#### LEPTONS IN THE PROTON

#### Luca Buonocore

University of Zurich, INFN-sezione di Napoli

in collaboration with P. Nason, F. Tramontano and G. Zanderighi

[L. Buonocore, P. Nason, F. Tramontano and G. Zanderighi, arXiv:2005.06477v1]



CERN, June 12th 2020



## **OUTLINE**

• Introduction and Motivations

- Lepton PDF formula and the LUXlep set
- Validation and (a bit of ) Pheno
- Conclusions

## **INTRODUCTION**

#### INTRODUCTION

 Hadrons are usually viewed as "broad band beams" of coloured particles (quarks and gluons). Hard processes described by factorisation formulae in terms of convolutions with partonic PDFs

$$\sigma(h_1 + h_2 \to V + X) = \sum_{ab} \int dx_1 dx_2 f_{a/h_1}(x_1, \mu_F) f_{b/h_2}(x_2, \mu_F) \hat{\sigma}_{ab \to V + X}(\hat{s}, \mu_R) + \dots$$

- Indeed hadrons are made of constituent/valence quarks and (soft and collinear) QCD radiation is copiously produced (sea of gluons and quark)
- Order of quark and gluon PDFs

$$(\alpha_s L)^k$$
  $\alpha_s (\alpha_s L)^k$   $L \equiv \ln \frac{Q^2}{\Lambda^2}$ 

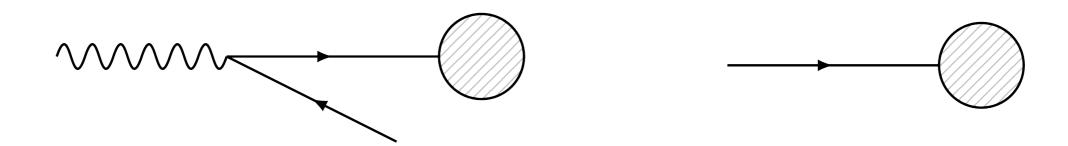
- $\circ$   $\Lambda$  is a characteristic **hadronic scale**.
- Since  $L \sim 1/\alpha_{\rm s}$ , all the contributions becomes relevant!

#### INTRODUCTION: MOTIVATIONS

- QCD radiation is certainly the dominant effect. When **electromagnetism** is taken into account, **photons and eventually leptons can be radiated** starting from quarks.
- Being down by **two powers** of electromagnetic  $\alpha$  (naive estimate), leptonic luminosities are indeed <u>very small</u> compared to the ones of the other partons inside the proton.
- It might be interesting to look at lepton initiated processes in hadronic colliders
  - in principle, all lepton-lepton combinations are available (and in a broad energy spectrum): potential to measure **exotic SM processes**
  - potential to look at exotic BSM physics
- LHC will take data for the next 15-20 years: **explore all of its possibilities!**

#### INTRODUCTION: MOTIVATIONS

In general, lepton PDFs do not open new production mechanisms



never-ending story 4F vs 5F

#### **PROs**

- Lepton masses very small (numerical instabilities). Potential large collinear  $\alpha \log \frac{Q}{m_\ell}$  effectively taken into account (and resummed in DGLAP)
- Smaller final state multiplicities
- Reduced hadronic activity (PS programs for lepton initiated processes required!)

# INTRODUCTION: MOTIVATIONS (PRE-LUX time)

 A crucial aspect which prevented so far to fully explore the phenomenology offered by lepton initiated processes is the lack of a precise determination of the lepton densities

# SINGLE LEPTOQUARK PRODUCTION AT HADRON COLLIDERS

J. Ohnemus<sup>1</sup>, S. Rudaz<sup>2</sup>, T.F. Walsh<sup>2</sup> and P.M. Zerwas<sup>3</sup>

 Physics Department, University of California, Davis CA 95616, USA
 School of Physics and Astronomy, University of Minnesota, Minneapolis MN 55455, USA

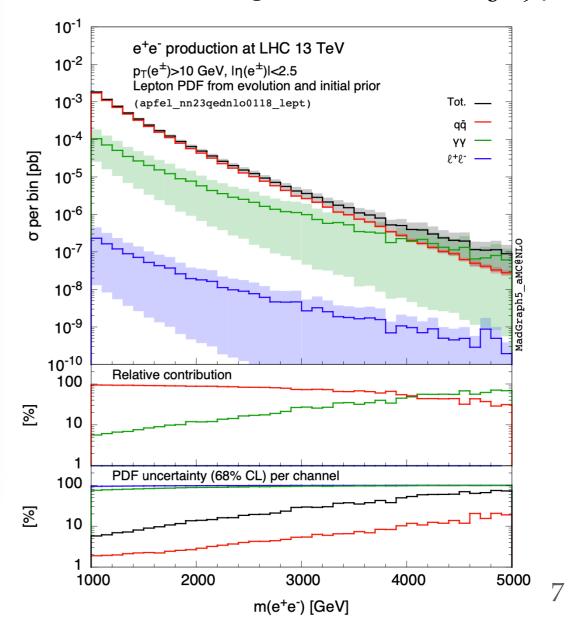
<sup>3</sup> Deutsches Elektronensynchrotron DESY, D-22603 Hamburg, FRG

#### Abstract

Leptoquarks can be produced in pairs by gluon–gluon fusion and quark–antiquark annihilation at hadron colliders. While HERA is the proper machine for single production of (eu) and (ed) type leptoquarks, the flavor species of  $(\mu u)$ ,  $(\mu d)$  and  $(\tau u)$ ,  $(\tau d)$  type leptoquarks can be produced at hadron colliders very efficiently. Besides exploiting gluon-quark collisions, leptoquarks can also be produced singly by colliding the quarks in one proton beam with leptons  $e, \mu, \tau$  generated by splitting photons which are radiated off the quarks in the other proton beam. For Yukawa couplings of the size  $\alpha$  leptoquark masses up to about 300 GeV can be generated at the Tevatron while the LHC can produce leptoquarks with masses up to about 3 TeV. [Leptoquarks involving heavy quarks can be produced singly at a lower rate, determined by the heavy flavor flux in the proton beam.]

1994 paper: very interesting, but almost forgotten...

[Bertone, Carrazza, Pagani, Zaro, JHEP 11 (2015) 194]



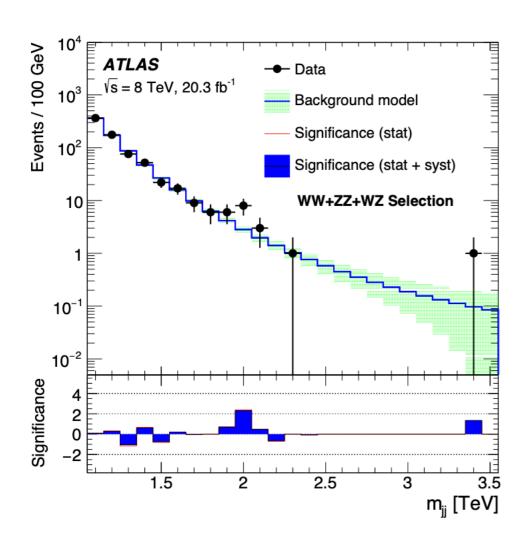
# LEPTON PDF FORMULA and THE LUXlep SET

#### The "LUX" APPROACH for the PHOTON PDF

LUX breakthrough in 2016-2017

[Manohar, Nason, Salam, Zanderighi, *Phys.Rev.Lett.* 117 (2016) 24, 242002] [Manohar, Nason, Salam, Zanderighi, *JHEP* 12 (2017) 046 ]

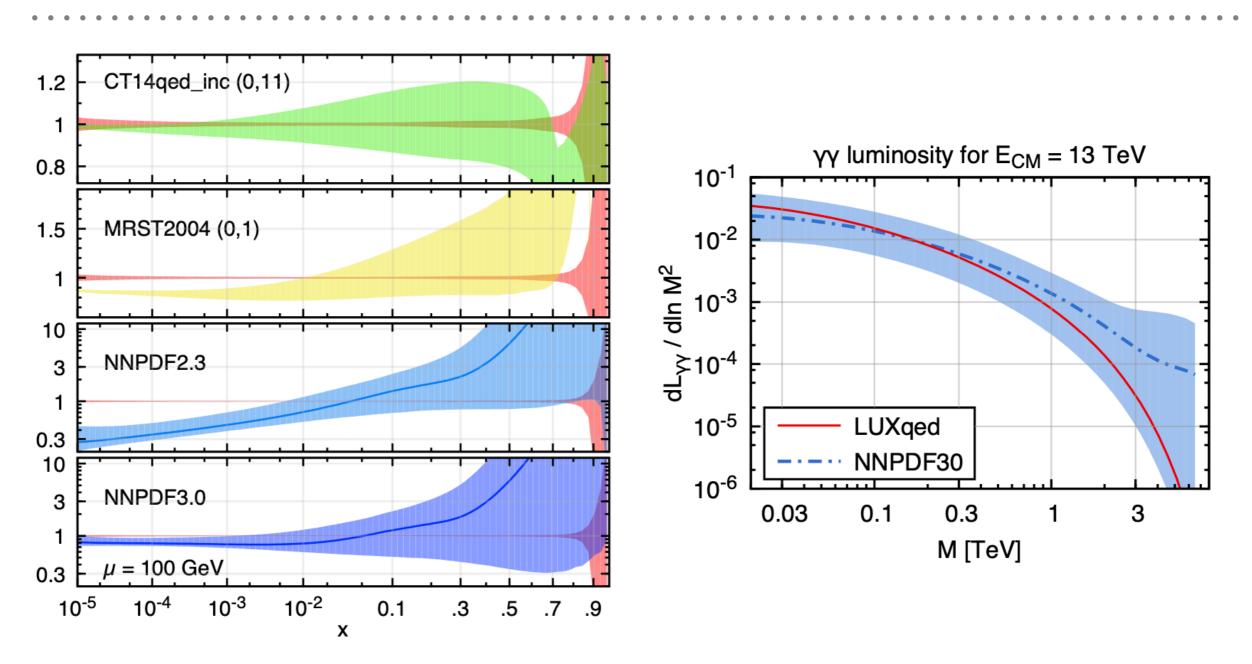
- determination of the photon density within ~5% uncertainty
- different motivations: uncertainty on the photon induced processes started to dominate the production of high mass objects



#### ATLAT boosted jets analysis (2015):

- 2 TeV excess in boson pair production
- Not confirmed in 13 TeV run
- The worry was that at very high scales gluon and quarks soften due to AP evolution.
- Photons mostly stay the same: importance of elastic contribution at low- $Q^2$

#### Before the "LUX" and after . . .



- Order of magnitude improvements in the knowledge of the photon content of proton (paradigmatic case: comparison with the "agnostic" photon of NNPDF30
- LUX approach can be used also for leptons (at order  $\alpha^2$ )!

#### THE LUX APPROACH in a NUTSHELL

- Relate the photon PDF to the electro-production structure functions and form factors for electron-proton scattering
  - physical ground: photon equivalent approximation and virtual quanta method, collinear factorisation
  - the computation can be systematically improved including higher order corrections to reach the desired accuracy goal
- Make use of the good quality data (already) available
  - electro-production structure functions measured in a wide range of energies
  - $\circ$  allow to **constrain** the photon PDF from **low- to high-** $Q^2$
- In general, no need to perform a global fit analysis
  - a new set which includes the photon can be produced starting from an existing one (which does not include it).

# PROs of LUX for lepton densities

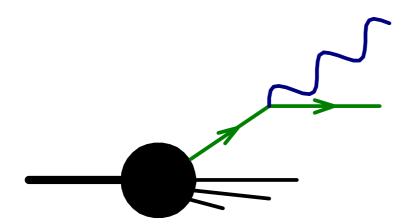
- In principle, lepton PDF can be generated radiatively from photon splitting into a lepton pair starting from their mass threshold.
- For electrons and muons, the mass threshold is smaller than the characteristic hadronic scales. LUX approach gives a correct initial condition taking into account low- $Q^2$  regime.
- Dominant mass contribution can be systematically taken into account.
- Control over the accuracy: systematic way to add corrections to a given order
- Better understanding of "radiative corrections" with leptons initiated processes (*what is NLO with mix QCD-QED partons?*)

# (PHYSICAL) COUNTING SCHEME for photons

Let's start from quark and gluon again ...

$$(\alpha_s L)^k$$
  $\alpha_s (\alpha_s L)^k$   $L \equiv \ln \frac{Q^2}{\Lambda^2}$ 

- The photon PDF is **down by a factor**  $\alpha L$  relatively to the quark density (the powers of  $(\alpha_s L)^k \sim 1$  are always understood in the following)
- $\alpha L$  is not of order one! This complicates the relative importance of the couplings



 $\alpha L$  (collinear-enhanced term)

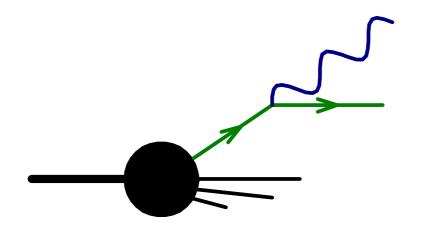
LO contribution

# (PHYSICAL) COUNTING SCHEME for photons

Let's start from quark and gluon again ...

$$(\alpha_s L)^k$$
  $\alpha_s (\alpha_s L)^k$   $L \equiv \ln \frac{Q^2}{\Lambda^2}$ 

- The photon PDF is **down by a factor**  $\alpha L$  relatively to the quark density (the powers of  $(\alpha_s L)^k \sim 1$  are always understood in the following)
- $\alpha L$  is not of order one! This complicates the relative importance of the couplings



 $\alpha$  (non-collinear-enhanced term)

down by  $1/L \sim \alpha_s$  wrt LO

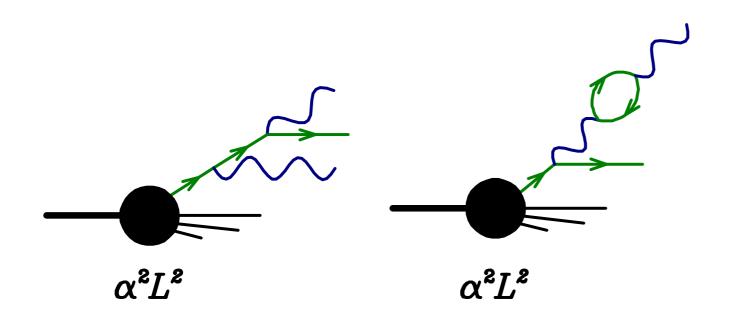
**NLO** contribution

# (PHYSICAL) COUNTING SCHEME for photons

Let's start from quark and gluon again ...

$$(\alpha_s L)^k$$
  $\alpha_s (\alpha_s L)^k$   $L \equiv \ln \frac{Q^2}{\Lambda^2}$ 

- The photon PDF is **down by a factor**  $\alpha L$  relatively to the quark density (the powers of  $(\alpha_s L)^k \sim 1$  are always understood in the following)
- $\alpha L$  is not of order one! This complicates the relative importance of the couplings



Assuming  $\alpha \sim \alpha_s^2$  down by  $\alpha L \sim \alpha_s$  wrt LO

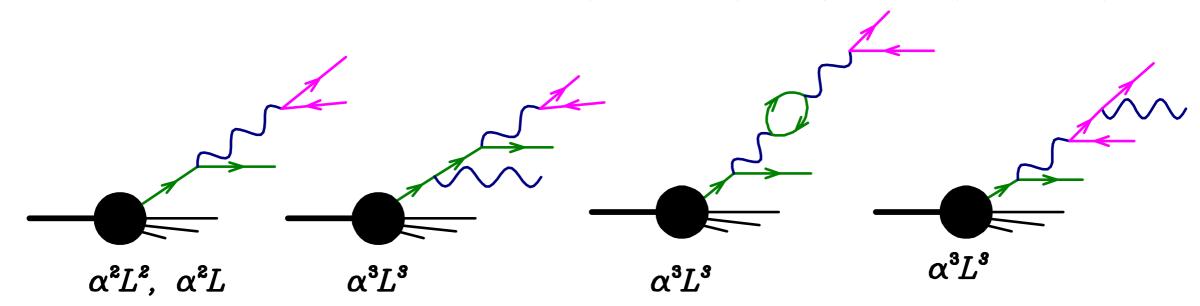
**NLO** contributions

# (PHYSICAL) COUNTING SCHEME for leptons

Photon PDF

 $f_{\gamma}: \begin{array}{cccc} & LO & NLO \\ \alpha L & \alpha & \alpha^2 L^2 & \dots \end{array}$ 

- Terms down by  $\alpha$  with respect to LO are neglected (NNLO in our counting)
- Similarly for lepton PDFs



#### APPLICATION TO THE LEPTON PDF CASE

• Consider a fictitious BSM scalar probe that couples only to leptons and allow a flavour changing transitions to a BSM heavy lepton L of mass M

$$\mathscr{L} \sim \phi_0 \bar{L}l + \text{h.c.}$$

1. The cross section can be computed as in **DIS** 

$$\sigma = \frac{1}{4p \cdot r} \int \frac{d^4q}{(2\pi)^4} \frac{1}{Q^4} L^{\mu\nu}(r,q)(4\pi) W_{\mu\nu}(p,q) \qquad p \qquad \qquad p$$

Leptonic Tensor

$$W_{\mu\nu}(p,q) = F_1 \left( -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \right) + \frac{F_2}{p \cdot q} \left( p_{\mu} - \frac{p \cdot q \ q_{\mu}}{q^2} \right) \left( p_{\nu} - \frac{p \cdot q \ q_{\nu}}{q^2} \right)$$

Hadronic Tensor (scattering of virtual photon)

$$Q^2 = -q^2 > 0, x_{bj} = \frac{Q^2}{2p \cdot q}$$

 $F_1(x_{bj}, Q^2), F_2(x_{bj}, Q^2)$  are the proton structure functions

## **DIS-like COMPUTATION**

Integration domain

$$\int \frac{dE_{\rm cm}^2}{2\pi} \frac{1}{4p \cdot r} \frac{1}{16\pi^2 E_{\rm cm}^2} \int_{x}^{1 - \frac{2xm_{\rm P}}{E_{\rm cm}}} dz \int_{\frac{m_{\rm P}^2 x^2}{1 - z}}^{\frac{E_{\rm cm}^2 (1 - z)}{z}} \frac{dQ^2}{Q^2} \qquad E_{\rm cm}^2 = (r - q)^2$$

logarithmic integral

• Sketch of the structure of the integral function  $\frac{dominated}{dominated}$  at low  $Q^2$ 

$$F_i \times P(Q^2, m_{\mathrm{P}}^2, m_{\ell}^2, \dots) \log \frac{M^2}{Q^2} + F_i \times R(Q^2, m_{\mathrm{P}}^2, m_{\ell}^2, \dots)$$
explicit logarithm of  $Q^2$ 

• P and R do not include logarithmic enhanced terms in  $Q^2$ 

#### DIS-like COMPUTATION

• Integration domain

$$\int \frac{dE_{\rm cm}^2}{2\pi} \frac{1}{4p \cdot r} \frac{1}{16\pi^2 E_{\rm cm}^2} \int_{x}^{1 - \frac{2xm_{\rm P}}{E_{\rm cm}}} dz \int_{\frac{m_{\rm P}^2 x^2}{1 - z}}^{\frac{E_{\rm cm}^2 (1 - z)}{z}} \frac{dQ^2}{Q^2} \qquad E_{\rm cm}^2 = (r - q)^2$$

Sketch of the structure of the integral function

$$F_i \times P(Q^2, m_{\rm P}^2, m_{\ell}^2, \dots) \log \frac{M^2}{Q^2} + F_i \times R(Q^2, m_{\rm P}^2, m_{\ell}^2, \dots)$$

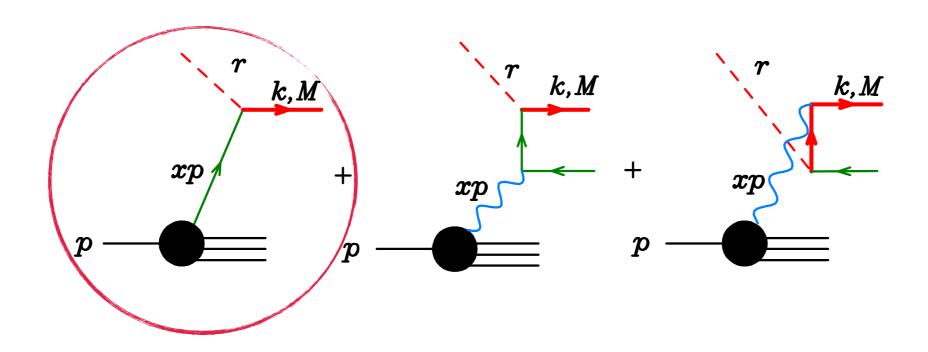
$$\frac{P}{Q^2} \times \frac{m_{\rm P}^2}{Q^2} \times \frac{m_{\ell}^2}{Q^2}$$

$$L \qquad \text{noting } \frac{m_{\rm P}^2}{m_{\ell}^2} \times \frac{m_{\ell}^2}{m_{\ell}^2} \times \frac{L}{m_{\ell}^2} \times \frac{m_{\ell}^2}{m_{\ell}^2} \times \frac{L}{m_{\ell}^2} \times \frac{L}{m_{\ell}^2}$$

19

## PARTON MODEL CALCULATION

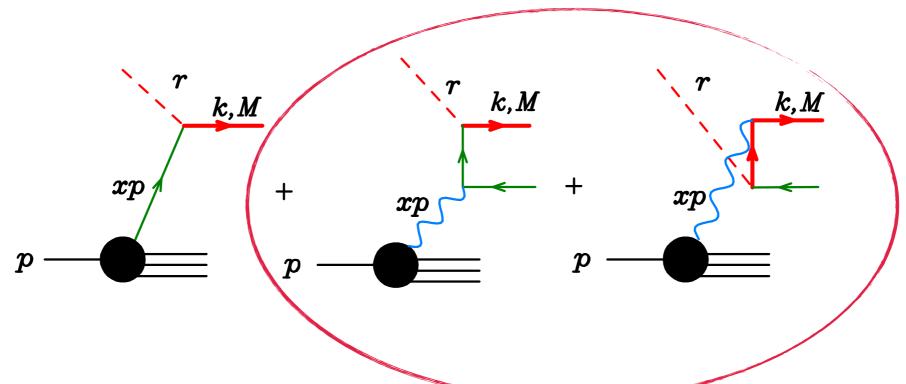
2. The cross section can be computed applying the collinear factorisation



$$\frac{\sigma}{\sigma_B} = \int dx f_{\ell}(x, \mu_F^2) \delta(Sx - M^2)$$

#### PARTON MODEL CALCULATION

2. The cross section can be computed applying the collinear factorisation



$$\begin{split} \frac{\sigma}{\sigma_B} &= \int dx f_{\ell}(x, \mu_F^2) \delta(Sx - M^2) \\ &+ \frac{\alpha}{2\pi} \frac{1}{M^2} \int_{\frac{M^2}{S}}^{1} dx f_{\gamma}(x, \mu_F^2) \left\{ z_{\ell} P_{l\gamma}(z_{\ell}) \left[ \log \frac{M^2}{\mu_F^2} + \log \frac{(1 - z_{\ell})^2}{z_{\ell}^2} \right] + 4 z_{\ell}^2 (1 - z_{\ell}) \right\} \end{split}$$

#### LEPTON PDF FORMULA

- Compare the two cross sections for the probe process
- **Retain** only terms that contribute within our accuracy

$$x_{\ell}f_{\ell}(x_{\ell},\mu_{F}^{2}) = \left(\frac{1}{2\pi}\right)^{2} \int_{x_{\ell}}^{1} \frac{\mathrm{d}x}{x} z_{\ell} \int_{x}^{1} \frac{\mathrm{d}z}{z} \int_{\frac{m_{p}^{2}x^{2}}{1-z}}^{\frac{\mu_{F}^{2}}{1-z}} \frac{\mathrm{d}Q^{2}}{Q^{2}} \alpha^{2}(Q^{2})$$

$$\left\{ P_{\ell\gamma}(z_{\ell}) \log \frac{\mu_{F}^{2}}{(1-z_{\ell})z_{\ell} \left(Q^{2} + \frac{m_{\ell}^{2}}{z_{\ell}(1-z_{\ell})}\right)} \left[ F_{2} \left(z P_{\gamma q}(z) + \frac{2m_{p}^{2}x^{2}}{Q^{2}}\right) - F_{L}z^{2} \right] + F_{2} \left[ 4(z-2)^{2} z_{\ell}(1-z_{\ell}) - (1+4z_{\ell}(1-z_{\ell})) z P_{\gamma q}(z) \right] \right\}.$$

$$P_{\gamma q}(z) = rac{1 + (1 - z)^2}{z}, \qquad P_{\ell \gamma}(z_{\ell}) = 1 - 2z_{\ell} + 2z_{\ell}^2$$
  $F_L \equiv \left(1 + rac{4x_{
m bj}^2 m_{
m p}^2}{Q^2}\right) F_2 - 2x_{
m bj} F_1$ 

#### LEPTON PDF FORMULA

- 3-fold integral (one integration can be performed analytically)
- $\alpha^3 L^3$  contributions **not included** here!

$$x_{\ell}f_{\ell}(x_{\ell},\mu_{F}^{2}) = \left(\frac{1}{2\pi}\right)^{2} \int_{x_{\ell}}^{1} \frac{\mathrm{d}x}{x} z_{\ell} \int_{x}^{1} \frac{\mathrm{d}z}{z} \int_{\frac{m_{p}^{2}x^{2}}{1-z}}^{\frac{\mu_{F}^{2}}{1-z}} \frac{\mathrm{d}Q^{2}}{Q^{2}} \alpha^{2}(Q^{2})$$

$$\left\{ P_{\ell\gamma}(z_{\ell}) \log \frac{\mu_{F}^{2}}{(1-z_{\ell})z_{\ell} \left(Q^{2} + \frac{m_{\ell}^{2}}{z_{\ell}(1-z_{\ell})}\right)} \left[ F_{2} \left(z P_{\gamma q}(z) + \frac{2m_{p}^{2}x^{2}}{Q^{2}}\right) - F_{L}z^{2} \right] + F_{2} \left[ 4(z-2)^{2} z_{\ell}(1-z_{\ell}) - (1+4z_{\ell}(1-z_{\ell})) z P_{\gamma q}(z) \right] \right\}.$$

Dominant mass effect given by the replacement

$$\log \frac{\mu_F^2}{(1 - z_\ell) z_\ell Q^2} \to \log \frac{\mu_F^2}{(1 - z_\ell) z_\ell \left(Q^2 + \frac{m_\ell^2}{z_\ell (1 - z_\ell)}\right)}$$

#### AP EQUATION

- By construction, the Lepton PDF formula is expected to satisfy a suitable Altarelli-Parisi equation.
- Taking the derivative with respect to the factorisation scale, we get

$$\frac{d}{d \ln \mu_F^2} f_{\ell} = \frac{\alpha(\mu_F^2)}{2\pi} P_{\ell\gamma} \otimes f_{\gamma} + \left(\frac{\alpha(\mu_F^2)}{2\pi}\right)^2 \sum_q P_{\ell q} \otimes f_q$$

$$\alpha \times \alpha L = \alpha^2 L \qquad \alpha^2 \times \mathcal{O}(1) = \alpha^2$$

- The first term is the dominant contribution to the DGLAP evolution. The second term is due to the NLO contributions included in our calculation. It is down by *L* with respect to the first contribution.
- Formally the second terms is a NNLO QED evolution kernel. Be aware, it should be included in DGLAP equations for the lepton pdf if NLO accuracy is required!

As a byproduct of our computation, we get an expression for the NNLO QED splitting kernel  $P_{\ell a}$  in agreement with [De Florian, Sborlini, Rodrigo, JHEP 10 (2016) 056]

#### AP EQUATION

- By construction, the Lepton PDF formula is expected to satisfy a suitable Altarelli-Parisi equation.
- Taking the derivative with respect to the factorisation scale, we get

$$\frac{d}{d \ln \mu_F^2} f_{\ell} = \frac{\alpha(\mu_F^2)}{2\pi} P_{\ell\gamma} \otimes f_{\gamma} + \left(\frac{\alpha(\mu_F^2)}{2\pi}\right)^2 \sum_q P_{\ell q} \otimes f_q$$

$$\alpha \times \alpha L = \alpha^2 L \qquad \alpha^2 \times \mathcal{O}(1) = \alpha^2$$

• Since we did not include  $\alpha^3 L^3$  terms, we miss the NLO contribution

$$\frac{\alpha(\mu_F^2)}{2\pi} P_{\ell\ell} \otimes f_{\ell}$$

$$\alpha \times \alpha^2 L^2 = \alpha^3 L^2 \sim \alpha^2$$

• We can restore its dominant contribution by solving suitable AP equations!

- The lepton PDF formula
  - can be computed numerically with high accuracy
  - requires experimental input for structure functions and form factors of the proton (fit + **uncertainties**) in both low- and high- $Q^2$  (from pdfs fit) regime (same data as for photon PDF)
  - allows the determination of the lepton densities at a given (almost) arbitrary scale. Sensitive to higher twist at low scales!
- To build a full grid, **use DGLAP** evolution (more efficient) starting from an already available pdf set! In principle,
  - use the lepton PDF formula to extract an initial condition for the lepton densities at a reference scale
  - solve the integro-differential DGLAP equations including all the relevant splitting kernels which contribute to the desired target accuracy:

$$\alpha_s$$
,  $\alpha_s \alpha$ ,  $\alpha^2$  ( $P_{\ell q}$  must be included!)

o make available the grid in a standard format (aka LHAPDF)

- In practice, it's a bit more involved
  - we use a input NNPDF31\_nlo\_as\_0118 luxqed
  - $\circ\,$  we rely on HOPPET as DGLAP solver (different evolution framework, it does not include the  $P_{\ell q}$  splitting)
  - we add missing  $\alpha^3 L^3$  contributions through evolution

- In practice, it's a bit more involved
  - we use a input NNPDF31\_nlo\_as\_0118 luxqed
  - $\circ$  we rely on HOPPET as DGLAP solver (different evolution framework, it does not include the  $P_{\ell q}$  splitting)
  - we add missing  $\alpha^3 L^3$  contributions through evolution



1. Choose a reference scale where the Lepton (and Photon) PDF are extracted with our formula

**Remarks**:  $\mu_{\text{ref}}$  cannot be arbitrarily small, otherwise too sensitive to power suppression terms in  $Q^2$  (higher twist)

- In practice, it's a bit more involved
  - we use a input NNPDF31\_nlo\_as\_0118 luxqed
  - $\circ$  we rely on HOPPET as DGLAP solver (different evolution framework, it does not include the  $P_{\ell q}$  splitting)
  - we add missing  $\alpha^3 L^3$  contributions through evolution



- 2. Choose a second scale  $\mu_{PDF}$  at which the original partons in NNPDF are loaded
- 3. Evolve from  $\mu_{PDF}$  to  $\mu_{ref}$  with all splitting turned on but the ones into leptons

**Remarks**: this is to avoid numerical instabilities due to the use of a different evolution program

- In practice, it's a bit more involved
  - we use a input NNPDF31\_nlo\_as\_0118 luxqed
  - $\circ$  we rely on HOPPET as DGLAP solver (different evolution framework, it does not include the  $P_{\ell q}$  splitting)
  - we add missing  $\alpha^3 L^3$  contributions through evolution

- 5. Add (replace) the lepton (photon) densities at  $\mu_{ref}$
- 6. Evolve down from  $\mu_{\text{ref}}$  to  $\mu_0$  with all splitting turned on but the  $P_{\ell\ell}$  which is responsible for the transition  $\ell \to \ell + \gamma$

**Remarks**: this matches our calculation of the lepton PDF

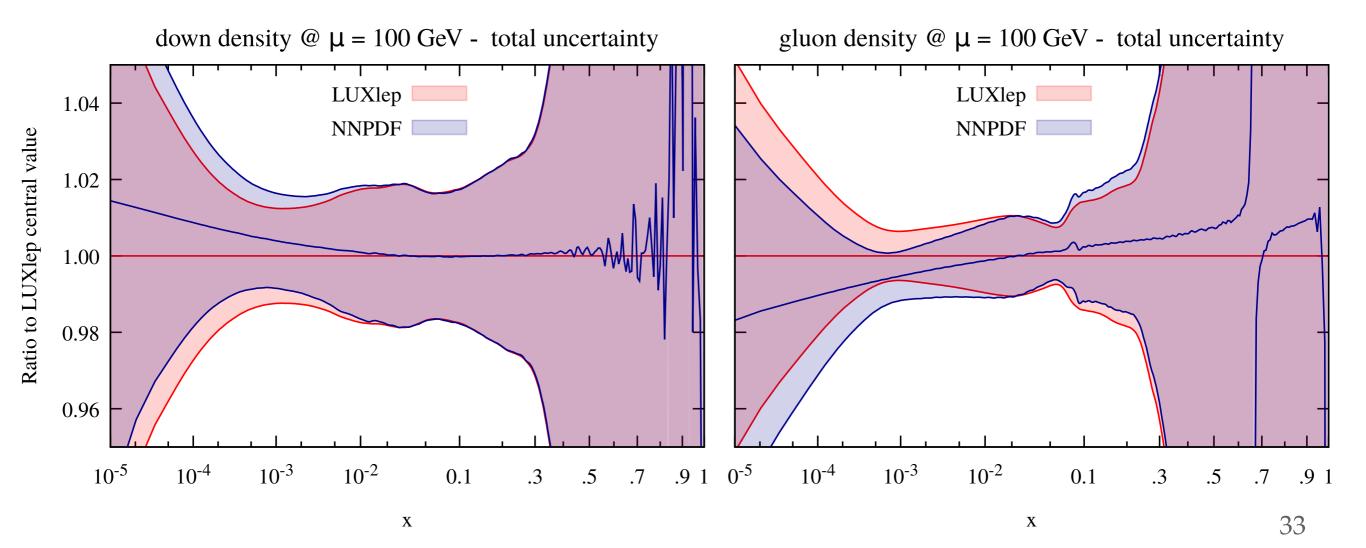
- In practice, it's a bit more involved
  - we use a input NNPDF31\_nlo\_as\_0118 luxqed
  - $\circ$  we rely on HOPPET as DGLAP solver (different evolution framework, it does not include the  $P_{\ell q}$  splitting)
  - we add missing  $\alpha^3 L^3$  contributions through evolution

7. Finally, evolve from  $\mu_0$  to all scales with the full set of splitting, including  $P_{\ell\ell}$  **Remarks**: in this way, we effectively take into account  $\alpha^3 L^3$  contributions

# **VALIDATION** and (a bit of) PHENO

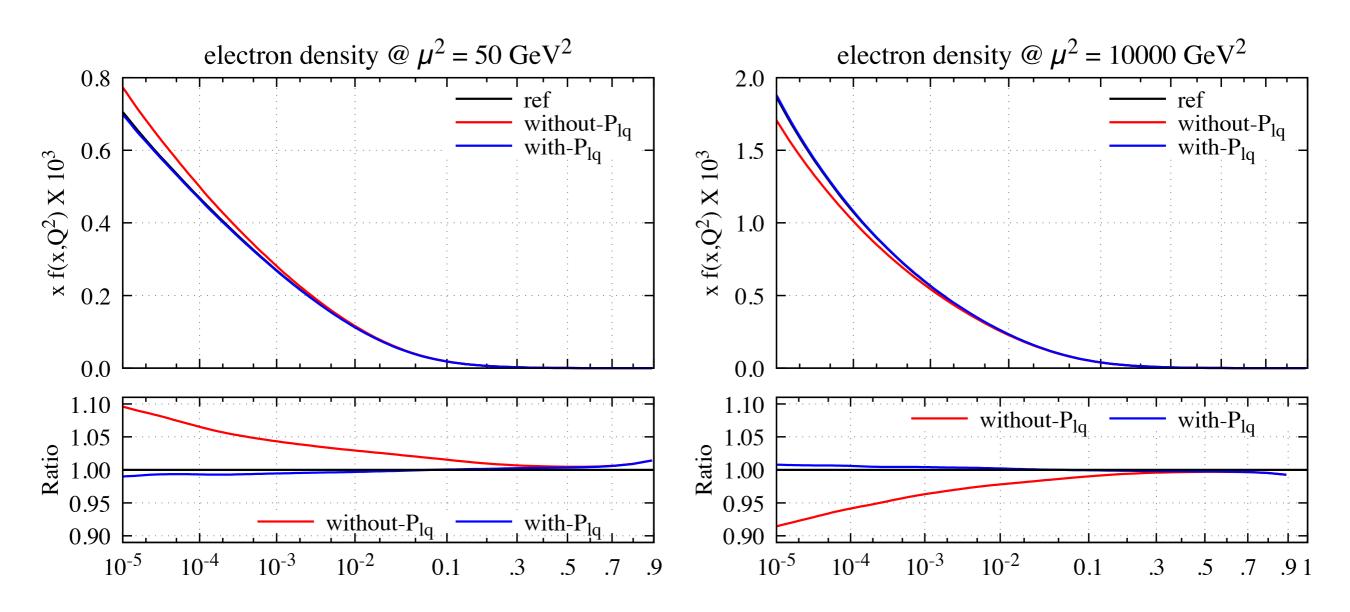
# **COMPARISON among QUARK/GLUON PFDs**

- Slightly differences (especially for gluons), within uncertainties
  - mainly due to the use of a different evolution framework (HOPPET)
  - effects of lepton PDF negligible on quark/gluon densities and momentum sum rule (sub per mille effect)



# IMPORTANCE of the $\mathcal{O}(\alpha^2)\,P_{\mathcal{C}q}$ -SPLITTING in DGLAP

• If not included, it leads to  $\mathcal{O}(10\%)$  differences in the small-x region, where its contribution is logarithmic enhanced



#### **UNCERTAINTIES on LEPTON DENSITIES**

#### We consider

- 6 variations on the fits used as input data for the proton structure functions and form factor (as in the photon PDF papers)
- a scale variation prescription to estimate the uncertainty due to missing higher orders
- replicas to take into account PDF uncertainties

**Procedure**: for each replica member *m* in the original NNPDF set

- 1. we apply our method to add leptons
- 2. we compute the correction

$$\Delta_{i}^{(m)}(x,\mu_{F}) = \sum_{j=1}^{7} \frac{f_{i,(j)}^{(0)}(x,\mu_{F}) - f_{i}^{(0)}(x,\mu_{F})}{f_{i}^{(0)}(x,\mu_{F})} f_{i}^{(m)}(x,\mu_{F}) \times R(m,j)$$

7 variations of the central set

Gaussian distributed random number with unit variance

#### **UNCERTAINTIES on LEPTON DENSITIES**

#### We consider

- 6 variations on the fits used as input data for the proton structure functions and form factor (as in the photon PDF papers)
- a scale variation prescription to estimate the uncertainty due to missing higher orders
- replicas to take into account PDF uncertainties

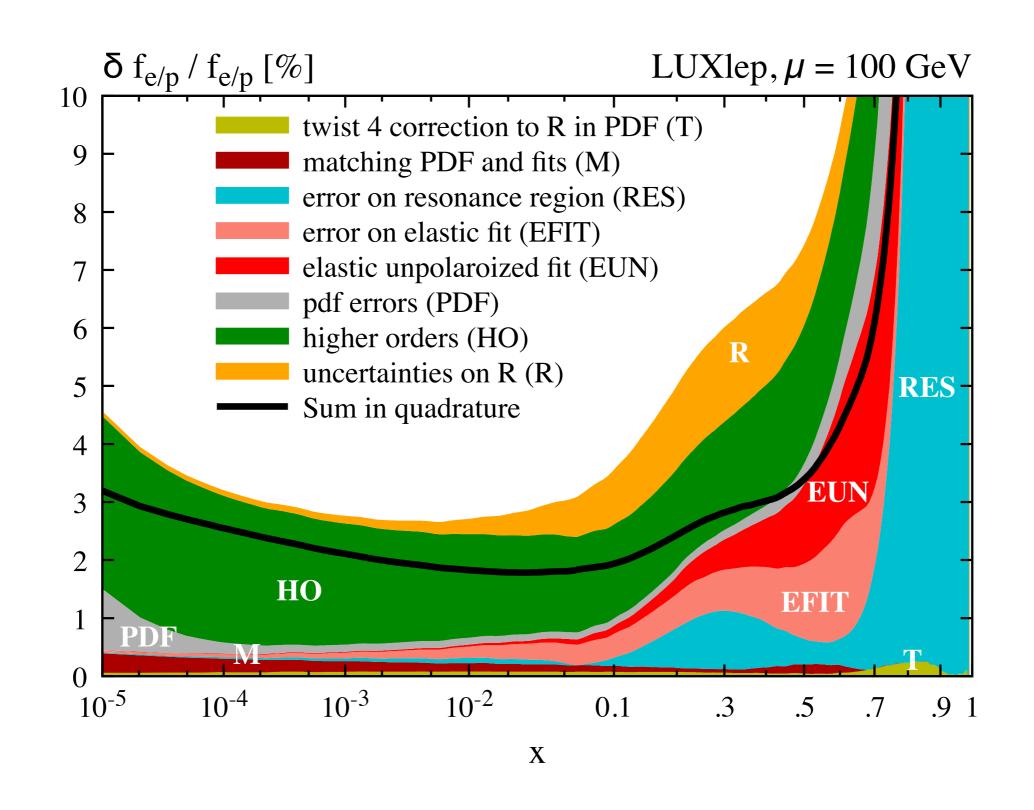
**Procedure**: for each replica member *m* in the original NNPDF set

- 1. we apply our method to add leptons
- 2. we compute the correction
- 3. we correct the replica as

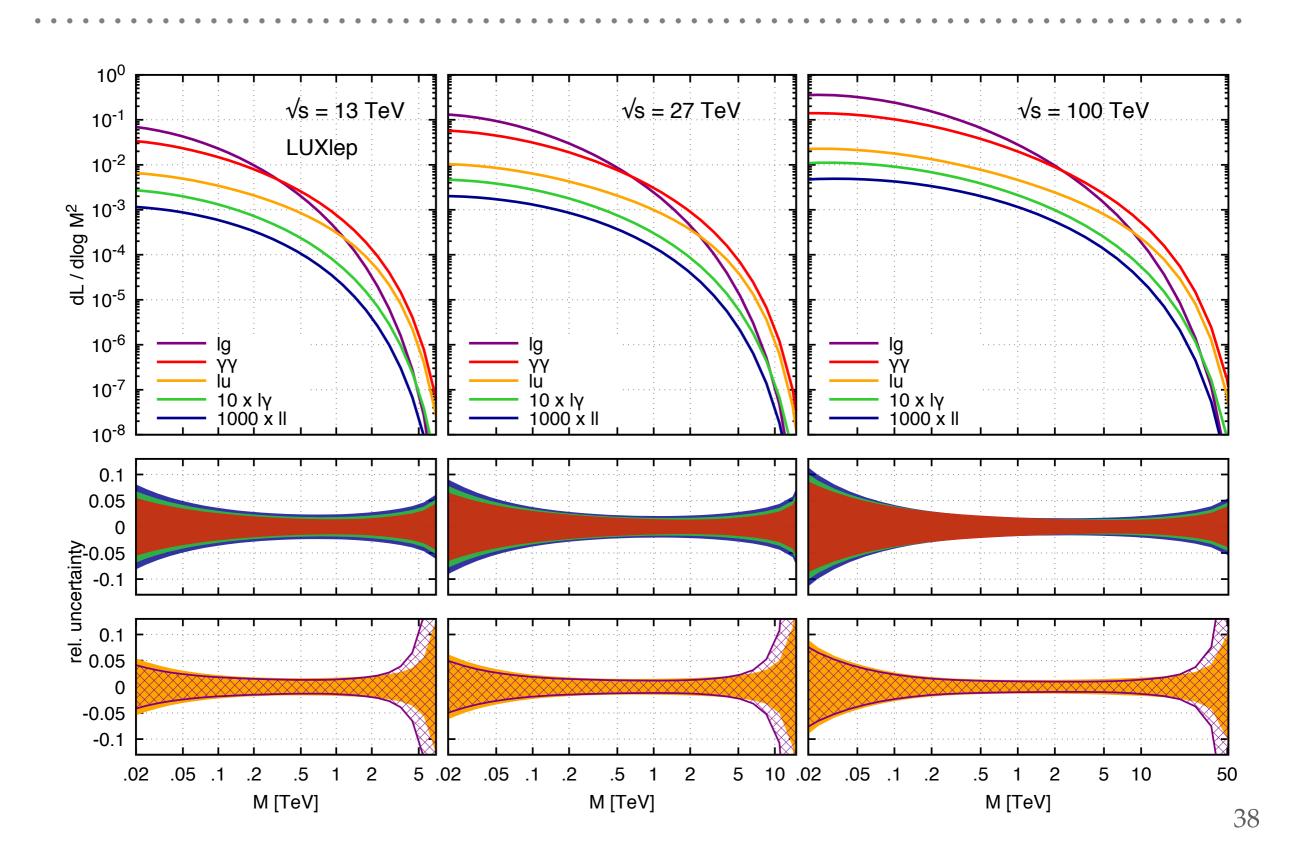
$$f_i^{(m)}(x,\mu_F) \to f_i^{(m)}(x,\mu_F) + \Delta_i^{(m)}(x,\mu_F) - \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \Delta_k^{(m)}(x,\mu_F)$$

so that the average of all replicas is not shifted.

## **UNCERTAINTIES on LEPTON DENSITIES**



## **LUMINOSITIES**



#### SM LEPTON-LEPTON SCATTERING

Lepton densities are very small, but lepton-lepton processes might be observed.
 Consider different flavours/same sign combination

$$p_{t,\ell} > 20 \text{GeV}, \quad |\eta_{\ell}| < 2.4$$

	$e^+\mu^-$	$e^+ au^-$	$\mu^+  au^-$	$e^+e^+$	$\mu^+\mu^+$	$ au^+ au^+$
$\sigma_{13{ m TeV}}$ [fb]	$0.29^{+0.13}_{-0.10}$	$0.18^{+0.11}_{-0.08}$	$0.16^{+0.10}_{-0.07}$	$0.24^{+0.10}_{-0.08}$	$0.19^{+0.09}_{-0.07}$	$0.08^{+0.06}_{-0.04}$
$\sigma_{27 { m TeV}} \ [{ m fb}]$	$0.53^{+0.25}_{-0.18}$	$0.34^{+0.21}_{-0.15}$	$0.30^{+0.20}_{-0.13}$	$0.439^{+0.19}_{-0.14}$	$0.33^{+0.17}_{-0.12}$	$0.14^{+0.12}_{-0.07}$

- We considered only WW background and we found it is negligible after requiring suitable signal-like cuts. Heavy flavour production background might be relevant
- A dedicated analysis **requires Shower Monte Carlo programs** for lepton initiated processes. (Current versions of Pythia and Herwig cannot do it. Richardson provided us a patch for Herwig).
- Since hardest radiation will be an opposite-charge lepton (from photon splitting), we expect a reduced hadron activity.

#### SM LEPTON-LEPTON SCATTERING

Lepton densities are very small, but lepton-lepton processes might be observed.
 Consider different flavours/same sign combination

$$p_{t,\ell} > 20 \text{GeV}, \quad |\eta_{\ell}| < 2.4$$

	$e^+\mu^-$	$e^+ au^-$	$\mu^+  au^-$	$e^+e^+$	$\mu^+\mu^+$	$ au^+ au^+$
$\sigma_{13{ m TeV}}$ [fb]	$0.29^{+0.13}_{-0.10}$	$0.18^{+0.11}_{-0.08}$	$0.16^{+0.10}_{-0.07}$	$0.24^{+0.10}_{-0.08}$	$0.19^{+0.09}_{-0.07}$	$0.08^{+0.06}_{-0.04}$
$\sigma_{27 { m TeV}}$ [fb]	$0.53^{+0.25}_{-0.18}$	$0.34^{+0.21}_{-0.15}$	$0.30^{+0.20}_{-0.13}$	$0.439^{+0.19}_{-0.14}$	$0.33^{+0.17}_{-0.12}$	$0.14^{+0.12}_{-0.07}$

- We considered only WW background and we found it is negligible after requiring suitable signal-like cuts. Heavy flavour production background might be relevant
- Quoted uncertainties refer to standard scale variation of the factorisation scale. They are bigger than PDF uncertainties.
- This corresponds to LO scale variation around a rather small scale.
- We expect a considerable reduction if "NLO" corrections are included. In particular, photon induced processes should be included.

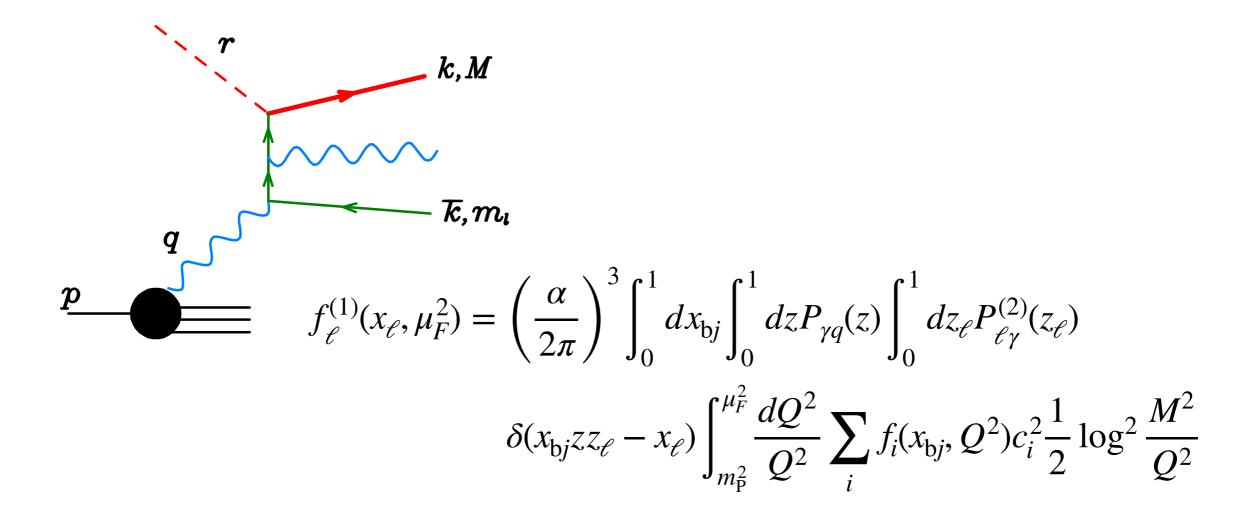
#### **CONCLUSIONs**

- Lepton densities in the proton can be modelled with high precision using the LUX approach
- Using NLO lepton PDFs, lepton initiated processes are pushed to a level of accuracy comparable to the ones involving coloured partons.
- NLO+PS implementations are within the reach. They are desirable to fully exploit the small hadron activity for background rejection.
- A comment upon NLO corrections to lepton initiated processes:  $\mathcal{O}(\alpha)$ -photon induced processes must be included as NLO QCD. In essence, this is due to the relative importance of photon and lepton densities.
- Despite the small cross sections
  - ◆ SM lepton initiated processes may be measurable
  - lepton initiated processes has the potential to enlarge New Physics sensitivity in hadron collisions
     Uli's talk!

## **BACKUP**

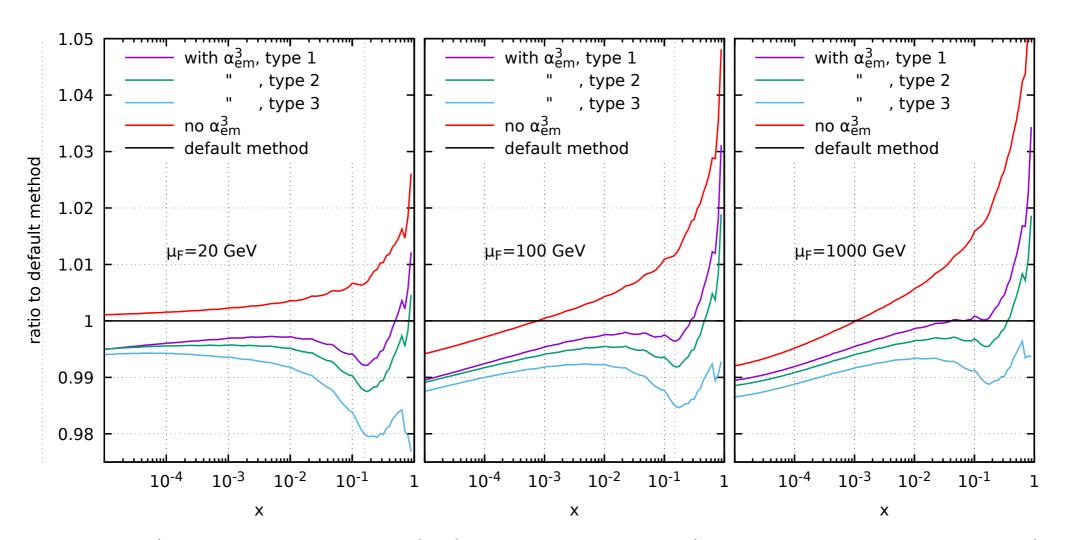
# $\mathcal{O}(\alpha^3 L^3)$ corrections

- We need to add only diagrams with photon emission off leptons
- The dominant contribution can be computed in the collinear approximations



two explicit logs and one from  $Q^2$  integration. Here  $P_{\ell\gamma}^{(2)}=P_{\ell\ell}\otimes P_{\ell\gamma}$ 

# $\mathcal{O}(\alpha^3 L^3)$ corrections – comparison with DGLAP



Our default method compared with three variants of the direct calculation of the  $\alpha^3 L^3$  terms. We conclude that

- The effect of the  $\alpha^3 L^3$  term is quite modest.
- There are large differences among the different method for its inclusion. This seems to indicate that sub-leading  $\alpha^3$  terms are not much smaller than the leading one.

# Luminosities for different leptons species

