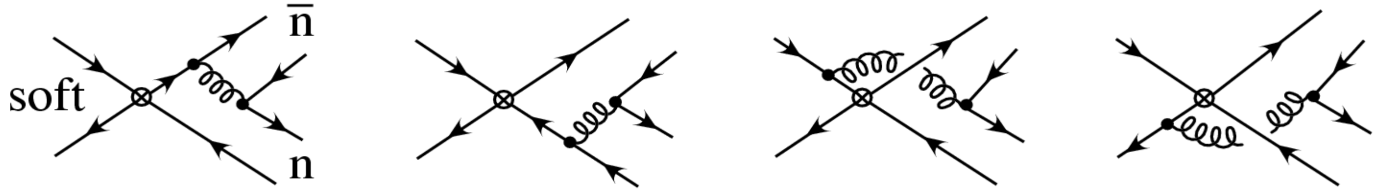
- Charm penguins: suppression of long distance part argued, not proven

  Lore: "long distance charm loops", "charming penguins", "$D\overline{D}$ rescattering" are the same (unknown) term; may yield strong phases and other surprises

- SCET: fit both ζ's and ζ_J's, calculate T's; QCDF: fit ζ's, calculate factorizable (2nd) terms perturbatively; PQCD: 1st line dominates and depends on $k_\perp$

Endpoint singularities (e.g., annihilation)

- Power suppressed $\mathcal{O}(\Lambda/E)$ corrections



Yields convolution integrals of the form: $\int_0^1 \mathrm{d}x\, \phi_\pi(x)/x^2\,, \quad \phi_\pi(x) \sim 6x(1-x)$

Singular if gluon near on-shell — one of the mesons near endpoint configuration

- KLS: first emphasized importance for strong phases and CPV [Keum, Li, Sanda]

Singularity regulated by k_T in $1/(m_b^2 x - k_T^2 + i\varepsilon)$, still sizable phases

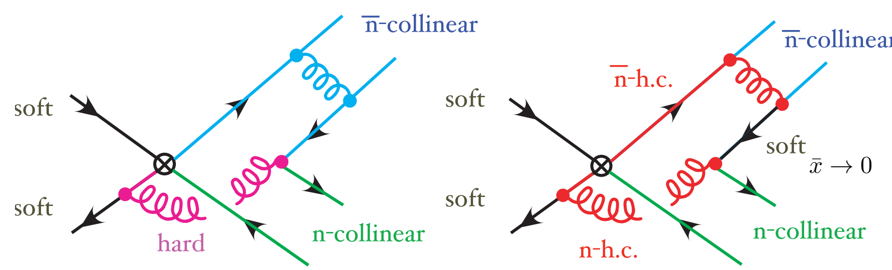
- BBNS: interpret as IR sensitivity $\Rightarrow$ model by complex parameters

$$\text{“}X_A\text{”} = \int_0^1 dx/x \to (1+\rho_A e^{i\varphi_A})\ln(m_B/500\,\mathrm{MeV})$$

[Beneke, Buchalla, Neubert, Sachrajda]

- SCET: singularity to do with double counting

Real & calculable at LO [Arnesen, ZL, Rothstein, Stewart]



Comparison of approaches

- For charmless two-body decays significant differences in details [Stewart @ FPCP'09]

	BPRS	BBNS	KLS
Expansion in $\alpha_s(\mu_i)$?	No	Yes	Yes
T, P if Singular convolution	N/A	New parameters	uses k_T
Annihilation	Real at "LO", complex "NLO"	Complex, new parameters	perturbative, large phases
Charm Loop?	Non-perturbative	Perturbative	Perturbative
Number of fit parameters	Most	Middle	N/A

- Many measurements are well described, some important issues remain...

Extracting α from $B \to \pi\pi$

- Until $\sim$1997 the hope was to determine α simply from:

$$\frac{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)}{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) + \Gamma(B^0(t) \to \pi^+\pi^-)} = S\sin(\Delta m\, t) - C\cos(\Delta m\, t)$$

$\arg\lambda_{\pi^+\pi^-} = (B\text{-mix} = 2\beta) + (\overline{A}/A = 2\gamma + \ldots)$ $\Rightarrow$ measures $\sin 2\alpha$ if amplitudes with one weak phase dominated — relied on expectation that $P/T = \mathcal{O}(\alpha_s/4\pi)$

$K\pi$ and $\pi\pi$ rates $\Rightarrow$ comparable amplitudes with different weak & strong phases

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- Isospin analysis:

Tree and penguin operators: $\Delta I = \frac{1}{2}, \frac{3}{2}$ terms; Bose statistics: $\pi\pi$ in $I = 0, 2$

$(u,\, d)$: I-spin doublet	$(\pi\pi)_{\ell=0}$	$\to$	$I_f = 0$	or	$I_f = 2$
other quarks and gluons: $I = 0$	(1×1)		$(\Delta I = \frac{1}{2})$		$(\Delta I = \frac{3}{2})$

[Note: γ, Z: mixtures of $I = 0, 1$, violate isospin and yield a (small) uncertainty]

$B \to \pi\pi$ results

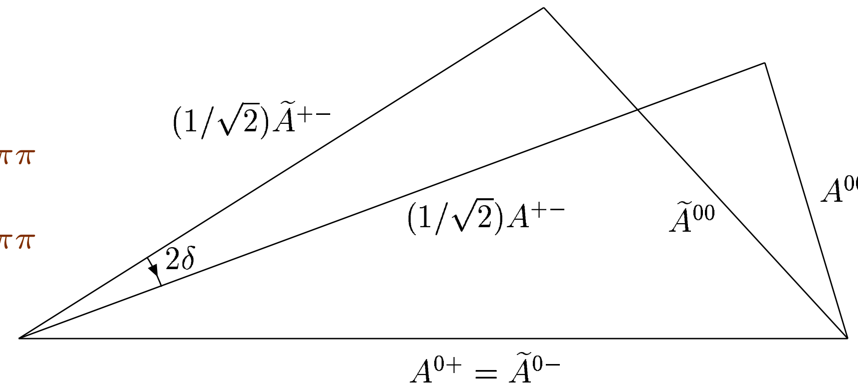
- Two amplitudes for B^+, B^0 and $B^-, \overline{B}^0$ decay:

$$A_{+-} = -\lambda_u(T + P_u) - \lambda_c P_c - \lambda_t P_t = e^{-i\gamma}\, T_{\pi\pi} - P_{\pi\pi}$$

$$\sqrt{2} A_{00} = \lambda_u(-C + P_u) + \lambda_c P_c + \lambda_t P_t = e^{-i\gamma}\, C_{\pi\pi} + P_{\pi\pi}$$

$$\sqrt{2} A_{-0} = -\lambda_u(T + C) = e^{-i\gamma}(T_{\pi\pi} + C_{\pi\pi})$$

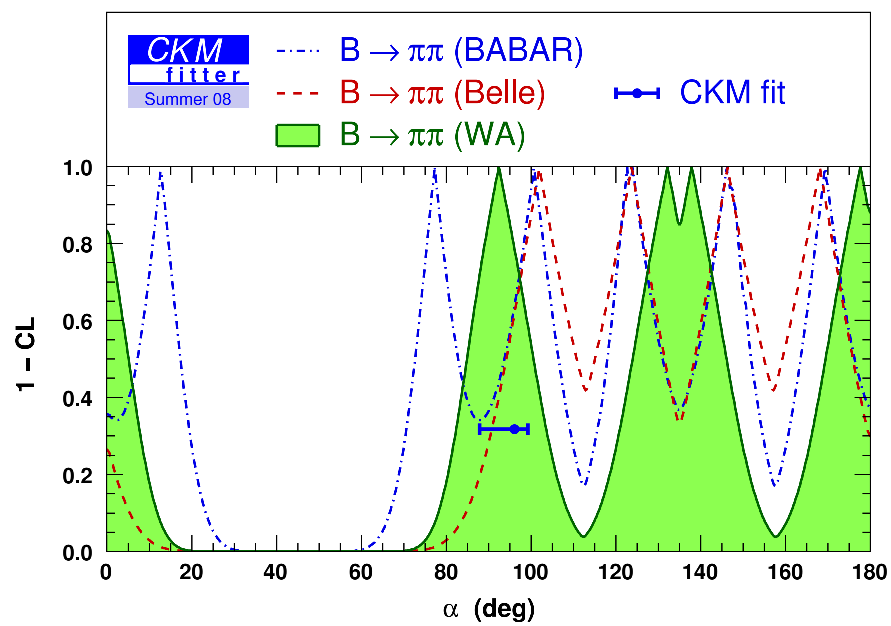
The 6 rates determine α & 5 hadronic parameters



- Need a lot more data — current bound:

$$\alpha - \alpha_{\rm eff} < 15^\circ \; (90\% \text{ CL})$$

- Far from limited by theoretical uncertainty



Puzzles in $B \to \pi\pi$ amplitudes

- Tension remains: BaBar: $C_{\pi^+\pi^-} = -0.25 \pm 0.08$, Belle: $C_{\pi^+\pi^-} = -0.55 \pm 0.09$
- Unexpected features of the data:

  $\mathcal{B}(B \to \pi^0\pi^0) = (1.55 \pm 0.19) \times 10^{-6}$: much bigger than earlier predictions

  $C_{\pi^0\pi^0} = -0.43 \pm 0.25$: expect opposite sign than $C^{(\mathrm{WA})}_{\pi^+\pi^-} = -0.38 \pm 0.06$, $(C \text{ or } T) \pm P$
- Problem: $|C/T|$ cannot be small because $\pi^0\pi^0$ rate is large

  We expect: $\arg(C/T) = \mathcal{O}(\alpha_s, \Lambda/m_b)$, P_u is calculable (small),

  Same sign for $C_{\pi^+\pi^-}$ and $C_{\pi^0\pi^0}$ implies some of:
  - $\arg(C/T)$ not small
  - P_u or P_{ew} not small / NP
  - annihilation not small
  - large fluctuations in the data
- Cannot do better than full isospin analysis, unless this is better understood

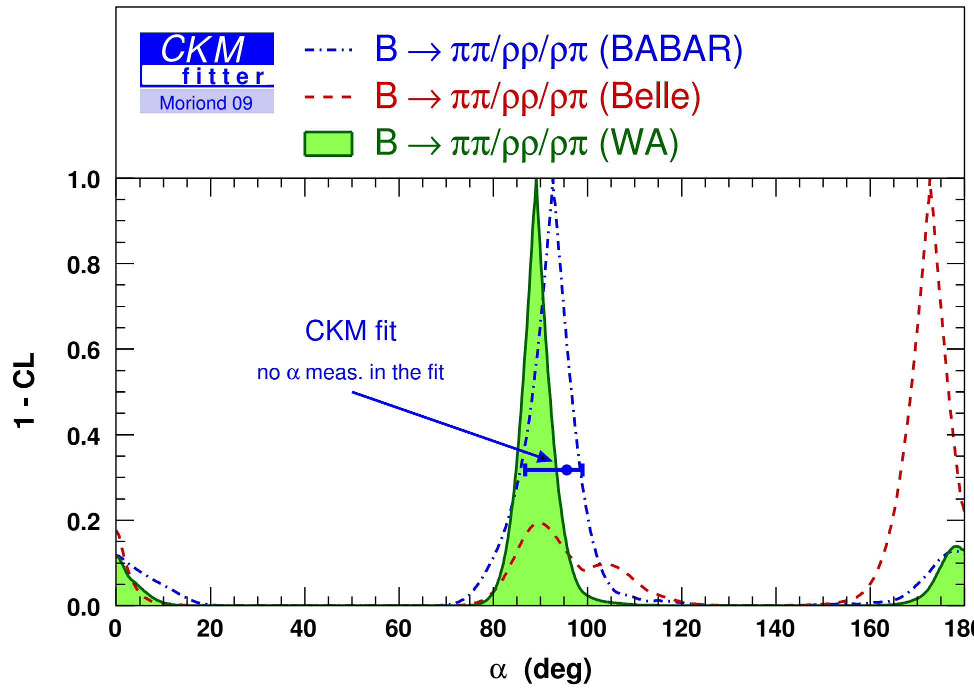
$B \to \rho\rho$: the best α at present

- $\rho\rho$ is mixture of CP even/odd (as all VV modes); data: $CP =$ even dominates
  Isospin analysis applies for each L, or in transversity basis for each $\sigma\ (= 0, \|, \perp)$

- Small rate $\mathcal{B}(B \to \rho^0\rho^0) = (0.73 \pm 0.28) \times 10^{-6}$ (90% CL) $\Rightarrow$ small penguin pollution
  $\frac{\mathcal{B}(B \to \pi^0\pi^0)}{\mathcal{B}(B \to \pi^+\pi^0)} \approx 0.28$ vs. $\frac{\mathcal{B}(B \to \rho^0\rho^0)}{\mathcal{B}(B \to \rho^+\rho^0)} \approx 0.03$

- Ultimately, more complicated than $\pi\pi$, $I = 1$ possible due to finite Γ_ρ, giving $\mathcal{O}(\Gamma_\rho^2/m_\rho^2)$ effects [can be constrained]

  $B \to \rho\rho$ isospin analysis: $\alpha = (90 \pm 5)^\circ$



- Also $B \to \rho\pi$ Dalitz plot analysis

- $\rho\rho$ mode dominates α determination for now, may change at a super B factory

Aside: amplituded ratios from $SU(3)$

- Simple example — compare: $B_d^0 \to \pi^0 K^0$ $(\bar{b} \to q\bar{q}\bar{s})$ vs. $B_s^0 \to \pi^0 \overline{K}^0$ $(\bar{b} \to q\bar{q}\bar{d})$

  $SU(3)$ flavor symmetry (in this case U-spin) implies amplitude relations:

  $$A(B_d^0 \to \pi^0 K^0) = V_{cb}^* V_{cs}\,(P_c - P_t + T_{c\bar{c}s}) + V_{ub}^* V_{us}\,(P_u - P_t + T_{u\bar{u}s}) \equiv P + T$$

  $$A(B_s^0 \to \pi^0 \overline{K}^0) = V_{cb}^* V_{cd}\,(P_c - P_t + T_{c\bar{c}s}) + V_{ub}^* V_{ud}\,(P_u - P_t + T_{u\bar{u}s}) = \lambda\, P + \lambda^{-1}\, T$$

- Assume B_d decay dominated by P, while B_s by T $\Rightarrow$ bound P/T from rates

  Caveats: no B_s data, often more complicated amplitude relations, octets / singlets

- Multi-state amplitude relations: generally weaker bounds, a simple & useful one:

  $$a(\pi^0 K_S) = \frac{1}{\sqrt{2}}\, b(K^+ K^-) - b(\pi^0 \pi^0)$$

  Gives: $|\xi_{\pi^0 K_S}| < 0.14$ — was useful to interpret earlier data

- In precision era, I doubt that $SU(3)$-based methods can establish presence of NP

The old / new $B \to K\pi$ puzzle

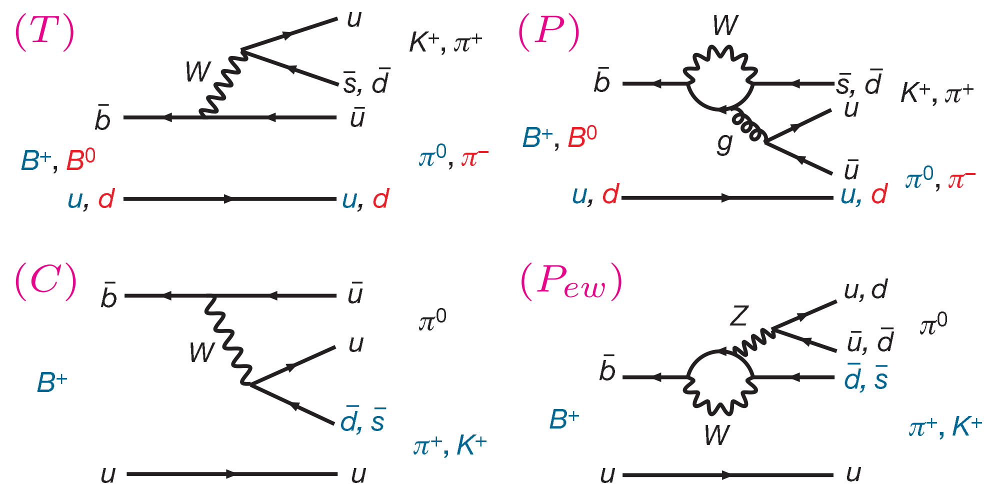
- **Q**: Have we seen new physics in CPV?

$$A_{K^+\pi^-} = -0.098 \pm 0.012 \quad (P+T)$$

$$A_{K^+\pi^0} = 0.050 \pm 0.025 \quad (P+T+C+A+P_{ew})$$

What is the reason for large difference?

$$A_{K^+\pi^0} - A_{K^+\pi^-} = 0.148 \pm 0.028 \quad (> 5\sigma)$$



(Annihilation not shown) [Belle, Nature 452, 332 (2008)]

SCET / factorization predicts: $\arg(C/T) = \mathcal{O}(\Lambda_{\rm QCD}/m_b)$ and $A + P_{ew}$ small

- **A**: huge fluctuation, breakdown of $1/m$ exp., missing something subtle, new phys.

- No similarly transparent problem with branching ratios, e.g., Lipkin sum rule looks OK by now:

$$2\,\frac{\bar\Gamma(B^- \to \pi^0 K^-) + \bar\Gamma(\overline{B}^0 \to \pi^0 \overline{K}^0)}{\bar\Gamma(B^- \to \pi^- \overline{K}^0) + \bar\Gamma(\overline{B}^0 \to \pi^+ K^-)} = 1.07 \pm 0.05 \qquad \text{(should be } \approx 1)$$

Summary

- Lots of progress for $|V_{cb}|$ and $|V_{ub}|$, determinations from exclusive decays largely in the hands of lattice QCD, room for progress in continuum — tension is troubling
- Theoretical tools for rare decays are similar, so developments often simultaneous
- Breakthroughs in understanding nonleptonic decays; unfortunately the best understood cases are not the most interesting to learn about weak scale physics
- More work & data needed to understand the expansions

  Why some predictions work at $\lesssim 10\%$ level, while others receive $\sim 30\%$ corrections

  Clarify role of charming penguins, chirally enhanced terms, annihilation, etc.
- Active field, experimental data stimulated lots of theory developments, expect more work & progress as LHCb and super-B provides challenges & opportunities