Newcastle University

Coherent and Incoherent Structures in Fuzzy Dark Matter Halos

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Gerasimos Rigopoulos

Nick Proukakis

IDM, Vienna, 19.07.2022

Newcastle University

Outline

- Fuzzy dark matter
- Halo structures & soliton core oscillation
- Vortices in FDM halos and granule size
- Conclusion

Fuzzy Dark Matter

- Ultralight mass, $m \approx 10^{-22}$ eV, gives $\lambda_{\mathrm{dB}} = h/mv$ in cosmological scale.

L. Hui, arXiv:2101.11735
Dentler et al., arXiv:2111.01199

Schrödinger-Poison equations (SPE)

$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + \mathrm{m}\Phi(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$

$$\nabla^2\Phi(\mathbf{r},t) = 4\pi G[\rho(\mathbf{r},t) - \bar{\rho}]$$

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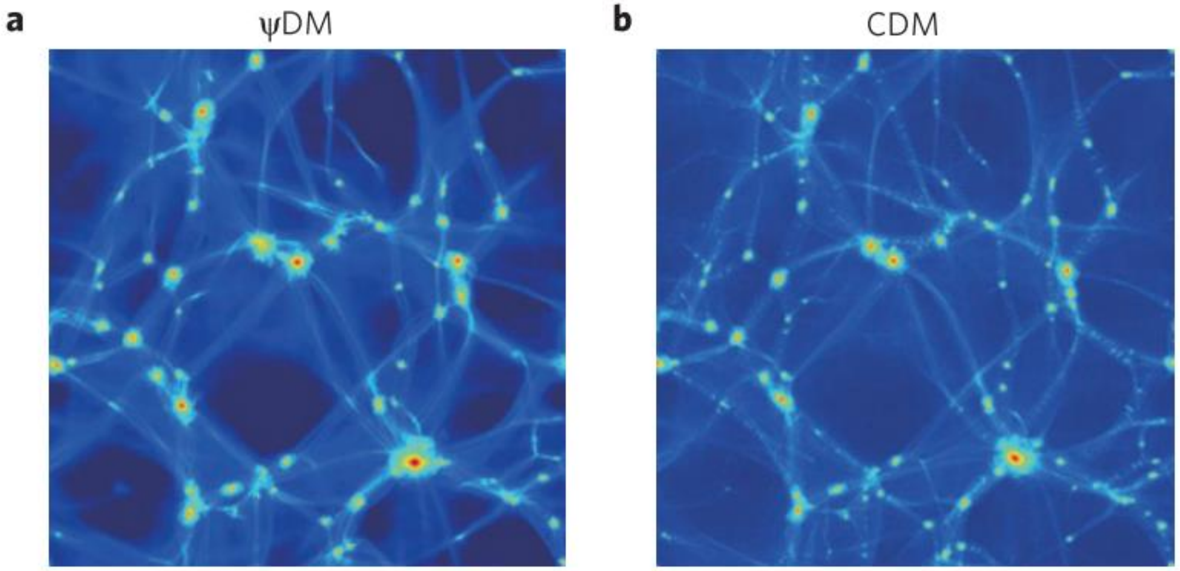

Schive et al., Nat.Phys. 10, 496 (2014)

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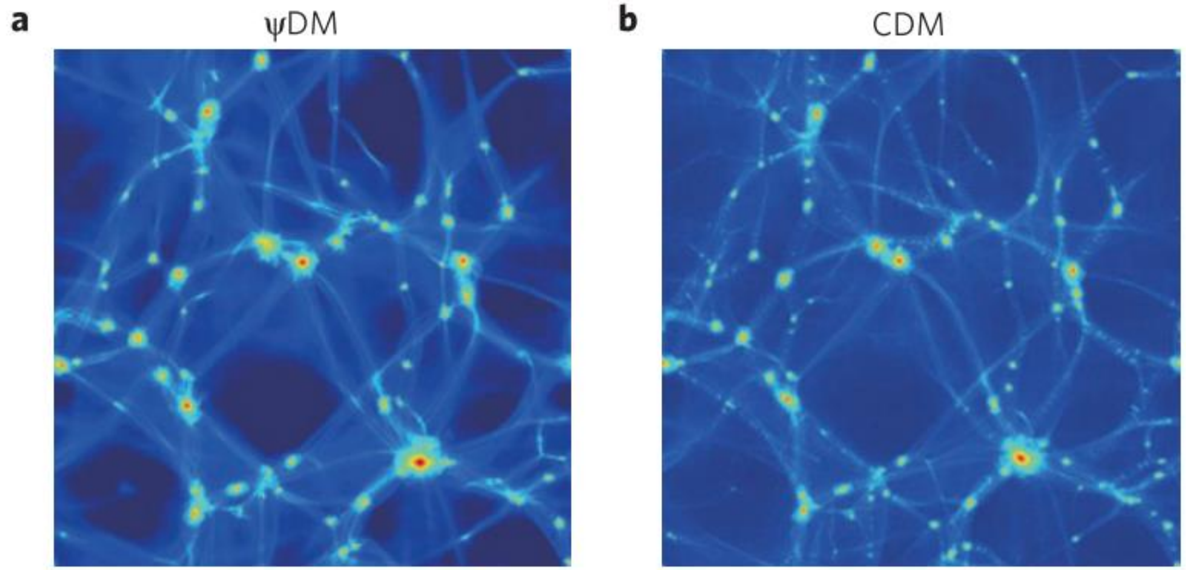

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Scaling invariant/symmetry

$$\{t, x, \Phi, \Psi, \rho\} \rightarrow \{\Lambda^2 t', \Lambda x', \Lambda^{-2}\Phi', \Lambda^{-2}\Psi', \Lambda^{-4}\rho'\}$$

$$\{t, x, \Phi, \Psi, \rho\} \rightarrow \{\alpha^{-1} t', x', \alpha^2\Phi', \alpha\,\Psi', \alpha^2\rho'\}$$

Ji & Sin, PRD, 50, 3655 (1994); Mocz et al., MNRAS 471, 4559 (2017)

Wavy/quantum features

- Wave interferences (fuzzy DM/ψDM/BECDM)

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$$\Psi(\mathbf{r},t) = \sqrt{\rho(\mathbf{r},t)}e^{i\varphi(\mathbf{r},t)}$$

$$\mathbf{v}(\mathbf{r},t) = \frac{\hbar}{m}\nabla\varphi(\mathbf{r},t)$$

Hydrodynamic description

$$\frac{\partial}{\partial t}\rho + \nabla\cdot(\rho\mathbf{v}) = 0$$

$$\frac{\partial}{\partial t}\mathbf{v} + \nabla\left[\frac{|\mathbf{v}|^2}{2} + \Phi - \frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right] = 0$$

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Superfluidity

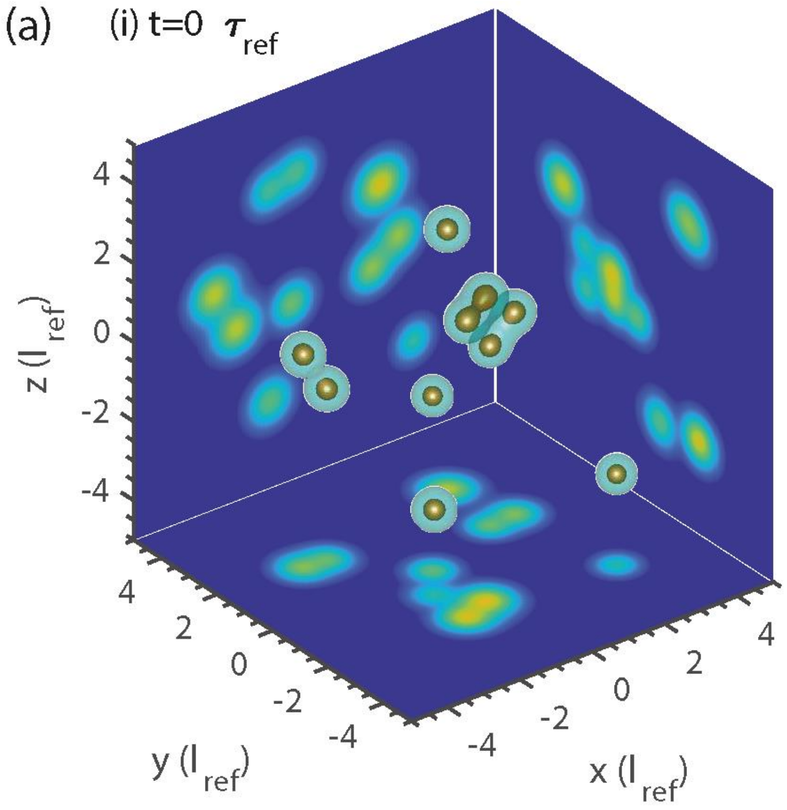
- Viscosity free
- Critical velocity (m-dependent Jeans scale)
- Irrotational

Halo structures

Soliton merger simulation

Mocz et al., MNRAS 471, 4559 (2017)
Chan et al., arXiv:2110.11882

Primary sample $(M = 100 M_{\mathrm{ref}})$



$$E_{\mathrm{ref}} = \hbar\sqrt{G\rho_{\mathrm{ref}}} \qquad \tau_{\mathrm{ref}} = (G\rho_{\mathrm{ref}})^{-1/2} \qquad l_{\mathrm{ref}} = \left(\frac{\hbar^2}{m^2 G\rho_{\mathrm{ref}}}\right)^{1/4}$$

$$\rho_{\mathrm{ref}} = 10^3 M_\odot \mathrm{kpc}^{-3}$$
$$m_{\mathrm{ref}} = 2.5 \times 10^{-23}\ \mathrm{eV}$$

$$M_{\mathrm{ref}} \approx 1.26 \times 10^9 M_\odot$$
$$\tau_{\mathrm{ref}} \approx 14.91\ \mathrm{Gyr}$$
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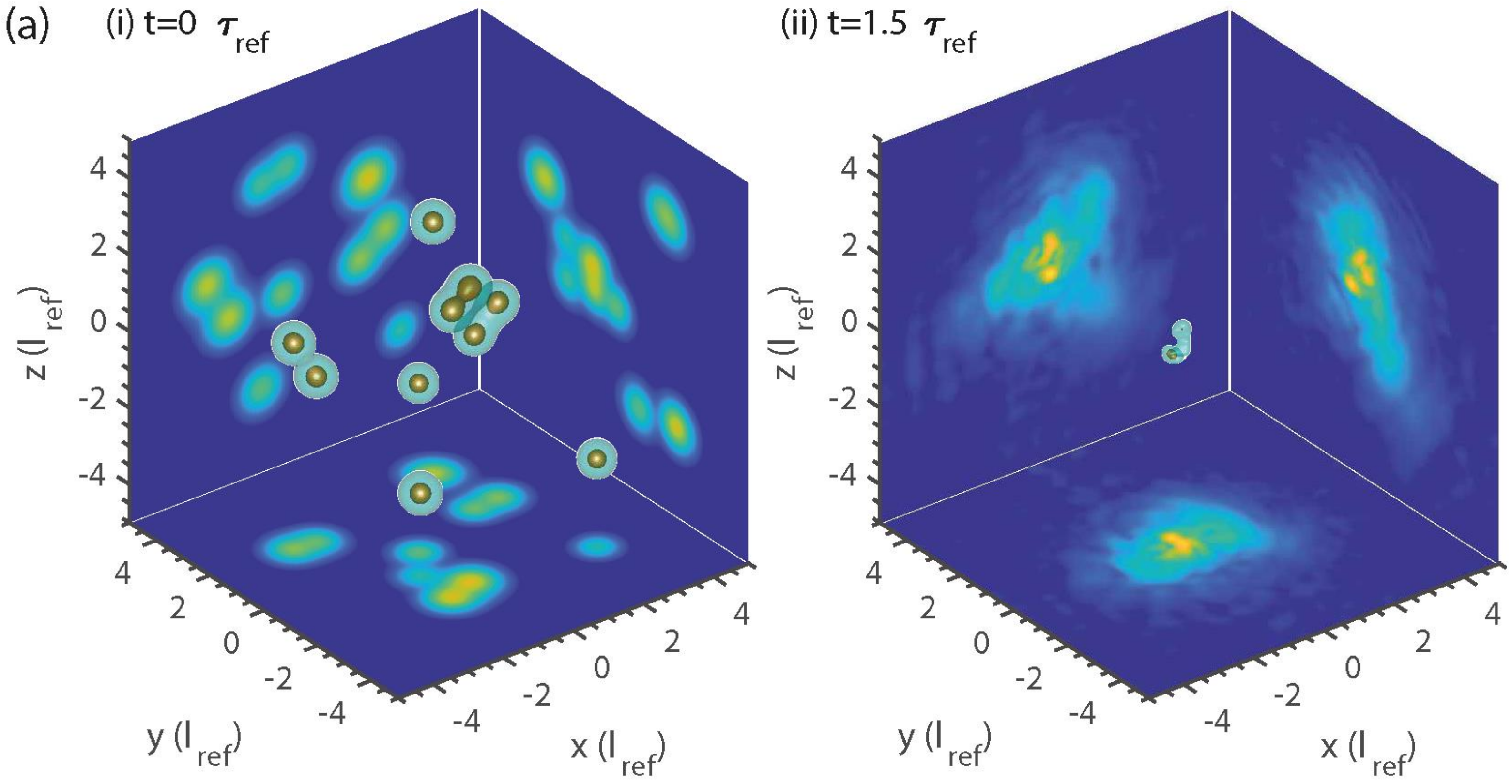

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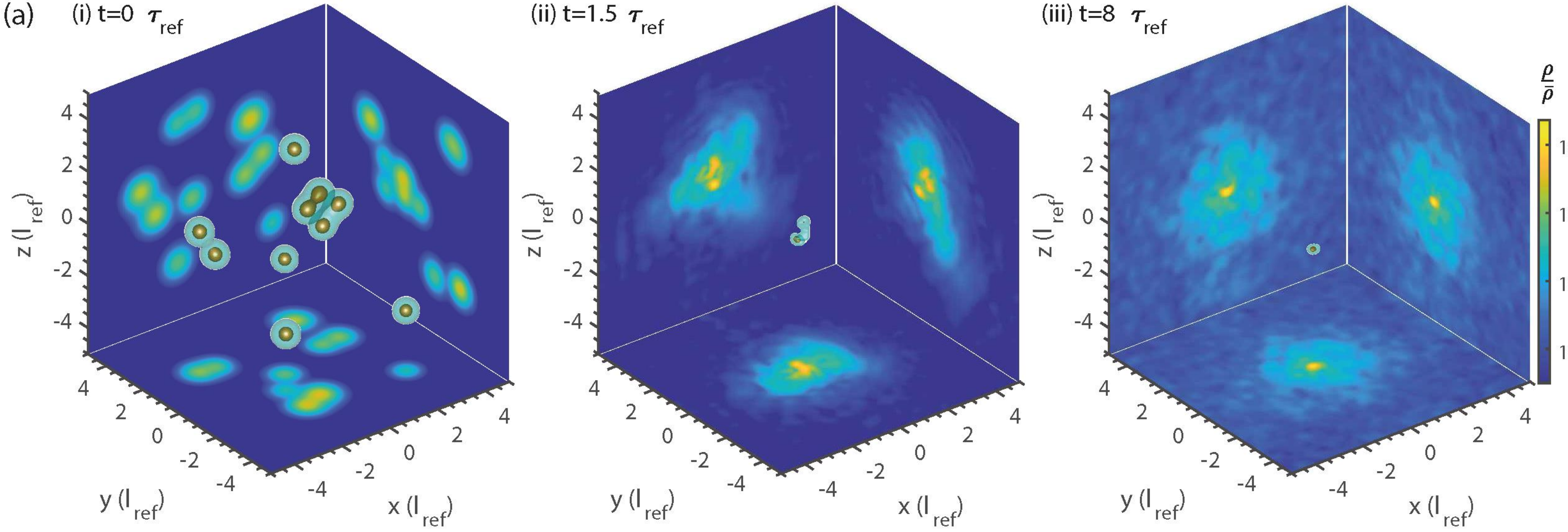

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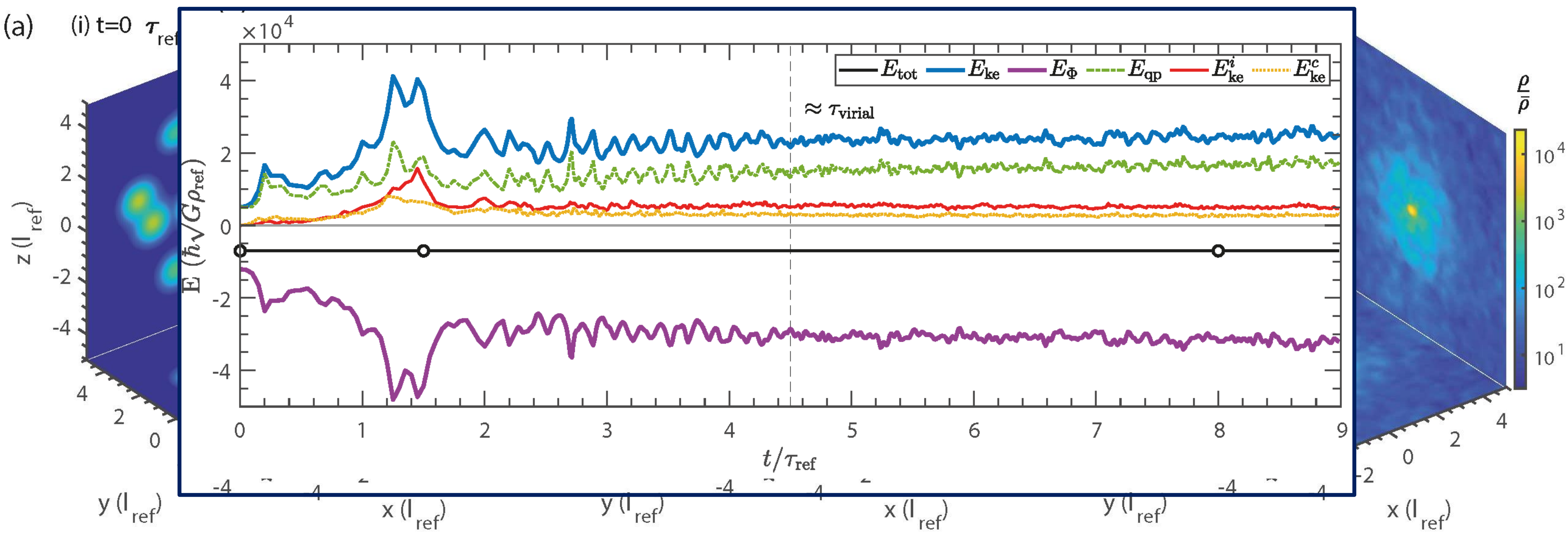

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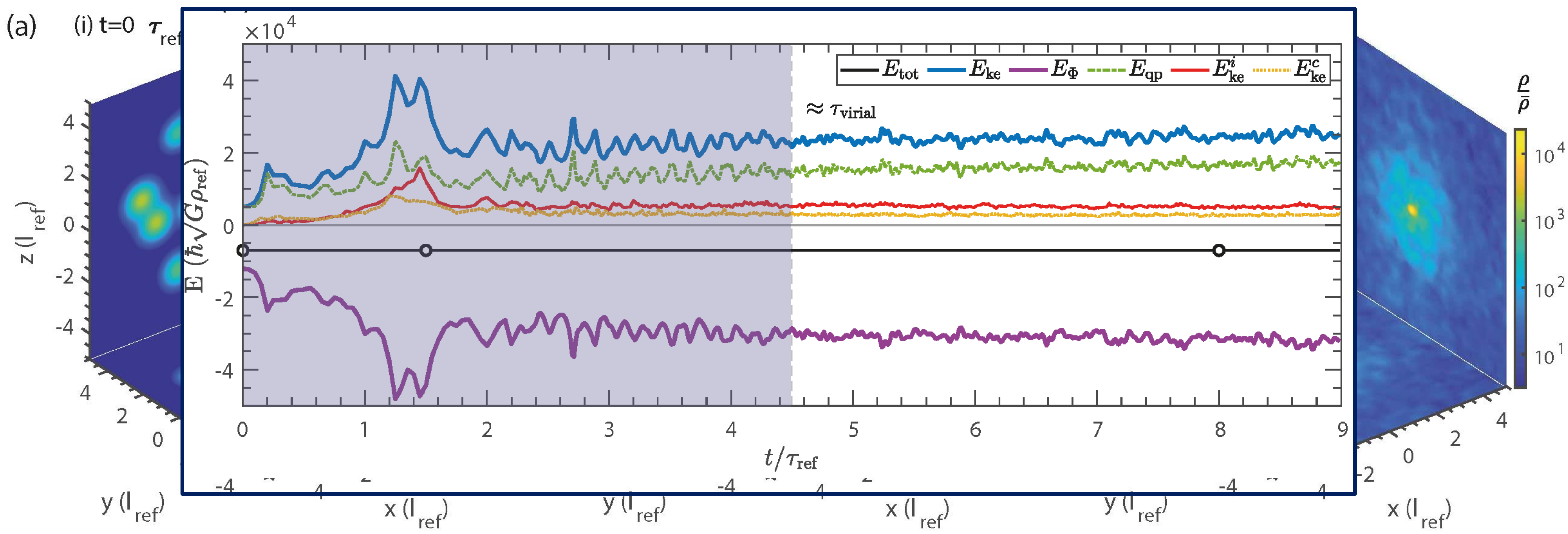

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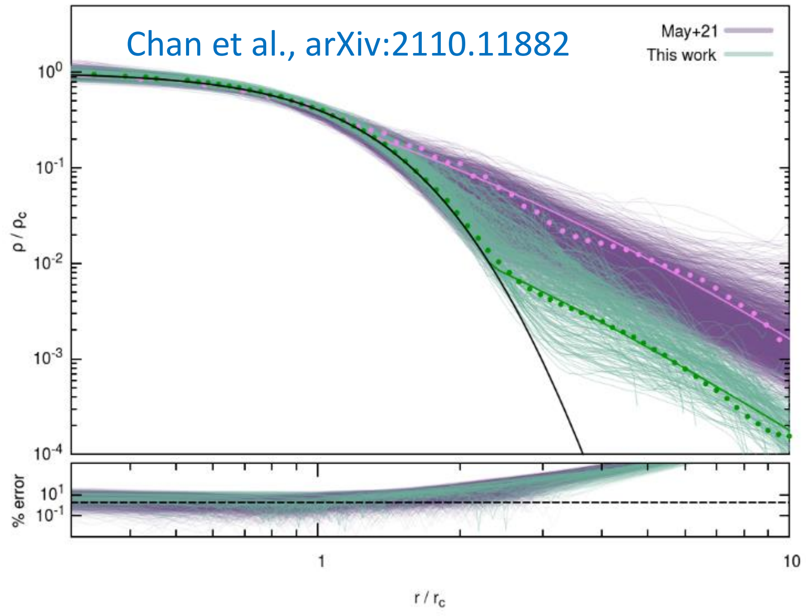
Halo structure

Cored-halo profile

$$\rho_{\mathrm{cNFW}}(r) = \begin{cases} \\ \rho_{\mathrm{NFW}}(r) & , r > r_t \end{cases}$$

$$\rho_{\mathrm{NFW}}(r) = \rho_h \left(\frac{r}{r_h}\right)^{-1} \left(1 + \frac{r}{r_h}\right)^{-2}$$

Navarro, Frenk &White, ApJ, 462, 563 (1996)



Halo structure

Cored-halo profile

$$\rho_{\mathrm{cNFW}}(r) = \begin{cases} \rho_{\mathrm{soliton}}(r) & , r \leq r_t \\ \rho_{\mathrm{NFW}}(r) & , r > r_t \end{cases}$$

$$\rho_{\mathrm{soliton}}(r) = \rho_c \left[1 - \lambda \left(\frac{r}{r_c}\right)^2\right]^{-8}$$

Schive et al., Nat.Phys. 10, 496 (2014); PRL 113, 261302 (2014)

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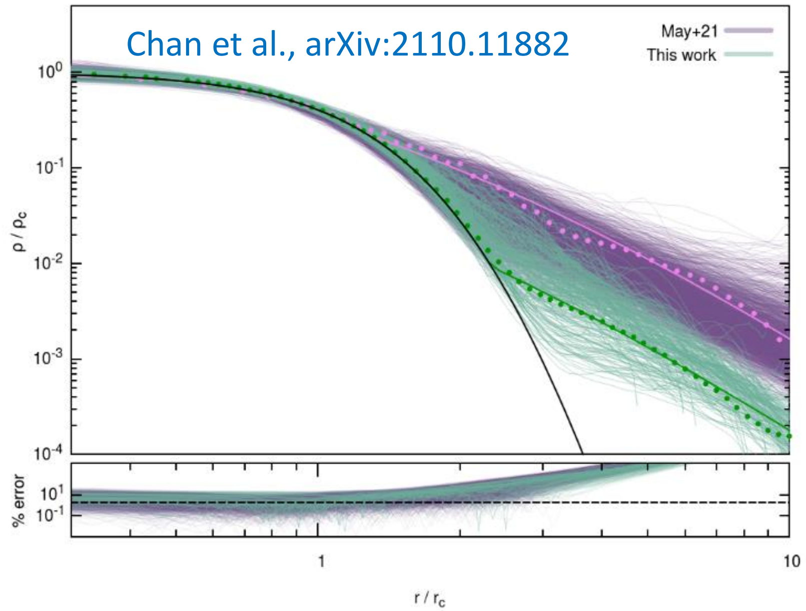
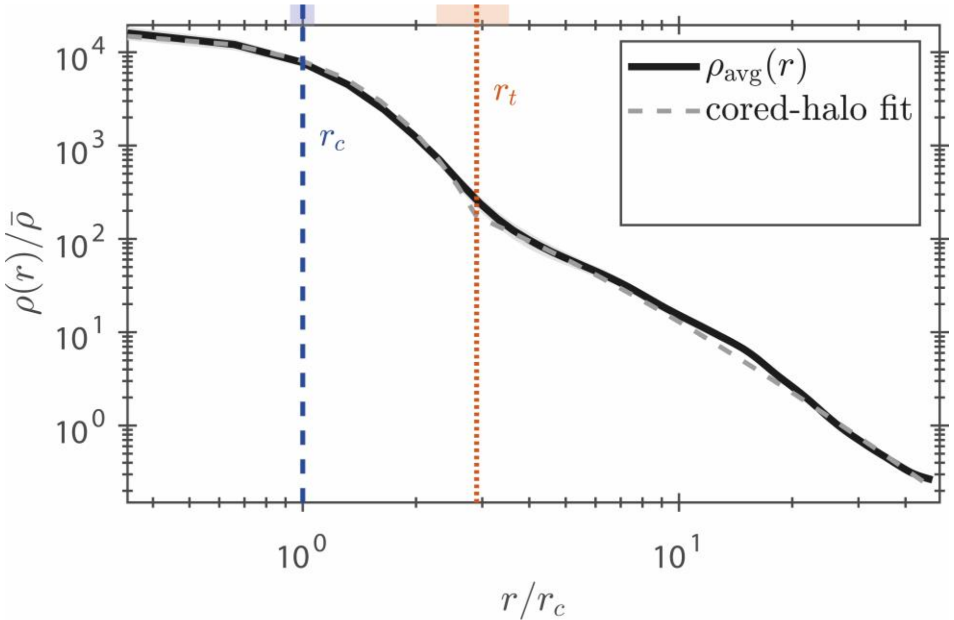
$\rho_h = \rho_h(\rho_c, r_c, r_t)$ to ensure the ρ_{cNFW} continuity

r_c, r_t, r_h are free parameters



Liu, Proukakis, Rigopoulos, 2207/08.XXXX



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Energy functional of the SP system

$$E = E[\Psi^*, \Psi] = \int d\mathbf{r}\, \Psi^* \left[-\frac{\hbar^2 \nabla^2}{2m^2} + \frac{1}{2}\Phi \right] \Psi$$

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Energetically preferred soliton solution, $\Psi(r) = \sqrt{\rho_{\mathrm{soliton}}(r)}$, for a fixed M_{soliton}

$$r_c = \left(\frac{11303\lambda^2}{128} \frac{\hbar^2}{m^2 G\pi} \right)^{\frac{1}{4}} \rho_c^{-\frac{1}{4}}$$

$$\approx 23.686 \left(\frac{2.5 \times 10^{-23}\mathrm{eV}}{m} \right)^{\frac{1}{2}} \left(\frac{10^3 M_{\odot} \mathrm{kpc}^{-3}}{\rho_c} \right)^{\frac{1}{4}} \mathrm{kpc}$$

$$M_{\mathrm{soliton}} = \frac{1107}{389} \frac{\sqrt{\lambda}\pi\hbar^2}{m^2 G} r_c^{-1}$$

$$\approx 3.68 \times 10^7 \left(\frac{2.5 \times 10^{-23}\mathrm{eV}}{m} \right)^2 \left(\frac{r_c}{\mathrm{kpc}} \right)^{\frac{1}{4}} \mathrm{M}_{\odot}$$

Liu, Proukakis, Rigopoulos, 2207/08.XXXX

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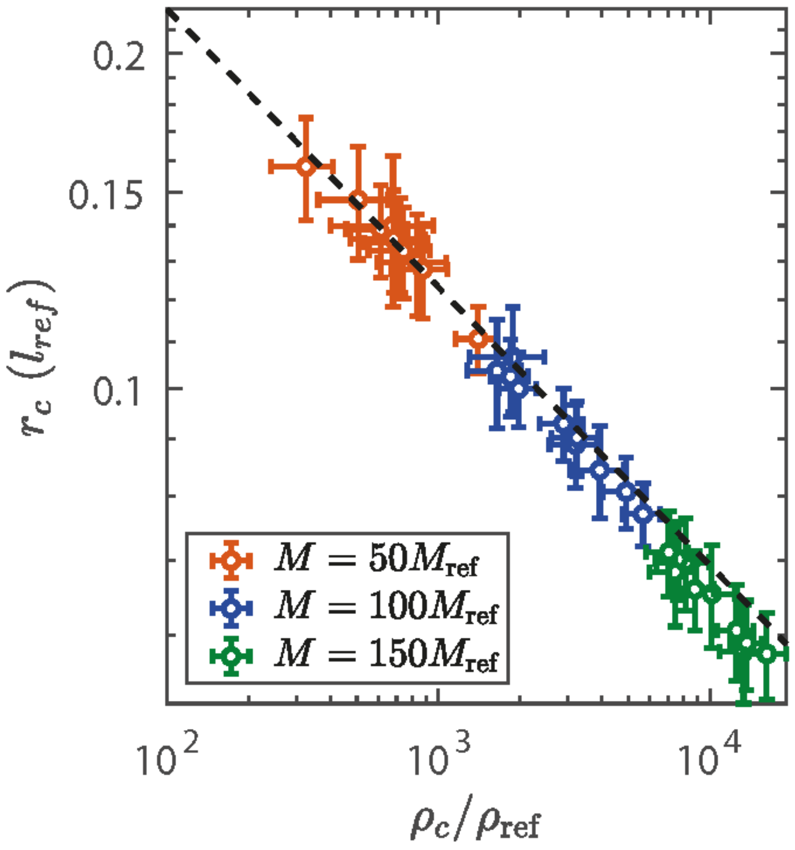


Liu, Proukakis, Rigopoulos, 2207/08.XXXX
Mocz et al., MNRAS 471, 4559 (2017); Chiang et al., PRD 103, 103019 (2021)

Halo structures: soliton core oscillations

Power spectrum

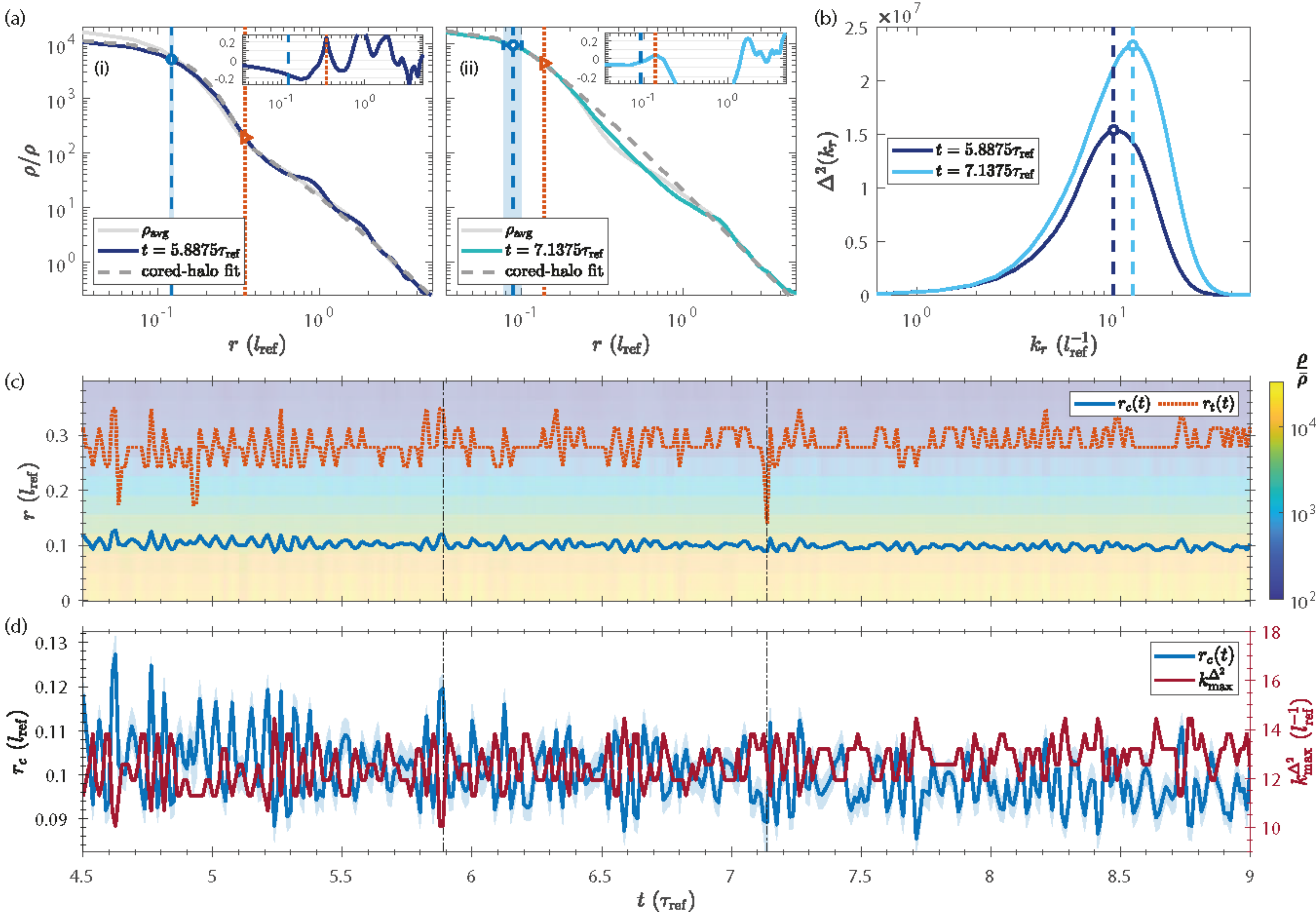
$$\Delta^2(k_r,t) = \frac{k_r^3}{(2\pi)^2}\int \frac{d\mathbf{\Omega_k}}{4\pi k_r^2} P(\mathbf{k},t)$$

$$P(\mathbf{k},t) = |\tilde{\eta}(\mathbf{k},t)|^2$$

$$\eta(\mathbf{r},t) = \frac{\rho(\mathbf{r},t)-\bar{\rho}}{\bar{\rho}}$$

$r_c - k_{\max}^{\Delta^2}$ is anti correlated

$$\omega \propto \rho_c^{1/2}$$



M. Indjin, P3D, El10, 21 July, 17:30

Liu, Proukakis, Rigopoulos, 2207/08.XXXX

Halo structures

Mode-mixing and nonlinear dynamic

Lin et al. PRD 97, 103523 (2018)

$$\Psi(\mathbf{r},t) = \sum_{nlm} a_{nlm} e^{iE_{nl}t/\hbar} \phi_{nlm}(\mathbf{r})$$

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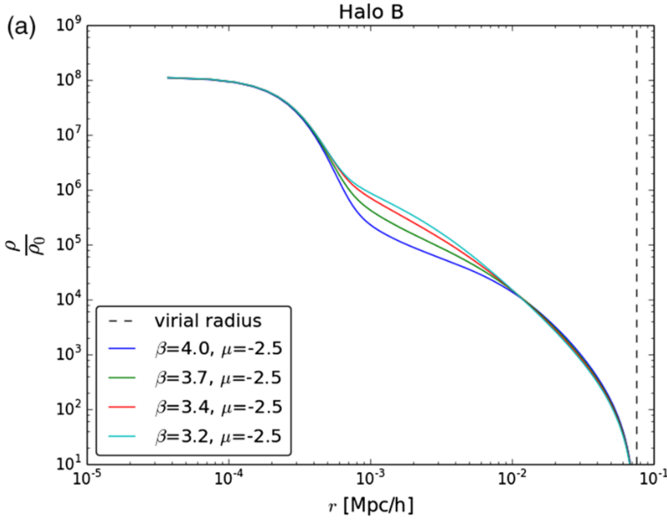
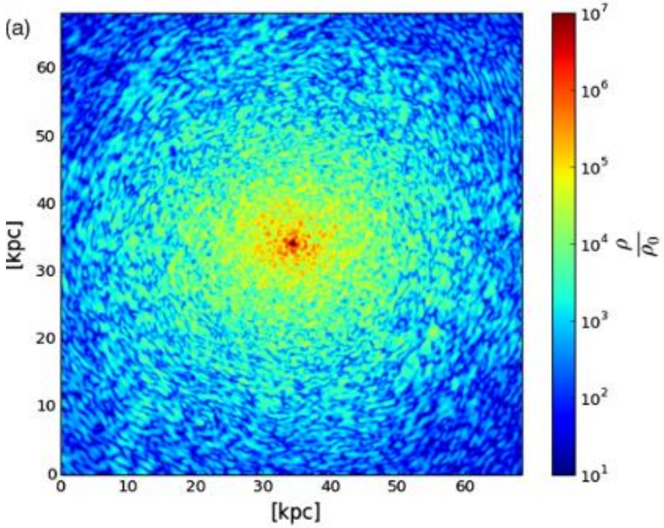
Random phase + Spherical-symmetry approximations

$$\nabla^2\bar{\Phi}(\mathbf{r}) = 4\pi G\bar{\rho}(r) = G\int d\boldsymbol{\Omega} \sum_{nlm} |a_{nlm}\phi_{nlm}(\mathbf{r})|^2$$





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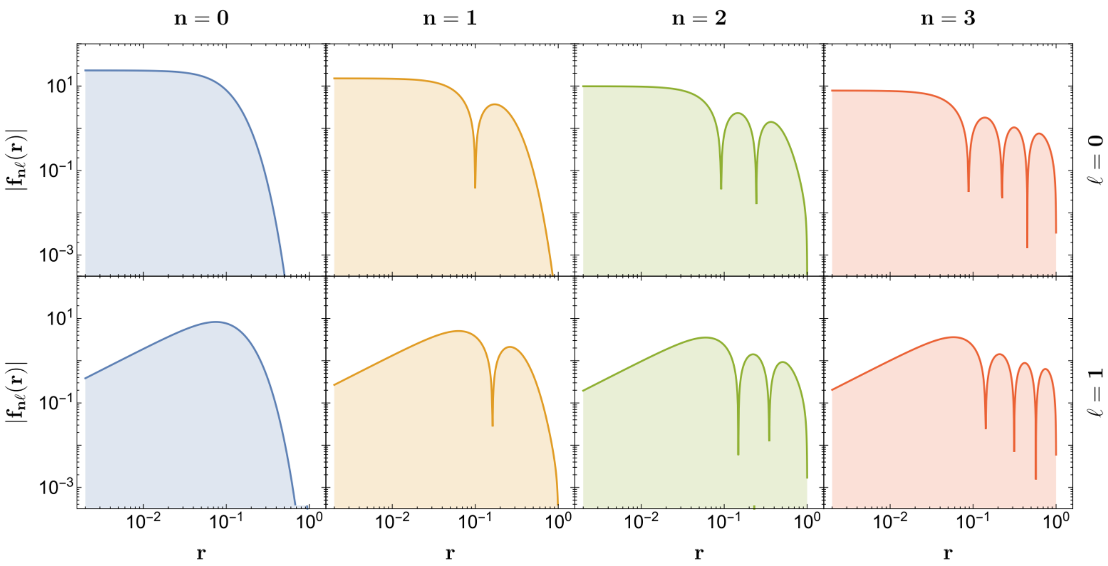
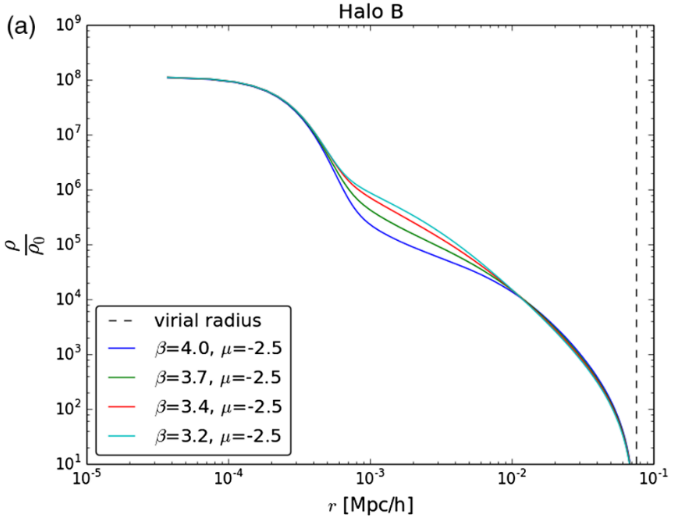
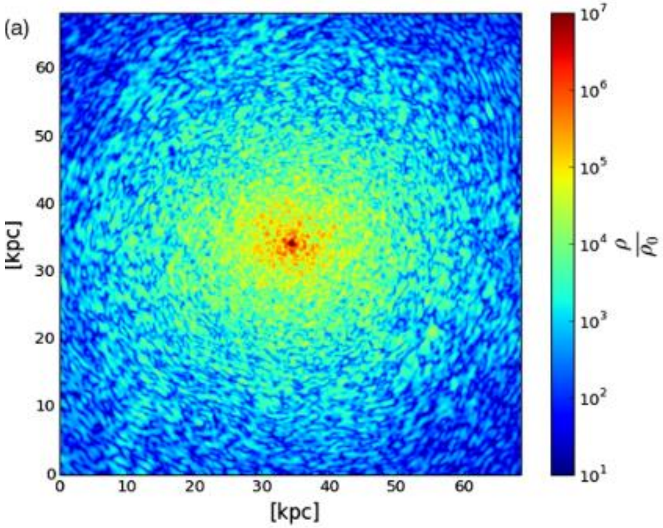
Zagorac et al., PRD 105, 103506







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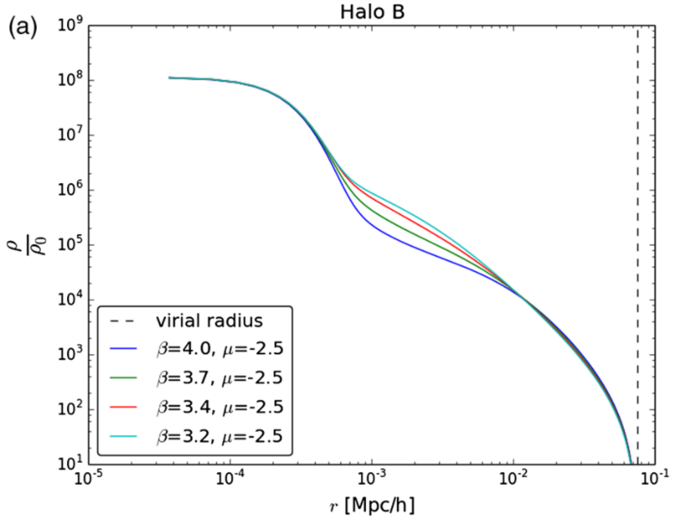

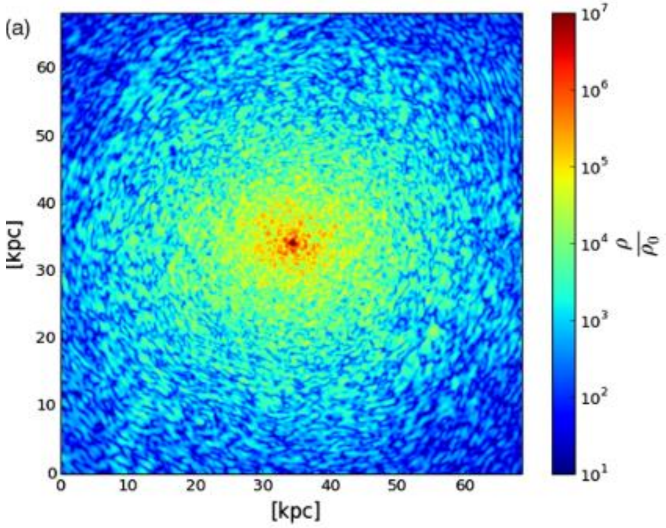

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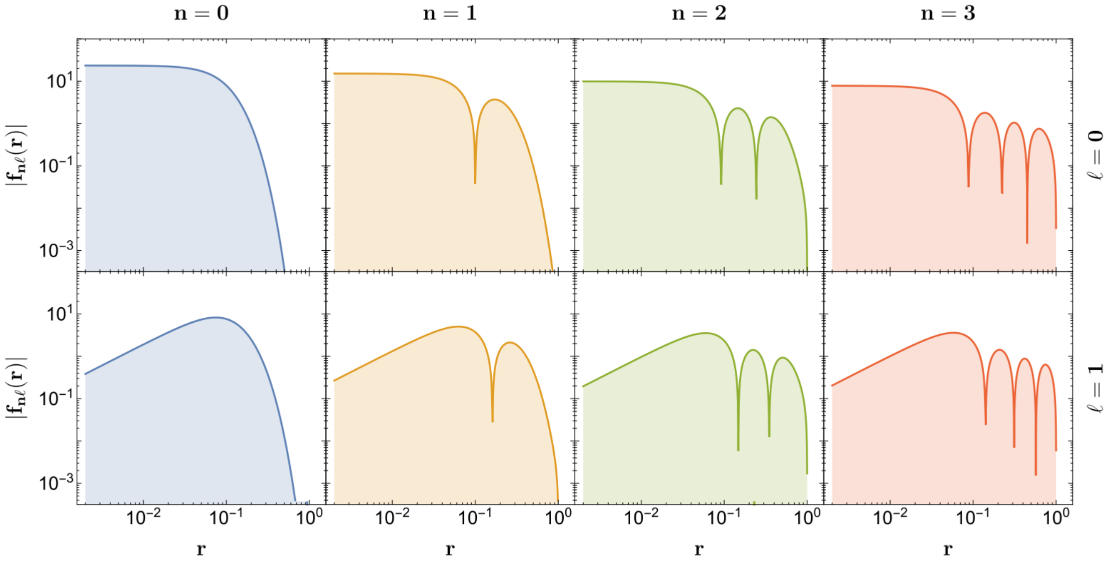

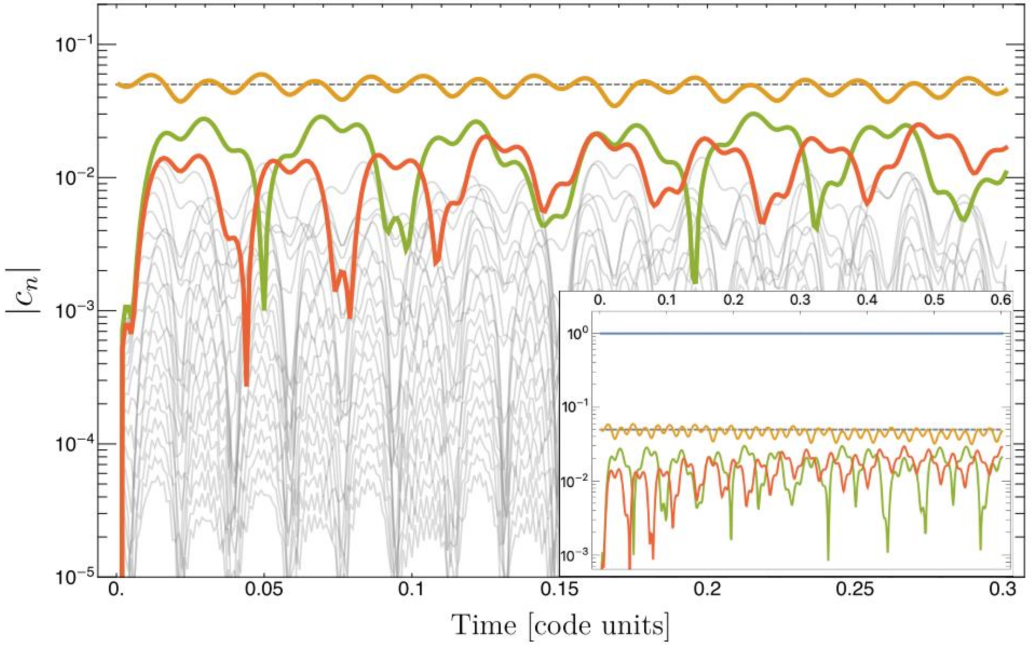

Halo structures

Coherence measures (Classical-field approach)

Liu, Proukakis, Rigopoulos, 2207/08.XXXX

Penrose-Onsager condensate mode (phase coherent)

$$\hat{\varrho}(\mathbf{r},\mathbf{r}')\Psi_{\mathrm{PO}}(\mathbf{r}) = N_{\mathrm{PO}}\Psi_{\mathrm{PO}}(\mathbf{r}), \hat{\varrho}(\mathbf{r},\mathbf{r}') = \langle \Psi^*(\mathbf{r}')\Psi(\mathbf{r})\rangle$$

Quasi-condensate (density coherent)

$$\rho_{\mathrm{qc}}(\mathbf{r}) = \sqrt{2\langle\rho(\mathbf{r})\rangle^2 - \langle\rho^2(\mathbf{r})\rangle}$$

Phase space density

$$c = \lambda_{dB}^3 n$$

First-order spatial correlation function

$$\mathrm{g}_1(\mathbf{r},\mathbf{r}') = \frac{\langle\Psi^*(\mathbf{r}')\Psi(\mathbf{r})\rangle}{\sqrt{\langle|\Psi(\mathbf{r}')|^2\rangle\langle|\Psi(\mathbf{r})|^2\rangle}}$$

Second-order spatial auto correlation function

$$\mathrm{g}_2(\mathbf{r}) = \frac{\langle\rho^2(\mathbf{r})\rangle}{\langle\rho(\mathbf{r})\rangle^2}$$

F. Dalfovo et al., Review of Mordern Physics 71, 463 (1999)
P. B. Blakie et al., Adv. Phys. 57, 363 (2008)
Liu et al., Comm.Phys. 1, 24 (2018) & PRR 2, 0333183 (2020)
Proukakis, PRA 74, 053617 (2006)

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Liu, Proukakis, Rigopoulos, 2207/08.XXXX

Penrose-Onsager condensate mode (phase coherent)

$$\hat{\varrho}(\mathbf{r},\mathbf{r}')\Psi_{\mathrm{PO}}(\mathbf{r}) = N_{\mathrm{PO}}\Psi_{\mathrm{PO}}(\mathbf{r}), \hat{\varrho}(\mathbf{r},\mathbf{r}') = \langle \Psi^*(\mathbf{r}')\Psi(\mathbf{r})\rangle$$

Quasi-condensate (density coherent)

$$\rho_{\mathrm{qc}}(\mathbf{r}) = \sqrt{2\langle\rho(\mathbf{r})\rangle^2 - \langle\rho^2(\mathbf{r})\rangle}$$

Phase space density

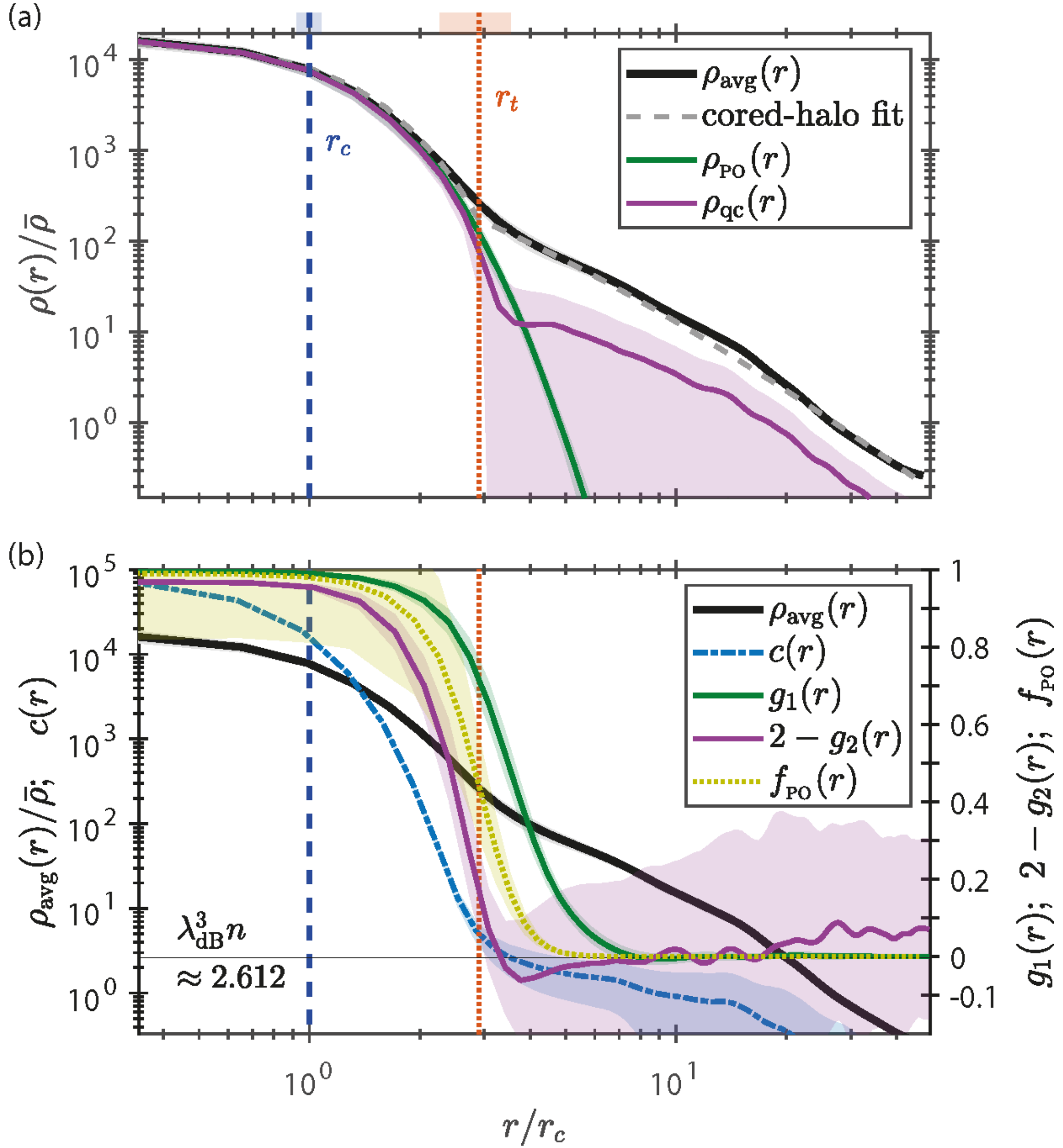
$$c = \lambda_{dB}^3 n$$

First-order spatial correlation function

$$\mathrm{g}_1(\mathbf{r},\mathbf{r}') = \frac{\langle \Psi^*(\mathbf{r}')\Psi(\mathbf{r})\rangle}{\sqrt{\langle|\Psi(\mathbf{r}')|^2\rangle\langle|\Psi(\mathbf{r})|^2\rangle}}$$

Second-order spatial auto correlation function

$$\mathrm{g}_2(\mathbf{r}) = \frac{\langle\rho^2(\mathbf{r})\rangle}{\langle\rho(\mathbf{r})\rangle^2}$$



F. Dalfovo et al., Review of Mordern Physics 71, 463 (1999)
P. B. Blakie et al., Adv. Phys. 57, 363 (2008)
Liu et al., Comm.Phys. 1, 24 (2018) & PRR 2, 0333183 (2020)
Proukakis, PRA 74, 053617 (2006)

Halo structures

Energy profiles

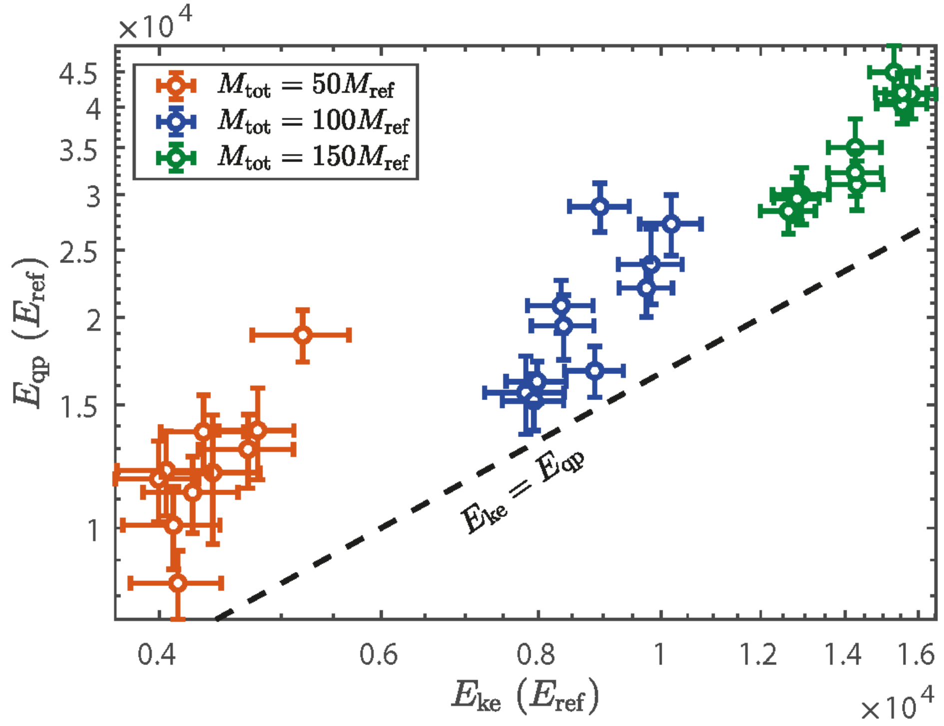
Mocz et al., MNRAS 471, 4559 (2017)

$$-\hbar^2\nabla^2/2\mathrm{m}^2 \qquad \Phi(\mathbf{r})\rho(\mathbf{r})/2$$

$$E = \int d\mathbf{r}\left\{\left[\varepsilon_{\mathrm{ke}}(\mathbf{r}) + \varepsilon_{\mathrm{qp}}(\mathbf{r})\right] + \varepsilon_{\Phi}(\mathbf{r})\right\}$$

$$\varepsilon_{\mathrm{ke}}(\mathbf{r}) = \frac{1}{2}\rho(\mathbf{r})|\mathbf{v}(\mathbf{r})|^2$$

$$\varepsilon_{\mathrm{qp}}(\mathbf{r}) = \frac{\hbar^2}{2m^2}\left|\nabla\sqrt{\rho(\mathbf{r})}\right|^2$$



Liu, Proukakis, Rigopoulos, 2207/08.XXXX

Quantum pressure energy is at least 2 times larger than the classical kinetic one.

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Mocz et al., MNRAS 471, 4559 (2017)

$$E = \int d\mathbf{r} \left\{ \left[\varepsilon_{\rm ke}(\mathbf{r}) + \underbrace{\varepsilon_{\rm qp}(\mathbf{r})}_{}\right] + \varepsilon_{\Phi}(\mathbf{r}) \right\}$$

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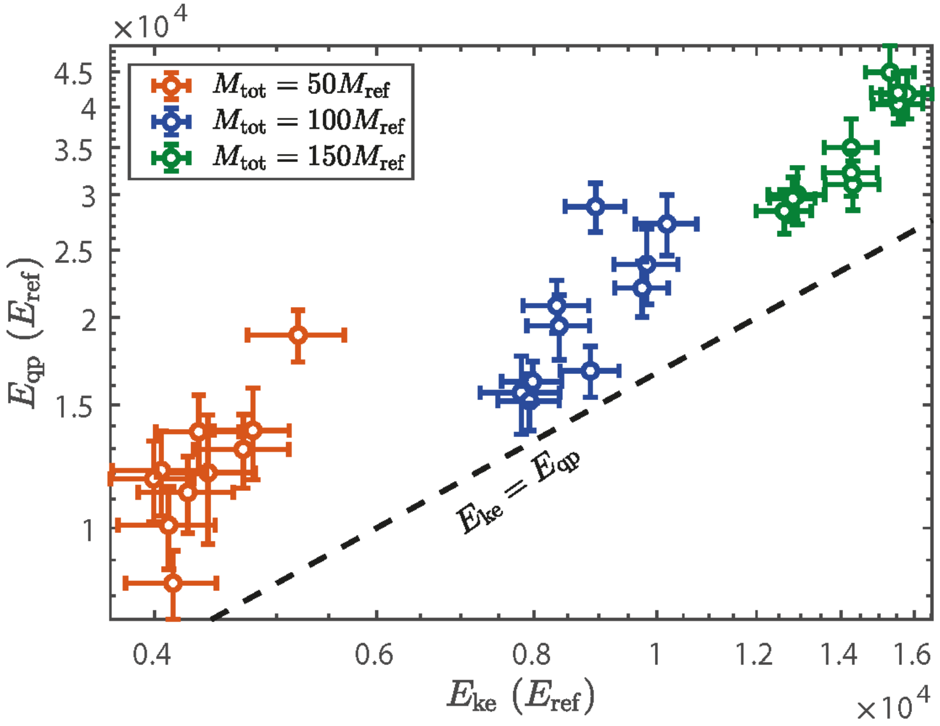

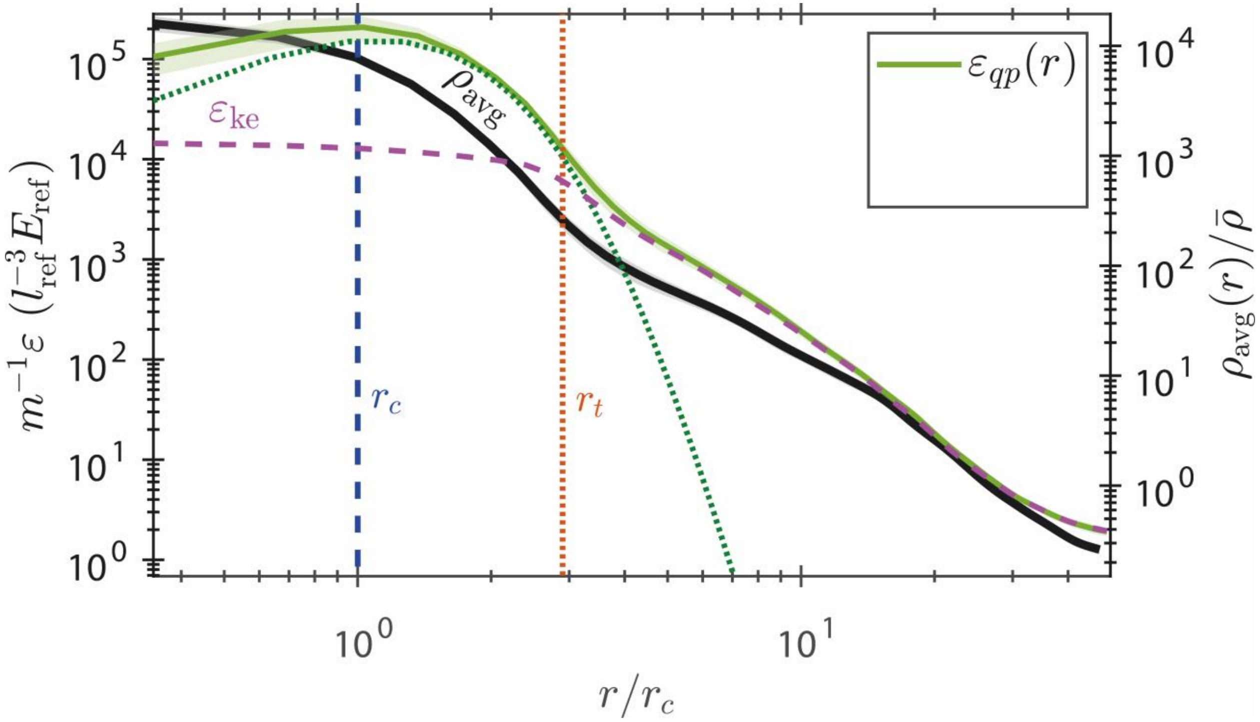

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$$\varepsilon_{\mathrm{ke}}(\mathbf{r}) = \frac{1}{2}\rho(\mathbf{r})|\mathbf{v}(\mathbf{r})|^2 = \frac{1}{2}|\mathbf{F}(\mathbf{r})|^2$$

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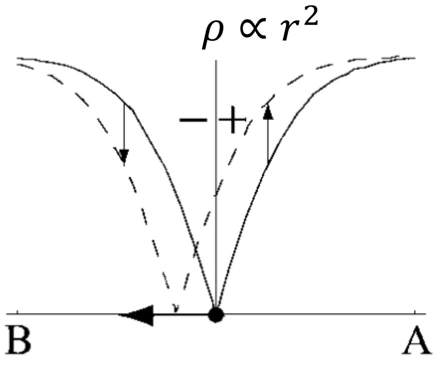
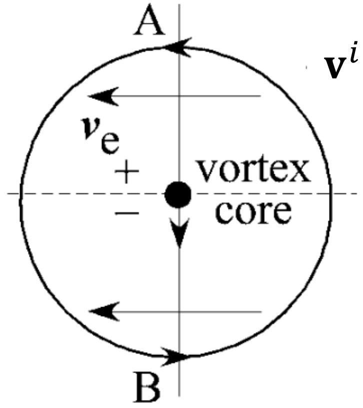
$$\varepsilon_{\mathrm{ke}}(\mathbf{r}) = \frac{1}{2}\left|\mathbf{F}^c(\mathbf{r}) + F^i(\mathbf{r})\right|^2 = \varepsilon_{\mathrm{ke}}^c + \varepsilon_{\mathrm{ke}}^i$$

Rotational/incompressible: $\nabla\cdot\mathbf{F}^i(\mathbf{r}) = \nabla\cdot\mathbf{v}^i(\mathbf{r}) = 0$

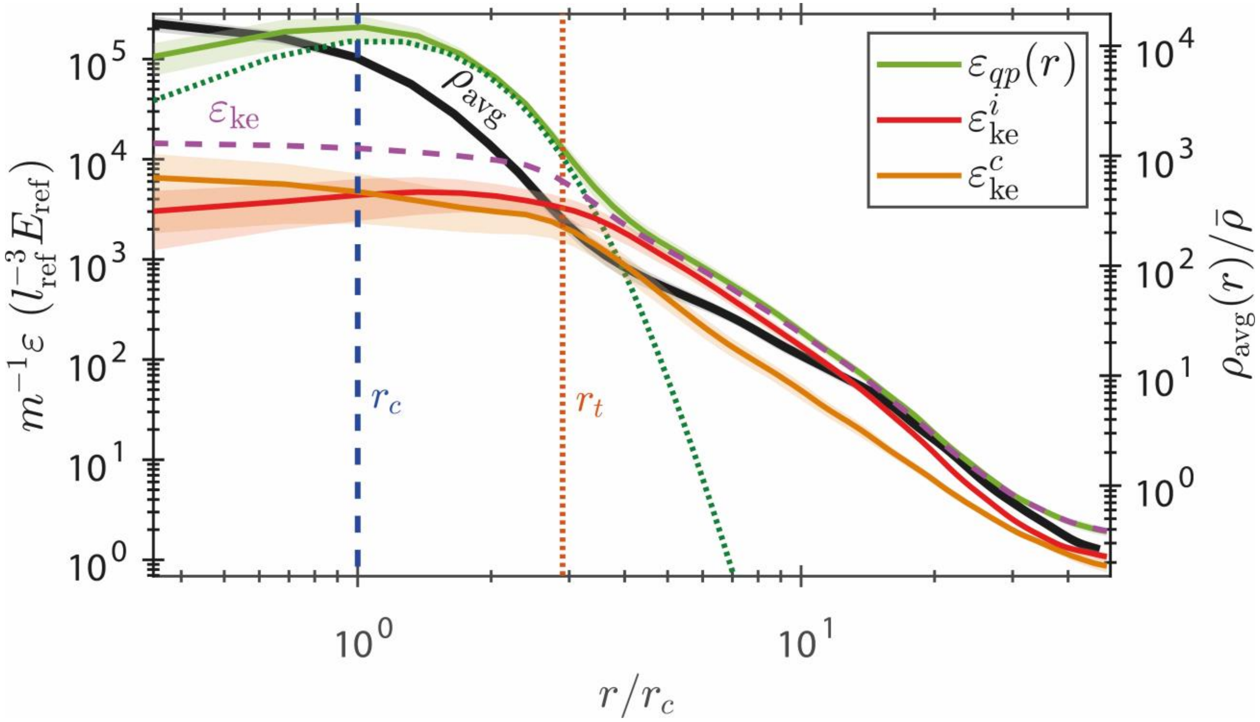
Irrotational/compressible: $\nabla\times\mathbf{F}^c(\mathbf{r}) = \nabla\times\mathbf{v}^c(\mathbf{r}) = 0$

Kobayashi & Tsubota., J. Phys. Soc. Jpn 74, 3248 (2005)





$\mathbf{v}^i = \nabla\varphi\hat{\boldsymbol{\phi}}$



Liu, Proukakis, Rigopoulos, 2207/08.XXXX

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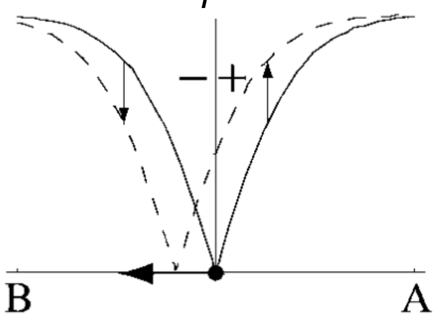
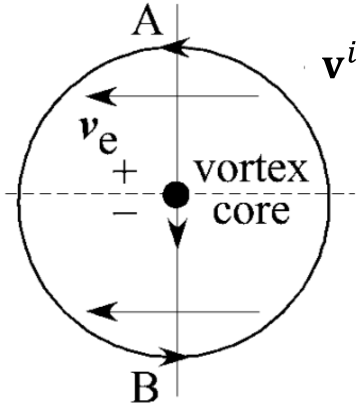
Kobayashi & Tsubota., J. Phys. Soc. Jpn 74, 3248 (2005)

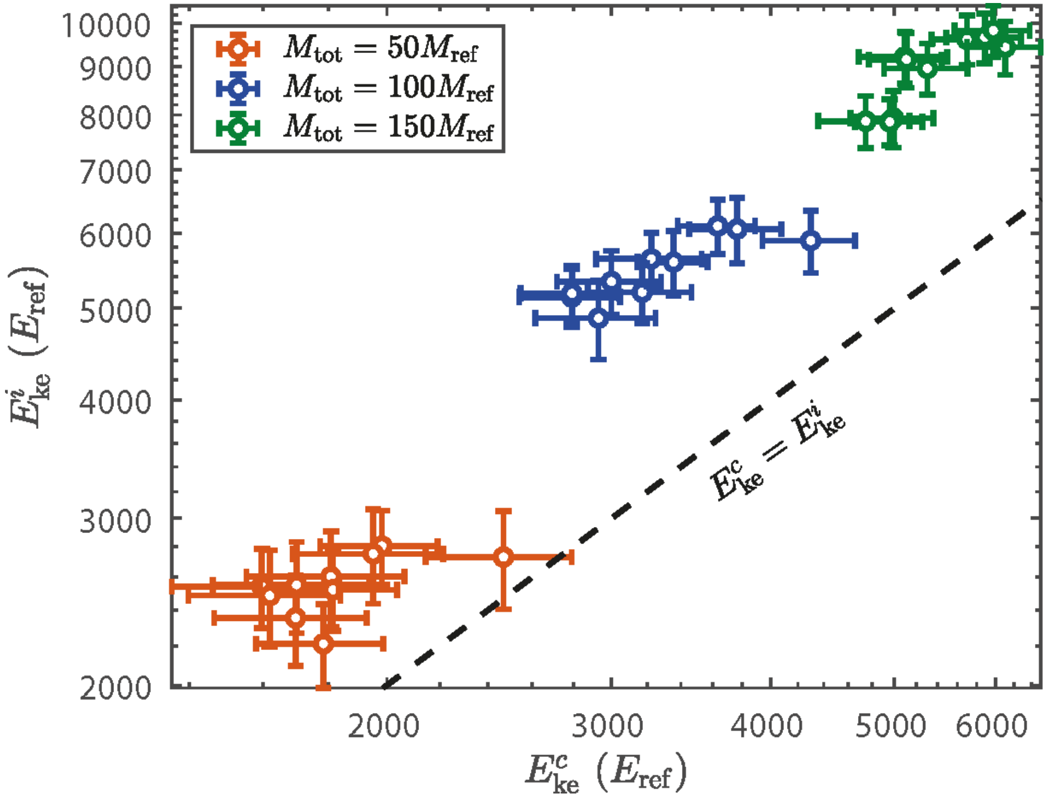
$\rho \propto r^2$





$$\mathbf{v}^i = \nabla\varphi\hat{\boldsymbol{\phi}}$$

$$\left|\mathbf{v}^i\right| = \frac{1}{\mathrm{r}}$$



Liu, Proukakis, Rigopoulos, 2207/08.XXXX

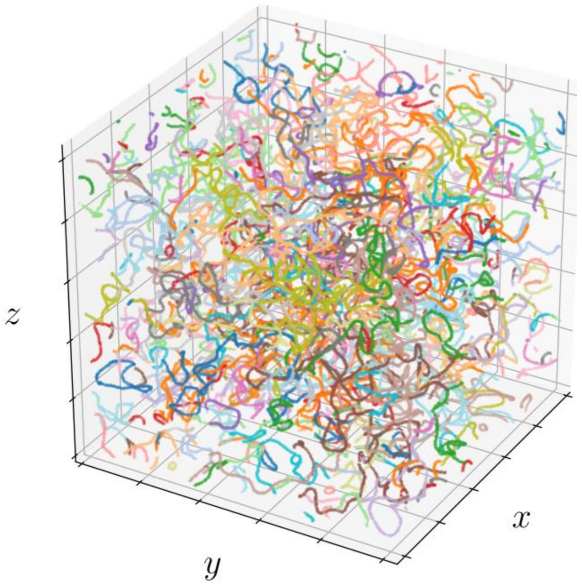
Vortices in FDM halos and granule size

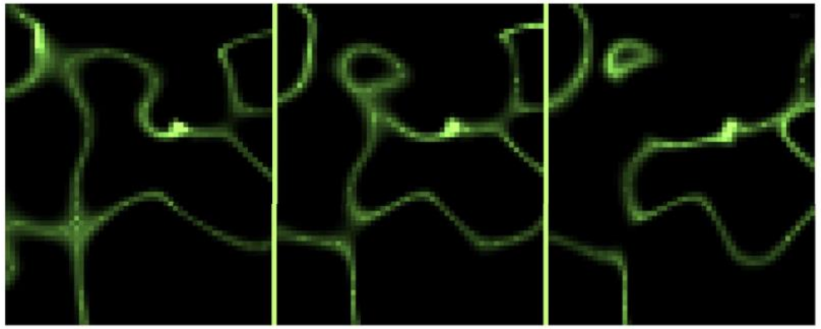
Vortical structure visualization

$$\left|\mathbf{v}^i(\mathbf{r})\right|^2 \approx \frac{\hbar^2}{m^2\Delta x^2} \approx 800\,\frac{l_{\mathrm{ref}}^2}{\tau_{\mathrm{ref}}^2}$$

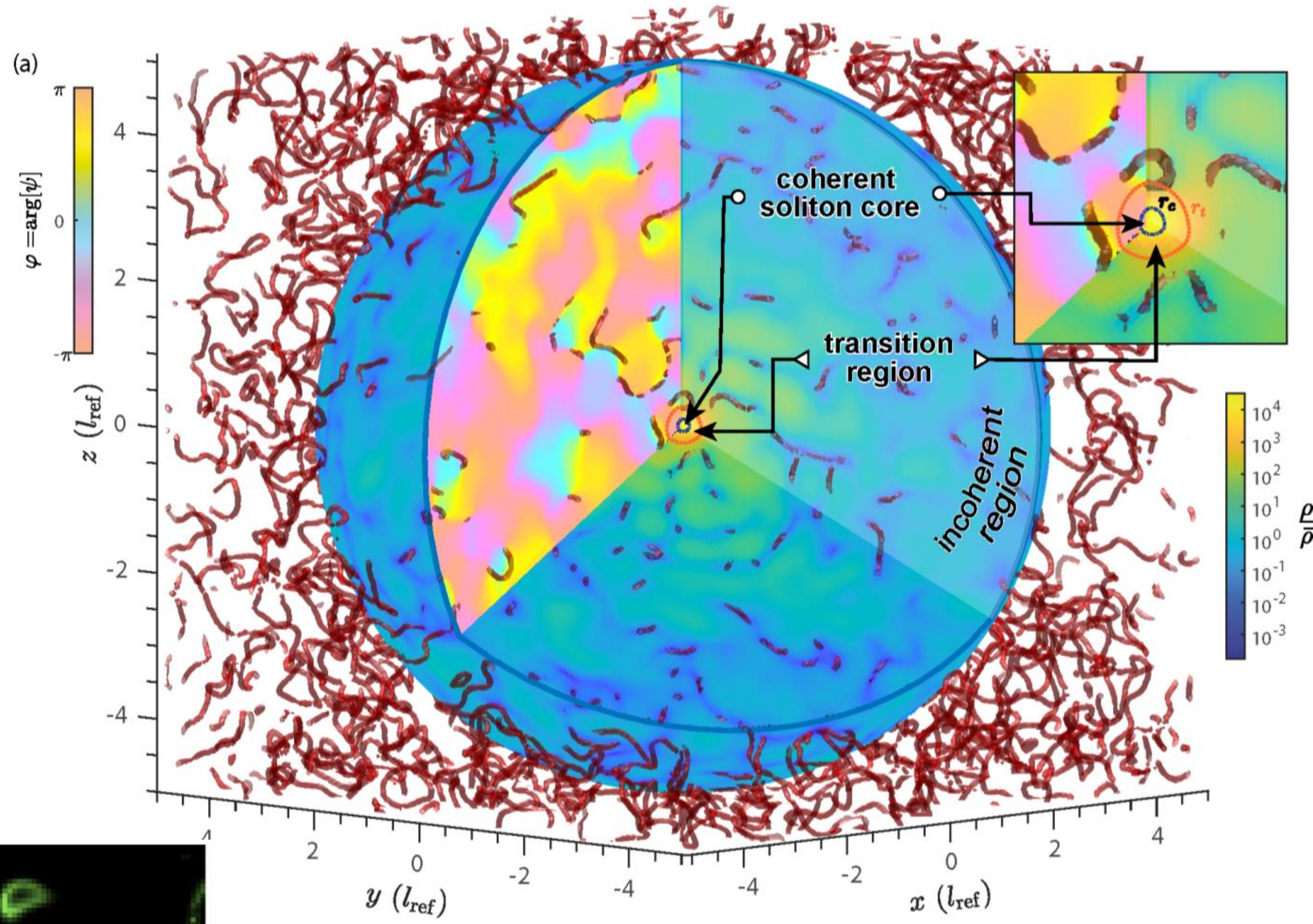
- Vortices are in closed loops.
- They are stretching, shrinking, reconnecting and tangling together

L. Hui et al., 2004.01188, JCOPAL (2021)





Mocz et al., MNRAS 471, 4559 (2017)



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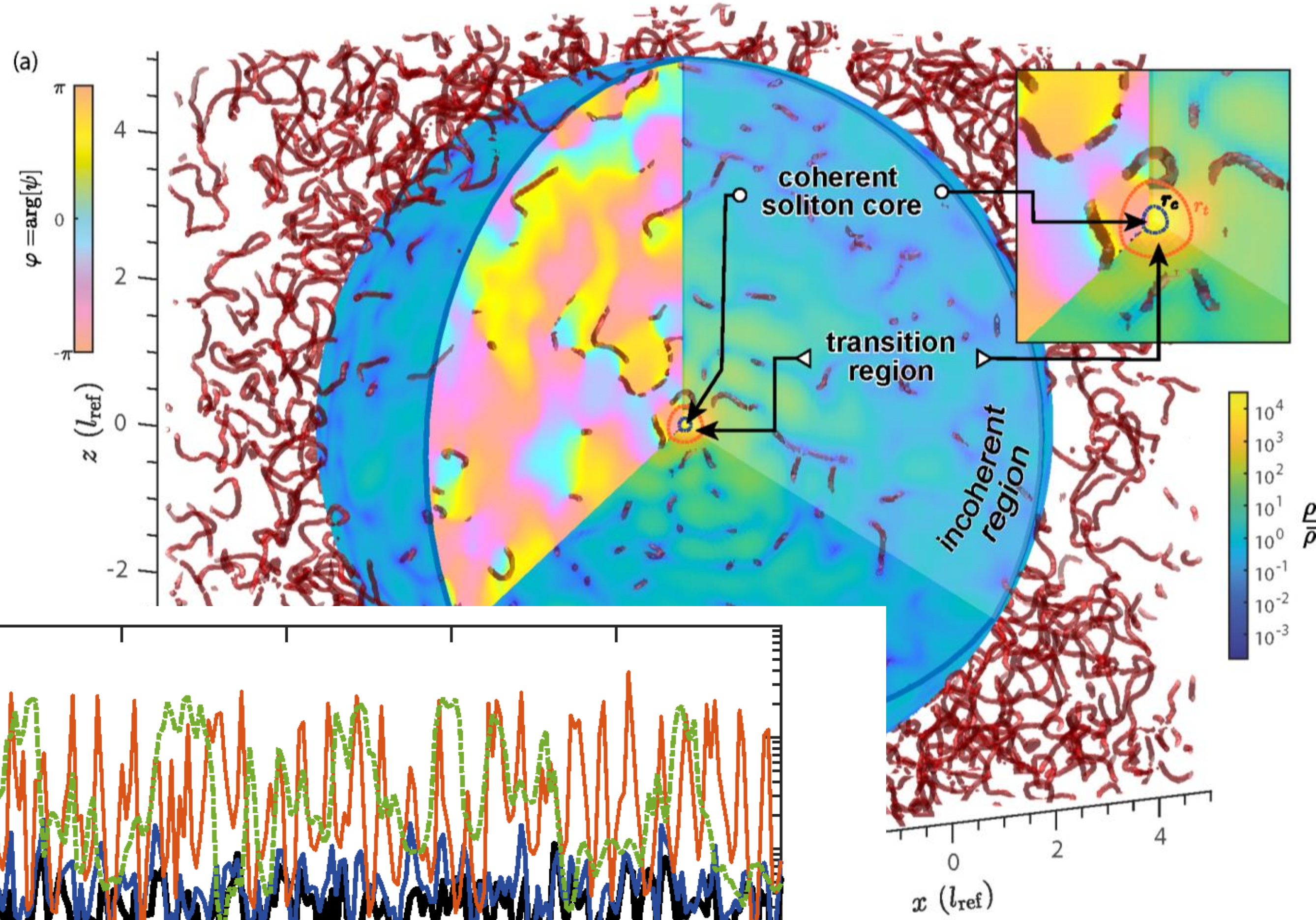

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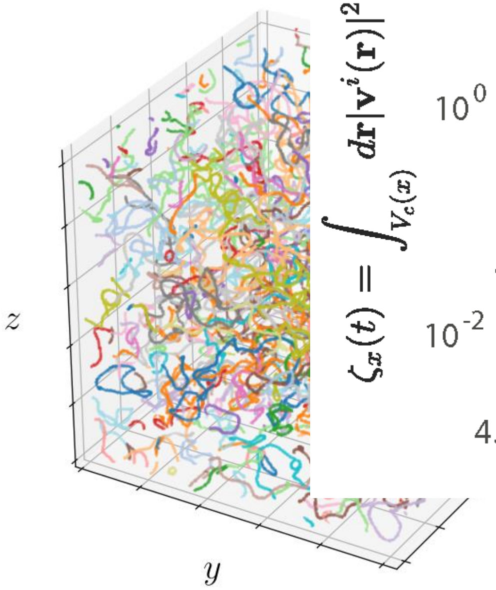

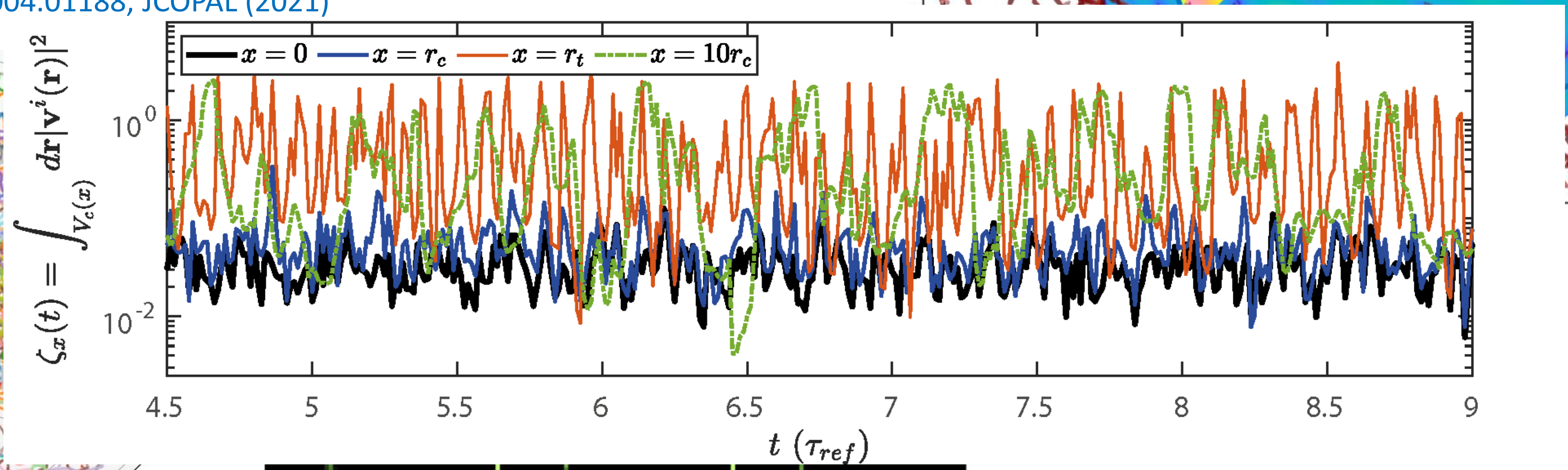

Mocz et al., MNRAS 471, 4559 (2017)

Liu, Proukakis, Rigopoulos, in preparation

Vortices in FDM halos and granule size

Vortex energy and granule power spectra

Vortex (Incompressible) energy spectrum

$$\tilde{\varepsilon}^{i}_{\mathrm{ke}}(k_r) \equiv \int d\Omega_k\, k^2 \varepsilon^{i}_{\mathrm{ke}}(\mathbf{r}, t)$$

Nore et al., Phys. of Fluids 9, 2644 (1997)
Stagg et al., PRA 94, 053632 (2016)

Quasi-classical turbulence, $k^{-5/3}$: Not seen

Mocz et al., MNRAS 471, 4559 (2017)

Vortex core structure, k^{-3}: $\rho\left|\mathbf{v}^i\right|^2 \approx$ Const. in large k

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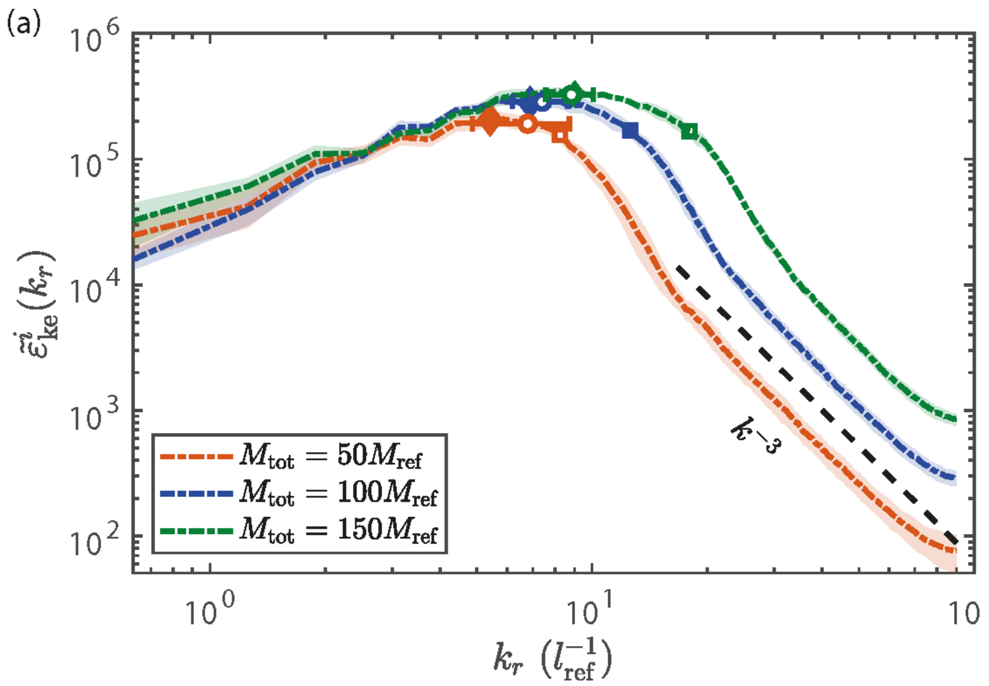
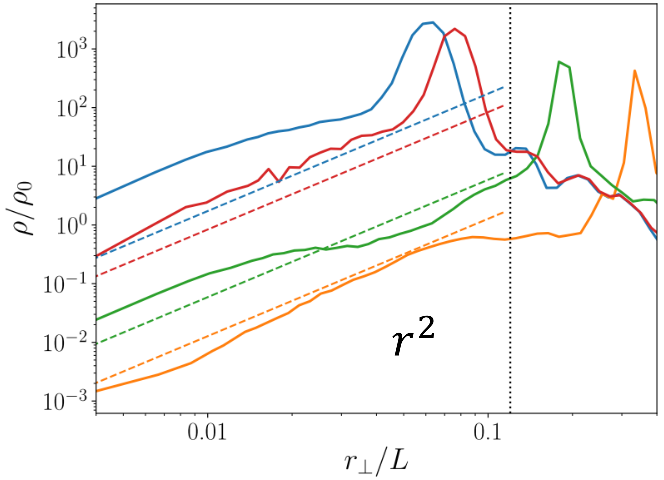
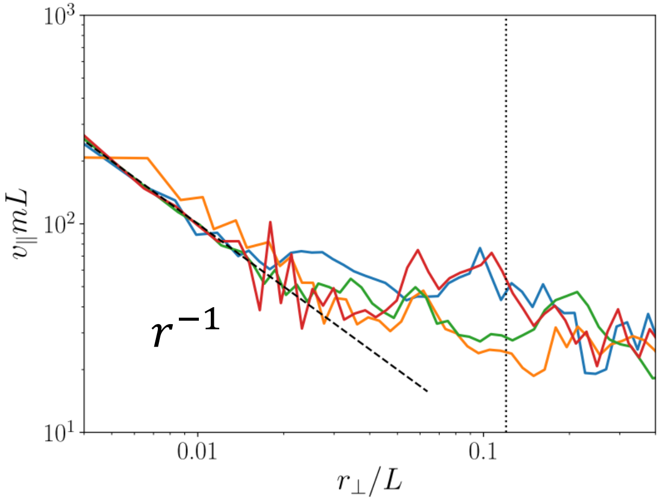
T. Chiueh et al., J. Phys. B 44, 115101 (2011)







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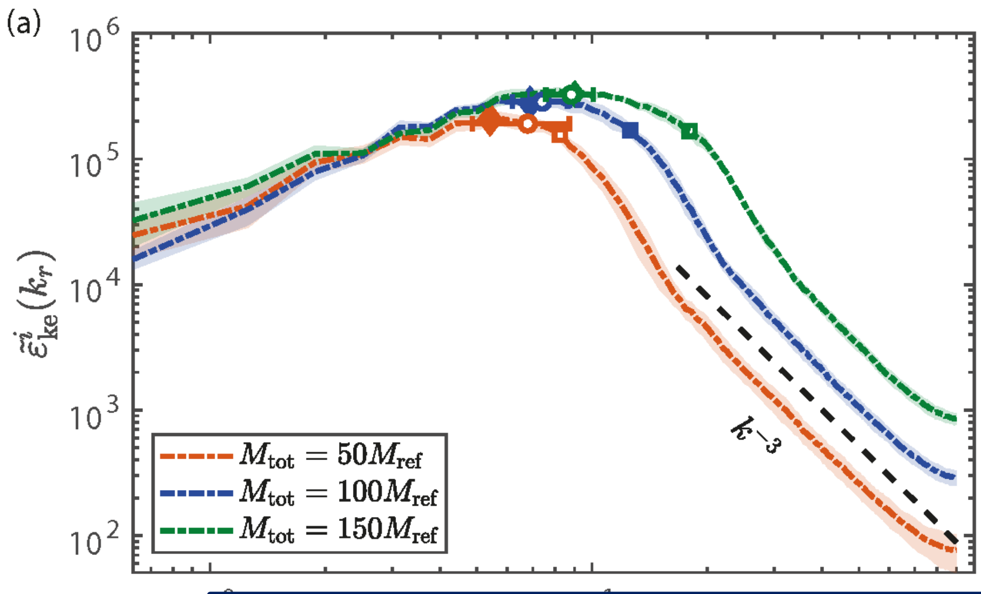

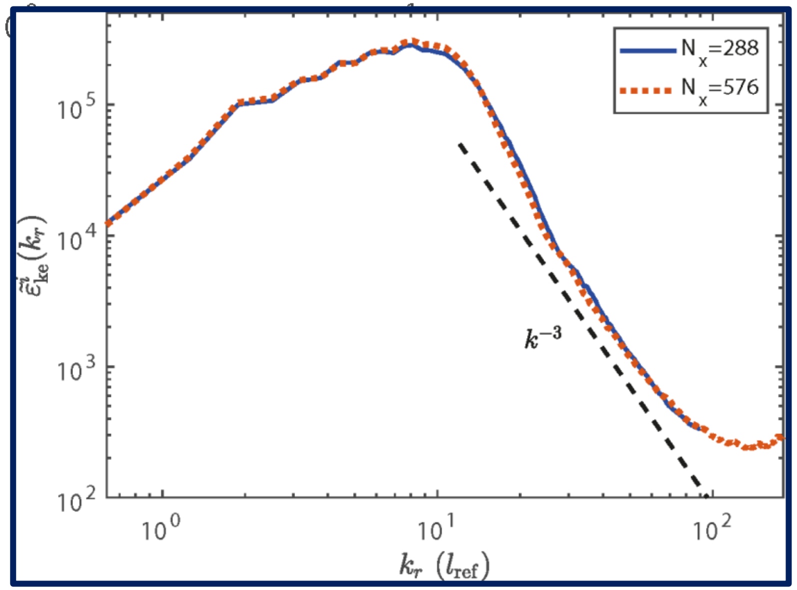

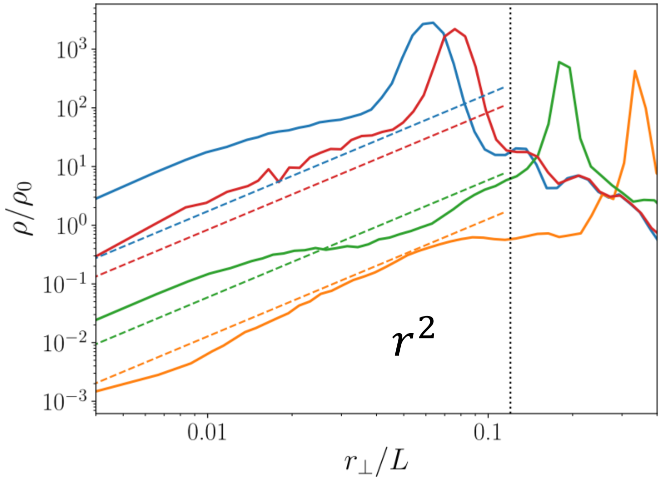

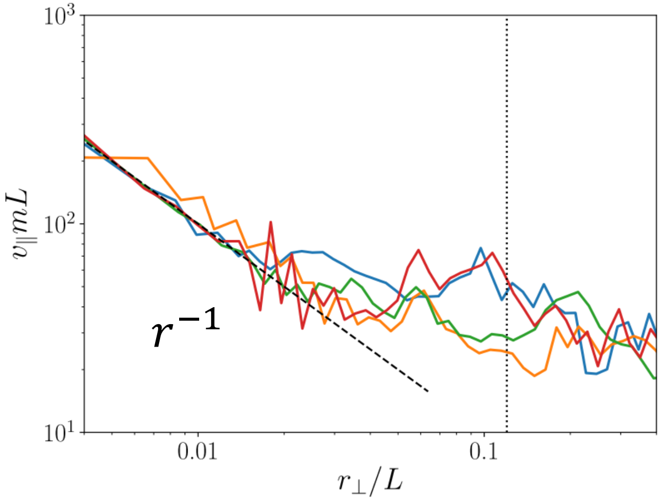

Liu, Proukakis, Rigopoulos, 2207/08.XXXX

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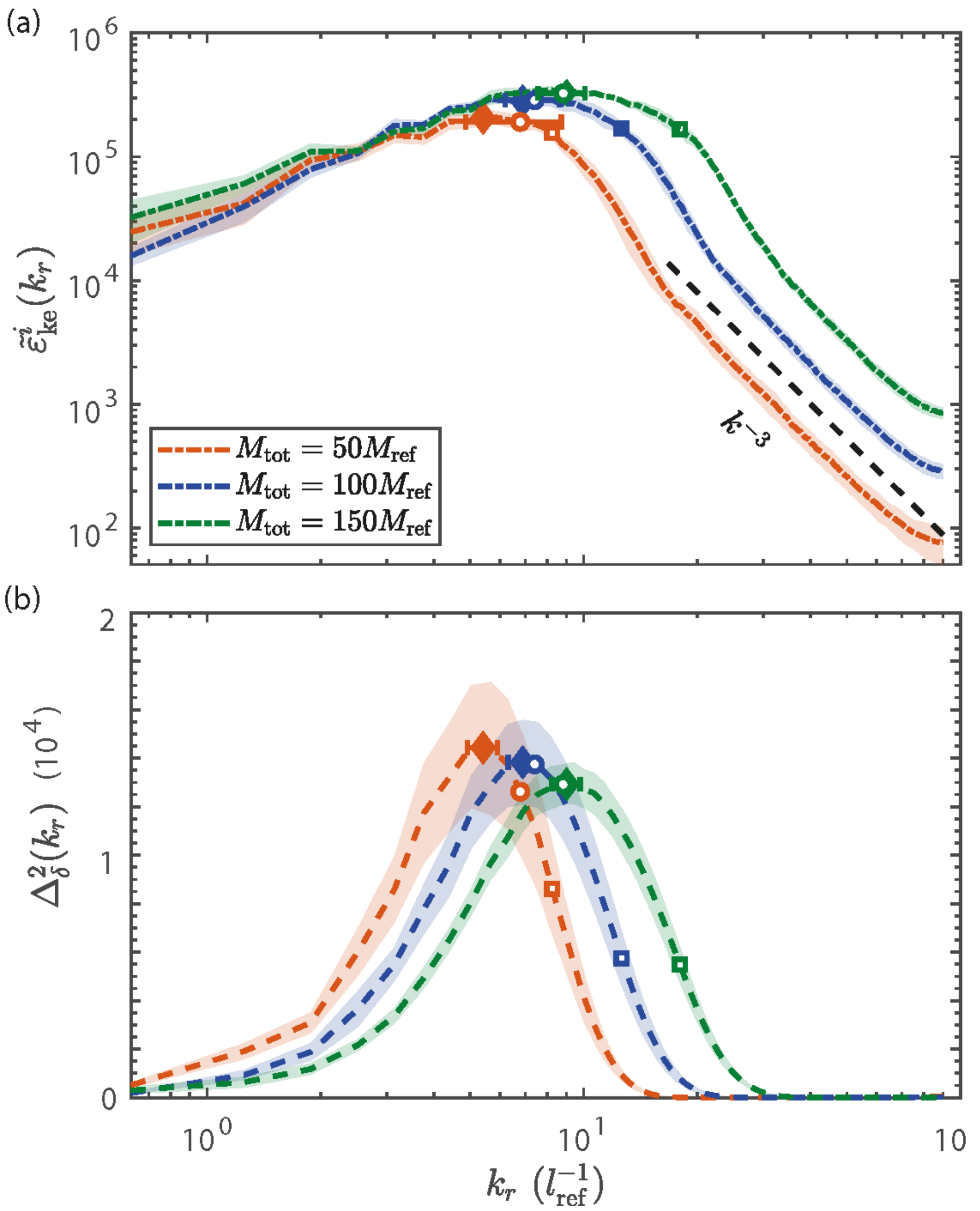
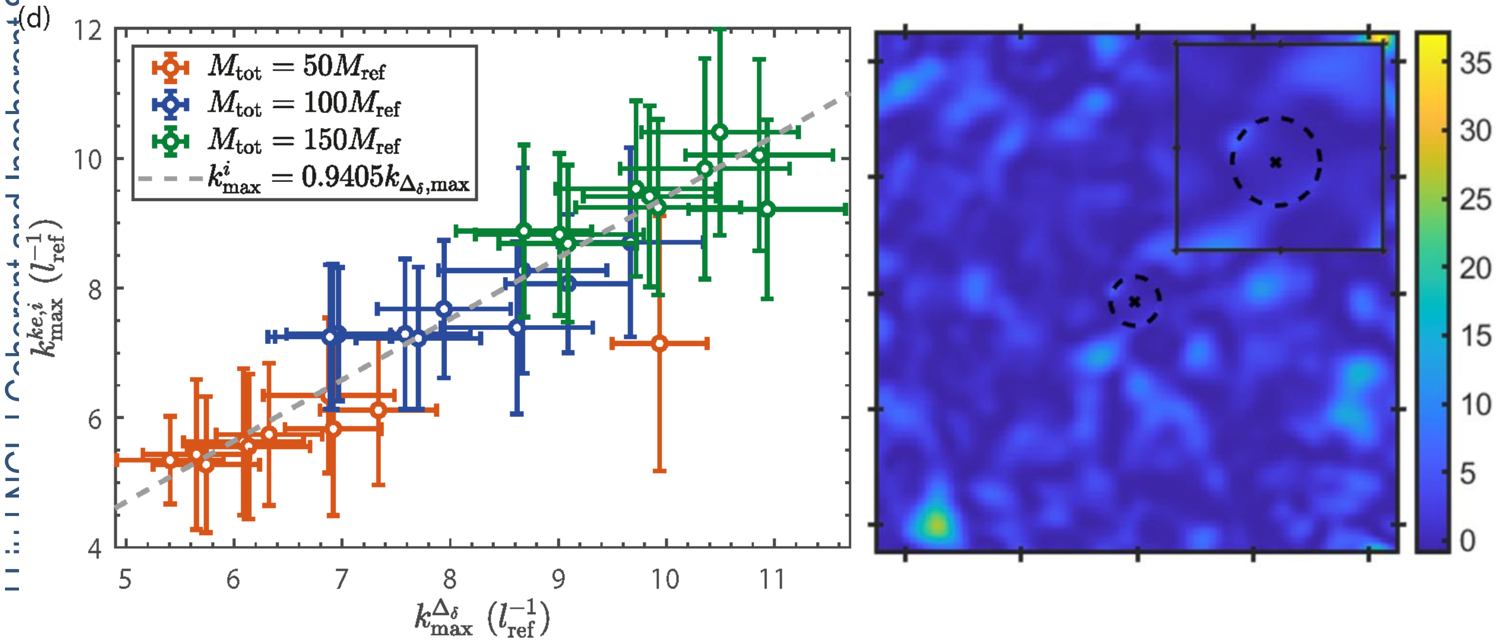
Granule power spectrum

$$\delta(\mathbf{r}, t) = \frac{\rho(\mathbf{r}, t) - \bar{\bar{\rho}}(\mathbf{r})}{\bar{\bar{\rho}}(\mathbf{r})}$$

Chan et al., MNRAS 478, 2686 (2018)
Lin et al. PRD 97, 103523 (2018)
Dutta Chowdhury et al. (2021)





Liu, Proukakis, Rigopoulos, 2207/08.XXXX

IDM, Vienna, 19.07.2022

Thanks for your attention

arXiv:2207/08.XXXXX

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Newcastle University

G. Rigopoulos

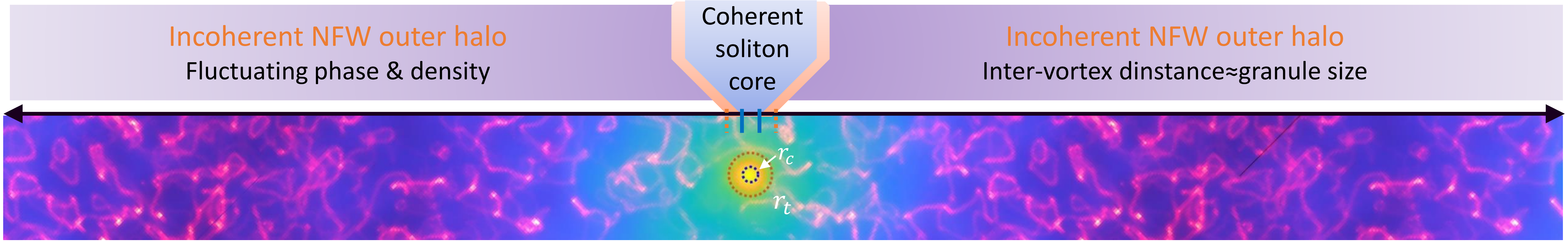
N. P. Proukakis

Milos Indjin [P3D, El10, 17:30, 21 July]
Luca Galantuchi, Alex Soto, Cora Uhlemann,
Alex Gough, Joachim Harnois-Deraps, etc

Fruitful discussions: H.-Y. Schive, T.-Z. Chiuech at NTU

Conclusion

- A FDM halo contains fully coherent soliton core in its centre.



- The radial halo profile can be fairly caught by the cored-halo fit with the parameters r_c and r_t.
- The soliton core oscillation can be reflected in the peak momentum of the power spectrum.
- Decomposing to the velocity fields shows looped vortical structures in the outer halo region;
- The vortex energy spectrum shows the FDM vortex core contains structure , $\rho \propto r^2$ and $v^i \propto r^{-1}$.
- The inter-vortex distances and the granule length scale are comparable,

Soliton core oscillation: Milos Indjin [P3D, El10, 17:30, 21 July]