Analytic methods in QCD
Perturbative QCD and effective Hamiltonian

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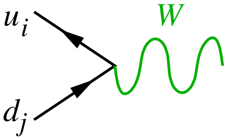
Contents

From weak to strong interactions for heavy flavours

Weak interaction and CKM-matrix

In the quark sector of the SM,

weak interaction not diagonal in mass eigenstates



$$\frac{g}{\sqrt{2}}\, \bar{u}_{Li} V_{ij} \gamma^\mu d_{Lj}\, W^+_\mu + \mathrm{h.c.}$$

with the Cabibbo-Kobayashi-Maskawa matrix:

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \simeq \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

3 generations of fermions $\Longrightarrow$ complex phase η, *CP*-violation in SM

“The” unitarity triangle

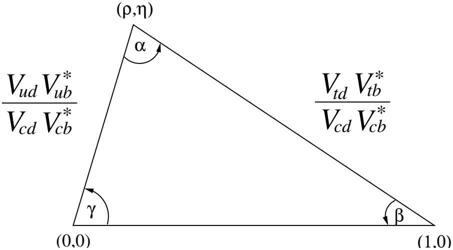
Unitarity of CKM matrix $\Longrightarrow$ relations between the matrix elements

B-meson triangle

$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

Terms of same size $O(A\lambda^3)$

$\Longrightarrow$ Large CP asymmetries



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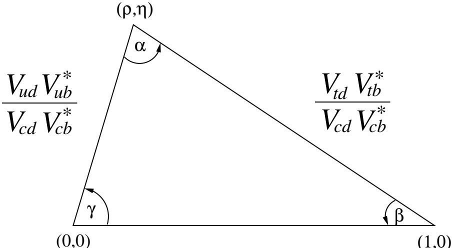

Idea : Overconstrain the CKM matrix, check its determination or find inconsistency related to new physics (BaBar, Belle, CDF/D0, LHCb…)

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For theorists at least, the answer is:

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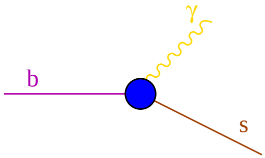

At the quark level, short-distance physics, described by electroweak part of lagrangian
[computed in perturbation theory]

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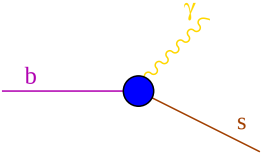

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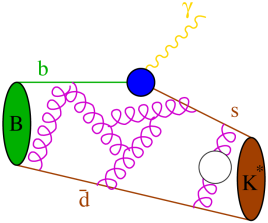


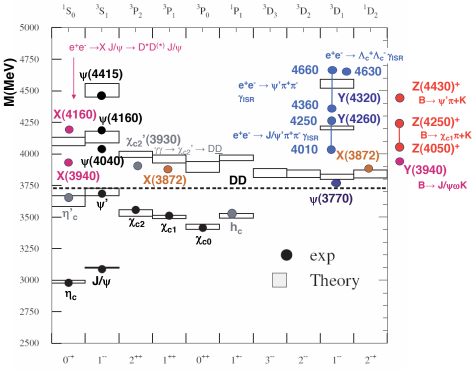
At the hadronic level, convoluted with long-distance physics, described by QCD
[new hadronic quantities]

$\Longrightarrow$We need to understand QCD
to extract information on weak interaction from heavy meson decays

Spectroscopy of heavy states

QCD needed to understand the spectrum and dynamics of

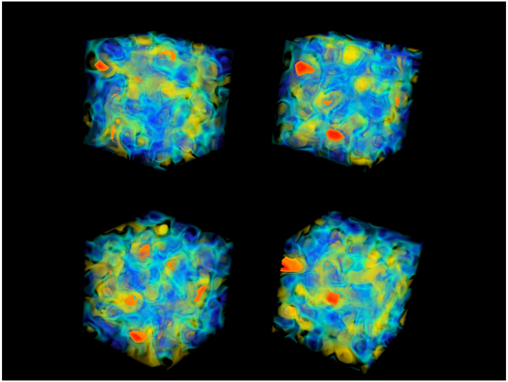
- Heavy-light states : $D, D_s, B, B_s \ldots$
- Heavy-heavy states : $J/\Psi, \eta_c, \chi_c, \Upsilon \ldots$



equivalent to hydrogen atom or positronium,

potential from strong interactions rather than electromagnetic

Elements of QCD



Free quarks

- Dirac equation: relativistic description of spin 1/2 fermion

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 \qquad \text{with} \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$$

in order to fulfill the on-shell conditions
$(\partial_\mu\partial^\mu + m^2)\psi = -(i\gamma^\mu\partial_\mu - m)(i\gamma^\nu\partial_\nu - m)\psi = 0 \longrightarrow E^2 - \vec{p}^2 = m^2$

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- 4×4 matrices $\gamma_{0,1,2,3}$: $\gamma^0 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix} \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$
  with Pauli matrices: $\vec{\sigma} = \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]$

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- Why 4×4 matrices ? spin 1/2 fermion has 4 degrees of freedom
  - 2 : Spin orientation (up or down)
  - 2 : Particle vs. antiparticle (2 spinors)

Colours

- Quark model : proton *uud*, neutron *udd*...
- Among states discovered in 50's
  $\Delta^{++}(J = 3/2, J_3 = 3/2) = u^\uparrow u^\uparrow u^\uparrow$
- But Δ is a fermion, with antisymmetric wave function (Pauli)

$\Longrightarrow$additional d.o.f. : colour (green, blue, red)

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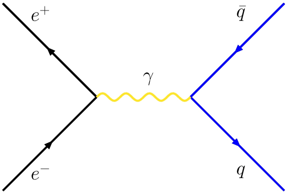
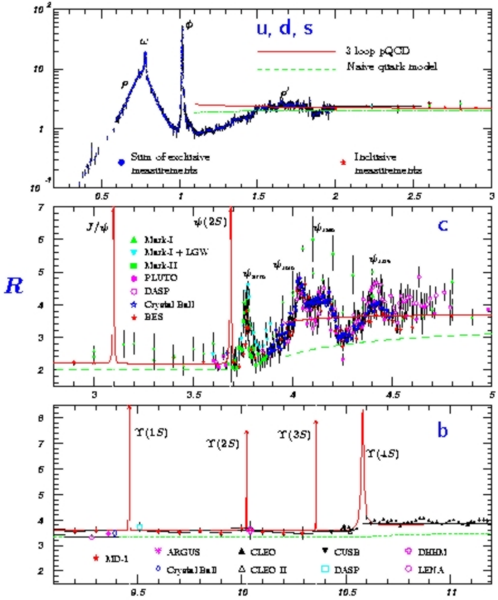


$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq \frac{\sum_q (e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq N_c \sum_q Q_q^2$$

varies when $q\bar{q}$ threshold crossed

3 colours



Resonances after each $q\bar{q}$ threshold, then asymptotic value with $N_c = 3$

QCD

Following similar train of thought to Quantum Electrodynamics

- QED: invariance of Maxwell equations under global redefinition of the phase $\psi(x) \to e^{i\alpha}\psi(x)$
- invariance of strong interaction under global redefinition of colour

$$q = \begin{pmatrix} q \\ q \\ q \end{pmatrix} \to Uq(x) = \exp[i\alpha_a T^a]q(x)$$

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parametrised by $a = 1 \ldots 8$ matrices 3×3 Gell-Mann matrices

$$T^a = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

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QCD Lagrangian

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- Covariant derivative : $D_\mu q = (\partial_\mu - ig_s G_\mu)q \to U(x)D_\mu q$
provided that $G_\mu \to U G_\mu U^\dagger - \frac{i}{g_s}(\partial^\mu U)U^\dagger$,

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- QCD Lagrangian for quarks : free + interaction

$$\mathcal{L}_D = \bar{q}(i\gamma^\mu D_\mu - m)q = \bar{q}(i\gamma^\mu\partial_\mu - m)q + g_s\bar{q}_\alpha T^a_{\alpha\beta}\gamma^\mu q_\beta G^a_\mu$$

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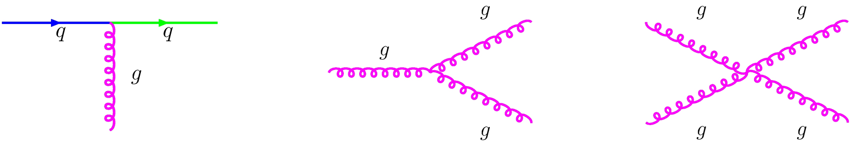
- QCD Lagrangian for gluons $\mathcal{L}_F = -\frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu} = -\frac{1}{2}Tr[G^{\mu\nu}G_{\mu\nu}]$

where $G^{\mu\nu}$ analogue of electromagnetic $F^{\mu\nu}$
$$G^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu - ig_s[G^\mu, G^\nu] \to UG^{\mu\nu}U^\dagger$$

QCD interactions

- $G^{\mu}_{\alpha\beta} = G^{\mu}_{a} T^{a}_{\alpha\beta}$ collects eight gluons
- No mass term (not gauge invariant), hence gluons are massless
- Interactions: q-q-g from $\mathcal{L}_D$, 3 gluons and 4 gluons from $\mathcal{L}_F$

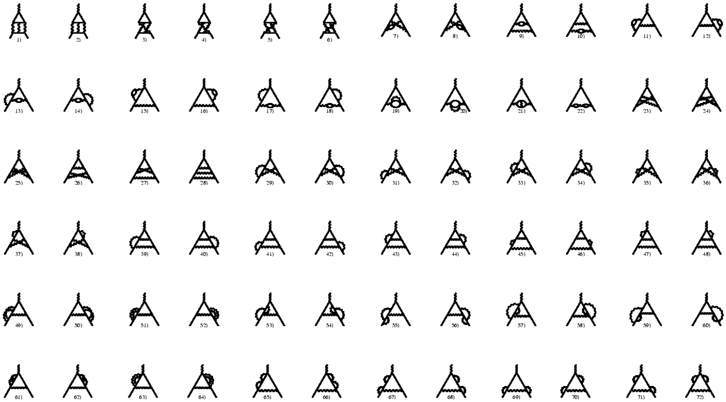


$g_s \gamma^{\mu} T^a$ $\quad$ $g_s f^{abc}$ $\quad$ $g_s^2 f_{abc} f_{ade}$

Differences from electromagnetism

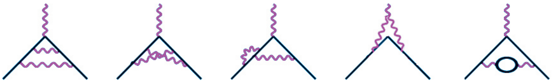
- Gluons themselves sensitive to strong interaction
- Universal coupling g_s (no “colour-electric charge”)

Perturbation theory



Perturbation theory

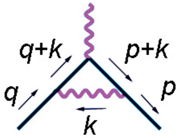
QCD computations can be treated as an expansion in powers of $\alpha_s = g_s^2/(4\pi)$ (each gluon exchange adds a power)



Quantum field theory

- Relativistic : creation/annihilation of pairs possible
  $\Longrightarrow$One can add loops of gluons and quarks
- Quantum : summation over all possible configurations
  $\Longrightarrow$Loop momenta not fixed (only sum up to external momenta)

Loops and renormalisation



$$g_s^3 \int d^4k \left[\gamma_\mu T^a \frac{1}{\not p + \not k - m}\gamma^\rho \frac{1}{\not q + \not k - m}\gamma^\mu T^a\right] \frac{1}{k^2}$$

where $\not k = k^\mu \gamma_\mu$

- Summation over all loop momenta, up to infinity
- Logarithmically divergent integral for $|k| \to \infty$: $\int \frac{d^4k}{k^4}$

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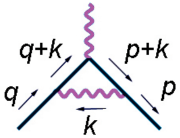

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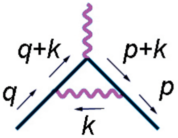

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$\Longrightarrow$Introduction of scale μ for coupling constant $g_s \times \mu^{\epsilon/2}$

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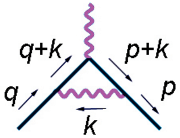

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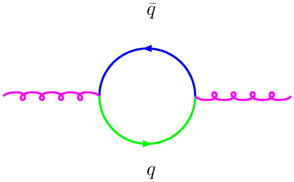
  $\Longrightarrow$Introduction of scale μ for coupling constant $g_s \times \mu^{\epsilon/2}$
- QCD renormalisable: at each order of pert. th., all divergences can be reabsorbed by redefinition of fundamental parameters

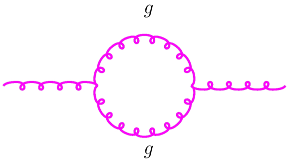
$$g_s^{(0)} \to g_s^{\mathrm{ren}}(\mu), \qquad m_q^{(0)} \to m_q^{\mathrm{ren}}(\mu) \ldots$$

- Remain of $1/\epsilon$ (cancelled by renorm): dependence on $\log \mu$

What about the potential between 2 quarks ?

gluon exchange between 2 quarks $\Longrightarrow$ vacuum polarisation





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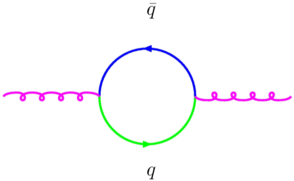

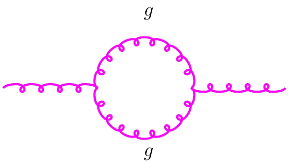

Pairs of virtual quarks and gluons from the vacuum

- modification of $\alpha_s = g_s^2/(4\pi)$ with the distance/energy

$$\frac{dg_s(\mu)}{d\log(\mu)} = \beta(g) = -\frac{g_s^3}{4\pi^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right] + \ldots$$

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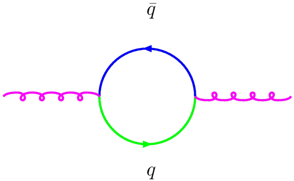

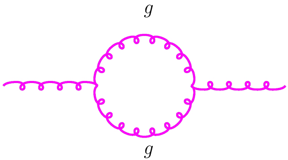

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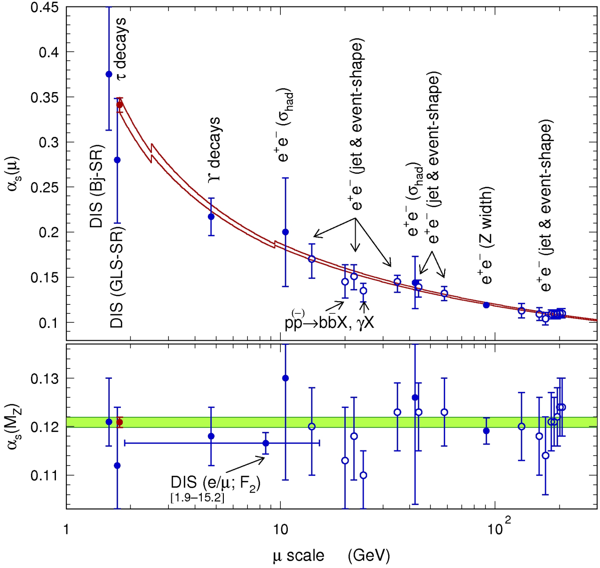
- pairs of quarks : α_s increases at small distances (large μ)
- pairs of gluons : α_s decreases at small distances
- in our world ($N_c = 3$, $N_f = 6$), the gluons win and $\beta < 0$!

α_s decrease at small distances

α_s at various scales



$\Longrightarrow$asymptotic freedom:
at large energies,
interactions (prop to g_s)
small perturbations

Correlatively,
at large distance/small energies,
no perturbation theory possible !

Explicit solution for α_s

- Dependence on μ (*renormalisation group equation or RGE*)

$$\frac{d\alpha_s(\mu)}{d\log\mu} = -2\beta_0\frac{\alpha_s^2}{4\pi} - 2\beta_1\frac{\alpha_s^3}{(4\pi)^2} + \dots$$

  - $\beta_0 = (11N_c - 2N_f)/3$ from 1-loop computation
  - $\beta_1 = (34N_c^2 - 10N_cN_f - 3(N_c^2 - 1)N_f/N_c)/3$ from 2 loops
  - $\log\mu$ dependence reflects divergences occuring at each loop

Explicit solution for α_s

- Dependence on μ (*renormalisation group equation or RGE*)

$$\frac{d\alpha_s(\mu)}{d\log\mu} = -2\beta_0\frac{\alpha_s^2}{4\pi} - 2\beta_1\frac{\alpha_s^3}{(4\pi)^2} + \dots$$

  - $\beta_0 = (11N_c - 2N_f)/3$ from 1-loop computation
  - $\beta_1 = (34N_c^2 - 10N_cN_f - 3(N_c^2-1)N_f/N_c)/3$ from 2 loops
  - $\log\mu$ dependence reflects divergences occuring at each loop

- Solution introduces a scale $\Lambda \simeq 250$ MeV (from experiment)

$$\frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\beta_0\log(\mu^2/\Lambda^2)} - \frac{\beta_1}{\beta_0^3}\frac{\log\log(\mu^2/\Lambda^2)}{\log^2(\mu^2/\Lambda^2)} + \dots$$

with $\log\mu$ dependence very well satisfied experimentally

Leading logarithms

- Keeping only first order in $d\alpha_s/d\log\mu$:

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 - \beta_0 \frac{\alpha_s(\mu_0)}{2\pi} \log(\mu_0/\mu)} = \alpha_s(\mu_0)\left[1 + \sum_{n=1}^{\infty}\left(\beta_0 \frac{\alpha_s(\mu_0)}{2\pi} \log \frac{\mu_0}{\mu}\right)^n\right]$$

- resummation of leading logs $\alpha_s^n(\mu_0)\log^n(\mu_0/\mu)$
  needed for $\mu \ll \mu_0$: $\alpha_s(\mu_0) \ll 1$ but $\alpha_s(\mu_0)\log(\mu_0/\mu) = O(1)$

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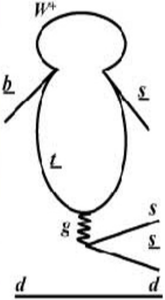
LO	1			
NLO	$\alpha_s(\mu_0)\log(\mu_0/\mu)$	$\alpha_s(\mu_0)$		
$NNLO$	$\alpha_s^2(\mu_0)\log^2(\mu_0/\mu)$	$\alpha_s^2(\mu_0)\log(\mu_0/\mu)$	$\alpha_s^2(\mu_0)$	
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
	Leading Logs	Next – to – Leading Logs	NNLL	$\ldots$
	RGE LO	RGE NLO	RGE NNLO	$\ldots$

Computing $d\alpha_s/d\log\mu$ at N^{k}LO in perturbation theory provides the resumation of N^k leading log contributions

Effective Hamiltonian



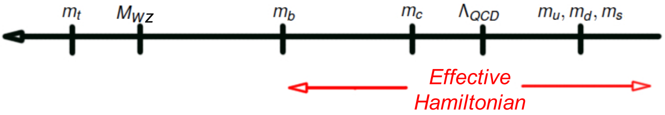
Making life slightly easier

Two different problems here due to mixture of strong/weak:

- Weak Lagrangian in terms of quarks, but hadronic final states
- Multi-scale problem $m_t, m_b, \Lambda_{QCD}, m_{light}$

Here scales of order m_b (or lower) !

so why not integrate out heavier degrees of freedom (t, W, Z) ?



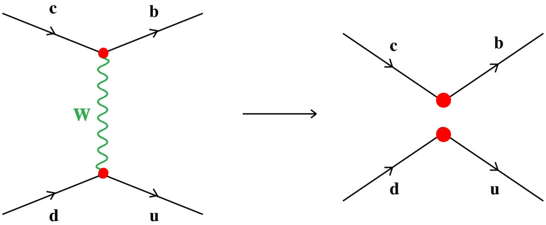
to get weak effective Hamiltonian $\mathcal{H}_{\rm eff}$

(still b, c, s, d, u, g and γ as dynamical particles)

Effective Hamiltonian

Fermi-like approach : μ separation between low and high energies

- Short distances : (perturbative) Wilson coefficients
- Long distances : local operator



$$V_{ud} V_{cb}^* \frac{G_F}{\sqrt{2}} \frac{m_W^2}{m_W^2 - p_W^2} \quad \bar{u}\gamma_\mu(1-\gamma_5)d\bar{b}\gamma^\mu(1-\gamma_5)c$$

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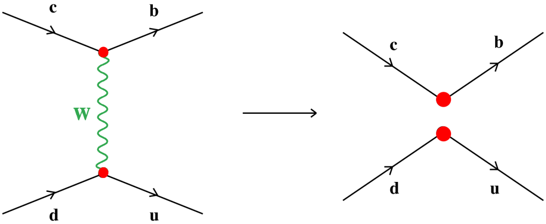

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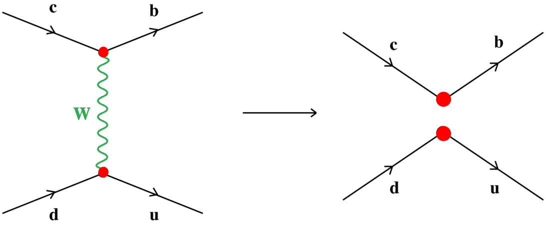
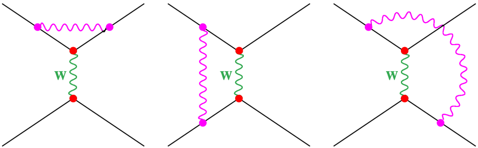


$$V_{ud} V_{cb}^* \frac{G_F}{\sqrt{2}} \bar{u}\gamma_\mu(1-\gamma_5)d\bar{b}\gamma^\mu(1-\gamma_5)c$$

$$\mathcal{A}(B \to H) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i\, C_i(\mu)\, \langle H|\mathcal{O}_i|B\rangle(\mu)$$

- λ_i collect CKM-matrix elements,
- $C_i(\mu)$ Wilson coefficients (physics above m_b)
- matrix-elements of local operators $\mathcal{O}_i$

QCD effects



When we take into account one (or more) gluons

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu)]$$

$$Q_1 = (\bar b_\alpha c_\beta)_{V-A} (\bar u_\beta d_\alpha)_{V-A} \qquad (\bar b c)_{V-A} = \bar b \gamma_\mu (1-\gamma_5) c$$

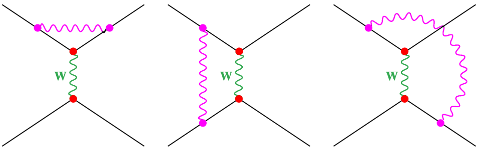
$$Q_2 = (\bar b_\alpha c_\alpha)_{V-A} (\bar u_\beta d_\beta)_{V-A}$$

- new colour structures (flipped indices α, β)
- divergences absorbed by renormalisation
- C_1 and C_2 calculable fonctions of μ as perturbative series in α_s
- Without QCD $C_1 = 0$, $C_2 = 1$, but with QCD ?

$\bar{b} \to \bar{c}\bar{d}u$ at one loop: fundamental theory

C high-energy part, independent of state :

take massless quarks, off-shell by $p^2 < 0$



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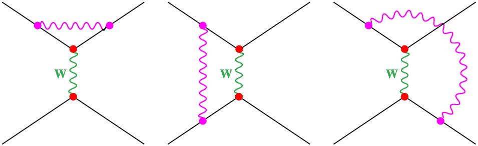


In "full" (SM) theory, taking into account quark renormalisation,

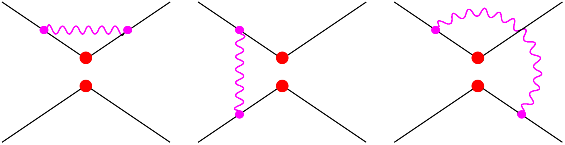
$$A^{(1)}_{full} = \frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud} \left[M_2 + \frac{3}{N_c} \frac{\alpha_s}{4\pi} \log \frac{M_W^2}{-p^2} M_2 - 3 \frac{\alpha_s}{4\pi} \log \frac{M_W^2}{-p^2} M_1 \right]$$

at leading logarithms, with the matrix elements

$$M_1 = \langle Q_1 \rangle^{LO} = (\bar{b}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$$
$$M_2 = \langle Q_2 \rangle^{LO} = (\bar{b}_\alpha c_\alpha)_{V-A} (\bar{u}_\alpha d_\alpha)_{V-A}$$

$\bar{b} \to \bar{c}\bar{d}u$ at one loop: effective theory

In the effective theory (effective Hamiltonian)



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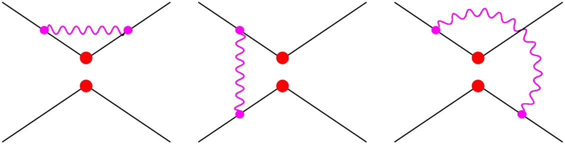

we obtain after quark-field renormalisation

$$\langle Q_1\rangle^{(0)} = M_1 + \frac{3}{N_c}\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{-p^2}\right)M_1 - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{-p^2}\right)M_2$$

$$\langle Q_2\rangle^{(0)} = M_2 + \frac{3}{N_c}\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{-p^2}\right)M_2 - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{-p^2}\right)M_2$$

- Effective theory more singular than fundamental theory ($1/\epsilon$, absorbed by renormalising operators of eff. Hamiltonian)
- Involve only low scales (p^2 and μ, but not M_W)

Matching and Wilson coefficients

Matching: C_1 and C_2 so that full and effective theory yield same result

$$A_{\text{full}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} [C_1(\mu)\langle Q_1(\mu)\rangle + C_2(\mu)\langle Q_2(\mu)\rangle]$$

At NLO in α_s, leading logarithms

$$C_1(\mu) = -3\frac{\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2} + O(\alpha_s^2), \qquad C_2(\mu) = 1 + \frac{3}{N_c}\frac{\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2} + O(\alpha_s^2)$$

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Matching performed separation of scales $-p^2 < \mu^2 < M_W^2$

$$\left(1+\alpha_s X \log\frac{M_W^2}{-p^2}\right) = \left(1+\alpha_s X \log\frac{M_W^2}{\mu^2}\right) \times \left(1+\alpha_s X \log\frac{\mu^2}{-p^2}\right)$$

$$\int_{-p^2}^{M_W^2} \frac{dk^2}{k^2} = \int_{\mu^2}^{M_W^2} \frac{dk^2}{k^2} + \int_{-p^2}^{\mu^2} \frac{dk^2}{k^2}$$

Resumming large logarithms

- At $\mu = m_b$ (separation between low and high energies)

$$
\begin{aligned}
C_1(\mu) &= -3\frac{\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2} + O(\alpha_s^2) = -0.3 + \ldots \\
C_2(\mu) &= 1 + \frac{\alpha_s}{4\pi}\log\frac{M_W^2}{\mu^2} + O(\alpha_s^2) = 1 + 0.1 + \ldots
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better to sum all leading-logs $\left(\alpha_s(\mu)\log\frac{M_W^2}{\mu^2}\right)^n$

$\Longrightarrow$use renormalisation group equation (dependence on μ)

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- Renormalising $\langle Q_i\rangle^{(0)} = Z_{ij}\langle Q_j\rangle,\ Z = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\begin{pmatrix} 3/N_c & -3 \\ -3 & 3/N_c \end{pmatrix}$

which is diagonal in $Q_\pm = \frac{Q_2 \pm Q_1}{2}$, $C_\pm = C_2 \pm C_1$:

$$Q_\pm^{(0)} = Z_\pm Q_\pm, \qquad C_\pm^{(0)} = Z_\pm^{-1} C_\pm$$

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} [C_+^{(0)} Q_+^{(0)} + C_-^{(0)} Q_-^{(0)}]$$

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Renormalisation group equation

- Renormalisation of Wilson coefficient: $C_\pm(\mu) = C_\pm^{(0)} Z_\pm(\alpha_s)$

  $C_\pm^{(0)}$ independent of μ, $Z_\pm$ function of μ through α_s, so

$$\frac{dC_\pm(\mu)}{d\log\mu} = \gamma_\pm(\mu) C_\pm(\mu) \qquad \gamma_\pm = \frac{1}{Z_\pm}\frac{dZ_\pm}{d\log\mu} = \pm\frac{\alpha_s(\mu)}{4\pi}\frac{6(N_c \mp 1)}{N_c}$$

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$$\longrightarrow C_\pm(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{\gamma_\pm^{(0)}}{\beta^{(0)}}} C_\pm(M_W) \qquad \gamma_\pm^{(0)} = \frac{6(N_c \mp 1)}{N_c}$$

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- Resumming leading logarithms in Wilson coefficients

$$C_+(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{6}{23}} \qquad C_-(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{-12}{23}}$$

Advantages of effective Hamiltonian

$A(B \to H) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i C_i(\mu) \langle Q_i \rangle(\mu)$

- Simplification of the problem, keeping only relevant d.o.f.
- Matching to fundamental theory at a high scale M_W and renormalisation of operators
  $\Longrightarrow$resummation of large logs (leading, next-to-leading...) in $C(\mu)$
- Easy implementation of New Physics (change C, new Q)
- Can be applied to any process, for instance $B_d \to \pi^+\pi^-$

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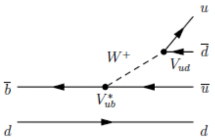

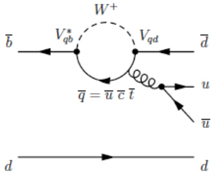

$V_{ud} V^*_{ub} (\bar{b}u)_{V-A} (\bar{u}d)_{V-A}$
Tree

$V_{qd} V^*_{qb} (\bar{b}d)_{V-A} \sum_q (\bar{q}q)_{V\pm A}$,
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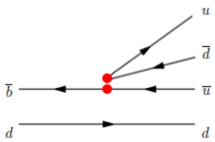

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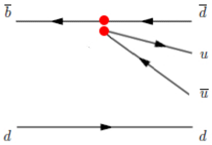

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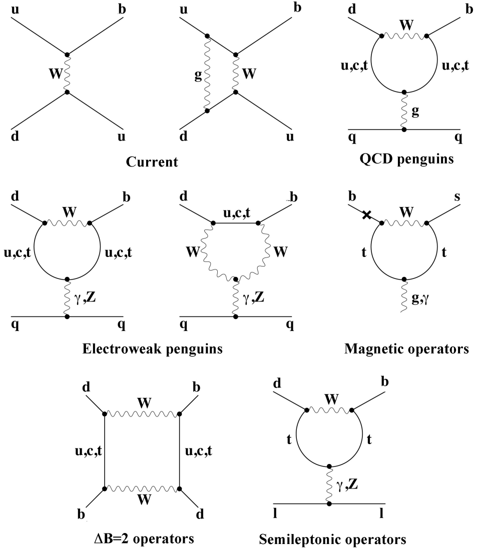
Operators of interest for heavy flavours

- Current-curent
  - $(\bar{b}u)_{V-A}(\bar{u}d)_{V-A}$,
  - $(\bar{b}_i u_j)_{V-A}(\bar{u}_j d_i)_{V-A}$



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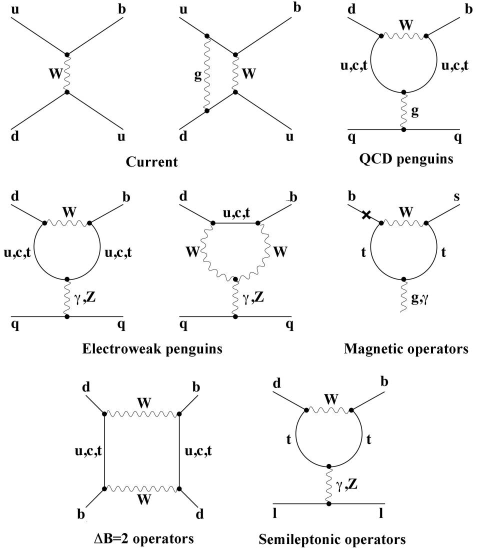

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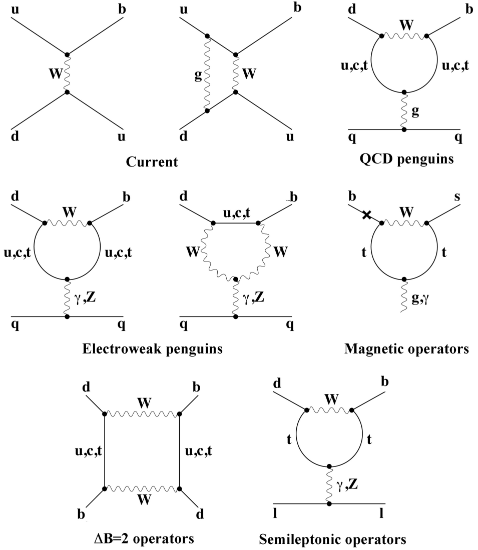

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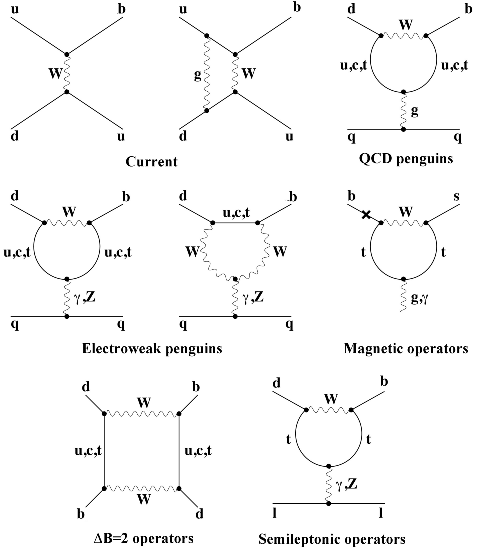
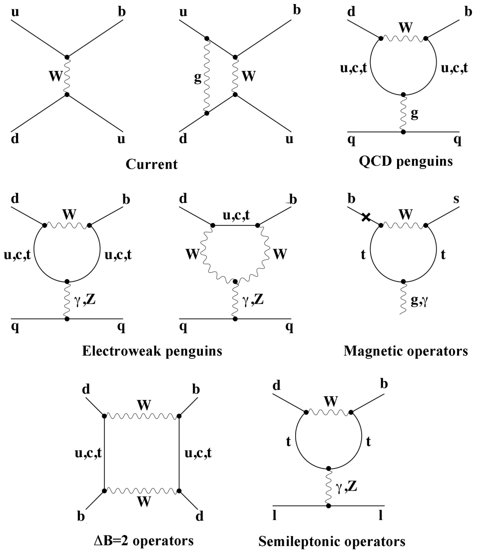
- Magnetic operators
  - $\frac{e}{8\pi^2} m_b \bar{s}\sigma^{\mu\nu}(1+\gamma_5)bF_{\mu\nu}$,
  - $\frac{g}{8\pi^2} m_b \bar{s}\sigma^{\mu\nu}(1+\gamma_5)bG_{\mu\nu}$



Buras et al.

Operators of interest for heavy flavours

- Current-curent
  - $(\bar{b}u)_{V-A}(\bar{u}d)_{V-A}$,
  - $(\bar{b}_i u_j)_{V-A}(\bar{u}_j d_i)_{V-A}$
- QCD penguins
  - $(\bar{b}d)_{V-A}\sum_q(\bar{q}q)_{V\pm A}$,
  - $(\bar{b}_i d_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V\pm A}$
- Electroweak penguins
  - $(\bar{b}d)_{V-A}\sum_q e_q(\bar{q}q)_{V\pm A}$,
  - $(\bar{b}_i d_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V\pm A}$
- Magnetic operators
  - $\frac{e}{8\pi^2} m_b \bar{s}\sigma^{\mu\nu}(1+\gamma_5)bF_{\mu\nu}$,
  - $\frac{g}{8\pi^2} m_b \bar{s}\sigma^{\mu\nu}(1+\gamma_5)bG_{\mu\nu}$
- $\Delta B = 2$ operators
  - $(\bar{b}d)_{V-A}(\bar{b}d)_{V-A}$



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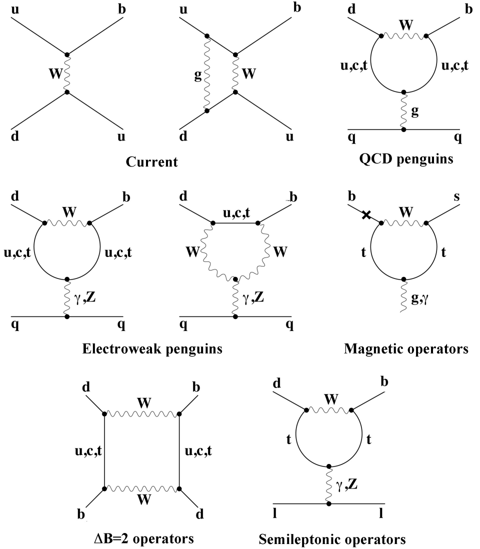
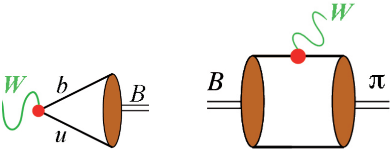
- $\Delta B = 2$ operators
  - $(\bar{b}d)_{V-A}(\bar{b}d)_{V-A}$
- Semileptonic operators
  - $(\bar{b}s)_{V-A}(\bar{e}e)_{V/A}$



Buras et al.

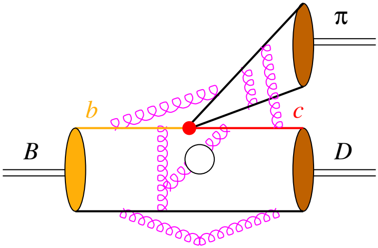
Hadronic quantities



Hadronic matrix elements

Effective Hamiltonian yields $A(B \to H) = \sum \lambda_i\, C_i(\mu) \langle H|\mathcal{O}_i|B\rangle(\mu)$

- above m_b, perturbative Wilson coefficients $C_i(\mu)$
- below m_b, operators yielding matrix elements $\langle H|\mathcal{O}_i|B\rangle(\mu)$



Strong interaction
in nonperturbative regime

How to compute $\langle H|\mathcal{O}_i|B\rangle$?

- Model building
- Lattice simulations
- Sum rules
- Light flavour symmetries (isospin, SU(3)...)
- Heavy flavour symmetries (HQET...)

Hadronic quantities

Describe hadronic matrix elements in terms of hadronic quantities

- simple (handled/computable theoretically if not perturbatively)
- universal (common to several processes)

$\Longrightarrow$Exploit Lorentz symmetry to simplify them whenever possible
$\Longrightarrow$The more mesons, the more complicated the quantity

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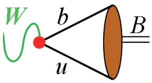
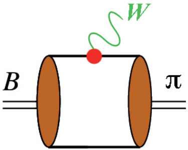
Decay constant
$\langle 0|\bar{u}\gamma_\mu\gamma_5 b|B^-(p)\rangle = ip_\mu F_B$ (real number)



- probability amplitude of hadronising quark pair into given hadron
- related (among others) to purely leptonic decay

$$\Gamma(B^- \to \ell\nu_\ell) \propto |V_{ub}|^2 F_B^2$$

Form factors



$$\langle \pi(p')|\bar{u}\gamma_\mu b|B(p)\rangle = (p+p')_\mu F_+(q^2) + (p-p')_\mu [F_0 - F_+](q^2)\frac{m_B^2 - m_\pi^2}{q^2}$$

- transition from meson to another through flavour change
- projection over available Lorentz stuctures $(p \pm p')_\mu$
- form factors $F_{+,0}$ scalar functions of $q^2 = (p-p')^2$
- more complicated for vector mesons, since polarisation available

$$\frac{d\Gamma(B \to \pi\ell\nu)}{d(q^2)} \propto |V_{ub}|^2 \times |F_+(q^2)|^2 \qquad (m_\ell \to 0)$$

General statements about form factors

Not much known, apart from structure of *S*cattering matrix

$$S_{\beta\alpha} = \langle \beta_{out} | \alpha_{in} \rangle = \langle \beta | \alpha \rangle$$

and its related *T*ransition matrix $S = 1 + iT$

$$\langle \beta | iT | \alpha \rangle = (2\pi)^4 \delta(\sum p_\alpha - \sum p_\beta) \cdot iA(\alpha \to \beta)$$

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Almost only one thing known for sure

from conservation of probability, S-matrix is unitary

$$\begin{aligned}(S^\dagger S)_{\gamma\alpha} &= \sum_\beta \langle \beta_{out} | \gamma_{in} \rangle^* \langle \beta_{out} | \alpha_{in} \rangle \\ &= \sum_\beta \langle \gamma_{in} | \beta_{out} \rangle \langle \beta_{out} | \alpha_{in} \rangle = \langle \gamma_{in} | \alpha_{in} \rangle = \delta(\alpha - \gamma)\end{aligned}$$

since sum over complet state of states $|\beta\rangle$

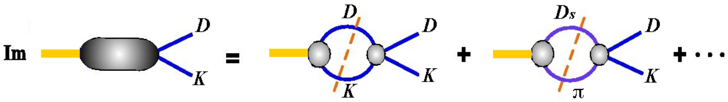
Cuts

Translation for *T*ransition matrix $S = 1 + iT$

$$S^\dagger S = 1 \Longrightarrow T - T^\dagger = iT^\dagger T$$

or in terms of amplitude

$$-i[A(\alpha \to \beta) - A^*(\alpha \to \beta)] = \sum_f A^*(\beta \to f) A(\alpha \to f)$$



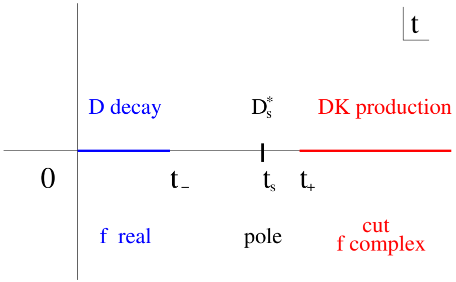
Form factors for $\alpha \to \beta$ acquire an imaginary part

- if there are (real) intermediate states f between α and β
- which depends on the value of the transfer momenta q^2

Analytic structure of a form factor

Taking for instance form factor describing $D \to K\ell\nu$

- Two physical regions, accessible to experiment
  - real for $t = q^2$ between m_ℓ^2 and $t_- = (m_D - m_K)^2$ $\qquad D \to K$ decay
  - complex for $t \geq (m_D + m_K)^2$ $\qquad W \to DK$ production
- Same form factor involved
  - Analytic function for almost every value of t in the complex plane
  - apart from poles for resonances (like D_s^*)
  - and cuts along the real axis due to imaginary part for open channels



Back to CP violation

Weak process = sum of several amplitudes $\lambda_i \, C_i(\mu) \, \langle H|\mathcal{O}_i|B\rangle(\mu)$

Complex amplitudes, with phases from

Weak part	CKM factor	Phase odd under CP
Strong part	Hadronic amplitude	Phase even under CP

Strong phases often important to extract SM parameters
from CP-violating observabless

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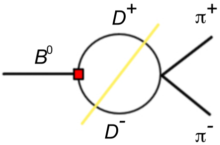
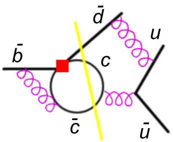
Two different ways of understanding the strong phases



Hadron level
Final state interaction



QCD level
Gluon exchanges

Conclusions

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Why strong interactions for heavy flavours

- Disentangle strong and weak interactions in decays
- Spectroscopy of heavy-light and heavy-heavy resonances

QCD

- Renormalisation yields running of strong coupling constant
- Asymptotic freedom: perturbation theory OK at high energies only

Effective Hamiltonian

- Disentangle the scales to integrate out perturbative high energies
- Necessary to resum effects of gluon exchanges (leading logs)

Hadronic quantities (decay constants, form factors, matrix elements)

- Well-defined quantities, related to low-energy strong interactions
- How to compute them from first principles ?

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Help from light- and heavy-quark symmetries
to describe hadronic part of heavy-flavour dynamics