Update on the studies of the emittance growth caused by the CC RF noise

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29/05/2020

Outline

- **Introduction**
- Overview of the theoretical model
- Reviewing theory and PyHEADTAIL with Sixtracklib simulations
- Tests with global Crab Cavity scheme
- Impact of colored noise

Introduction

- **High-Luminosity LHC** project aims at the upgrade of the LHC machine in order to increase the yearly luminosity production with proton beams.

- **Crab Cavities (CC)** will help improve the luminosity by providing a rotational kick either side of IP1 and IP5 (ATLAS & CMS) which will restore effective head-on collisions.

- The transverse kick to the bunch e.g. for the vertical plane, is given by:

$$\boxed{\Delta p_y = \frac{eV_{CC}}{E_b}\sin(kz + \phi_{CC})}$$

where *Vcc* is the CC voltage in V, *Eb* the beam energy in eV, *k* is the wavenumber (ω_{CC}/c), *z* is the longitudinal position of the particle wrt to the synchronous particle in m and φ_{CC} the phase of the CC.

- The CCs are expected to induce **emittance growth by residual noise** on the beam introduced by the **CC RF control**.

- To study this emittance growth 2 prototype CCs at 400.789 MHz have been installed into the SPS and were tested in 2018.

Motivation

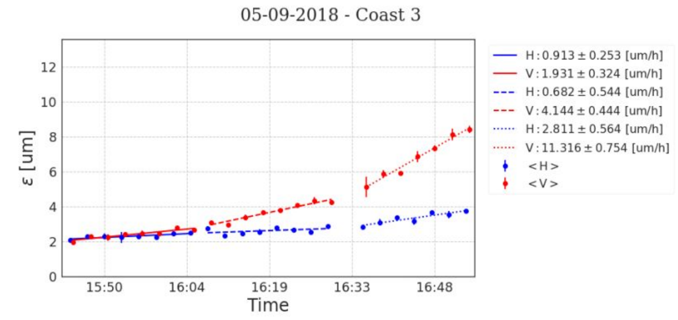
- A theoretical relationship between the crab cavity phase and amplitude noise and the transverse emittance growth has been developed and it was validated through simulations with PyHEADTAIL (PhysRevSTAB.18.101001).

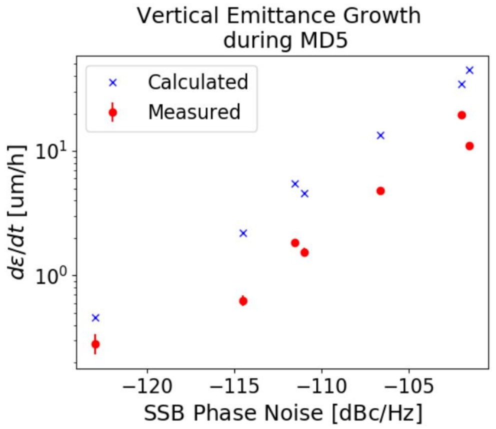
- SPS tests in 2018: MD5 → emittance growth at 270 GeV with levels of noise injected on the crab cavities. Phase noise was dominant.

    - Top figure: Example coast from MD5. The three different time spans correspond to different levels of the injected noise. Larger emittance growth for stronger noise.

    - The measured growth is **lower** by a factor 2-3 than predicted from analytical calculations (bottom figure).

    - **The goal of the studies presented here is to examine the reason for the discrepancy between the simulations/theory and the experimental results.**





Measured (Wire Scan) and calculated during the coasts with different noise levels.

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Modeling Crab Cavity noise

- The transverse kick to the bunch due to CC noise is given by:

$$\boxed{\Delta p_y = \frac{eV_{CC}}{E_b}(1+\Delta A_n)\sin(kz+\phi_{CC}+\Delta\phi_n)}$$

where $\boldsymbol{\Delta A_n}$ the relative **amplitude noise** and $\boldsymbol{\Delta \varphi_n}$ the **phase noise**.

- It is shown (PhysRevSTAB.18.101001) that the for small ΔA_n and $\Delta \varphi_n$ the transverse kick can be written as:

$$\boxed{\Delta p_y = \frac{eV_{CC}}{E_b}\cos(kz+\phi_{CC})\Delta\phi_n + \frac{eV_{CC}}{E_b}\sin(kz+\phi_{CC})\Delta A_n}$$

which means that that if the phase and amplitude noise spectra are independent, **the two cases can be treated separately.**

Emittance growth due to CC RF noise

- For a number of turns which is significantly larger than the noise decoherence time:

- **Phase noise**

$$\boxed{\frac{d\epsilon_y}{dt} = \beta_{CC} \left(\frac{eV_{CC} f_{rev}}{2E_b} \right)^2 C_{\Delta\phi(\sigma_\phi)} \int_{-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} S_{\Delta\phi}[(k \pm \bar{\nu_b}) f_{rev}] \rho(\nu_b) d\nu_b}$$

* PhysRevSTAB.18.101001

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Beam parameters

Correction term. Decreases with bunch length.

Overlap between phase noise spectrum and betatron tune distribution.

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- **Amplitude noise**

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Emittance growth due to CC RF noise

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Correction term. Increases with bunch length.

Overlap between phase noise spectrum and synchro-betatron tune distribution.

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Methodology

- **It is possible that both PyHEADTAIL simulations and theory miss some beam dynamics that could explain this difference.**

- Therefore we run the simulations using a different, more complete, simulation code to cross check the results.
  - Use of **Sixtracklib** which is a non-linear single-particle tracking library.

- We start from **basic sanity checks** by first simulating **white noise and a local CC scheme**, which allow us to only model the CC momentum kicks due to noise (slide 6). Thus:

- The momentum kick on a particle every turn due to **phase noise** alone is modeled as follows:

$$p_{y1} = p_{y0} + A \cdot \xi \cdot \cos(2\pi \cdot 400.789 \cdot 10^6 \cdot z/\beta c)$$

- The momentum kick on a particle every turn due to **amplitude noise** alone is modeled as follows:

$$p_{y1} = p_{y0} + A \cdot \xi \cdot \sin(2\pi \cdot 400.789 \cdot 10^6 \cdot z/\beta c)$$

  where β is the relativistic beta, z the longitudinal position in m, A the strength of the noise and ξ a random number drawn from a normal distribution with mean 0 and standard deviation 1.

- This sequence of uncorrelated random kicks on the beam corresponds to **white noise kicks.**

Simulation Parameters - I

- Both amplitude and phase noise are modeled with **A = 1e-8** such as we get a fast growth and observe a significant effect in our simulation time (1e5 turns ~ 2.5 s in SPS).
  - Specifically we expect about **22 nm/s** and **24 nm/s** for **amplitude** and **phase** noise respectively.
- Some consideration is needed regarding the **tune spread values**.
  - The emittance growth as predicted by the theoretical model depends on the tune distribution. However, in the case of white noise the $S\Delta_{\varphi,A}(\nu_b f_{rev})$ is constant within the betatron spread and therefore **the effect of noise is independent of the actual tune distribution**.
  - For the theoretical model to be valid the simulation time should be much longer than the inverse betatron tune spread. This means that for the simulation time of 2.5 s the **rms betatron tune spread needs to be >>> 6e-6.**
- The momentum kicks are applied at the location of CC2 in SPS.

Simulation Parameters - II

Unless stated otherwise we use the following values for these parameters:

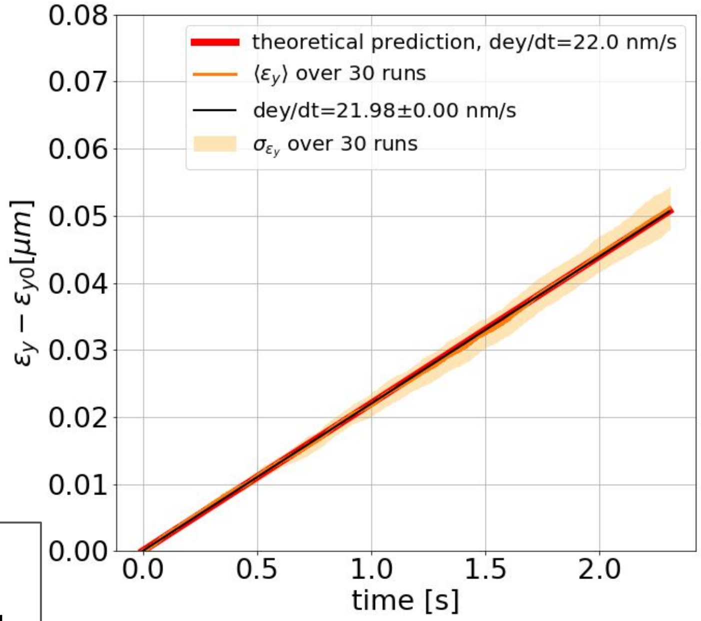
Beam energy	270 GeV
Horizontal betatron tune, ν_x	26.13
Vertical betatron tune, ν_x	26.18
Synchrotron tune, ν_s	0.00349
Normalised horizontal emittance, ϵ_x	$2\mu m$
Normalised vertical emittance, ϵ_y	$2\mu m$
Horizontal beta function at CC2, β_x	29 m
Vertical beta function at CC2, β_y	76 m
rms bunch length, σ_z	15.5 cm

- The initial distribution of 1e5 particles follows a gaussian both in transverse and longitudinal planes.

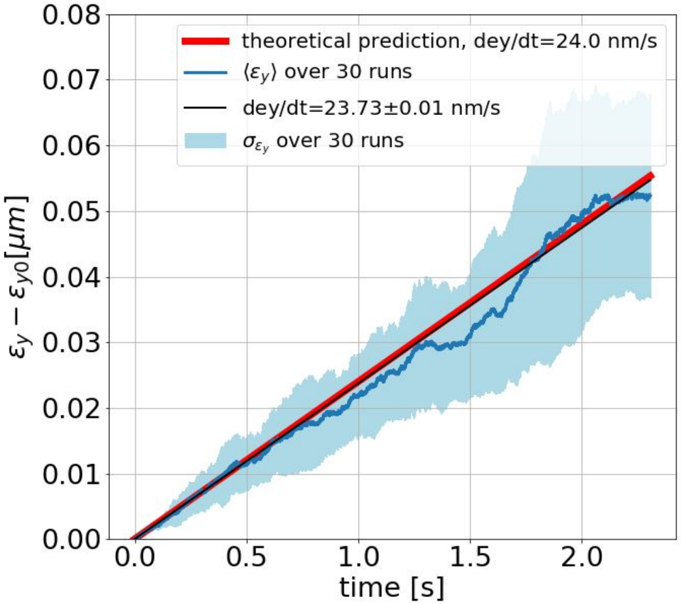
Growth rate with local CC scheme

- The average normalized emittance growth rate **agrees very well** with the theoretically predicted rate both for amplitude and phase noise.
- The spread, $\sigma_{\varepsilon y}$, is expected due to the way the noise kicks are implemented.

Amplitude noise



Phase noise



Qpx = -1.74
Qpy = -0.78
ΔQyrms ~2ε-4

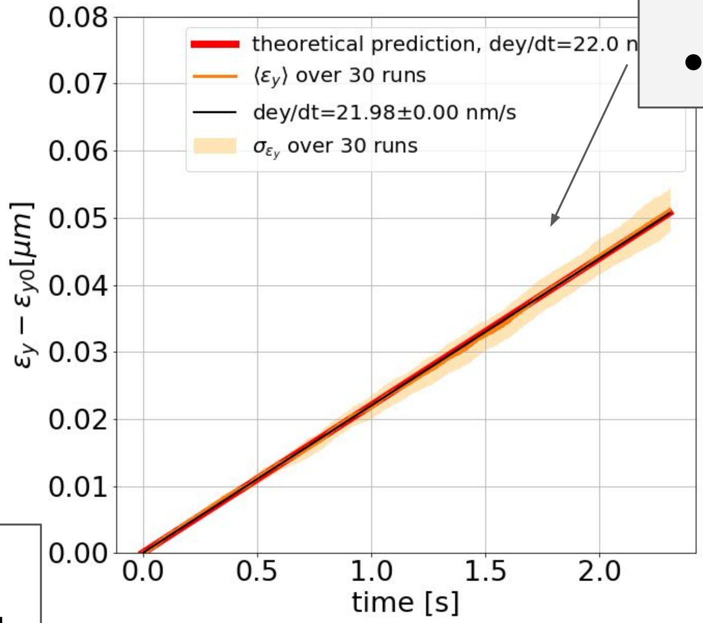
Growth rate with Local CC scheme

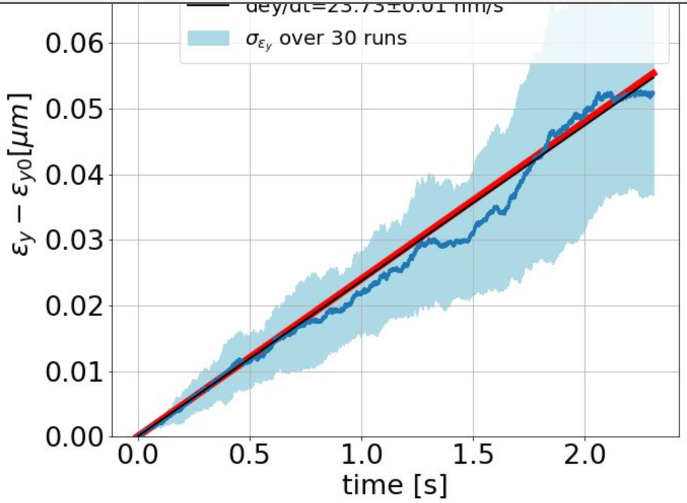
- The average normalized emittance growth rate **agrees very well** with the theoretically predicted rate both for amplitude and phase noise.

- The spread, σεy, is expected due to the wa

- The very small spread over the different runs, in the amplitude noise case, in Sixtracklib simulations seems suspicious.
- The reason for this is not yet identified.

Amplitude noise





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Noise on the Crab Cavity element in Sixtracklib

- The growth rates obtained from Sixtracklib simulations for white noise kick and a local CC scheme agreed with the theoretically expected rates.

- We move on by performing simulations with a **global CC scheme** which is the realistic case during the MDs.

- For this study the use of a **"real" CC element** is required.

  - In Sixtracklib, the CC element is represented by an **RFMultipole** element which has the properties of an RF-cavity and of a magnet of arbitrary order oscillating at a certain frequency. To simulate the vertical CC kick we implement it as a **modulating (skew) dipole**.

- The voltage or the phase of the CC element is updated every turn such as it includes the noise contribution. As presented before the noise is drawn from a normal distribution with mean 0 and given standard deviation equals to the strength of the noise

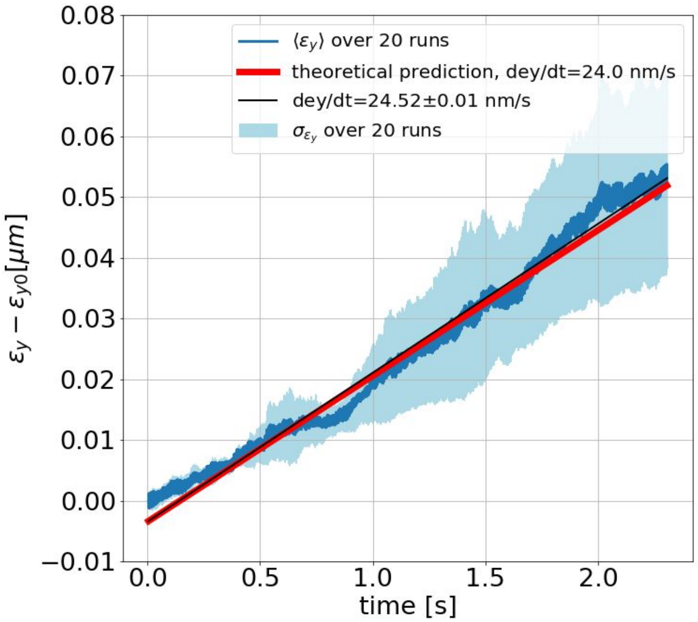
- Previous studies (link) showed that the CCs themselves do not cause any emittance growth.

  - Thus, in the presence of noise **we expect no difference in the growth rate between the local and global CC scheme.**

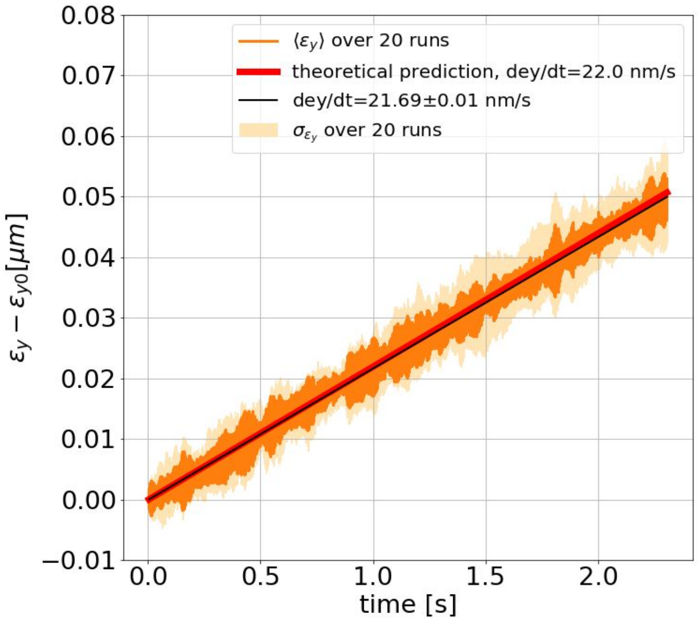
Growth rate with Global CC scheme

- Qpx = Qpy = 2 (~ MD 2018), VCC1 = 0MV, VCC1 = 1MV, ramped up in 200 turns to avoid emittance blow up.
- The average normalized emittance growth rate, for A=1e-8, **agrees very well** with the theoretically predicted rate both for amplitude and phase noise.

Amplitude noise



Phase noise



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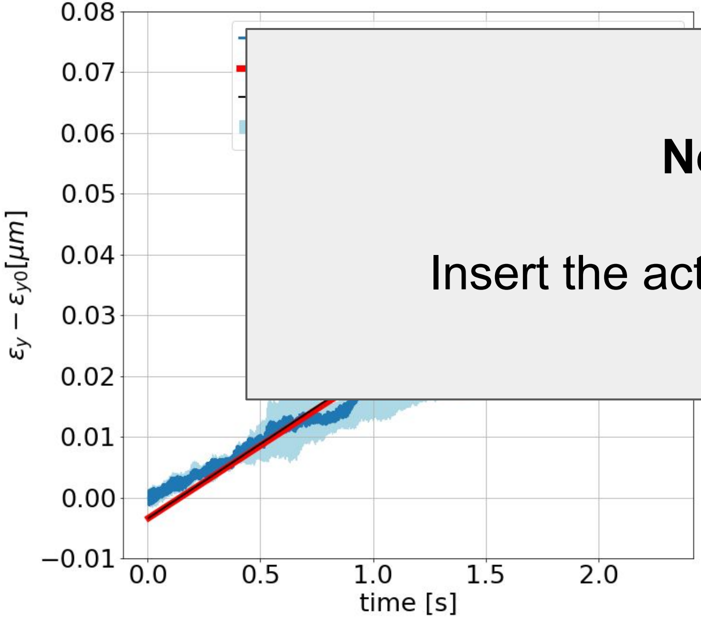
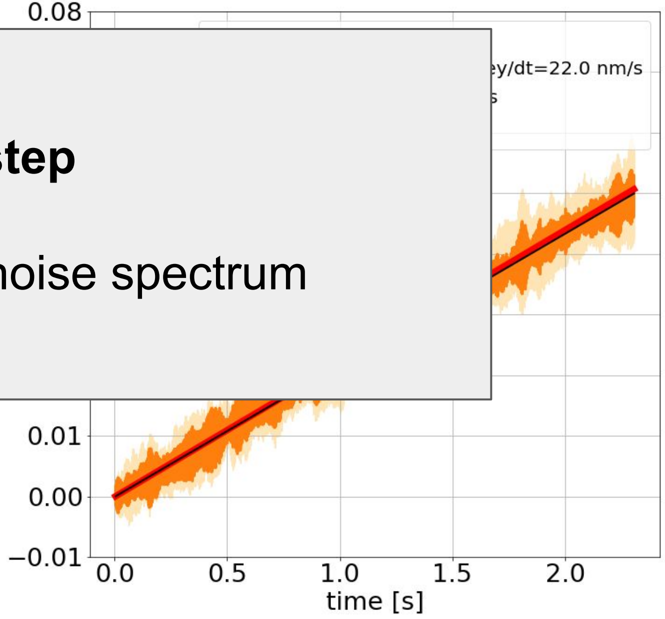
Amplitude noise



Phase noise



Next step

Insert the actual noise spectrum

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Colored phase noise in the Crab Cavities

- The crab cavity noise so far is assumed to be *white noise*.

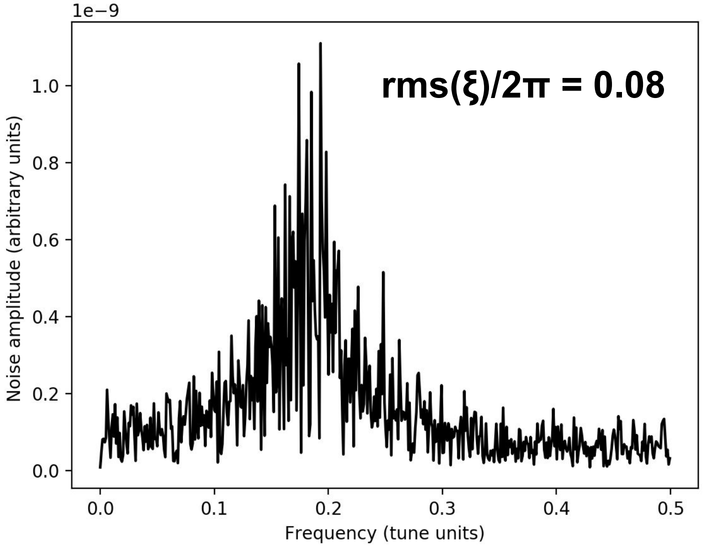
- To investigate the effect of colored noise on the emittance growth rate we change the phase of the crab cavity every turn by:
  $\phi_{\text{noise},t} = \phi_0 \cos(\psi_t)$, where *t* is the turn number and φ_0 is a constant (which corresponds to the strength of the noise *A*).

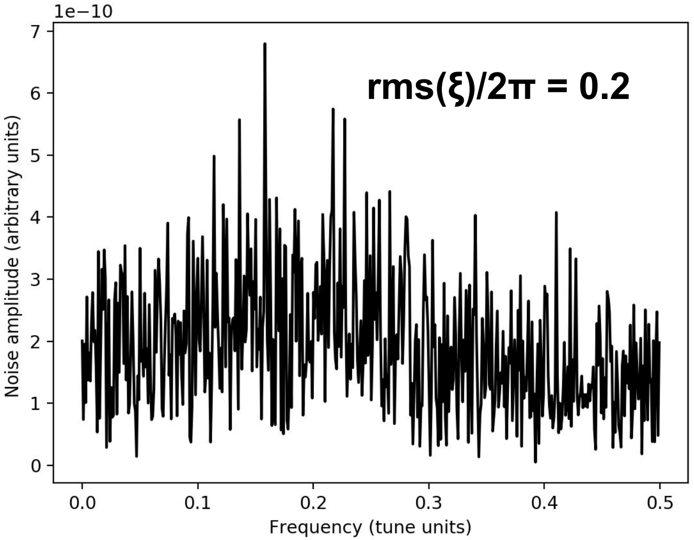
  The phase ψ_t increases on each turn according to $\psi_{t+1} = \psi_t + \Delta\psi_0 + \xi_t$, where $\Delta_{\psi 0}$ is a constant which determines the position of the **peak** of the spectrum (in our case the betatron frequency Qy) and ξ_t is taken from a set of random variables with normal distribution and given standard deviation ***rms(ξ)***, which actually determines the **width of the spectrum.**

- We use a local CC scheme, with the two CCs cancelling each other out. The phase noise is added only in the second CC. With this configuration we achieve to study the clear effect of the noise kick.

Examples of the noise spectrum

Phase noise around the betatron frequency, Qy = 0.18





Controlling the tune spread

- Unlike the case of white noise, now the $S\Delta_{\varphi,A}(\nu_b f_{rev})$ may not be constant within the betatron spread in which case **the effect of noise will depend on the actual tune distribution**.
  - Note that this may not be a realistic scenario considering the noise that was injected during the MDs and the tune spread in SPS.
  - However, we perform the studies mostly for academic purposes.

- **The goal of these studies is to investigate the impact of the peaked phase noise around the betatron frequency for different values of tune spread.**

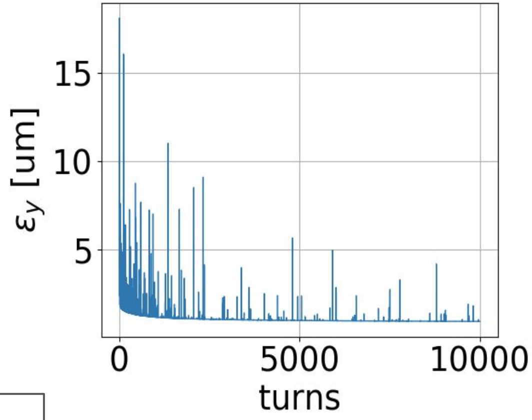
- The optimal way to control the tune spread is to use the **Landau octupole families** of the SPS. Their strengths are chosen such as they result to the requested value of $\boldsymbol{\alpha_{yy}}$ **(detuning coefficient)**. Additionally, we request $\boldsymbol{\alpha_{xy}=0}$ such to simplify the computation of the tune spread in the vertical plane. The matching is done with PTC.

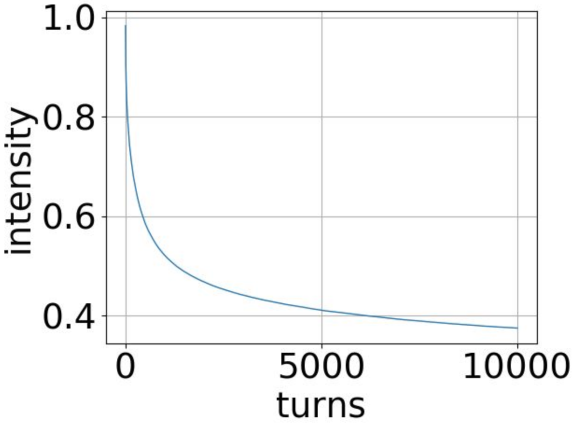
- To be noted that the Landau octupole families in SPS are arranged in a symmetric way around the lattice so they suppress resonances in first order.

Upper limits of the αyy coefficient - I

- Even though the octupoles don’t excite strong resonances there is an *upper limit* on the αyy value we can request for.

- Large αyy values result to **losses**.
  - Large tune spread: particles reach some integer or half integer resonances. As a result of the diffusion the particles reach very high amplitudes (“spikes” in the emittance evolution, which is also decreasing) and eventually they are lost (intensity decrement - right plot).





Qpx = Qpy = 0
σz = 1.55 mm
VCC1=VCC2=1MV

Fig: Example of emittance (left) and intensity (right) evolution for αyy > 1e6 (matching with Landau SPS octupoles). No noise kick is applied. Aperture 1m

Upper limits of the αyy coefficient - II

- The table below shows the upper limits of the αyy values, which are identified such as **no significant beam loss** occurs.

Table: Upper limits on αyy requested values and resulted tune spread

	αyy (1/m)	αxy (1/m)	αxx (1/m)	rms(ΔQy)
σz = 15.5 cm	+- 3e5	0	** No constraint. Usually final value around -αyy	0.0042

- The rms(ΔQy) is computed (backup slides) including the contribution of the transverse detuning with amplitudes due to octupoles and the of the 2nd order chromaticity which becomes large when powering the octupoles. We consider the case where the **linear chromaticity is set to zero**.

$$rms(\Delta Q_y) = \sqrt{rms(\Delta Q_{y,oct})^2 + rms(\Delta Q_{y,chroma})^2}$$

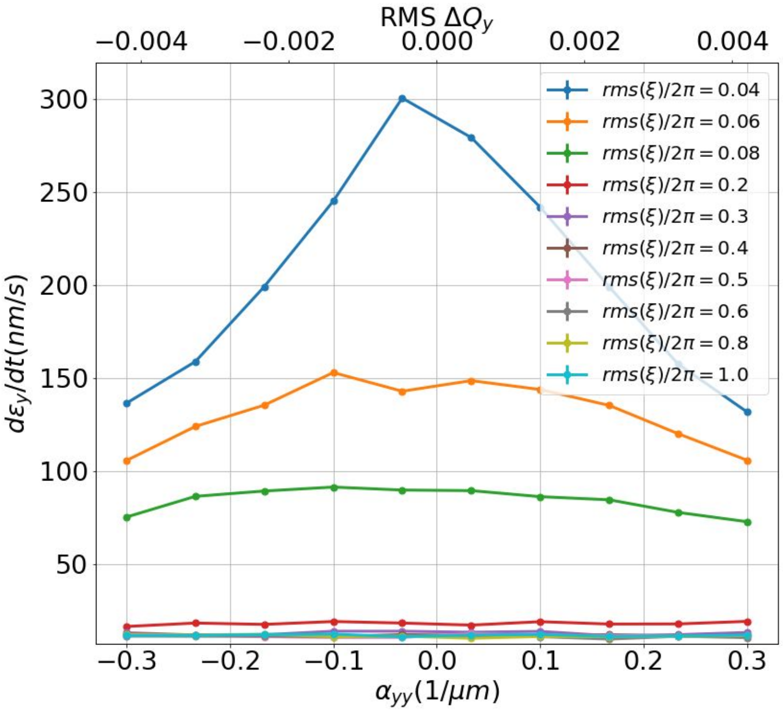
- The tune spread introduced from the 2nd order chromaticity is 2 or more orders of magnitudes smaller than the one from the octupoles. From here onwards, **when we refer to rms tune spread we won't include the chromatic contribution, unless it's stated otherwise**.

Dependence of emittance growth on the noise spectrum

- Simulations were performed over a large range of rms(ξ) values for a phase noise around the betatron frequency and **strength of 1e-8**.

- Observed dependence of the growth rate on the **rms(ξ) values**. Decreases for larger rms(ξ).

- Observed dependence of the growth rate on the **rms(ΔQy).** Decreases for larger rms(ΔQy).
    - Saturation of the dependence for rms(ξ)/2π > 0.2.

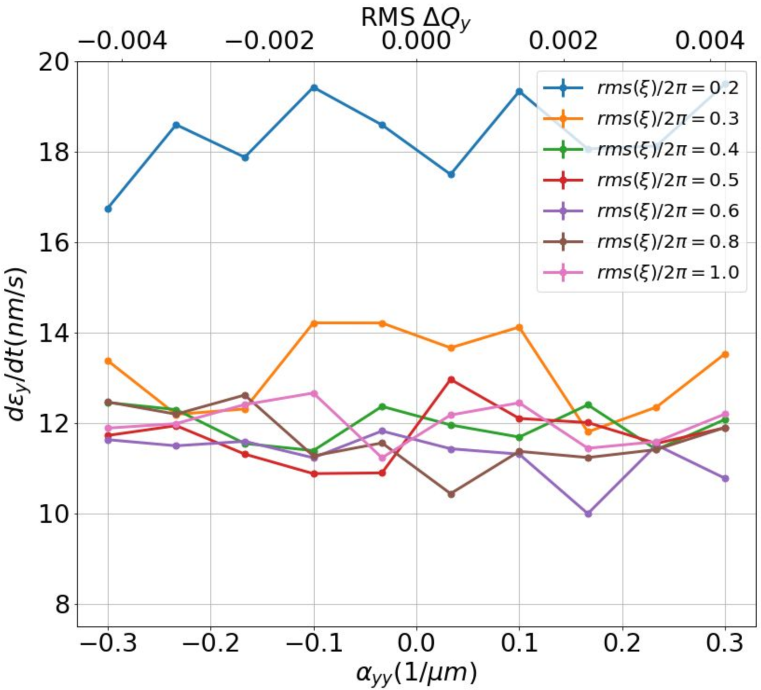


Qpx = Qpy = 0
σz = 15.5 cm
VCC1=VCC2=1MV
A = 1e-8

Regime where there is no dependence on the tune spread

- A closer look in the regime where the dependence on the tune spread is saturated - rms(ξ)/2π > 0.2.

- Way smaller rate than 24 nm/s which is expected in the presence of white noise.
- The dependence on the rms(ξ) seems to saturate for rms(ξ)/2π > 0.3.
- Growth rates are not averaged over many runs. Thus they are not yet representative.
    - This explains also the observed fluctuation of the computed rates.



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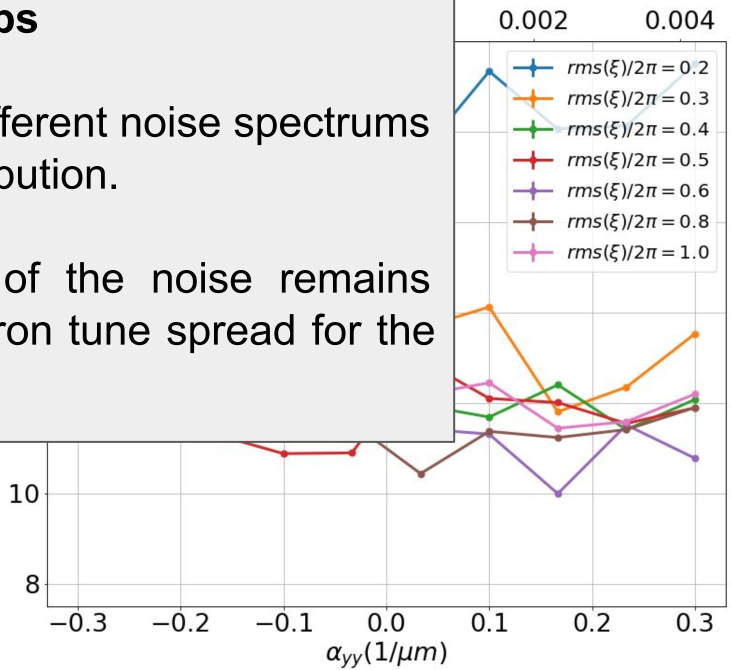
of the computed rates.

Next steps

1) Test the overlap of the different noise spectrums with the actual tune distribution.

2) Test whether the PSD of the noise remains constant within the betatron tune spread for the different rms(ξ) values.



Qpx = Qpy = 0
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VCC1=VCC2=1MV
A = 1e-8

Summary and next steps

- The simulations' results with **Sixtracklib** with local and global CC scheme in the presence of amplitude and phase noise are in **good agreement with the theory** (by Baudrenghien and Mastoridis).
  - Realistic machine conditions according to the CC MDs of 2018 but the magnet multipole errors of SPS lattice. However, previous studies (link) indicate that no significant impact should be expected.
  - **Proceed with inserting the actual noise spectrum**.
- Preliminary studies with Sixtracklib show that the **type of noise spectrum** may be one possible factor for the observed discrepancy between the theory and the measurements.
- Further studies are currently in progress, to obtain better understanding including different tune distributions, the impact of noise PSD and other effects.
- On another note, studies aiming to investigate the difference between definition of emittance between the measured and the calculated will be performed.

Thank you for your attention!

Backup slides

Emittance in the theoretical model PhysRevSTAB.18.101001

- The transverse geometric emittance is defined as:

$$\epsilon = \frac{1}{2}(E[(\boldsymbol{x} - E[\boldsymbol{x}])^2] + E[(\boldsymbol{p} - E[\boldsymbol{p}])^2]) = E[(\boldsymbol{x} - E[\boldsymbol{x}])^2] = E[(\boldsymbol{p} - E[\boldsymbol{p}])^2] \approx E[\boldsymbol{x}^2]$$

as E[x] i very small at all times and therefore much smaller than E[x^2].

Emittance computation

- The normalised transverse emittance is computed from the turn by turn data as follows:

$$uu = \langle u^2 \rangle - \frac{\langle u \cdot \delta \rangle^2}{\langle \delta^2 \rangle}$$

$$uup = \langle u \cdot up \rangle - \frac{\langle u \cdot \delta \rangle \langle up \cdot \delta \rangle}{\langle \delta^2 \rangle}$$

$$upup = \langle up^2 \rangle - \frac{\langle up \cdot \delta \rangle^2}{\langle \delta^2 \rangle}$$

$$\epsilon = \beta\gamma\sqrt{uu \cdot upup - uup^2}$$

, where (u, up) → (x, px) or (y, py) δ the momentum deviation of the particle and β,γ the relativistic β, γ.

Emittance growth due to CC RF Phase noise

* PhysRevSTAB.18.101001

- For a number of turns which is significantly larger than the noise decoherence time (= decoherence of betatron oscillations?1/betatron frequency?) :

- **Phase noise :** Depends on the overlap between phase noise spectrum and betatron tune distribution.

$$\frac{d\epsilon_y}{dt} = \beta_{CC}\left(\frac{eV_{CC}f_{rev}}{2E_b}\right)^2 C_{\Delta\phi(\sigma_\phi)}\int_{-\infty}^{+\infty}\sum_{k=-\infty}^{+\infty} S_{\Delta\phi}[(k\pm\nu_b)f_{rev}]\rho(\nu_b)d\nu_b$$

, where βcc the beta function at the location of the CC in m, frev is the revolution frequency in Hz, vb the betatron tune and ρ(vb) its pdf over all particles. SΔφ is the PSD of the noise at every betatron sideband in rad^2/Hz. σφ is the rms longitudinal bunch line density in rad at the CC frequency. CΔφ is the the correction term due to bunch length. It decreases with the bunch length.

- **Amplitude noise :** Depends on the overlap between phase noise spectrum and synchrobetatron tune distribution. CΔA is the the correction term due to bunch length. It increases with the bunch length. d S ΔA is the (relative) amplitude noise power spectral density (with units of 1/Hz).

$$\frac{d\epsilon_y}{dt} = \beta_{CC}\left(\frac{eV_{CC}f_{rev}}{2E_b}\right)^2 C_{\Delta A(\sigma_\phi)}\int_{-\infty}^{+\infty}\sum_{k=-\infty}^{+\infty} S_{\Delta A}[(k\pm\bar{\nu}_b\pm\bar{\nu}_s)f_{rev}]\rho(\nu_b)d\nu_b$$

Computing the total rms tune spread

- Below you can find the calculations from which we obtain the rms tune spread for the αyy values of upper and lower limit.
- rms(ΔQy)_oct = αyy*2*rms(Jy) = 0.014 and 0.0042 and , for αyy = 1e6 and 3e5 respectively.
  where the rms(Jy) computed as the rms of the <u>initial</u> actions of all particles computed as below, for the location of the 2nd CC.

$$J_x = \frac{x_N^2 + px_N^2}{2}$$

$$x_N = \frac{x}{\sqrt{\beta(s)}}$$

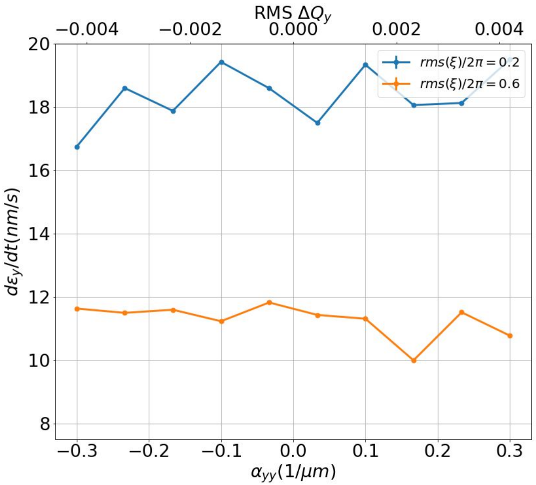
$$px_N = px \cdot \sqrt{\beta(s)} + x \frac{\alpha(s)}{\sqrt{\beta(s)}}$$

- rms(ΔQy)_chrom = $Q_{py}^{(2)}$ rms(δ)^2 ~ -1.5e-4, -5.3e-8
  As $Q_{py}^{(2)}$ = -13282.7307 and -3891.301526 for αyy = 1e6 and 3e5 respectively.
  rms(δ) = 2.1e-4, 2.6e-6 and for σz =15.5 cm and σz =1.55mm respectively.
- Therefore the total tune spread in the vertical plane for the upper limits of αyy values is computed from (1):
  - for σz = 15.5 cm : rms(ΔQy) ~ 0.0042
  - for σz = 1.55 mm: rms(ΔQy) ~ 0.014
- We can see that the <u>contribution of the 2nd order chromaticity</u> to the total tune spread is <u>negligible</u>, due to the small values of δ.

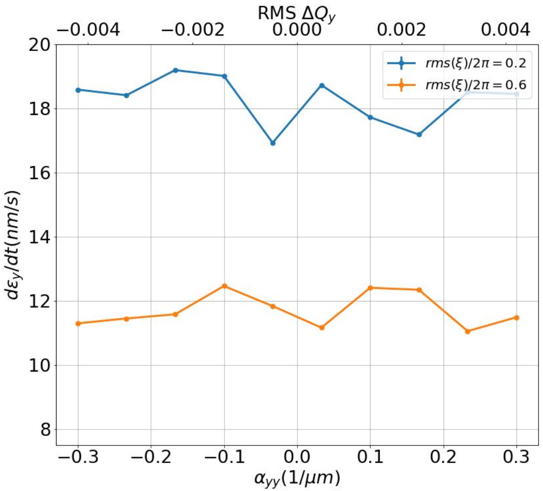
Non-zero linear chromaticity

- The studies are repeated with the **realistic chromaticity** settings of the 2018 CC MDs: **Qpx=Qpy=2**.
  - Two values of rms(ξ) tested in the area where there is no dependence on the tune spread.
  - Therefore, <u>no significant impact on the rate</u> is expected, which is validated from the simulations.

<u>Qpx = Qpy =0</u>



<u>Qpx = Qpy =2</u>



Qpx = Qpy = 2
σz = 15.5 cm
VCC1=VCC2=1MV
A = 1e-8