Effects of the gravitational Chern-Simons term during axion-SU(2) inflation

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Zooming in on Axions in the Early Universe, June 2020, CERN

Based on 2003.05931

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The Basics of the Model

Spectator axion-SU(2) Gauge Field

inflation action | Spectator sector

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{\varphi} - \frac{1}{2}(\partial\chi)^2 - U(\chi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\lambda_1 \chi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$

Maleknejad - Sheikh-Jabbari 2011, Adshead - Wyman 2012, Dimastrogiovanni - Fasiello - Tolley 2012, Namba - Dimastrogiovanni - Peloso 2013, Maleknejad - Sheikh-Jabbari 2013, Maleknejad - Sheikh-Jabbari - Soda 2013, Dimastrogiovanni - Peloso 2013, Adshead - Martinec -Wyman 2013, Dimastrogiovanni - Fasiello - Fujita 2016, Agrawal - Fujita - Komatsu 2017, Caldwell - Devulder 2017, Lozanov - Maleknejad - Komatsu 2018, Domcke et al 2018, …

$$ds^2 = a^2(\tau)\Big[- d\tau^2 + (1 + \tilde{h}_+(\tau, z))dx^2 + (1 - \tilde{h}_+(\tau, z))dy^2 + 2\tilde{h}_\times(\tau, z)dxdy + dz^2\Big]$$

Slow-roll background solution

$$A^a_0 = 0, \quad A^a_i = a(t)Q(t)\delta^a_i$$

Maleknejad - Sheikh-Jabbari 2011

Gauge tensor perturbations

$$\delta A^1_i = (\tilde{t}_+, \tilde{t}_\times, 0) \quad \delta A^2_i = (\tilde{t}_\times, -\tilde{t}_+, 0)$$

$$\tilde{t}_{L,R} \equiv \frac{1}{\sqrt{2}}(\tilde{t}_+ \pm i\tilde{t}_\times)$$

See the talks by Emma Dimastrogiovanni and Tomo Fujita

E.O.M. of GWs

Vacuum vs. sourced

1. Homogeneous solutions : GWs from vacuum fluctuations

$$\tilde{h}''_{ij} + 2\mathcal{H}\tilde{h}'_{ij} + k^2\tilde{h}_{ij} = 0$$

2. Inhomogeneous solutions : GWs from sources, e.g. gauge fields

$$\tilde{h}''_{ij} + 2\mathcal{H}\tilde{h}'_{ij} + k^2\tilde{h}_{ij} = 16\pi G a^2 \pi_{ij}$$

$\mathcal{H} \equiv a'/a$

E.O.M. of Tensor Modes

Axion-SU(2) Gauge Fields

$$\partial_x^2 h_A + (1 - \frac{2}{x^2}) h_A = \mathcal{O}(t_A)$$

$$\partial_x^2 t_A + \left[1 + \frac{2}{x^2}\left(m_Q\xi_1 - sx(m_Q + \xi_1)\right)\right] t_A = \mathcal{O}(h_A)$$

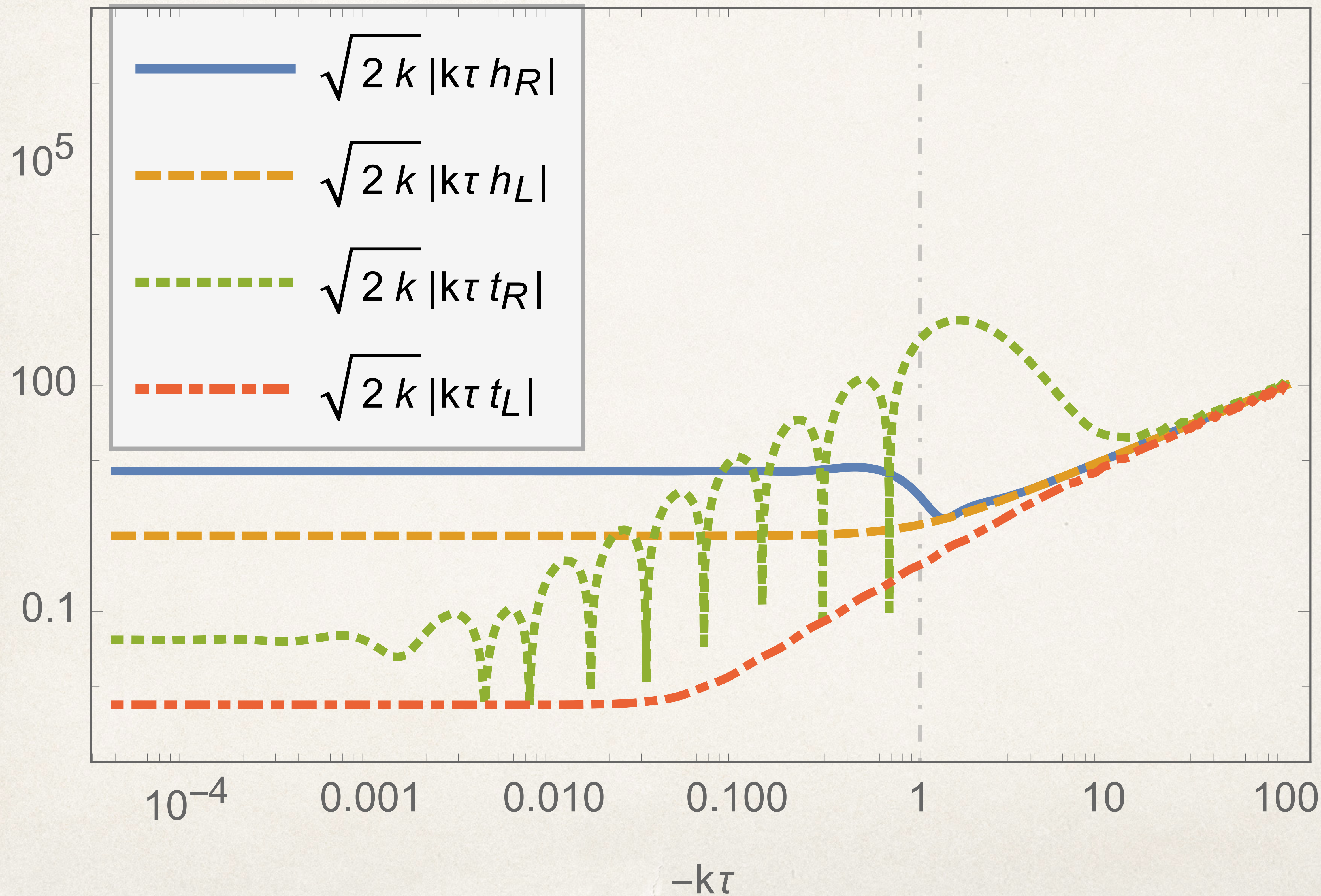
$s = -1, 1$

Gauge tensors split and one is amplified.

$h_{ij} \equiv a\frac{M_{pl}}{\sqrt{2}}\tilde{h}_{ij}, \quad t_i^a = \sqrt{2}a\tilde{t}_i^a \qquad \tilde{h}_{L,R} \equiv \frac{1}{\sqrt{2}}(\tilde{h}_+ \pm i\tilde{h}_\times) \qquad A = L, R \qquad m_Q \equiv \frac{g_A Q}{H}, \quad \xi_1 \equiv \frac{\lambda_1 \dot{\chi}}{2fH} \qquad x \equiv -k\tau$

Chiral Gravitational Waves

Parity is spontaneously broken by the background!



Symmetry demands more!

We have to consider all of parity-violating terms that arise from the same physics!

$$\chi F\tilde{F} \longleftrightarrow \chi R\tilde{R}$$

Witten 1984, Choi 1997, Lue - Wang - Kamionowski 1999, Choi - Hwang - Hwang 2000

Symmetry demands more!

Add the Gravitational Chern-Simons term

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{\varphi} + \mathcal{L}_{SPEC} + \mathcal{L}_{GCS}$$

$$\mathcal{L}_{GCS} = \frac{\lambda_2 \chi}{4f} R\tilde{R}$$

E.O.M of GWs

Vacuum, sourced and modified

1. Homogeneous solutions : GWs from vacuum fluctuations

$$\tilde{h}''_{ij} + 2\mathcal{H}\tilde{h}'_{ij} + k^2\tilde{h}_{ij} = 0$$

2. Inhomogeneous solutions : GWs from sources, e.g. gauge fields

$$\tilde{h}''_{ij} + 2\mathcal{H}\tilde{h}'_{ij} + k^2\tilde{h}_{ij} = 16\pi G a^2 \pi_{ij}$$

3. Inhomogeneous solutions + Gravitational Chern-Simons term

$$\left(1 - s\frac{\xi_2 k}{\mathcal{H}}\right)h''_A + 2\mathcal{H}h'_A + \left(1 - s\frac{\xi_2 k}{\mathcal{H}}\right)k^2 h_A = 16\pi G a^2 \pi_{ij}$$

$$\xi_1 \equiv \frac{\lambda_1 \dot{\chi}}{2fH}, \quad \xi_2 \equiv \frac{1}{2}\frac{\lambda_2 \dot{\chi}}{2fH}\left(\frac{H}{M_{pl}}\right)^2 \quad s = -1, 1 \quad A = L, R \quad \mathcal{H} \equiv a'/a$$

E.O.M of Tensor Modes

Axion-SU(2) Gauge fields + Gravitational Chern-Simons

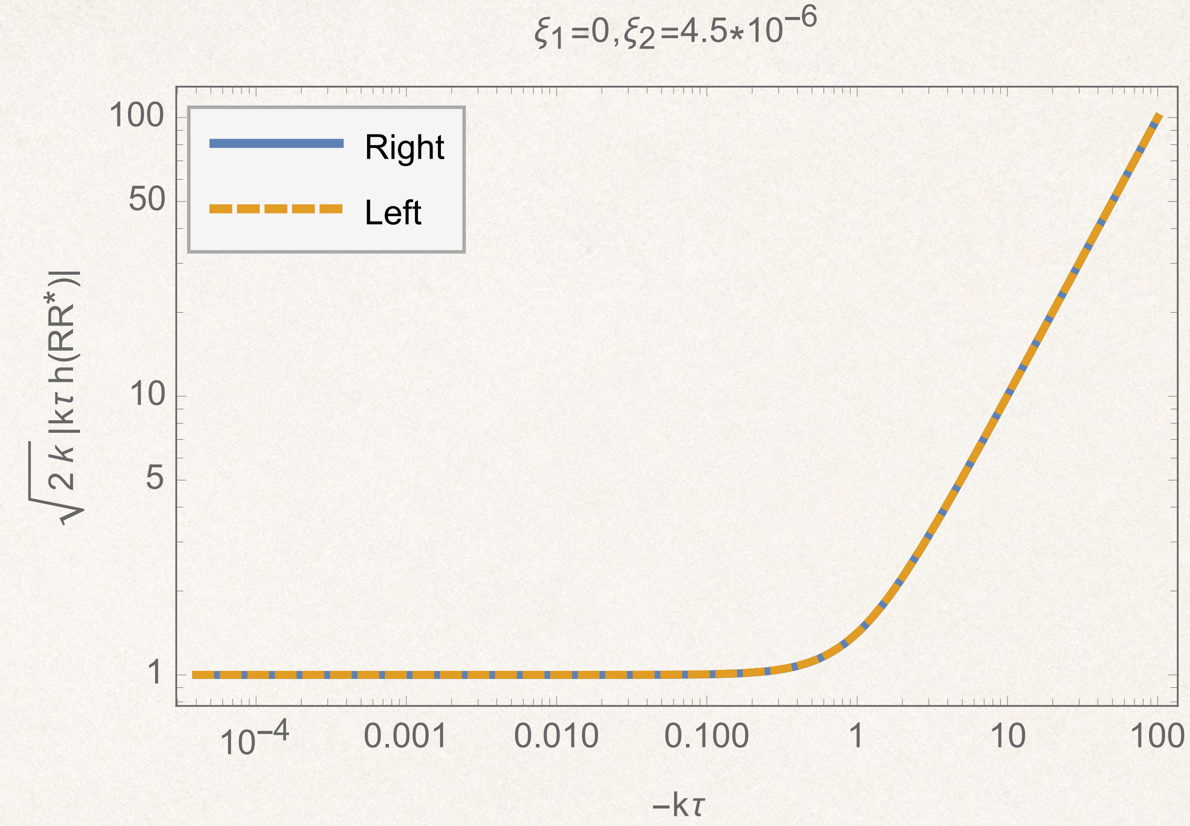
$$\partial_x^2 t_A + \left[1 + \frac{2}{x^2}\left(m_Q\xi_1 - sx(m_Q + \xi_1)\right)\right] t_A = \mathcal{O}(h_A)$$

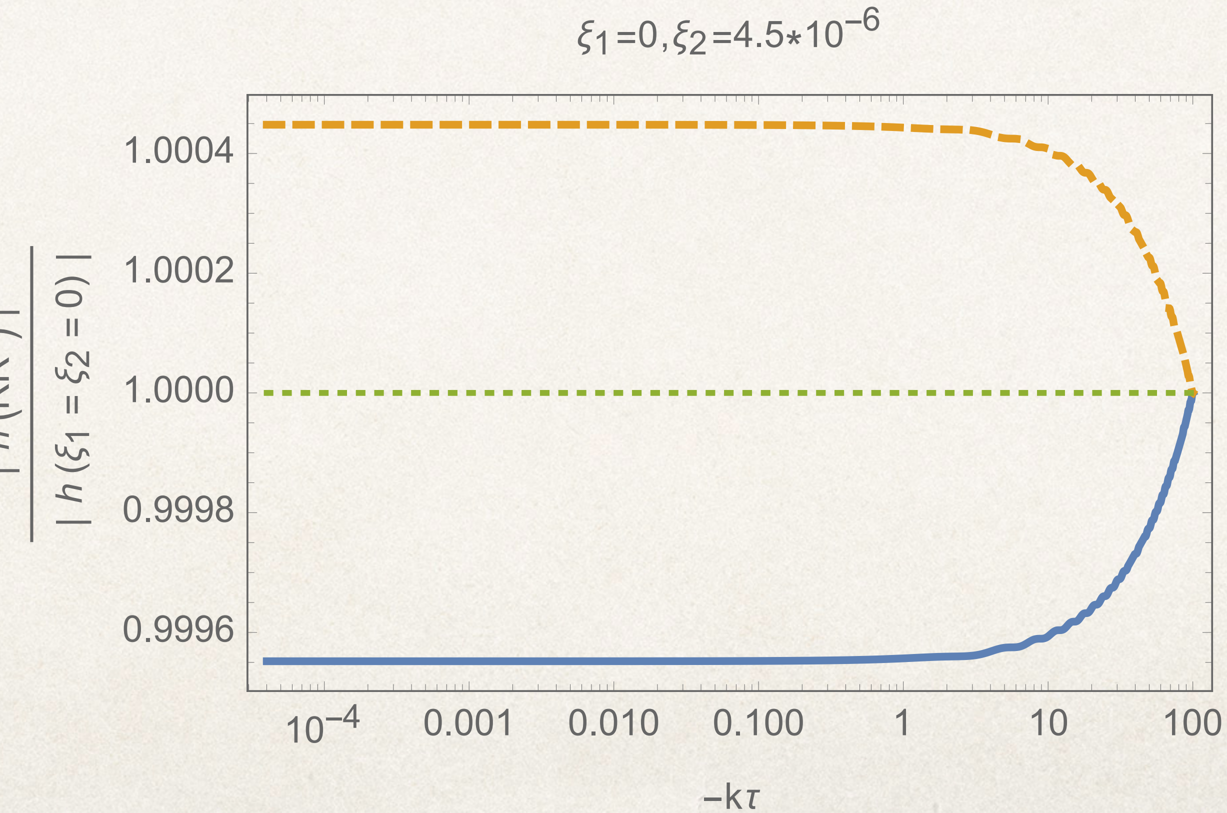
$$\left(1 - s\xi_2 x\right)\left[\partial_x^2 h_A + (1 - \frac{2}{x^2})h_A\right] - 2s\xi_2\partial_x h_A = \mathcal{O}(t_A)$$

$s = -1, 1$

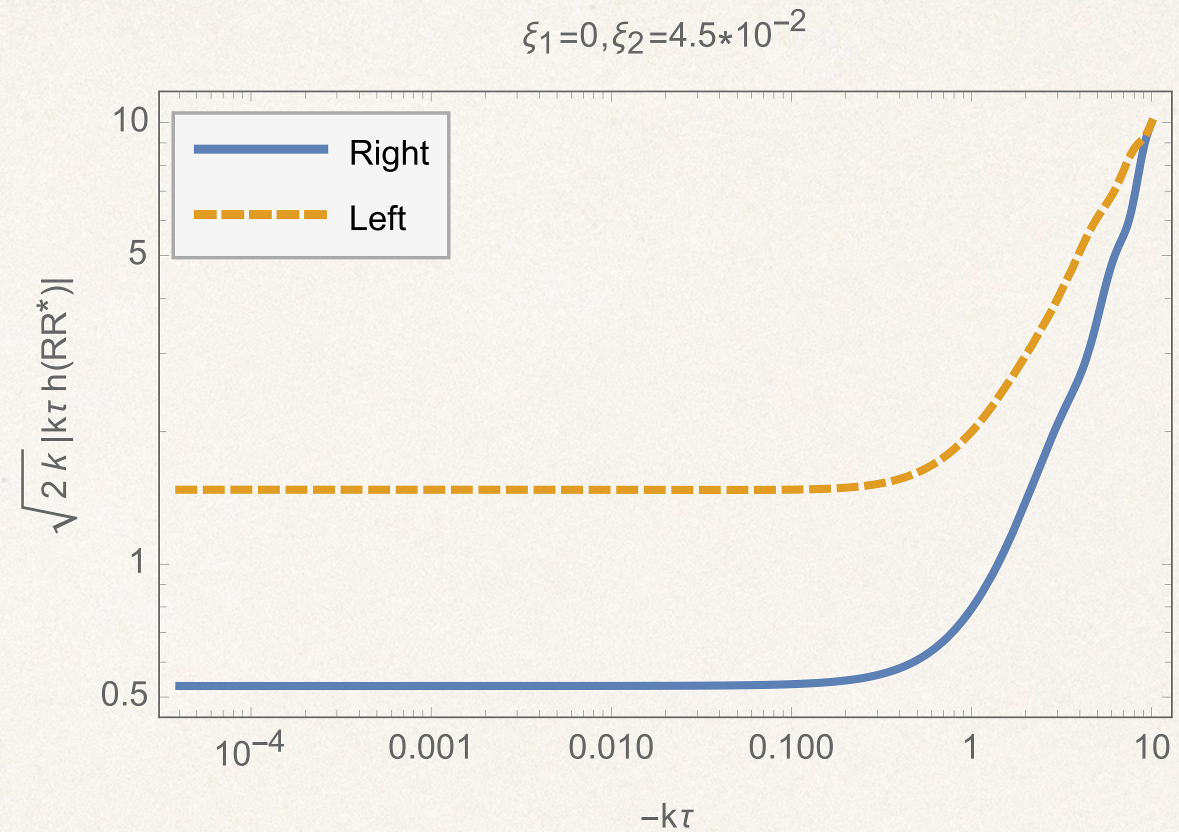
$h_{ij} \equiv a\frac{M_{pl}}{\sqrt{2}}\tilde{h}_{ij}, \quad t_i^a = \sqrt{2}a\tilde{t}_i^a \quad m_Q \equiv \frac{g_A Q}{H}, \quad \xi_1 \equiv \frac{\lambda_1\dot{\chi}}{2fH}, \quad \xi_2 \equiv \frac{1}{2}\frac{\lambda_2\dot{\chi}}{2fH}\left(\frac{H}{M_{pl}}\right)^2 \quad A = L, R \quad x \equiv -k\tau$

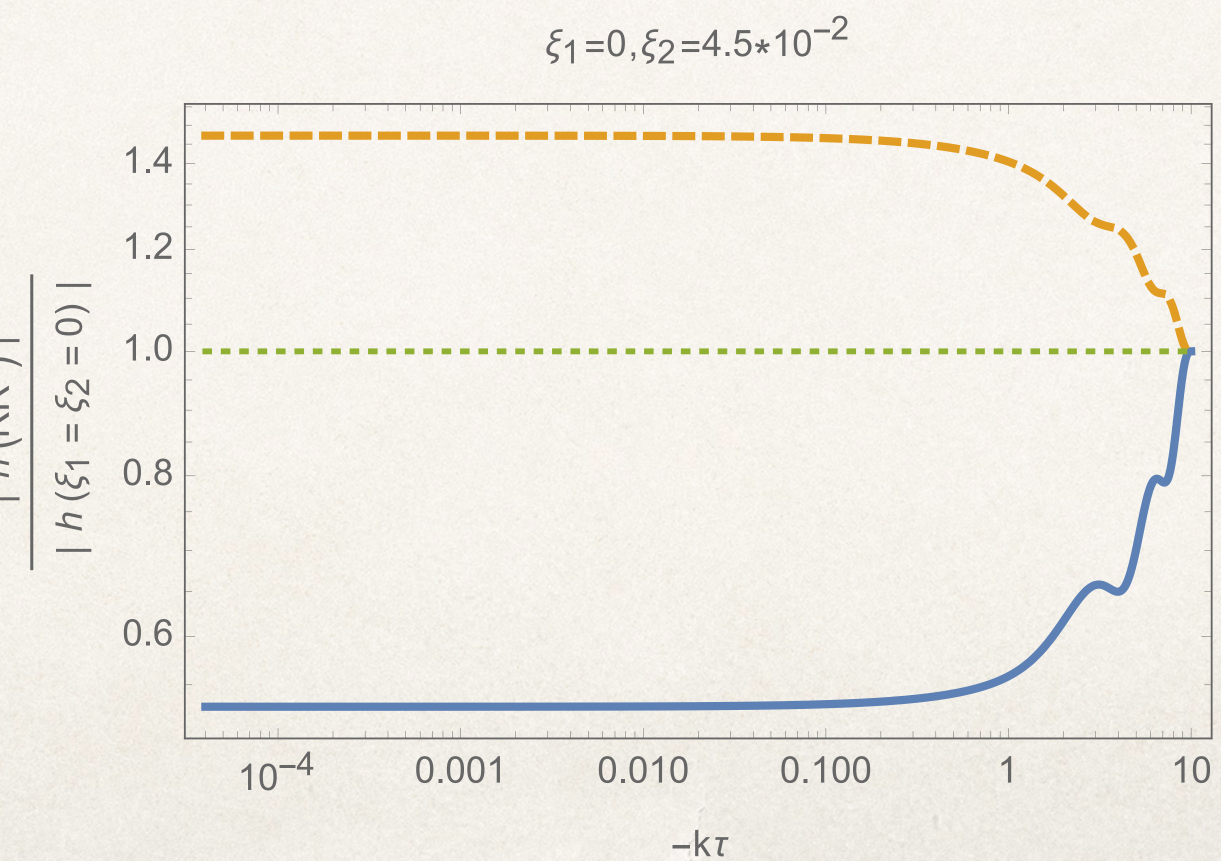
Without $F\tilde{F}$





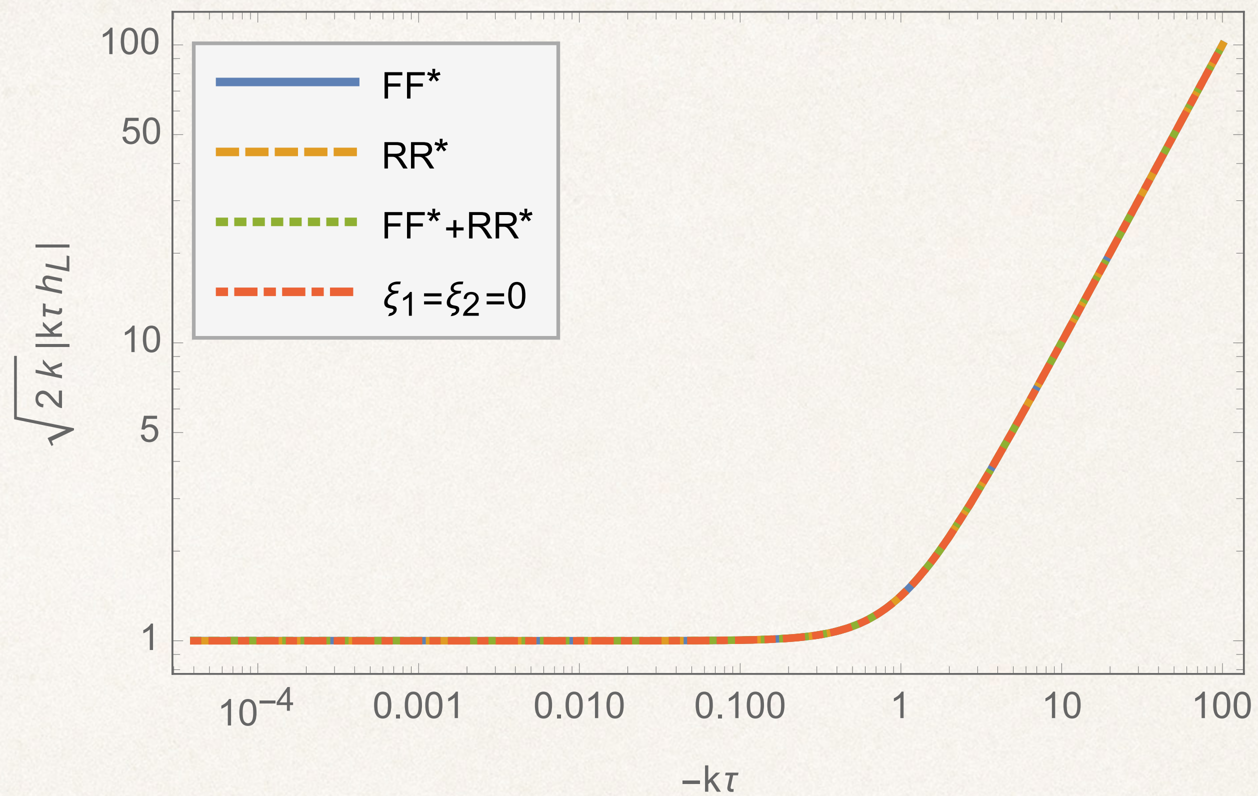
Without $\mathbf{F\tilde{F}}$



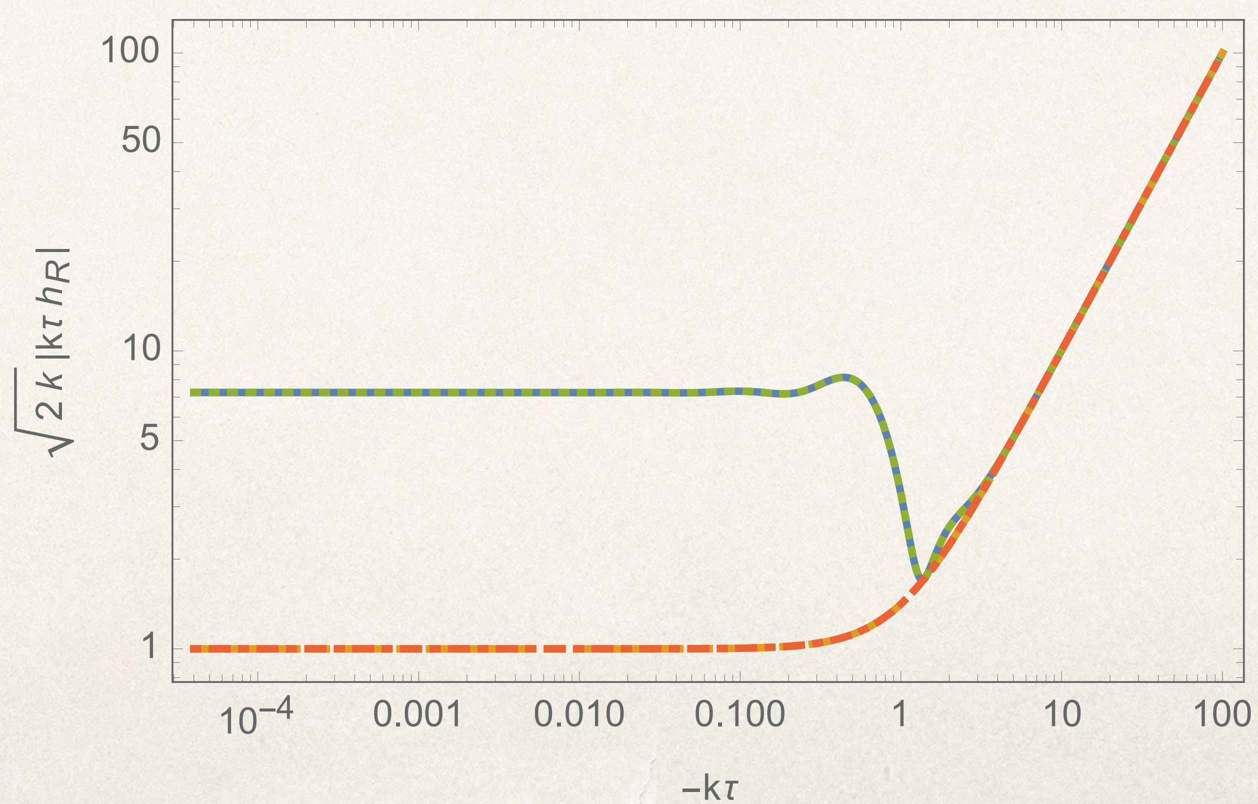


With $F\tilde{F}$

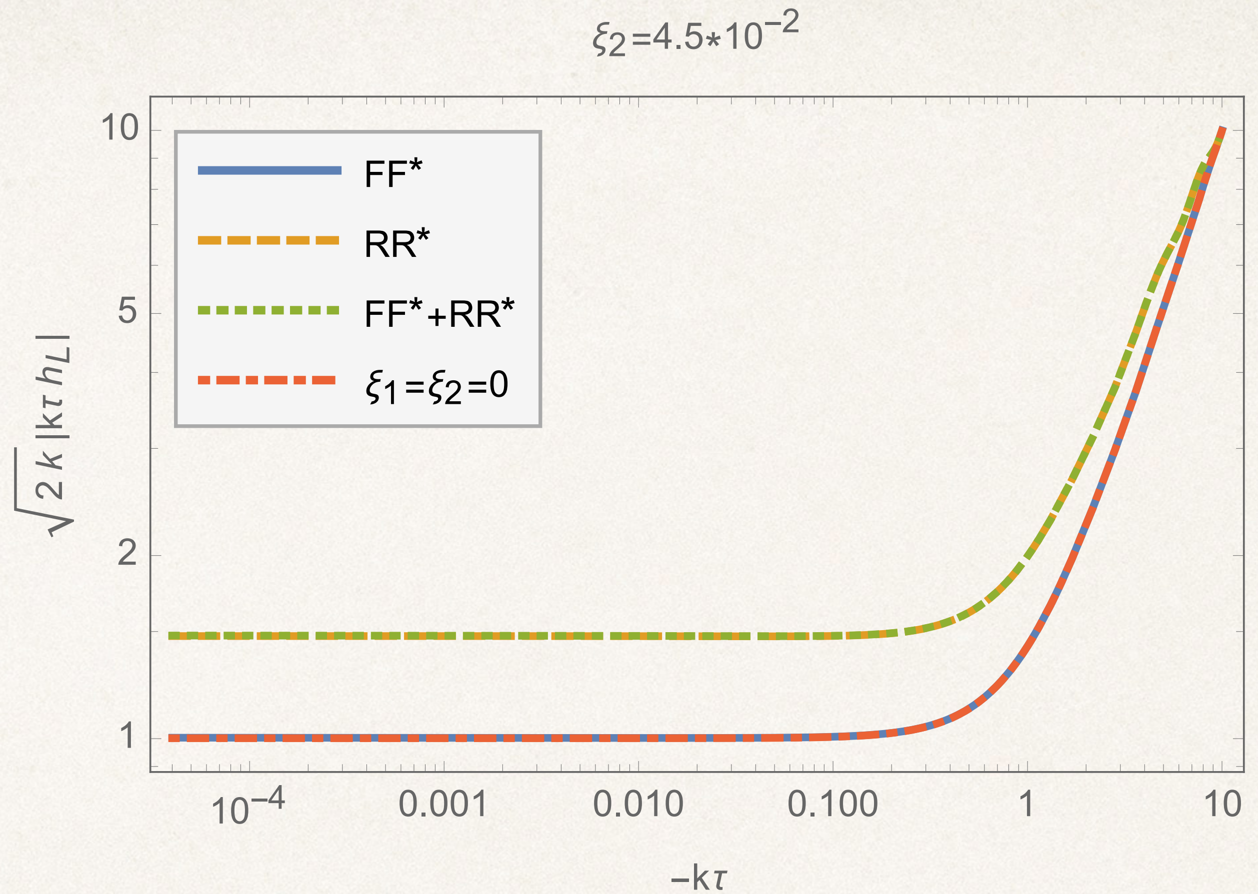
$\xi_2 = 4.5 * 10^{-6}$

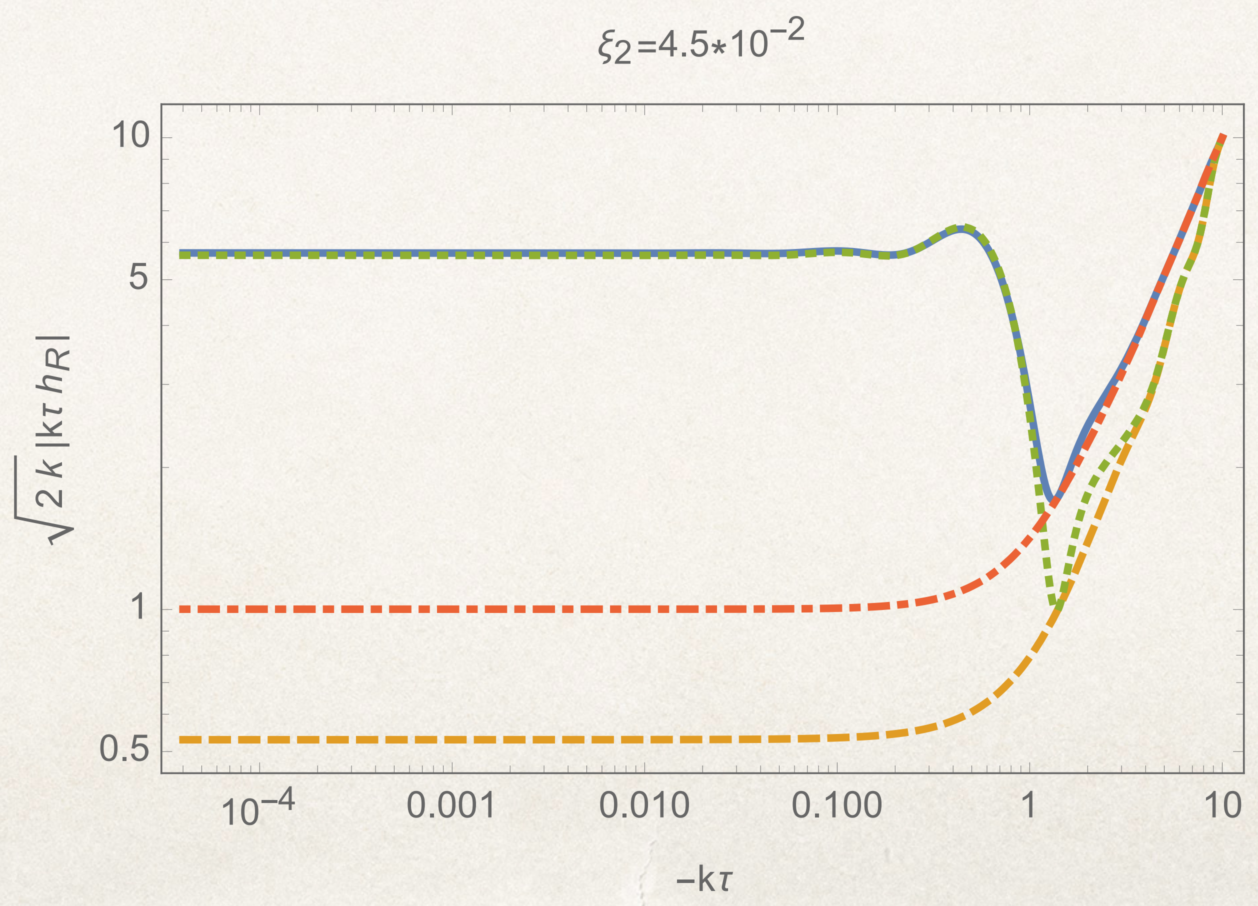


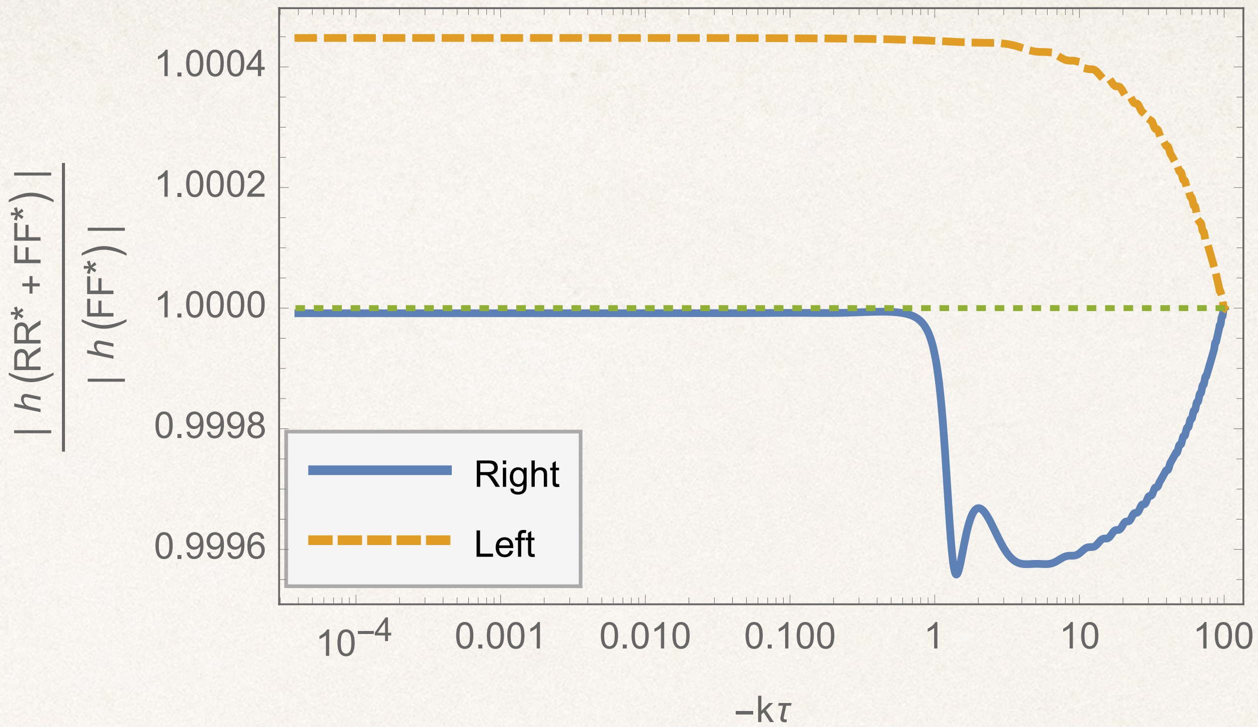
$\xi_2 = 4.5 * 10^{-6}$

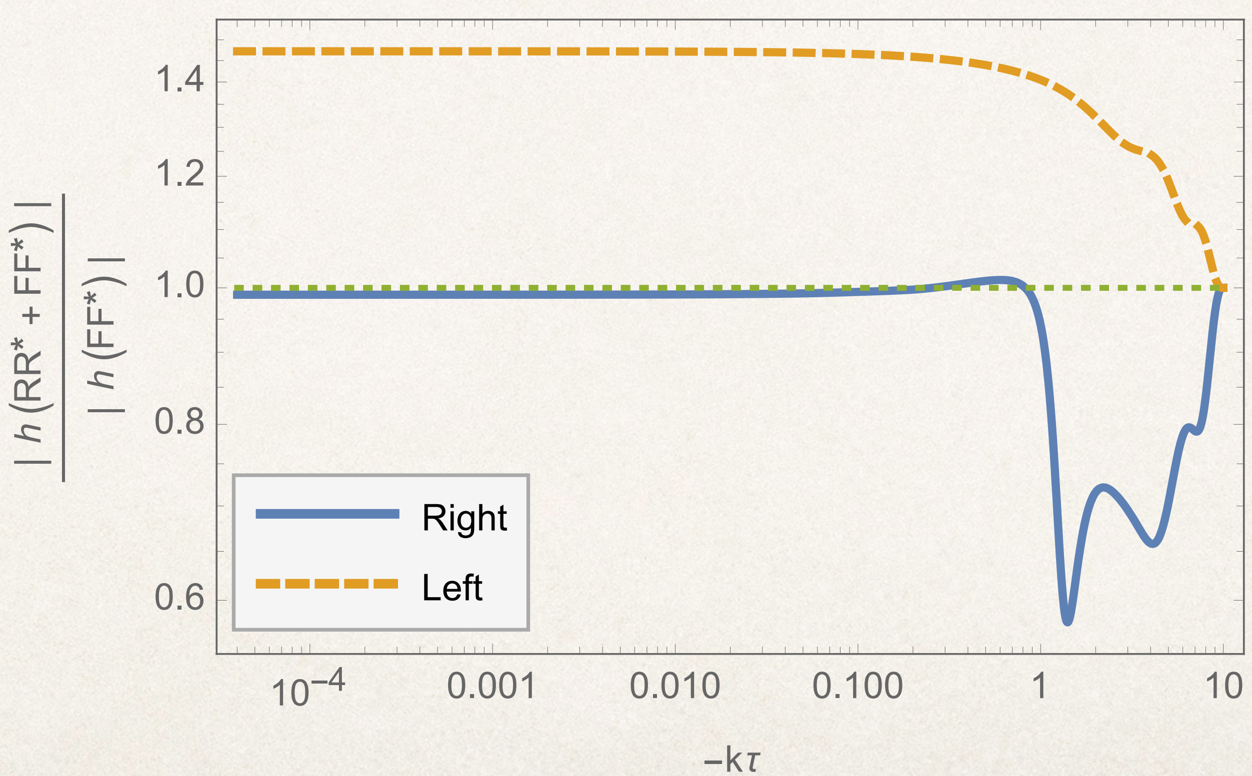


With $\mathbf{F\tilde{F}}$






1.0004
1.0002
1.0000
0.9998
0.9996
| h (RR* + FF*) |
| h (FF*) |
Right
Left
10^−4
0.001
0.010
0.100
1
10
100
−kτ


1.4
1.2
1.0
0.8
0.6
| h (RR* + FF*) |
| h (FF*) |
Right
Left
10^−4
0.001
0.010
0.100
1
10
−kτ

What About Ghosts?

$$\left(1 - s\xi_2 x\right)\left[\partial_x^2 h_A + (1 - \frac{2}{x^2})h_A\right] - 2s\xi_2\partial_x h_A = \mathcal{O}(t_A)$$

For s = 1 in physical coordinates: $(1 - \frac{\xi_2}{H}k_{phy})$

The sign of the kinetic term in the equation of motion for the right-handed helicity becomes negative!

Instability Analysis

This is an effective description so we can put a cut-off Λ for our theory to prevent these instabilities.

$$(1 - \frac{\xi_2}{H} k_{phy})$$

$$\downarrow$$

$$\boxed{\frac{\xi_2}{H} \Lambda < 1}$$

Constraints on Free Parameters

$$\lambda_2 = 2\xi_2 \frac{\lambda_1}{\xi_1} \left(\frac{M_{pl}}{H} \right)^2$$

Conservative case:

$$\Lambda = M_{pl}$$

$$\xi_2 < H/M_{pl}$$

$$\lambda_2 < 2\left(\frac{\lambda_1}{\xi_1} \right) \left(\frac{M_{pl}}{H} \right)$$

More radical case:

$$\Lambda = 20H$$

$$\xi_2 < 1/20$$

$$\lambda_2 < \left(\frac{1}{10} \right) \left(\frac{\lambda_1}{\xi_1} \right) \left(\frac{M_{pl}}{H} \right)^2$$

Constraints on Free Parameters

Conservative case:

$$\Lambda = M_{pl}$$

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More radical case:

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$$\lambda_2 < \left(\frac{1}{10}\right)\left(\frac{\lambda_1}{\xi_1}\right)\left(\frac{M_{pl}}{H}\right)^2$$

$\sim \mathcal{O}(10) \gtrsim 1.5 \times 10^9$

No stringent constraints on λ_2

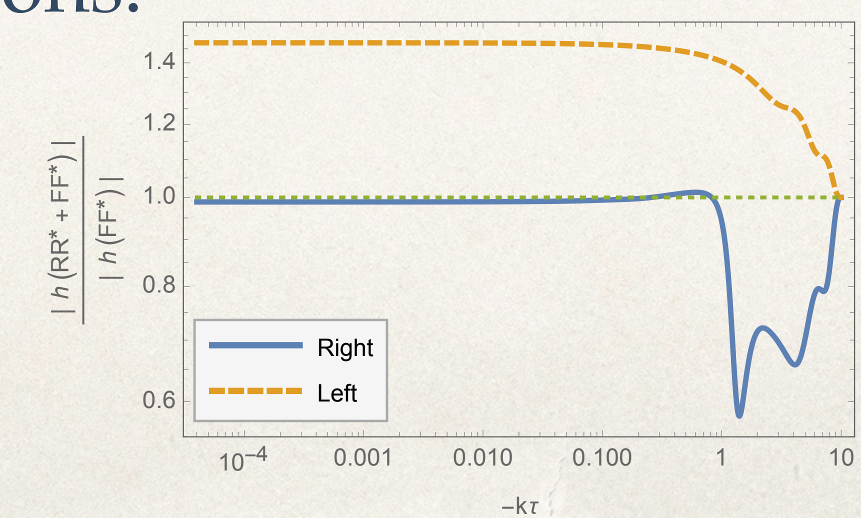
Conclusion

The inflation models with the spectator axion-SU(2) sector have unique signatures that are testable with the next generation of CMB experiments.

The inflation models with the spectator axion-SU(2) sector remain phenomenologically viable in the presence of the gravitational Chern-Simons term.

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{\varphi} + \mathcal{L}_{SPEC} + \mathcal{L}_{GCS}$$

The effect of the gravitational Chern-Simons term on chiral gravitational waves can be as large as fifty percent amplification for the left-handed helicity mode functions.



Thank You!