Level 2 in SCETlib.

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Factorization at Small q_T

At leading order in q_T/Q (leading power), q_T spectrum factorizes into hard, collinear, and soft contributions (here: $x_{a,b} \equiv (Q/E_{\rm cm})e^{\pm Y}$)

$$\frac{d\sigma^{(0)}}{dQ\,dY\,dq_T^2} = \sum_{a,b} H_{ab}(Q^2,\mu)[B_a B_b S](Q^2, x_a, x_b, \vec{q}_T, \mu)\Big[1 + \mathcal{O}\Big(\frac{q_T^2}{Q^2}, \frac{\Lambda_{\rm QCD}^2}{Q^2}\Big)\Big]$$

$$[B_a B_b S] = \int d^2\vec{k}_a\, d^2\vec{k}_b\, d^2\vec{k}_s\, \delta^{(2)}(\vec{q}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s) \times B_a(x_a, \vec{k}_a, \mu, \nu/Q)\, B_b(x_b, \vec{k}_b, \mu, \nu/Q)\, S(\vec{k}_s, \mu, \nu)$$

- Most general form with no hard-coded choices yet

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$$\begin{aligned}[B_a B_b S] &= \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s\, \delta^{(2)}(\vec{q}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s) \\ &\quad \times B_a(x_a, \vec{k}_a, \mu, \nu/Q)\, B_b(x_b, \vec{k}_b, \mu, \nu/Q)\, S(\vec{k}_s, \mu, \nu) \\ &\equiv \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{q}_T}\, \tilde{B}_a(x_a, b_T, \mu, \nu/Q)\, \tilde{B}_b(x_b, b_T, \mu, \nu/Q)\, \tilde{S}(b_T, \mu, \nu) \\ &\equiv \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{q}_T}\, \tilde{f}_a(x_a, b_T, \mu, \zeta_a)\, \tilde{f}_b(x_b, b_T, \mu, \zeta_b)\end{aligned}$$

(where $\zeta_{a,b} \propto \omega_{a,b}^2$ with $\zeta_a\zeta_b = Q^4$ plays the role of ν)

- Most general form with no hard-coded choices yet
  - All 3 forms are still *completely* equivalent

Schematic Resummation Structure.

$$\mathrm{d}\sigma^{(0)} = H(Q,\mu) \times B(p_T,\mu,\nu/Q)^2 \otimes S(p_T,\mu,\nu/p_T)$$

$$\ln^2\frac{p_T}{Q} = 2\ln^2\frac{Q}{\mu} + 2\ln\frac{p_T}{\mu}\ln\frac{\nu}{Q} + \ln\frac{p_T}{\mu}\ln\frac{\mu\, p_T}{\nu^2}$$

- All-order logarithmic structure is encoded in μ, ν dependence
  - μ, ν dependence *exactly* cancels *at each order*
  - This is how differential equations (RGEs, Collins-Soper eq.) that govern μ, ν dependence are derived

- Resummation follows from solving RGEs, and evolving each function from some starting scales μ_i, ν_i to common (but arbitrary) μ, ν

$$H(\mu) = H(\mu_H) \times U_H(\mu_H, \mu)$$
$$B(\mu,\nu) = B(\mu_B,\nu_B) \otimes U_B(\mu_B,\nu_B;\mu,\nu)$$
$$S(\mu,\nu) = S(\mu_S,\nu_S) \otimes U_B(\mu_S,\nu_S;\mu,\nu)$$

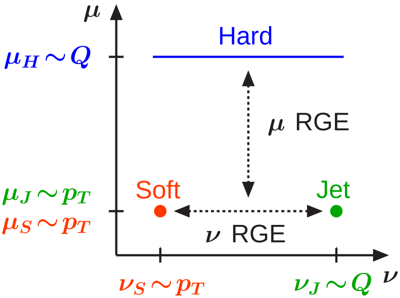
  - Arbitrary μ, ν still cancel *exactly* = RGE consistency (path independence)

Schematic Resummation Structure.

$$\mathrm{d}\sigma^{(0)} = H(\mu_H) \times U_H(\mu_H, \mu) \times [B(\mu_B, \nu_B) \otimes U_B(\mu_B, \nu_B; \mu, \nu)]^2 \otimes S(\mu_S, \nu_S) \otimes U_B(\mu_S, \nu_S; \mu, \nu)$$

- *Boundary conditions* $H(\mu_H)$, $B(\mu_B, \nu_B)$, $S(\mu_S, \nu_S)$ can (must) be calcluated in (log-free) fixed order, so at

$$\mu_H \sim Q$$
$$\mu_B \sim p_T\,, \qquad \nu_B \sim Q$$
$$\mu_S \sim p_T\,, \qquad \nu_S \sim p_T$$



- RGE really sums logs of ratios

$$\ln\frac{\mu_B}{\mu_H} \sim \ln\frac{\mu_S}{\mu_H} \sim \ln\frac{\nu_S}{\nu_B} \sim \ln\frac{p_T}{Q}$$

- Choice of boundary scales does matter
  - Determine precise form of resummed logarithms (“resummation” scales)
  - Their dependence only cancels *to the order* boundary conditions are calculated (in $\alpha_s(\mu_i)$, so μ_i are also “renormalization” scales)

Complete RGE System.

In virtuality scale μ

$$\mu \frac{\mathrm{d}H(Q,\mu)}{\mathrm{d}\mu} = \gamma_H(Q,\mu)\, H(Q,\mu)$$

$$\mu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_B(\mu,\nu)\, B(\vec{p}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_S(\mu,\nu)\, S(\vec{p}_T,\mu,\nu)$$

and rapidity scale ν

$$\nu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = -\frac{1}{2}\int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, B(\vec{p}_T - \vec{k}_T,\mu,\nu)$$

$$\nu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = \int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, S(\vec{p}_T - \vec{k}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\gamma_\nu(\vec{k}_T,\mu) = \nu \frac{\mathrm{d}}{\mathrm{d}\nu}\gamma_S(\mu,\nu)\delta(\vec{k}_T) = -4\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)]\delta(\vec{k}_T)$$

- plus evolution equations for $\alpha_s(\mu)$ and PDFs(μ)
- plus consistency relations between different anomalous dimensions γ_i which encode RGE consistency

Limiting scale choices.

- Solving RGE system for q_T distribution is (surprisingly) difficult
  - Exact distributional solution in q_T space is equivalent (up to different boundary terms) to solving RGE in b_T space with *canonical* b_T *scales* $(b_0 = 2e^{-\gamma_E})$

$$\mu_H = Q\,, \quad \mu_B = b_0/b_T\,, \quad \nu_B = Q\,, \quad \mu_S = \mu_\nu = \nu_S = b_0/b_T$$

  - Quite nontrivial statement, proven in [Ebert, FT; 1611.08610]
  - This is level 1

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- Where is the usual μ_R "renormalization" scale? Fixed-order limit:

$$\mu_H \equiv \mu_B \equiv \mu_S \equiv \mu_R\,, \qquad \nu_B \equiv \nu_S$$

  - Exactly turns off resummation and gives back FO result (at leading power) $\mathrm{d}\sigma^{(0)} = H(\mu_R) B(\mu_R)^2 S(\mu_R) = \mathrm{d}\sigma^{\mathrm{FO}(0)}(\mu_R)$
  - This is what needs to happen for $q_T \sim Q$ (or more precisely when power expansion in q_T/Q is no longer justified)
  - This is what we add in level 2 (turning off resummation)

- What about μ_F (PDF "factorization" scale)? I've ignored it for simplicity $B_i(x_a, \mu_B) = f_i(x_a, \mu_B) \otimes [1 + \mathcal{O}(\alpha_s(\mu_B))]$, so $\mu_F \equiv \mu_B \to \mu_R$ for $q_T \sim Q$

Profile Scales.

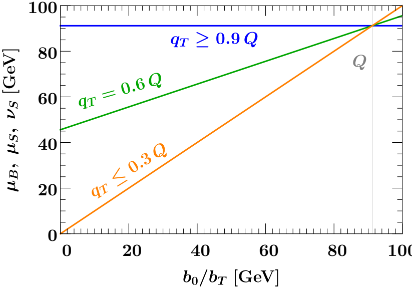
- Everything determined (only) by μ_i, ν_i choices: Use *profile scales*
  [Lustermans, Michel, FT, Waalewijn, 1901.03331]

$$\mu_H = \nu_B = \mu_{\rm FO} = Q$$

$$\mu_B, \mu_S, \nu_S \equiv \mu_{\rm prof}(q_T, b_T) = \mu_{\rm FO}\, f_{\rm prof}\Big(\frac{q_T}{Q}, \frac{b_0}{b_T Q}\Big) \begin{cases} = b_0/b_T & q_T \ll Q \\ \to \mu_{\rm FO} & q_T \to Q \end{cases}$$

  - Key point: Resummation turn off for $q_T \to Q$ *does not* alter correct (canonical) resummation at $q_T \ll Q$
  - Plus nonpert. cutoff prescription for $b_0/b_T \lesssim 1\,\mathrm{GeV}$ (as at level 1) (freeze-out, local b^*, global b^*)

- Canonical (res. on) $\to$ FO (res. off)
  - Transition driven by q_T/Q
    (b_T is just means to an end, we want to predict physical q_T spectrum not the b_T spectrum)
  - Transition points are chosen based on relative size of leading-power vs. nonsingular (power) corrections



Uncertainties via Scale Variations.

Assuming (pretending) that scale variations make some degree of sense

$$\boldsymbol{\Delta}_{\mathbf{total}} = \sqrt{\boldsymbol{\Delta}^2_{\mathbf{resum}} + \boldsymbol{\Delta}^2_{\mathbf{FO}} + \boldsymbol{\Delta}^2_{\mathbf{match}} + \boldsymbol{\Delta}^2_{\boldsymbol{\Lambda}}}$$

- “Resummation” $\boldsymbol{\Delta}_{\mathbf{resum}}$: Max envelope of profile scale variations
  - 36 variations: chosen such that all possible scale ratios get probed and changed by factor 2 (but not 4) for $q_T \to 0$
- ”Fixed-order” $\boldsymbol{\Delta}_{\mathbf{FO}}$: Max envelope of $\mu_{\mathbf{FO}}$ by fator of 2
  - Keeps all resummed scale ratios invariant
- “Matching” $\boldsymbol{\Delta}_{\mathbf{match}}$: Max envelope of varying transition points
  - 4 variations: Start and end of transition up and down
- “Nonpert. cutoff” $\boldsymbol{\Delta}_{\boldsymbol{\Lambda}}$: Max envelope of cutoff variation
  - Rough guesstimate to cover missing nonpert. (would cancel cutoff dep.)
- Rationale/interpretation:
  - Think of each as a (somewhat) independent “source” $\to$ add in quadrature
  - Within each: Arbitrary knobs all probing the same thing $\to$ take envelope

Matching to Fixed Order.

Matching to fixed order essentially comes for free now (well, for given σ^{FO})

$$\sigma = \underbrace{\sigma^{(0)}(\mu_H, \mu_B, \nu_B, \mu_S, \nu_S)} + \underbrace{\Big[\sigma^{\mathrm{FO}}(\mu_{\mathrm{FO}}) - \sigma^{(0)}(\mu_i, \nu_i \equiv \mu_{\mathrm{FO}})\Big]}$$

$$\equiv \sigma^{\mathrm{resum}}(\mu_H, \mu_B, \nu_B, \mu_S, \nu_S) + \sigma^{\mathrm{nons}}(\mu_{\mathrm{FO}})$$

- σ^{resum} and σ^{nons} are *separately* scale independent
  - They should be because for $q_T \ll Q$ they are *independent* pert. series
  - Using profile scales to steer resummation inside σ^{resum} automatically keeps them cleanly separated, in particular, σ^{nons} is pure fixed order and does not depend on any resummation details
- σ^{nons} is (must be) power-suppressed by q_T/Q for $q_T \ll Q$
  - This requires $\sigma^{(0)}$ and σ^{FO} to have consistent orders, i.e., order of boundary conditions in $\sigma^{(0)}$ must match order of σ^{FO}