Equilibration of the chiral asymmetry due to finite electron mass

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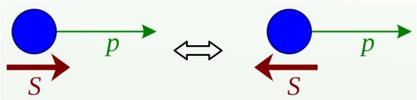

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Outline

Motivation: Chiral asymmetry

- In Standard Model left and right fermions have different properties: **L** – doublets, **R** – singlets.
- Many models of New Physics are also sensitive to the chirality.
- This may generate at some point the **chiral asymmetry** – excess of the fermions with one chirality over another ones.

Question: ***How fast does this asymmetry disappear?***

This can have an impact on different phenomena:

◇ generation of cosmological magnetic fields;

M. Joyce and M. Shaposhnikov, PRL **79**, 1193 (1997).

A. Boyarsky, J. Fröhlich and O. Ruchayskiy, PRL **108**, 031301 (2012).

D.E. Kharzeev, Prog. Part. Nucl. Phys. **75**, 133 (2014).

◇ baryon asymmetry of the Universe – ?

◇ Dark Matter production – ?

Chirality and mass

We consider the electron-positron plasma at temperatures $m_e \ll T \ll T_{EW}$ and only with EM interactions included.

$$H_{QED} = \int d^3\mathbf{x} \Big[\underbrace{-i\overline{\psi}(\gamma\cdot\nabla)\psi + (E^2+B^2)/2 + e\bar{\psi}\gamma^\mu\psi A_\mu}_{H_0} + \underbrace{m_e\bar{\psi}\psi}_{H_m} \Big]$$

$$N_5 = \int d^3\mathbf{x}\ \psi^\dagger\gamma^5\psi, \quad [N_5, H_0] = 0, \quad [N_5, H_m] = -2m_e \int d^3\mathbf{x}\, \overline{\psi}\gamma^5\psi \neq 0$$

Real fermions are **massive** that is why the chirality is undefined for them. There are two ways to consider the evolution of initial chiral asymmetry:

- To use the methods of nonequilibrium QFT (Schwinger-Keldysh formalism);
- To treat mass m_e as a perturbation.

In our work, we follow the second approach.

Definition of the chirality flipping rate

At classical level

$$\partial_\mu j_5^\mu = 2im_e\overline{\psi}\gamma^5\psi$$

$$\frac{dN_5}{dt} = 2im_e\int d^3\mathbf{x}\,\overline{\psi}\gamma^5\psi$$

In quantum field theory

$$\partial_\mu j_5^\mu = 2im_e\overline{\psi}\gamma^5\psi - \frac{e^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\frac{d}{dt}\underbrace{\left[N_5 + \frac{\alpha}{\pi}\mathcal{H}\right]}_{Q_5} = 2im_e\int d^3\mathbf{x}\,\overline{\psi}\gamma^5\psi$$

- For $m_e = 0$, the total fermion number N and the chiral charge Q_5 are **conserved** separately.
- For $m_e \neq 0$,

$$\boxed{\frac{d\langle Q_5\rangle}{dt} = -\Gamma_{\rm flip}\langle Q_5\rangle}$$

$\Gamma_{\rm flip}$ can be found perturbatively in m_e and α.

Relation to axions

- It is argued in the literature that the chiral anomaly is associated with a new pseudoscalar degree of freedom – **axion**

  J. Fröhlich and B. Pedrini, Int. Conf. Math. Phys. (2000)

  A. Boyarsky, J. Fröhlich and O. Ruchayskiy, PRD **92**, 043004 (2015)

- When fermions have been integrated out,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \alpha^2 \frac{a}{M} F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

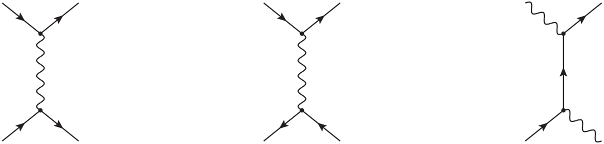
- In the spatially homogeneous case,

$$\partial_0 a \propto (N_R - N_L) \propto (\mu_R - \mu_L)$$

  and EoM for such a system are just the well-known Maxwell equations (with CME) + anomaly equation.

Naive estimate for chirality flipping rate

The lowest order processes in QED equilibrating the particles' momenta are e^-e^-, e^-e^+, **and Compton scatterings**:



They are accompanied by the subleading processes where the chirality of the fermion is flipped. The probability of this event is proportional to m_e^2:

$$\boxed{\Gamma_{\rm flip}^{\rm naive} \sim \frac{m_e^2}{\langle p^2 \rangle}\Gamma_{\rm coll} \sim \frac{m_e^2}{(3T)^2}\alpha^2 T.}$$

This expression was widely used in the literature as a natural estimate for the chirality flipping rate in electron-positron plasma.
Our main result: **this estimate is not correct**.

Boltzmann kinetic equation

$$\boxed{\dot{f}^a(\mathbf{k}) = -\mathcal{C}^a[f]}$$

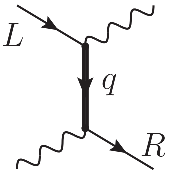
Appeared as a classical tool, but can be applied to quantum systems,

- if their quasiparticle description is well justified;
- if the effects of quantum statistics are taken into account;
- if the collision integral $\mathcal{C}^a[f]$ is calculated using the Feynman rules of the underlying QFT.

For $2 \leftrightarrow 2$ processes $a(k) + b(p) \leftrightarrow c(k') + d(p')$, the collision integral has the form

$$\mathcal{C}^a[f] = \sum_{proc.} \int_{k',p,p'} |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(k + p - k' - p') \times \\ \times [\underbrace{f^a f^b (1 \pm f^c)(1 \pm f^d)}_{\text{“loss”}} - \underbrace{(1 \pm f^a)(1 \pm f^b) f^c f^d}_{\text{“gain”}}].$$

Compton scattering $e_L + \gamma \leftrightarrow \gamma + e_R$



Assumptions:

- Spatially homogeneous electron-positron plasma.
- Classical EM field is absent.
- Characteristic timescale $\Gamma_{\rm coll}^{-1} \ll t \ll \Gamma_{\rm flip}^{-1}$.

The distribution function of left/right electrons – Fermi-Dirac distribution

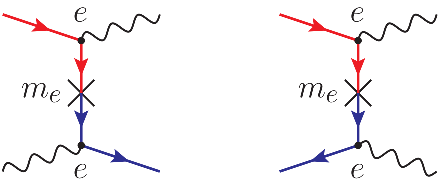
$$f_{L,R}(k) = n_F(\epsilon_k - \mu_{L,R}), \qquad \mu_{R,L} = \mu \pm \mu_5.$$

The chemical potentials $\mu_{L,R}$ slowly evolve in time because of the chirality flipping processes (μ_5 evolves). Then, to the leading order

$$LHS = \dot{f}_R - \dot{f}_L = -\frac{2}{T} n_F(\epsilon_k)[1 + n_F(\epsilon_k)]\dot{\mu}_5 \propto \dot{\mu}_5 \propto \frac{d}{dt}\langle Q_5 \rangle,$$

$$RHS = -\mathcal{C}_R[f] + \mathcal{C}_L[f] \propto [\text{"loss"} - \text{"gain"}] \propto \mu_5 \propto \langle Q_5 \rangle.$$

Thus, $\frac{d}{dt}\langle Q_5 \rangle = -\Gamma_{\rm flip}\langle Q_5 \rangle$ indeed holds.



Matrix elements of the Compton scattering and annihilation processes:

$$|\mathcal{M}|^2 = \frac{8m_e^2 e^4 \epsilon_k \epsilon_p (1 - \cos\theta_{kp})}{|(q-\Sigma)^2|^2}, \qquad \Sigma \propto \frac{e^2 T^2}{q}.$$

Substituting this to the collision integral, we finally get the expression for the chirality flipping rate

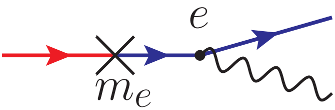
$$\Gamma_{\text{flip}} = \frac{3\pi^3}{2} m_e^2 \alpha^2 T \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{(1-\cos^2\theta_{kq})/q}{|(q-\Sigma)^2|^2} \propto m_e^2 \alpha^2 \times \frac{1}{q_{IR}^2}.$$

Numerically,

$$\boxed{\Gamma_{\text{flip}} \approx 0.24 \frac{\alpha m_e^2}{T}.}$$

Lower order processes - ?

Since we treat the electron mass m_e as a perturbation, the asymptotic states are massless. This allows for some lower-order processes to occur.



For massless particles, such collinear processes of bremsstrahlung, absorption and annihilation become possible.
Thus, the estimate for $\Gamma_{\rm flip}$ would be also to the same order:

$$\boxed{\Gamma_{\rm flip} \propto \frac{m_e^2}{T}\alpha.}$$

However, this contribution is very sensitive to the deformations of the electron's dispersion relation (e.g., due to the thermal corrections in plasma).

Chirality flipping rate from linear response formalism 1

Linear response formalism: $H = H_0 + H'$, $\langle A(\mathbf{x},t)\rangle_0 = 0$

$$\boxed{\langle A(\mathbf{x},t)\rangle = -i\int_{-\infty}^{t}\left\langle\left[\tilde{A}(\mathbf{x},t),\tilde{H}'(t')\right]\right\rangle_0 dt'}$$

Here tilda denotes an operator in Heisenberg representation of H_0.
In our case, we have $H = H_0 + H_m$, $\langle \dot{Q}_5\rangle_0 = 0$.
Heisenberg equation for Q_5 reads

$$\dot{Q}_5 = \frac{1}{i}[Q_5, H_m] = 2im_e\int d^3\mathbf{x}\,\overline{\psi}\gamma^5\psi.$$

Applying linear response formula to our system, we obtain:

$$\boxed{\langle \dot{Q}_5\rangle = 2m_e^2\int d^3\mathbf{x}d^3\mathbf{y}\int_{-\infty}^{t}dt'\left\langle\left[\overline{\psi}(\mathbf{x},t)\gamma^5\psi(\mathbf{x},t),\ \overline{\psi}(\mathbf{y},t')\psi(\mathbf{y},t')\right]\right\rangle_0}$$

Chirality flipping rate in the linear response formalism 2

It is convenient to introduce the retarded Green's function

$$G_{\rm ret}(\omega,\mathbf{q}) = -i\int d^4x\, e^{i\omega t - i\mathbf{q}\cdot\mathbf{x}}\theta(t)\left\langle\left[\bar{\Psi}_L(t,\mathbf{x})\Psi_R(t,\mathbf{x}),\, \bar{\Psi}_R(0,0)\Psi_L(0,0)\right]\right\rangle.$$

Then, the rate of change of the chiral charge per unit volume

$$\boxed{\frac{1}{V}\left\langle\frac{dQ_5}{dt}\right\rangle = -4m_e^2\Im m\left[G_{ret}(\omega=-2\mu_5,\mathbf{q}=0)\right]}.$$

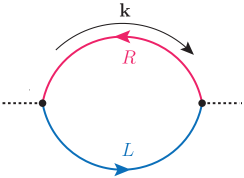
The chirality flipping rate can be determined as

$$\boxed{\Gamma_{\rm flip} = \frac{12m_e^2}{T^2}\left[\frac{\partial}{\partial\mu_5}\Im m G_{\rm ret}(-2\mu_5,0)\right]\Bigg|_{\mu_5=0}}$$

In practice: **Matsubara** $G_M(i\Omega_n,\mathbf{q}) \xrightarrow{i\Omega_n\to\omega+i0}$ **Retarded** $G_{ret}(\omega,\mathbf{q})$

Zeroth order in α

Trivial case: EM interaction is switched off.



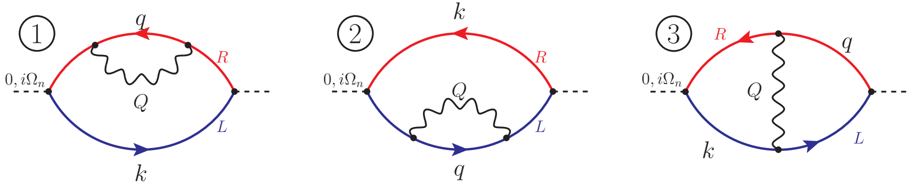
The computation is straightforward and it gives the chirality flipping rate

$$\Gamma^{(0)}_{\text{flip}} = \frac{12\pi m_e^2}{T^3}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\cosh^2\frac{k}{2T}}\delta(2k) = 0.$$

In fact, this means that the spontaneous flip of chirality is impossible for the free particle.

In this process, incoming and outgoing particles cannot be simultaneously on shell for any nonzero momentum because of the **angular momentum conservation**.

In the first order in α there are contributions from three diagrams:



Divergences:

- Infrared divergences from the region $Q \to 0$ (IR photon) – are canceled out when summing **all** three diagrams in 1st order.
- Collinear divergences in 1 and 2 diagrams, when $k \to 0$. Are **not** canceled out and lead to logarithmically divergent result.

$$\boxed{\Gamma_{\text{flip}} = \frac{3}{4}\frac{m_e^2}{T}\alpha \int \frac{dk}{k}\frac{1}{\cosh^2\frac{k}{2T}} \approx \frac{3}{4}\frac{m_e^2}{T}\alpha \ln \frac{T}{\varepsilon}}$$

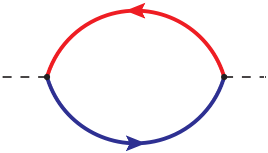
Here ε is the IR regularization parameter.

Self-energy resummation

There are two reasons for this divergence:

- × product of 3 propagators with the same momentum;
- × self-energy insertion is itself singular $\Sigma \propto 1/k$ as $k \to 0$.

Obviously, the divergence would become even more severe if we added more loops on upper and/or lower line.
That is why the **resummation** is required.

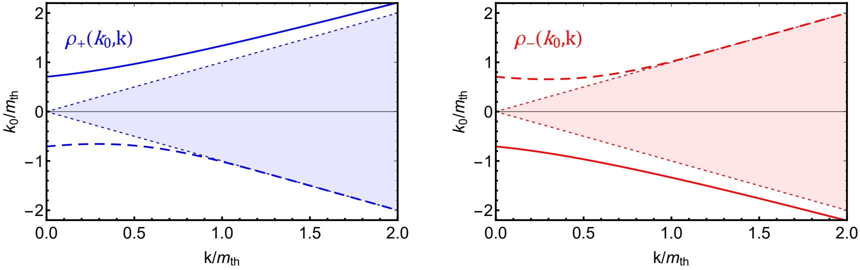


$$\Gamma_{\rm flip}^{1,2} = \frac{6m_e^2}{T^3} \int \frac{d^4k}{(2\pi)^4} \frac{1}{\mathrm{ch}^2 \frac{k^0}{2T}} \rho_+(k^0,\mathbf{k})\rho_-(k^0,\mathbf{k})$$

$\rho_\pm = -2\Im m G_\pm(k^0,k)$ is the spectral density of particles with positive/negative ratio of chirality to helicity.

Electron spectral density in plasma: HTL

$$\rho_\pm(k_0,k) = 2\pi\left[Z_\pm(k)\delta(k^0-\epsilon_\pm(k)) + Z_\mp(k)\delta(k^0+\epsilon_\mp(k))\right] + \rho_\pm^{(\mathrm{LD})}(k_0,k)$$

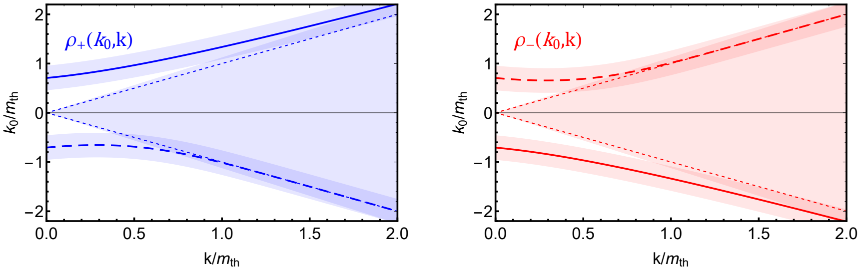


HTL self-energy resummation brings new features:

- Usual branch in spectrum acquires a gap $eT/(2\sqrt{2})$ at $k=0$. For large momenta the dispersion is $\epsilon_+(k) \approx \sqrt{k^2+m_{\mathrm{th}}^2}$, $m_{\mathrm{th}} = eT/2$.
- New branch appears with the opposite helicity. However, it exists only at soft momenta $k \sim eT$ and is exponentially suppressed for $k \sim T$.
- The spectral density includes the continuous ("incoherent") contribution for $k_0^2 < k^2$.

Electron spectral density in plasma: beyond HTL

$$\rho_\pm(k_0,k) = 2\pi\left[Z_\pm(k)\delta_\gamma(k^0-\epsilon_\pm(k)) + Z_\mp(k)\delta_\gamma(k^0+\epsilon_\mp(k))\right] + \rho_\pm^{(\mathrm{LD})}(k_0,k)$$



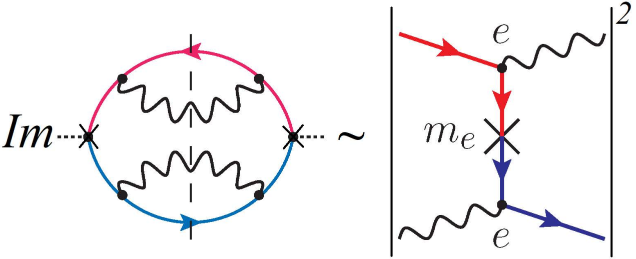
HTL result does not include the effects of collisions.

- Finite lifetime of quasiparticles $\tau^{-1} = \gamma = \frac{e^2}{4\pi}\log e^{-1}T$.
- Broadening of the poles: $\delta(\epsilon) \to \delta_\gamma(\epsilon)$.
- Washing out the bounds of continua.

Overlapping continua

The first contribution to Γ_{flip} comes from the overlap of incoherent contributions of ρ_+ and ρ_-.

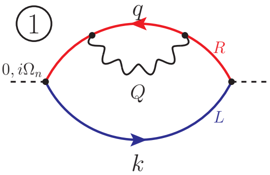
- Appears from the self-energy resummation of **both** lines.
- Exists already in **HTL** approximation.
- Beyond HTL (washing out the bounds) – higher order corrections.

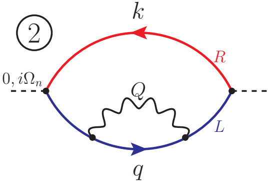


$$\Gamma_{\text{flip}}^{\text{cont}} \approx 0.24 \frac{\alpha m_e^2}{T}$$ – exactly what we had in kinetic approach.

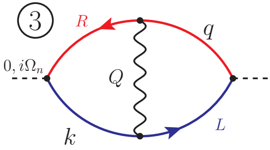
Overlap of the pole with continuum

We have the **quasiparticle pole** of one chirality overlapping with the **continuum** of another chirality. This corresponds to well-known graphs





But now, the particles have modified dispersion relations and finite lifetime.



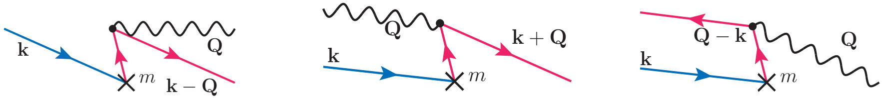
At this stage, also the 3d graph must be added.

Collinear $1 \leftrightarrow 2$ processes in plasma

We have already calculated these diagrams. The result is

$$\boxed{\Gamma_{\rm flip}^{\rm coll} = \frac{3}{4}\frac{m_e^2}{T}\alpha\int\frac{dk}{k}\frac{1}{\cosh^2\frac{k}{2T}} \approx \frac{3}{4}\frac{m_e^2}{T}\alpha\ln\frac{T}{\varepsilon}}$$

Here ε is the lowest possible energy scale at which the collinear processes are allowed.



We can violate a bit the energy conservation law in the collision event:

$$\epsilon_+(k) \pm \epsilon_+(q) \pm \omega_t(Q) \lesssim 2\gamma.$$

$$\epsilon_+(k) \approx k + \frac{m_{\rm th}^2}{2k}, \quad \omega_t(Q) \approx Q + \frac{m_\gamma^2}{2Q}, \qquad m_{\rm th}, m_\gamma \sim eT.$$

$$\varepsilon \simeq \frac{m_{\rm th}^2}{\gamma} \sim \frac{T}{\ln e^{-1}} \quad \Rightarrow \quad \boxed{\Gamma_{\rm flip}^{\rm coll} \approx \frac{3}{4}\frac{m_e^2}{T}\alpha\ln\ln e^{-1}}$$

Final result for Γ_{flip}

Full leading order result for the chirality flipping rate reads as

$$\boxed{\Gamma_{\text{flip}} = \frac{m_e^2}{T}\alpha\left[\frac{3}{4}\ln\ln\alpha^{-1} - 0.25 + \frac{1.12}{\ln\alpha^{-1}} + \mathcal{O}\left(\frac{1}{(\ln\alpha^{-1})^2}\right)\right]}$$

For $\alpha = 1/137$,

$$\Gamma_{\text{flip}} \approx 1.17\alpha\frac{m_e^2}{T} \approx 8.5\times 10^{-3}\frac{m_e^2}{T}.$$

The naive estimate was

$$\Gamma_{\text{flip}}^{\text{naive}} \approx \frac{\alpha^2}{9}\frac{m_e^2}{T} \approx 5.9\times 10^{-6}\frac{m_e^2}{T}.$$

Summary

Conclusions:

- Chirality flipping rate in electron-positron plasma was calculated in the leading order in mass m_e and EM coupling constant α.
- Compton scattering and annihilation processes contribute to this rate, however, the IR divergences in corresponding matrix elements change the parametric dependence on α.
- In addition, the nearly collinear processes of bremsstrahlung, absorption, and annihilation become allowed because of the finite lifetime of the quasiparticles.
- The final result is $\Gamma_{\text{flip}} \approx 1.17\alpha\frac{m_e^2}{T}$, which is 3 orders of magnitude larger than the naive estimates in the literature.

Issues to be addressed in further investigations:

* Other types of interactions also must be included (e.g., weak reactions, inverse Higgs decay).
* The model in which the chiral asymmetry is created must be added.
* The consequences for magnetogenesis, BAU and DM must be studied.

We are grateful to Artem Ivashko, Sasha Gamayun, Kyrylo Bondarenko, Sasha Monin, and Mikhail Shaposhnikov for valuable discussions.

Thank you for your attention!

Appendix

Averaging in grand canonical ensemble

$$\boxed{\langle \dot{Q}_5 \rangle = 2m_e^2 \int d^3\mathbf{x} d^3\mathbf{y} \int_{-\infty}^{t} dt' \left\langle \left[\overline{\psi}(\mathbf{x},t)\gamma^5\psi(\mathbf{x},t), \overline{\psi}(\mathbf{y},t')\psi(\mathbf{y},t') \right] \right\rangle_0}$$

$$\mathcal{O}_H(\mathbf{x},t) = e^{iHt}\mathcal{O}_S(\mathbf{x})e^{-iHt} = e^{iKt}\underbrace{e^{it(\mu N+\mu_5 Q_5)}\mathcal{O}_S(\mathbf{x})e^{-it(\mu N+\mu_5 Q_5)}}_{\tilde{\mathcal{O}}_S(\mathbf{x},t)}e^{-iKt} = \tilde{\mathcal{O}}_K(\mathbf{x},t)$$

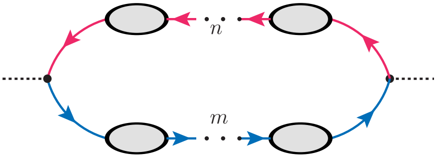
$$\left\langle \frac{dQ_5}{dt} \right\rangle = 2m^2 \int d^3x d^3y \int_{-\infty}^{\infty} dt' \theta(t-t') \times$$
$$\times \left\{ e^{-2i\mu_5(t-t')} \left\langle \left[W(x,t), W^\dagger(y,t') \right] \right\rangle - e^{2i\mu_5(t-t')} \left\langle \left[W^\dagger(x,t), W(y,t') \right] \right\rangle \right\}.$$

$$G_{ret}(x-y; t-t') = -i\theta(t-t') \left\langle \left[W(x,t), W^\dagger(y,t') \right] \right\rangle, \quad W(x,t) = \overline{\Psi}_L(x,t)\Psi_R(x,t).$$

$$\boxed{\frac{1}{V} \left\langle \frac{dQ_5}{dt} \right\rangle = -4m^2 \Im m \left[G_{ret}(q=0; \omega = -2\mu_5) \right]}$$

Need for resummation

Let us consider the general case with arbitrary number of self-energy insertions in the loop



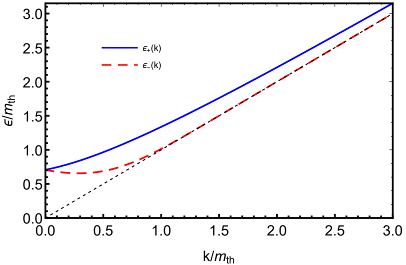
We have

- $n + m + 2$ propagators with the same momentum k;
- $n + m$ self-energy insertions behaving $\Sigma \propto 1/k$ as $k \to 0$;
- one integration $d^4 k$.

$$\Im m\, G_{\text{ret}}^{n,m} \propto \int d^4 k \frac{(1/k)^{n+m}}{k^{(n+m+2)}} \propto \int \frac{dk}{k^{2n+2m-1}} \sim \varepsilon^{2(1-n-m)}.$$

Higher orders lead to more severe divergences – resummation is needed.

M. Le Bellac, "Thermal Field Theory", (Cambridge University Press, 1996).



$$\epsilon_\pm(k \ll m_{\rm th}) \approx \frac{m_{\rm th}}{\sqrt{2}} \pm \frac{k}{3},$$

$$\epsilon_+(k \gg m_{\rm th}) \approx \sqrt{k^2 + m_{\rm th}^2} \approx k + \frac{m_{\rm th}^2}{2k},$$

$$\epsilon_-(k \gg m_{\rm th}) \approx k\left[1 + 2e^{-\frac{4k^2}{m_{\rm th}^2} - 1}\right].$$

Incoherent contribution to the spectral density (HTL):

$$\rho_\pm^{LD}(k^0,\mathbf{k}) = \frac{\frac{\pi m_{\rm th}^2}{2k}\left(1 \mp \frac{k^0}{k}\right)\theta(k^2 - k_0^2)}{\left[(k^0 \mp k)\left(1 \pm \frac{m_{\rm th}^2}{4k^2}\ln\left|\frac{k+k^0}{k-k^0}\right|\right) \mp \frac{m_{\rm th}^2}{2k}\right]^2 + \left[\frac{\pi m_{\rm th}^2}{4k}\left(1 \mp \frac{k^0}{k}\right)\right]^2}.$$

Contribution of collinear processes

Overlap of the quasiparticle pole with the incoherent part:

$$\Gamma_{\rm flip}^{(\rm pole)} = \frac{6m_e^2}{\pi^2 T^3}\int_0^\infty \frac{k^2 dk}{\mathrm{ch}^2\frac{k}{2T}}\int_{-\infty}^{\infty} dx\, \delta_{\gamma_e}(x)\rho_-(\epsilon_+(k)+x,k),$$

where the spectral density far from the shell reads as

$$\rho_-(\epsilon_+(k)+x,k) \approx -\frac{1}{2k^2}\Im m\Sigma_-(\epsilon_+(k)+x+i0,k).$$

Finally, we get

$$\Gamma_{\rm flip}^{(\rm pole,\, collinear)} = \frac{3m_e^2\alpha}{T^3}\int_0^\infty \frac{dk}{\mathrm{ch}^2\frac{k}{2T}}\int\frac{d^3\mathbf{Q}}{(2\pi)^3}\sum_{\lambda',\lambda''=\pm}\frac{\lambda''}{Q}\left(\mathrm{cth}\frac{\lambda'' Q}{2T}+\mathrm{th}\frac{k-\lambda'' Q}{2T}\right)\times$$

$$\times\underbrace{\delta_{2\gamma}\left(\epsilon_+(k)-\lambda'\epsilon_+(\mathbf{k}-\mathbf{Q})-\lambda''\omega_t(Q)\right)}_{\text{approximate energy conservation}}\left(1+\lambda'\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{Q})}{k|\mathbf{k}-\mathbf{Q}|}\right).$$

Comparison to other results