The Sensitivity of Spin-Precession Axion Experiments

arXiv: 22XX.XXXXX
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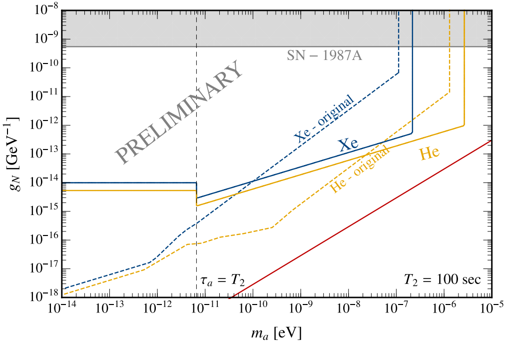
QUANTUM UNIVERSE

Overview

- Consider Nuclear Magnetic Resonance (NMR) detection of axion-like particle dark matter
- Revisit the calculation and find previously overlooked effects
- Updated projections for the CASPEr experiment: improvement at high masses, reduction at low masses. Analysis applies to CASPEr electric too, but discuss only CASPEr wind



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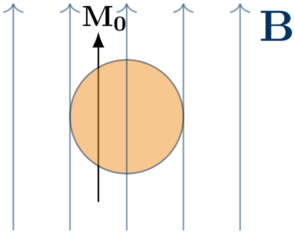
Axion field acts as a pseudo-magnetic field
DM can be detected by NMR experiments!

[Graham, Rajendran,'13]
[Budker,Graham, Ledbetter,Rajendran,'14]

Nuclear Magnetic Resonance Basics



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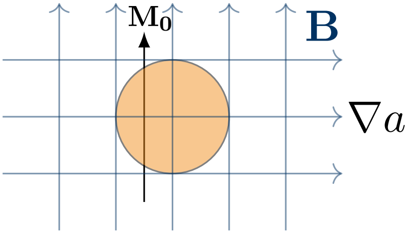

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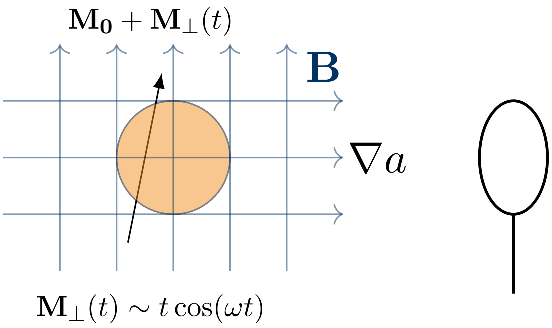

Axion-Induced NMR: Bloch Equations

> One can start from Schrödinger equation for spins to find magnetization....or use the Bloch Equations:

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B} - \frac{M_x \hat{\mathbf{x}} + M_y \hat{\mathbf{y}}}{T_2} - \frac{(M_z - M_0)\hat{\mathbf{z}}}{T_1}$$

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> Perturbative expansion in g_n & decouple the equations

$$\ddot{M}_x + \frac{2}{T_2}\dot{M}_x + \omega_0^2 M_x = 2 g_N M_0 \omega_0 (\hat{x} \cdot \nabla a) - 2 g_N M_0 \frac{d}{dt}(\hat{y} \cdot \nabla a)$$

Axion-Induced NMR: Master Equation

- Take Discrete Fourier Transform of Bloch equation:

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The Cosmological Factor encodes information on the axion model

Axion DM Models

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> Jumping Phase Model

$$\nabla a(t) = a_0 \vec{k}(t) \cos(\omega_a t + \phi(t))$$

wavenumber $\vec{k}$ & phase ϕ sampled every coherence time $\tau_a \sim Q_a/m_a$

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- If the total sampling time $T << \tau_a$, both models collapse to

$$\nabla a(t) \sim \vec{k} a_0 \cos(\omega_a t + \phi)$$

Axion DM Models & Master Equation

$$S_x(\omega) = \begin{cases} \frac{\rho_{DM} g_N^2 M_0^2 T_2^2}{m_a} \delta_{k,k_s} & \text{when } \tau_a < T_2 \\ \frac{\rho_{DM} g_N^2 M_0^2 v_{DM}^2 T_2^3}{12} \delta_{k,k_s} & \text{when } T_2 < \tau_a \end{cases}$$

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In both regimes, the PSD is sharply peaked and power is primarily in a single bin

This is in contrast to previous work, where signal was assumed to be in multiple bins

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$$\lambda_{SP}(\omega) = \frac{\gamma^2 n}{4\pi V T_2} \frac{1}{T_2^{-2} + (\omega_0 - \omega)^2}$$

[Braun, König,'06]
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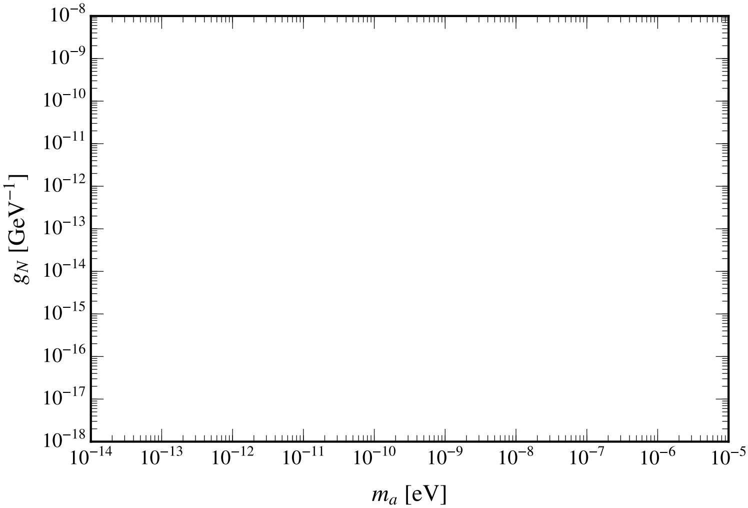
> These are fed into a likelihood framework & 95% limits are derived via the Asimov dataset

$$\tilde{\Theta} = 2 \sum_{k=1}^{N-1} \left[\left(1 - \frac{\lambda_B^k}{\lambda_S \delta_{k,k_s} + \lambda_B^k} \right) - \ln \left(1 + \frac{\lambda_S \delta_{k,k_s}}{\lambda_B^k} \right) \right]$$

Updated Projections



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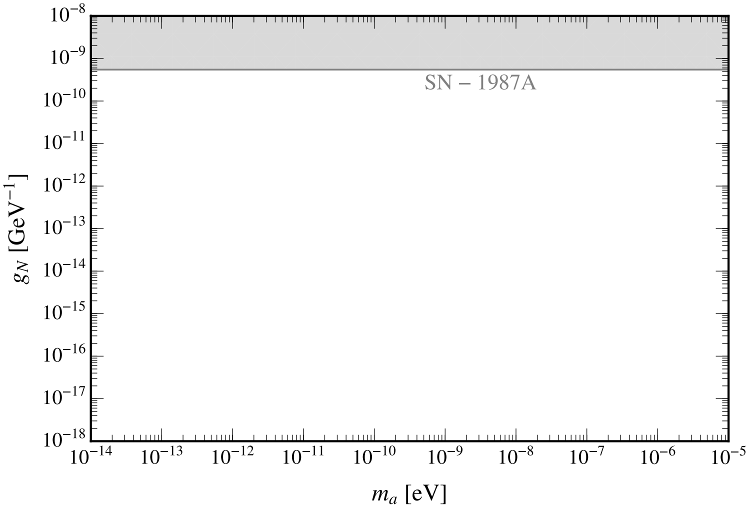

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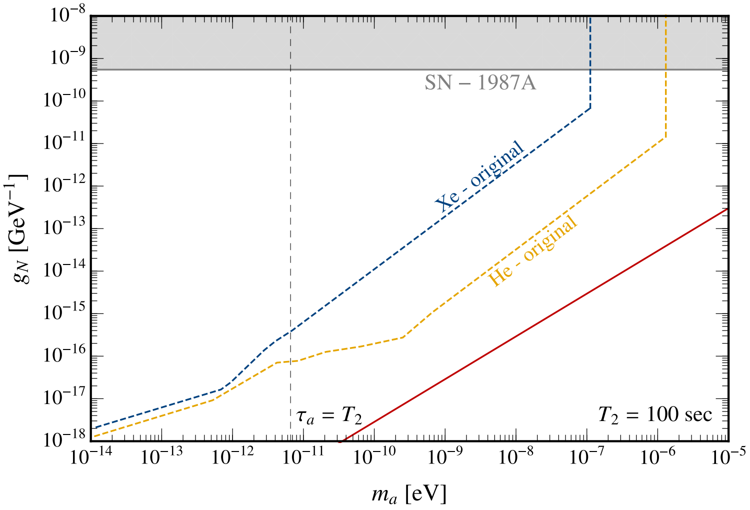

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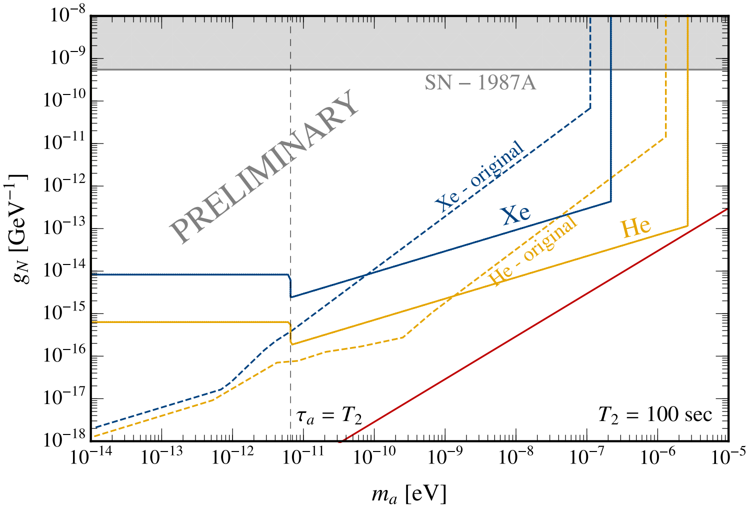

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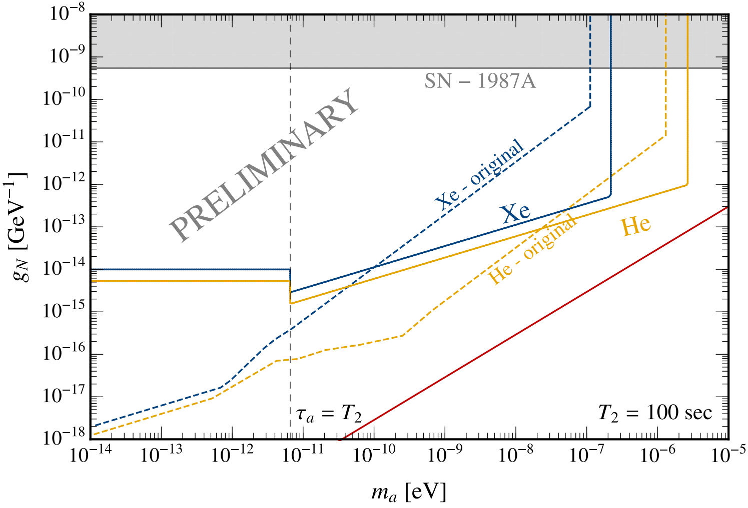

Thank you!

Contact

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