PAUL SCHERRER INSTITUT

TOP MASS SCHEME AND SCALE UNCERTAINTIES

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I Introduction

II Gluon Fusion

III $H^* \to \gamma\gamma$

IV $gg \to HH$

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I *INTRODUCTION*

- $\sigma(gg \to H) = \left(54.72^{+4.3\%}_{-6.5\%}(TH) \pm 3.2\%(PDF, \alpha_s)\right)\ pb$ @ $\sqrt{s} = 14$ TeV
  Anastasiou,. . .
- uncertainties: PDF+α_s, renormalization/factorization scale
  top/bottom masses: $\sim \pm 0.8\%$ $\leftarrow$ scale/scheme dependence
- pole mass $\leftrightarrow$ $\overline{\text{MS}}$ mass:

$$\overline{m}_t(m_t) = \frac{m_t}{\kappa(m_t)}$$

$$\kappa(m_t) = 1 + \frac{4}{3}\frac{\alpha_s(m_t)}{\pi} + 10.9\left(\frac{\alpha_s(m_t)}{\pi}\right)^2 + 107.1\left(\frac{\alpha_s(m_t)}{\pi}\right)^3$$

$$\overline{m}_t(\mu_t) = \overline{m}_t(m_t)\frac{c\left[\alpha_s(\mu_t)/\pi\right]}{c\left[\alpha_s(m_t)/\pi\right]}$$

$$c(x) = \left(\frac{7}{2}x\right)^{\frac{4}{7}}\left[1 + 1.398x + 1.793\,x^2 - 0.6834\,x^3\right]$$

$m_t = 172.5$ GeV $\qquad \overline{m}_t(\overline{m}_t) = 163.0$ GeV $\qquad M_H/4 < \mu_t < M_H$

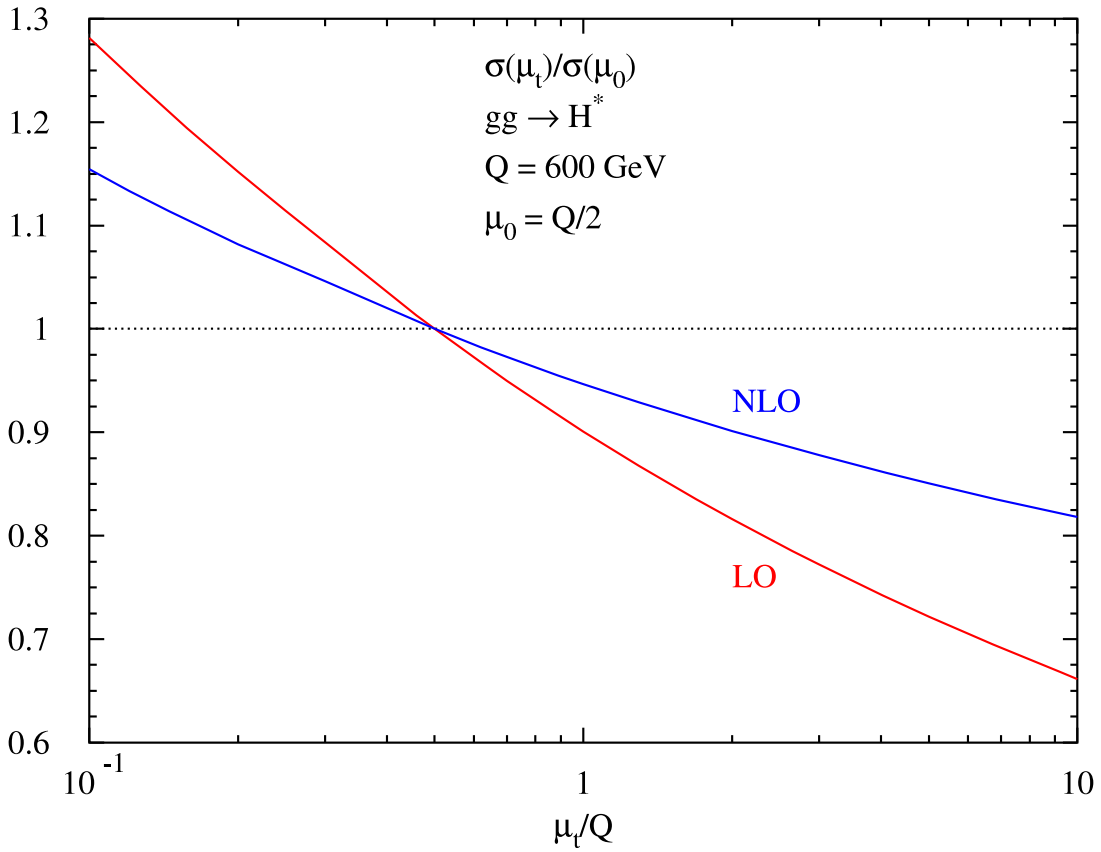
$\sigma(gg \to H)_{LO} = 18.43^{+0.8\%}_{-1.1\%}$ pb $\qquad \sigma(gg \to H)^{QCD}_{NLO} = 42.17^{+0.4\%}_{-0.5\%}$ pb

II *GLUON FUSION*

- off-shell Higgs production: $gg \to H^*$

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma_H}{dQ^2} + \frac{d\sigma_{int}}{dQ^2} + \frac{d\sigma_{cont}}{dQ^2}$$

$$\frac{d\sigma_H}{dQ^2} = \frac{Q}{\pi} \frac{\sigma(gg \to H^*) \times \Gamma(H^* \to X)}{(Q^2 - M_H^2)^2 + M_H^2 \Gamma_H^2}$$



m_t scheme/scale uncertainties only:

- LO:

$\sigma(gg \to H^*)|_{Q=125\ \text{GeV}} = 18.43^{+0.8\%}_{-1.1\%}\ \text{pb}, \qquad \sigma(gg \to H^*)|_{Q=300\ \text{GeV}} = 4.88^{+23.1\%}_{-1.1\%}\ \text{pb}$

$\sigma(gg \to H^*)|_{Q=400\ \text{GeV}} = 4.94^{+1.2\%}_{-1.8\%}\ \text{pb}, \qquad \sigma(gg \to H^*)|_{Q=600\ \text{GeV}} = 1.13^{+0.0\%}_{-26.2\%}\ \text{pb}$

$\sigma(gg \to H^*)|_{Q=900\ \text{GeV}} = 0.139^{+0.0\%}_{-36.0\%}\ \text{pb}, \quad \sigma(gg \to H^*)|_{Q=1200\ \text{GeV}} = 0.0249^{+0.0\%}_{-41.1\%}\ \text{pb}$

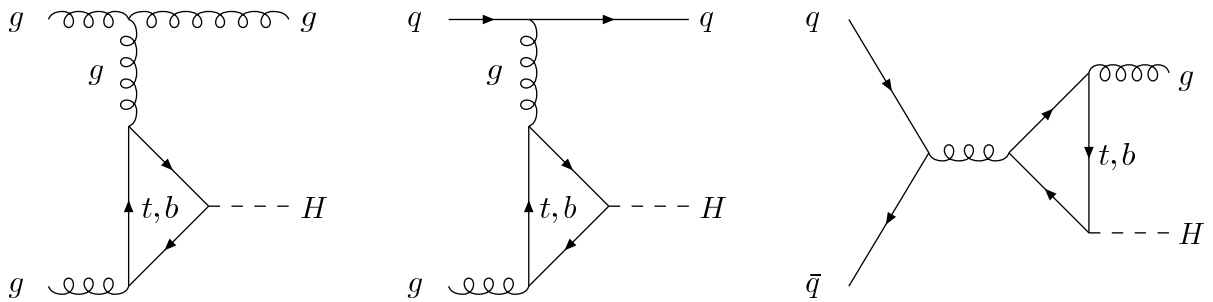
- NLO QCD:

$\sigma(gg \to H^*)|_{Q=125\ \text{GeV}} = 42.17^{+0.4\%}_{-0.5\%}\ \text{pb}, \qquad \sigma(gg \to H^*)|_{Q=300\ \text{GeV}} = 9.85^{+7.5\%}_{-0.3\%}\ \text{pb}$

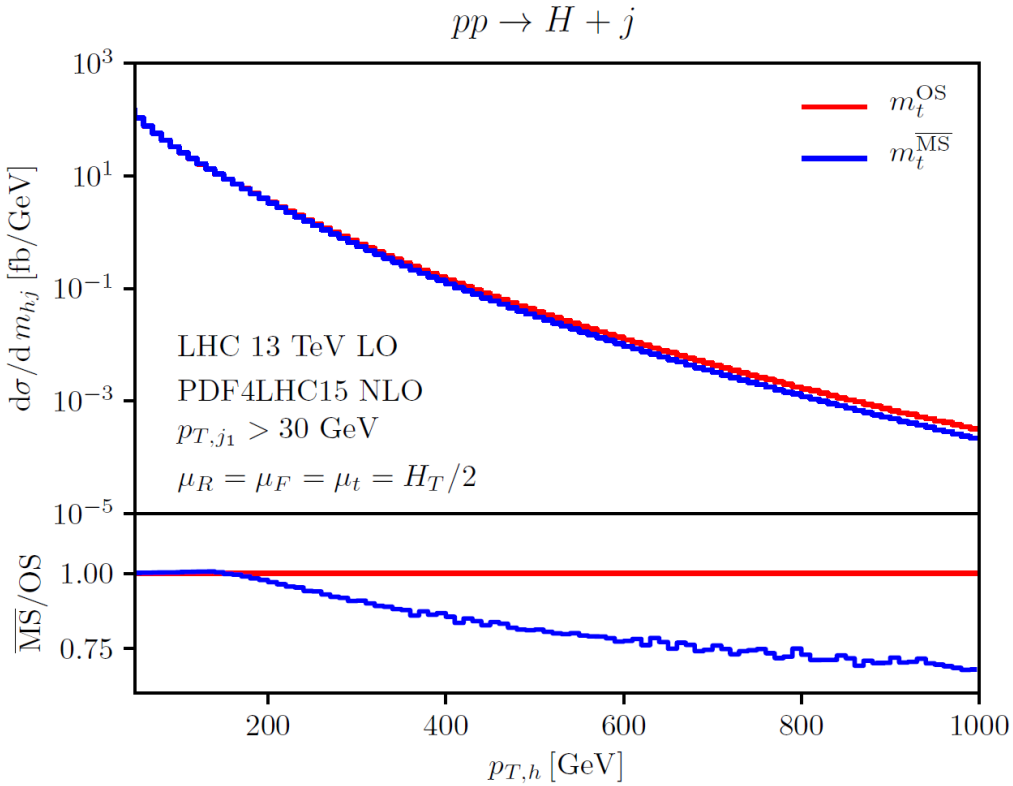
$\sigma(gg \to H^*)|_{Q=400\ \text{GeV}} = 9.43^{+0.1\%}_{-0.9\%}\ \text{pb}, \qquad \sigma(gg \to H^*)|_{Q=600\ \text{GeV}} = 1.97^{+0.0\%}_{-15.9\%}\ \text{pb}$

$\sigma(gg \to H^*)|_{Q=900\ \text{GeV}} = 0.230^{+0.0\%}_{-22.3\%}\ \text{pb}, \quad \sigma(gg \to H^*)|_{Q=1200\ \text{GeV}} = 0.0402^{+0.0\%}_{-26.0\%}\ \text{pb}$

- Higgs + jet production: $gg \to H + j$



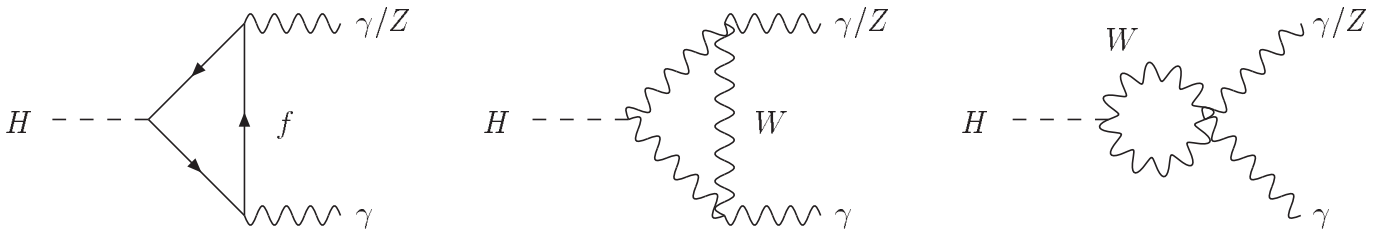
LO: $\mu_t = H_T/2 = (\sqrt{M_H^2 + p_T^2} + p_{Tj})/2$



→ NLO? Jones, Kerner, Luisoni

Jones, S.

III $H^* \to \gamma\gamma$



$$BR(H \to \gamma\gamma, Z\gamma) \lesssim 2 \times 10^{-3}$$

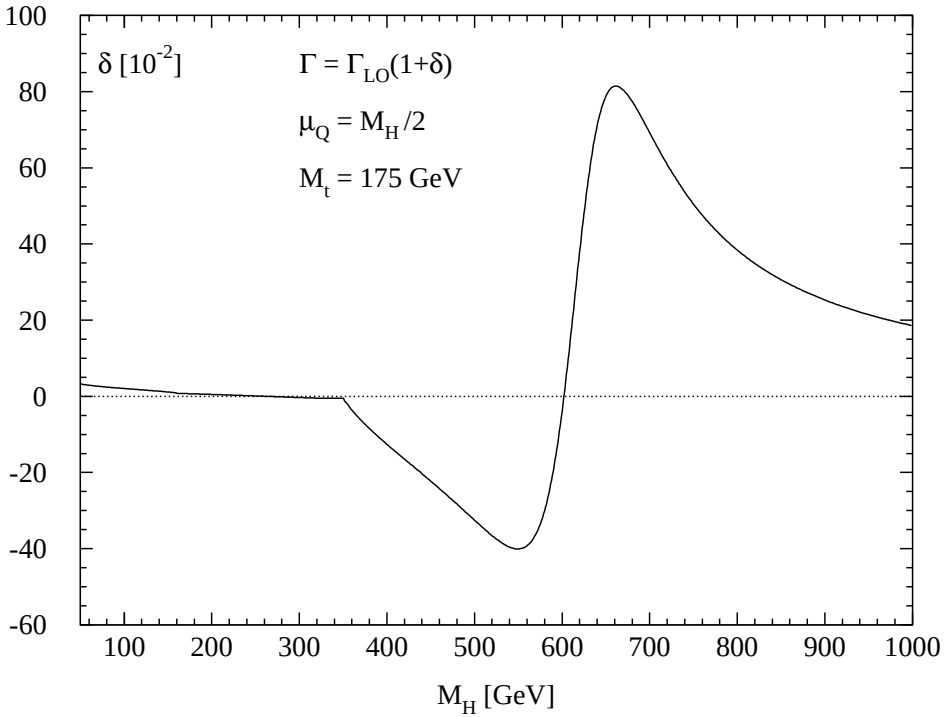
- $W - t$ destructive interference
- QCD corrections: $\lesssim$ 3% in intermediate mass range

Zheng, Wu
Djouadi, S., Zerwas
Melnikov, Yakovlev
Inoue,. . .

- elw. corr.: $\lesssim \mathcal{O}(10\%)$

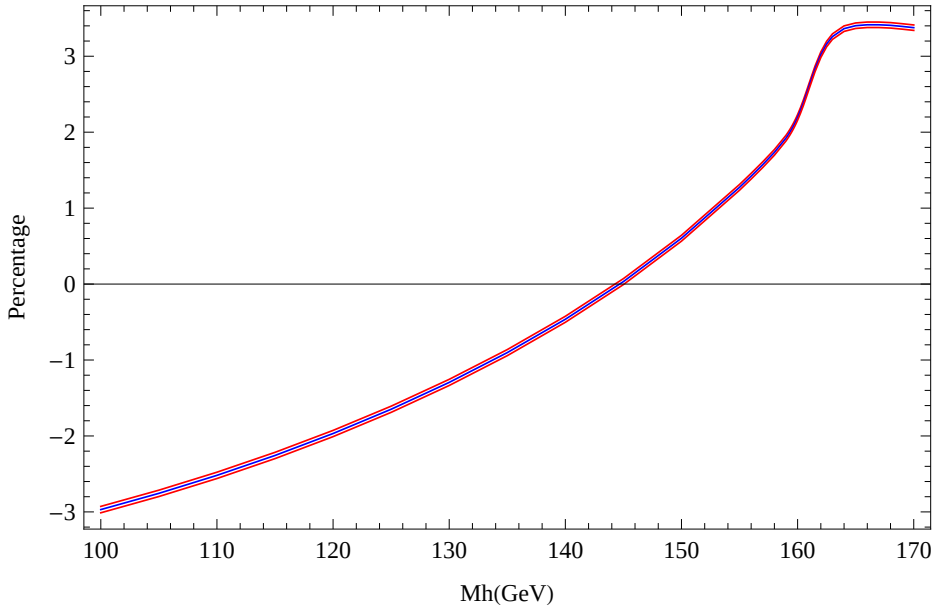
Aglietti, Bonciani, Degrassi, Vicini
Degrassi, Maltoni
Actis, Passarino, Sturm, Uccirati

$H \to \gamma\gamma$



QCD

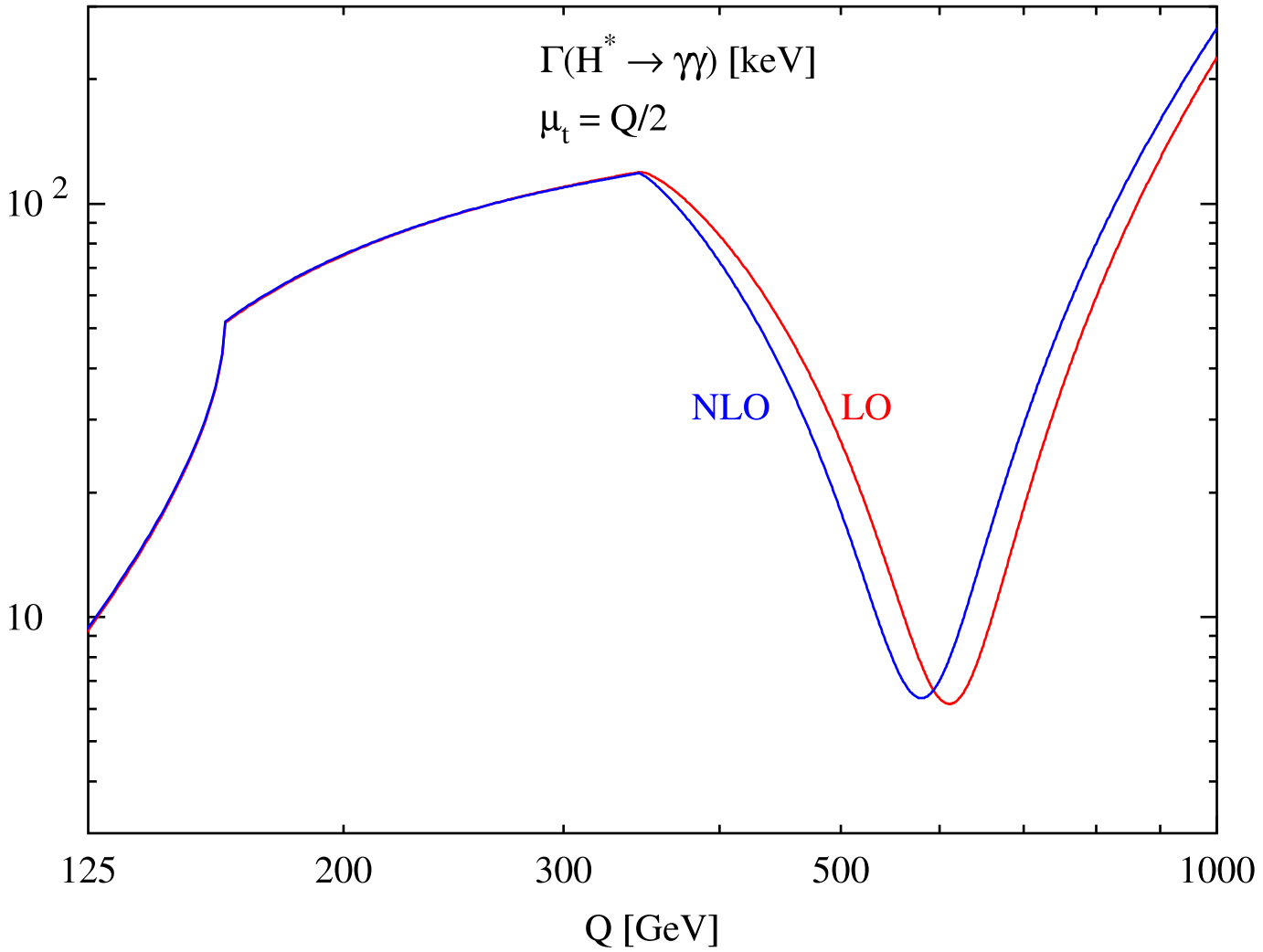
S., Djoaudi, Zerwas



elw

Actis, Passarino, Sturm, Uccirati

- running mass: $m_t(\mu_t) = \kappa(m_t)\overline{m}_t(\mu_t) \leftarrow$ HDECAY



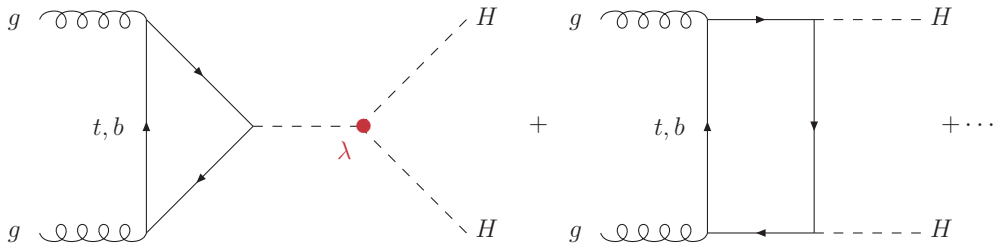
- $\mu_t = Q/4 \ldots Q$ @ NLO:

$$\Gamma(H^* \to \gamma\gamma)|_{Q=125\ \text{GeV}} = 9.43^{+0.1\%}_{-0.4\%}\ \text{keV}, \qquad \Gamma(H^* \to \gamma\gamma)|_{Q=300\ \text{GeV}} = 109.4^{+0.5\%}_{-2.2\%}\ \text{keV}$$

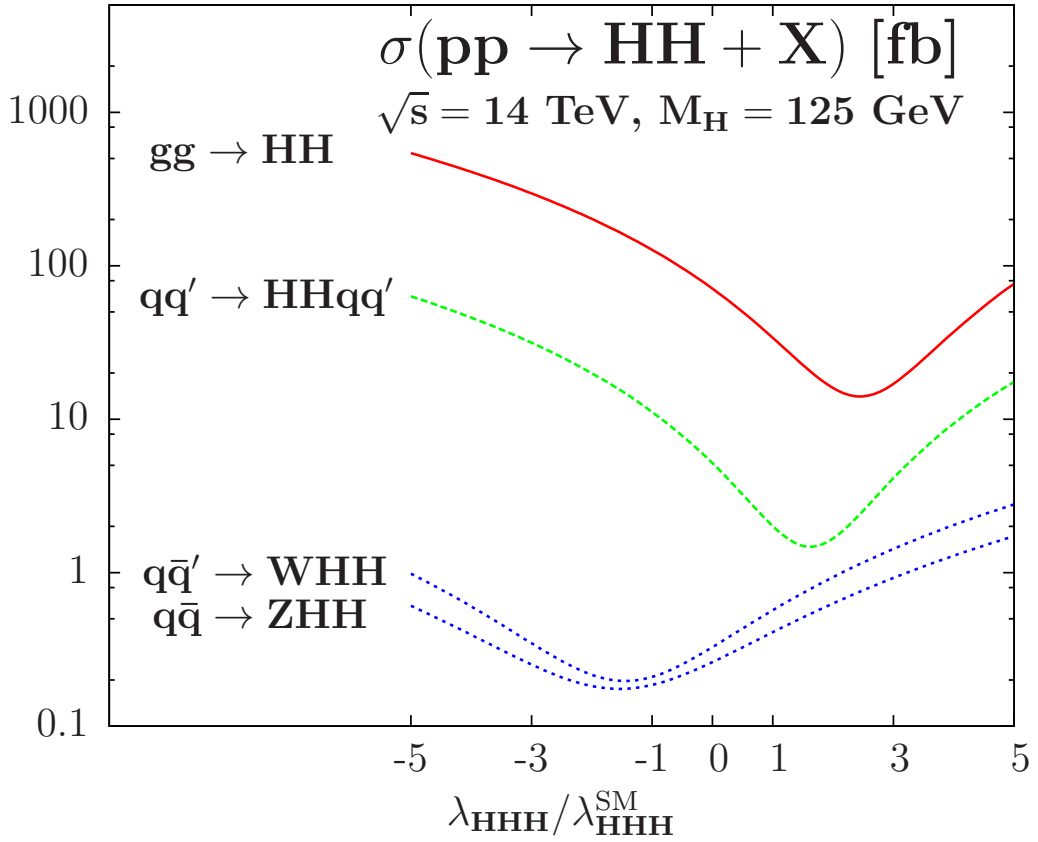
$$\Gamma(H^* \to \gamma\gamma)|_{Q=400\ \text{GeV}} = 72.3^{+9.9\%}_{-35\%}\ \text{keV}, \qquad \Gamma(H^* \to \gamma\gamma)|_{Q=600\ \text{GeV}} = 7.03^{+156\%}_{-35\%}\ \text{keV}$$

$$\Gamma(H^* \to \gamma\gamma)|_{Q=900\ \text{GeV}} = 158.7^{+16\%}_{-1.5\%}\ \text{keV}, \qquad \Gamma(H^* \to \gamma\gamma)|_{Q=1200\ \text{GeV}} = 572.3^{+3.4\%}_{-0\%}\ \text{keV}$$

IV $gg \to HH$



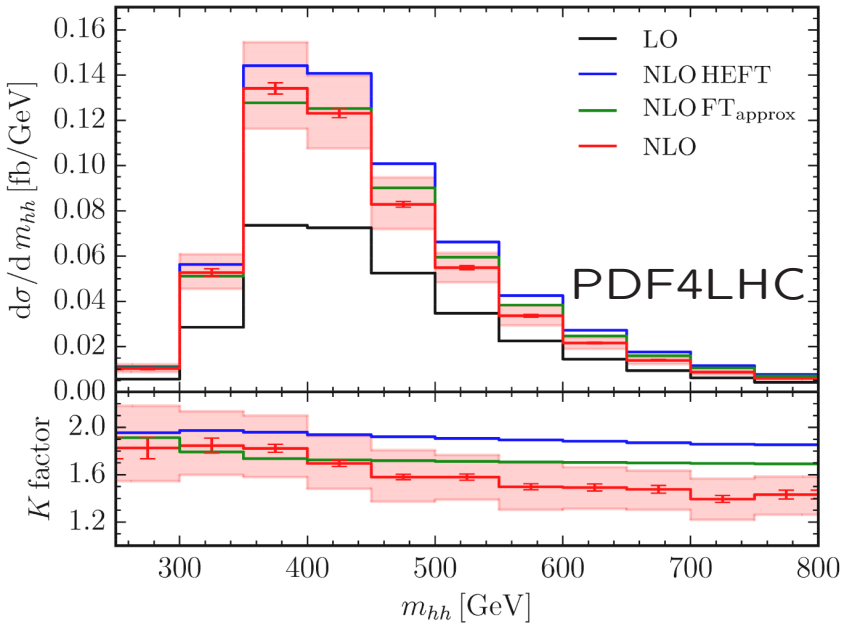
- third generation dominant: $t\,(b)$



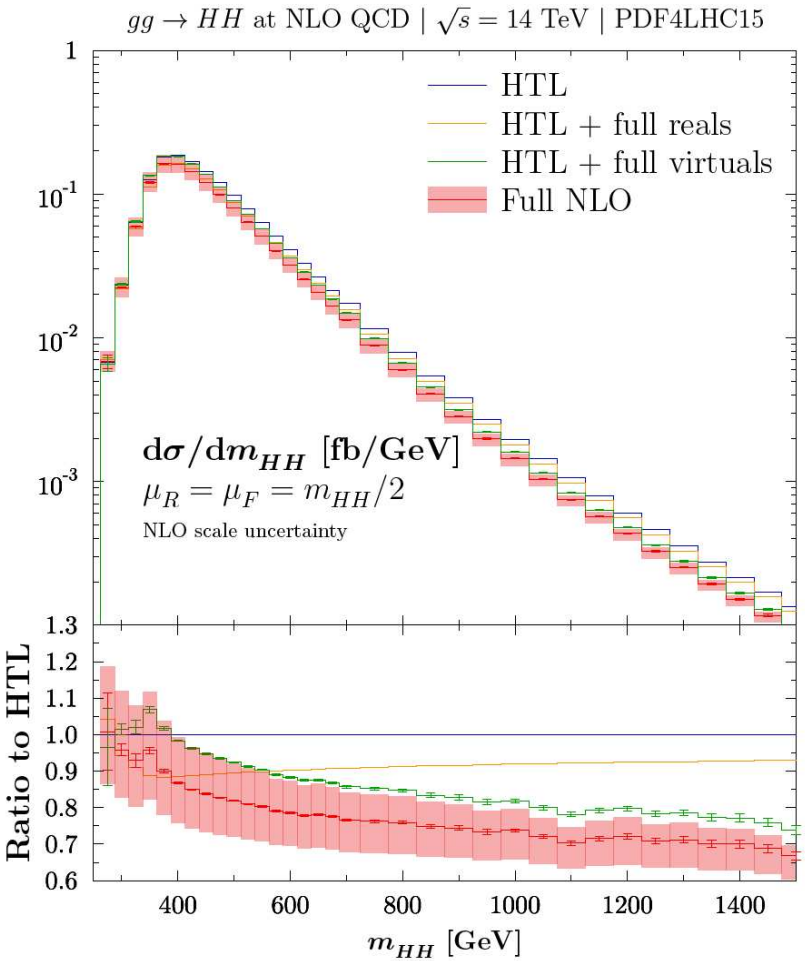
$$gg \to HH:$$

$$\frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$

Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, S.



Borowka, Greiner, Heinrich, Jones, Kerner
Schlenk, Schubert, Zirke



Baglio, Campanario, Glaus,
Mühlleitner, Ronca, S., Streicher

$$\sigma_{NLO} = 32.91(10)^{+13.8\%}_{-12.8\%}\ fb \qquad 32.81(7)^{+13.5\%}_{-12.5\%}\ fb$$

$$\sigma^{HTL}_{NLO} = 38.75^{+18\%}_{-15\%}\ fb \qquad 38.66^{+18\%}_{-15\%}\ fb$$

$$m_t = 173\ \text{GeV} \qquad 172.5\ \text{GeV}$$

$\Rightarrow$ -15% mass effects on top of LO

uncertainties due to m_t

- use m_t, $\overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q \rightarrow$ uncertainty = envelope:

$$\frac{d\sigma(gg \to HH)}{dQ}|_{Q=300\ \text{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \text{fb/GeV},$$

$$\frac{d\sigma(gg \to HH)}{dQ}|_{Q=400\ \text{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \text{fb/GeV},$$

$$\frac{d\sigma(gg \to HH)}{dQ}|_{Q=600\ \text{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \text{fb/GeV},$$

$$\frac{d\sigma(gg \to HH)}{dQ}|_{Q=1200\ \text{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \text{fb/GeV}$$

- bin-by-bin interpolation:

$$\sigma(gg \to HH) = 32.81^{+4\%}_{-18\%}\ \text{fb}$$

- why a dynamical scale $\sim Q$?
  large momentum expansion ($\hat{s} = Q^2 \gg m_t^2$), two FF:

← Davies, Mishima, Steinhauser, Wellmann

pole mass m_t:

$$\Delta F_{1,mass} \rightarrow \frac{\alpha_s}{\pi}\left\{2F_{1,LO}\log\frac{m_t^2}{\hat{s}} + \frac{m_t^2}{\hat{s}}G_1(\hat{s},\hat{t})\right\},$$

$$\Delta F_{2,mass} \rightarrow \frac{\alpha_s}{\pi}\left\{2F_{2,LO}\log\frac{m_t^2}{\hat{s}} + \frac{m_t^2}{\hat{s}}G_2(\hat{s},\hat{t})\right\}$$

$\overline{\text{MS}}$ mass $\overline{m}_t(\mu_t)$:

$$\Delta F_{1,mass} \rightarrow \frac{\alpha_s}{\pi}\left\{2F_{1,LO}\left[\log\frac{\mu_t^2}{\hat{s}} + \frac{4}{3}\right] + \frac{\overline{m}_t^2(\mu_t)}{\hat{s}}G_1(\hat{s},\hat{t})\right\},$$

$$\Delta F_{2,mass} \rightarrow \frac{\alpha_s}{\pi}\left\{2F_{2,LO}\left[\log\frac{\mu_t^2}{\hat{s}} + \frac{4}{3}\right] + \frac{\overline{m}_t^2(\mu_t)}{\hat{s}}G_2(\hat{s},\hat{t})\right\}$$

$\Rightarrow$ scale $\mu_t \sim Q$ preferred at large Q

- renormalization/factorization scale uncertainties @ NLO:

$$\begin{aligned}
\sqrt{s} = 13 \text{ TeV}: \quad \sigma_{tot} &= 27.73(7)^{+13.8\%}_{-12.8\%} \text{ fb} \\
\sqrt{s} = 14 \text{ TeV}: \quad \sigma_{tot} &= 32.81(7)^{+13.5\%}_{-12.5\%} \text{ fb} \\
\sqrt{s} = 27 \text{ TeV}: \quad \sigma_{tot} &= 127.0(2)^{+11.7\%}_{-10.7\%} \text{ fb} \\
\sqrt{s} = 100 \text{ TeV}: \quad \sigma_{tot} &= 1140(2)^{+10.7\%}_{-10.0\%} \text{ fb}
\end{aligned}$$

- m_t scale/scheme uncertainties @ NLO:

$$\begin{aligned}
\sqrt{s} = 13 \text{ TeV}: \quad \sigma_{tot} &= 27.73(7)^{+4\%}_{-18\%} \text{ fb} \\
\sqrt{s} = 14 \text{ TeV}: \quad \sigma_{tot} &= 32.81(7)^{+4\%}_{-18\%} \text{ fb} \\
\sqrt{s} = 27 \text{ TeV}: \quad \sigma_{tot} &= 127.8(2)^{+4\%}_{-18\%} \text{ fb} \\
\sqrt{s} = 100 \text{ TeV}: \quad \sigma_{tot} &= 1140(2)^{+3\%}_{-18\%} \text{ fb}
\end{aligned}$$

- how to combine them? $\rightarrow$ envelope $\sim$ linear sum (rel. err.)

- renormalization/factorization scale uncertainties @ NNLO$_{FTapprox}$:

$$\begin{aligned}
\sqrt{s} = 13 \text{ TeV}: \quad \sigma_{tot} &= 31.05^{+2.2\%}_{-5.0\%} \text{ fb} \\
\sqrt{s} = 14 \text{ TeV}: \quad \sigma_{tot} &= 36.69^{+2.1\%}_{-4.9\%} \text{ fb} \\
\sqrt{s} = 27 \text{ TeV}: \quad \sigma_{tot} &= 139.9^{+1.3\%}_{-3.9\%} \text{ fb} \\
\sqrt{s} = 100 \text{ TeV}: \quad \sigma_{tot} &= 1224^{+0.9\%}_{-3.2\%} \text{ fb}
\end{aligned}$$

- HO corrections: dominated by universal S+V+C corrections

$\Rightarrow \sim$ rescaling of rel. m_t scale/scheme uncertainties

final combined ren./fac. scale and m_t scale/scheme unc. @ NNLO$_{FTapprox}$:

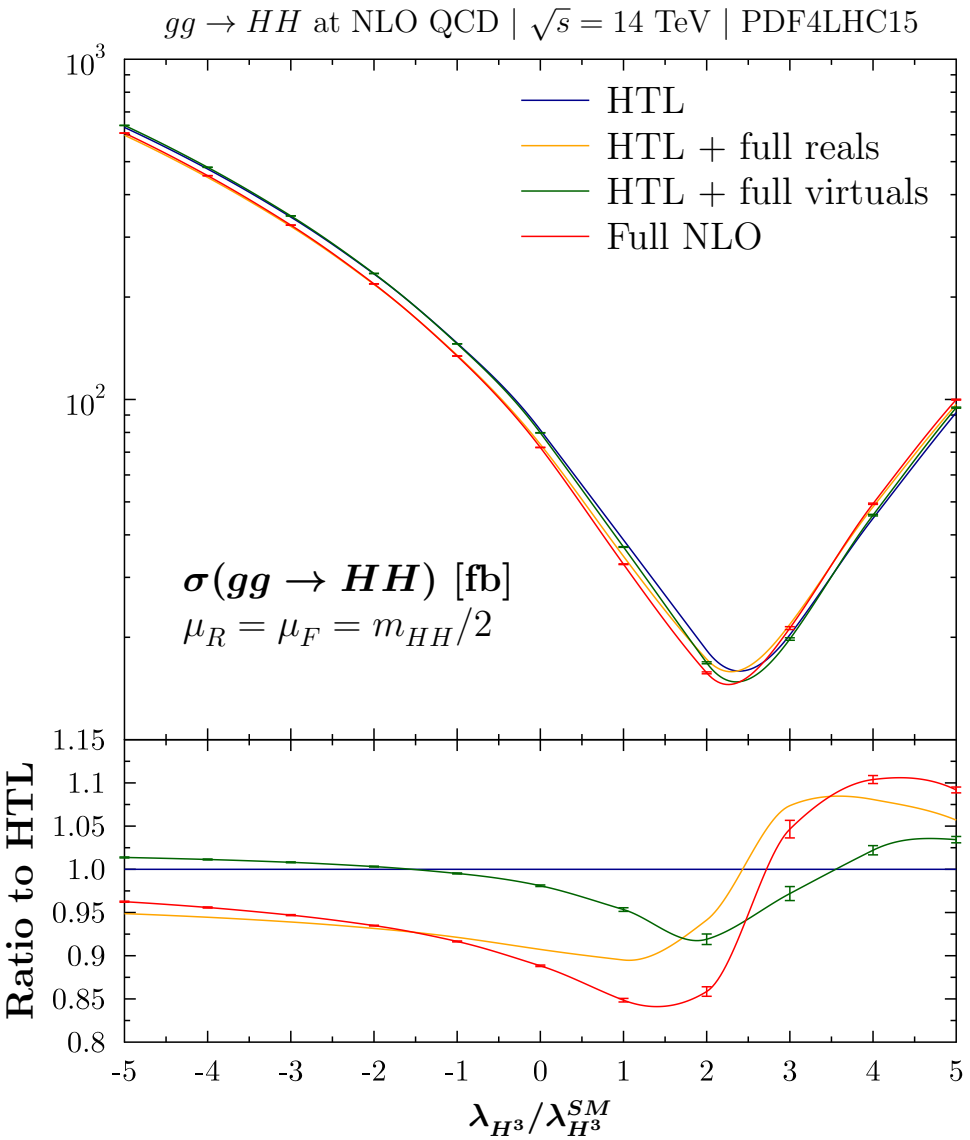
$$\sqrt{s} = 13 \text{ TeV}: \quad \sigma_{tot} = 31.05^{+6\%}_{-23\%} \text{ fb}$$

$$\sqrt{s} = 14 \text{ TeV}: \quad \sigma_{tot} = 36.69^{+6\%}_{-23\%} \text{ fb}$$

$$\sqrt{s} = 27 \text{ TeV}: \quad \sigma_{tot} = 139.9^{+5\%}_{-22\%} \text{ fb}$$

$$\sqrt{s} = 100 \text{ TeV}: \quad \sigma_{tot} = 1224^{+4\%}_{-21\%} \text{ fb}$$

λ dependence



Baglio, Campanario, Glaus, Mühlleitner, Ronca, S.

- m_t scale/scheme uncertainties @ NLO:

$$\begin{aligned}
\kappa_\lambda = -10: \quad \sigma_{tot} &= 1438(1)^{+10\%}_{-6\%}\ \text{fb} \\
\kappa_\lambda = -5: \quad \sigma_{tot} &= 512.8(3)^{+10\%}_{-7\%}\ \text{fb} \\
\kappa_\lambda = -1: \quad \sigma_{tot} &= 113.66(7)^{+8\%}_{-9\%}\ \text{fb} \\
\kappa_\lambda = 0: \quad \sigma_{tot} &= 61.22(6)^{+6\%}_{-12\%}\ \text{fb} \\
\kappa_\lambda = 1: \quad \sigma_{tot} &= 27.73(7)^{+4\%}_{-18\%}\ \text{fb} \\
\kappa_\lambda = 2: \quad \sigma_{tot} &= 13.2(1)^{+1\%}_{-23\%}\ \text{fb} \\
\kappa_\lambda = 2.4: \quad \sigma_{tot} &= 12.7(1)^{+4\%}_{-22\%}\ \text{fb} \\
\kappa_\lambda = 3: \quad \sigma_{tot} &= 17.6(1)^{+9\%}_{-15\%}\ \text{fb} \\
\kappa_\lambda = 5: \quad \sigma_{tot} &= 83.2(3)^{+13\%}_{-4\%}\ \text{fb} \\
\kappa_\lambda = 10: \quad \sigma_{tot} &= 579(1)^{+12\%}_{-4\%}\ \text{fb}
\end{aligned}$$

- renormalization/factorization scale uncertainties @ NNLO$_{FTapprox}$:

$$\begin{aligned}
\kappa_\lambda = -10: \quad \sigma_{tot} &= 1680^{+3.0\%}_{-7.7\%}\ \text{fb} \\
\kappa_\lambda = -5: \quad \sigma_{tot} &= 598.9^{+2.7\%}_{-7.5\%}\ \text{fb} \\
\kappa_\lambda = -1: \quad \sigma_{tot} &= 131.9^{+2.5\%}_{-6.7\%}\ \text{fb} \\
\kappa_\lambda = 0: \quad \sigma_{tot} &= 70.38^{+2.4\%}_{-6.1\%}\ \text{fb} \\
\kappa_\lambda = 1: \quad \sigma_{tot} &= 31.05^{+2.2\%}_{-5.0\%}\ \text{fb} \\
\kappa_\lambda = 2: \quad \sigma_{tot} &= 13.81^{+2.1\%}_{-4.9\%}\ \text{fb} \\
\kappa_\lambda = 2.4: \quad \sigma_{tot} &= 13.10^{+2.3\%}_{-5.1\%}\ \text{fb} \\
\kappa_\lambda = 3: \quad \sigma_{tot} &= 18.67^{+2.7\%}_{-7.3\%}\ \text{fb} \\
\kappa_\lambda = 5: \quad \sigma_{tot} &= 94.82^{+4.9\%}_{-8.8\%}\ \text{fb} \\
\kappa_\lambda = 10: \quad \sigma_{tot} &= 672.2^{+4.2\%}_{-8.5\%}\ \text{fb}
\end{aligned}$$

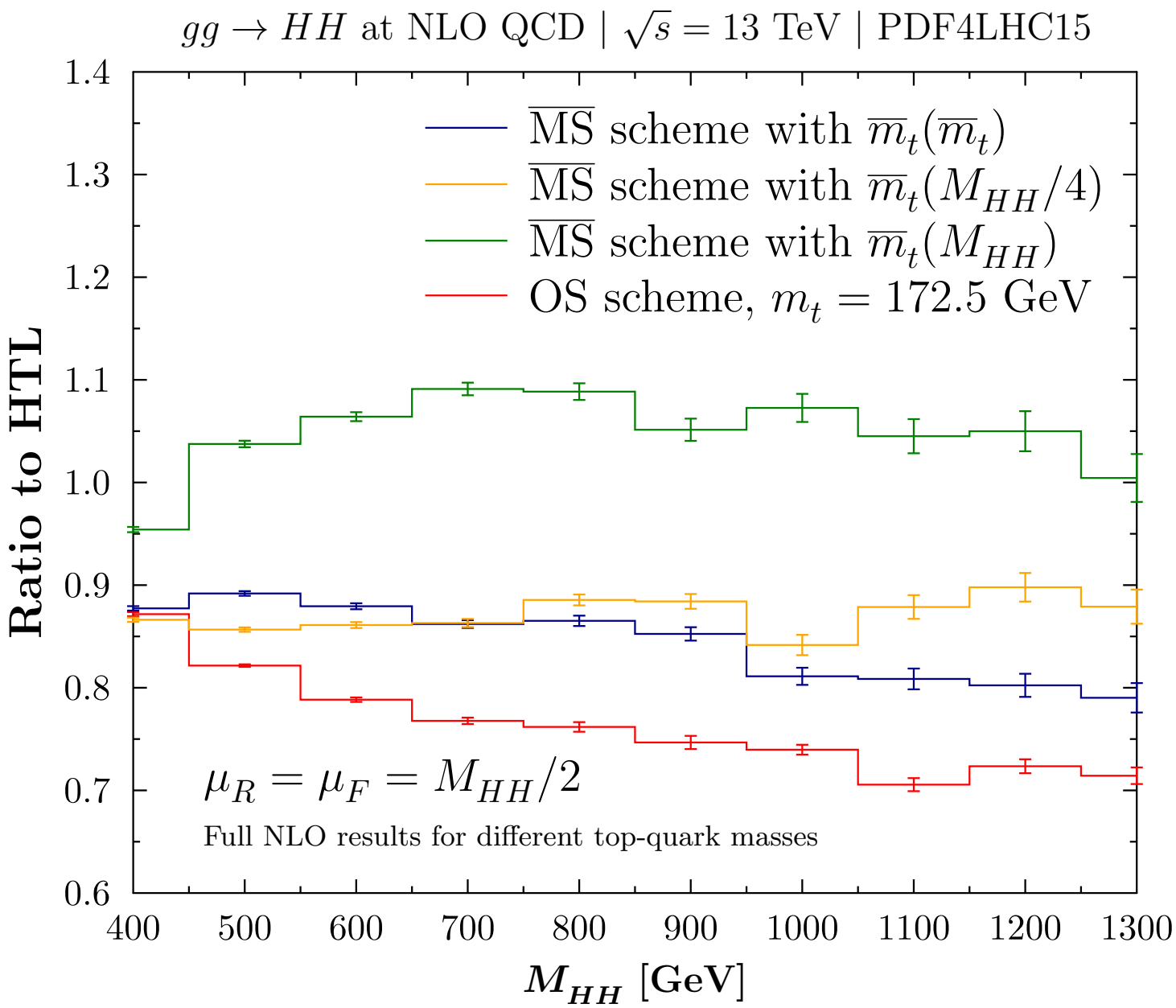
- final combined uncertainties @ NNLO$_{FTapprox}$:

$$
\begin{aligned}
\kappa_\lambda = -10: \quad \sigma_{tot} &= 1680^{+13\%}_{-14\%}\ \text{fb} \\
\kappa_\lambda = -5: \quad \sigma_{tot} &= 598.9^{+13\%}_{-15\%}\ \text{fb} \\
\kappa_\lambda = -1: \quad \sigma_{tot} &= 131.9^{+11\%}_{-16\%}\ \text{fb} \\
\kappa_\lambda = 0: \quad \sigma_{tot} &= 70.38^{+8\%}_{-18\%}\ \text{fb} \\
\kappa_\lambda = 1: \quad \sigma_{tot} &= 31.05^{+6\%}_{-23\%}\ \text{fb} \\
\kappa_\lambda = 2: \quad \sigma_{tot} &= 13.81^{+3\%}_{-28\%}\ \text{fb} \\
\kappa_\lambda = 2.4: \quad \sigma_{tot} &= 13.10^{+6\%}_{-27\%}\ \text{fb} \\
\kappa_\lambda = 3: \quad \sigma_{tot} &= 18.67^{+12\%}_{-22\%}\ \text{fb} \\
\kappa_\lambda = 5: \quad \sigma_{tot} &= 94.82^{+18\%}_{-13\%}\ \text{fb} \\
\kappa_\lambda = 10: \quad \sigma_{tot} &= 672.2^{+16\%}_{-13\%}\ \text{fb}
\end{aligned}
$$

III *CONCLUSIONS*

- scale and scheme uncertainties due to m_t relevant for large momenta
- significant uncertainties for Higgs production @ large p_T
- significant uncertainties for off-shell Higgs production and decays (heavy BSM Higgs bosons)
- Higgs pair production: m_t effects on top of LO $\sim -15\%$ for σ_{tot} [larger for distributions]
- factorization/renormalization scale uncertainties @NNLO$_{FTapprox}$ $\lesssim 5\%$
- uncertainties due to m_t scale/scheme choice sizeable $\lesssim 20\%$ $\rightarrow$ linear combination of rel. uncertainties
- analogous issues in $gg \rightarrow H + 2jet, ZH, ZZ$ etc.

BACKUP SLIDES



Baglio, Campanario, Glaus, Mühlleitner, Ronca, S.

$$\sigma_{\mathrm{NLO}}(pp \to HH + X) = \sigma_{\mathrm{LO}} + \Delta\sigma_{\mathrm{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$$\begin{aligned}
\sigma_{\mathrm{LO}} &= \int_{\tau_0}^1 d\tau\, \frac{d\mathcal{L}^{gg}}{d\tau}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s) \\
\Delta\sigma_{\mathrm{virt}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau\, \frac{d\mathcal{L}^{gg}}{d\tau}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s)\; C \\
\Delta\sigma_{gg} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau\, \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{M^2}{\tau s} \right. \\
&\qquad \left. + d_{gg}(z) + 6[1 + z^4 + (1-z)^4] \left(\frac{\log(1-z)}{1-z} \right)_+ \right\} \\
\Delta\sigma_{gq} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s (1-z)^2} + d_{gq}(z) \right\} \\
\Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z}\, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s)\; d_{q\bar{q}}(z)
\end{aligned}$$

$$C \to \pi^2 + \frac{11}{2} + C_{\triangle\triangle}, \quad d_{gg} \to -\frac{11}{2}(1-z)^3, \quad d_{gq} \to \frac{2}{3}z^2 - (1-z)^2, \quad d_{q\bar{q}} \to \frac{32}{27}(1-z)^3$$

- m_t scale/scheme uncertainties at LO:

$$\frac{d\sigma(gg \to HH)}{dQ}|_{Q=300\ \text{GeV}} = 0.01656^{+62\%}_{-2.4\%}\ \text{fb/GeV}$$
$$\frac{d\sigma(gg \to HH)}{dQ}|_{Q=400\ \text{GeV}} = 0.09391^{+0\%}_{-20\%}\ \text{fb/GeV}$$
$$\frac{d\sigma(gg \to HH)}{dQ}|_{Q=600\ \text{GeV}} = 0.02132^{+0\%}_{-48\%}\ \text{fb/GeV}$$
$$\frac{d\sigma(gg \to HH)}{dQ}|_{Q=1200\ \text{GeV}} = 0.0003223^{+0\%}_{-56\%}\ \text{fb/GeV}$$

$$
\begin{aligned}
F_i &= F_{i,LO} + \Delta F_i \\
\Delta F_i &= \Delta F_{i,HTL} + \Delta F_{i,mass}
\end{aligned}
$$

- pole mass:

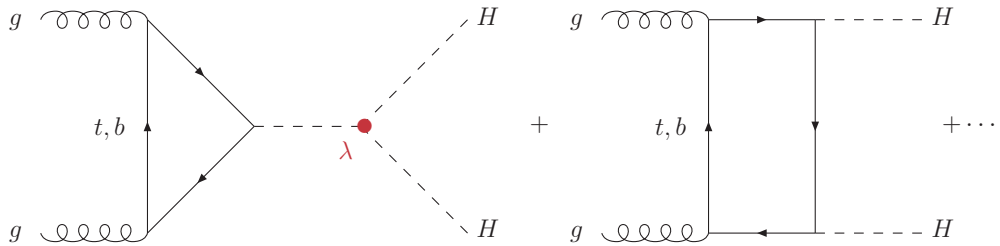
$$
\begin{aligned}
F_{1,LO} &\rightarrow 4\frac{m_t^2}{\hat{s}} \\
F_{2,LO} &\rightarrow -\frac{m_t^2}{\hat{s}\hat{t}(\hat{s}+\hat{t})}\{(\hat{s}+\hat{t})^2 L_{1ts}^2 + \hat{t}^2 L_{ts}^2 + \pi^2[(\hat{s}+\hat{t})^2 + \hat{t}^2]\}
\end{aligned}
$$

- $\overline{\mathrm{MS}}$ mass:

$$
\begin{aligned}
F_{1,LO} &\rightarrow 4\frac{\overline{m}_t^2(\mu_t)}{\hat{s}} \\
F_{2,LO} &\rightarrow -\frac{\overline{m}_t^2(\mu_t)}{\hat{s}\hat{t}(\hat{s}+\hat{t})}\{(\hat{s}+\hat{t})^2 L_{1ts}^2 + \hat{t}^2 L_{ts}^2 + \pi^2[(\hat{s}+\hat{t})^2 + \hat{t}^2]\}
\end{aligned}
$$

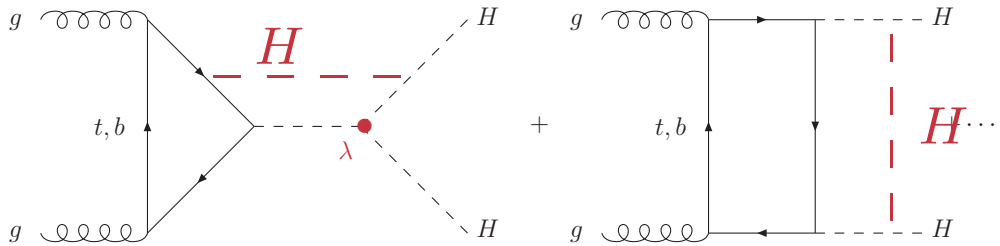
- different scales for y_t in triangle (Q) and box (M_H) diagrams?

$\longrightarrow$ has to hold at all orders



- different scales for y_t in triangle (Q) and box (M_H) diagrams?

$\longrightarrow$ has to hold at all orders



elw. corrections

$\Rightarrow$ same scales in all diagrams

M_HH	mt(M_HH/4)	mt(M_HH/2)	mt(M_HH)
125	189.209370262526	176.772460597358	166.501914700149
260	176.139964023672	165.972836934324	156.889554725476
275	175.247098219568	165.224863654266	156.188624671063
300	173.888433241807	164.084218616097	155.118481503625
350	171.556916171559	162.101622772544	153.272150436136
375	170.543285547792	161.158290295641	152.465560631846
400	169.611142167793	160.289697463114	151.721739637882
500	166.501914700149	157.384965182267	149.226383426185
600	164.084218616097	155.118481503625	147.270941230420
700	162.101622772544	153.272150436136	145.672596390682
800	160.289697463114	151.721739637882	144.326704798025
900	158.737886290123	150.390138497802	143.168060367441
1000	157.384965182267	149.226383426185	142.153427561240
1100	156.188624671063	148.195135247933	141.252743160739
1200	155.118481503625	147.270941230420	140.444302478362
1300	154.152026867353	146.434896300904	139.711950260189
1400	153.272150436136	145.672596390682	139.043354388391
1500	152.465560631846	144.972828986822	138.428898934501