Polarization Computation Challanges

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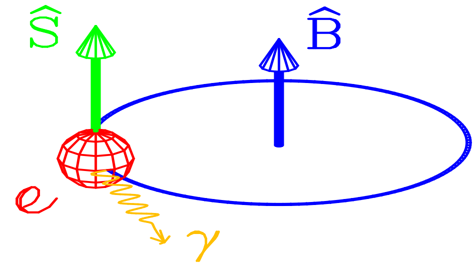
Radiative polarization

Sokolov-Ternov effect (1963): a small amount of the radiation emitted by a $e^{\pm}$ moving in the field is accompanied by *spin flip*.

Slightly different probabilities $\longrightarrow$ *self polarization!*



- Equilibrium polarization

$$\vec{P}_{\mathrm{ST}} = \hat{y} P_{\mathrm{ST}} \qquad |P_{\mathrm{ST}}| = \frac{|n^+ - n^-|}{n^+ + n^-} = \frac{8}{5\sqrt{3}} = 92.4\%$$

e^- polarization is anti-parallel to $\vec{B}$, while e^+ polarization is parallel to $\vec{B}$.

- Build-up rate

$$\tau_{\mathrm{ST}}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \hbar}{m_0} \frac{\gamma^5}{|\rho|^3} \quad \rightarrow \quad \tau_p^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \hbar\, \gamma^5}{m_0 C} \oint \frac{ds}{|\rho|^3} \text{ for an } \textit{actual} \text{ ring}$$

The spin motion is described by the Thomas-BMT equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

In the laboratory frame and MKS units

$$\vec{\Omega}(s) = -\frac{e}{m_0}\left[\left(a + \frac{1}{\gamma}\right)\vec{B} - \frac{a\gamma}{\gamma+1}\vec{\beta}\cdot\vec{B}\vec{\beta} - \left(a + \frac{1}{\gamma+1}\right)\frac{\vec{\beta}}{c^2} \times \vec{E}\right]$$

with $\vec{\beta} \equiv \vec{v}/c$ and a=0.0011597 ($e^{\pm}$).

On the design closed orbit of a planar ring, there is a periodic solution, $\hat{n}_0(s)=\hat{y}$ everywhere and therefore an $e^{\pm}$ beam may get polarized through Sokolov-Ternov mechanism, and polarization is vertical. If on the design orbit there are not only vertical fields (tilted dipoles, vertically misaligned quads etc) there is still a periodic solution, $\hat{n}_0(s)$. In this case the initially aligned to $\hat{n}_0$ spin of particle will deviate from $\hat{n}_0$ anytime the particle sees a field $\hat{B} \nparallel \hat{n}_0$: stochastic photon emission (almost instantaneous) leads to spin diffusion, Sokolov-Ternov effect being too slow for counteracting.

Derbenev and Kondratenko (1973) included spin diffusion by using a semiclassical approach:

$$\vec{P}_{\mathrm{DK}} = \hat{n}_0 \frac{8}{5\sqrt{3}} \frac{\oint ds < \frac{1}{|\rho|^3} \hat{b} \cdot (\hat{n} - \frac{\partial \hat{n}}{\partial \delta}) >}{\oint ds < \frac{1}{|\rho|^3} \left[1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18}(\frac{\partial \hat{n}}{\partial \delta})^2\right] >} \qquad \hat{b} \equiv \hat{v} \times \dot{\hat{v}}/|\hat{v}|$$

periodic solution to T-BMT eq. on c.o.

randomization of particle spin directions due to photon emission ($\delta \equiv \delta E/E$)

Polarization rate

$$\tau_{\mathrm{DK}}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \gamma^5 \hbar}{m_0 C} \oint ds < \frac{1}{|\rho|^3} \left[1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18}\left(\frac{\partial \hat{n}}{\partial \delta}\right)^2\right] >$$

These formulas involve averaging over ring and *beam distribution*.

There have been some disputes about the meaning of the quantity $\hat{\boldsymbol{n}}$ and $\partial\hat{\boldsymbol{n}}/\partial\boldsymbol{\delta}$ in the original paper.

Many authors (S. Mane, K. Yokoya, D. P. Barber, G. Hoffstätter, M. Vogt...) have contributed to give a rigorous mathematical definition of $\hat{\boldsymbol{n}}(\vec{\boldsymbol{u}};\boldsymbol{s})$ and its derivative:

- $\hat{\boldsymbol{n}}$ is understood as an *invariant spin field* i.e. a solution of T-BMT eq. satisfying the condition $\hat{\boldsymbol{n}}(\vec{\boldsymbol{u}};\boldsymbol{s})=\hat{\boldsymbol{n}}(\vec{\boldsymbol{u}};\boldsymbol{s}+\boldsymbol{C})$.
- The term $\partial\hat{\boldsymbol{n}}/\partial\boldsymbol{\delta}$ quantifies the depolarizing effects resulting from the trajectory perturbations due to photon emission. A particle emitting a photon will jump from one point $\vec{\boldsymbol{u}}$ in phase space to a new one, with a different $\hat{\boldsymbol{n}}$.

The computation of Derbenev-Kondratenko expressions in the general case of a non-perfectly planar machine is tricky and codes attempting to evaluate them have limitations.

Tools for radiative polarization computation

- By linearizing orbit and spin motion it is possible to calculate polarization “analytically” in terms of one turn maps. This formalism has been developed at the end of the 70s by A. Chao (SLIM) and K. Yokoya. A thick lenses version of SLIM is D. P. Barber SLICK and J. Kewisch SITF.
  - Only linear resonances!
- S. R. Mane wrote SMILE (middle 80s) handling fully 3D spin motion in *perturbation* theory. This approach required large computing time and had convergence problem at high energy.
- SODOM by K. Yokoya (1992) computes Derbenev-Kondratenko $\hat{n}$ and $\partial\hat{n}/\partial\delta$ using Fourier expansions. It has similar issues as SMILE.

Instead of trying to evaluate Derbenev-Kondratenko expression, a *statistical* approach is used in SITROS by J. Kewisch (1982). Orbital motion is up to 2d order and spin motion is not linearized. An initially fully polarized beam is tracked. Polarization evolves as

$$P(t) = P_\infty(1 - \mathrm{e}^{-t/\tau_p}) + P(0)\mathrm{e}^{-t/\tau_p}$$

In the presence of depolarizing effects it is

$$\frac{1}{\tau_p} \simeq \frac{1}{\tau_{\mathrm{BKS}}} + \frac{1}{\tau_{\mathrm{d}}} \qquad \text{and} \qquad P_\infty \simeq \frac{\tau_p}{\tau_{\mathrm{BKS}}} P_{\mathrm{BKS}}$$

known from optics

For an initially fully polarized beam, $P(0)=P_{\mathrm{BKS}}$, for small t it is

$$P(t) \simeq P_{\mathrm{BKS}}\mathrm{e}^{-t/\tau_d}$$

Once τ_d is known one can compute τ_p and P_∞.

SITROS

For speeding up computations, the ring is divided into sections were elements are lumped together:

- Maps describing the linear elements include damping.
- Sextupoles are included as thin lenses.
- Stochastic photon emission takes place at the end of each section.
  - Photons are emitted according to a centered gaussian distribution with unit variance (white noise).
  - The photon energy is scaled with a strength factor initially evaluated from the theoretical beam energy spread and adjusted during tracking by monitoring the bunch length.

Orbital tracking :

- The particle trajectory is described by the 6+21=27 dimension vector

$$\vec{X}^t \equiv (x, x', y, y', \ell, \delta, x^2, xx' \ldots \ell\delta, \ell^2)$$

- The 6×27[a] transport matrices are computed by launching 72 particles with orthogonal starting coordinates ("ray tracing method")

$$\vec{X}^t_{1,2} = (\pm\sigma_x, 0, ...0) \quad \vec{X}^t_{3,4} = (0, \pm\sigma_y, 0, ...0)$$

$$.....\vec{X}^t_{64,67} = (0, 0, ... \pm \sigma_{y'}, 0, \pm\sigma_\delta) \quad \vec{X}^t_{68,72} = (0, 0, ... \pm \sigma_\ell, \pm\sigma_\delta)$$

plus 6 particles for computing the 6d closed orbit.

- The resulting transport matrix is not symplectic (radiation damping!). For HERAe M. Böge showed that the damping times evaluated by tracking were the same as the analytical ones.

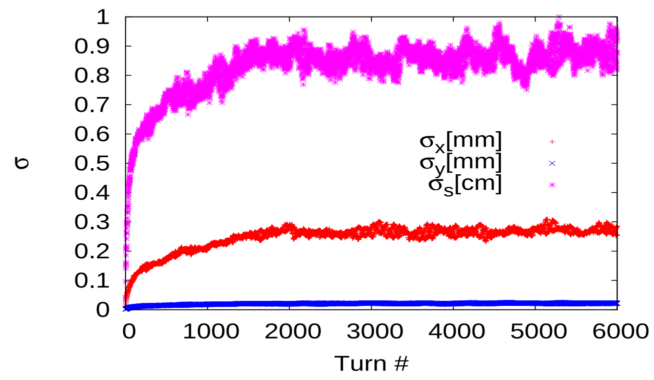
- The calculation of the transport matrices for the spin uses spinor formalism. It follows the same logic as the orbital tracking.

SITROS, updated by M. Böge and M. Berglund, has been used for evaluating the expected polarization in HERAe.

Agreement with observations was good, with exception of beam-beam effects.

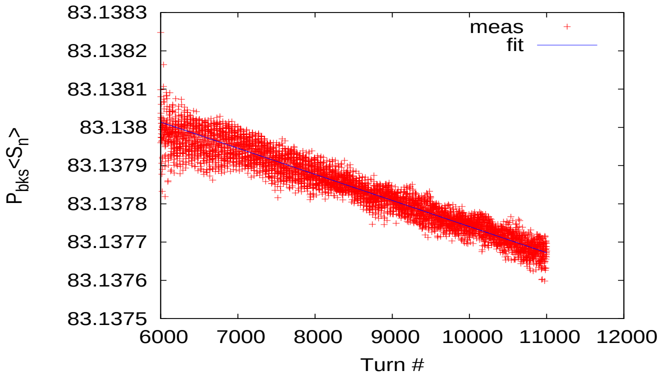
[a] once $(x, x', y, y', \ell, \delta)$ are known also their combinations are.

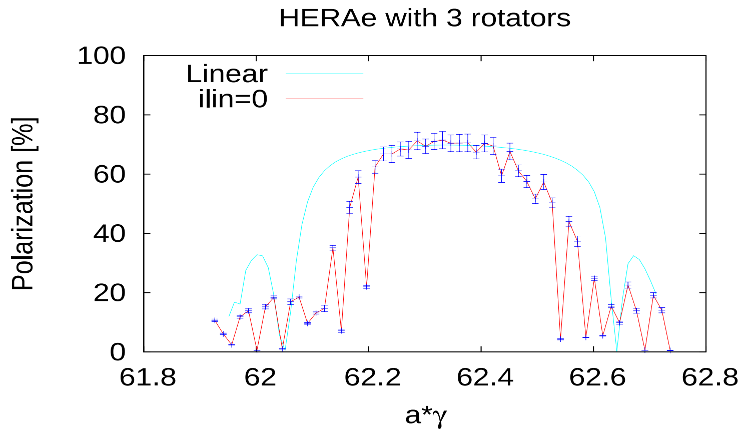
An examples for HERAe II - ideal optics



Beam size at Hermes

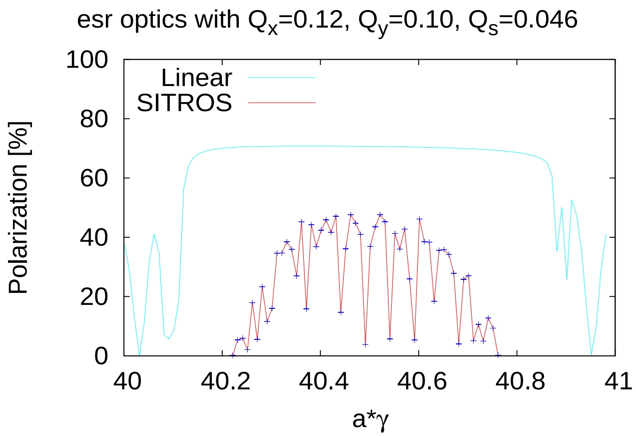
	σ_x [mm]	σ_y [mm]	σ_ℓ [mm]
Analytic	0.288	0.036	8.651
Tracking	0.265	0.022	8.603





EIC electron storage ring

Ideal optics



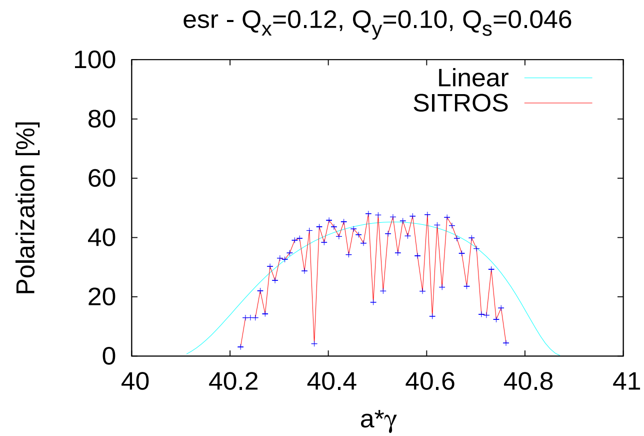
Beam size at IP

	σ_x (mm)	σ_y (μm)	σ_ℓ (mm)
Analytic (SITF)	0.110	0.276	8.434
Tracking (SITROS)	0.106	4.4	8.524

In presence of misalignments and corrections

Beam size at IP

	σ_x (mm)	σ_y (μm)	σ_ℓ (mm)
Analytic	0.111	1.758	8.543
Tracking	0.107	2.044	8.357



Some limitations

- SITROS reads files in PETROS format; PETROS was used for simulating closed orbit distortion and correction for HERAe.
- It allows only magnet offsets and tilts around $\hat{\boldsymbol{s}}$.
  - It can handle strength errors for quadrupoles and dipoles.
- Does not allow thin lenses.
- It allows only rectangular bending magnets; the option DOEDGE allows to consider the edge focusing for all magnets.
- No overlap of magnets is allowed.

These limitations make difficult to use the program when more sophisticated errors and corrections are simulated through more modern codes.

FCCee latest polarization simulations

T. Charles misalignments:

	IR Quads	other Quads	Sexts
δx (μm)	50	100	100
δy (μm)	50	100	100
$\delta\theta$ (μrad)	50	100	100

- BPMs are supposed perfectly aligned to the near-by quadrupole and perfectly calibrated.
- Tune shift and coupling are corrected by 1204 normal + 1204 skew *thin lenses* quadrupoles.

SITROS can't treat thin lenses $\longrightarrow$ replaced by 5 mm long quadrupoles, in lack of more space. Code edited for dropping

- magnets shorter than 10 mm in emittance and damped transport matrix calculation;
- quadrupole component of misaligned sextupoles in the closed orbit calculation (for compatibility with MADX).

For some seeds the thin lenses substitution went well:

Seed 13, with radiation, B_w=0

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (pm)	ϵ_y (pm)
MADX (thin)	23	22	276.4	0.04
MADX (thick)	35	22	278.4	0.04

Seed 13, with radiation , B_w for $\tau_{10\%}$=1.7 h

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (nm)	ϵ_y (pm)
MADX (thin)	23	22	239.7	0.114
MADX (thick)	35	22	241.5	0.114

Seed 1, with radiation, B_w=0

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (pm)	ϵ_y (pm)
MADX (thin)	35	21	278.2	0.366
MADX (thick)	35	21	280.2	0.375

Seed 1, with radiation , B_w for $\tau_{10\%}$=1.7 h

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (pm)	ϵ_y (pm)
MADX (thin)	35	21	242.8	0.281
MADX (thick)	35	21	244.6	0.288

Seed 1, with radiation and 8 wigglers

	Q_x	Q_y	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (nm)	ϵ_y (pm)
MADX (thick)	0.1457	0.2181	34.9	21.5	0.245	0.288
SITF	0.1459	0.2175	34.4	20.7	0.231	10.3 (*)

(*) Due to CV798 ! Dropping it is ϵ_y=0.34 pm. Why MADX gives 0.288 pm?

Seed 13, with radiation and 8 wigglers

	Q_x	Q_y	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (nm)	ϵ_y (pm)
MADX (thick)	0.1447	0.2097	35.2	22.1	0.241	0.112
SITF	0.1447	0.2099	35.2	21.3	0.231	0.394

For some seeds the substitution with 5 mm lenses did not work (even within MADX!).

Seed 17, 45 GeV

	x_{rms} (μm)	y_{rms} (μm)	J_x	J_y	J_s	ϵ_x (nm)	ϵ_y (pm)
MADX (thin)	34.3	21.7	1.001	1.000	1.998	0.240	0.14
MADX (thick)	35.4	23.3	1.200	1.395	1.402	0.234	84.5

Summary

- SITROS approach is at the moment the only viable method for radiative polarization computation in actual storage rings.
- Spin tracking could be implemented in more modern codes as MADX or Bmad to take advantage of their larger capabilities in simulating mis-alignments and corrections.

However SITROS will be still very important for benchmarking new codes!