

QCD sur réseau: L'interaction forte sur le grill et sur une grille

ECOLE DE GIF

Besse, 6-10 septembre 2010

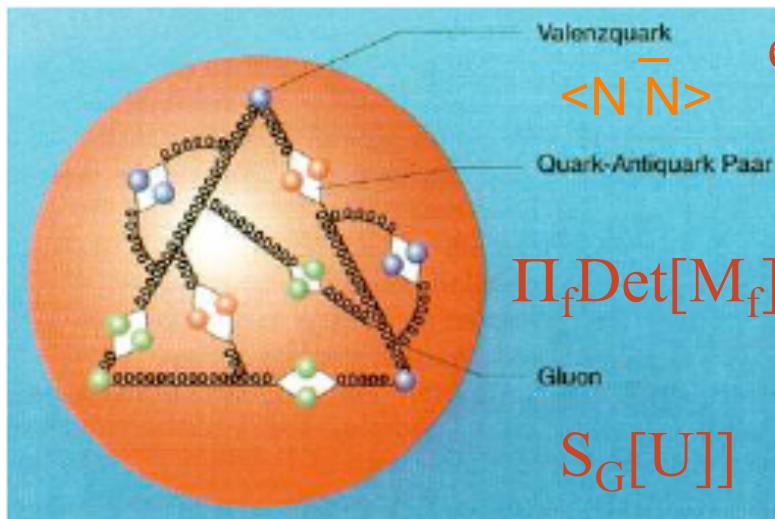
- ✓ The plateau method, nucleon mass.
- ✓ B physics and extrapolation to the b mass
- ✓ Electromagnetic radius,
- ✓ Rho decay

http://www.th.u-psud.fr/page_perso/Pene/GIF_2010/index.html

Example: nucléon propagator

$$\int dU \exp[(-6/g^2) S_G[U]] \Pi_f \text{Det}[M_f] O/Z ; Z = \int dU \exp[(-6/g^2) S[U]]$$

où $S[U] = S_G[U] + T_R \{\log [M_f]\}$



$$\exp[(-6/g^2) S_G[U]] \Pi_f \text{Det}[M_f] N(x)N(y)/Z$$

G.Schierholz

We will compute the nucleon propagator from the quarks propagating **in a background field**: every one of the gauge configuration ensemble selected by Monte-Carlo. The average over the ensemble will give **the interaction generated by the Fields $U_\mu(x)$ and the dynamical quarks**. The ensemble has been generated by Monte-Carlo according to the chosen lattice action.

We never compute a diagram

$$\langle 0 | N_a(x) N_a(y) | 0 \rangle = \int dU \exp[(-6/g^2) S_G[U]] \Pi_f \text{Det}[M_f] N_a(x) \bar{N}_a(y) / Z$$

$$Z = \int dU \exp[(-6/g^2) S_G[U]] \Pi_f \text{Det}[M_f] \approx \sum_i N_a^i(x) N_a^i(y)$$

$\bar{N}_a(y)$ creates a nucleon and resonances N^*

$N_a(x)$ annihilates the nucleon and resonances N^*

$$N_a \equiv \bar{\epsilon}^{ijk} (u^i C \gamma_5 d^j) u_a^k \quad \bar{N}_a \equiv \epsilon^{ijk} (\bar{u}^i C \gamma_5 \bar{d}^j) \bar{u}_a^k$$

This is an **interpolating field** for the nucleon

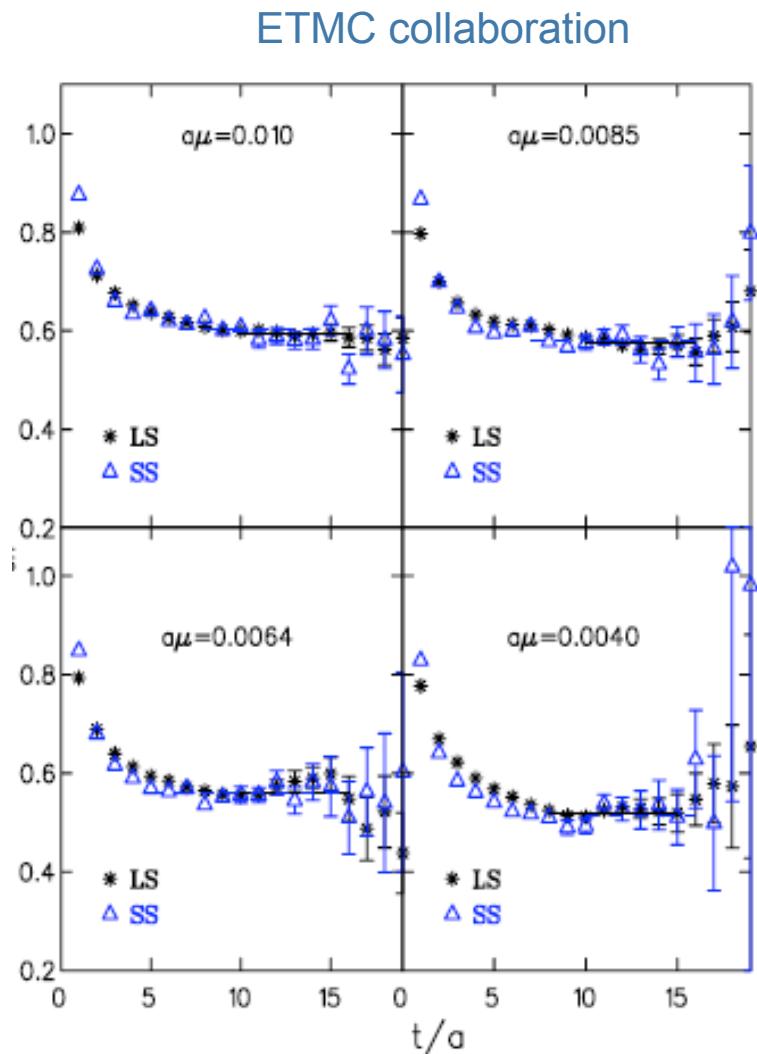
$$C = i \gamma_2 \gamma_0$$

$$G_{2ab}(t_x, \vec{p}') \equiv \sum_{\vec{x}} e^{-i\vec{p}'\vec{x}} \langle 0 | N_a(x) \bar{N}_b(0) | 0 \rangle , \quad \sum_n |n\rangle \langle n| = 1$$

$$G_{2ab}(t_x, \vec{p}') = V_3 \sum_n \langle 0 | N_a(0) | n \rangle \langle n | \bar{N}_b(0) | 0 \rangle e^{-E_n t_x} ,$$

For large t_x , only the state with minimal energy contributes. The exponential slope E_0 of G_2 at large time yields the energy of a nucleon of momentum p

In practice, we compute the average, over the sample of gauge configurations, of the product of three quark propagators computed in the background of these gauge configurations. The three quark propagators spin-color indices are combined according to the combination of quark fields in the nucleon interpolating fields.



For every configuration of the random sample, we compute the quark propagator. We then combine three propagators with the tensors ϵ^{ijk} et $C \gamma_5$ (cf preceding)

The propagator of a quark from x to y is the inverse matrix: $S(x,y)=M_f^{-1}(x,y)$

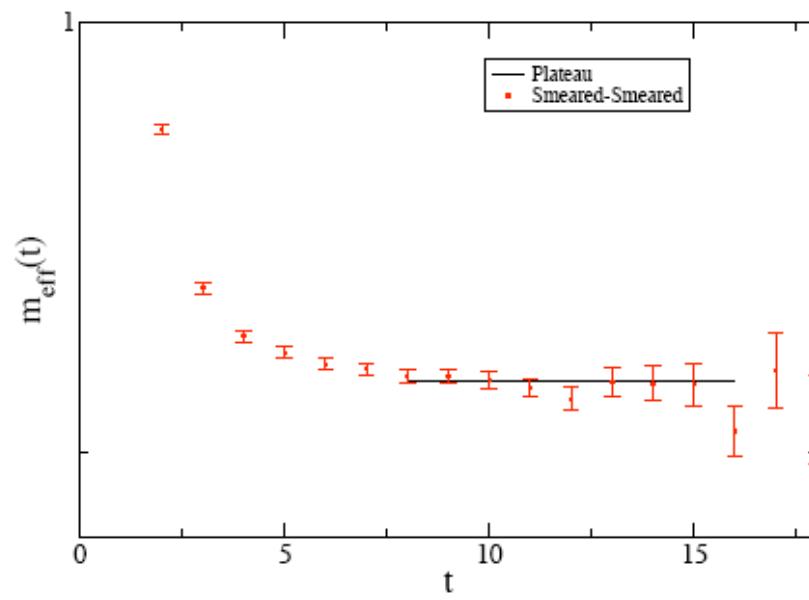
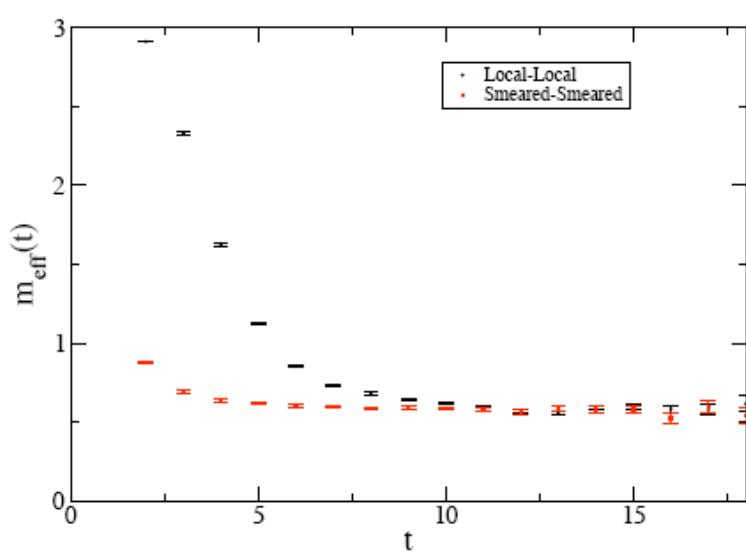
Inversion in the space of sites, colors and spins



One sees that the nucleon
is isolated after a time of
 $\sim 0.5-1.$ fm/c

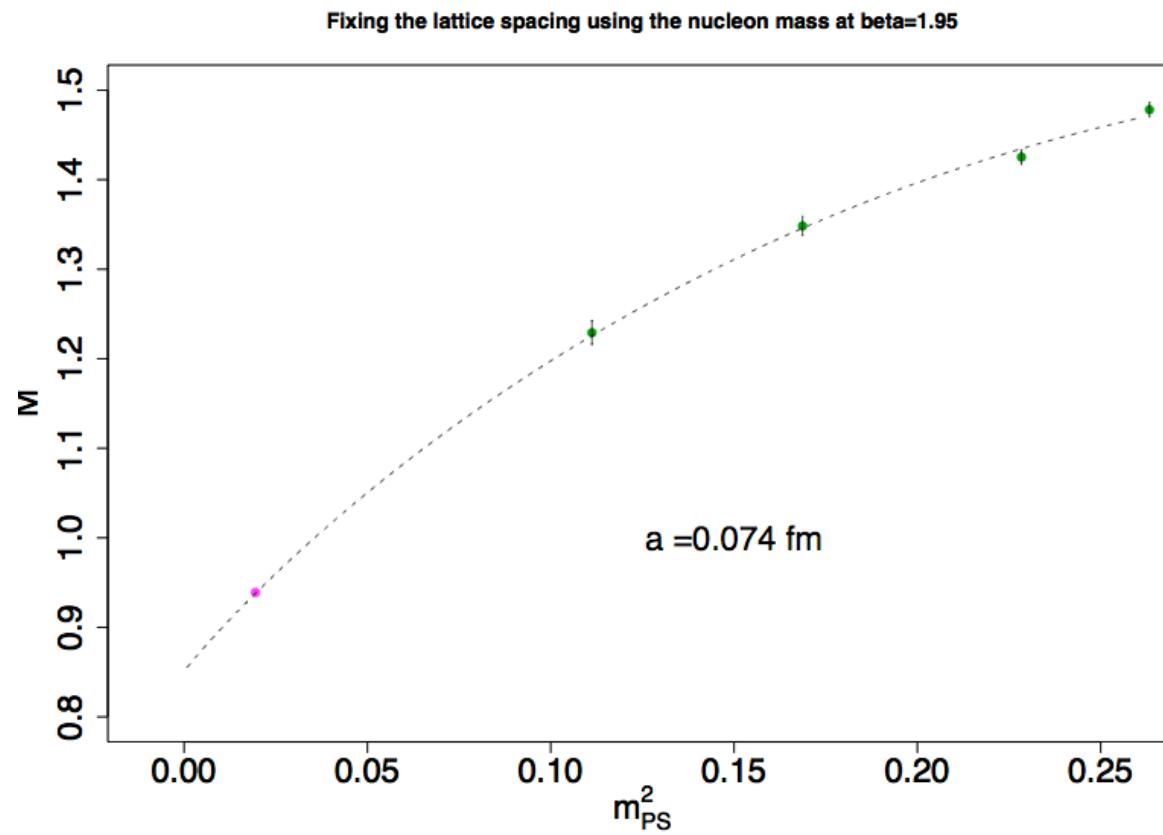
Nf=2 twisted quarks $a=0.0087$ fm

$$aM(t_x) = \log[G_2(t_x)/G_2(t_x+a)]$$



ETMC collaboration

Grenoble, V.Drach,



La masse du pion
Sert à estimer la
masse des quarks
Légers :

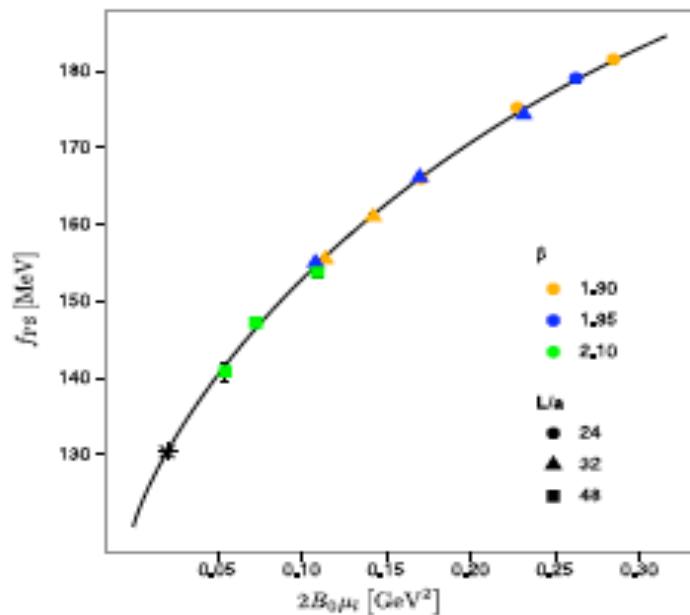
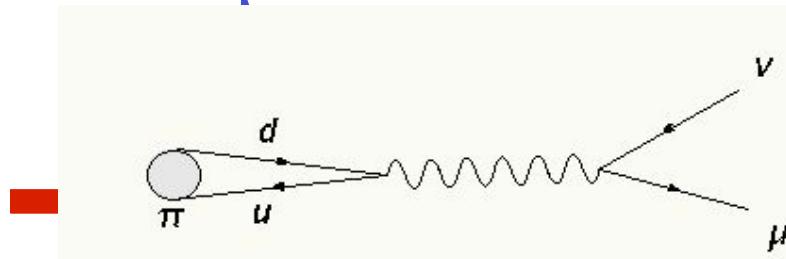
$$m_\pi^2 \propto m_q$$

formule de fit:

$$\dots_N = m_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3$$

Quatre quarks dynamiques (de la mer): u,d,s,c

$\pi \rightarrow \mu\nu$



Herdoiza, Reker, lattice 2010

Les pions meurent aussi

Le taux de désintégration $\pi \rightarrow \mu\nu$ est proportionnel à des facteurs connus multipliés par une constante f_π^2 qui dépend de l'interaction forte.

La valeur de f_π dépend de la masse des quarks u et d selon une loi connue la «théorie effective chirale» valable quand les masses des quarks sont suffisamment petites

$$a=0.06, 0.08, 0.09 \text{ fm}$$

	$N_f = 2 + 1 + 1$	$N_f = 2$
$\bar{\ell}_3$	3.70(27)	3.50(31)
$\bar{\ell}_4$	4.67(10)	4.66(33)
f_π/f_0	1.076(3)	1.076(9)

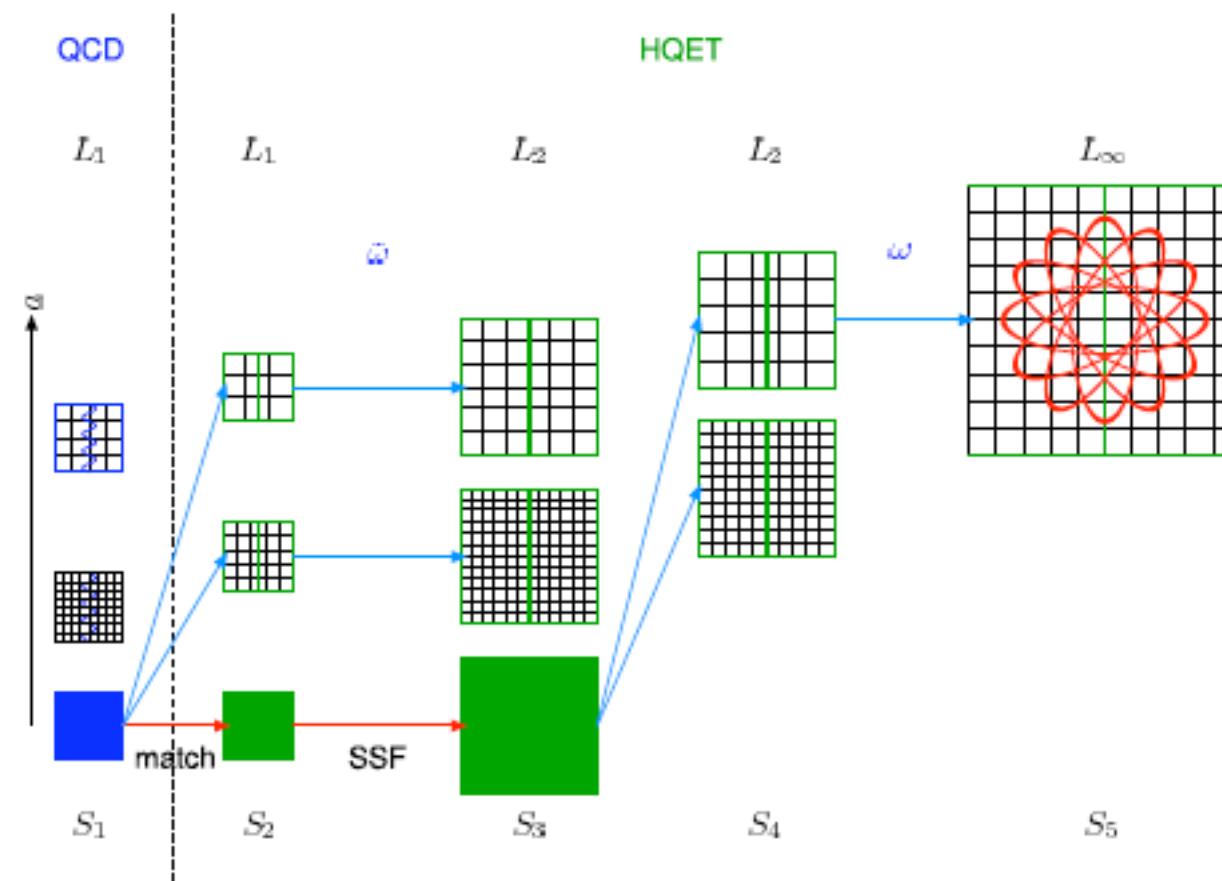
Festival des Saveurs *Lourdes*



La masse du b: $a mb \gg 1$, gros effets de discréétisation:

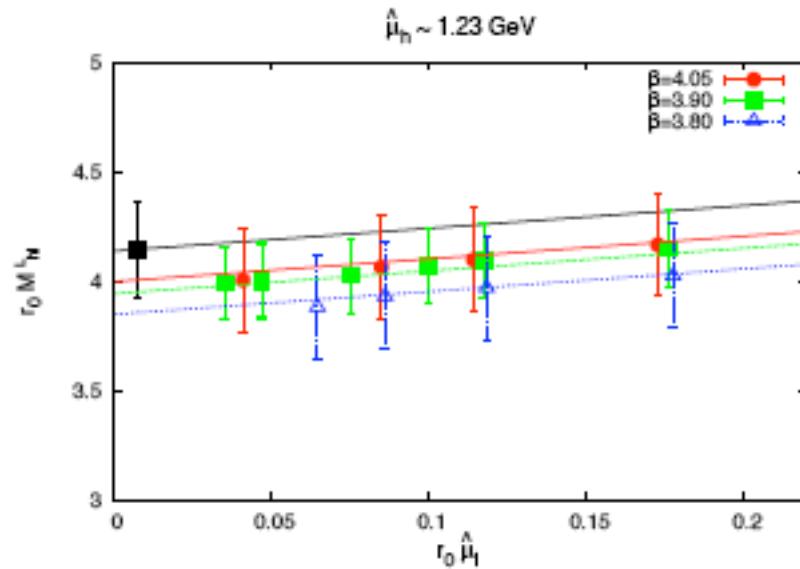
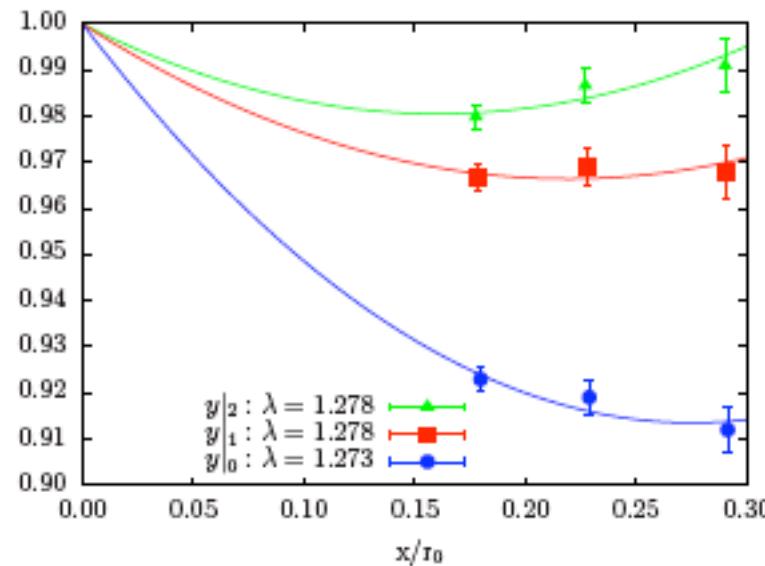
Step scaling method: calculer des rapports de quantités pour des mailles (et des volumes) de plus en plus grands dans un rapport constant 2.

groupe ALPHA



La masse du b: a mb >> 1, gros effets de discréétisation:
 Un nouvelle méthode, calculer des rapports de quantités
 pour des masses dans un rapport constant (1.2) depuis
 le charme jusqu'au B

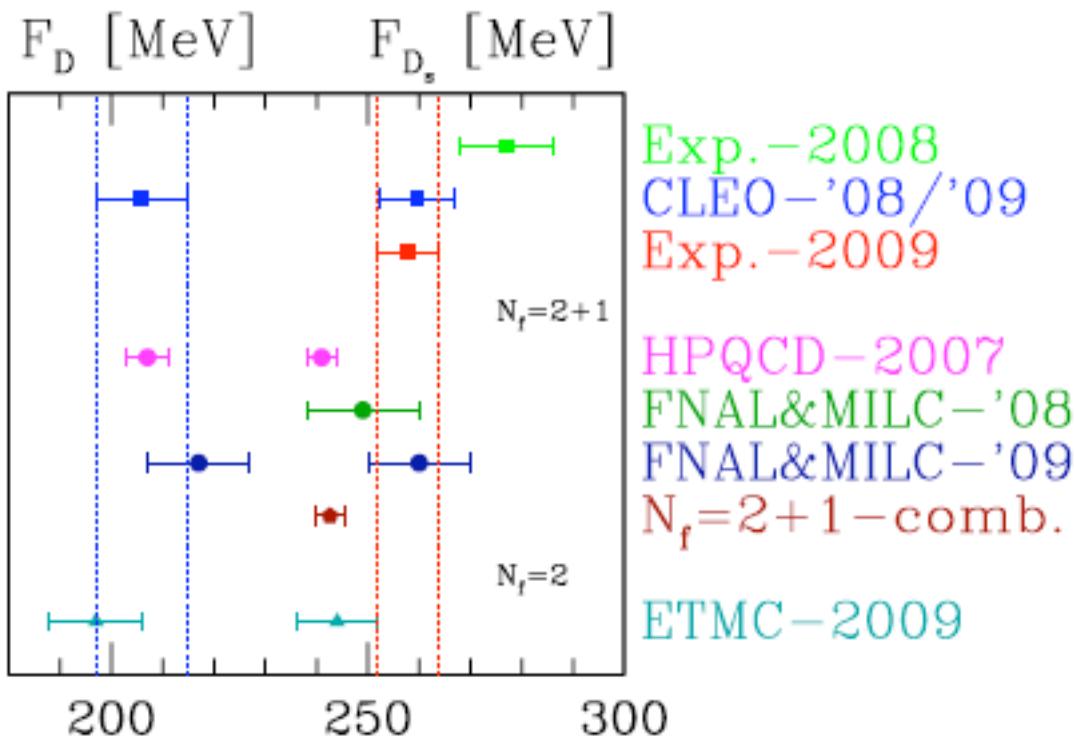
ETMC, Blossier et al., JHEP1004(2010)049



Results for $N_f = 2$ maximally twisted mass Wilson fermions:

$$\overline{m}_b^{\overline{\text{MS}}} (\overline{m}_b) = 4.63(27) \text{ GeV} \quad F_B = 194(16) \text{ MeV} \quad F_{B_s} = 235(12) \text{ MeV}$$

Problème avec F_{D_s} ?

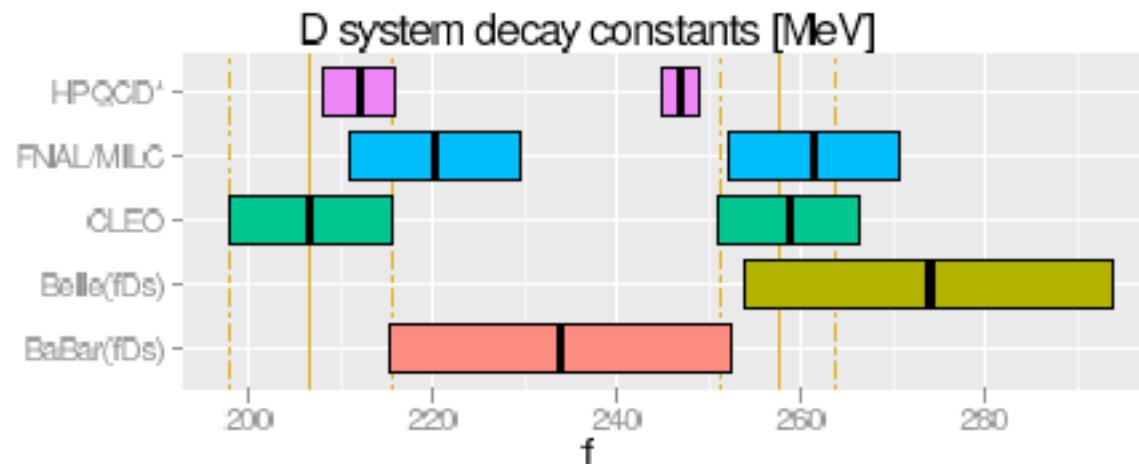


Heitger, lattice 2010

Among the possible explanations for the discrepancy between experiment and lattice:

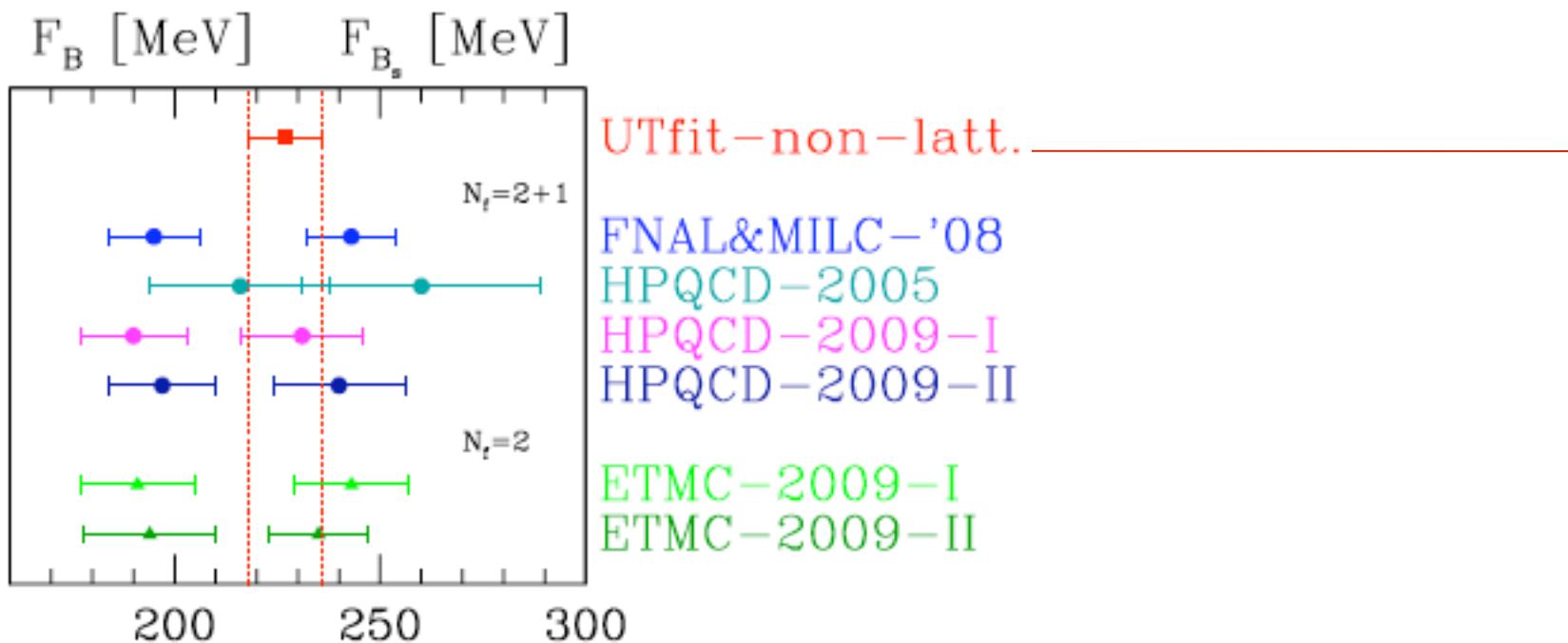
- ▶ Experimental issues ?
- ▶ Systematic effect, e.g., discret. error missed ?
- ▶ Tension = Hint of new physics in the flavour sector ?

After many updates



"Puzzle" seems to disappear:

No conclusive evidence for New Physics in the charm quark sector yet, but the $D_{(s)}$ leptonic decays will continue to help constraining SM extensions



Goal of lattice computations:

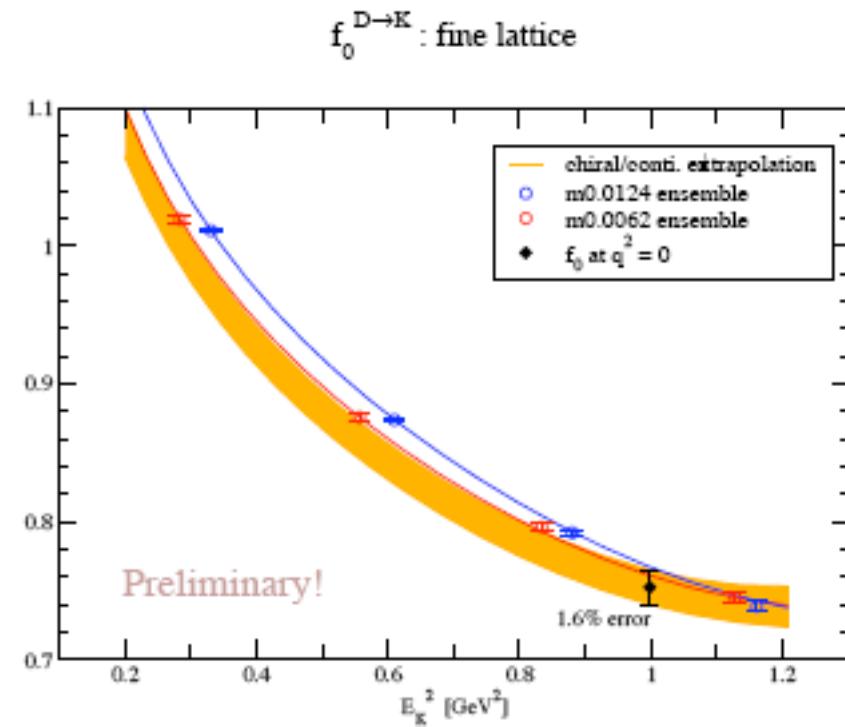
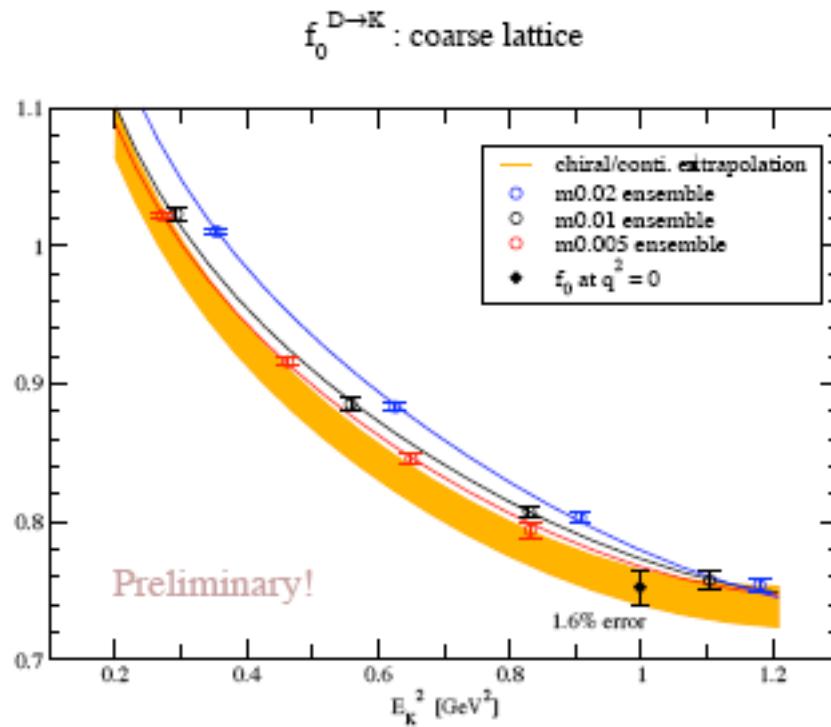
$O(10\%) \rightarrow O(3\%)$ errors; better control of α - and mass effects, NP renormalization

Independent determination of $|V_{cs}|, |V_{cd}|$; holds $|V_{ud}| \approx |V_{cs}|$ actually ?

$D \rightarrow \pi \ell \nu_\ell$ for massless leptons

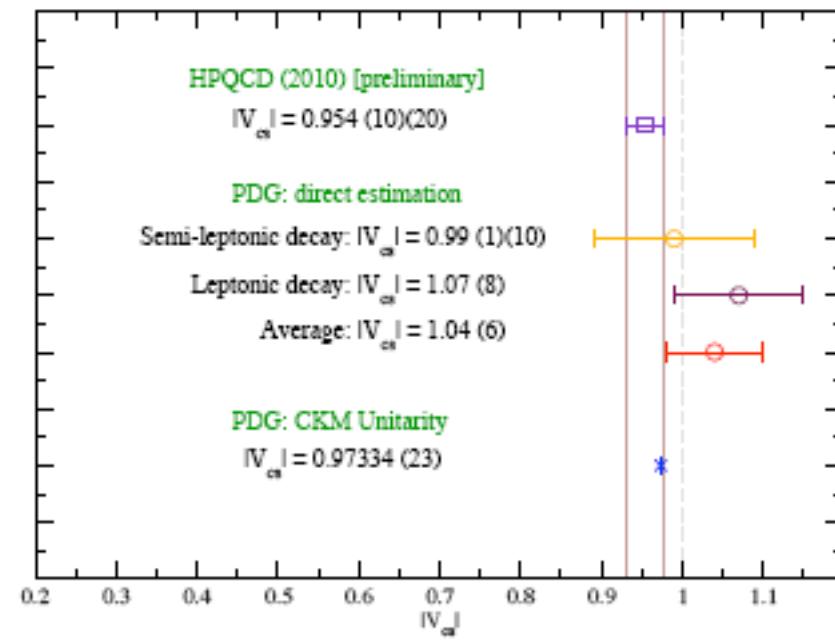
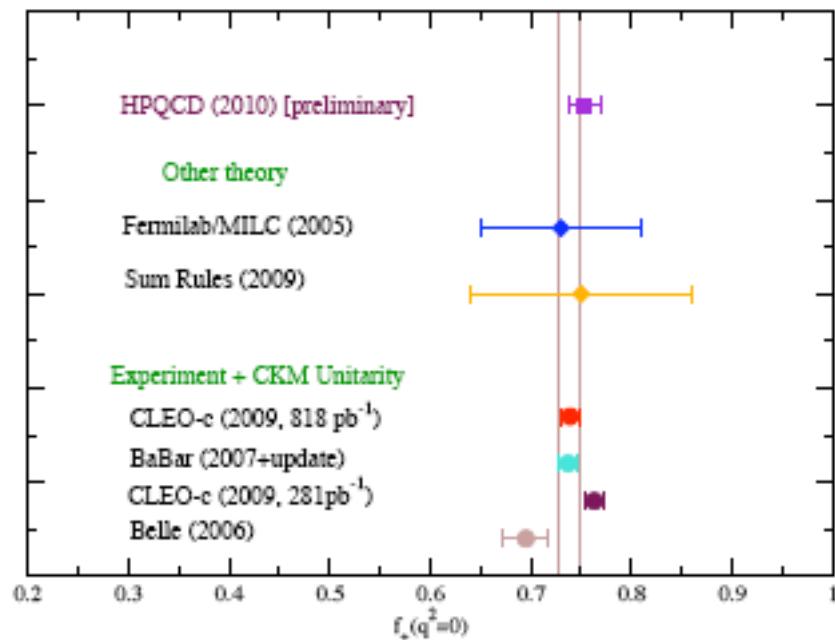
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3 m_D^3} [(m_D^2 + m_\pi^2 - q^2)^2 - 4m_D^2 m_\pi^2]^{\frac{3}{2}} |f_+(q^2)|^2 |V_{cd}|^2$$

The same for $D \rightarrow K \ell \nu$, with $V_{cd} \rightarrow V_{cs}$, results from HPQCD:



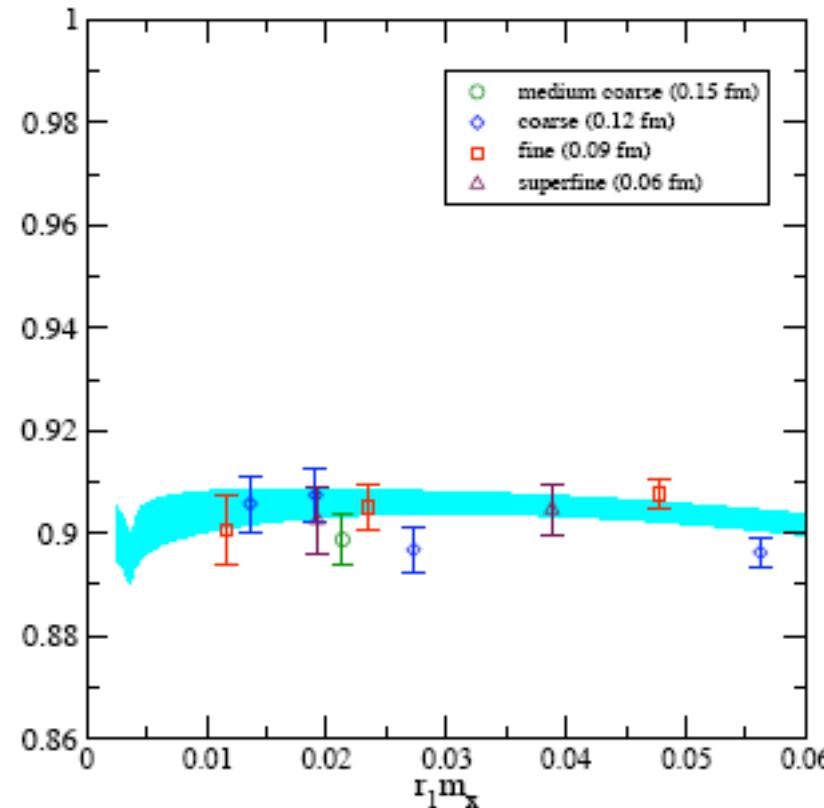
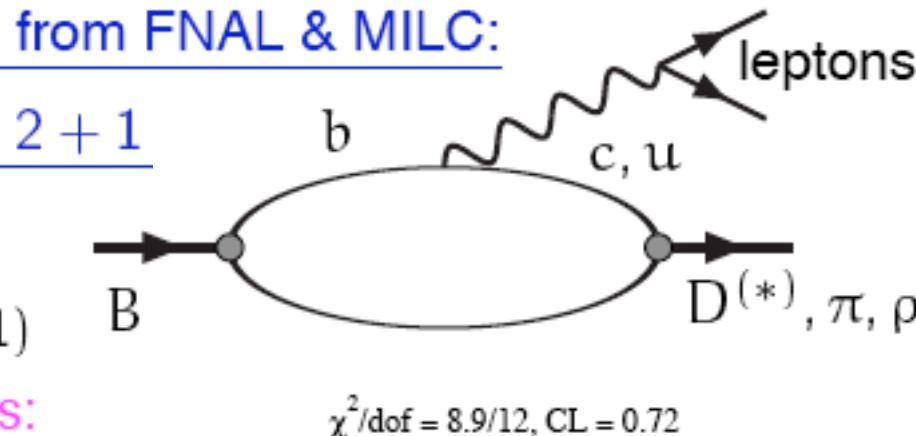
$$f_+(q^2 = 0) = 0.753(12)(10) \text{ [(stat)(syst)]}$$

$$|V_{cs}| = 0.954(10)(20) \text{ [(exp)(lat)]}$$



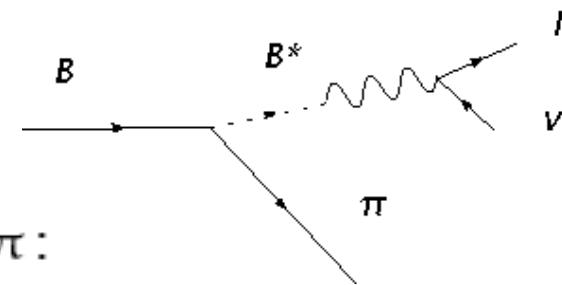
Status of $B \rightarrow D^* \ell \nu_\ell$ for $N_f = 2 + 1$

- Zero recoil \Rightarrow just $F(1) \equiv h_A(1)$
- **Double ratios of matrix elements:**
Cancellations of stat. errors and renormalization, left perturbative matching uncertainty small
- $a \approx (0.06 - 0.15) \text{ fm}$, quadrupled statistics
- $F_{\text{blind}} F(1) = 0.8949(51)(88)(72)(93)(50)(30)$
(errors due to statistics, $g_{D^* D \pi}$, chiral)



$$\langle B^0(\pi) | d\gamma_\mu \gamma_5 u | B^{*+} \rangle \Rightarrow B \rightarrow \pi l \bar{\nu}$$

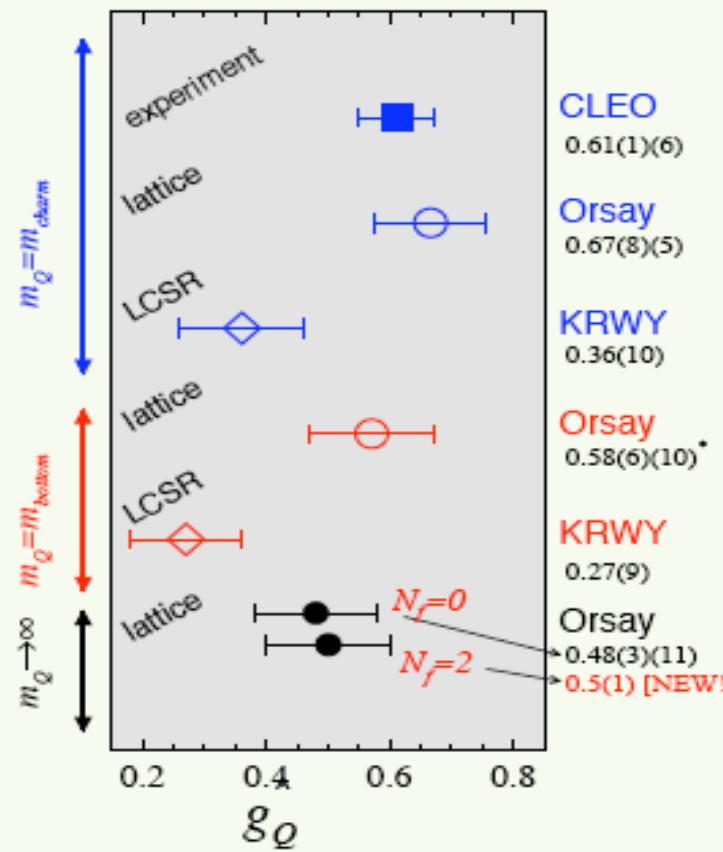
$$F(q^2) \propto g_{B^* B \pi} / (q^2 - M_{B^*}) + \dots$$



Matrix element for the strong decay $B^* \rightarrow B \pi$:

$$\langle B^0(p) \pi^+(q) | B^{*+}(p') \rangle \equiv -g_{B^* B \pi}(q^2) q_\mu \eta^\mu(p') (2\pi)^4 \delta(p' - p - q)$$

Selection of previous results



$N_f = 0$ lattice
and light cone QCD sum rules results

[compilation by Bećirević et al. @ Lattice 2005]

$N_f = 2$ results:

► $g^{\text{stat}} = 0.516(5)_{\text{stat}}(31)_{\chi}(28)_{\text{PT}}(28)_a$
[Ohki et al, 2008]

► $g^{\text{stat}} = 0.44(3)^{+0.07}_{-0.00}$
[Bećirević et al. et al, 2009]

$B \rightarrow D^{**} l\nu$

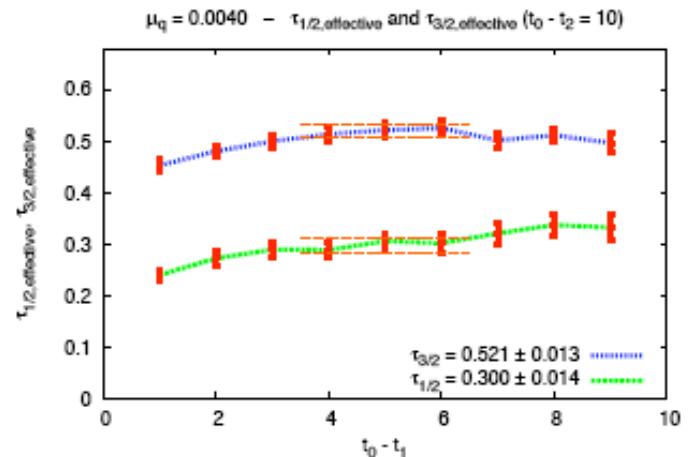
the 1/2 3/2 puzzle ???

Quark model thesis of V.Morenas

TABLE II: Results of the $D^{(*)}\pi^+$ pair invariant mass study. $\mathcal{B}(\text{mode}) = \mathcal{B}(B \rightarrow D^{**} l\nu) \times \mathcal{B}(D^{**} - D^{(*)}\pi^+)$. The first error is statistical and the second is systematic.

	Mode	Yield	$\mathcal{B}(\text{mode}), \%$	Signif.
1/2 →	$B^+ \rightarrow D_0^{*0} \ell^+ \nu$	102 ± 19	$0.24 \pm 0.04 \pm 0.06$	5.4
3/2 →	$B^+ \rightarrow D_2^{*0} \ell^+ \nu$	94 ± 13	$0.22 \pm 0.03 \pm 0.04$	8.0
1/2 →	$B^0 \rightarrow D_0^{*-} \ell^+ \nu$	61 ± 22	$0.20 \pm 0.07 \pm 0.05$	2.6
			< 0.4 @ 90% C.L.	
3/2 →	$B^0 \rightarrow D_2^{*-} \ell^+ \nu$	68 ± 13	$0.22 \pm 0.04 \pm 0.04$	5.5
1/2 →	$B^+ \rightarrow D_1^{*0} \ell^+ \nu$	-5 ± 11	< 0.07 @ 90% C.L.	
3/2 →	$B^+ \rightarrow D_3^{*0} \ell^+ \nu$	81 ± 13	$0.42 \pm 0.07 \pm 0.07$	6.7
3/2 →	$B^+ \rightarrow D_1^{*0} \ell^+ \nu$	35 ± 11	$0.18 \pm 0.06 \pm 0.03$	3.2
1/2 →	$B^0 \rightarrow D_1^{*-} \ell^+ \nu$	4 ± 8	< 0.5 @ 90% C.L.	
3/2 →	$B^0 \rightarrow D_1^{*-} \ell^+ \nu$	20 ± 7	$0.54 \pm 0.19 \pm 0.09$	2.9
			< 0.9 @ 90% C.L.	
3/2 →	$B^0 \rightarrow D_3^{*-} \ell^+ \nu$	1 ± 6	< 0.3 @ 90% C.L.	

$j^{\mathcal{P}}$	$J^{\mathcal{P}}$
$(1/2)^- \equiv S$	$0^- \equiv B, D$
	$1^- \equiv B^*, D^*$
$(1/2)^+ \equiv P_-$	$0^+ \equiv D_0^* \equiv D_0^{1/2}$
	$1^+ \equiv D_1' \equiv D_1^{1/2}$
$(3/2)^+ \equiv P_+$	$1^+ \equiv D_1 \equiv D_1^{3/2}$
	$2^+ \equiv D_2^* \equiv D_2^{3/2}$



Belle 2007

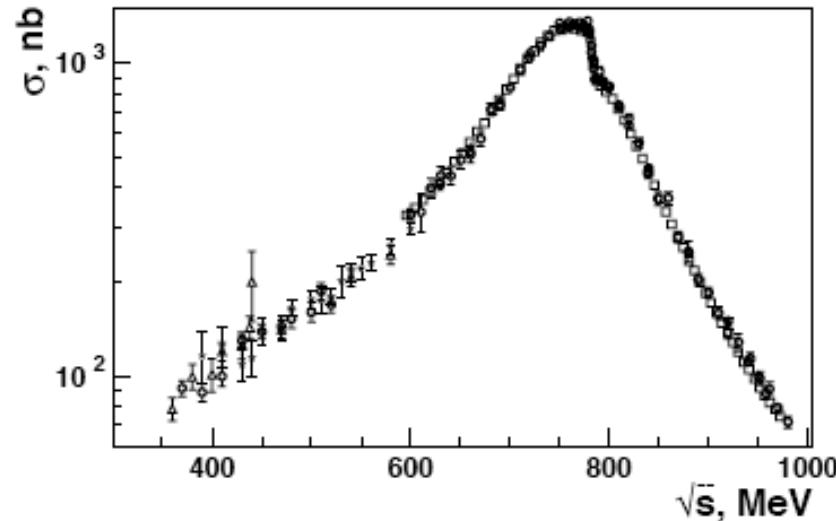
$$\tau_{1/2}^{\text{phys}}(1) = 0.297(26), \quad \tau_{3/2}^{\text{phys}}(1) = 0.528(23).$$

$\tau_i(w) = \tau_i(1)[1 + \hat{\tau}'_i(w-1)]$, and the following relation: $\hat{\tau}'_{1/2} = \hat{\tau}'_{3/2} + 0.5$ [18]. A simultaneous fit to the w -distributions for D_0^* and D_2^* gives $\hat{\tau}'_{3/2} = -1.8 \pm 0.3$. Using the measured branching ratios of $B \rightarrow D_{0,2}^* \ell \nu$, we also calculate $\tau_{3/2}(1) = 0.75$ and $\tau_{1/2}(1) = 1.28$. All parameters are in agreement with expectations except for $\tau_{1/2}(1)$, which is larger than predicted due to the large value of $\mathcal{B}(B \rightarrow D_0^* \ell \nu)$.

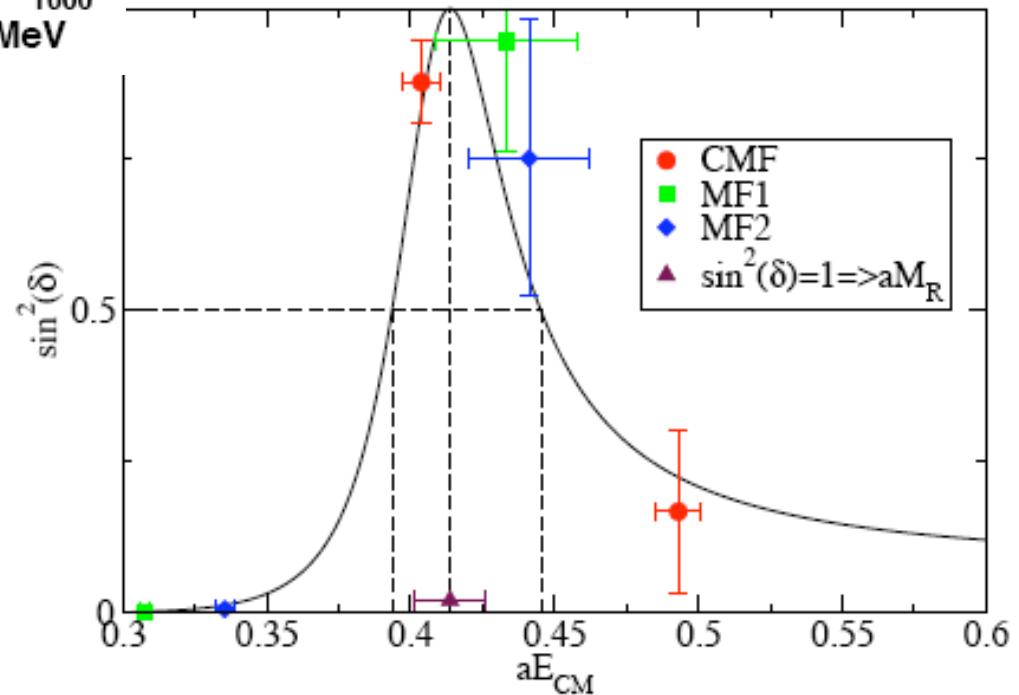
Blossier, OP, Wagner
Using a « trick »
Leibovich et al
PRD57,306 (96)

La désintégration annoncée Du méson ρ

Rho decay (new)

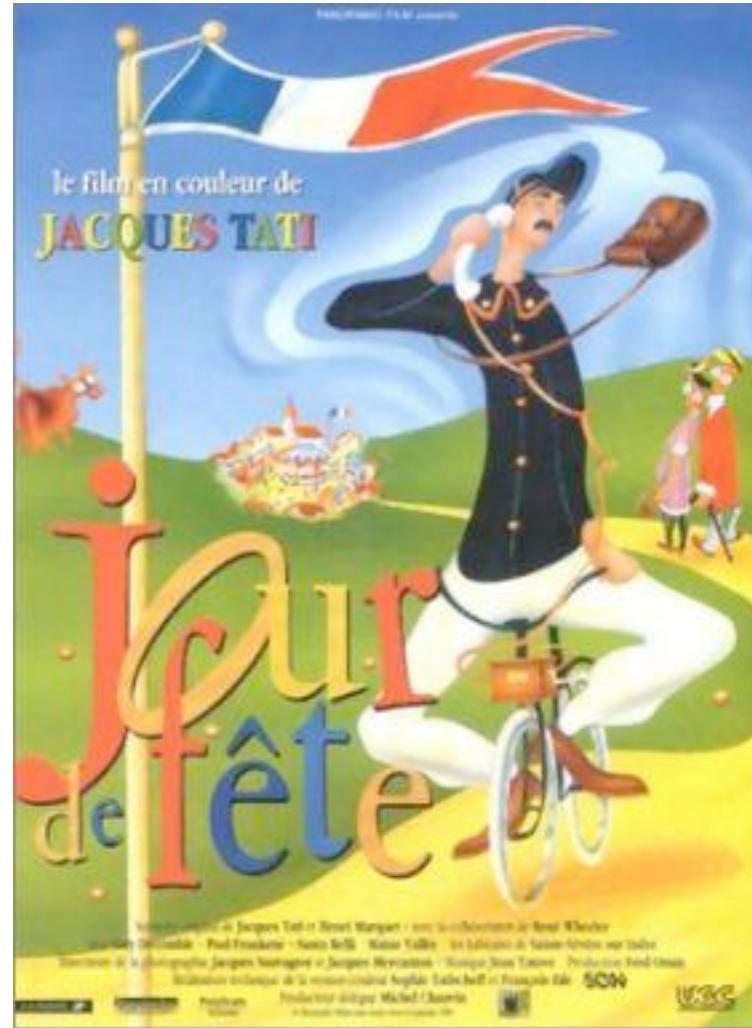


Experiment



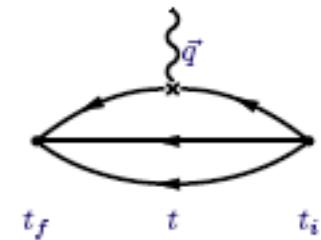
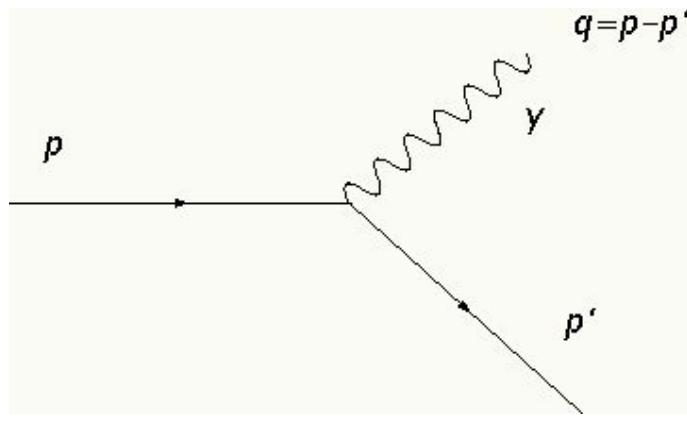
Lattice, ETMC
Xu Feng, Dru Renner

Un facteur en forme



Et maintenant
Des facteurs de forme

Nucleon form factors



Pour calculer le facteur de forme (FF) d'un courant on utilise une fonction de Green à trois points: champ interpolant qui crée le nucléon, le courant dont on cherche le FF et le champ interpolant qui annihile.

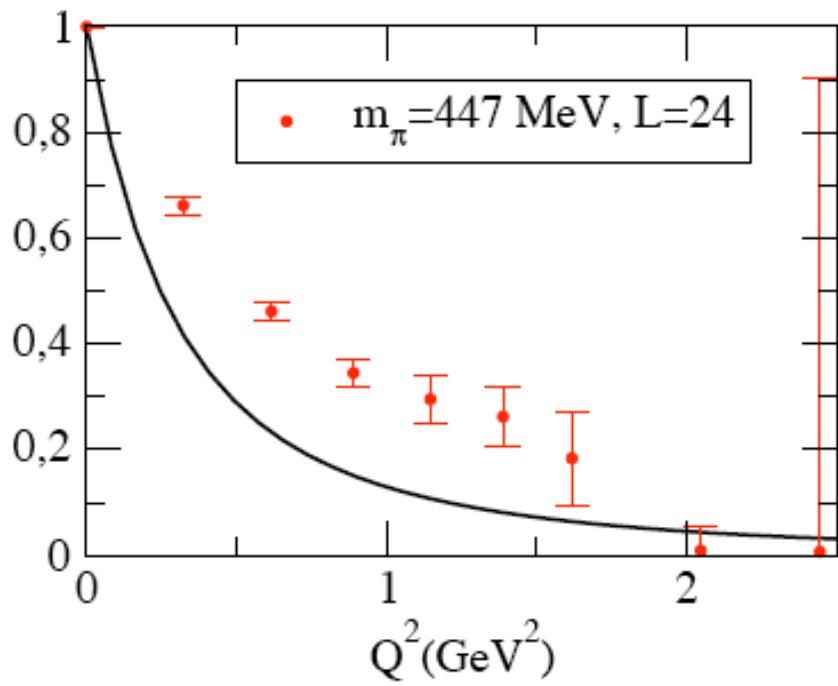
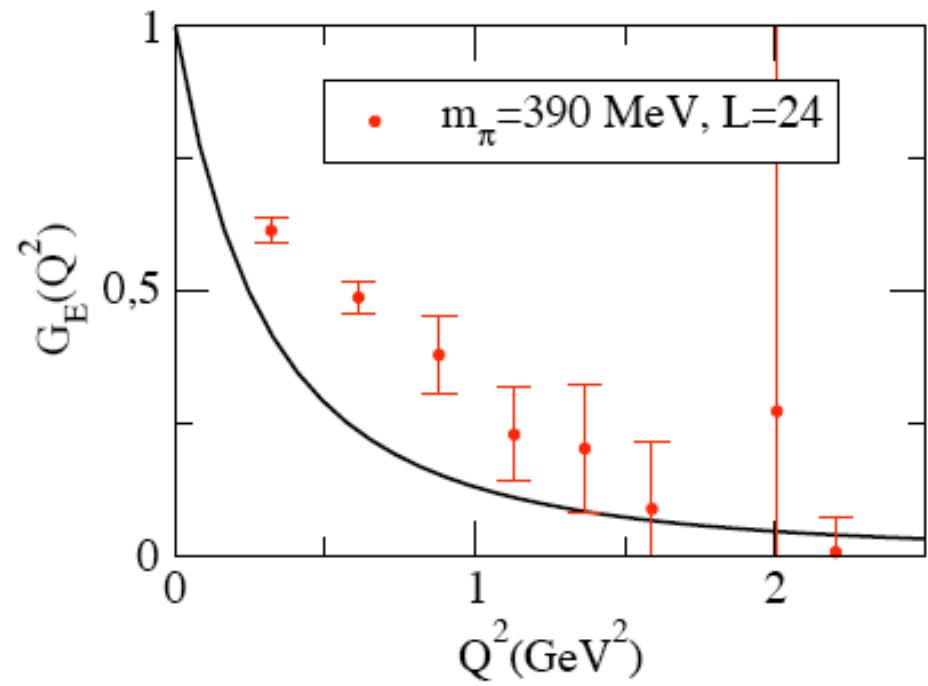
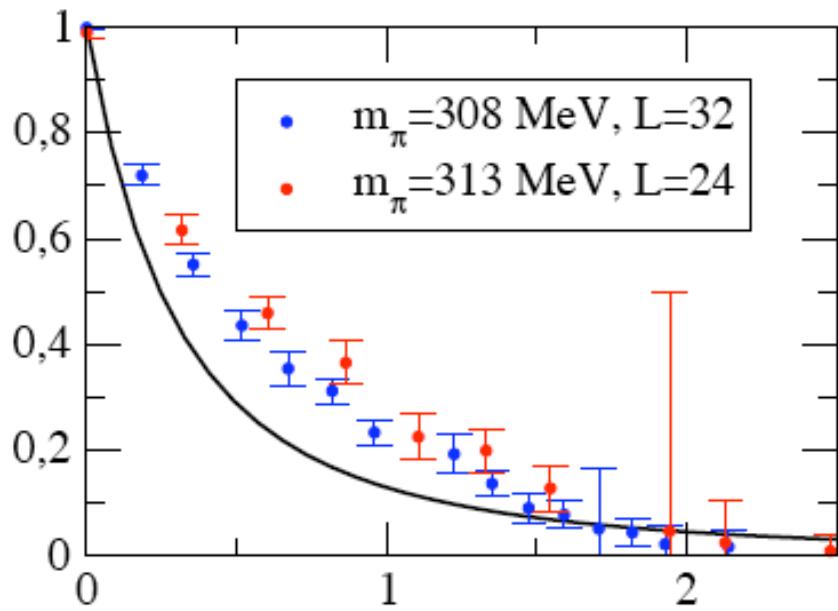
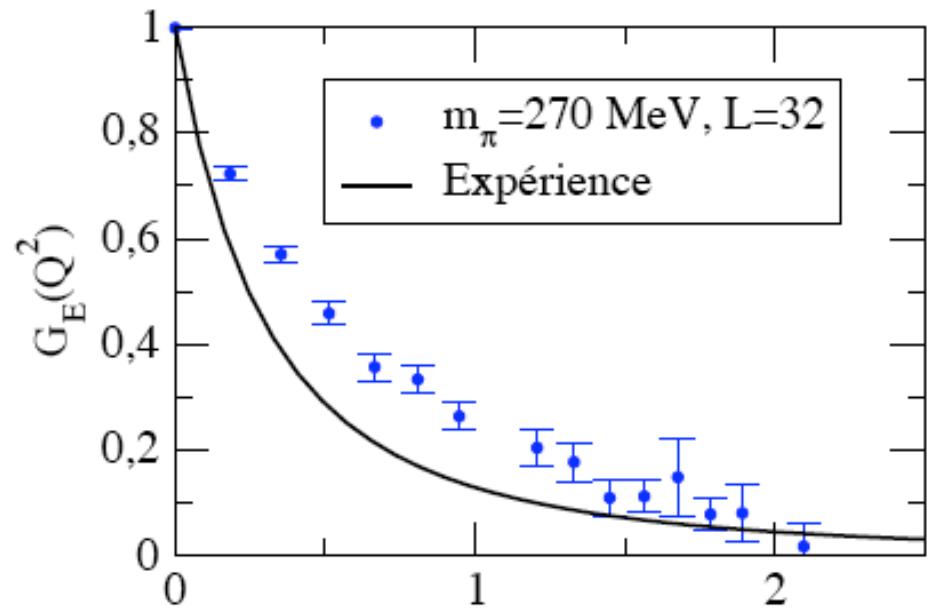
For nucleons, there are two independent Lorentz scalar functions of q^2 defining the form factor. We can use F_1, F_2 or G_E, G_M

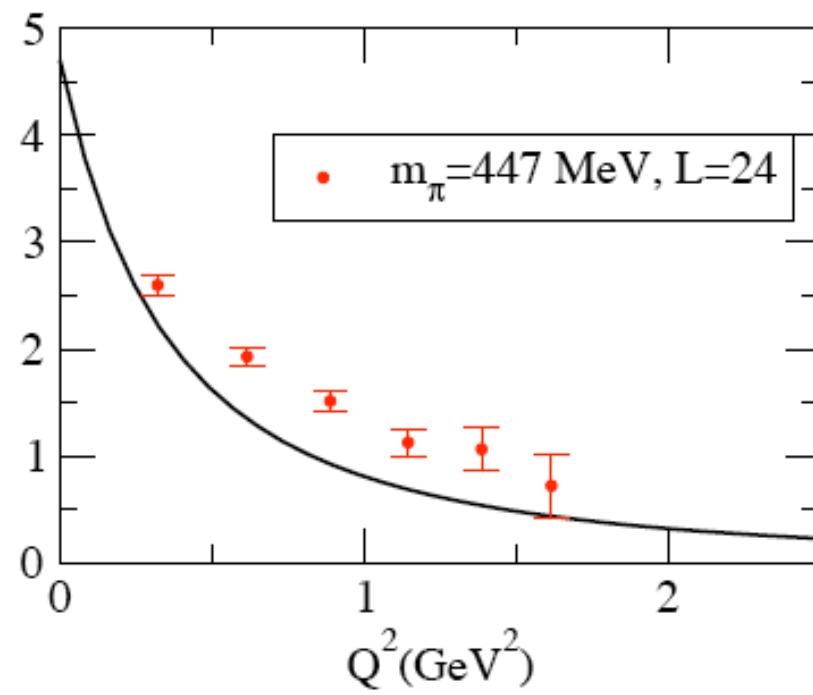
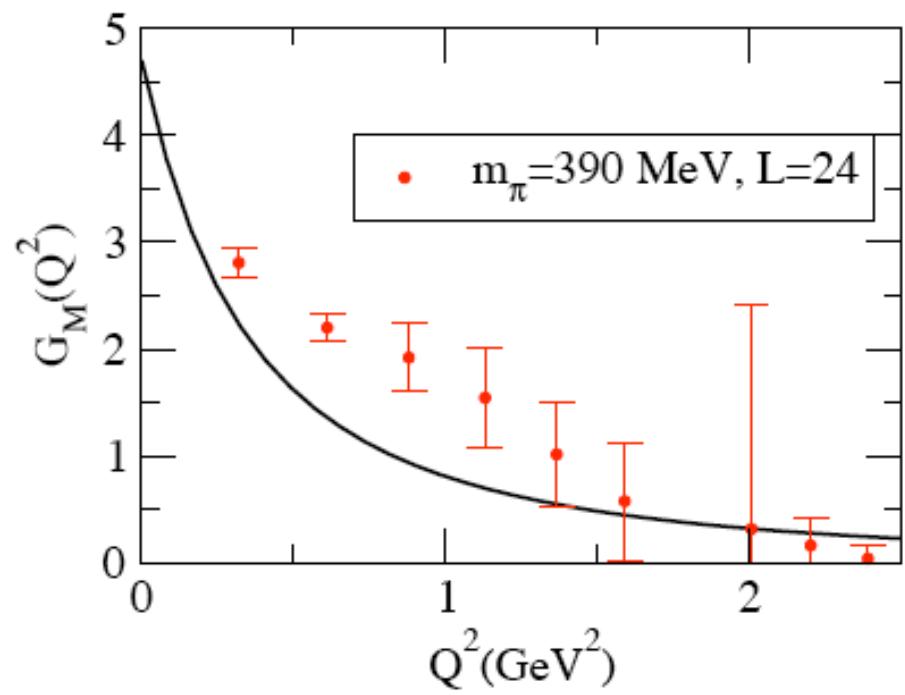
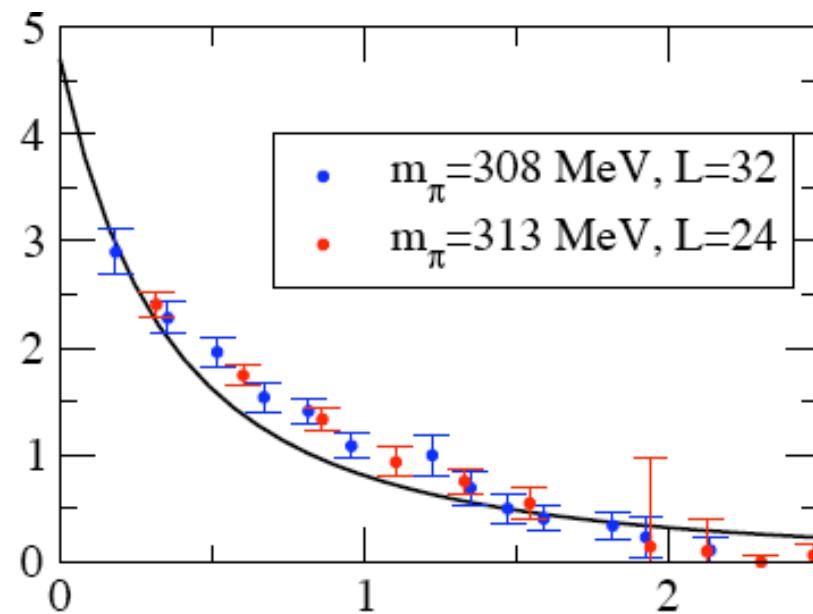
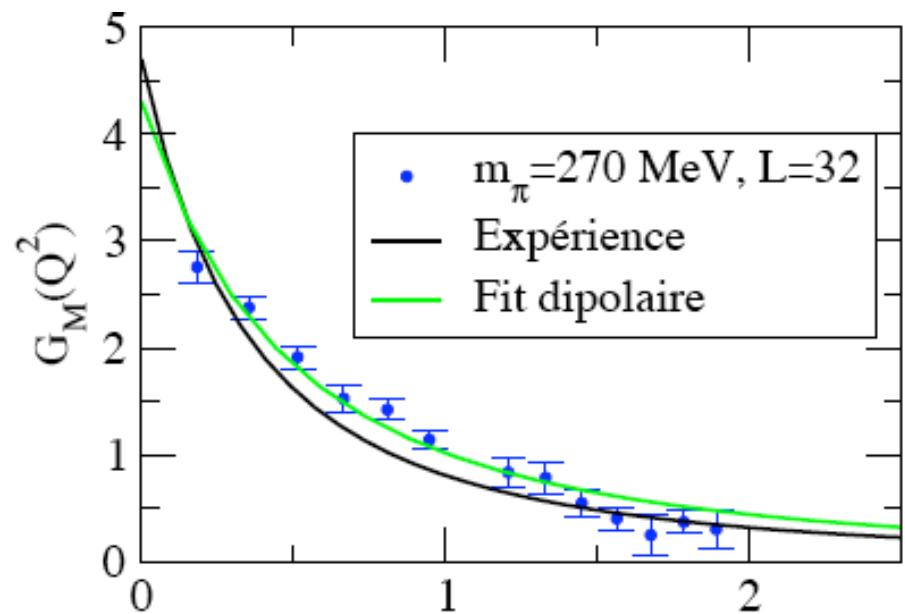
$$\langle N(p_f, s_f) | V_\mu(0) | N(p_i, s_i) \rangle = \sqrt{\frac{m_N^2}{E_{N(\vec{p}_f)} E_{N(\vec{p}_i)}}} \bar{u}(p_f, s_f) \mathcal{O}_\mu u(p_i, s_i)$$

$$\mathcal{O}_\mu = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} F_2(q^2)$$

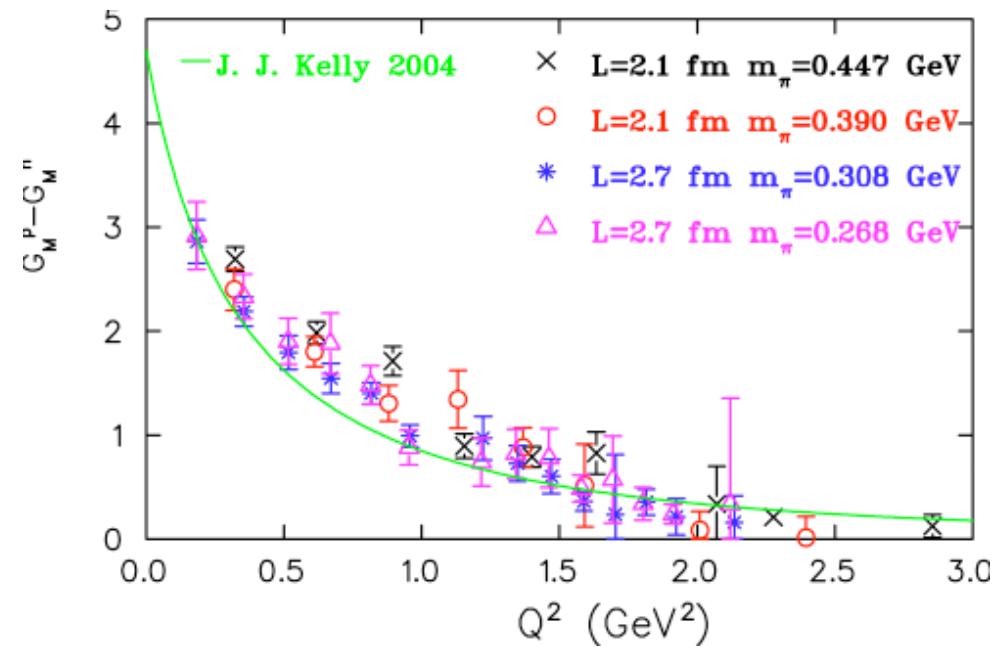
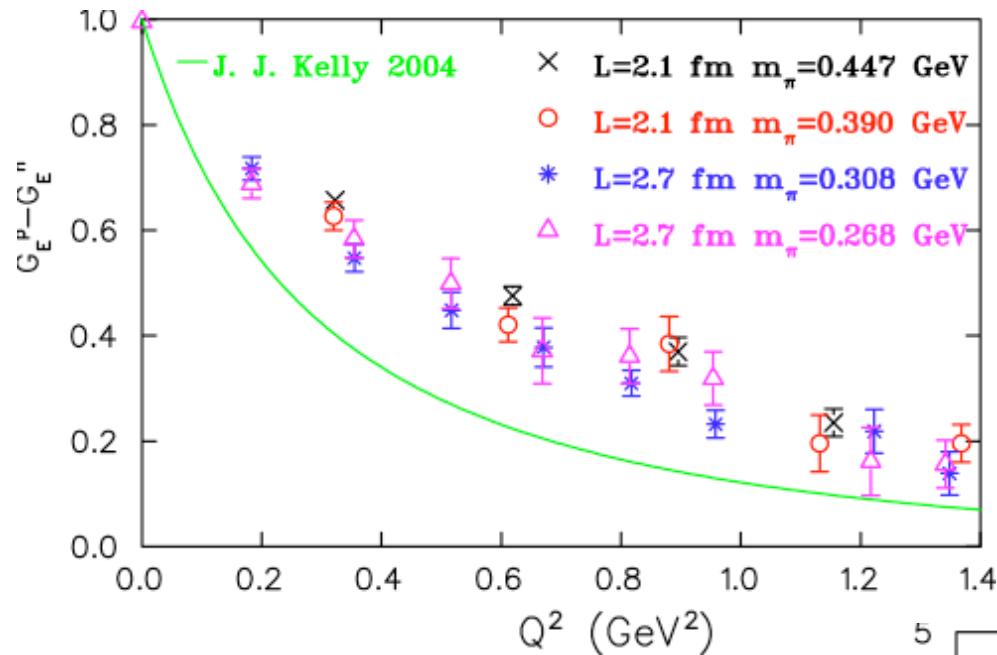
$$G_E(q^2) = F_1(q^2) - \frac{q^2}{(2m_N)^2} F_2(q^2) \quad V_\mu^{EM}(x) = \frac{2}{3} \bar{u}(x) \gamma_\mu u(x) - \frac{1}{3} \bar{d}(x) \gamma_\mu d(x)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$





C. Alexandrou » , lat09 (ETMC)



P.Guichon
Using a formula
From A.Thomas

$$\text{fit} = -(1+5g_A^2) \text{Log}[m_\pi^2/(m_\pi^2 + \mu^2)] / (4\pi f_\pi^2) / (1+c_2 m_\pi^2)$$

$$\mu = 0.56 \text{ GeV}, c_2 = -1 \text{ GeV}^{-2}$$

