LATTICE FORMULATION OF AXION-INFLATION:

APPLICATION TO PREHEATING

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IFIC, Valencia, Spain

Zooming in on Axions in the Early Universe, CERN, 22-26 June 2020

TWO TOPICS

Part I: LATTICE FORMULATION OF $\phi F\tilde{F}$

Part II: PREHEATING AFTER AXION-INFLATION

TWO TOPICS

Part I: LATTICE FORMULATION OF $\phi F\tilde{F}$

[1] arXiv:1705.09629 (NPB 926 (2018) 544)
[2] arXiv:1707.09967 (JHEP 04 (2018) 026)
[3] arXiv:1904.11892 (JHEP 10 (2019) 142)

Part II: PREHEATING AFTER AXION-INFLATION

with M. Shaposhnikov .... [1,2,3]
A. Florio .... [3]

TWO TOPICS

Part I: LATTICE FORMULATION OF $\phi F\tilde{F}$

Part II: PREHEATING AFTER AXION-INFLATION

[4] arXiv:1812.03132 (JCAP 06 (2019) 002)
[5] Fun with Inflation (in progress) 2020

with	J. R. Canivete Cuissa …….	[4]
	J. Lizarraga …………………..	[5]
and	M. Peloso ……………………..	[5]
	J. Urrestilla …………………..	[5]

Part 1

LATTICE FORMULATION of $\phi F\tilde{F}$

LATTICE FORMULATION of $\phi F\tilde{F}$

Motivation ?

LATTICE FORMULATION of $\phi F\tilde{F}$

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + C$ on couplings to other fields

LATTICE FORMULATION of $\phi F\tilde{F}$

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$$\boxed{\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + V_{\cancel{\text{shift}}}(\phi) + \frac{c_\psi}{f}\partial_\mu \phi\, \bar{\psi}\, \gamma^\mu\, \gamma_5\, \psi + \frac{\alpha}{f}\phi\, F_{\mu\nu}\, \tilde{F}^{\mu\nu}}$$

fermions gauge fields

(derivative couplings)

LATTICE FORMULATION of $\phi F\tilde{F}$

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breaks shift-symm

fermions

gauge fields

(derivative couplings)

With shift symmetry, $\Delta V \propto V_{\cancel{\text{shift}}}$ $\Rightarrow$ **Protected against radiative corrections !**

LATTICE FORMULATION of $\phi F\tilde{F}$

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fermions　　gauge fields

(derivative couplings)

$= \partial_\mu K^\mu$

topological term

$[\phi\partial_\mu K^\mu = K^\mu\partial_\mu\phi]$

LATTICE FORMULATION of $\phi F\tilde{F}$

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fermions gauge fields

(derivative couplings)

Not the QCD axion;

LATTICE FORMULATION of $\phi F\tilde{F}$

Axion-Inflation

Shift symmetry $\phi \to \phi + C$

[J. Cook, L. Sorbo (arXiv:1109.0022)] [N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]

$$V(\varphi) + \frac{\phi}{\Lambda} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

inflaton ϕ = pseudo-scalar axion

$\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$

$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

The rolling inflaton excites the gauge field(s)

LATTICE FORMULATION of $\phi F\tilde{F}$

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The rolling inflaton excites the gauge field(s)

Photon: 2 helicities

$$\vec{A}(\tau, \mathbf{x}) = \sum_{\lambda=\pm}\int \frac{d^3k}{(2\pi)^{3/2}}\left[\vec{\epsilon}_\lambda(\mathbf{k})a_\lambda(\mathbf{k})A_\lambda(\tau,\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}\right]$$

$$\vec{A}'' - \nabla^2\vec{A} - \frac{\alpha}{f}\phi'\,\vec{\nabla}\times\vec{A} = 0 \quad \Rightarrow \quad \left[\frac{\partial^2}{\partial\tau^2} + k^2 \pm \frac{2k\xi}{\tau}\right]A_\pm(\tau, k) = 0,$$

$$\boxed{\xi \equiv \frac{\alpha\dot{\phi}}{2fH}}$$

LATTICE FORMULATION of $\phi F\tilde{F}$

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Chiral instability

$$A_+(\tau, k) \cong \frac{1}{\sqrt{2k}}\left(\frac{k}{2\xi a H}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}}$$

$$\boxed{\xi \equiv \frac{\alpha\dot{\phi}}{2fH}}$$

LATTICE FORMULATION of $\phi F\tilde{F}$

Axion-Inflation

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The rolling inflaton excites the gauge field(s)

$$A_+ \propto e^{\pi\xi}\,, \quad |A_-| \ll |A_+|$$

A_+ exponentially amplified,

A_- has no amplification

Gauge field excitation is chiral !

Then … Primordial non-Gaussianities, primordial black holes, μ distortions, primordial magnetic fields, baryon asymmetry and Gravitational waves (GW)

LATTICE FORMULATION of $\phi F\tilde{F}$

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The rolling inflaton excites the gauge field(s)

Gauge field excitation creates chiral GWs !

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\rm TT}_{ij} \propto \{E_iE_j + B_iB_j\}^{TT}$$

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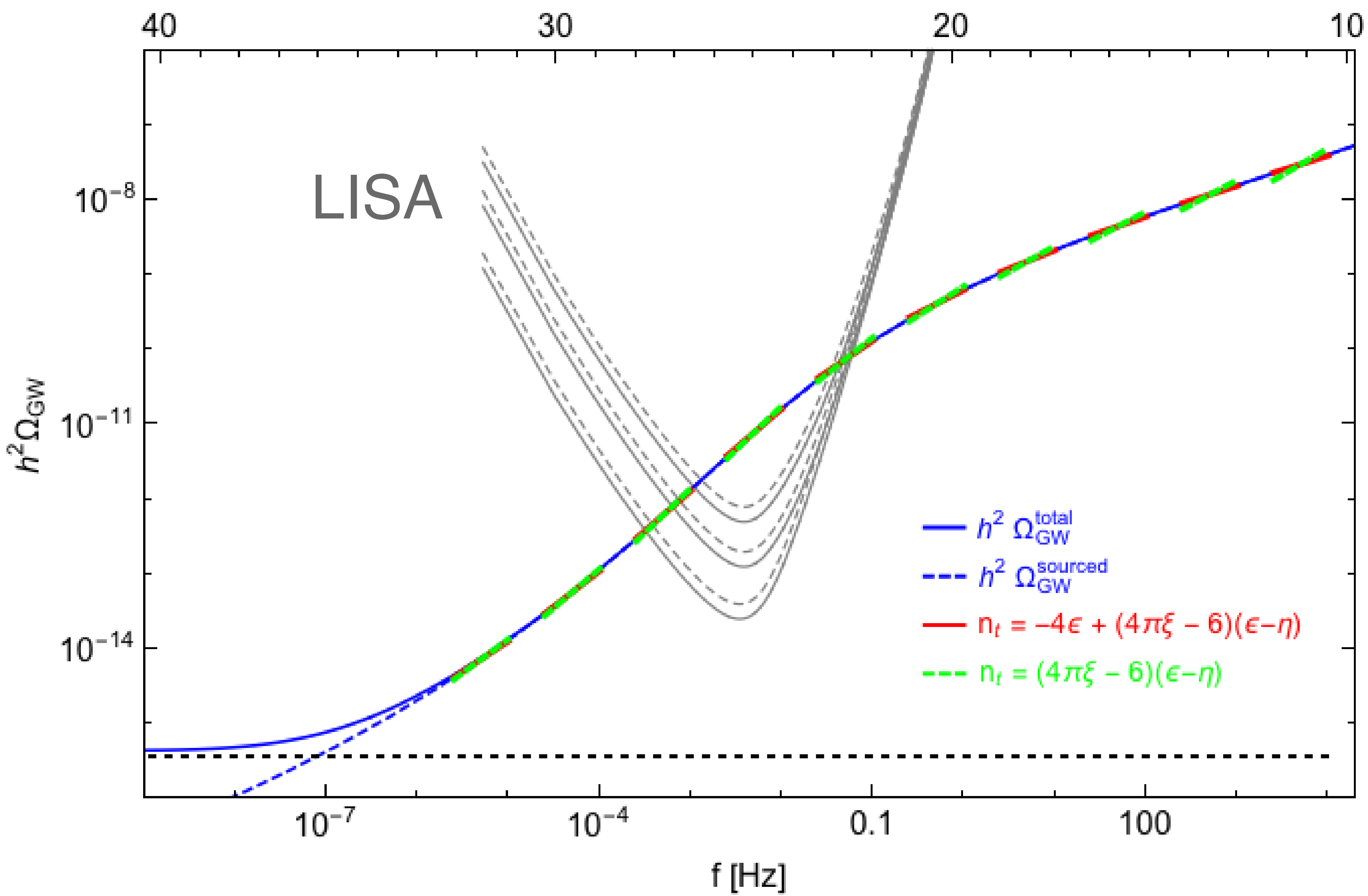
GW one-chirality only

A_μ **Chiral**

LATTICE FORMULATION of $\phi F\tilde{F}$

Axion-Inflation

GW energy spectrum today



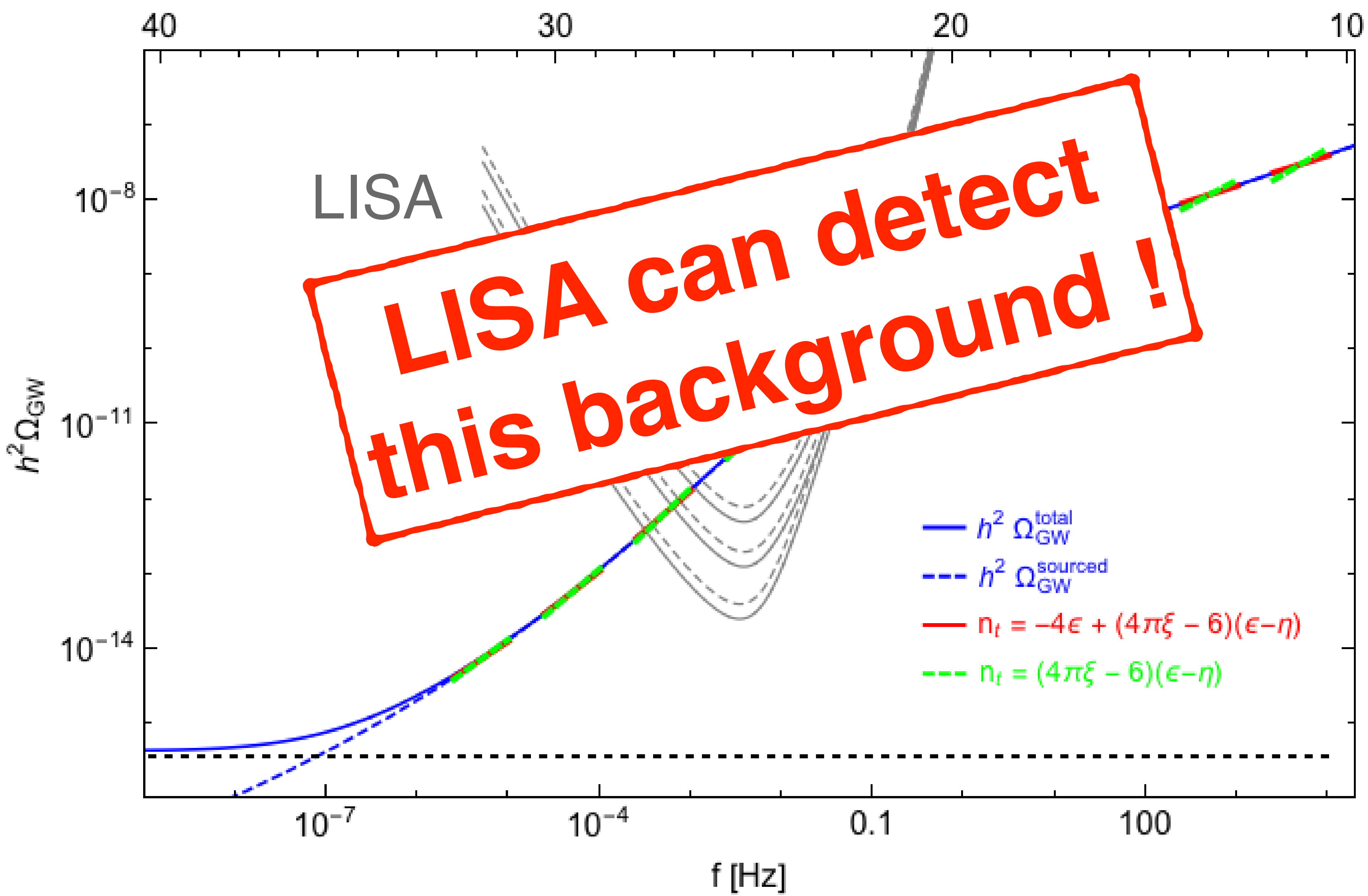
Gauge fields source a Blue-Tilted + Chiral + Non-G GW background

Bartolo et al 2016

LATTICE FORMULATION of $\phi F\tilde{F}$

Axion-Inflation

GW energy spectrum today



Gauge fields source a Blue-Tilted + Chiral + Non-G GW background

Bartolo et al 2016

Implementation

LATTICE FORMULATION of $\phi F\tilde{F}$

Continuum

$$S = \int d^4x\sqrt{-g}\left(\underbrace{\frac{1}{2}m_{\rm pl}^2 R}_{\substack{\text{gravity}\\ \text{(GR)}}} - \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2}_{\text{Inflaton}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Gauge}} + \underbrace{\frac{\phi}{4\Lambda}F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{Interaction}}\right)$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\nu\ , \quad \tilde{F}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\ , \quad \epsilon^{0123} \equiv \frac{1}{\sqrt{-g}} = \frac{1}{a^3(t)}$$

LATTICE FORMULATION of $\phi F\tilde{F}$

Continuum

EoM

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

$$\pi_\phi \equiv \dot{\phi}\,, \qquad E_i \equiv \dot{A}_i - \partial_i A_0\,, \qquad B_i \equiv \epsilon_{ijk}\partial_j A_k\,,$$

LATTICE FORMULATION of $\phi F\tilde{F}$

Continuum

EoM

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E} - \frac{\phi}{a\Lambda}\underbrace{\left(\dot{\vec{B}} - \vec{\nabla}\times\vec{E}\right)}_{=0}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B} - \frac{\phi}{a\Lambda}\underbrace{\vec{\nabla}\cdot\vec{B}}_{=0}\,,$$ (Gauss Law)

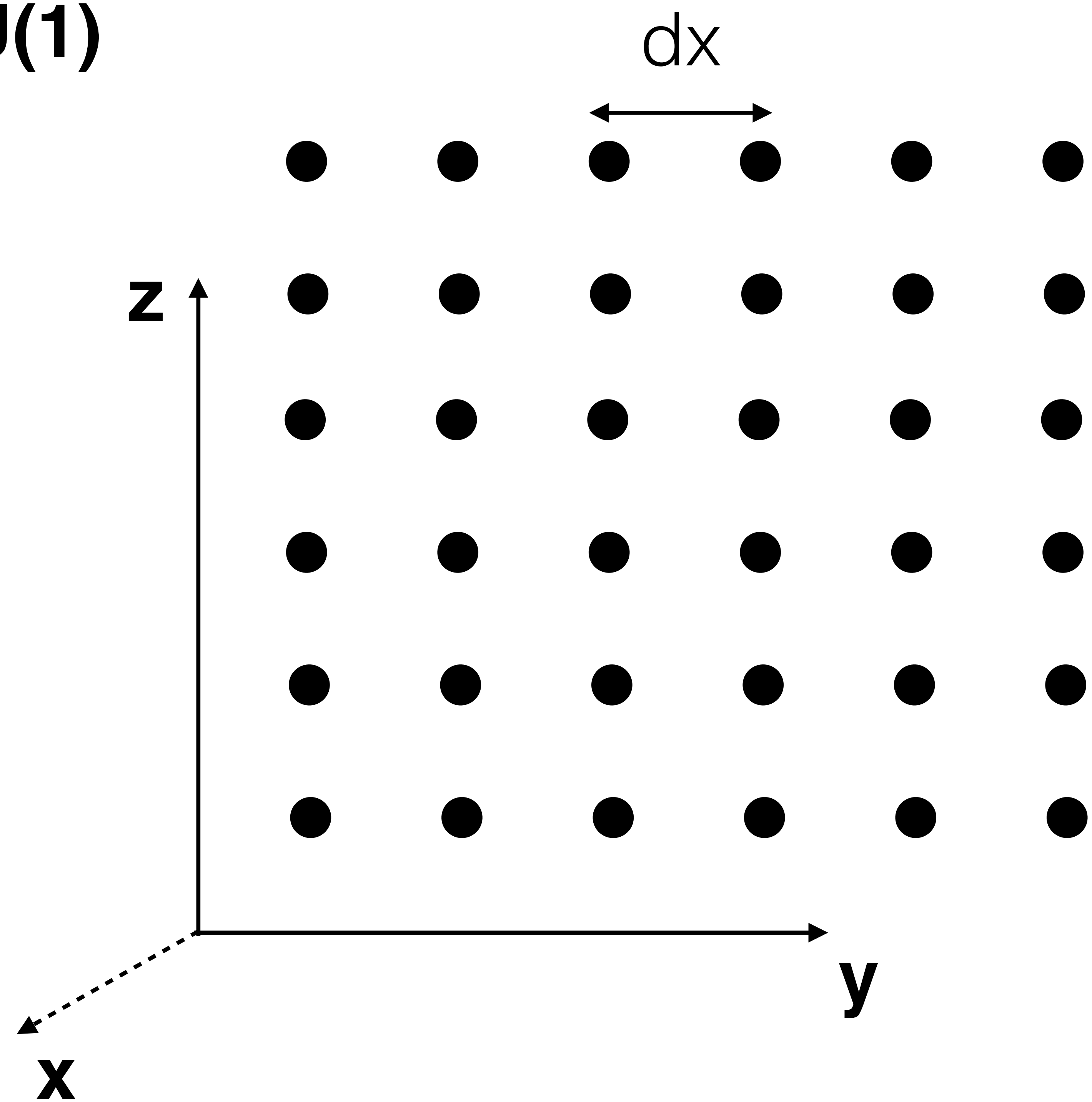
$$\partial_\mu(\sqrt{-g}\tilde{F}^{\mu\nu}) = 0 \iff \begin{cases} \dot{\vec{B}} - \vec{\nabla}\times\vec{E} = 0 \\ \vec{\nabla}\cdot\vec{B} = 0 \end{cases}$$

(Bianchi Identities)

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

U(1)



Comoving
Coordinates

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

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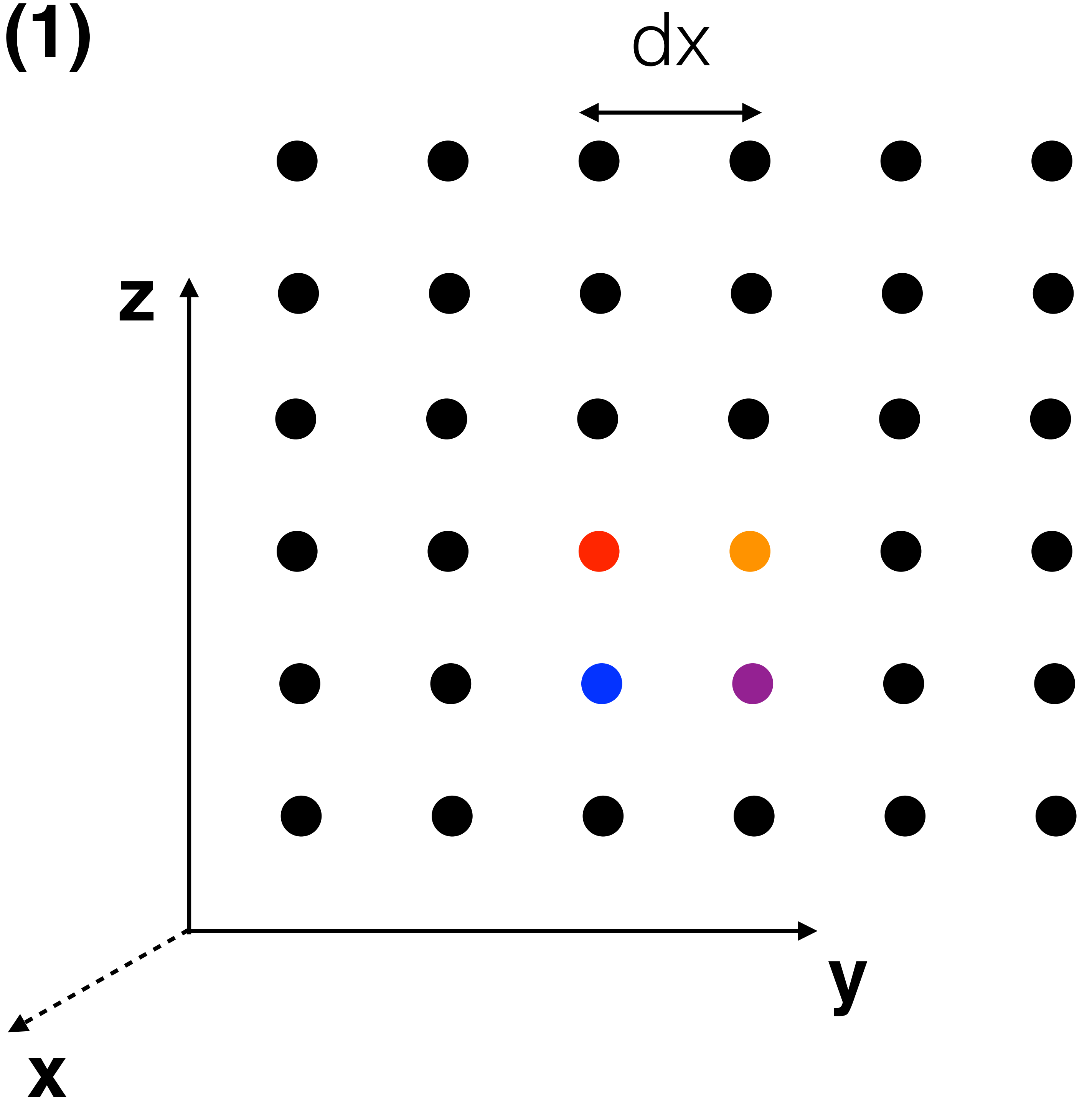

Comoving Coordinates

LATTICE FORMULATION of $\phi F\tilde{F}$

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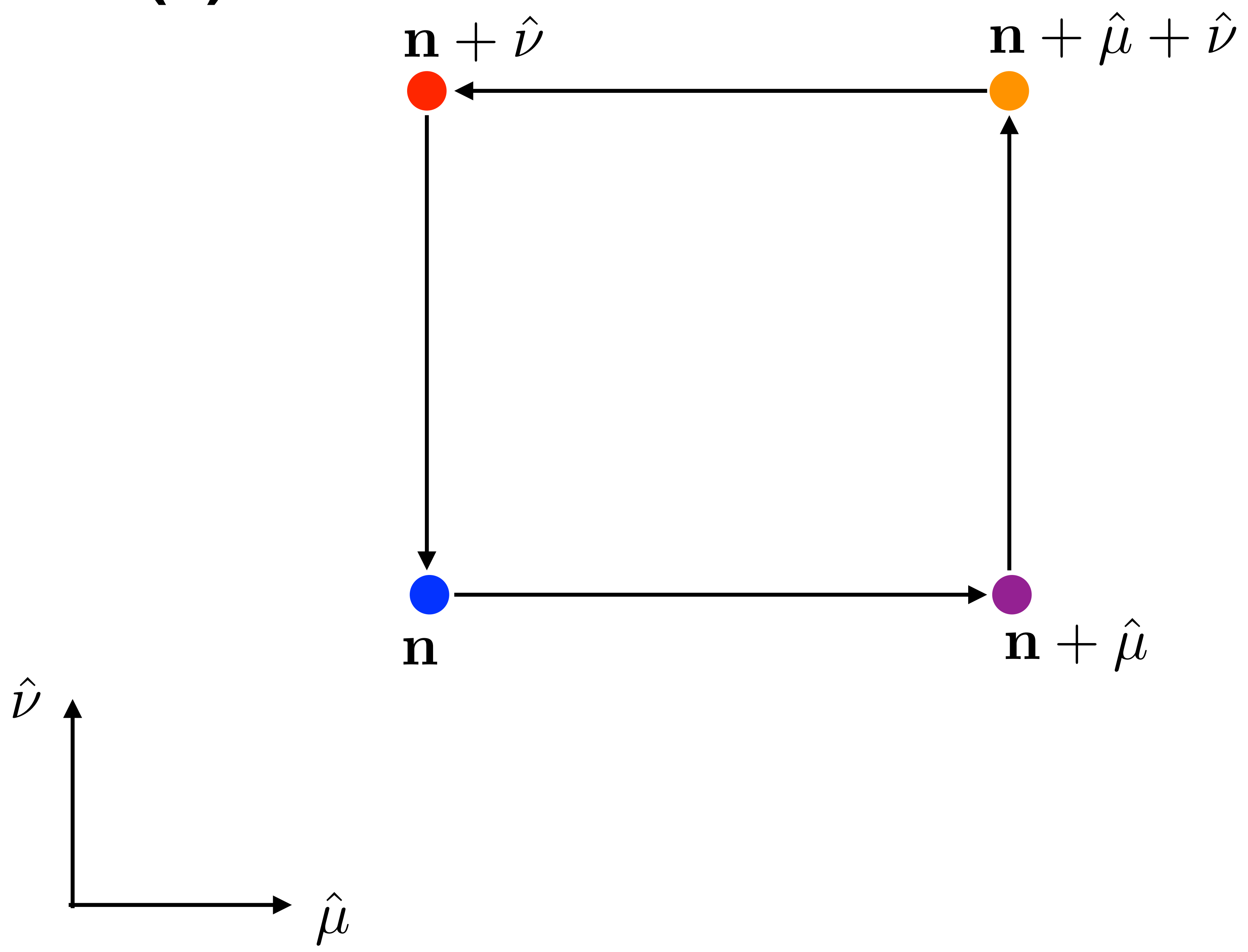

LATTICE FORMULATION of $\phi F\tilde{F}$

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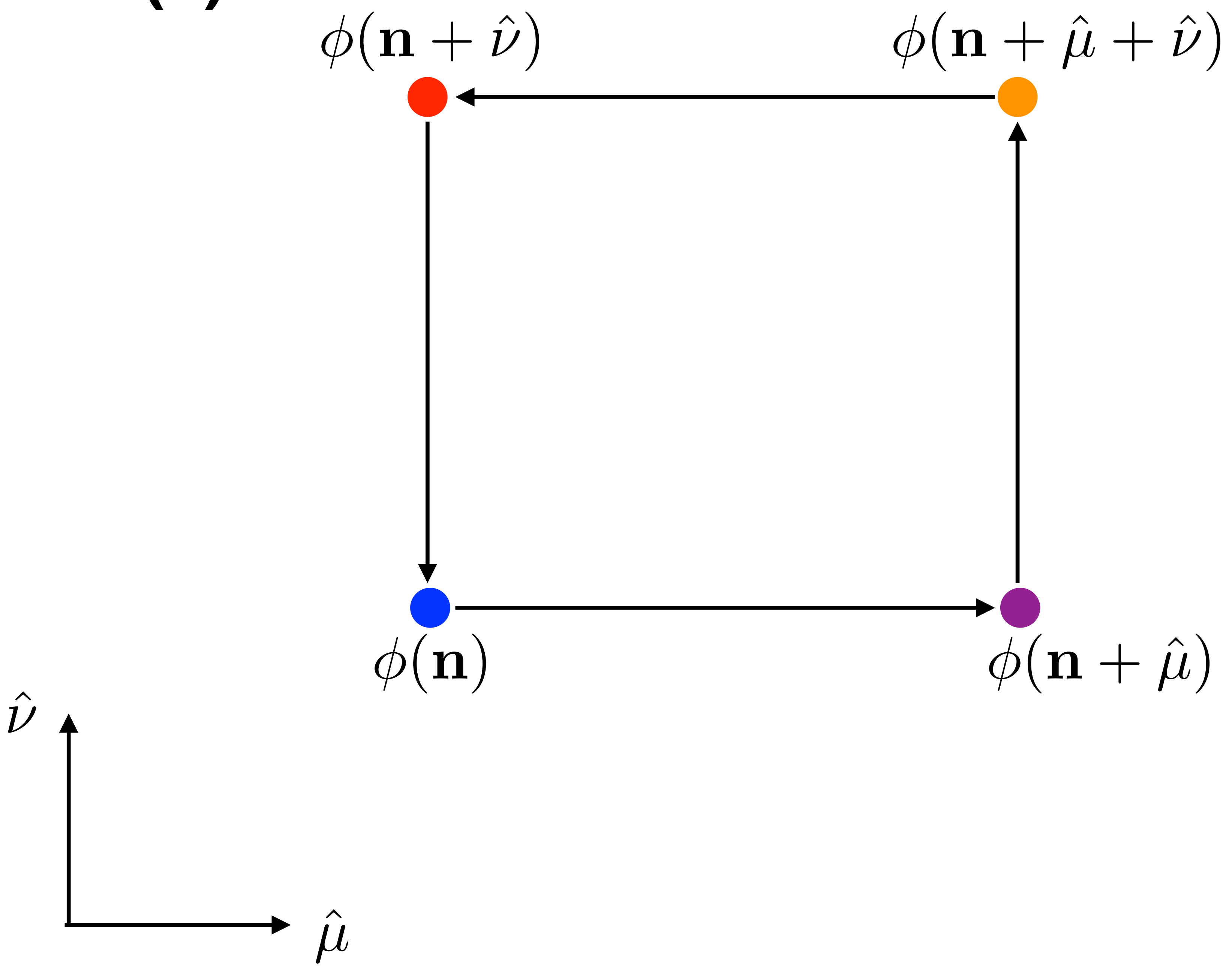

LATTICE FORMULATION of $\phi F\tilde{F}$

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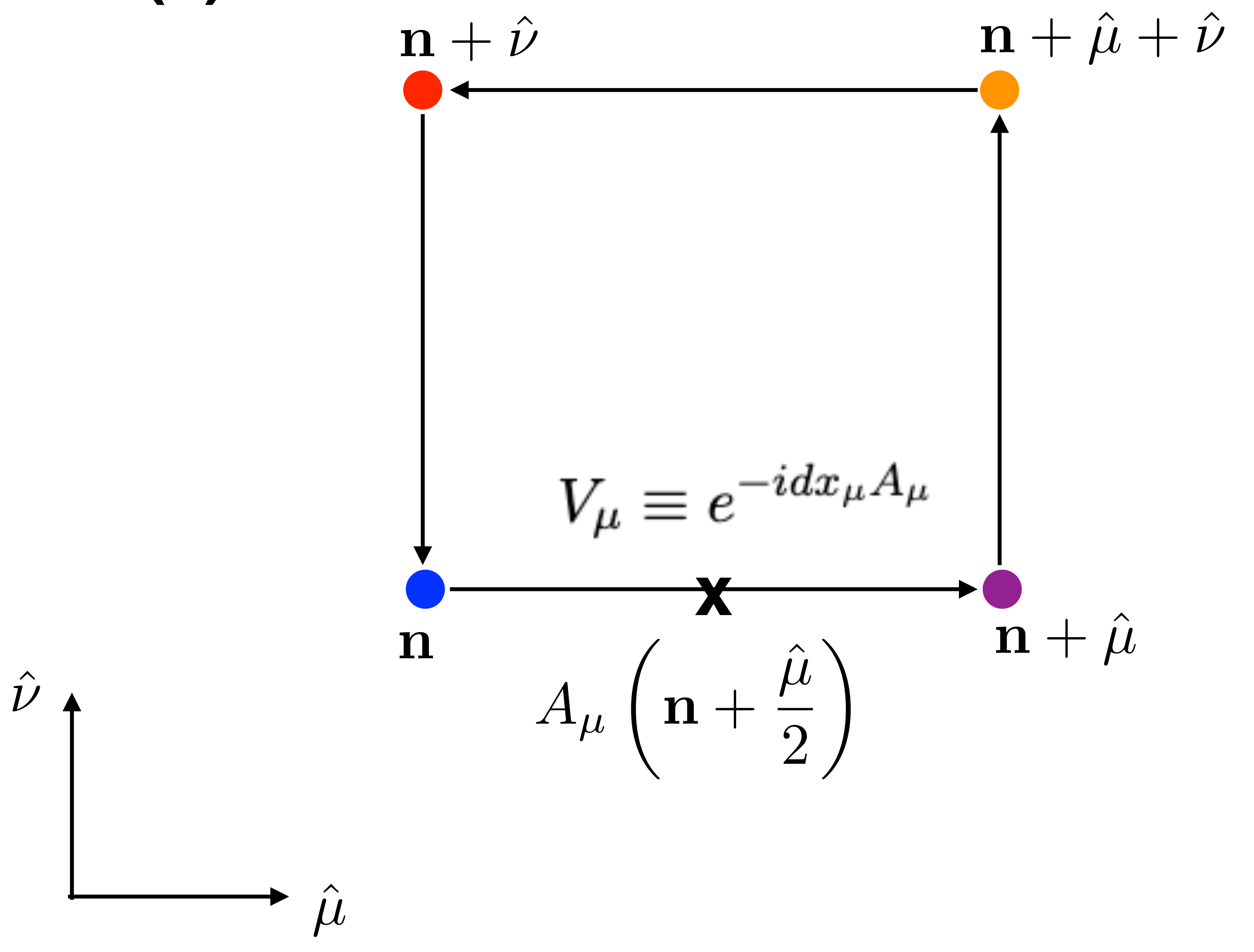

LATTICE FORMULATION of $\phi F\tilde{F}$

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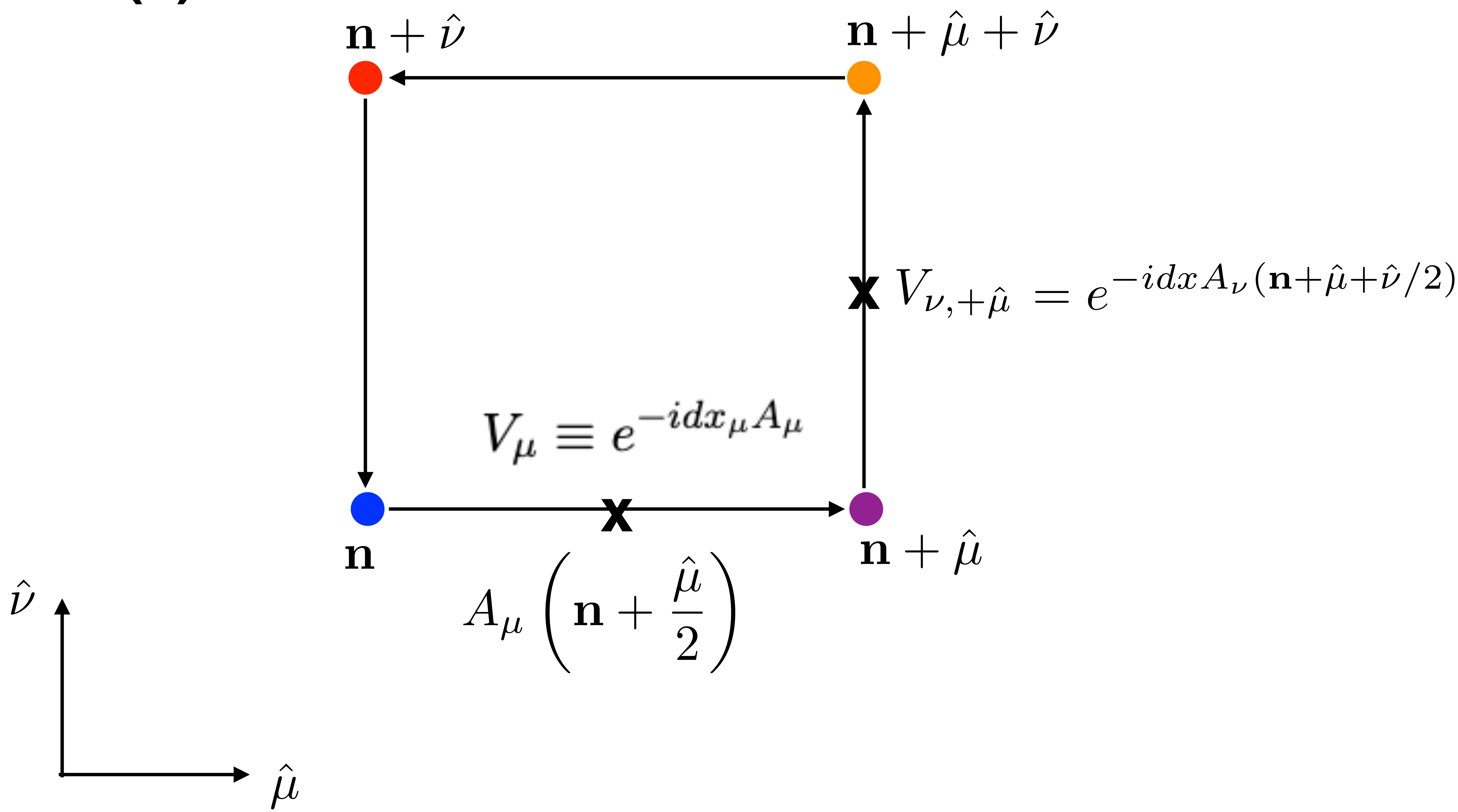

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

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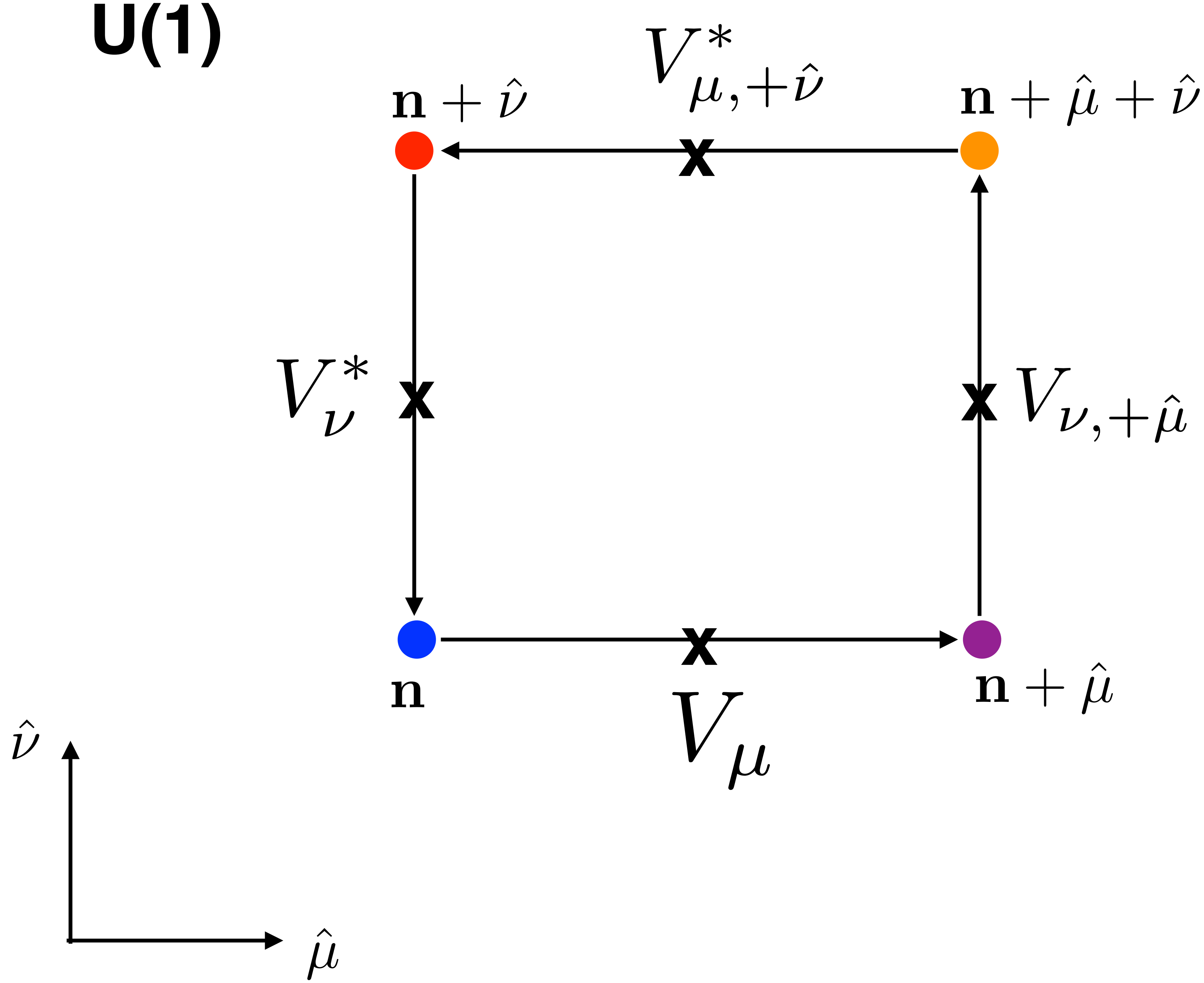
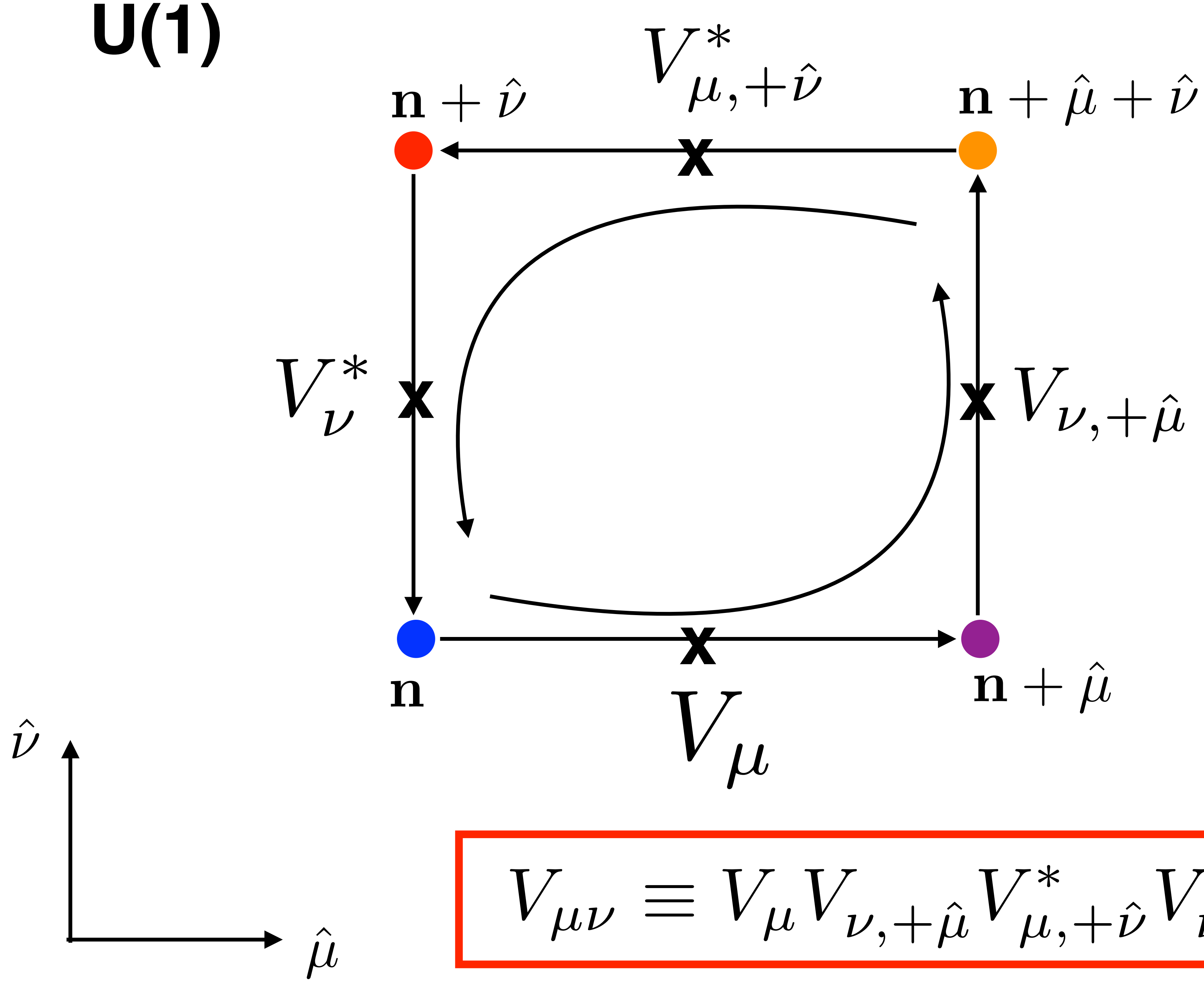


LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

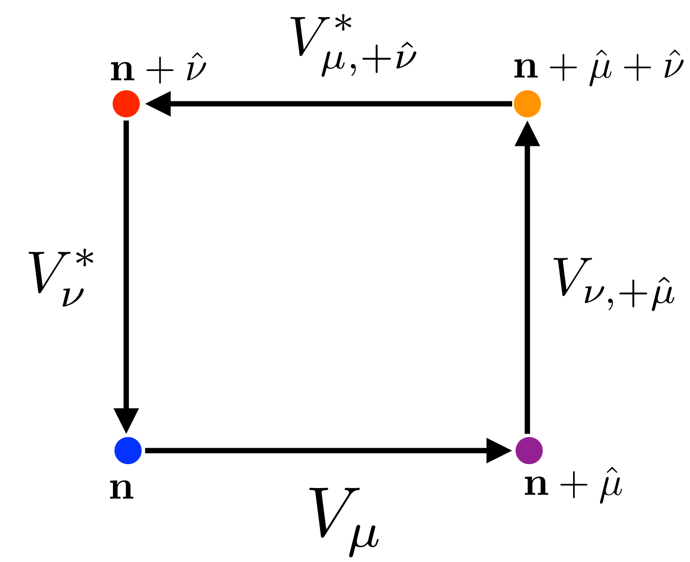
U(1)



$$\boxed{V_{\mu\nu} \equiv V_\mu V_{\nu,+\hat{\mu}} V^*_{\mu,+\hat{\nu}} V^*_{\nu}}$$ (**Plaquette**)

LATTICE FORMULATION of $\phi F\tilde{F}$

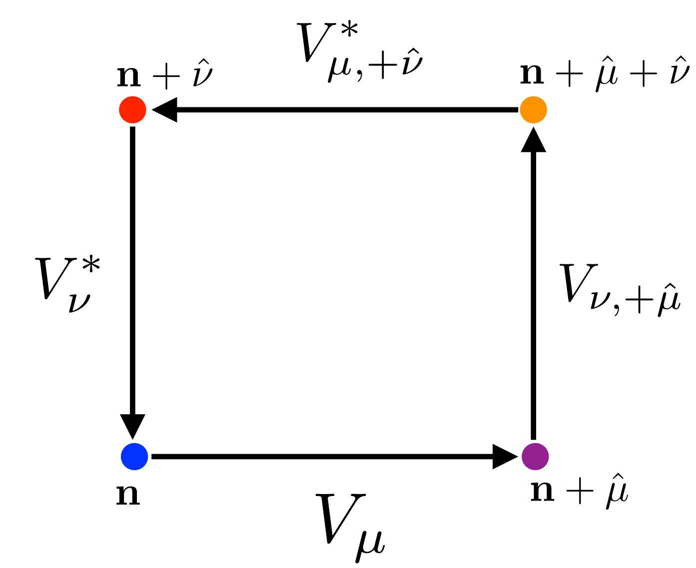
'Latticesizing'



$$V_{\mu\nu} \equiv V_\mu V^*_{\nu,+\hat{\mu}} V^*_{\mu,+\hat{\nu}} V^*_{\nu} \simeq e^{-idx_\mu dx_\nu [F_{\mu\nu}+\mathcal{O}(\delta x)]}$$ (Plaquette)

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'



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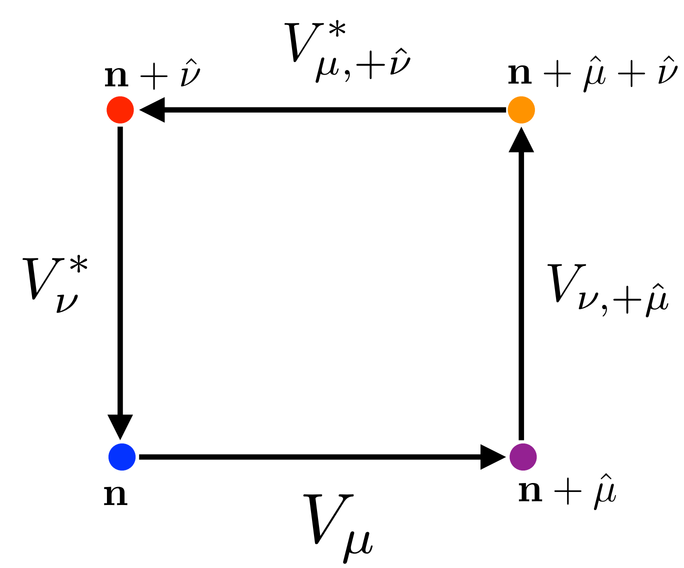
$$\begin{cases} \mathcal{R}e\{V_{\mu\nu}\} \longrightarrow 1 - \frac{1}{2}dx_\mu^2 dx_\nu^2 F_{\mu\nu}^2 + \mathcal{O}(\delta x^5)\,, \quad \mathbf{l} = \mathbf{n} + \frac{1}{2}\hat\mu + \frac{1}{2}\hat\nu \\ \\ \mathcal{I}m\{V_{\mu\nu}\} \longrightarrow -dx_\mu dx_\nu F_{\mu\nu} + \mathcal{O}(\delta x^3)\,, \quad \mathbf{l} = \mathbf{n} + \frac{1}{2}\hat\mu + \frac{1}{2}\hat\nu \end{cases}$$

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

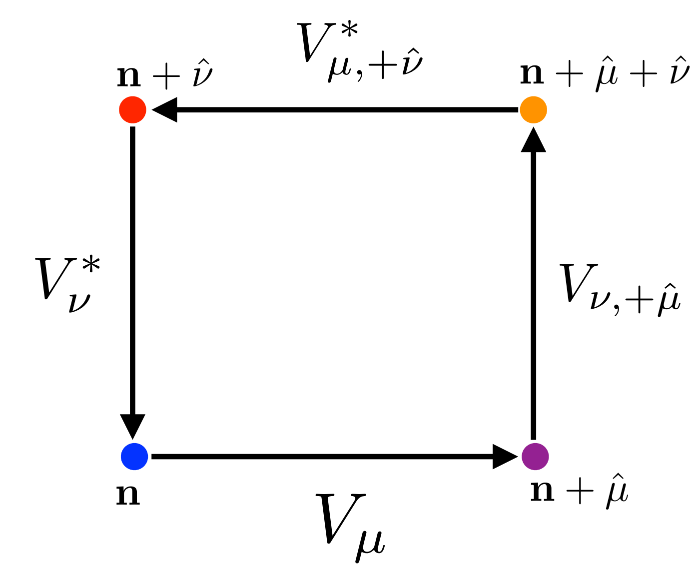


$$V_{\mu\nu} \equiv V_\mu V^*_{\nu,+\hat{\mu}} V^*_{\mu,+\hat{\nu}} V^*_{\nu} \simeq e^{-idx_\mu dx_\nu [F_{\mu\nu}+\mathcal{O}(\delta x)]} \quad \text{(Plaquette)}$$

$$\left\{\begin{array}{l} \sum_n \frac{1}{4}F^2_{\mu\nu} \cong -\frac{1}{2}\sum_n \frac{\mathcal{R}e\{V_{\mu\nu}\}}{dx^2_\mu dx^2_\nu} = -\frac{1}{4}\sum_n \frac{(V_{\mu\nu}+V^*_{\mu\nu})}{dx^2_\mu dx^2_\nu} + \mathcal{O}(\delta x^2) \\ \sum_n \frac{1}{4}F^2_{\mu\nu} \simeq \sum_n \frac{1}{4}\frac{\mathcal{I}m^2\{V_{\mu\nu}\}}{dx^2_\mu dx^2_\nu} = -\sum_n \frac{1}{4}\frac{(V_{\mu\nu}-V^*_{\mu\nu})^2}{dx^2_\mu dx^2_\nu} + \mathcal{O}(\delta x^2) \end{array}\right] \quad \text{(Compact)}$$

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'



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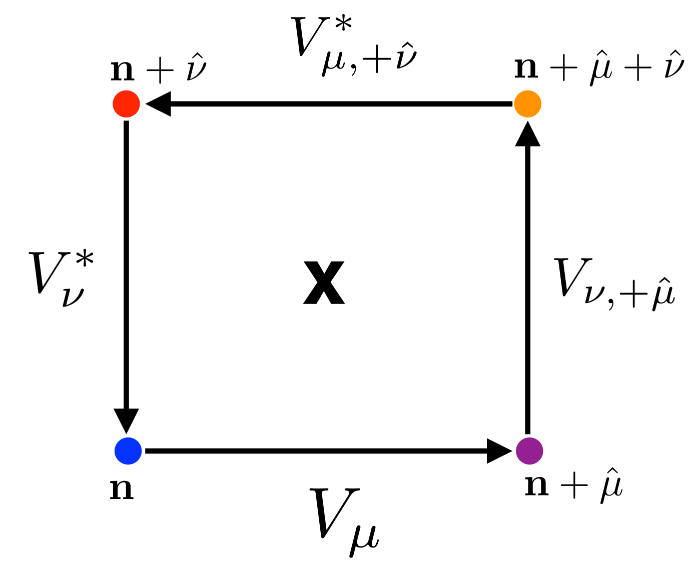
$$\sum_n \frac{1}{4}F^2_{\mu\nu} \simeq \frac{1}{4}\sum_n(\Delta^+_\mu A_\nu - \Delta^+_\nu A_\mu)^2 + \mathcal{O}(\delta x^2)$$ (**Non-Compact**)

(**Abelian only!**)

Finite difference Operator

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'



$$\sum_n \tfrac{1}{4}F^2_{\mu\nu} \simeq \tfrac{1}{4}\sum_n(\Delta^+_\mu A_\nu - \Delta^+_\nu A_\mu)^2 + \mathcal{O}(\delta x^2)$$

(Plaquette Non-Compact)

Finite difference Operator

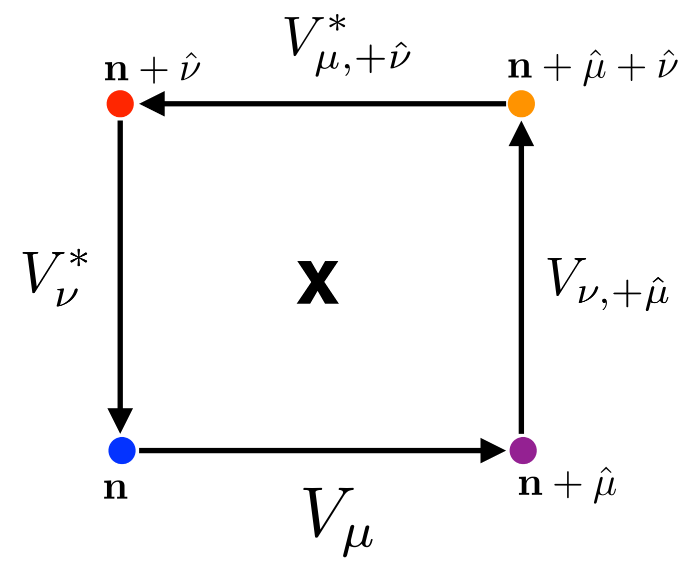
$$[\nabla^\pm_\mu f] = \frac{\pm f(\mathbf{n}\pm\hat{\mu}) \mp f(\mathbf{n})}{\delta x^\mu} \longrightarrow \begin{cases} \partial_i \mathbf{f}(\mathbf{x})\big|_{\mathbf{x}\equiv\mathbf{n}\delta x} + \mathcal{O}(\delta x)\,. \\ \partial_i \mathbf{f}(\mathbf{x})\big|_{\mathbf{x}\equiv(\mathbf{n}\pm\hat{\mu}/2)\delta x^\mu} + \mathcal{O}(\delta x^2)\,. \end{cases}$$

$$F_{\mu\nu} \equiv (\Delta^+_\mu A_\nu - \Delta^+_\nu A_\nu)$$

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\,\alpha\,\vec{E}\vec{B}$$



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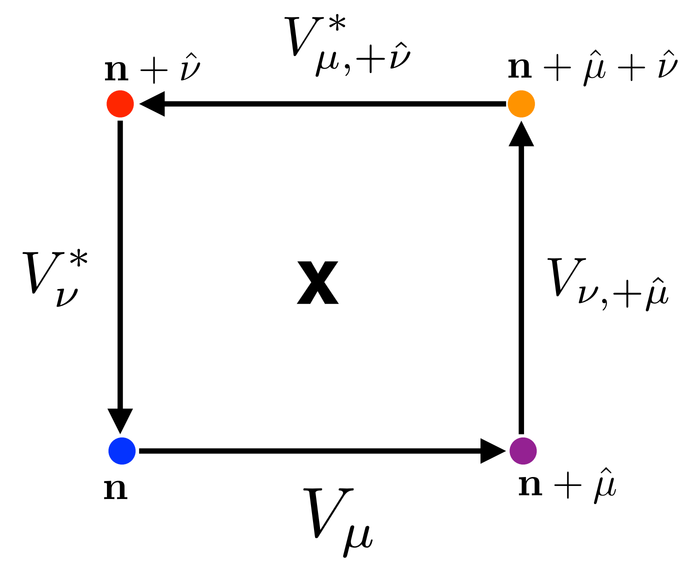

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(I) $$S_{ac}^{L(1)} \propto \sum_{\vec{n},n_o}\alpha\sum_i E_i B_i = \sum_{\vec{n},n_o}\alpha\sum_i(\Delta_o^+A_i - \Delta_i^+A_o)\epsilon_{ijk}\Delta_j^+A_k$$

$$\left[F_{\mu\nu} \equiv (\Delta_\mu^+A_\nu - \Delta_\nu^+A_\nu)\right]$$

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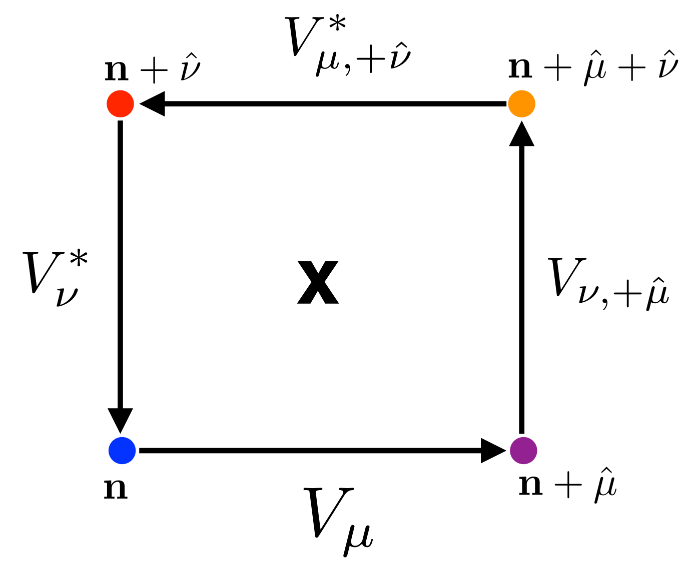


(I) $$S_{ac}^{L(1)} \propto \sum_{\vec{n},n_o}\alpha\sum_i E_i B_i = \sum_{\vec{n},n_o}\alpha\sum_i(\Delta_o^+ A_i - \Delta_i^+ A_o)\epsilon_{ijk}\Delta_j^+ A_k$$

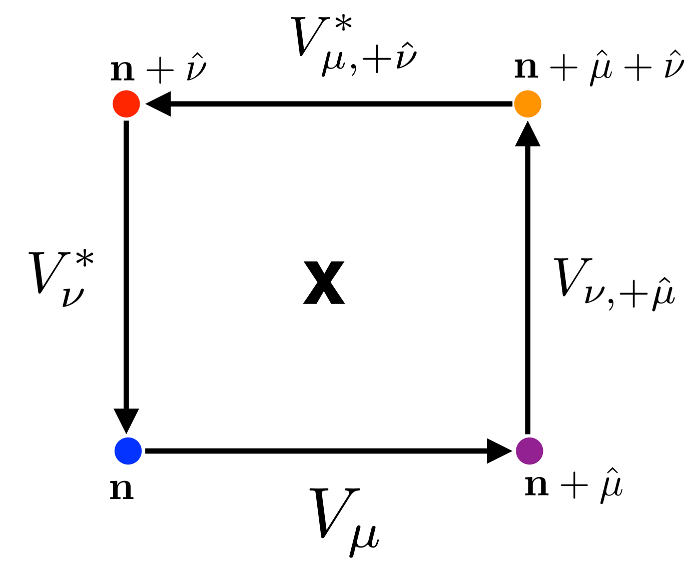
$$\left[\,F_{\mu\nu} \equiv (\Delta_\mu^+ A_\nu - \Delta_\nu^+ A_\nu)\,\right]$$

EOM: $$\begin{cases} [\Delta_o^- B_i - (\nabla^- \times \vec{E})_i] \neq 0 \\ \sum \Delta_i^- B_i \neq 0 \end{cases}$$

(**Violation of Bianchi Identities**)

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'



$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\, \alpha\, \vec{E}\vec{B}$$

(I) $$S^{L(1)}_{ac} \propto \sum_{\vec{n},n_o} \alpha \sum_i E_i B_i = \sum_{\vec{n},n_o} \alpha \sum_i \underbrace{(\Delta^+_o A_i - \Delta^+_i A_o)}_{\text{@ semi-integer times}}\underbrace{\epsilon_{ijk}\Delta^+_j A_k}_{\text{@ integer times}}$$

EOM: $$\begin{cases} [\Delta^-_o B_i - (\nabla^- \times \vec{E})_i] \neq 0 \\ \sum \Delta^-_i B_i \neq 0 \end{cases}$$

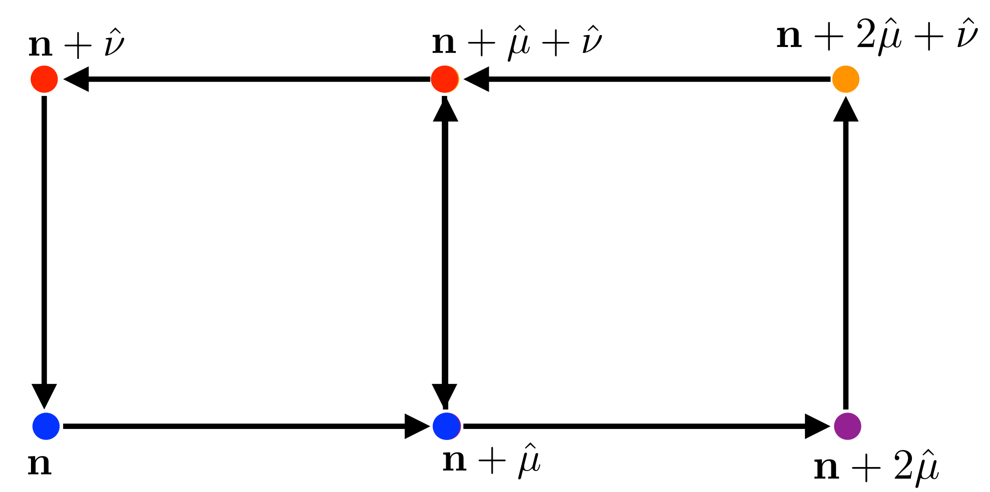
(**Violation of Bianchi Identities**)

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\,\alpha\,\vec{E}\vec{B}$$

(II) $F_{\mu\nu} \equiv (\Delta^+_\mu A_\nu - \Delta^+_\nu A_\nu)$



$$E_i^{(2)} \equiv \frac{1}{2}(E_i + E_{i,-i})(l)\Big|_{l\equiv n+\frac{\hat{0}}{2}}$$

$$E_i^{(4)} \equiv \frac{1}{4}(E_i + E_{i,-i} + E_{i,-0} + E_{i,-i-0})(l)\Big|_{l\equiv n}$$

$$B_i^{(4)} \equiv \frac{1}{4}(B_i + B_{i,-j} + B_{i,-k} + B_{i,-j-k})(l)\Big|_{l\equiv n}$$

$$E_i^{(8)} \equiv \frac{1}{2}\left(E_i^{(4)} + E_{i,+i}^{(4)}\right), \qquad B_i^{(8)} \equiv \frac{1}{2}\left(B_i^{(4)} + B_{i,+i}^{(4)}\right)$$

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

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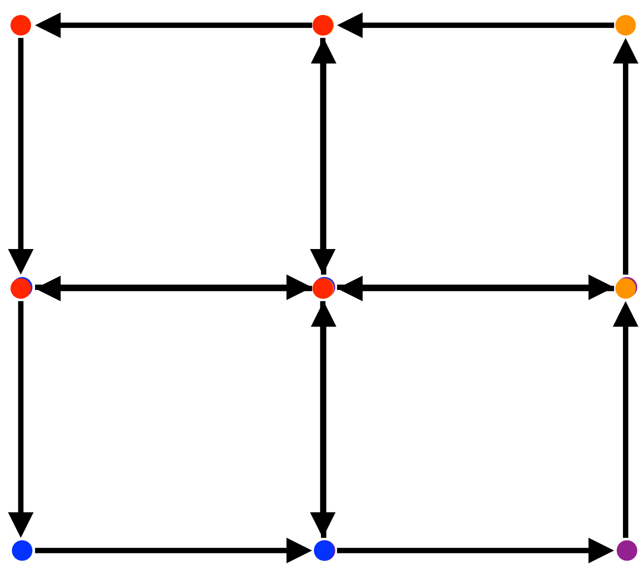
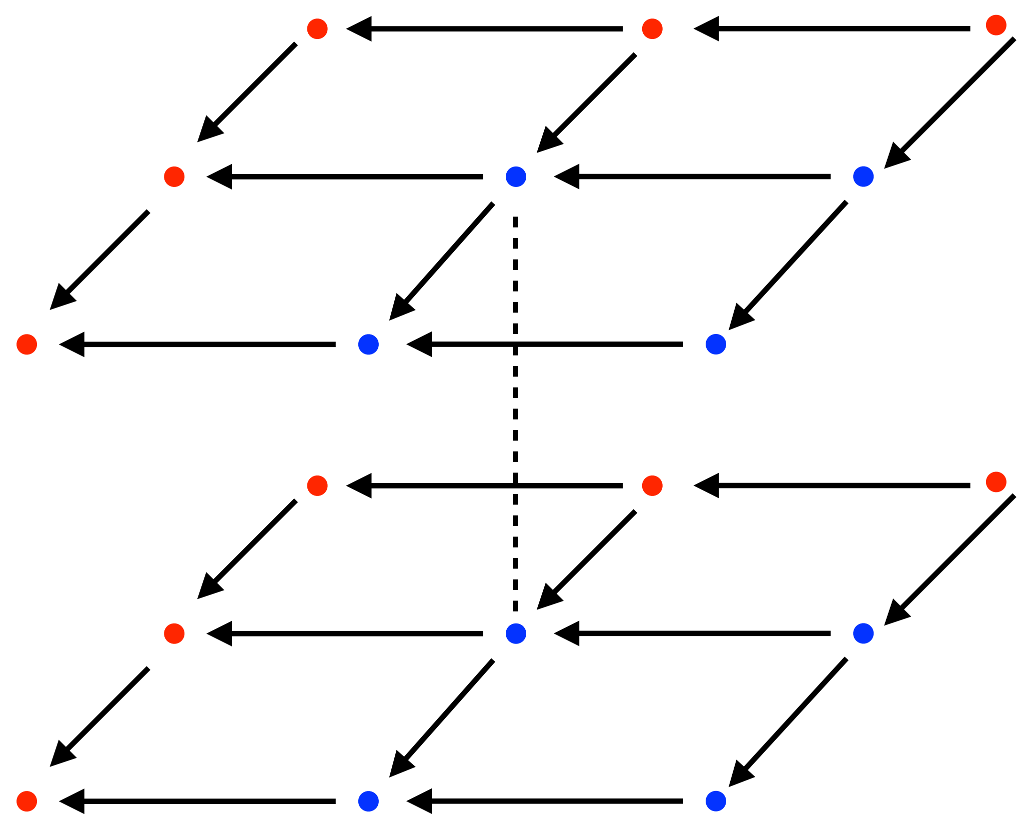


LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\,\alpha\,\vec{E}\vec{B}$$



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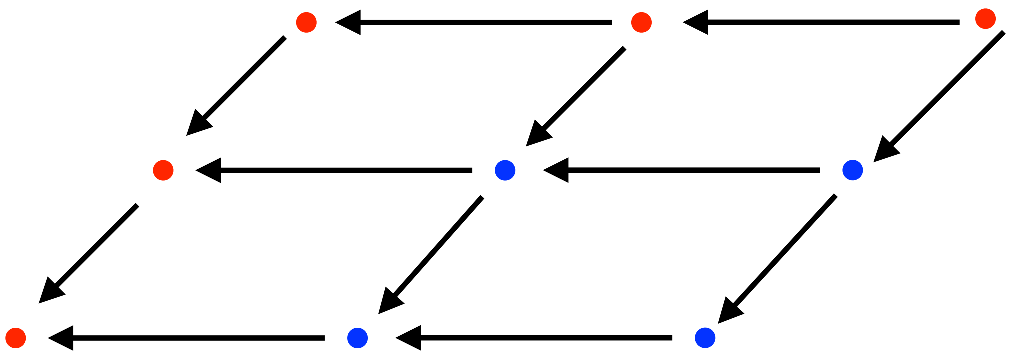
LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\,\alpha\,\vec{E}\vec{B}$$

(II) $S_{ac}^{L(2)} \propto \sum_{\vec{n},n_o} \alpha \sum_i E_i^{(4)} B_i^{(4)}$



EOM:
$$\begin{cases} [\sum_{j,k}\epsilon_{ijk}(\Delta_j^+ + \Delta_j^-)E_k^{(8)} - (\Delta_o^+ + \Delta_o^-)B_i^{(8)}] = 0 \quad ✓ \\ \text{resembles } (\epsilon_{ijk}\partial_j E_k - \partial_o B_i) = 0 \end{cases}$$

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\, \alpha\, \vec{E}\vec{B}$$

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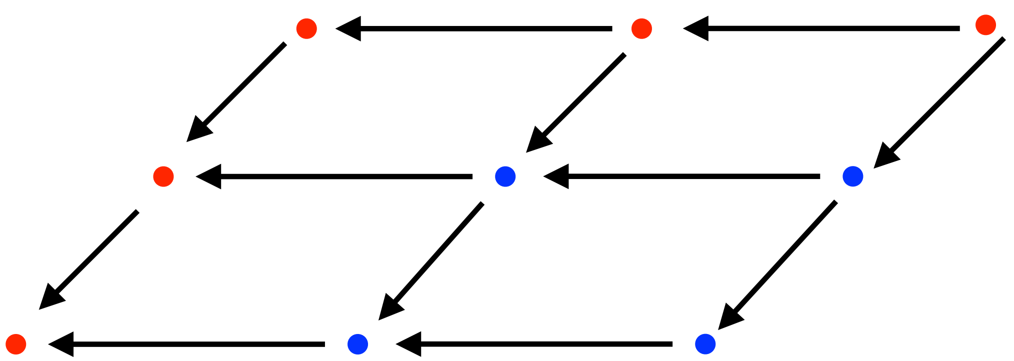

EOM: $\left\{ \begin{array}{l} \sum_i \Delta_i^-(B_i^{(4)} + B_{i,+i}^{(4)}) = 0 \ \checkmark \\ \text{resembles } \partial_i B_i = 0 \end{array} \right.$

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\,\alpha\,\vec{E}\vec{B}$$

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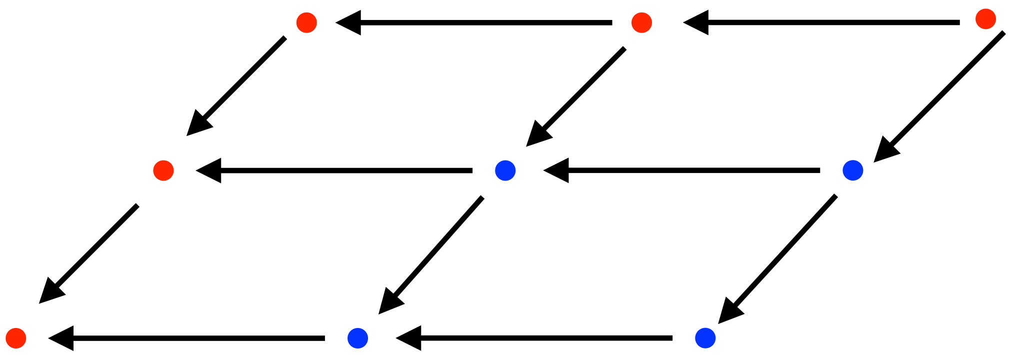


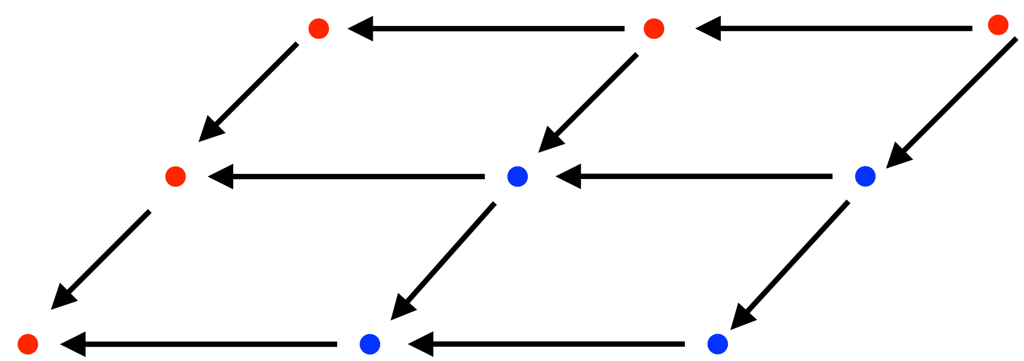
EOM: $\Bigg\{$ **Iterative scheme is Inconsistent !** (One cannot advance one variable as a function of previous ones)

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\,\alpha\,\vec{E}\vec{B}$$

(II) $S_{ac}^{L(2)} \propto \sum_{\vec{n},n_o} \alpha \sum_i E_i^{(4)} B_i^{(4)}$



$$E_i^{(4)} \equiv \tfrac{1}{2}(E_i^{(2)} + E_{i,-0}^{(2)})$$

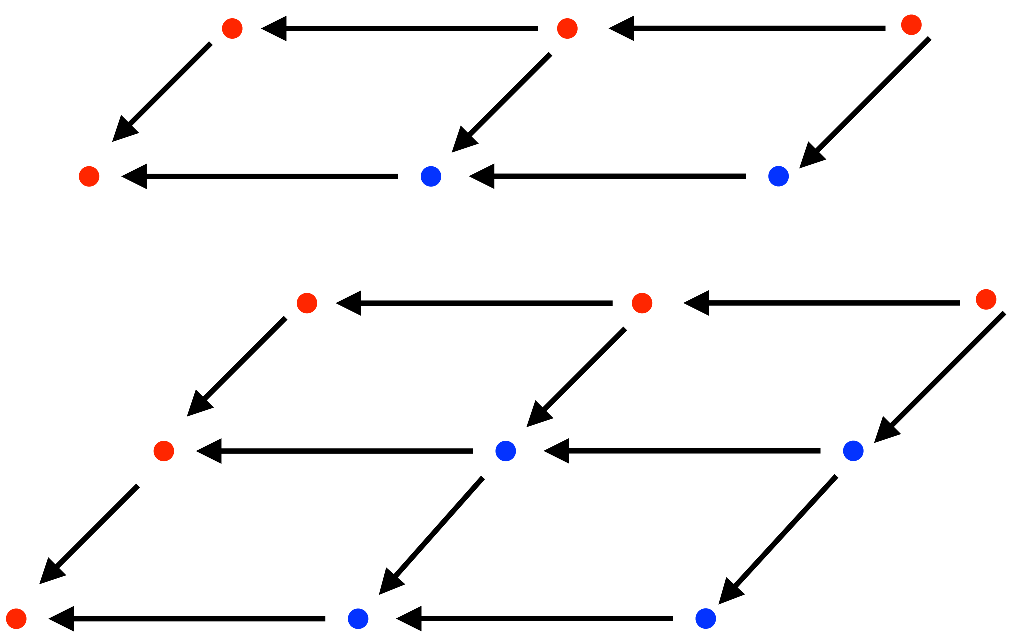
EOM: $\Big\{$ **Iterative scheme is Inconsistent !** (One cannot advance one variable as a function of previous ones)

LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda} \; ; \; S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\, \alpha\, \vec{E}\vec{B}$$

(III) $S_{ac}^{L(3)} \propto \sum_{\vec{n},n_o} \alpha \sum_i E_i^{(2)} B_i^{(4)}$



LATTICE FORMULATION of $\phi F\tilde{F}$

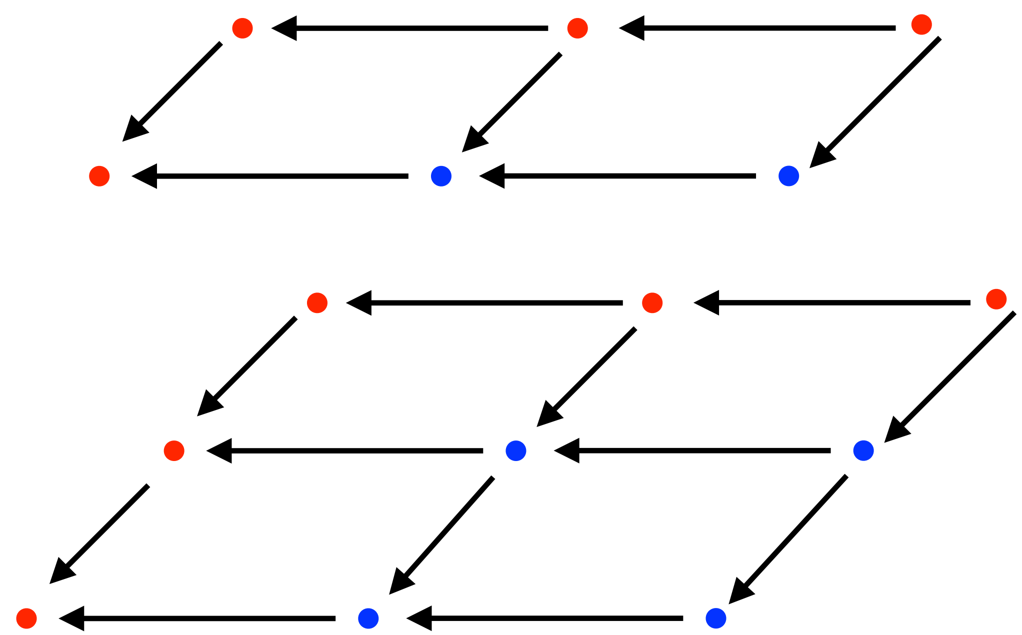
'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\,\alpha\,\vec{E}\vec{B}$$

(III) $$S_{ac}^{L(3)} \propto \sum_{\vec{n},n_o}\alpha\sum_{i}E_i^{(2)}B_i^{(4)}$$

@ semi-integer times

@ integer times

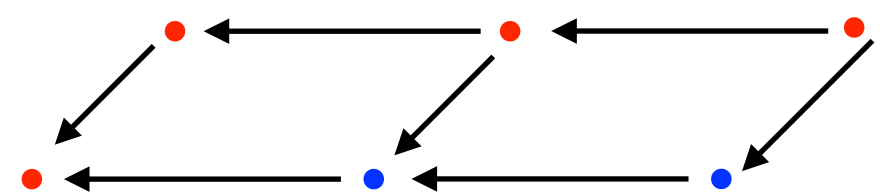


It won't work !

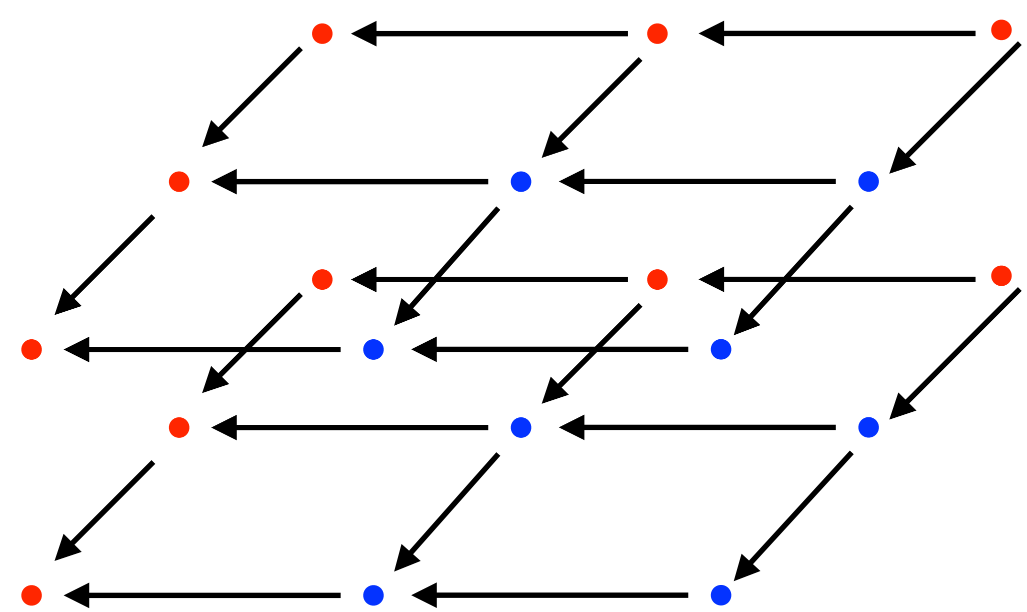
LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda} \;;\; S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\, \alpha\, \vec{E}\vec{B}$$



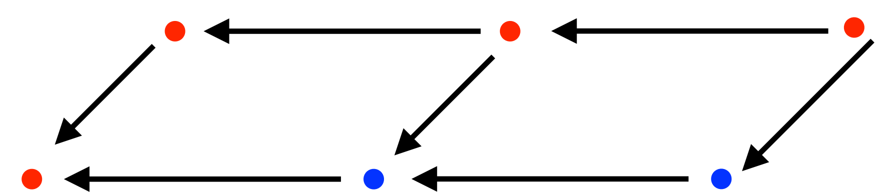
(IV) $$S_{ac}^{L(4)} \propto \sum_{\vec{n},n_o} \alpha \sum_i E_i^{(2)}(B_i^{(4)} + B_{i+0}^{(4)})$$



LATTICE FORMULATION of $\phi F\tilde{F}$

'Latticesizing'

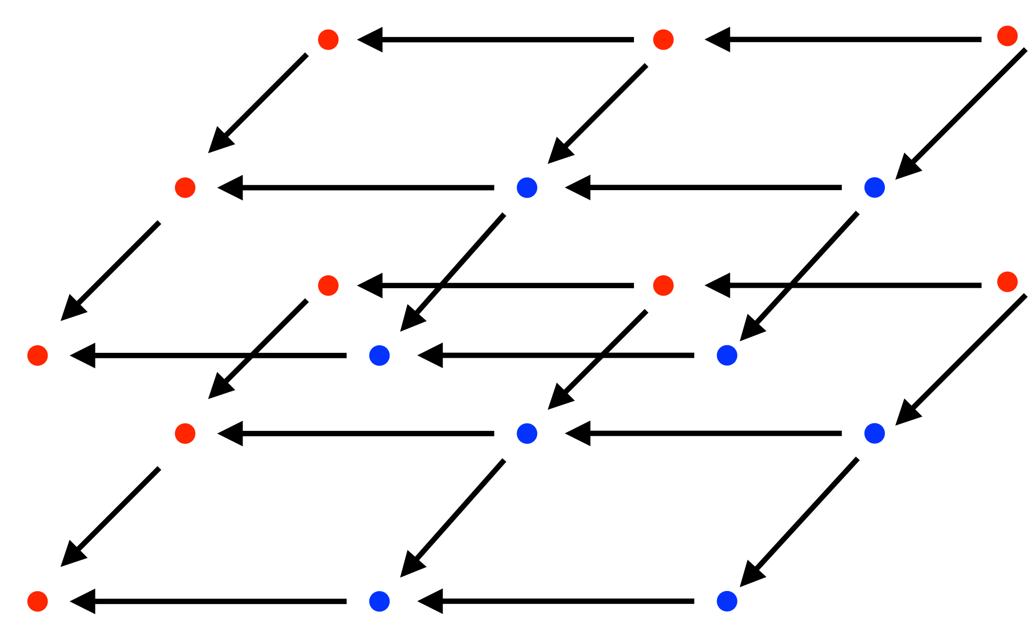
$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\,\alpha\,\vec{E}\vec{B}$$



(IV) $$S_{ac}^{L(4)} \propto \underbrace{\sum_{\vec{n},n_o} \alpha \sum_i E_i^{(2)}(B_i^{(4)} + B_{i+0}^{(4)})}$$

@ semi-
integer times

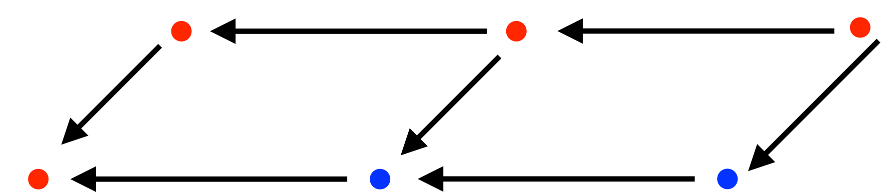
Axion need to live at semi-integer times !



LATTICE FORMULATION of $\phi F\tilde{F}$

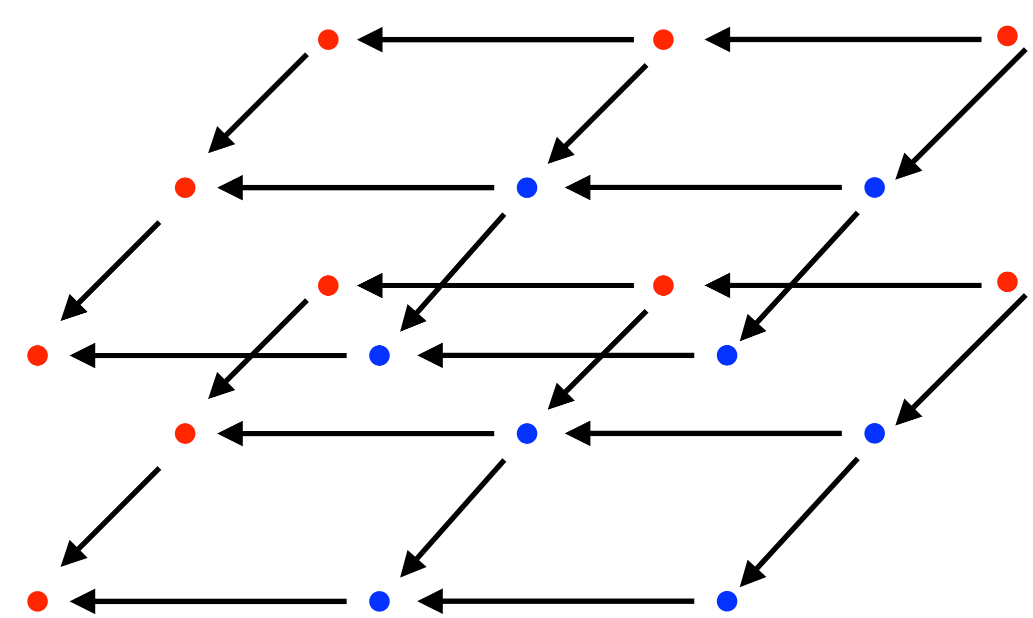
'Latticesizing'

$$\alpha \equiv \frac{\phi}{\Lambda}\ ;\ S_{ac} \equiv \frac{1}{4\pi^2}\int d^4x\, \alpha\, \vec{E}\vec{B}$$



(IV) $$S_{ac}^{L(4)} \propto \underbrace{\sum_{\vec{n},n_o} \alpha \sum_i E_i^{(2)}(B_i^{(4)} + B_{i+0}^{(4)})}_{\text{@ semi-integer times}}$$

Bianchi Identities fine !



LATTICE FORMULATION of $\phi F\tilde{F}$

$$
\begin{aligned}
S_L = \Delta t \Delta x^3 \sum_{t,\vec{n}} \Bigg\{ & \frac{1}{2} a^3 \left(\Delta_0^- \phi_{+\frac{\hat{0}}{2}}\right)^2 - \frac{1}{2} a_{+\frac{\hat{0}}{2}} \left(\Delta_i^+ \phi_{+\frac{\hat{0}}{2}}\right)^2 - \frac{1}{2} a^3_{+\frac{\hat{0}}{2}} m^2 \phi^2_{+\frac{\hat{0}}{2}} \\
& + \frac{1}{2} a_{+\frac{\hat{0}}{2}} \sum_i (\Delta_0^+ A_i - \Delta_i^+ A_0)^2 - \frac{1}{4a} \sum_{i,j} \left(\Delta_i^+ A_j - \Delta_j^+ A_i\right) \\
& + \frac{\phi}{\Lambda} \sum_i \frac{1}{2} E_i^{(2)} \left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right) \Bigg\},
\end{aligned}
$$

Lattice action

LATTICE FORMULATION of $\phi F\tilde{F}$

$$S_L = \Delta t \Delta x^3 \sum_{t,\vec{n}} \Bigg\{ \ \frac{1}{2} a^3 \left(\Delta_0^- \phi_{+\frac{\hat{0}}{2}}\right)^2 - \frac{1}{2} a_{+\frac{\hat{0}}{2}} \left(\Delta_i^+ \phi_{+\frac{\hat{0}}{2}}\right)^2 - \frac{1}{2} a^3_{+\frac{\hat{0}}{2}} m^2 \phi^2_{+\frac{\hat{0}}{2}}$$
$$+ \frac{1}{2} a_{+\frac{\hat{0}}{2}} \sum_i (\Delta_0^+ A_i - \Delta_i^+ A_0)^2 - \frac{1}{4a} \sum_{i,j} \left(\Delta_i^+ A_j - \Delta_j^+ A_i\right)$$
$$+ \frac{\phi}{\Lambda} \sum_i \frac{1}{2} E_i^{(2)} \left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right) \Bigg\},$$

Lattice action

1. Lattice Gauge Inv: $A_\mu \to A_\mu + \Delta_\mu^+ \alpha$
2. Cont. Limit to $\mathcal{O}(dx^2)$

LATTICE FORMULATION of $\phi F\tilde{F}$

$$S_L = \Delta t \Delta x^3 \sum_{t,\vec{n}} \Big\{ \frac{1}{2}a^3\left(\Delta_0^-\phi_{+\frac{\hat{0}}{2}}\right)^2 - \frac{1}{2}a_{+\frac{\hat{0}}{2}}\left(\Delta_i^+\phi_{+\frac{\hat{0}}{2}}\right)^2 - \frac{1}{2}a^3_{+\frac{\hat{0}}{2}}m^2\phi^2_{+\frac{\hat{0}}{2}}$$
$$+ \frac{1}{2}a_{+\frac{\hat{0}}{2}}\sum_i(\Delta_0^+A_i - \Delta_i^+A_0)^2 - \frac{1}{4a}\sum_{i,j}\left(\Delta_i^+A_j - \Delta_j^+A_i\right)$$
$$+ \frac{\phi}{\Lambda}\sum_i \frac{1}{2}E_i^{(2)}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)\Big\},$$

Lattice action

1. Lattice Gauge Inv: $A_\mu \to A_\mu + \Delta_\mu^+\alpha$
2. Cont. Limit to $\mathcal{O}(dx^2)$
3. Lattice Bianchi Identities:

$$\begin{cases} \sum_i \Delta_i^-(B_i^{(4)} + B_{i,+i}^{(4)}) = 0 \\ (\Delta_0^+ + \Delta_0^-)(B_i^{(4)} + B_{i,+i}^{(4)}) = \\ \qquad = \sum_{j,k}\epsilon_{ijk}(\Delta_j^+ + \Delta_j^-)(E_k^{(4)} + E_{k,+k}^{(4)}) \end{cases}$$

LATTICE FORMULATION of $\phi F\tilde{F}$

$$S_L = \Delta t \Delta x^3 \sum_{t,\vec{n}} \Big\{ \; \frac{1}{2} a^3 \left(\Delta_0^- \phi_{+\frac{\hat{0}}{2}}\right)^2 - \frac{1}{2} a_{+\frac{\hat{0}}{2}} \left(\Delta_i^+ \phi_{+\frac{\hat{0}}{2}}\right)^2 - \frac{1}{2} a^3_{+\frac{\hat{0}}{2}} m^2 \phi^2_{+\frac{\hat{0}}{2}}$$
$$+ \frac{1}{2} a_{+\frac{\hat{0}}{2}} \sum_i (\Delta_0^+ A_i - \Delta_i^+ A_0)^2 - \frac{1}{4a} \sum_{i,j} \left(\Delta_i^+ A_j - \Delta_j^+ A_i\right)$$
$$+ \frac{\phi}{\Lambda} \sum_i \frac{1}{2} E_i^{(2)} \left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right) \Big\},$$

Lattice action

1. Lattice Gauge Inv: $A_\mu \to A_\mu + \Delta_\mu^+ \alpha$
2. Cont. Limit to $\mathcal{O}(dx^2)$
3. Lattice Bianchi Identities: $\begin{cases} \sum_i \Delta_i^- (B_i^{(4)} + B_{i,+i}^{(4)}) = 0 \\ (\Delta_0^+ + \Delta_0^-)(B_i^{(4)} + B_{i,+i}^{(4)}) = \\ \qquad = \sum_{j,k} \epsilon_{ijk} (\Delta_j^+ + \Delta_j^-)(E_k^{(4)} + E_{k,+k}^{(4)}) \end{cases}$
4. Topological Term: $(F_{\mu\nu}\tilde{F}^{\mu\nu})_L \equiv \sum_i \frac{1}{2} E_i^{(2)} \left(B_i^{(4)} + B_{i,+0}^{(4)}\right) = \Delta_\mu^+ K^\mu$ **(CS current)**

   $[\, F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu \,]$

Exact Shift Sym. on the lattice !

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$
$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\ ,$$ (Gauss Law)

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$
$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\ ,$$ (Gauss Law)

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Constraint)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat 0}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat 0}{2}} - a^3_{+\frac{\hat 0}{2}}m^2\phi_{+\frac{\hat 0}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}\underline{E^{(2)}_{i,+\frac{\hat 0}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat 0}\right)}$$

$$\Delta_0^-\left(a_{+\frac{\hat 0}{2}}E_{i,+\frac{\hat 0}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$
$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat 0}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat 0}{2}}\right\}$$

$$a_{+\frac{\hat 0}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat 0}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat 0}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat 0}\right)_{\pm i}\ ,$$ (Gauss Law)

$$\dot\pi_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec\nabla^2\phi - m^2\phi + \underline{\frac{1}{a^3\Lambda}\vec E\cdot\vec B},$$

$$\dot{\vec E} = -H\vec E - \frac{1}{a^2}\vec\nabla\times\vec B - \frac{1}{a\Lambda}\pi_\phi\vec B + \frac{1}{a\Lambda}\vec\nabla\phi\times\vec E$$

$$\vec\nabla\cdot\vec E = -\frac{1}{a\Lambda}\vec\nabla\phi\cdot\vec B$$ (Gauss Constraint)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)$$

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \underline{\frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)}\right)}$$

$$+\frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i}\,,$$ (Gauss Law)

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \underline{\frac{1}{a\Lambda}}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Constraint)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)$$

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)}\right)$$

$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\underline{\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i}\,, \quad \text{(Gauss Law)}$$

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \underline{\frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B} \quad \text{(Gauss Constraint)}$$

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

Continuum

EoM

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Constraint)

LATTICE FORMULATION of $\phi F\tilde{F}$

Continuum

EoM

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Constraint)

$$\frac{\ddot{a}}{a} = \frac{-1}{6m_{\rm pl}^2}(3\overline{p}+\overline{\rho})\ , \qquad \rho \equiv \frac{1}{2}\pi_\phi^2 + \frac{1}{2a^2}\sum_i(\partial_i\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\left(\sum_i\frac{E_i^2}{a^2} + \sum_i\frac{B_i^2}{a^4}\right)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3m_{\rm pl}^2}\overline{\rho}\ , \qquad p \equiv \frac{1}{3a^2}\sum_j T_{jj} = \frac{1}{2}\pi_\phi^2 - \frac{1}{6a^2}\sum_i(\partial_i\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{6}\left(\sum_i\frac{E_i^2}{a^2} + \sum_i\frac{B_i^2}{a^4}\right)$$

(Hubble law)

LATTICE FORMULATION of $\phi F\tilde{F}$

Lattice Formulation

EoM

$$\begin{aligned}
\Delta_0^+\left(a^3\pi_\phi\right) &= a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}} m^2\phi_{+\frac{\hat{0}}{2}} \\
&\quad + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)\,, \\
\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right) \\
&\quad + \frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\} \\
a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\,, \quad \text{(Gauss Law)}
\end{aligned}$$

LATTICE FORMULATION of $\phi F\tilde{F}$

Lattice Formulation

EoM

$$
\begin{aligned}
\Delta_0^+\left(a^3\pi_\phi\right) &= a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} \\
&\quad + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right), \\
\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right) \\
&\quad + \frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^{\pm}\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^{\pm}\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\} \\
a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^{\pm}\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}, \quad \text{(Gauss Law)}
\end{aligned}
$$

Expansion

$$
\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 = \frac{a^2}{3m^2_{\rm pl}}\rho_L\,,
$$

$$
\Delta_0^-\Delta_0^+ a_{+\hat{0}/2} = -\frac{a_{+\hat{0}/2}}{6m^2_{\rm pl}}(\rho_L + 3p_L)_{+\hat{0}/2}
$$

LATTICE FORMULATION of $\phi F\tilde{F}$

Lattice Formulation

EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right),$$

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^-B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)}\right) + \frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i},\quad \text{(Gauss Law)}$$

Expansion

$$\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 = \frac{a^2}{3m_{\rm pl}^2}\rho_L\,,$$

$$\Delta_0^-\Delta_0^+ a_{+\hat{0}/2} = -\frac{a_{+\hat{0}/2}}{6m_{\rm pl}^2}(\rho_L + 3p_L)_{+\hat{0}/2}$$

$$\bar{H}^{\rm kin} = \frac{1}{N^3}\sum_{\vec{n}}\frac{\pi_\phi^2}{2}\,,\qquad \bar{H}^{\rm grad} = \frac{1}{N^3}\sum_{\vec{n}}\frac{1}{2}\sum_i(\Delta_i^+\phi_{+\frac{\hat{0}}{2}})^2\,,\qquad \bar{H}^{\rm pot} = \frac{1}{N^3}\sum_{\vec{n}}\frac{1}{2}m^2\phi^2_{+\frac{\hat{0}}{2}}$$

$$\bar{H}^E = \frac{1}{N^3}\sum_{\vec{n}}\sum_i\frac{1}{2}E^2_{i,+\frac{\hat{0}}{2}}\,,\qquad \bar{H}^B = \frac{1}{N^3}\sum_{\vec{n}}\sum_{i,j}\frac{1}{4}(\Delta_i^+A_j - \Delta_j^+A_i)^2\,,$$

$$\rho_L = \bar{H}^{\rm kin} + \frac{1}{a^2}\frac{1}{2}(\bar{H}^{\rm grad}_{-\hat{0}/2} + \bar{H}^{\rm grad}_{+\hat{0}/2}) + \frac{1}{2}(\bar{H}^{\rm pot}_{-\hat{0}/2} + \bar{H}^{\rm pot}_{+\hat{0}/2}) + \frac{1}{a^2}\frac{1}{2}(\bar{H}^E_{-\hat{0}/2} + \bar{H}^E_{+\hat{0}/2}) + \frac{1}{a^4}\bar{H}^B\,,$$

$$(\rho_L + 3p_L)_{+\hat{0}/2} = 2(\bar{H}^{\rm kin} + \bar{H}^{\rm kin}_{+\hat{0}}) - 2\bar{H}^{\rm pot}_{+\hat{0}/2} + \frac{2}{a^2_{+\hat{0}/2}}\bar{H}^E + \frac{1}{a^4_{+\hat{0}/2}}(\bar{H}^B + \bar{H}^B_{+\hat{0}})\,,$$

Part 2

Axion-Inflation Preheating

Axion-Inflation Preheating

Initial Condition: 1-2 efolds before end Inflation

Preparation: Much before, Bunch-Davies Vacuum @ sub-Hubble

$$A_+(k_i, t_i) \simeq \frac{e^{-i\omega_i t_i}}{\sqrt{2a(t_i)\omega_i}} \equiv \frac{1}{\sqrt{2k_i}}\left[\cos(\omega_i t_i) - i\sin(\omega_i t_i)\right]$$

$$\dot{A}_+(k_i, t_i) \simeq -i\frac{\omega_i}{\sqrt{2k_i}}e^{-i\omega_i t_i} \equiv -\frac{1}{a(t_i)}\sqrt{\frac{k_i}{2}}\left[\sin(\omega_i t_i) + i\cos(\omega_i t_i)\right]$$

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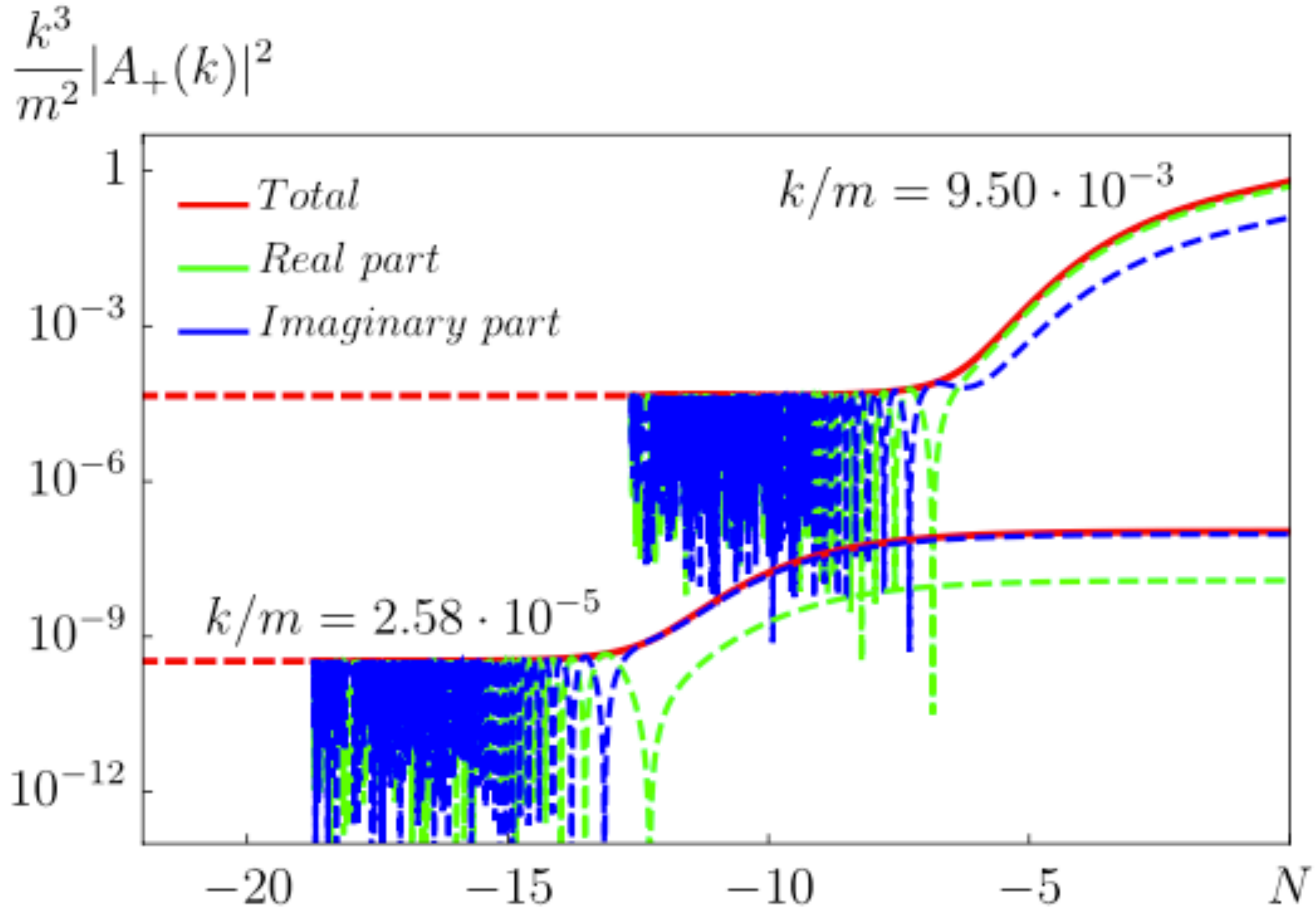
$$\dot{A}_+(k_i, t_i) \simeq -i\frac{\omega_i}{\sqrt{2k_i}} e^{-i\omega_i t_i} \equiv -\frac{1}{a(t_i)}\sqrt{\frac{k_i}{2}} \left[\sin(\omega_i t_i) + i\cos(\omega_i t_i)\right]$$

$$\boxed{\ddot{A}_\pm(t,k) + \left(\frac{\dot{a}}{a}\right)\dot{A}_\pm(t,k) + \left[\frac{k^2}{a^2} \pm \left(\frac{k\dot{\phi}}{a\Lambda}\right)\right] A_\pm(t,k) = 0}$$

Axion-Inflation Preheating

Initial Condition: 1-2 efolds before end Inflation

Preparation: Much before, Bunch-Davies Vacuum @ sub-Hubble



Axion-Inflation Preheating

Initial Condition: 1-2 efolds before end Inflation

Preparation: Much before, Bunch-Davies Vacuum @ sub-Hubble

$$\ddot{A}_\pm(t,k) + \left(\frac{\dot{a}}{a}\right)\dot{A}_\pm(t,k) + \left[\frac{k^2}{a^2} \pm \left(\frac{k\dot{\phi}}{a\Lambda}\right)\right]A_\pm(t,k) = 0$$

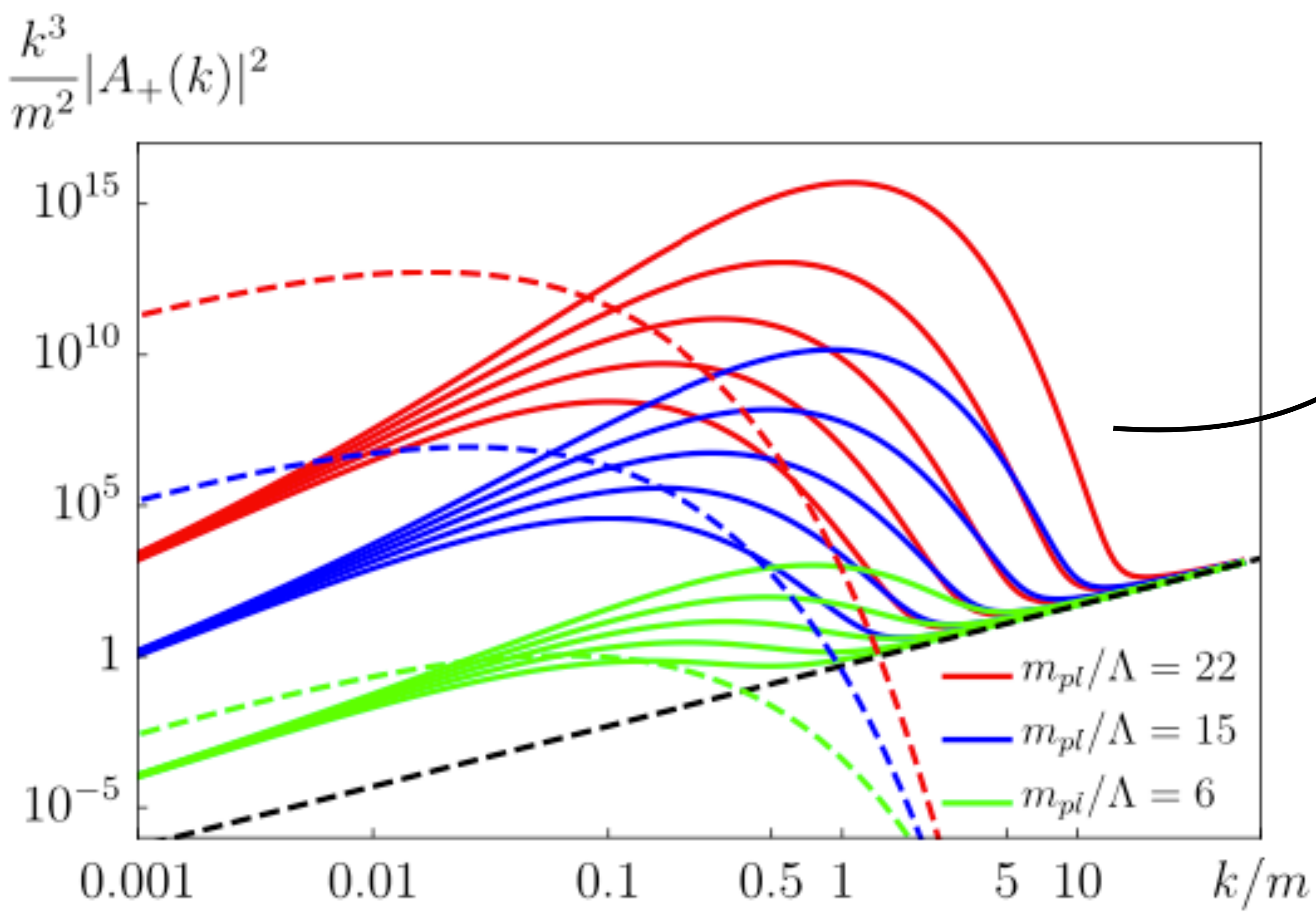
If dominates:
Deep inside
Hubble radius
(Oscillations @ inflation)

If dominates:
Tachyonic IR
Instability
Both at Inflation and preheating !

Axion-Inflation Preheating

Initial Condition: 1-2 efolds before end Inflation

Preparation: Much before, Bunch-Davies Vacuum @ sub-Hubble

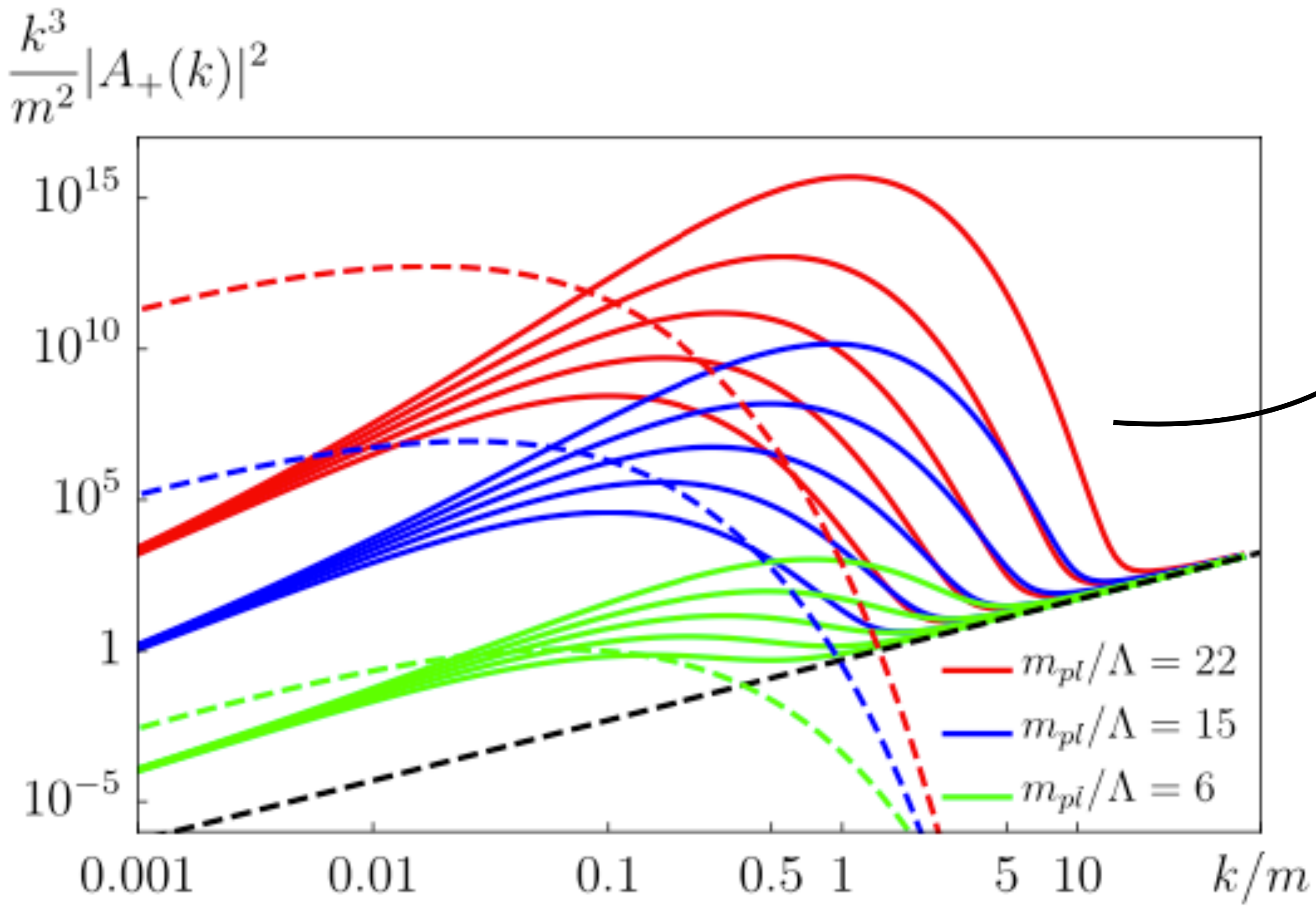


Backreaction-less Solutions

Axion-Inflation Preheating

Initial Condition: 1-2 efolds before end Inflation

Preparation: Much before, Bunch-Davies Vacuum @ sub-Hubble



Backreaction-less Solutions

Random Numbers: Create 3d random realization of $A_\mu(\mathbf{k})$

Axion-Inflation Preheating

Initial Condition: 1-2 efolds before end Inflation

Preparation: Much before, Bunch-Davies Vacuum @ sub-Hubble

Random Numbers: Create 3d random realization of $A_\mu(\mathbf{k})$

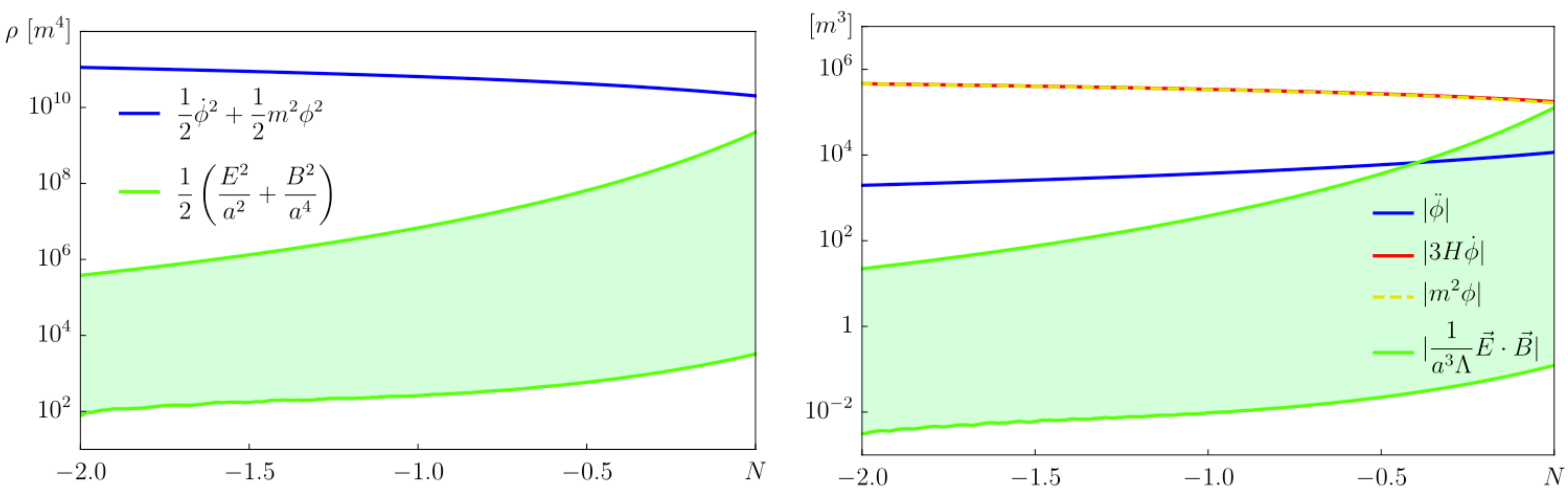
Axion-Inflation Preheating

Initial Condition: 1-2 efolds before end Inflation

Preparation: Much before, Bunch-Davies Vacuum @ sub-Hubble

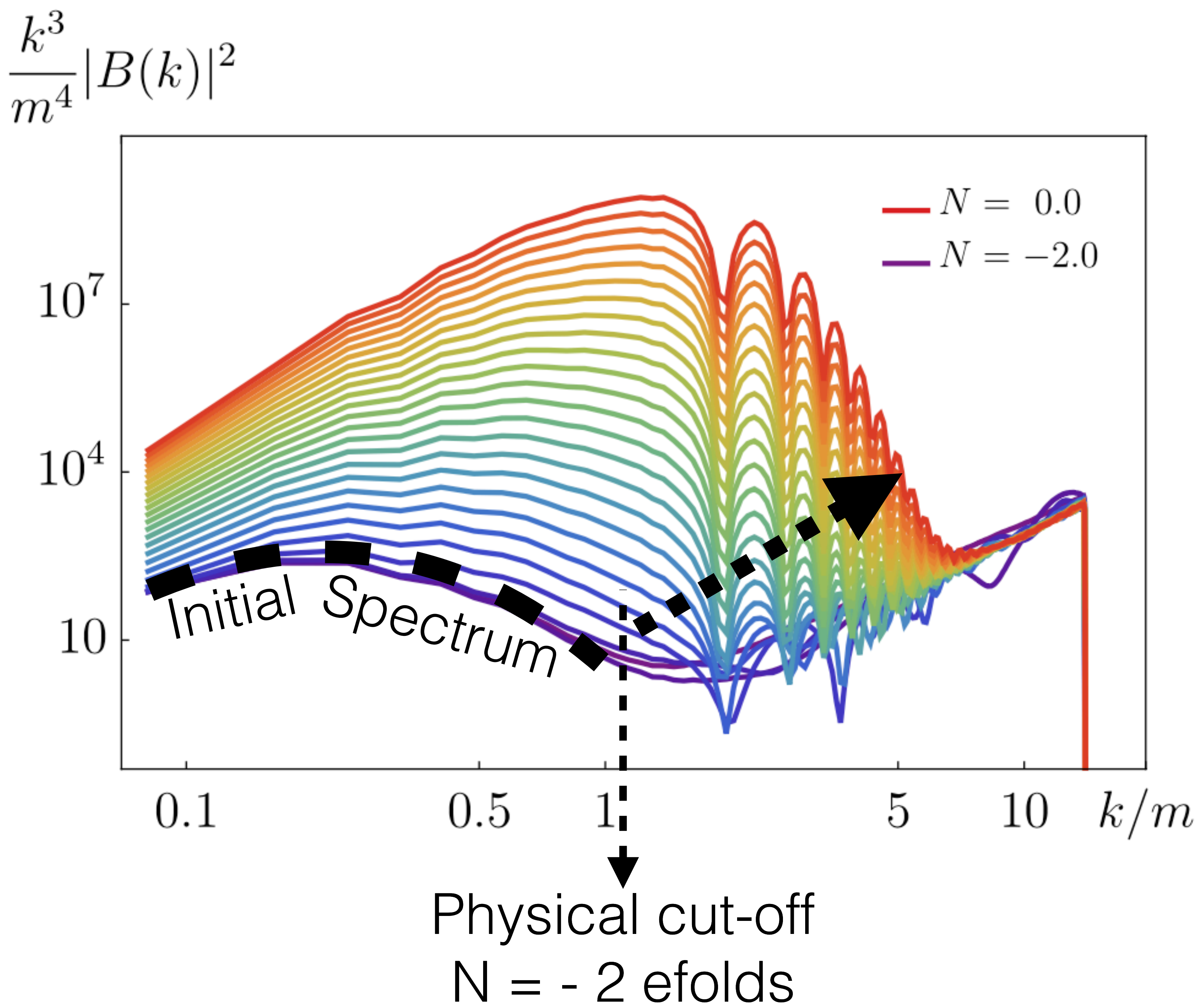
Random Numbers: Create 3d random realization of $A_\mu(\mathbf{k})$

Sub-dominance: Gauge fields must be completely negligible



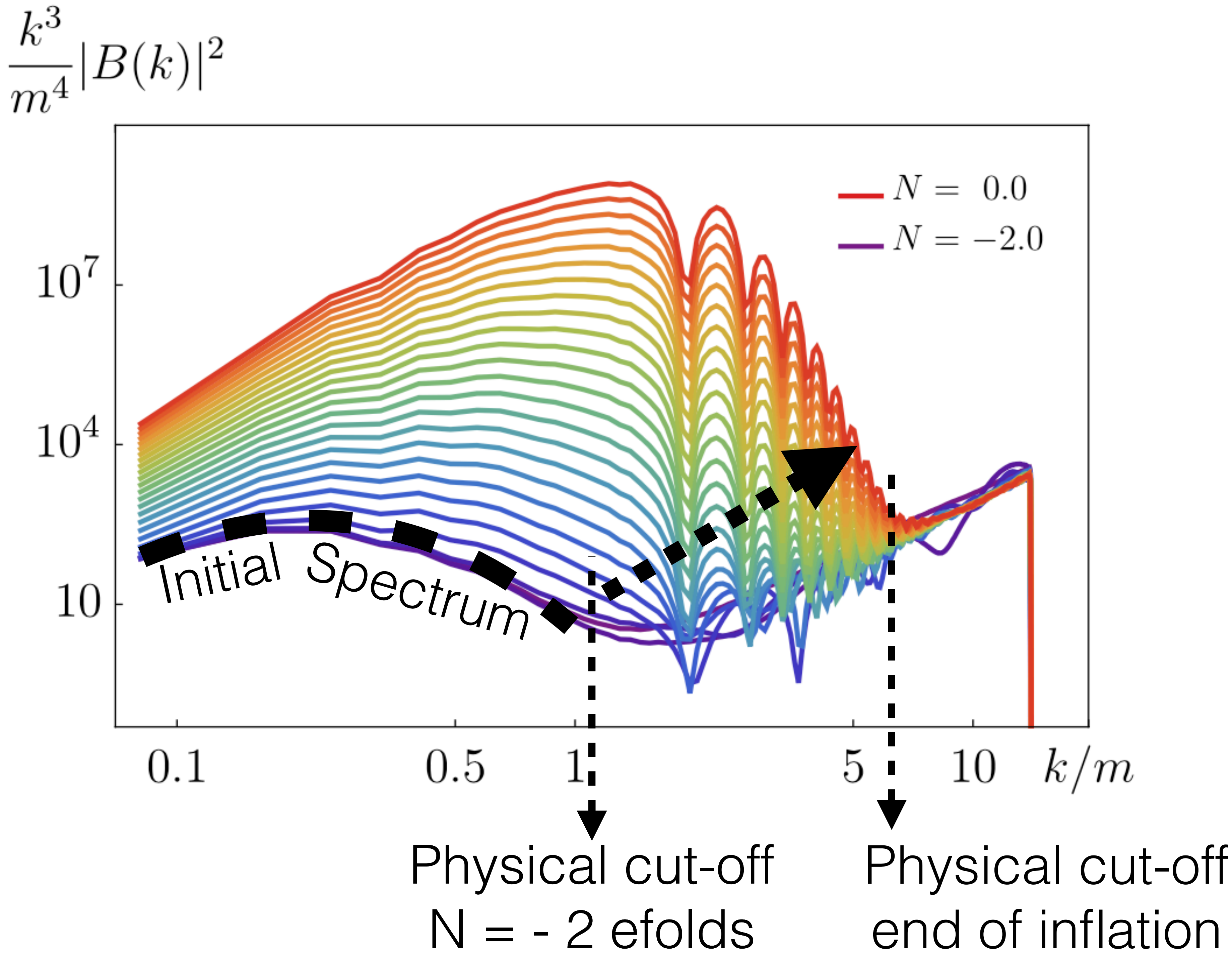
Axion-Inflation Preheating

Initial Cut-off Problem: Separate IR from UV modes



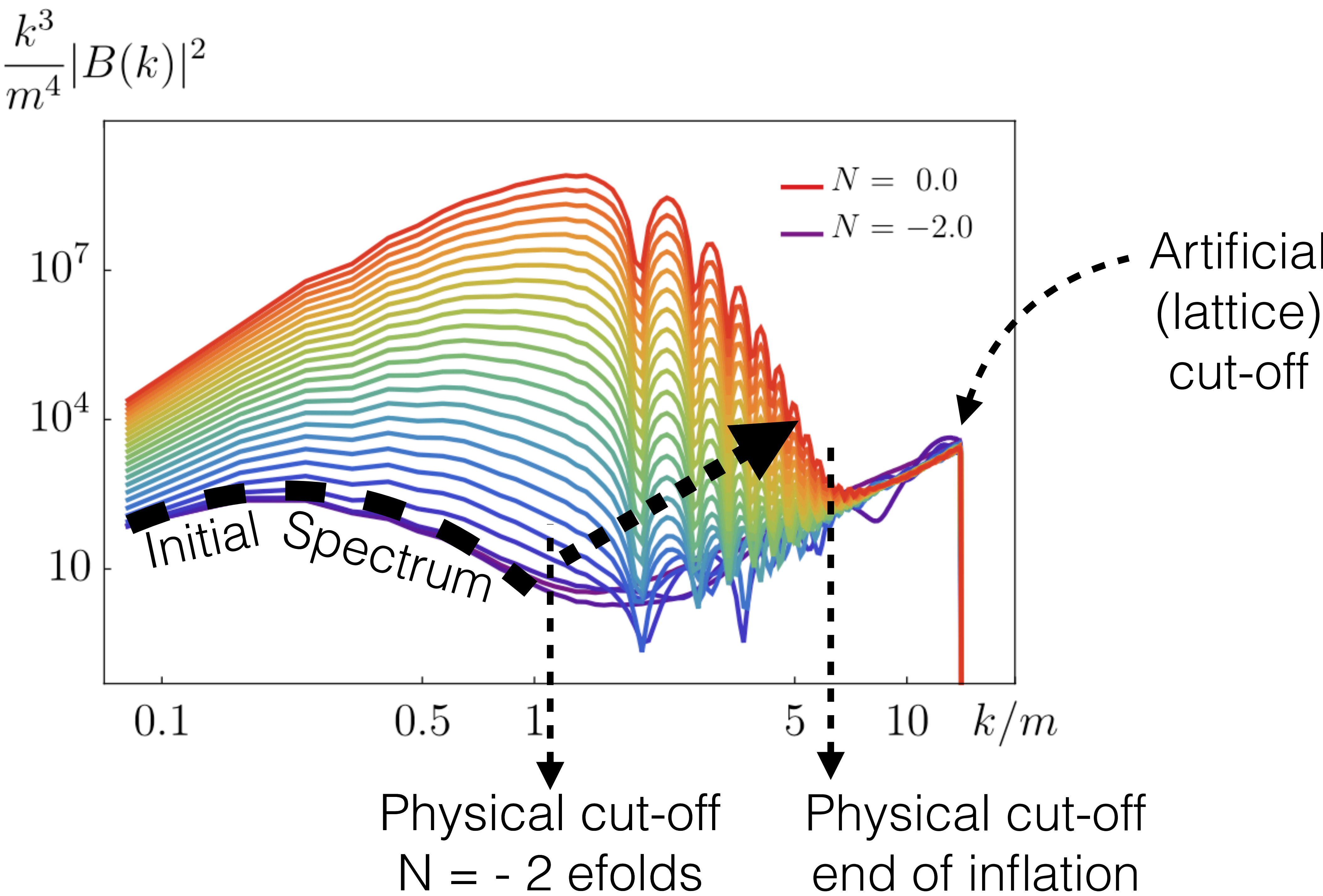
Axion-Inflation Preheating

Initial Cut-off Problem: Separate IR from UV modes



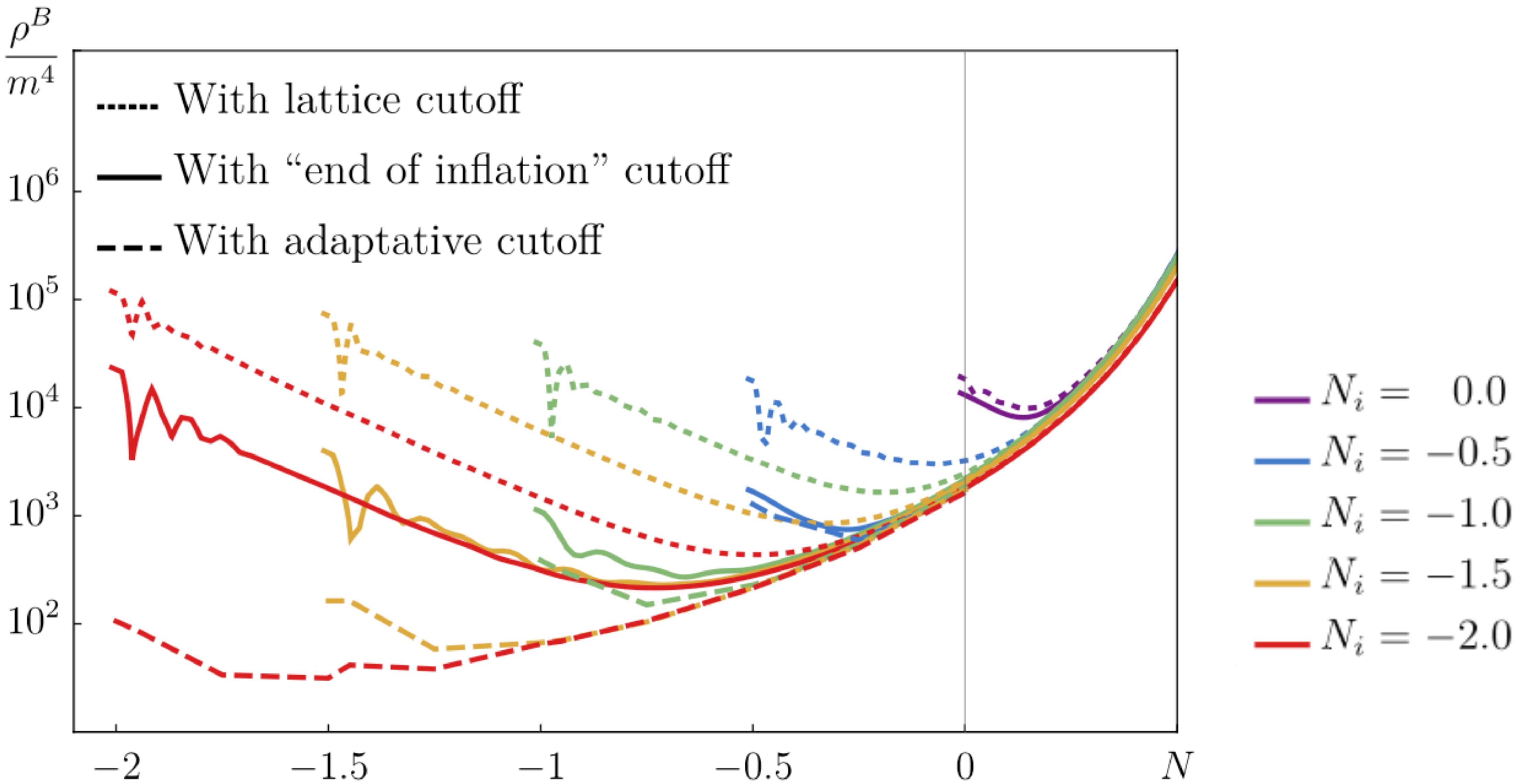
Axion-Inflation Preheating

Initial Cut-off Problem: Separate IR from UV modes



Axion-Inflation Preheating

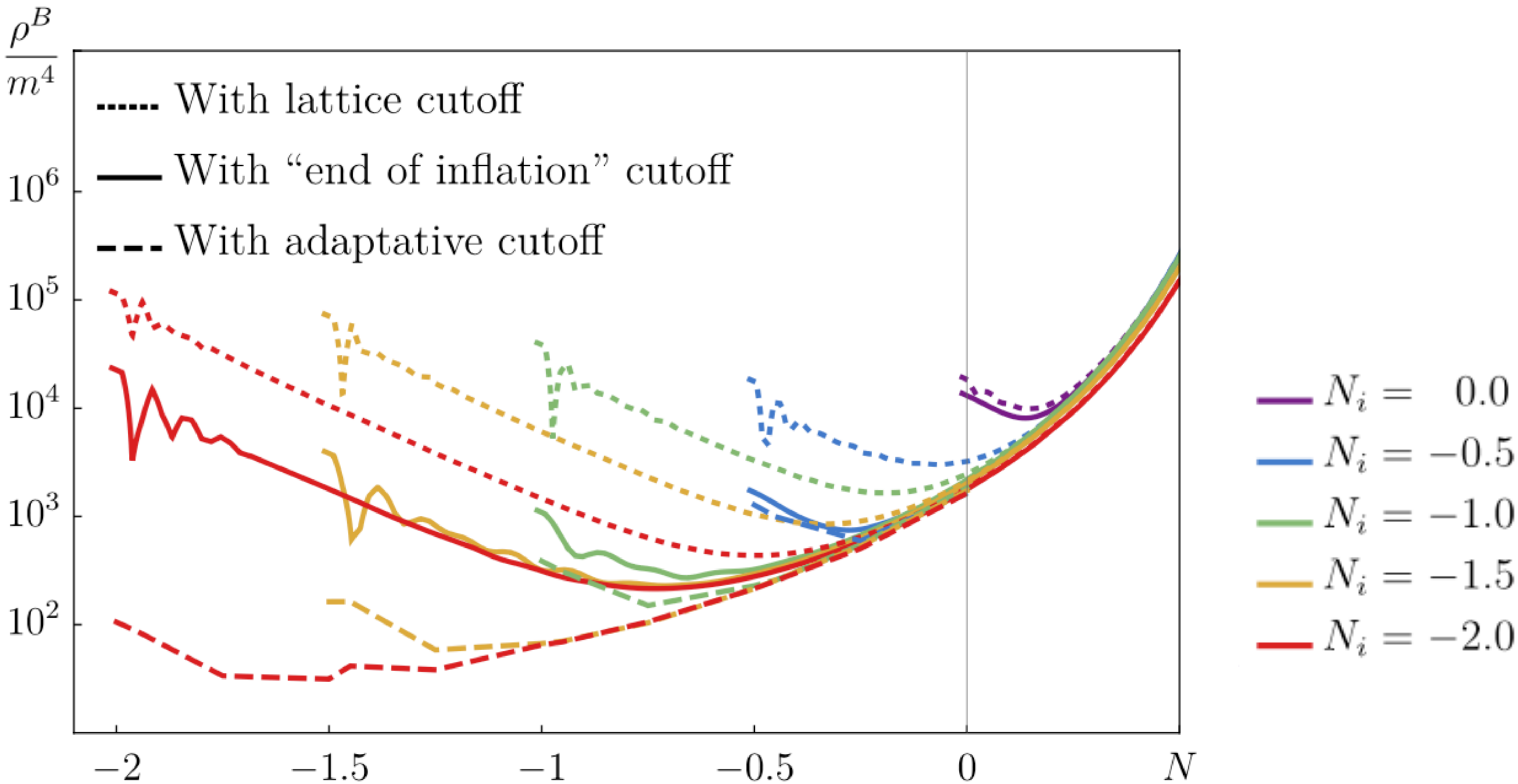
Trajectories should overlap: Starting at different times



Lattice cut-off: Trajectories don't overlap ✗

Axion-Inflation Preheating

Trajectories should overlap: Starting at different times

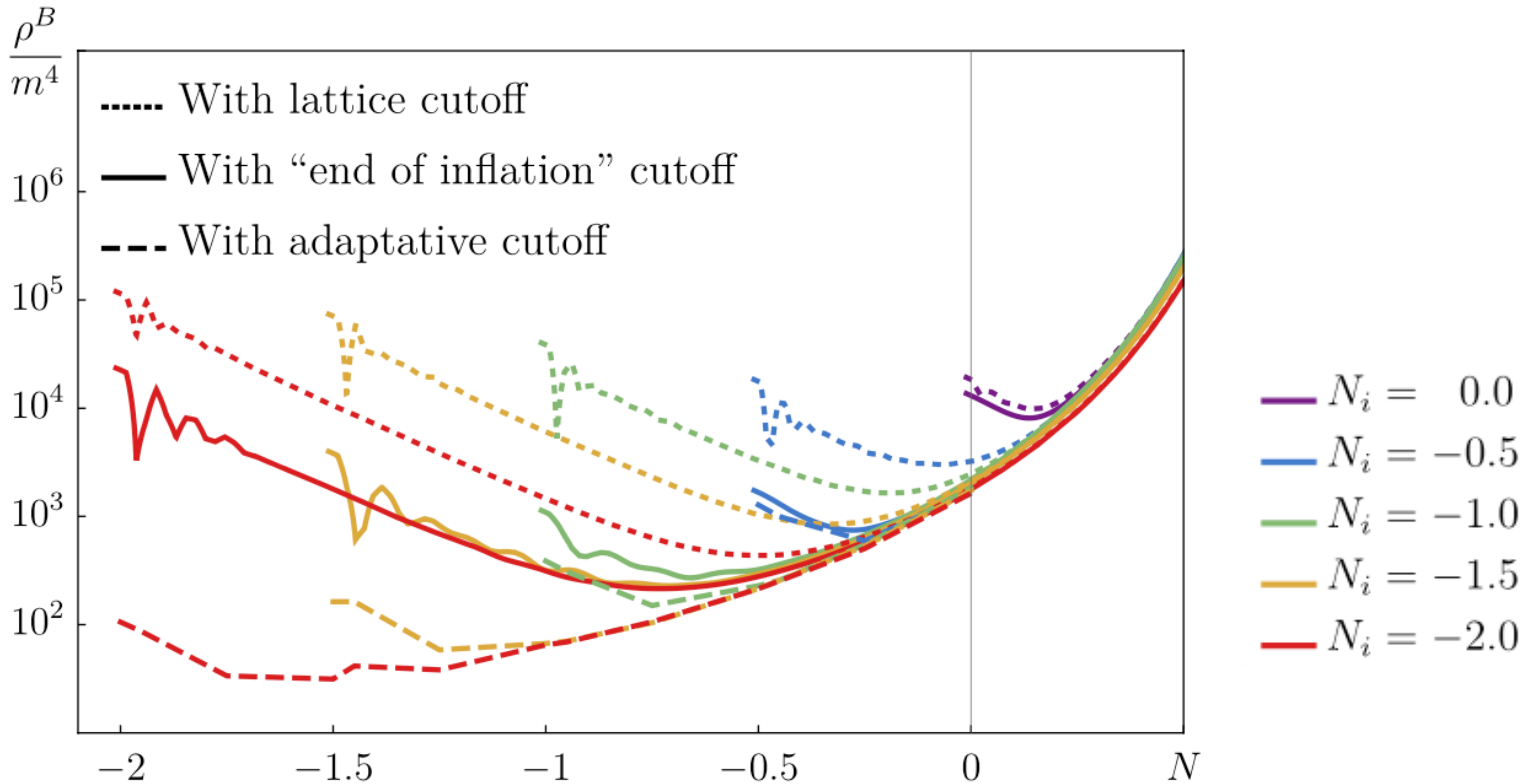


Lattice cut-off: Trajectories don't overlap 

End inflation cut-off: Trajectories overlap, excess energy in the UV 

Axion-Inflation Preheating

Trajectories should overlap: Starting at different times



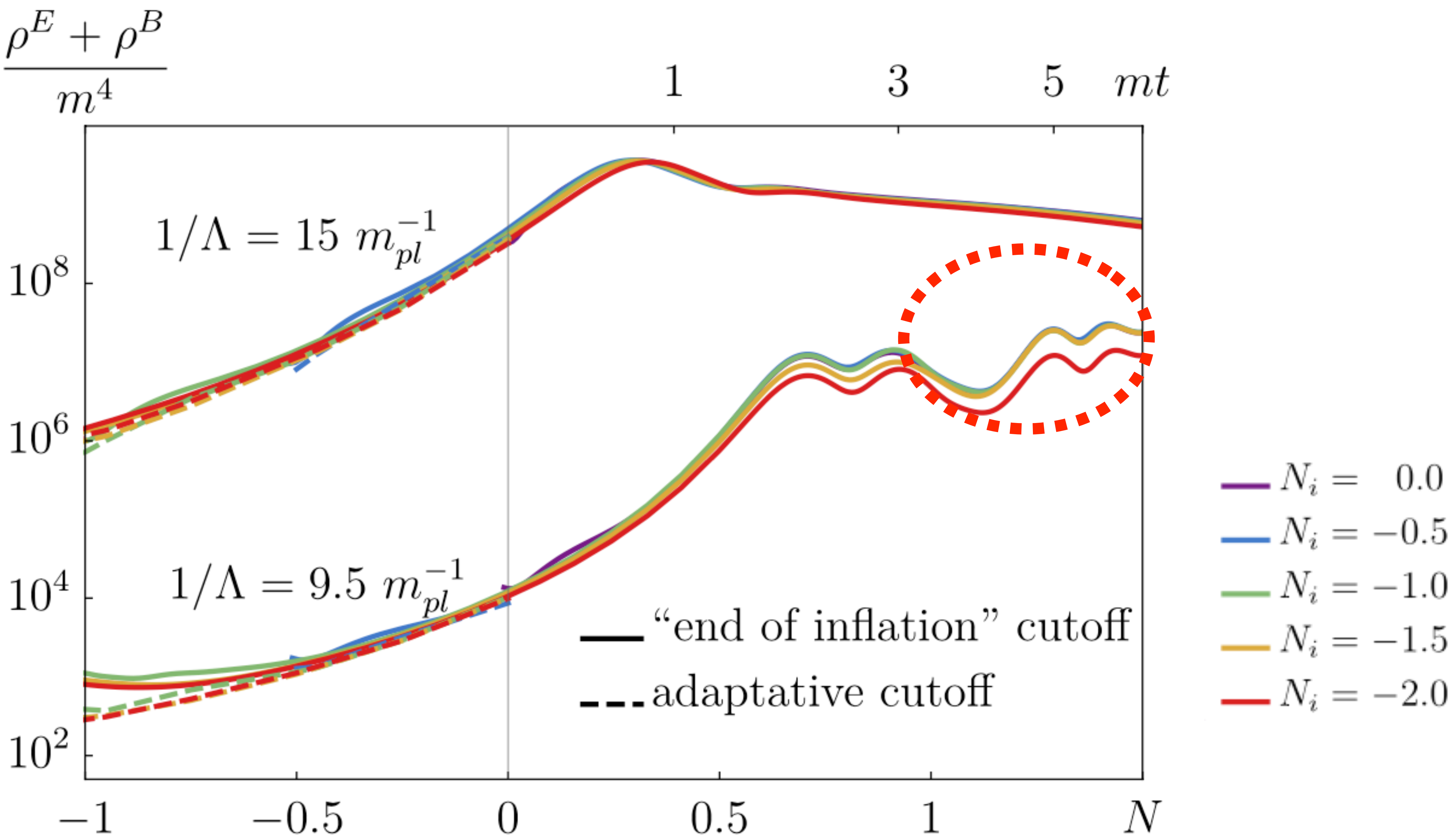
Lattice cut-off: Trajectories don't overlap

End inflation cut-off: Trajectories overlap, excess energy in the UV

Adaptative cut-off: Trajectories overlap, excess UV energy removed !

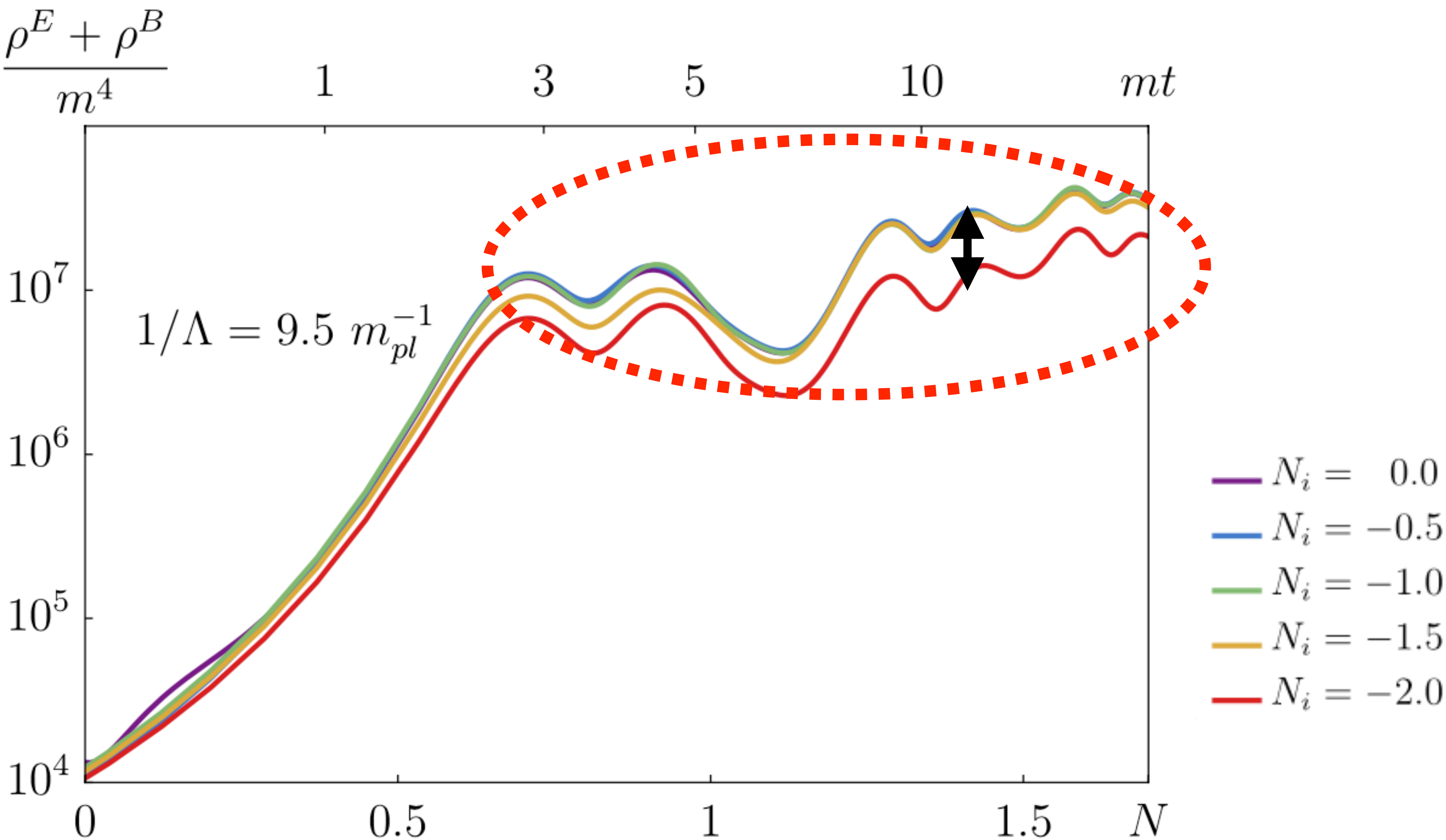
Axion-Inflation Preheating

Choice of initial moment: how many efolds before inflation ends ?



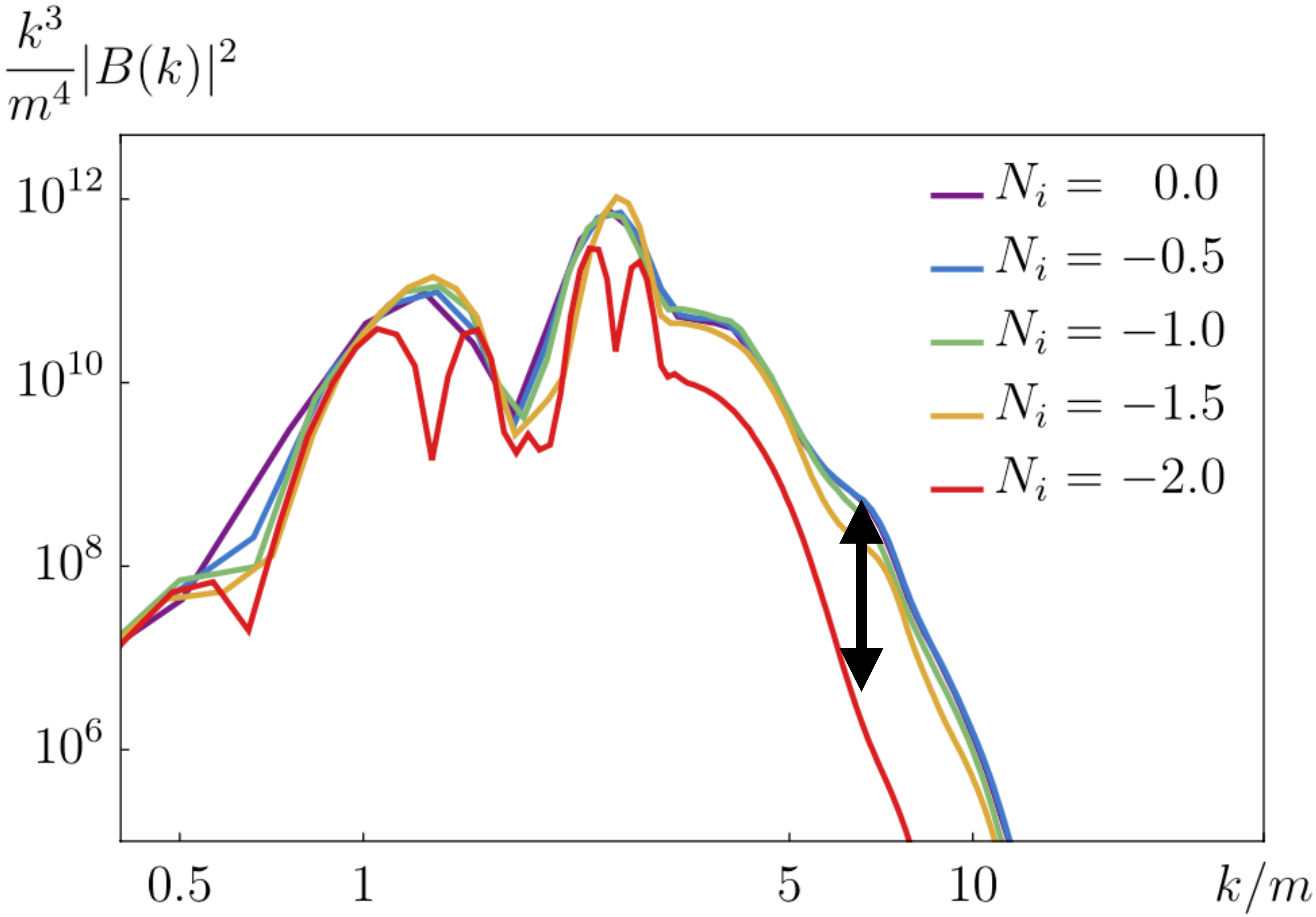
Axion-Inflation Preheating

Choice of initial moment: how many efolds before inflation ends ?



Axion-Inflation Preheating

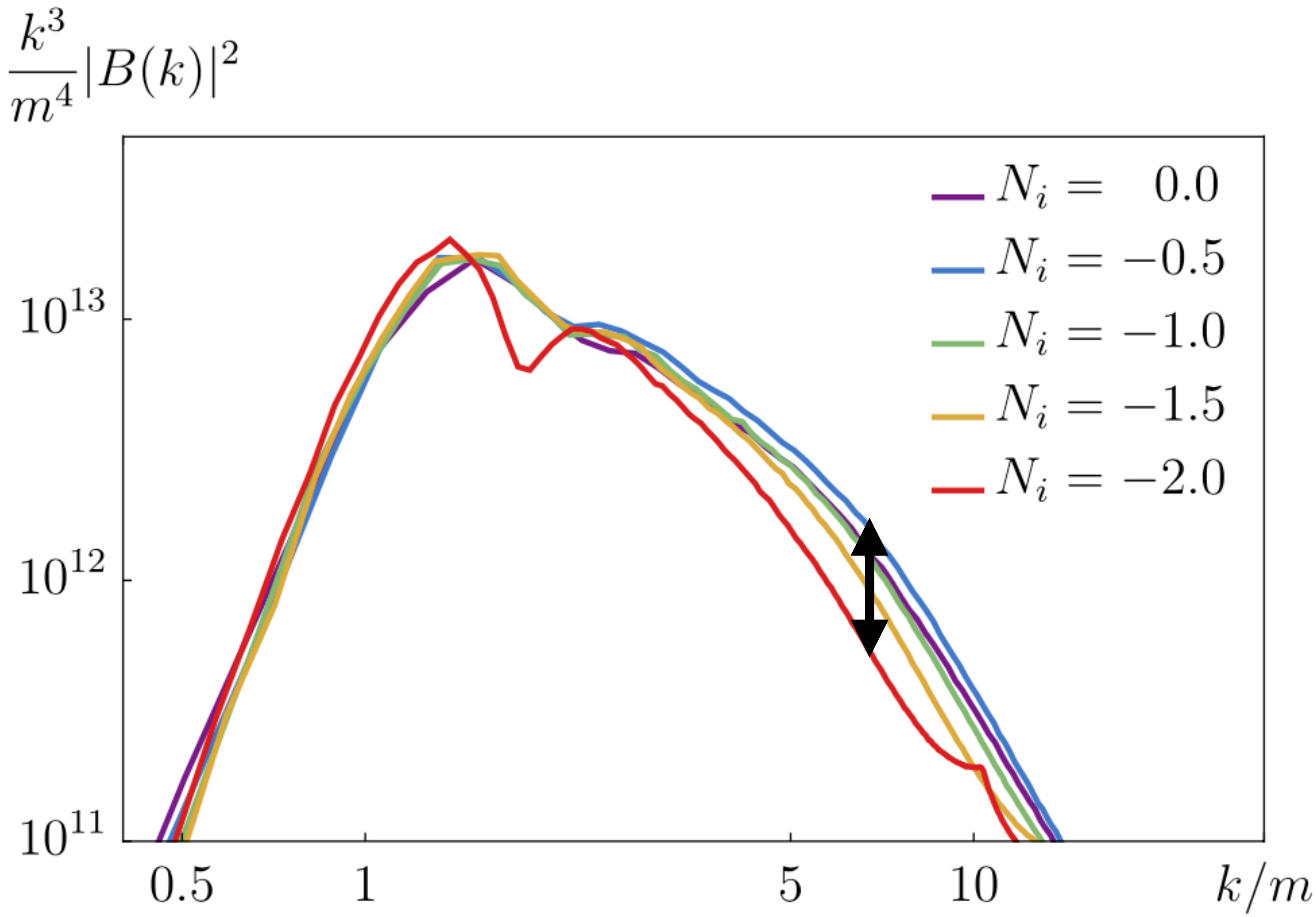
Choice of initial moment: how many efolds before inflation ends ?



$$1/\Lambda = 9.5\, m_{\rm pl}^{-1}$$

Axion-Inflation Preheating

Choice of initial moment: how many efolds before inflation ends ?



$$1/\Lambda = 15\, m_{\rm pl}^{-1}$$

Axion-Inflation Preheating

Overlapping trajectories

Gauge energy negligible

IR-UV finite range (N = 256)

Choice of Couplings:

$1/\Lambda \sim 6\, m_{\rm pl}^{-1} - 15\, m_{\rm pl}^{-1}$

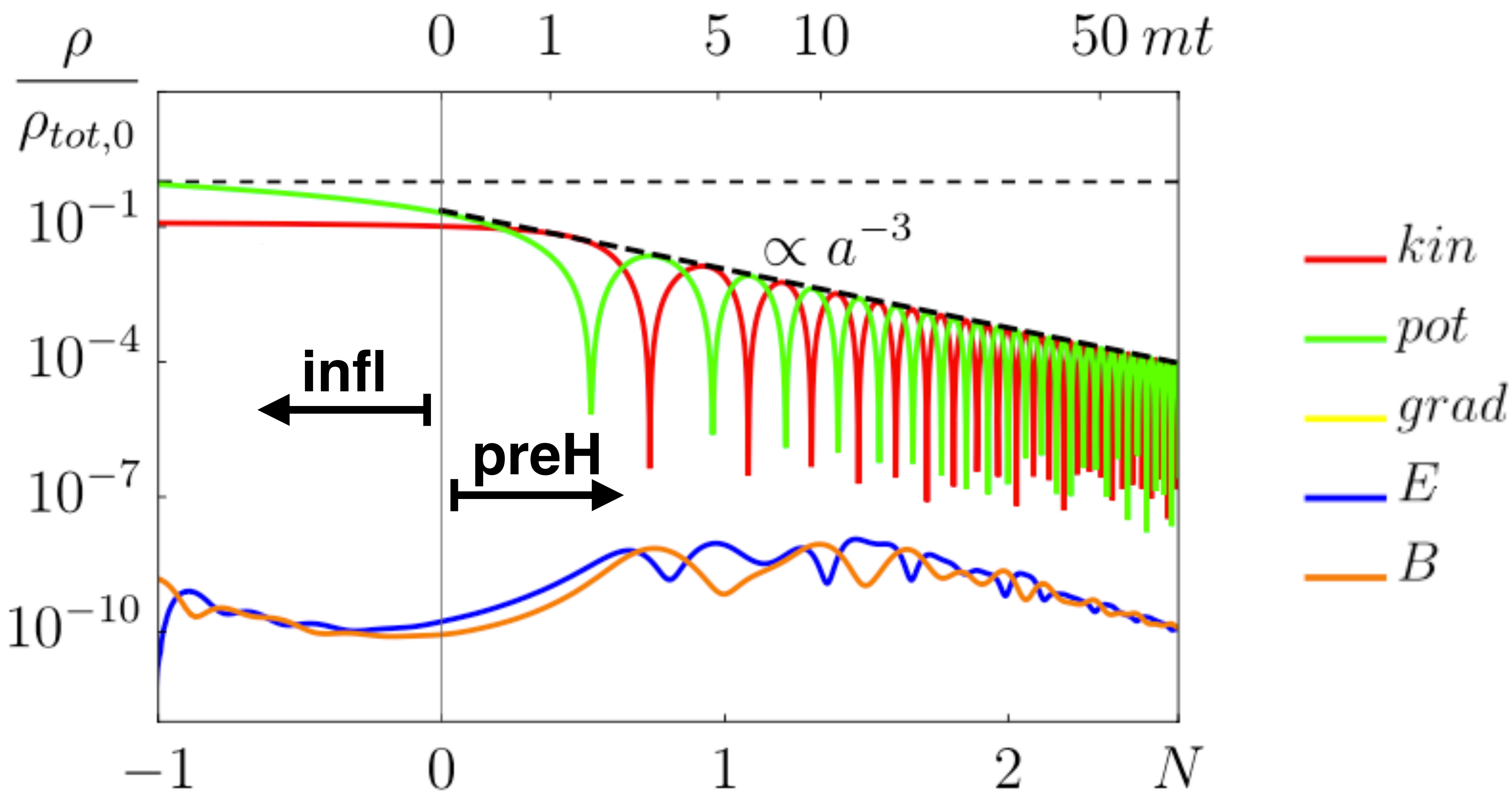
Initial moment:

N = -1.5 — -1.0

Let’s see the dynamics !

Axion-Inflation Preheating

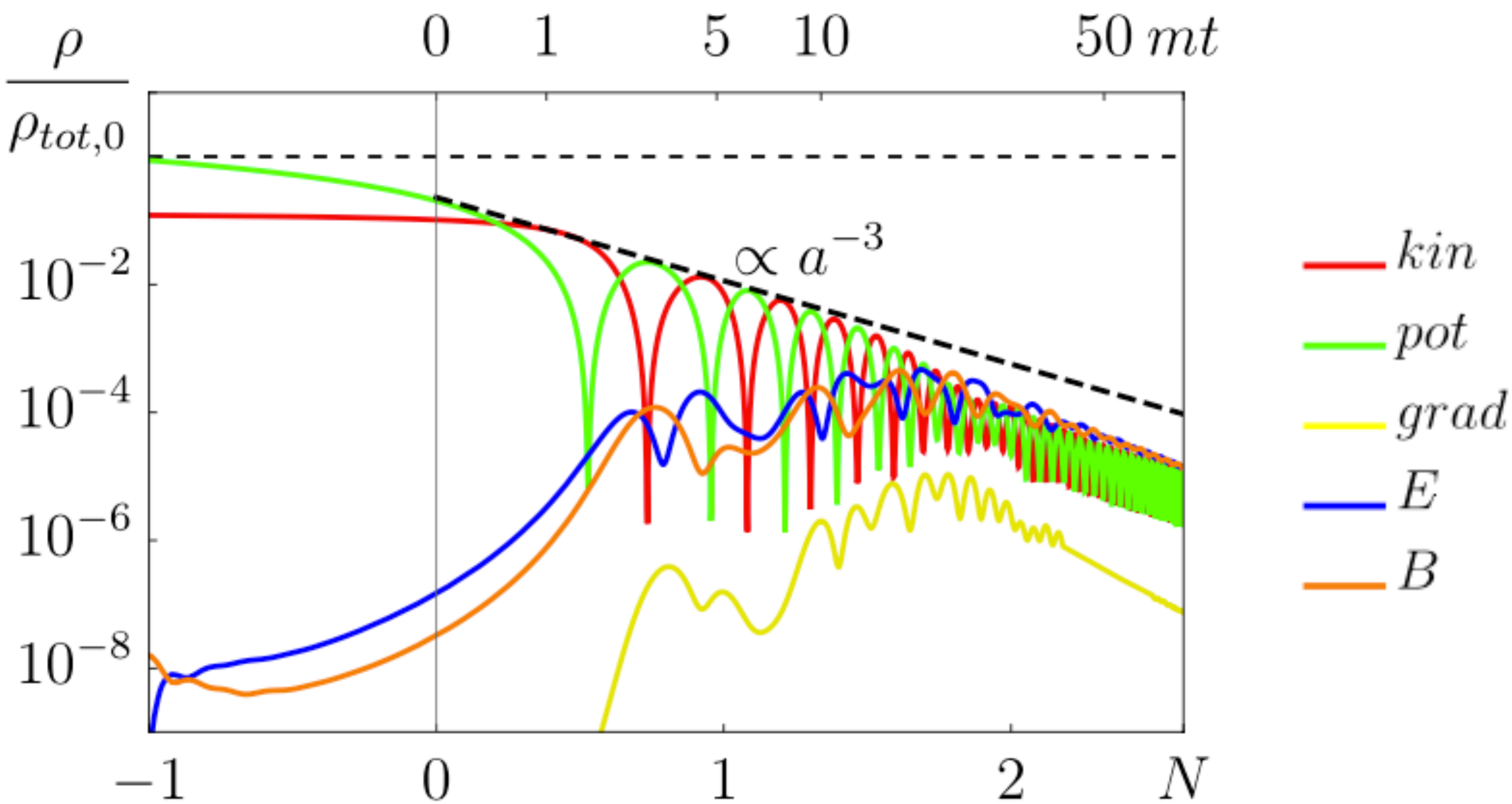
Energy densities



(a) $1/\Lambda = 6\,m_{\rm pl}^{-1}$ (**Weak coupling**)

Axion-Inflation Preheating

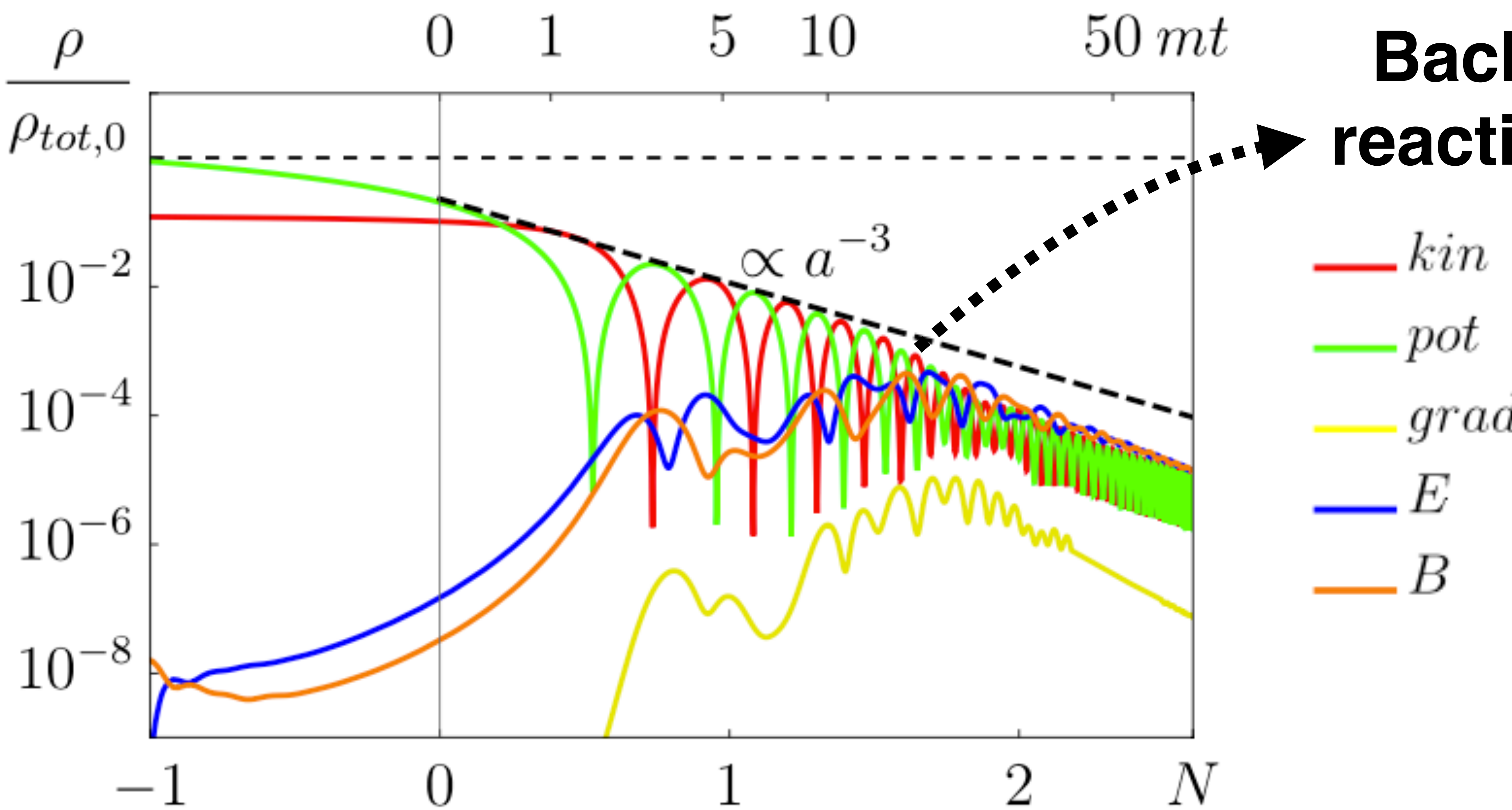
Energy densities



(b) $1/\Lambda = 9.5\, m_{\rm pl}^{-1}$ (**Intermediate coupling**)

Axion-Inflation Preheating

Energy densities



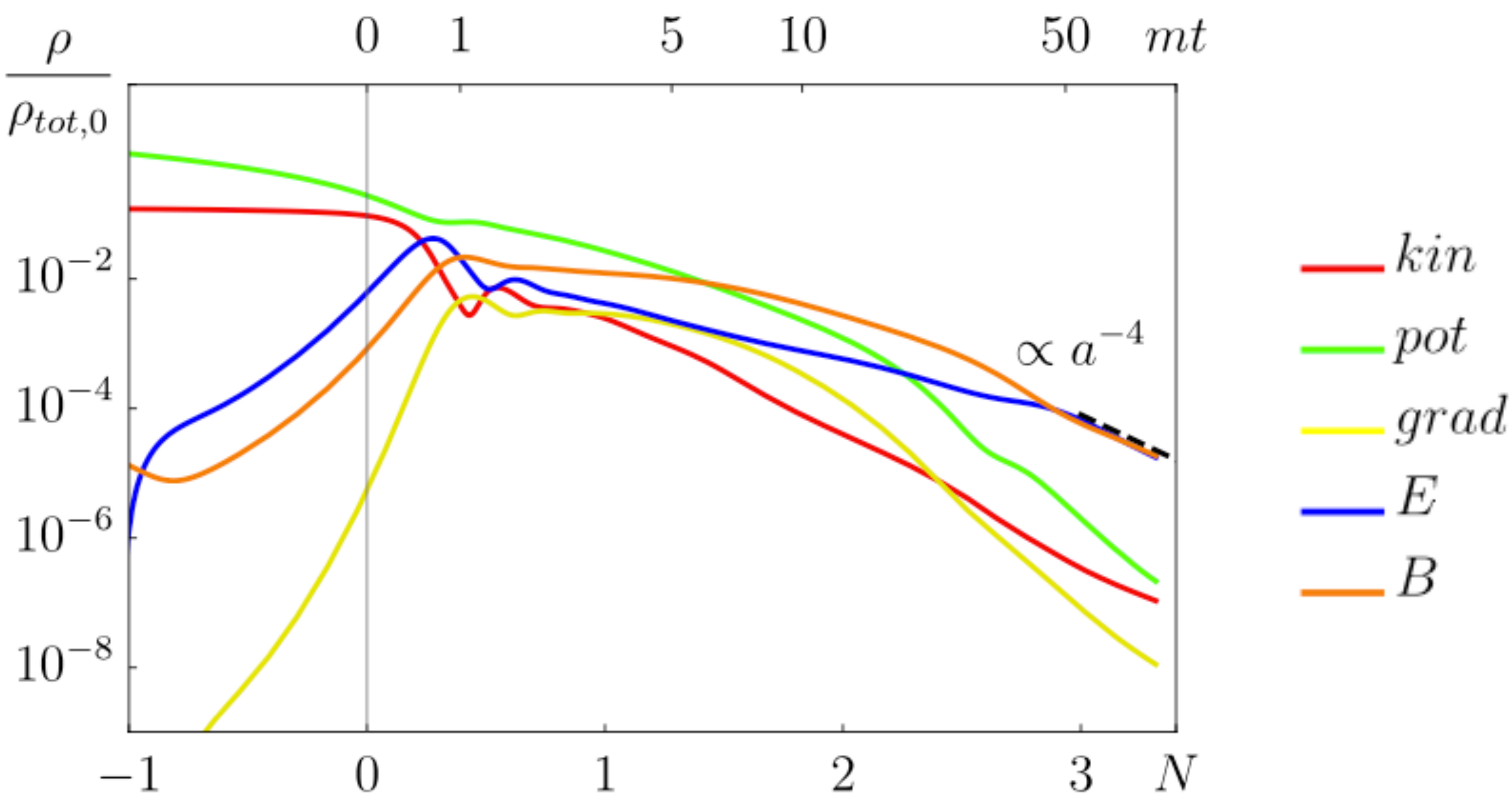
Back-reaction

(b) $1/\Lambda = 9.5\, m_{\rm pl}^{-1}$

(**Intermediate coupling**)

Axion-Inflation Preheating

Energy densities

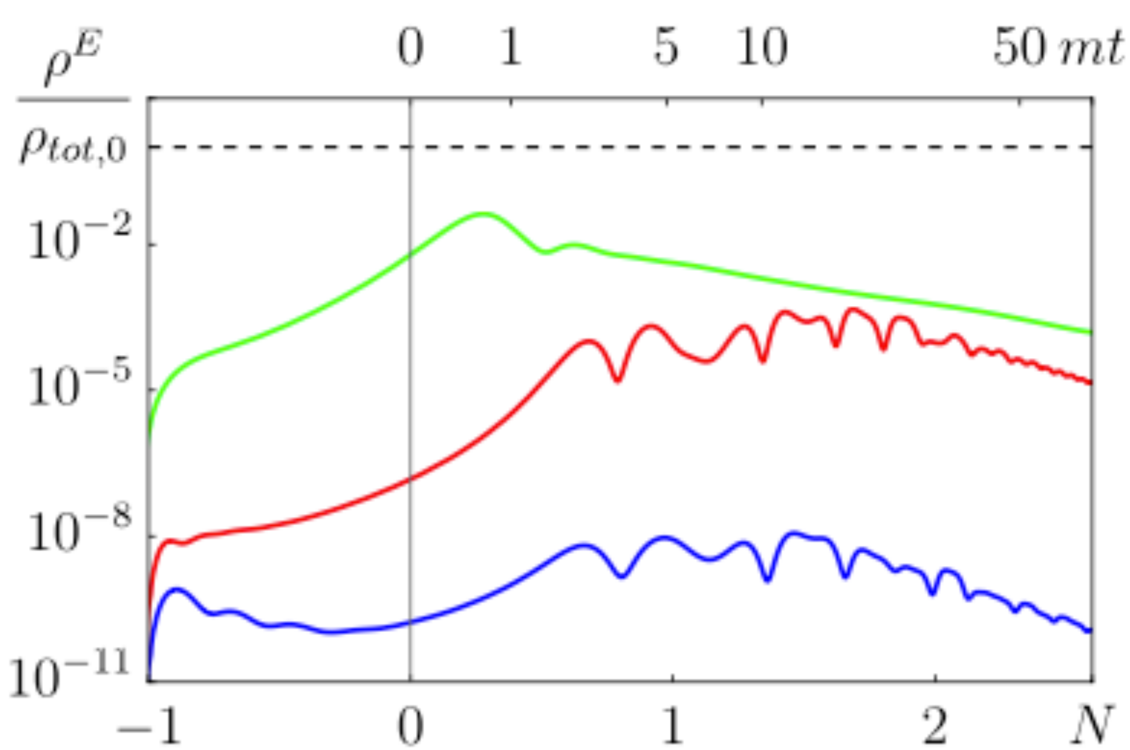


(c) $1/\Lambda = 15\, m_{\rm pl}^{-1}$

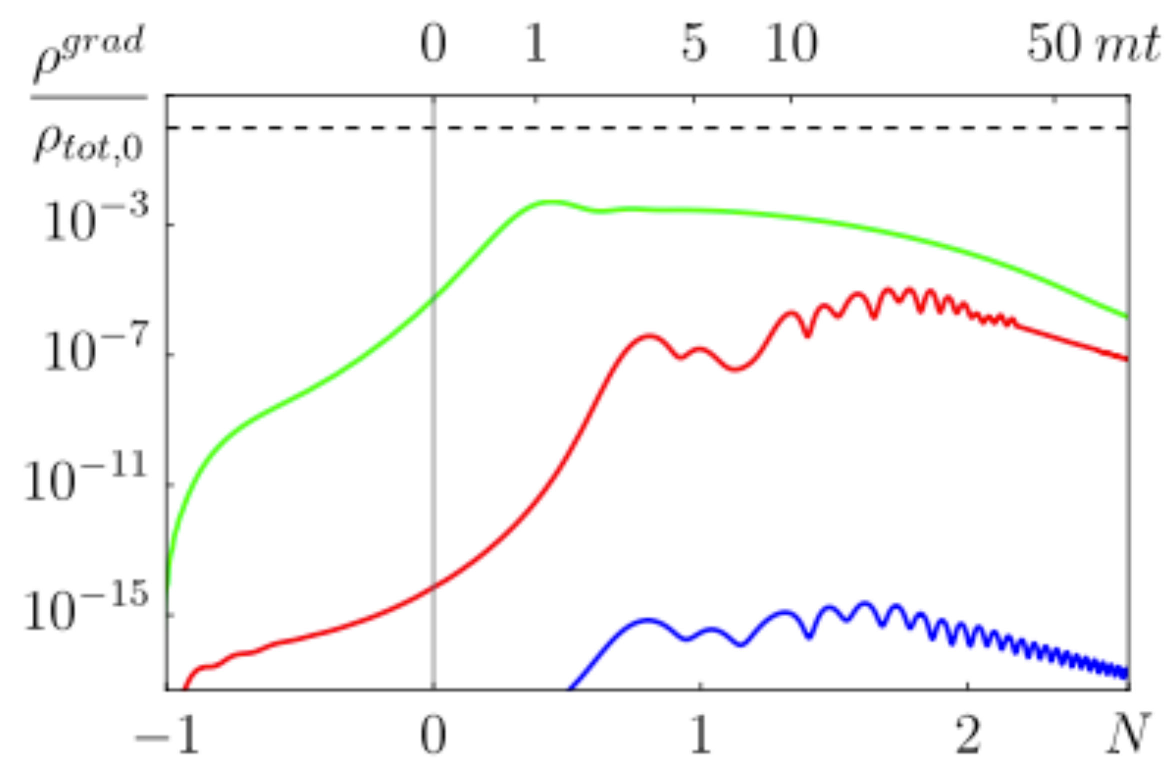
(**Strong coupling**)

Axion-Inflation Preheating

Energy densities



Electric gauge fld energy



Axion gradient energy

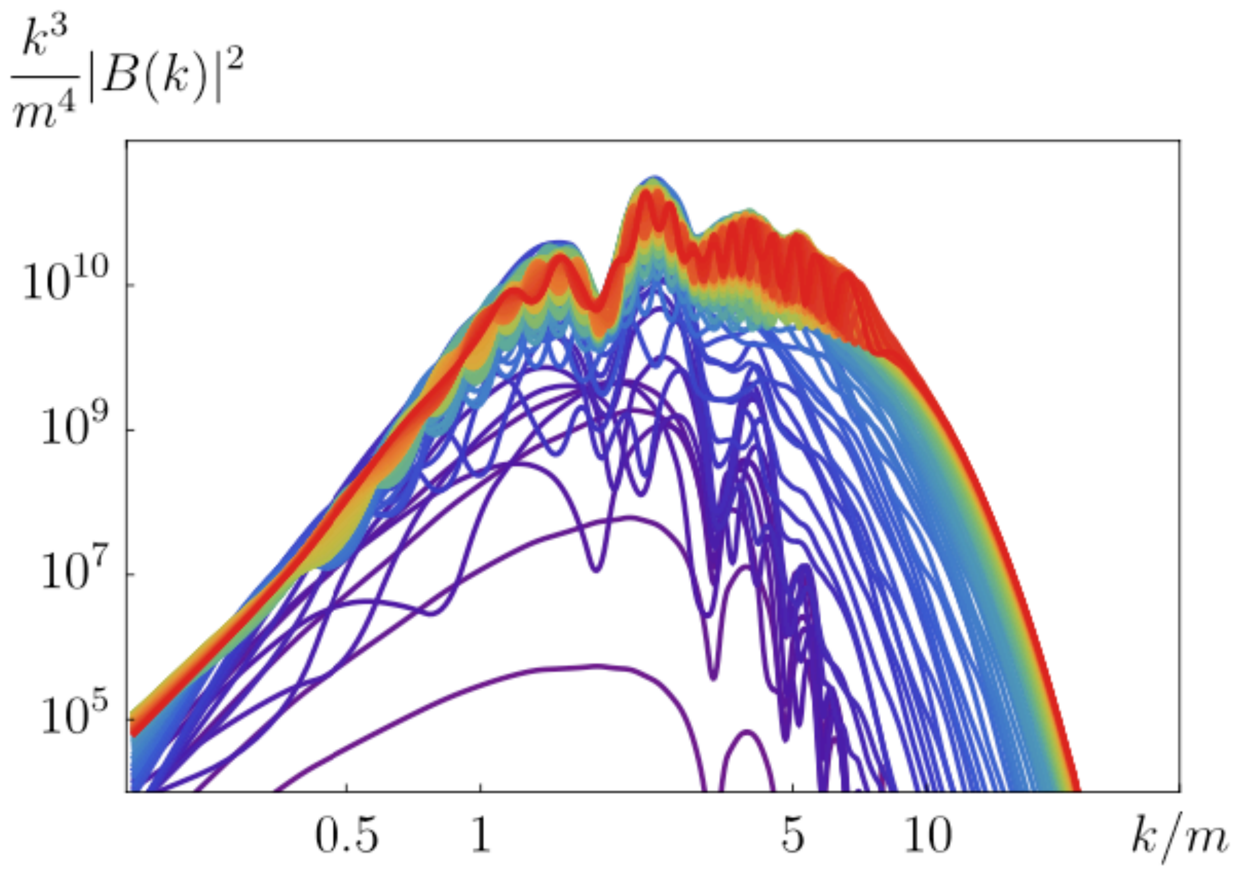
$1/\Lambda = 15\ m_{pl}^{-1}$ (strong)

$1/\Lambda = 9.5\ m_{pl}^{-1}$ (mild)

$1/\Lambda = 6\ m_{pl}^{-1}$ (weak)

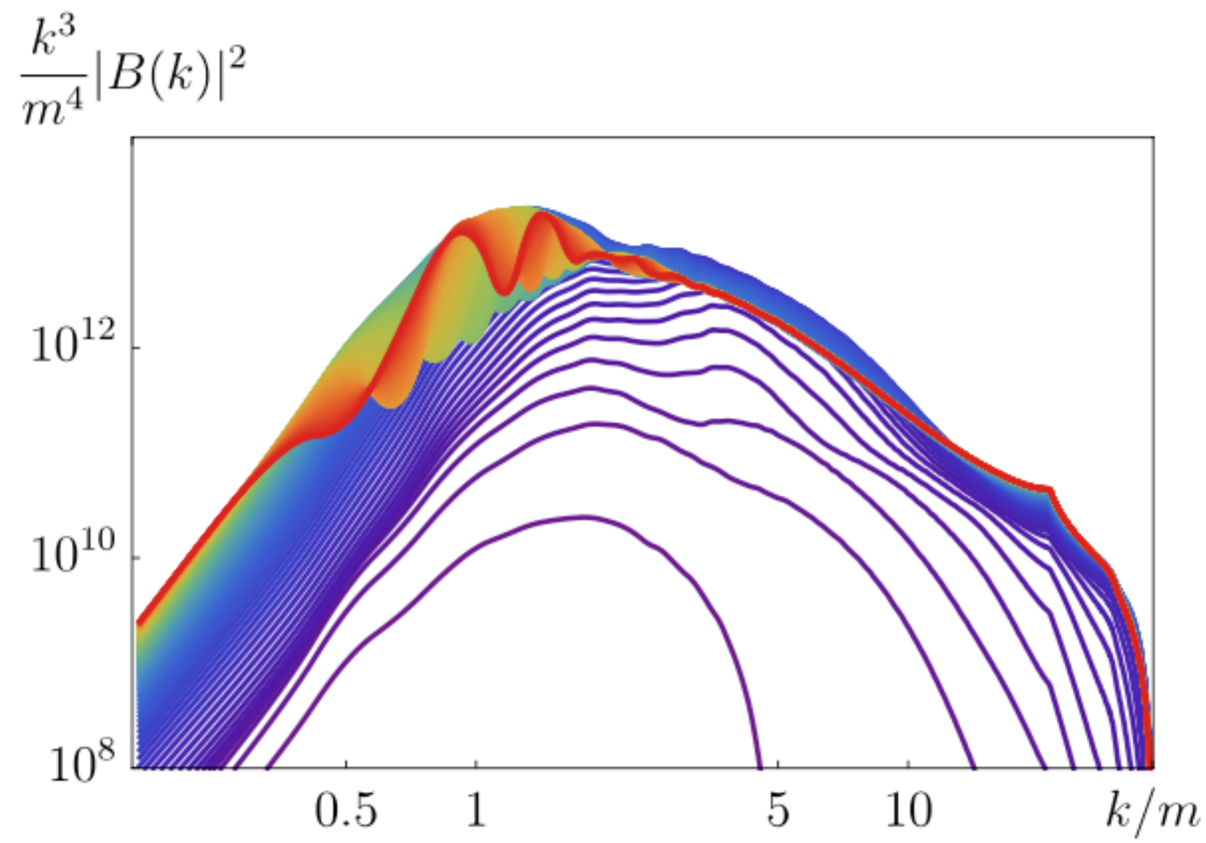
Axion-Inflation Preheating

Energy Spectra



(b) $1/\Lambda = 9.5\, m_{\rm pl}^{-1}$

(mild)



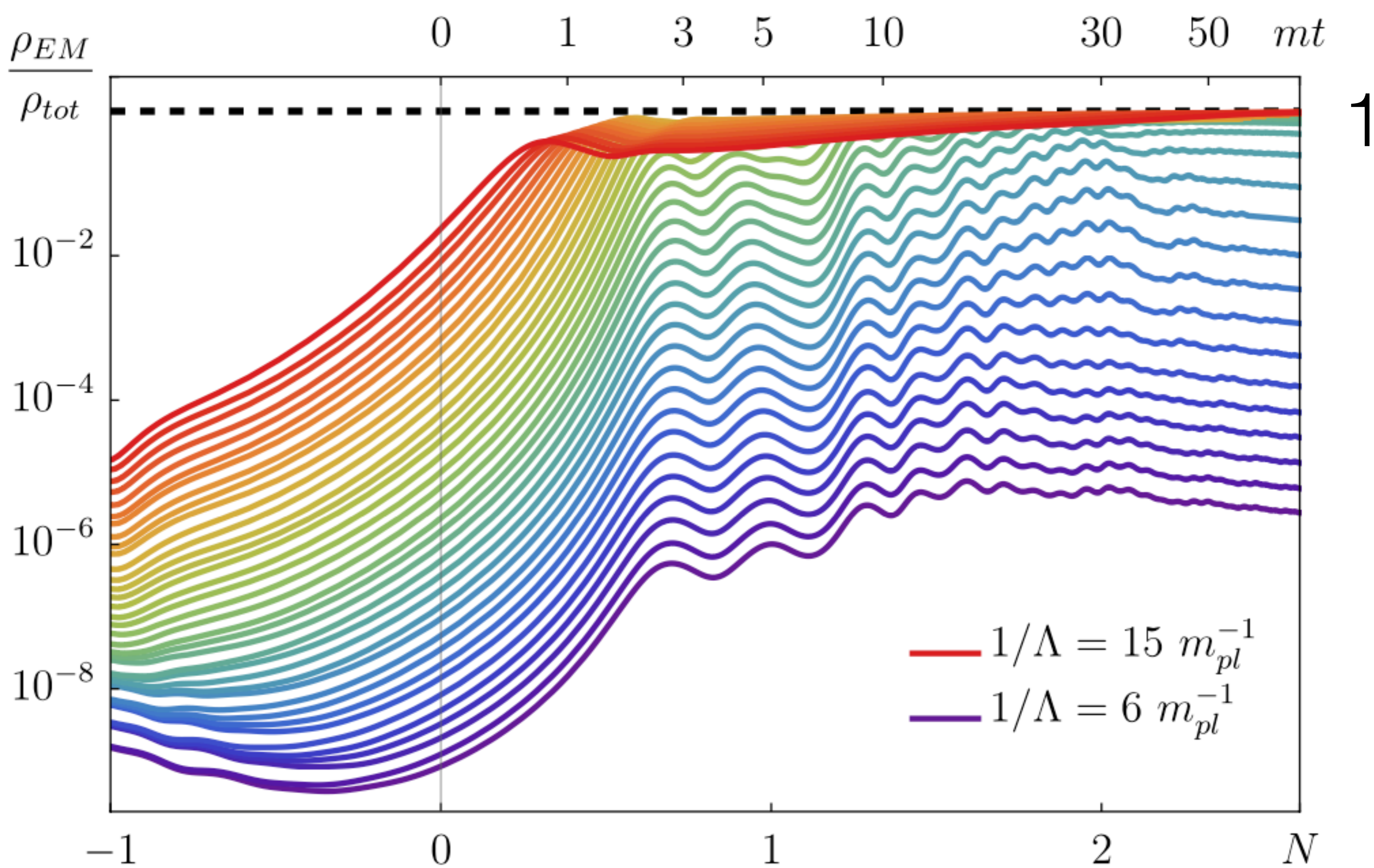
(c) $1/\Lambda = 15\, m_{\rm pl}^{-1}$

(strong)

Axion-Inflation Preheating

Energy Transfer (reheating efficiency)

Ratio EM-energy To total



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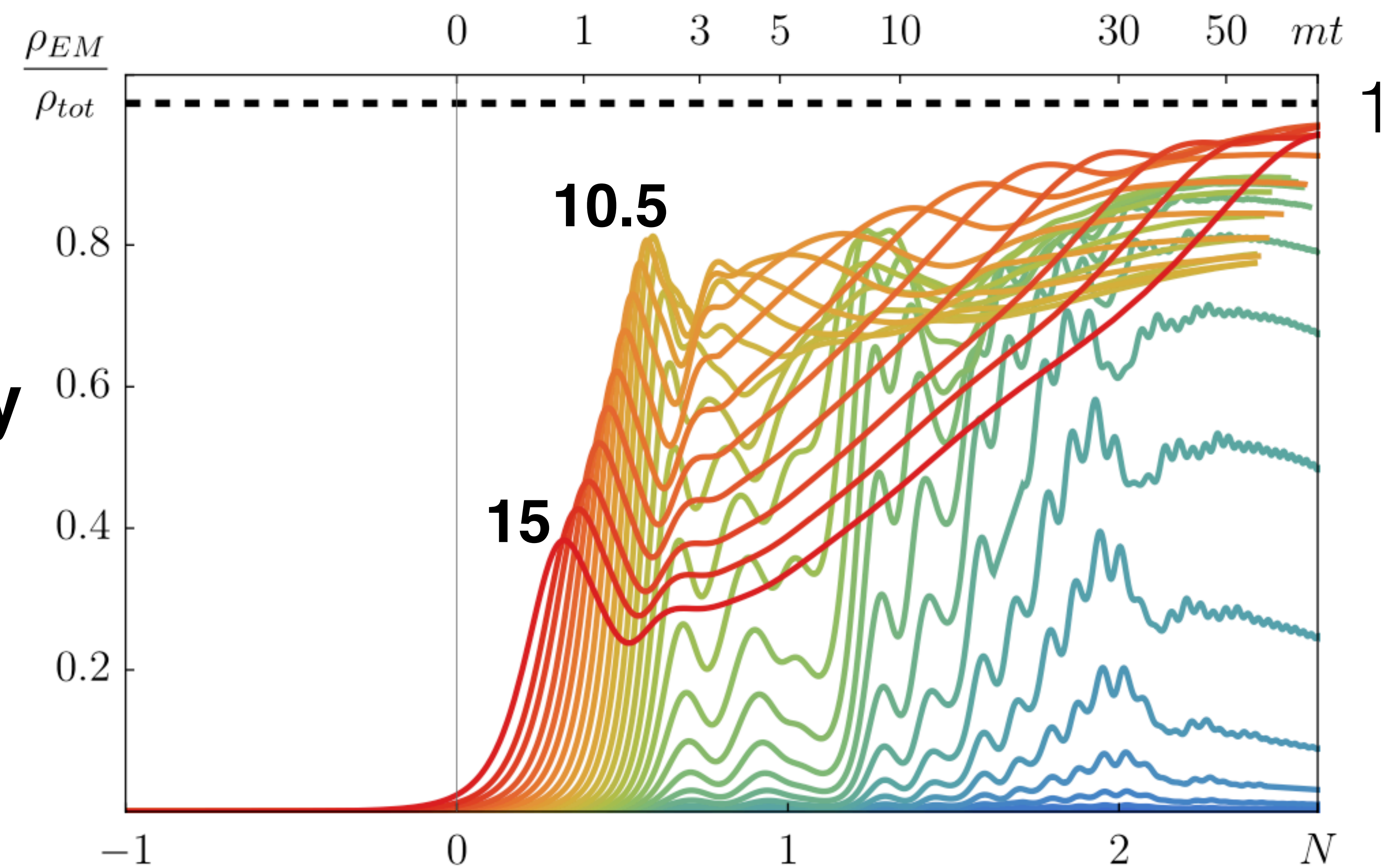

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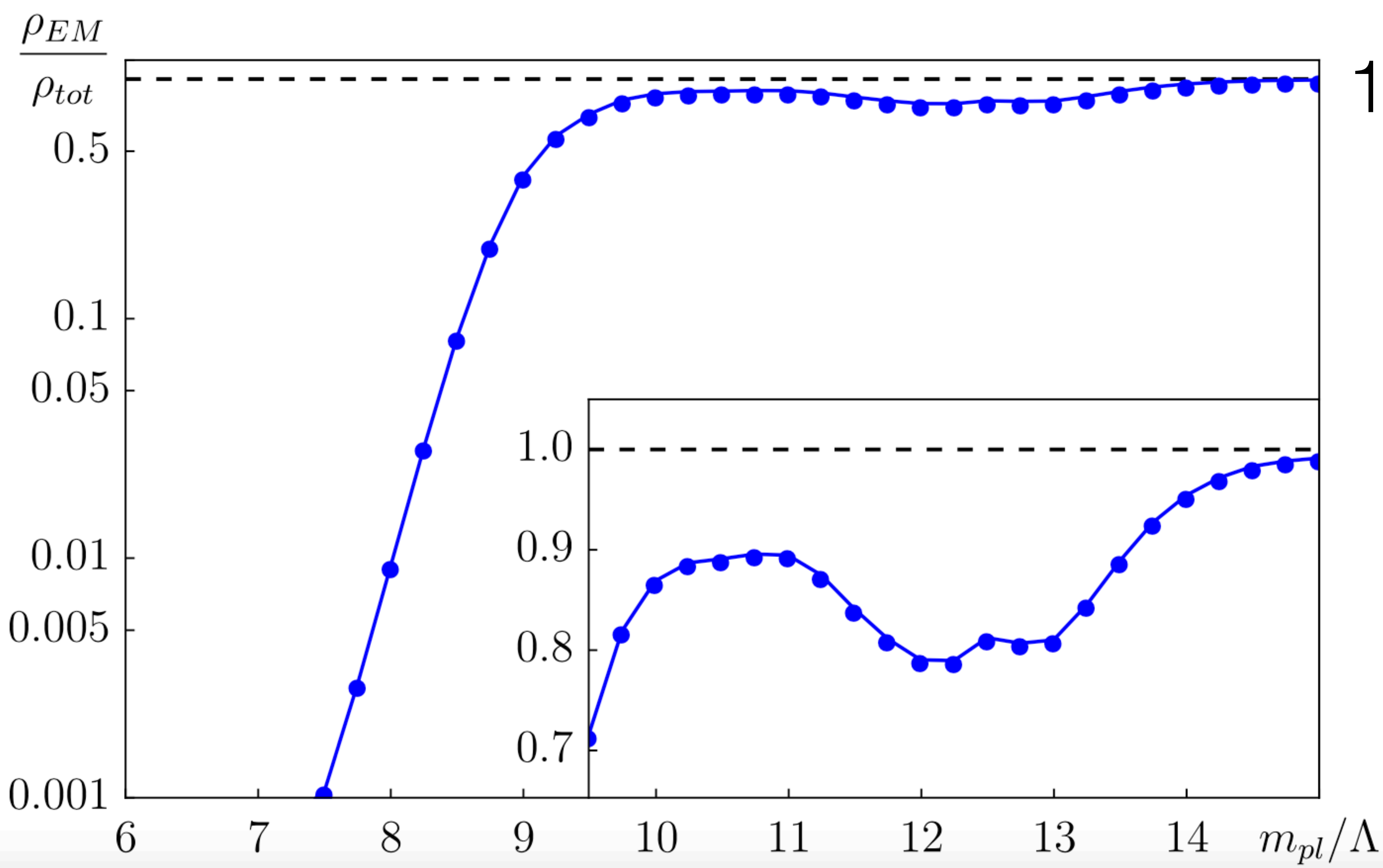


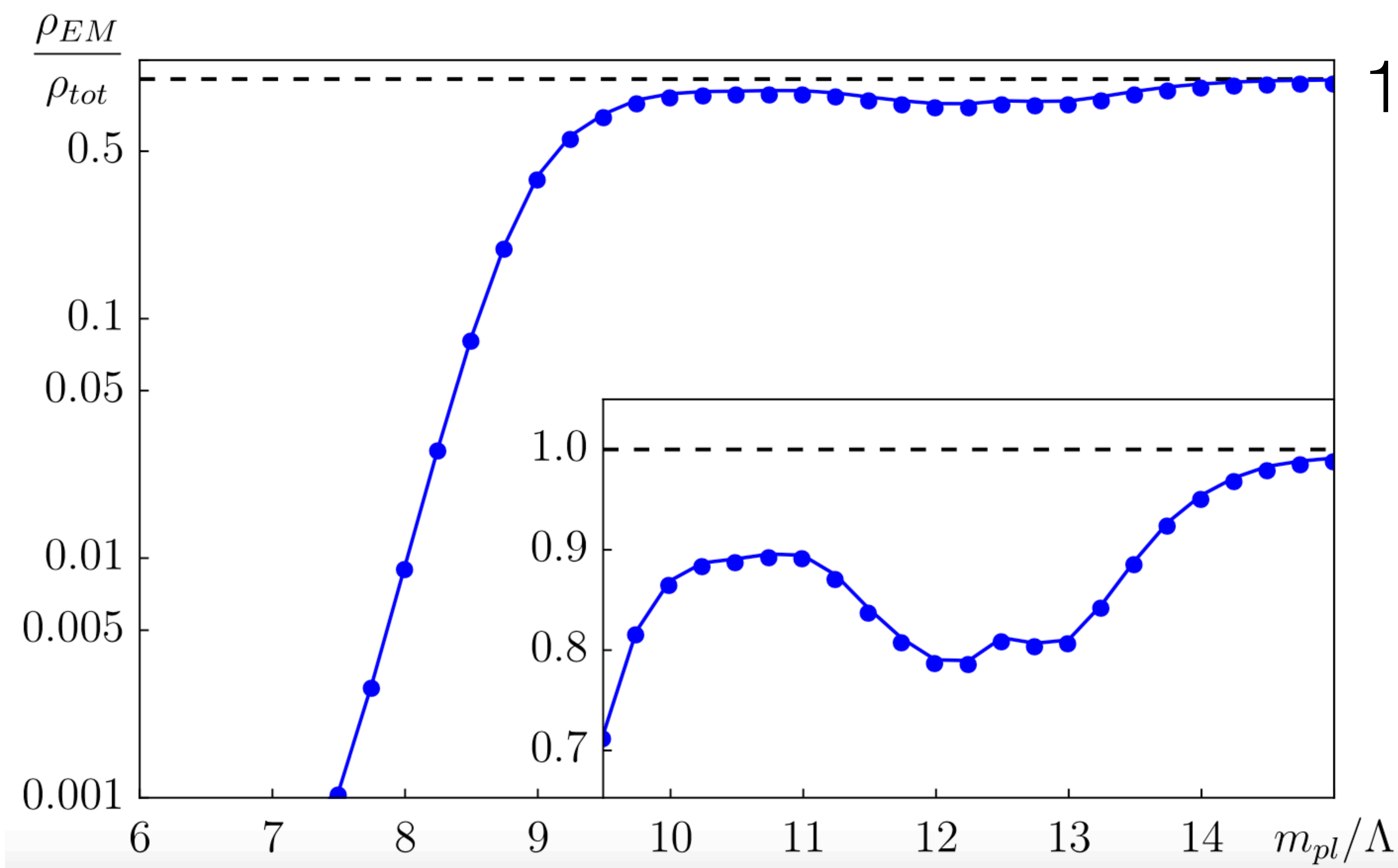
$1/\Lambda \gtrsim 9.5\, m_{\rm pl}^{-1}$: Supercritical couplings

More than ~50% energy in Gauge fields

Axion-Inflation Preheating

Energy Transfer (reheating efficiency)

Ratio EM-energy To total



We reproduce very similar behaviour to Adshead et al 2015 (1502.06506)

Part 3

Conclusions

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   - Gauge inv,
   - O(dx^2),
   - Bianchi ID,
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* RELATED TOPICS
  - axion-inflation and GWs (Dimastrogiovanni / Fujita talks)
  - GW from preheating (Pieroni's talk)

Part 4

Muchas gracias
por vuestra atención !