Self-similar solutions for Fuzzy Dark Matter

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Introduction

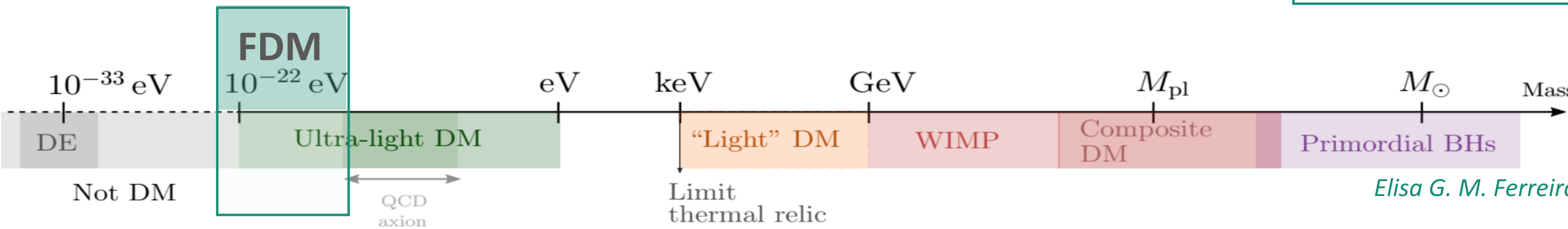
Fuzzy dark matter (FDM)

- DM particles not detected.
- CDM has tensions at small scales.
- Alternative to CDM N-Body simulations.

FDM Soliton

$$\nabla\left(\Phi_{\mathrm{N}}+\Phi_{\mathrm{Q}}\right)=0$$

Alternative scenarios → Scalar field dark matter → Solitons → Flat density profile.



Elisa G. M. Ferreira (2020)

FDM Action: $$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{m^2}{2}\phi^2\right]$$

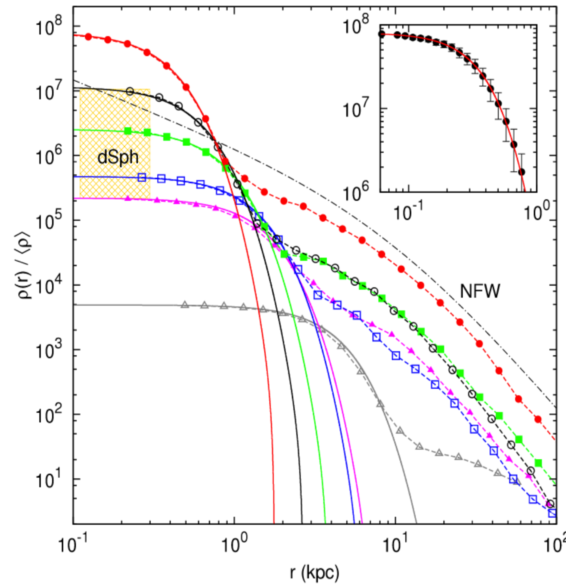
Fluid picture: Continuity, Euler and Poisson

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{v})=0,$$
$$\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}=-\nabla\left(\Phi_{\mathrm{N}}+\Phi_{\mathrm{Q}}\right)$$
$$\nabla^2\Phi_{\mathrm{N}}=4\pi\rho.$$

Quantum pressure

$$\Phi_{\mathrm{Q}}=-\frac{\epsilon^2}{2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}.$$

$\epsilon=\frac{T}{mL^2}\sim\frac{\lambda_{DB}}{L}$ $\quad\lambda_{DB}\sim 0.5\ kpc$



Radial density profiles of haloes in ψDM model

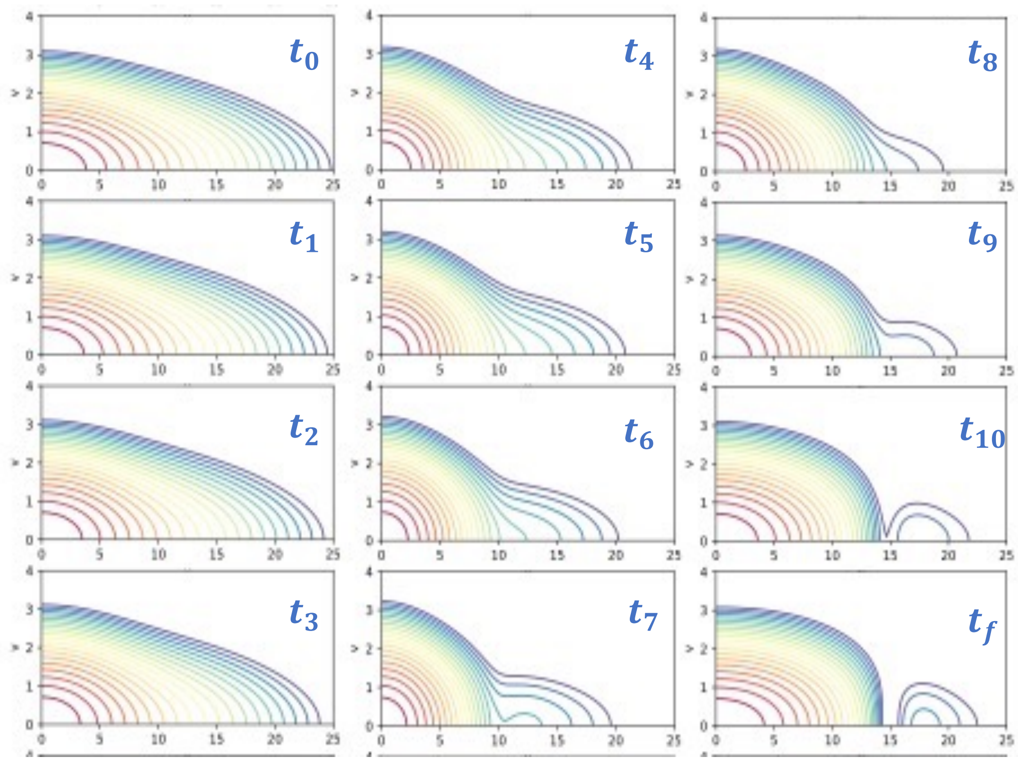
Schive, Chiueh, and Broadhurst (2014)

Motivation of this work

1. **Go beyond the static solitons** by investigating **dynamical self-similar solutions.**
2. Understand physical processes: **gravitational cooling.**
3. Understand **comparison with self similar solutions for CDM.**

Dynamics: 3D Numerical simulations

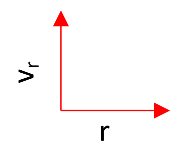
FDM out of equilibrium, ε=1



R.Galazo-Garcia

Husimi Phase-space distribution

Axis:



Gravitational Cooling

E.Seidel, W.M.Suen(1994)

F.S Guzmán, L.A. Ureña-López (2004)

Self-similar solutions for FDM

Cosmological Self-similar solutions

SELF-SIMILAR ANSTATZ

$$\rho = t^{-2} f\left(\frac{r}{\sqrt{t}}\right), \quad v = t^{-1/2} g\left(\frac{r}{\sqrt{t}}\right), \quad \Phi_{\rm N} = t^{-1} h\left(\frac{r}{\sqrt{t}}\right)$$

Continuity, Euler and Poisson

PERTURBATIONS AROUND THE EXPANDING COSMOLOGICAL BACKGROUND

$$\rho = \bar{\rho}(1+\delta), \quad \vec{v} = \bar{\vec{v}} + \vec{u}, \quad \Phi_{\rm N} = \bar{\Phi}_{\rm N} + \varphi_{\rm N},$$

Einstein de-Sitter Universe: $a \propto t^{2/3}$

Self-similar form ✓

COMOVING FLUID EQUATIONS

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla_x \cdot [(1+\delta)\vec{u}] = 0,$$

$$\frac{\partial \vec{u}}{\partial t} + \frac{1}{a}(\vec{u}\cdot\nabla_x)\vec{u} + H\vec{u} = -\frac{1}{a}\nabla_x(\varphi_{\rm N} + \Phi_{\rm Q}),$$

$$\nabla_x^2 \varphi_{\rm N} = \frac{2}{3}\frac{\delta}{a},$$

SPHERICAL SELF-SIMILAR SOLUTIONS

$$\delta(x,t) = \hat{\delta}(\eta), \quad u(x,t) = \epsilon^{1/2} t^{-1/2}\, \hat{u}(\eta),$$

$$\varphi_{\rm N}(x,t) = \epsilon t^{-1}\, \hat{\varphi}_{\rm N}(\eta), \quad \Phi_{\rm Q}(x,t) = \epsilon t^{-1}\, \hat{\Phi}_{\rm Q}(\eta),$$

$$\delta M(x,t) = \epsilon^{3/2} t^{-1/2}\, \delta\hat{M}(\eta),$$

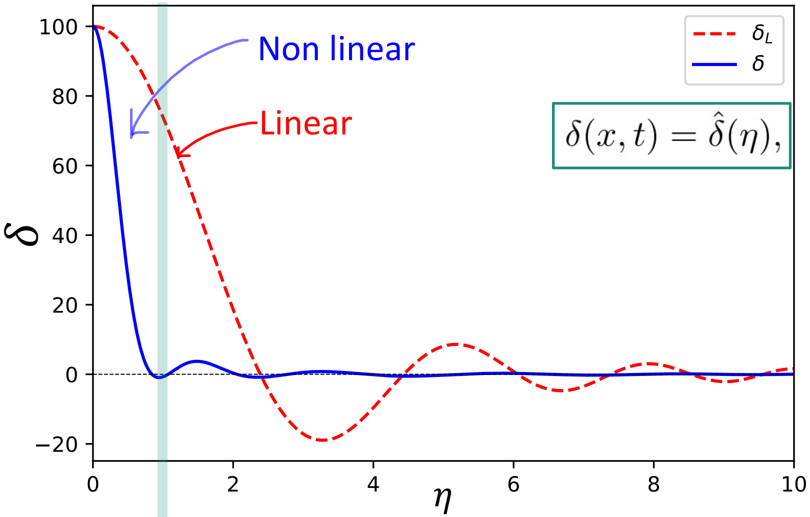
SCALING VARIABLE

$$\eta = \frac{t^{1/6} x}{\epsilon^{1/2}} = \frac{r}{\sqrt{\epsilon t}}.$$

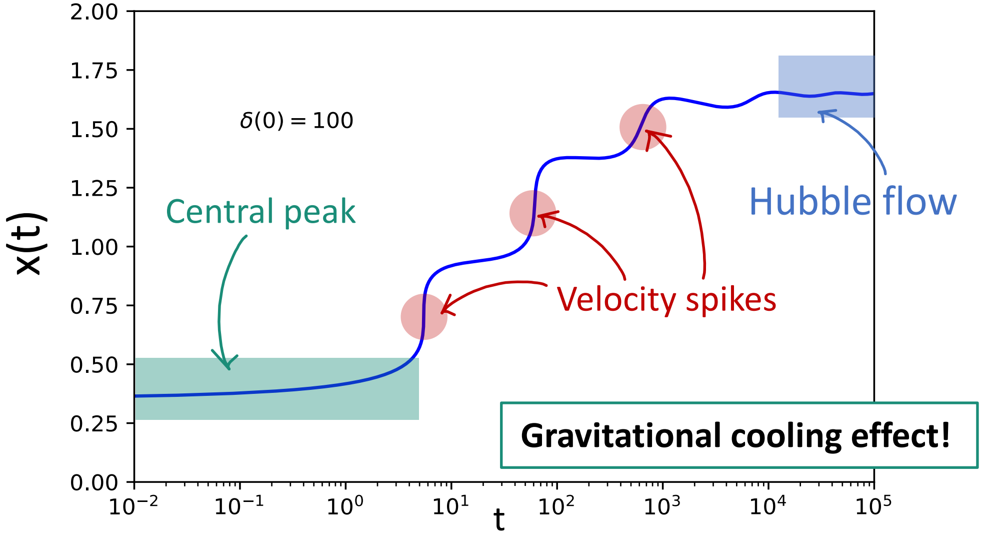
1) Linear regime
2) Non-linear regime

Non-linear regime: Overdensity, $\delta(0) = 100$

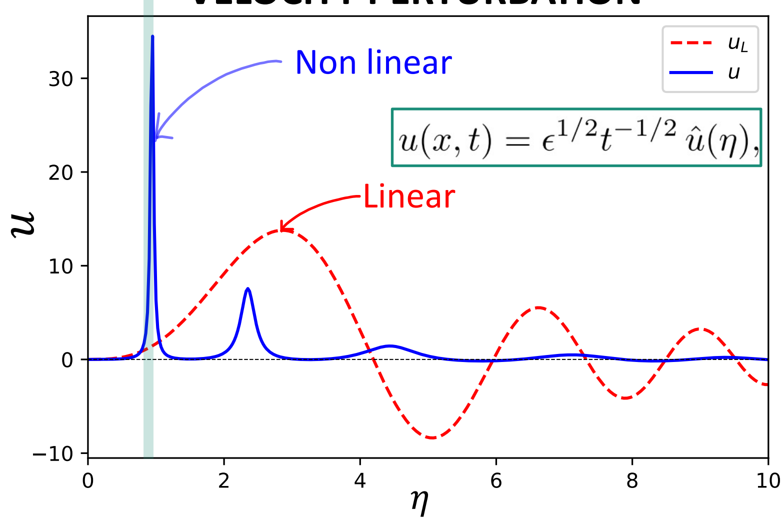
DENSITY PERTURBATION



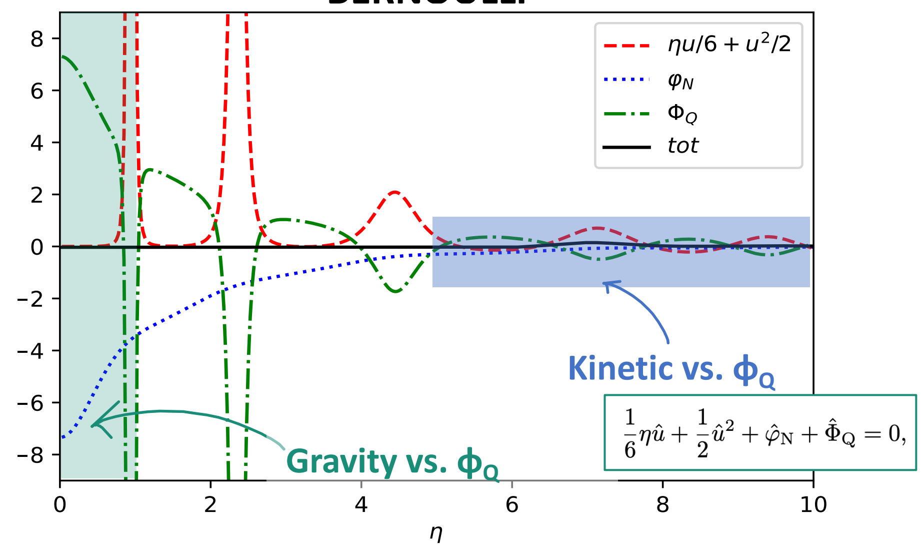
TRAJECTORY



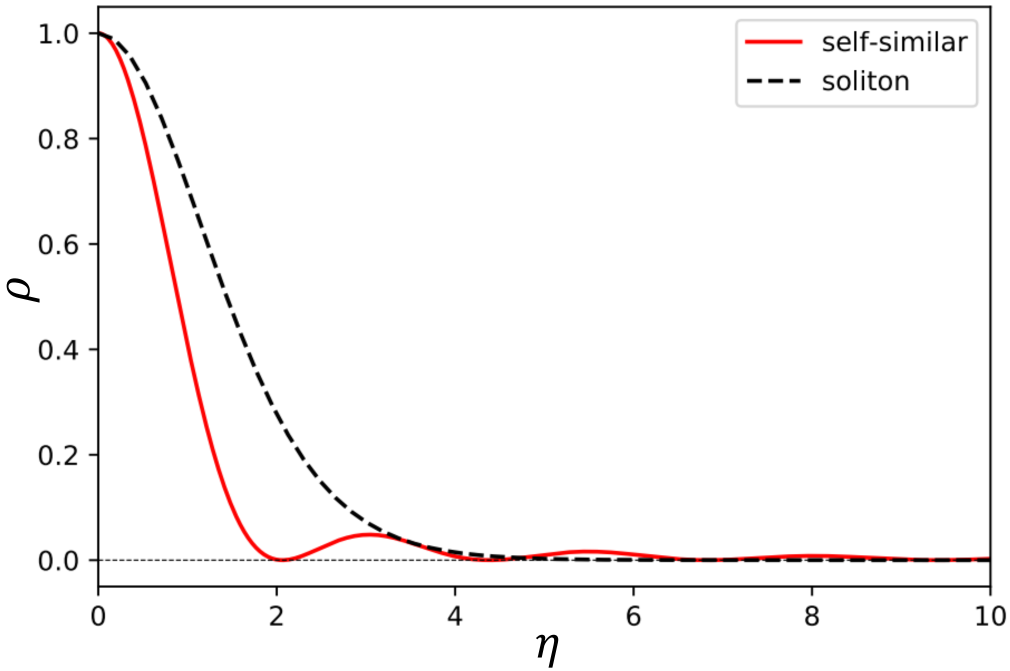
VELOCITY PERTURBATION



BERNOULLI



High-density asymptotic limit



- The **BKGD density becomes negligible** as compared with the central density.
- The inner profile converge to a limiting shape that obeys the scaling law:

$$\{\eta, \psi, \rho, M\} \to \{\lambda^{-1}\eta, \lambda^2\psi, \lambda^4\rho, \lambda M\}$$

- The central peak of the **self-similar solution is narrower than the soliton peak**.
- **The shape of the central peak of the self-similar profile does not converge to the soliton equilibrium**. → **kinetic effects** (dominate near the boundary of the central peak).

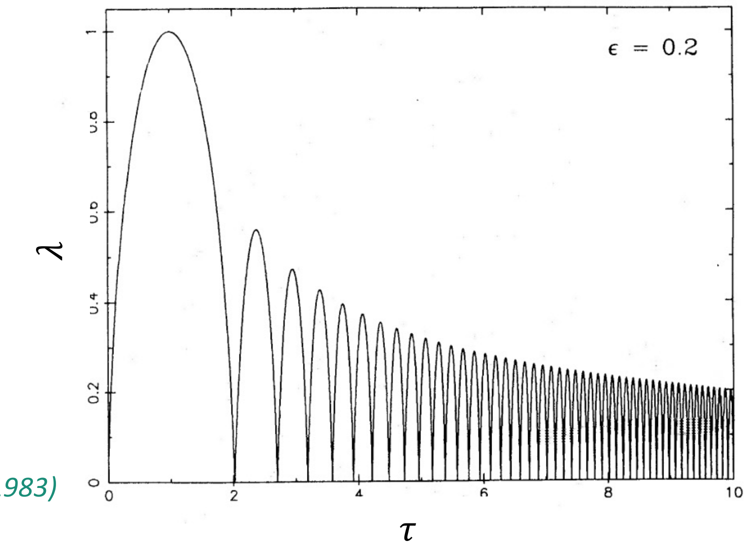
Comparison with CDM self-similar solutions & Conclusions

Comparison: CDM vs FDM

CDM

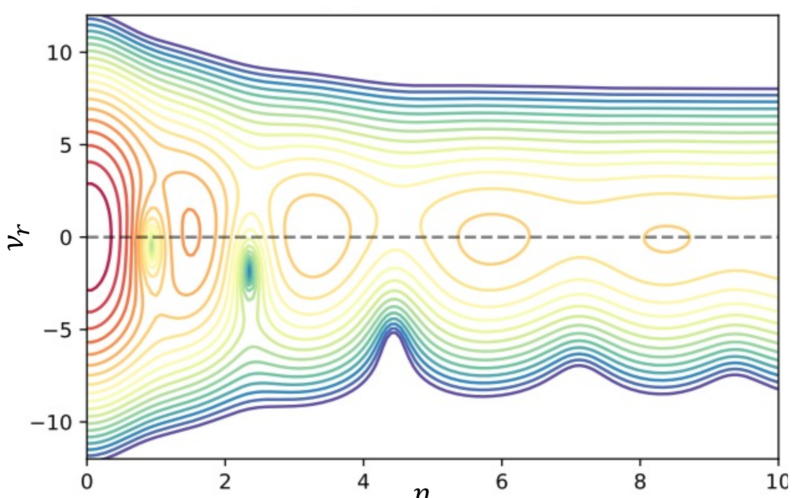
- **Transition from the linear regime to the non-linear regime.**
- **Gravitational collapse→** Virial equilibrium in the inner nonlinear core.
- **The size grows** with time, in **physical and comoving** coordinates.

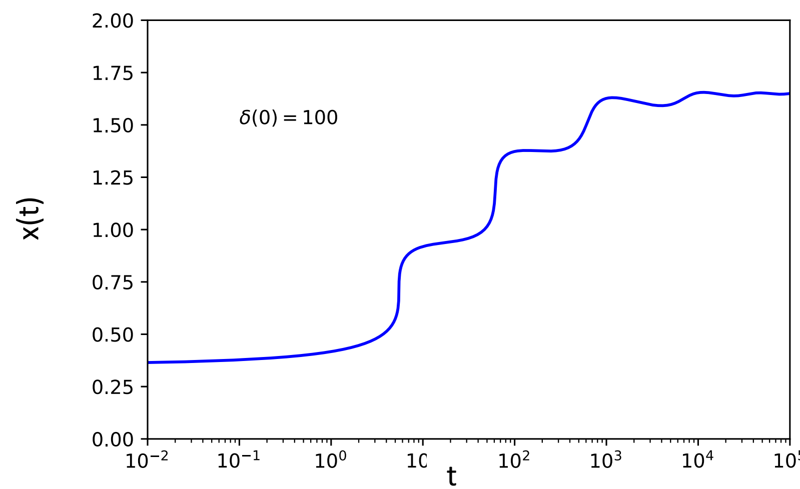
Bertschinger, E.(1985) Fillmore, J. A. and Goldreich, P. (1983)



FDM

- **No transition from the linear to the non-linear regime.**
- **Gravitational cooling**.
- **The size grows in physical** coordinates but **shrinks in comoving** coordinates.





Self-similar solutions for Fuzzy Dark Matter

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Thank you for your attention

Raquel Galazo García

Institut de Physique Théorique (IPhT)

ICTP Summer School on Cosmology, July 5, 2022

Upcoming work

- **New numerical studies** at **galactic scales** to compare **different scalar field dark matter models (SFDM).**
- Extend self-similar solutions.
- **SFDM cosmological simulations** to study how **the formation of large scale structure is modified compared to the CDM scenario.**

Back up: Introduction

FDM small scale

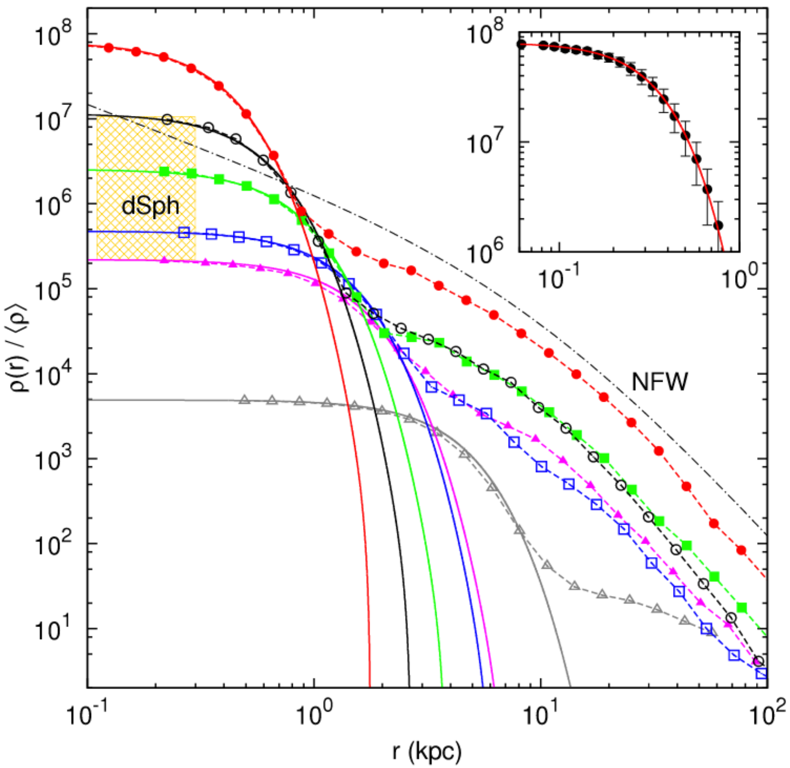
Hydrostatic equilibrium

$$\Phi_{\rm N} + \Phi_{\rm Q} = \alpha,$$

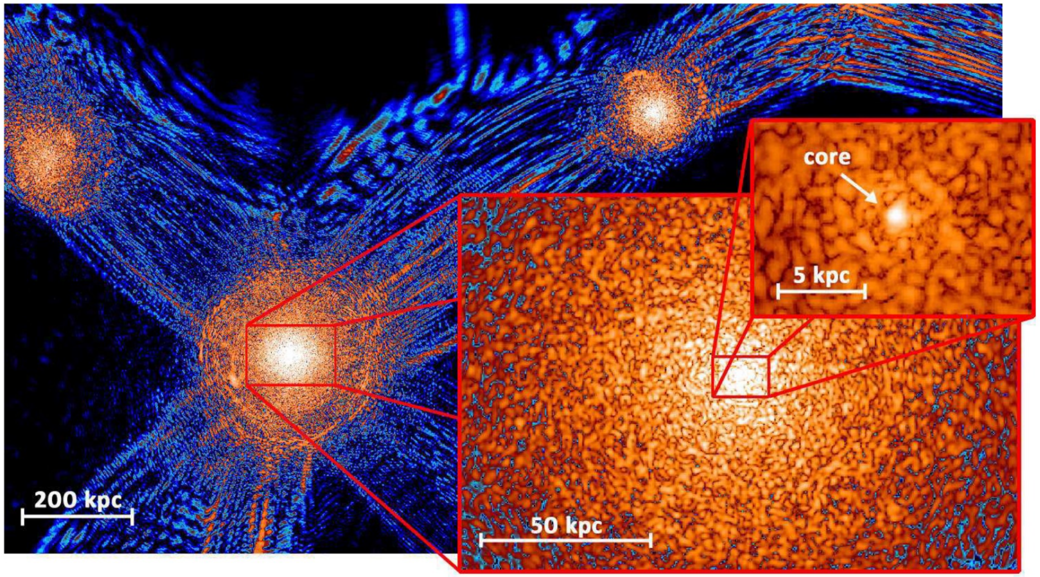
Soliton profile

$$\epsilon^2 \nabla^2 \psi_{\rm sol} = 2(\Phi_{\rm N} - \alpha)\psi_{\rm sol}.$$

$$\nabla^2 \Phi_{\rm N} = 4\pi \psi_{\rm sol}^2$$



Radial density profiles of haloes formed in the ψDM model

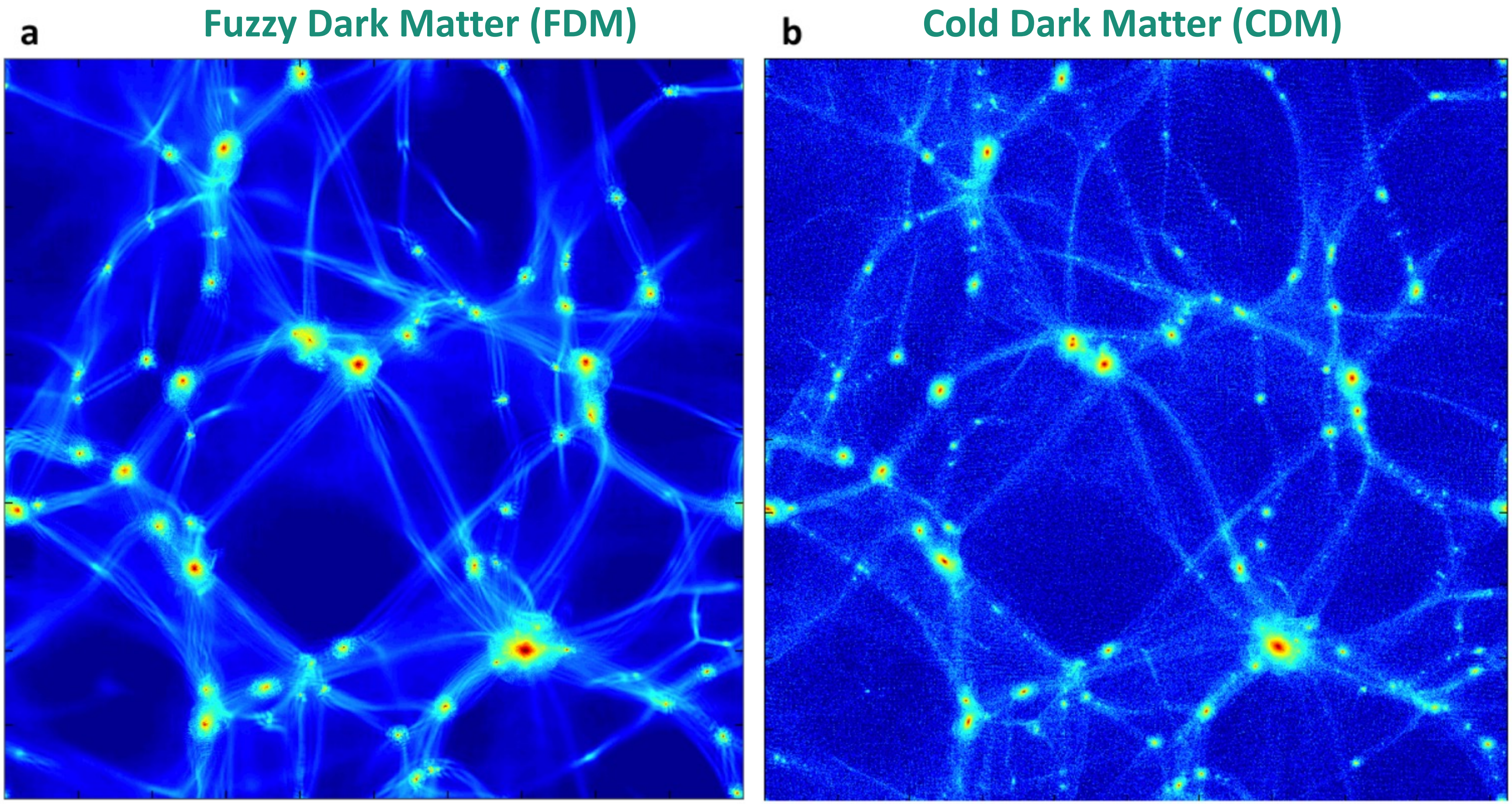


A slice of density field of ψDM simulation on various scales at z=0.1

Schive, Chiueh, and Broadhurst (2014)

FDM large scale distribution

Recover the success of CDM large scale distribution of filaments and voids.



Schive, Chiueh, and Broadhurst (2014)

Dynamics of Fuzzy Dark Matter

$m \sim 10^{-22}\ eV$ De Broglie wavelength $\sim 0.5 kpc$

Action:

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2}{2}\phi^2\right]$$

SCHRÖDINGER-POISSON SYSTEM (SP)

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi + m\Phi_{\rm N}\,\psi,$$

$$\nabla^2\Phi_{\rm N} = 4\pi\mathcal{G}_{\rm N}\rho, \quad \rho = m\psi\psi^*$$

SP system scaling law

$$\{t,\vec{r},\Phi_{\rm N},\psi,\rho\} \to \{\lambda^{-2}t,\lambda^{-1}\vec{r},\lambda^2\Phi_{\rm N},\lambda^2\psi,\lambda^4\rho\}\,.$$

Non-Relativistic regime:

$$\phi = \frac{1}{\sqrt{2m}}\left(\psi\exp^{-imt} + \psi^*\exp^{imt}\right)$$

$|\ddot{\psi}| \ll m|\dot{\psi}|$ Factor-out the fast time oscillation of $\boldsymbol{\phi}$

HYDRODYNAMICAL PICTURE

$$\psi = \sqrt{\rho}\,e^{iS/\epsilon}, \quad \vec{v} = \nabla S,$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0,$$

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\nabla\left(\Phi_{\rm N} + \Phi_{\rm Q}\right)$$

$$\nabla^2\Phi_{\rm N} = 4\pi\rho.$$

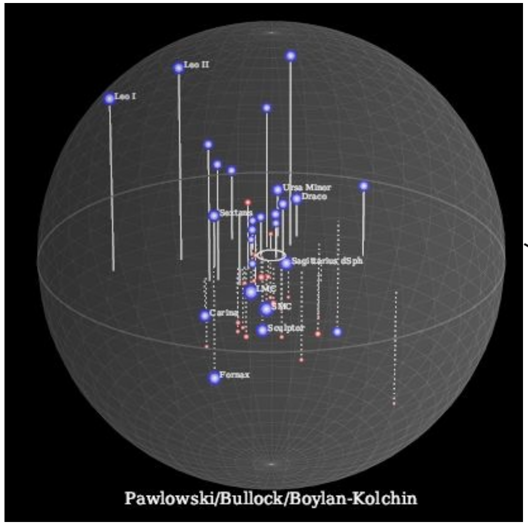
Quantum pressure

$$\Phi_{\rm Q} = -\frac{\epsilon^2}{2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}.$$

$$\epsilon = \frac{T}{mL^2} \sim \frac{\lambda_{DB}}{L}$$

FDM Motivation 1) Explanation to CDM small-scales tensions

Missing satellite problem



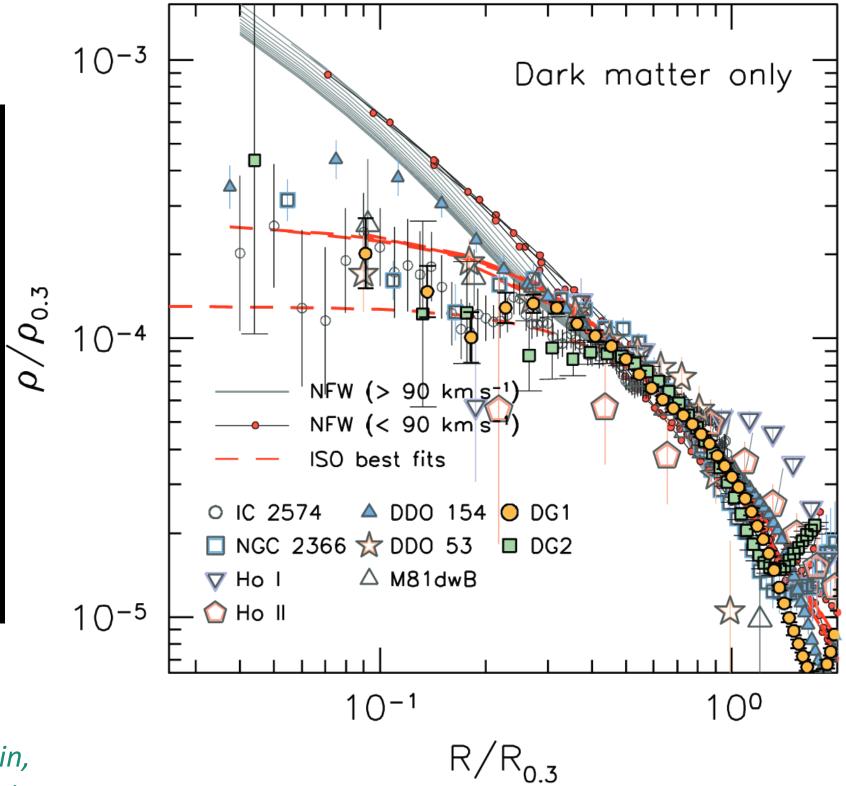
Predicted ΛCDM substructure

Simulation by V. Robles and T. Kelley and collaborators.

Known Milky Way satellites

James S. Bullock, M. Boylan-Kolchin, M. Pawlowski

Core/cusp problem



Density profiles observations and simulations

Antonino Popolo, Morgan Le Delliou (2017)

FDM Motivation 2) Alternative to CDM N-Body simulations

CDM: A classical collisionless fluid is governed by the **Vlasov-Poisson equations**. $f = f(\boldsymbol{x}, \boldsymbol{p}, t)$

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{ma^2} \cdot \boldsymbol{\nabla}_x f - m(\boldsymbol{\nabla}_x \phi) \cdot \boldsymbol{\nabla}_p f = 0, \qquad \nabla_x^2 \phi = 4\pi G \bar{\rho}(t)\, a^2 \delta(\boldsymbol{x}, t),$$

$$\rho = \bar{\rho}(t)[1 + \delta(\boldsymbol{x}, t)], \qquad \rho = \int f(\boldsymbol{x}, \boldsymbol{p}, t)\, \mathrm{d}^3 p.$$

- **Wigner** quasi-probability distribution: **link the Schrödinger** wave function ψ to a function f in phase space.

$$f_{\mathrm{W}}(\vec{r}, \vec{v}) = \int \frac{d\vec{r}\,'}{(2\pi)^3} e^{i\vec{v}\cdot\vec{r}\,'} \psi\left(\vec{r} - \frac{\epsilon}{2}\vec{r}\,'\right) \psi^*\left(\vec{r} + \frac{\epsilon}{2}\vec{r}\,'\right),$$

- **Husimi** representation: **smoothing of the Wigner** distribution by a Gaussian filter of width σx and σp in x and p space.

$$f_{\mathrm{H}}(\vec{r}, \vec{v}) = \int \frac{d\vec{r}\,' d\vec{v}\,'}{(2\pi\epsilon)^3 \sigma_r^3 \sigma_v^3} e^{-(\vec{r}-\vec{r}\,')^2/(2\epsilon\sigma_r^2) - (\vec{v}-\vec{v}\,')^2/(2\epsilon\sigma_v^2)} \times f_{\mathrm{W}}(\vec{r}\,', \vec{v}\,'),$$

- Husimi is a positive-semidefinite function → No fast oscillations of Wigner

FDM comoving Vlasov equation

$$\frac{\partial f_{\mathrm{W}}}{\partial t} + \frac{\vec{p}}{a^2} \cdot \frac{\partial f_{\mathrm{W}}}{\partial \vec{x}} - \vec{\nabla}_x \varphi_{\mathrm{N}} \cdot \frac{\partial f_{\mathrm{W}}}{\partial \vec{p}} + \mathcal{O}(\epsilon) = 0.$$

Kaiser (1993)
C. Uhlemann,
M. Kopp, & T. Haugg (2014)

FDM Motivation 2) Alternative to CDM N-Body simulations

FDM comoving Vlasov equation

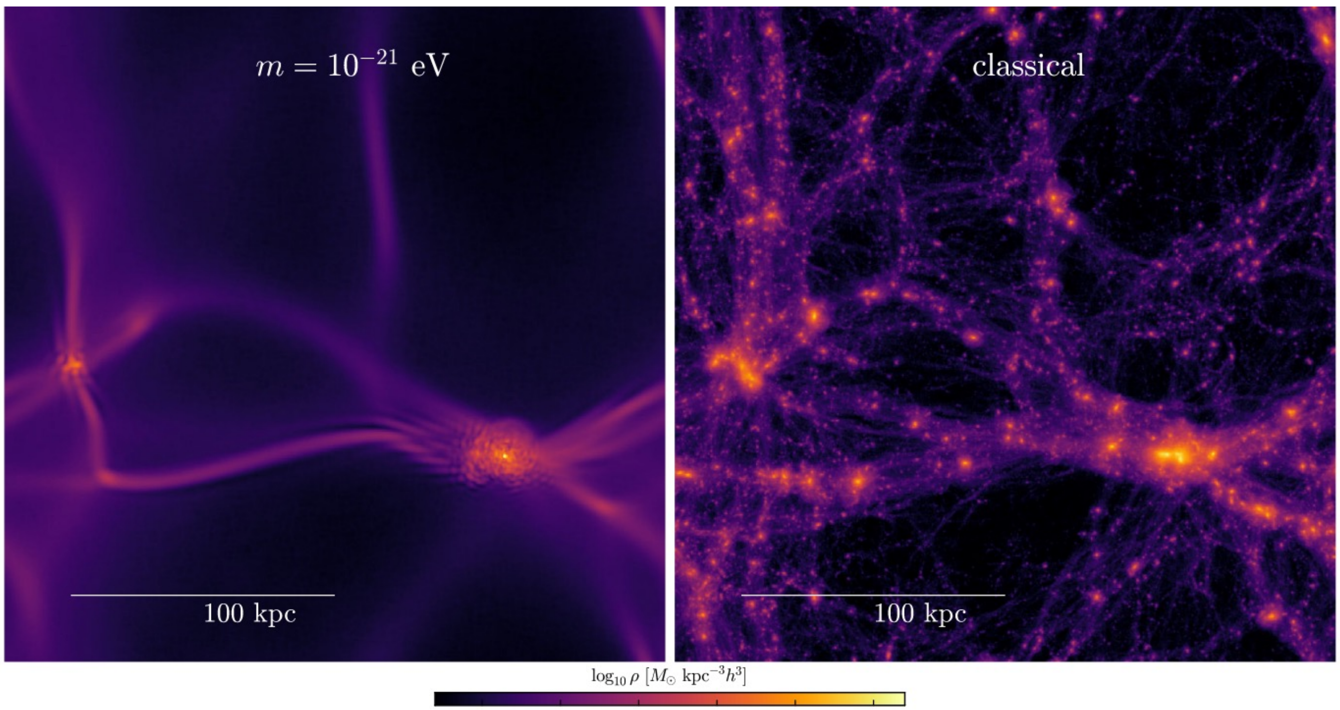
$$\frac{\partial f_{\rm W}}{\partial t} + \frac{\vec{p}}{a^2} \cdot \frac{\partial f_{\rm W}}{\partial \vec{x}} - \vec{\nabla}_x \varphi_{\rm N} \cdot \frac{\partial f_{\rm W}}{\partial \vec{p}} + \mathcal{O}(\epsilon) = 0.$$

CDM comoving Vlasov equation

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{ma^2} \cdot \boldsymbol{\nabla}_x f - m(\boldsymbol{\nabla}_x \phi) \cdot \boldsymbol{\nabla}_p f = 0,$$

Kaiser (1993)

Link the Schrödinger wave function ψ to a function f in phase space.



Cosmological simulation at z=3, evolved either as CDM (VP eq) or as FDM (SP)

Philip Mocz and Lachlan Lancaste,Anastasia Fialkov and Fernando Becerra, Pierre-Henri Chavanis (2018).

Back up: Self-similar solutions for FDM

Cosmological Self-similar solutions

FLUID PICTURE

$$\rho = t^{-2} f\left(\frac{r}{\sqrt{t}}\right), \quad v = t^{-1/2} g\left(\frac{r}{\sqrt{t}}\right), \quad \Phi_{\rm N} = t^{-1} h\left(\frac{r}{\sqrt{t}}\right)$$

Continuity, Euler and Poisson

COSMOLOGICAL BACKGROUND

Einstein de-Sitter Universe: matter era & the scale factor : $a \propto t^{2/3}$

$$\bar{\rho} = \frac{1}{6\pi t^2}, \quad \bar{v} = \frac{2r}{3t}, \quad \bar{\Phi}_{\rm N} = \frac{r^2}{9t^2}.$$

Self-similar form ✓

Comoving spatial coordinates $\vec{x} = \vec{r}/a$

PERTURBATIONS AROUND THE EXPANDING BACKGROUND

$$\rho = \bar{\rho}(1+\delta), \quad \vec{v} = \bar{\vec{v}} + \vec{u}, \quad \Phi_{\rm N} = \bar{\Phi}_{\rm N} + \varphi_{\rm N},$$

Comoving Self-similar solutions: Scaling variable

$\rho = \bar{\rho}(1+\delta), \quad \vec{v} = \bar{\vec{v}} + \vec{u}, \quad \Phi_{\rm N} = \bar{\Phi}_{\rm N} + \varphi_{\rm N},$ → Substituting into the Euler, Poisson and continuity eq.

COMOVING FLUID EQUATIONS

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla_x \cdot [(1+\delta)\vec{u}] = 0,$$
$$\frac{\partial \vec{u}}{\partial t} + \frac{1}{a}(\vec{u}\cdot\nabla_x)\vec{u} + H\vec{u} = -\frac{1}{a}\nabla_x(\varphi_{\rm N} + \Phi_{\rm Q}),$$
$$\nabla_x^2 \varphi_{\rm N} = \frac{2}{3}\frac{\delta}{a},$$

Quantum Pressure

$$\Phi_{\rm Q} = -\frac{\epsilon^2}{2a^2}\frac{\nabla_x^2\sqrt{\rho}}{\sqrt{\rho}}.$$

Spherical self-similar solutions will be of the form:

$$\delta(x,t) = \hat{\delta}(\eta), \quad u(x,t) = \epsilon^{1/2} t^{-1/2}\,\hat{u}(\eta),$$
$$\varphi_{\rm N}(x,t) = \epsilon t^{-1}\,\hat{\varphi}_{\rm N}(\eta), \quad \Phi_{\rm Q}(x,t) = \epsilon t^{-1}\,\hat{\Phi}_{\rm Q}(\eta),$$
$$\delta M(x,t) = \epsilon^{3/2} t^{-1/2}\,\delta\hat{M}(\eta),$$

Self-similar ansatz ✓

$$v = t^{-1/2} g\left(\frac{r}{\sqrt{t}}\right)$$
$$\rho = t^{-2} f\left(\frac{r}{\sqrt{t}}\right) \quad \Phi_{\rm N} = t^{-1} h\left(\frac{r}{\sqrt{t}}\right)$$

Where the mass perturbation inside the radius r :

$$\delta M(r) = 4\pi \int_0^r dr\, r^2 \delta\rho(r) = \frac{2}{3}\int_0^x dx\, x^2 \delta(x),$$

SCALING VARIABLE

$$\eta = \frac{t^{1/6} x}{\epsilon^{1/2}} = \frac{r}{\sqrt{\epsilon t}}.$$

Scaling variable

Spherical self-similar solutions:

$$\delta(x,t) = \hat{\delta}(\eta), \quad u(x,t) = \epsilon^{1/2} t^{-1/2}\, \hat{u}(\eta),$$
$$\varphi_{\mathrm{N}}(x,t) = \epsilon t^{-1}\, \hat{\varphi}_{\mathrm{N}}(\eta), \quad \Phi_{\mathrm{Q}}(x,t) = \epsilon t^{-1}\, \hat{\Phi}_{\mathrm{Q}}(\eta),$$
$$\delta M(x,t) = \epsilon^{3/2} t^{-1/2}\, \delta\hat{M}(\eta),$$

SCALING VARIABLE

$$\eta = \frac{t^{1/6} x}{\epsilon^{1/2}} = \frac{r}{\sqrt{\epsilon t}}.$$

$$M = \bar{M} + \delta M = \epsilon^{3/2} t^{-1/2} \left[\frac{2}{9}\eta^3 + \delta\hat{M}(\eta) \right].$$

- The size **grows as $\sim\sqrt{t}$ in physical units** but more slowly than the scale factor, → shrink **as $t^{-1/6}$** in comoving units.
- The associated **mass decreases as $M \sim 1/\sqrt{t}$**. (≠CDM: grow both in comoving size and in mass.)

Back up: Linear regime

Linear regime: Fourier space

FOURIER SPACE

$$\ddot{\delta}_L + \frac{4}{3t}\dot{\delta}_L - \frac{2}{3t^2}\delta_L + \frac{\epsilon^2 k^4}{4t^{8/3}}\delta_L = 0.$$

FDM Growing and decaying modes: $D_{\pm}(k,t)$

$$D_+(k,t) = t^{-1/6} J_{-5/2}\left(\frac{3}{2}\epsilon k^2 t^{-1/3}\right),$$
$$D_-(k,t) = t^{-1/6} J_{5/2}\left(\frac{3}{2}\epsilon k^2 t^{-1/3}\right).$$

CDM Growing and decaying modes: $D_{\pm}(k,t)$

- Semi-classical limit, $\epsilon \to 0$, or on large scales $k \to 0$ →

$$D_+(k,t) \propto t^{2/3} \propto a$$
$$D_-(k,t) \propto t^{-1}$$

- At late times → CDM behaviour → damping of Φ_Q term $t^{-8/3}$
- For $\epsilon \neq 0$, Φ_Q → Acoustic waves: $D_+(k,t) \sim \cos(3\epsilon k^2 t^{-1/3}/2)$ $D_-(k,t) \sim \sin(3\epsilon k^2 t^{-1/3}/2)$

To recover the **BKGD** density on large scales, we keep the decaying mode:

$$\delta_L(x,t) = 1 + \frac{\eta^4}{45} - \frac{8\eta^2}{9\pi}\, {}_2F_3\left(-\frac{1}{2}, 2; \frac{3}{2}, \frac{5}{4}, \frac{7}{4}; -\frac{\eta^4}{144}\right)$$

Linear regime: Real space

REAL SPACE

$$\delta_L^{(4)} + \frac{4}{\eta}\delta_L^{(3)} + \frac{\eta^2}{9}\delta_L'' + \frac{\eta}{3}\delta_L' - \frac{8}{3}\delta_L = 0,$$

FDM 4 independent linear modes

$$\delta_{L1} = 45 + \eta^4, \quad \delta_{L2} = \frac{1}{\eta}\,{}_2F_3\left(-\frac{5}{4}, \frac{5}{4}; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{\eta^4}{144}\right),$$

$$\delta_{L3} = \eta\,{}_2F_3\left(-\frac{3}{4}, \frac{7}{4}; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; -\frac{\eta^4}{144}\right),$$

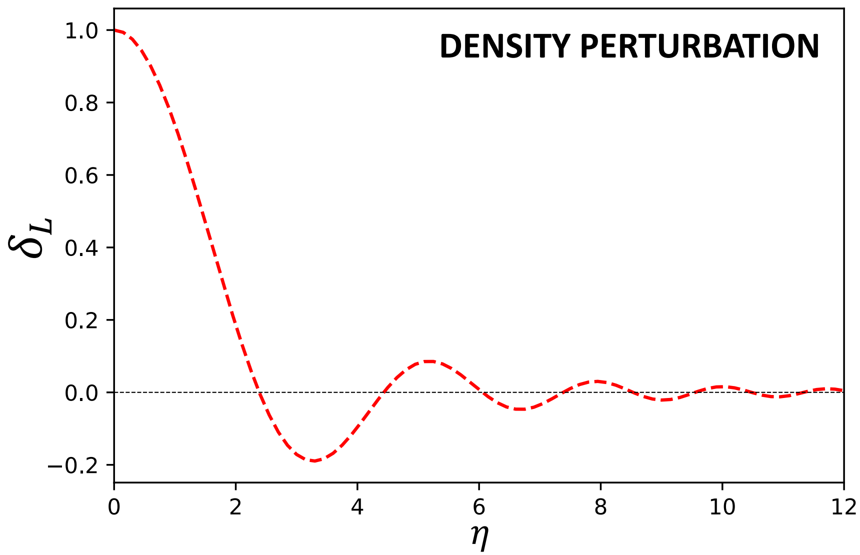
$$\delta_{L4} = \eta^2\,{}_2F_3\left(-\frac{1}{2}, 2; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -\frac{\eta^4}{144}\right).$$

- Smooth solution at $\eta = 0$
- Satisfy the boundary conditions at infinity with $\delta(0) = 1$

$$\delta_L = -\frac{8}{9\pi}\left(\delta_{L4} - \frac{\pi}{40}\delta_{L1}\right)$$

✓ **Recover Fourier solution**

Linear regime self-similar solution



SCALING VARIABLE

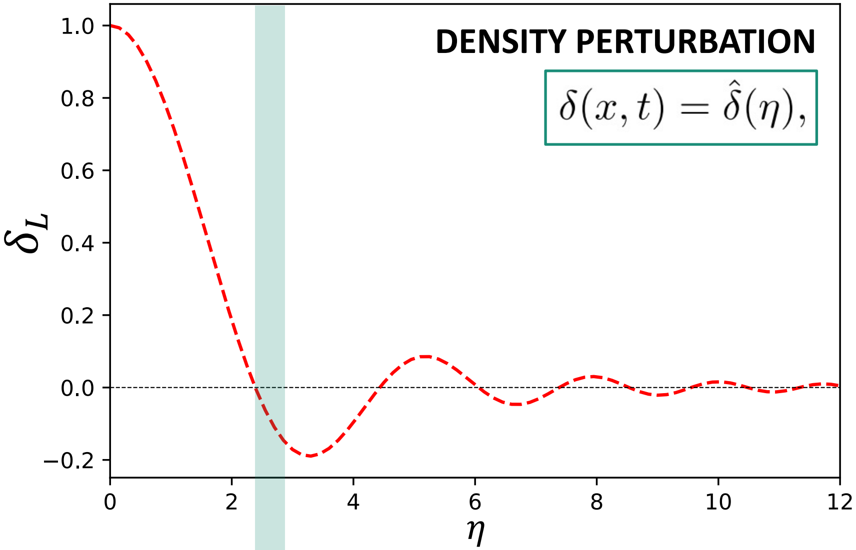
$$\eta = \frac{t^{1/6} x}{\epsilon^{1/2}} = \frac{r}{\sqrt{\epsilon t}}.$$

$$\delta(x,t) = \hat{\delta}(\eta),$$

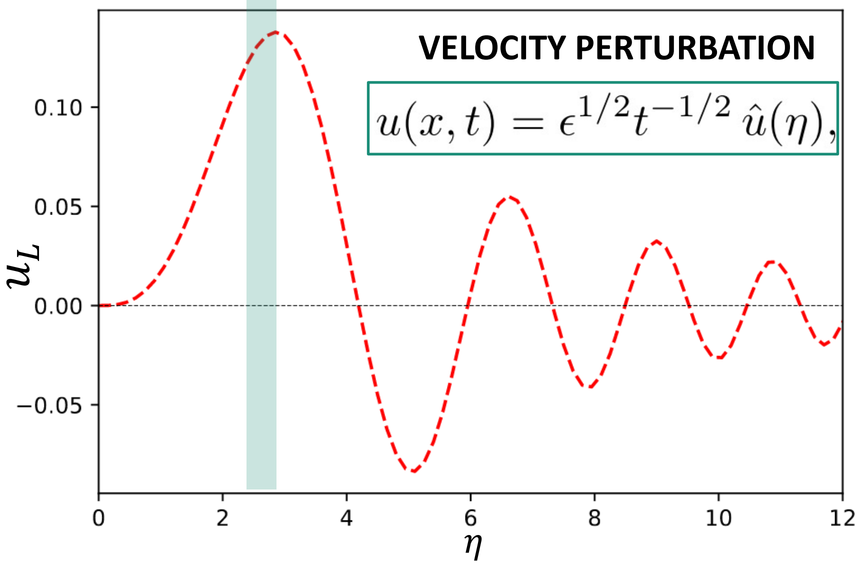
$$\delta_L(x,t) = 1 + \frac{\eta^4}{45} - \frac{8\eta^2}{9\pi}\, {}_2F_3\left(-\frac{1}{2}, 2; \frac{3}{2}, \frac{5}{4}, \frac{7}{4}; -\frac{\eta^4}{144}\right)$$

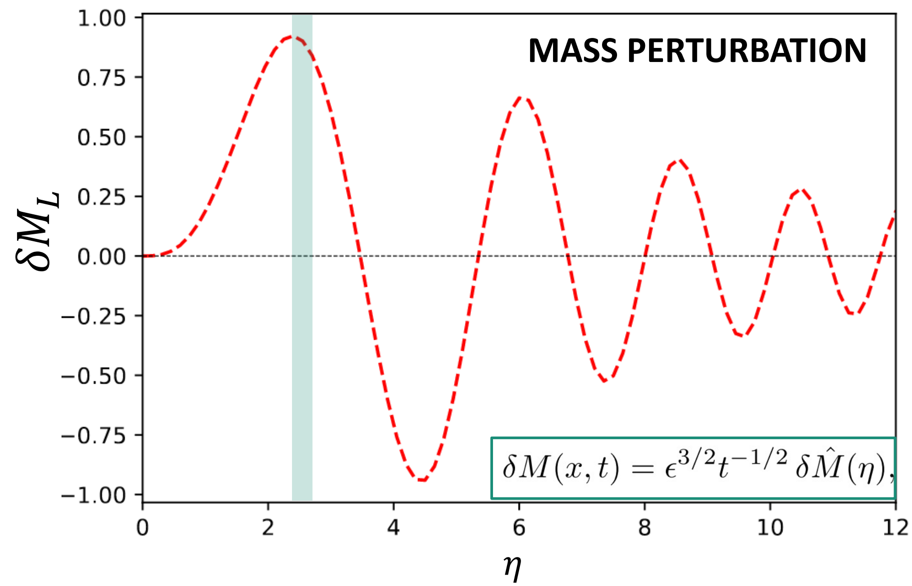
- **Central peak much higher** than the next peaks.
- **Amplitude of δ_L does not grow with time and remains constant. (≠ CDM)**
  - It is stable and **does not reach the nonlinear regime at late times**.
  - **To reach the nonlinear regime → start with a large nonlinear perturbation.**
- As time grows, self-similar solution **grows in physical** coordinates but **shrinks in comoving** coordinates.

I) Small perturbations: Linear regime, $\delta(0) = 1$



- **FDM: constant amplitude. ($\neq$ CDM)**
- **FDM: grows in physical** coordinates but **shrinks in comoving** coordinates. **($\neq$ CDM)**
- **FDM** : Fields with **oscillations ,** Φ_Q **. ($\neq$ CDM)**





Back up: Non-linear regime

Closed equation over δM

Euler equation in terms of η :

$$\frac{1}{6}(\hat{u}+\eta\hat{u}')+\hat{u}\hat{u}'+\hat{\varphi}'_{\rm N}+\hat{\Phi}'_{\rm Q}=0,$$

The fields in terms of η :

$$\hat{\Phi}_{\rm Q}=-\frac{1}{2\eta^2\sqrt{1+\hat{\delta}}}\frac{d}{d\eta}\left(\eta^2\frac{d}{d\eta}\sqrt{1+\hat{\delta}}\right) \qquad \hat{\delta}=\frac{3}{2\eta^2}\delta\hat{M}'$$

Poisson equation in terms of η :

$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2\frac{d\hat{\varphi}_{\rm N}}{d\eta}\right)=\frac{2}{3}\hat{\delta},$$

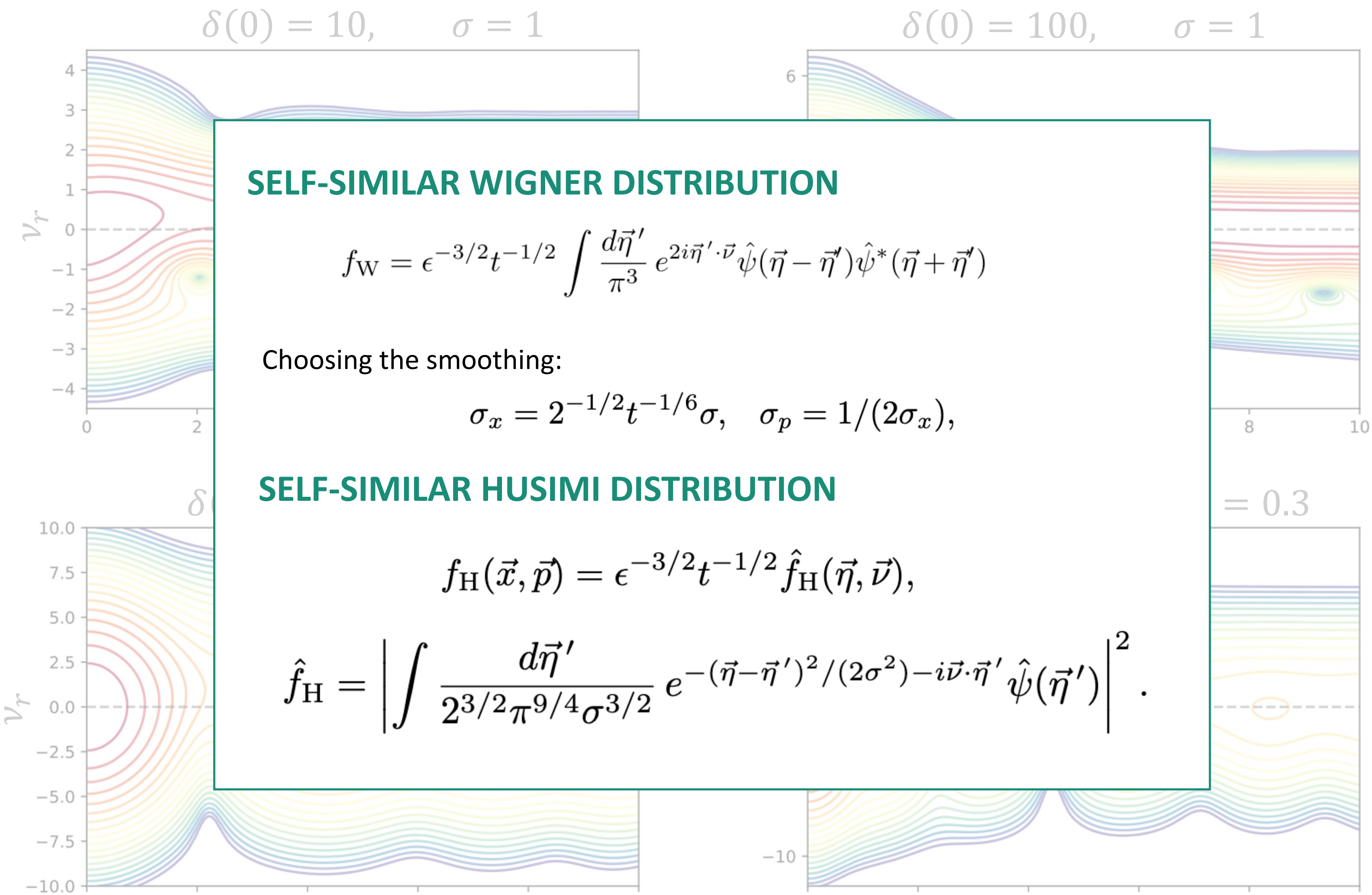
Integrating the continuity equation:

$$\hat{u}=\frac{3\delta\hat{M}-\eta\delta\hat{M}'}{4\eta^2+6\delta\hat{M}'}.$$

CLOSED NON LINEAR EQUATION OVER δM

$$\begin{aligned}
&9(2\eta^3+3\eta\delta\hat{M}')^2\delta\hat{M}^{(4)}-(144\eta^5+216\eta^3\delta\hat{M}'+108\eta^4\delta\hat{M}''\\
&\quad+162\eta^2\delta\hat{M}'\delta\hat{M}'')\delta\hat{M}^{(3)}+(4\eta^8+288\eta^4+36\eta^5\delta\hat{M}\\
&\quad-216\eta^2\delta\hat{M}'+324\eta^3\delta\hat{M}''+81\eta^2\delta\hat{M}^2+81\eta^2\delta\hat{M}''^2)\delta\hat{M}''\\
&\quad-3(4\eta^7+96\eta^3+180\eta^4\delta\hat{M}+243\eta^2\delta\hat{M}\delta\hat{M}'-3\eta^3\delta\hat{M}'^2\\
&\quad+108\delta\hat{M}\delta\hat{M}'^2)\delta\hat{M}'-12\eta^3(7\eta^3-9\delta\hat{M})\delta\hat{M}=0.
\end{aligned}$$

Self-similar Husimi phase-space distribution $\hat{f}_H\,(\eta; v_r; v_t = 0)$



SELF-SIMILAR WIGNER DISTRIBUTION

$$f_{\rm W} = \epsilon^{-3/2} t^{-1/2} \int \frac{d\vec{\eta}\,'}{\pi^3}\, e^{2i\vec{\eta}\,'\cdot\vec{\nu}}\, \hat{\psi}(\vec{\eta}-\vec{\eta}')\hat{\psi}^*(\vec{\eta}+\vec{\eta}')$$

Choosing the smoothing:

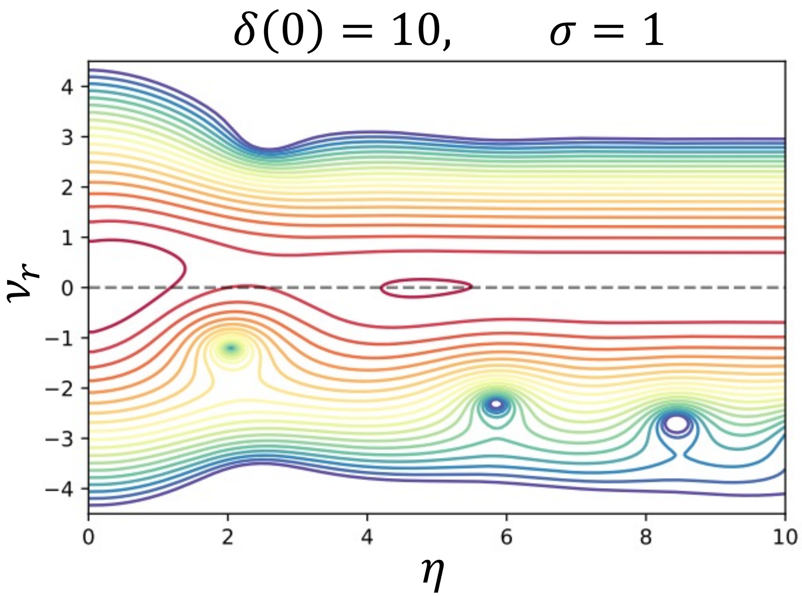
$$\sigma_x = 2^{-1/2} t^{-1/6}\sigma, \quad \sigma_p = 1/(2\sigma_x),$$

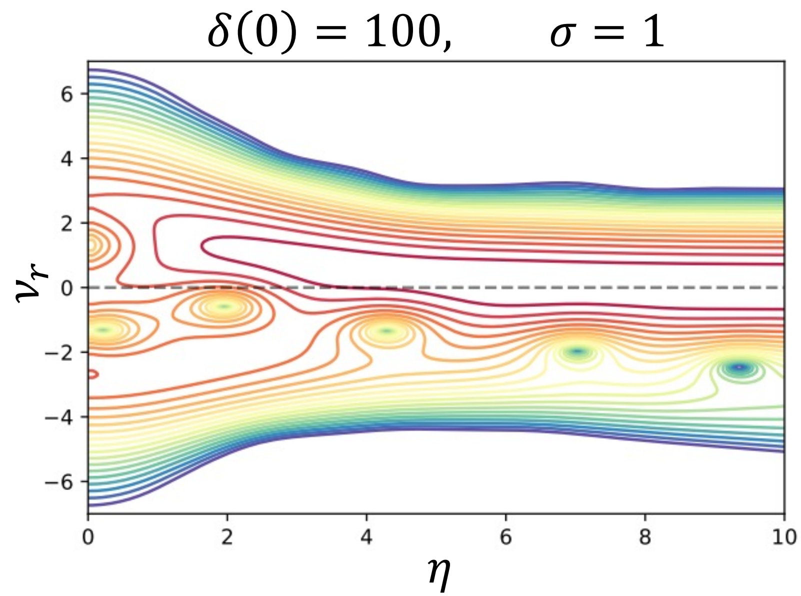
SELF-SIMILAR HUSIMI DISTRIBUTION

$$f_{\rm H}(\vec{x},\vec{p}) = \epsilon^{-3/2} t^{-1/2} \hat{f}_{\rm H}(\vec{\eta},\vec{\nu}),$$

$$\hat{f}_{\rm H} = \left| \int \frac{d\vec{\eta}\,'}{2^{3/2}\pi^{9/4}\sigma^{3/2}}\, e^{-(\vec{\eta}-\vec{\eta}\,')^2/(2\sigma^2) - i\vec{\nu}\cdot\vec{\eta}\,'}\, \hat{\psi}(\vec{\eta}\,') \right|^2 .$$

Radial Husimi phase-space distribution $\hat{f}_H\,(\eta; v_r; v_t = 0), \sigma = 1$



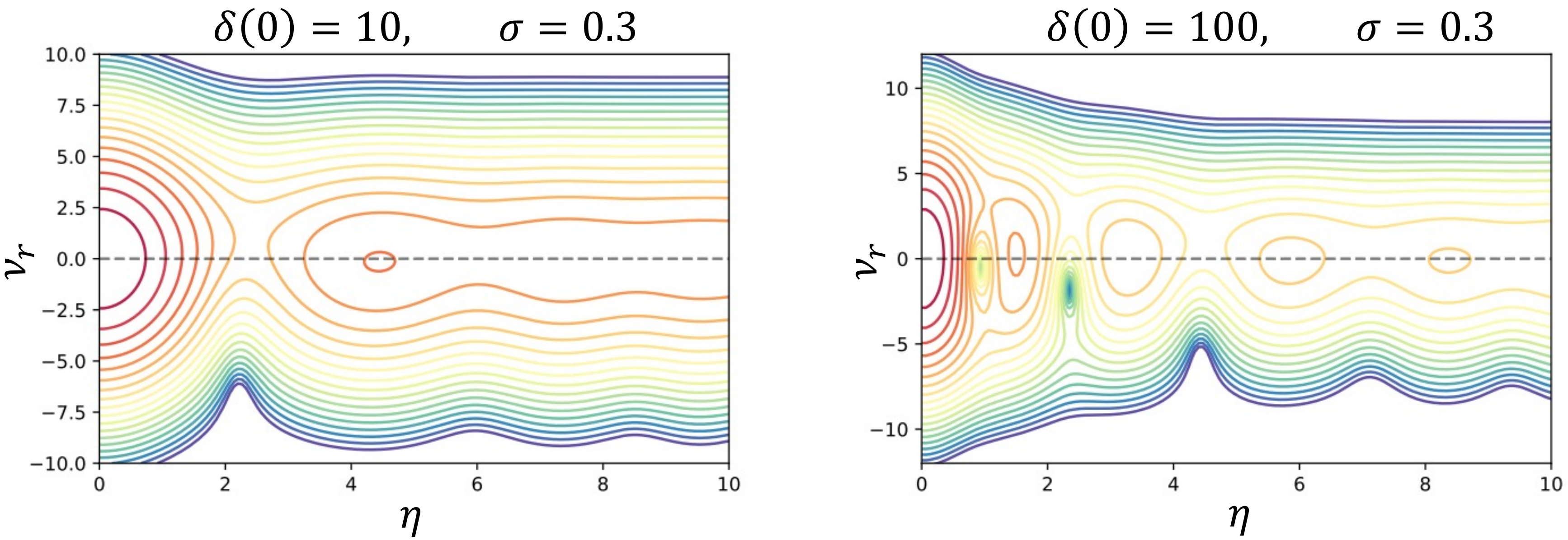


- As the spatial coarsening $\sigma_x \propto \sigma$ increases → the velocity coarsening, $\sigma_p \propto 1/\sigma$ decreases : **Heisenberg uncertainty principle.**

- **Velocity asymmetries but spatial profile is smoothed out.**
- At large distance, the profile → **cosmological BKGD.**

- **The coarsening $\sigma = 1$ is no longer sufficient to separate the first few peaks.**
- Artificial interferences between these peaks and to a Husimi distribution that is difficult to interpret and far from the semiclassical expectations

Radial Husimi phase-space distribution $\hat{f}_H\,(\eta; v_r; v_t = 0), \sigma = 0.3$



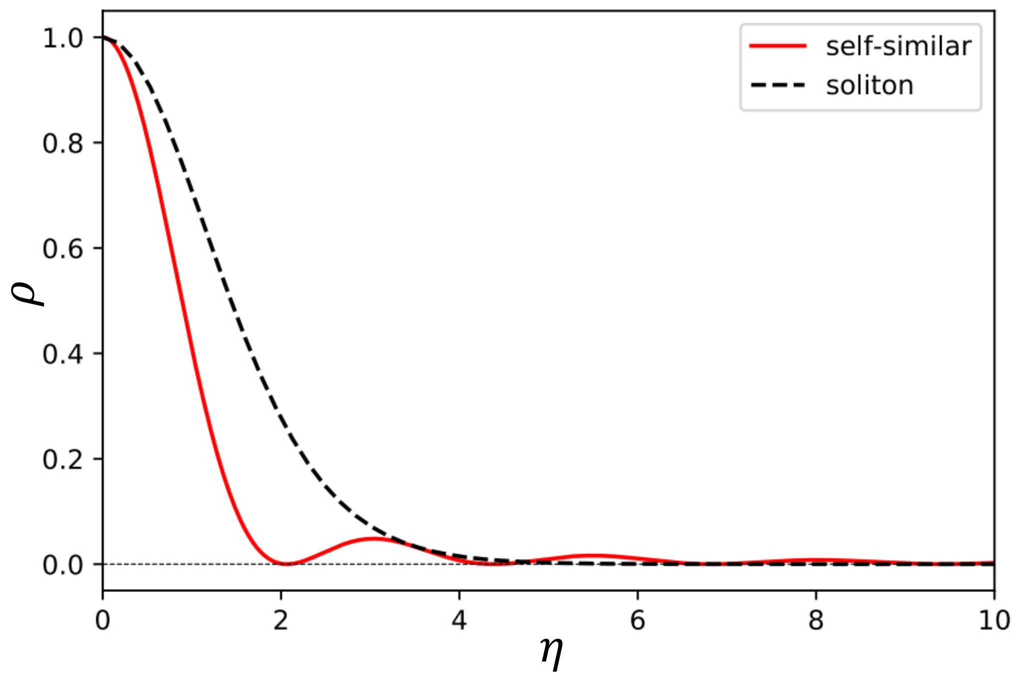
- **Well-defined peaks increasing** with δ(0) and preserves **signs of the density fluctuations.**
- **At large distance→ the cosmological BKGD.**

- More **faithful representation**: Sequence of scalar-field clumps
- **COST: This erases most of the information about the velocity field.**
- Different choices of σ→ different pictures →Difficult to relate to the underlying dynamics

- **The hydrodynamical mapping clearer picture of the dynamics.**

Back up: High-density asymptotic limit

High-density asymptotic limit



- The BKGD density becomes negligible as compared with the central density.
- The inner profile converge to a limiting shape that obeys the scaling law:

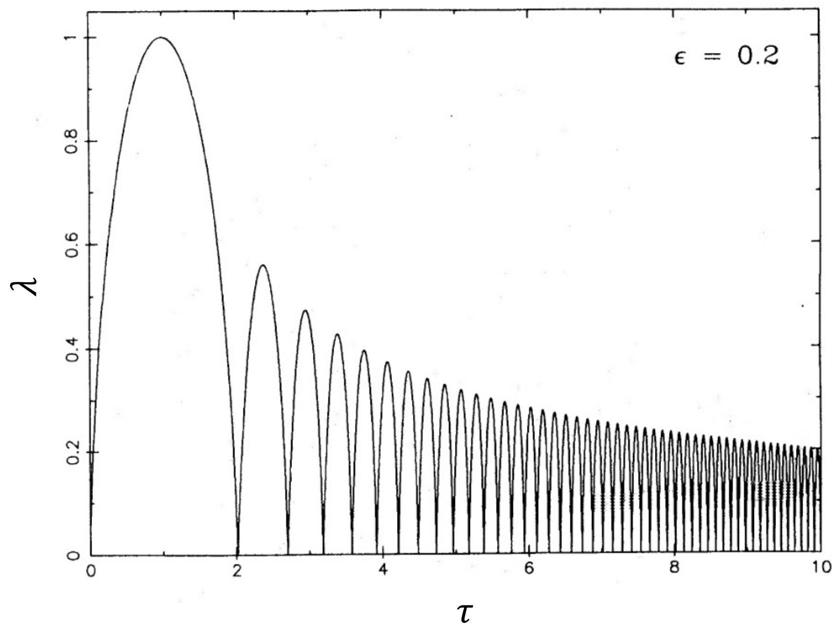
$$\{\eta, \psi, \rho, M\} \rightarrow \{\lambda^{-1}\eta, \lambda^2\psi, \lambda^4\rho, \lambda M\}$$

- The central peak of the **self-similar solution is narrower than the soliton peak**.
- **The shape of the central peak of the self-similar profile does not converge to the soliton equilibrium**. → **kinetic effects** (dominate near the boundary of the central peak).

CDM self-similar solutions vs FDM self-similar solutions

CDM spherical collapse

- **Density contrast grows from linear regime with a power-law profile → non linear regime**.
- Non-linear effects modify the shape of the profile in the inner regions→ **At small radii it takes again a power-law form.**
- Gravitational instability and increasingly distant shells collapse→ **Stabilize at a fixed fraction of the turn-around radius→ (gravity balanced by velocity dispersion). → Virial equilibrium in the inner nonlinear core.**
- **Mass and a radius that grow with time, both in physical and comoving coordinates**.



Bertschinger, E.(1985)
Fillmore, J. A. and Goldreich, P. (1983)

CDM self-similar solutions vs FDM self-similar solutions

- **Density amplitude does not grow with time.** → Balance φ_N vs Φ_Q → No transition from the linear to non-linear regime.→ **No gravitational collapse.**

- **Oscillations of the fields.**

- **The profile remains linear on all scales and at all times or non-linear at the center.**

- At large distance→ **oscillations around the BCKG**→ 2 additional linear modes.
- **Acoustic-like oscillations**→ transport information from small to large scales.

- Matter moves though clumps : **Gravitational cooling**.

- **FDM self-similar solutions disappear in the semi classical limit $\epsilon \to 0$, as their size and mass decrease as $\epsilon^{1/2}$and $\epsilon^{3/2}$**