Lepto-Axiogenesis

Raymond Co

Leinweber Center for Theoretical Physics
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↓

William I. Fine Theoretical Physics Institute
University of Minnesota

Online CERN Axion Workshop June 24^{th} 2020

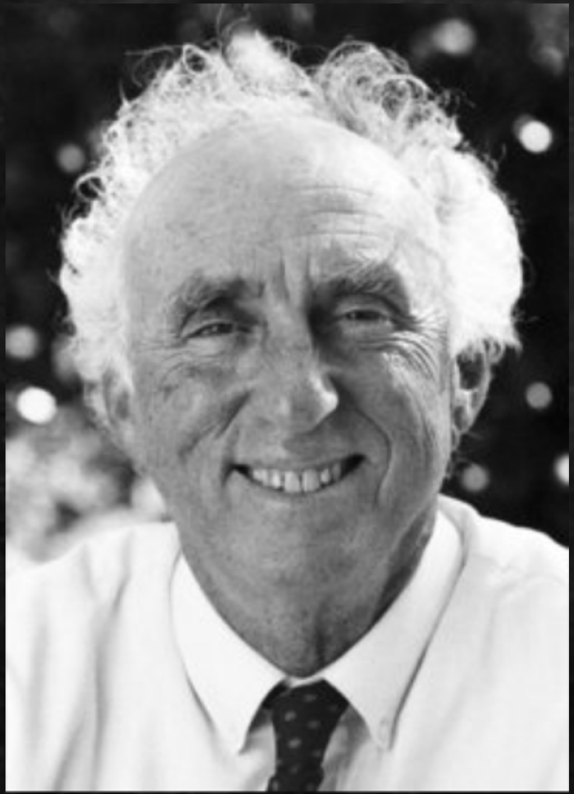
Collaborators:

arXiv: 1910.02080 Keisuke Harigaya Phys. Rev. Lett. 124, 111602 (2020)

arXiv: 1910.14152 Lawrence Hall, Keisuke Harigaya Phys. Rev. Lett. accepted

arXiv: 2006.05687 Nicolas Fernandez, Akshay Ghalsasi, Lawrence Hall, Keisuke Harigaya

Today



Martin L. Perl

1995 Nobel Prize in Physics

"for the discovery of the tau lepton"

born on

June 24th 1927

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https://www.nobelprize.org/prizes/physics/1995/perl/biographical/

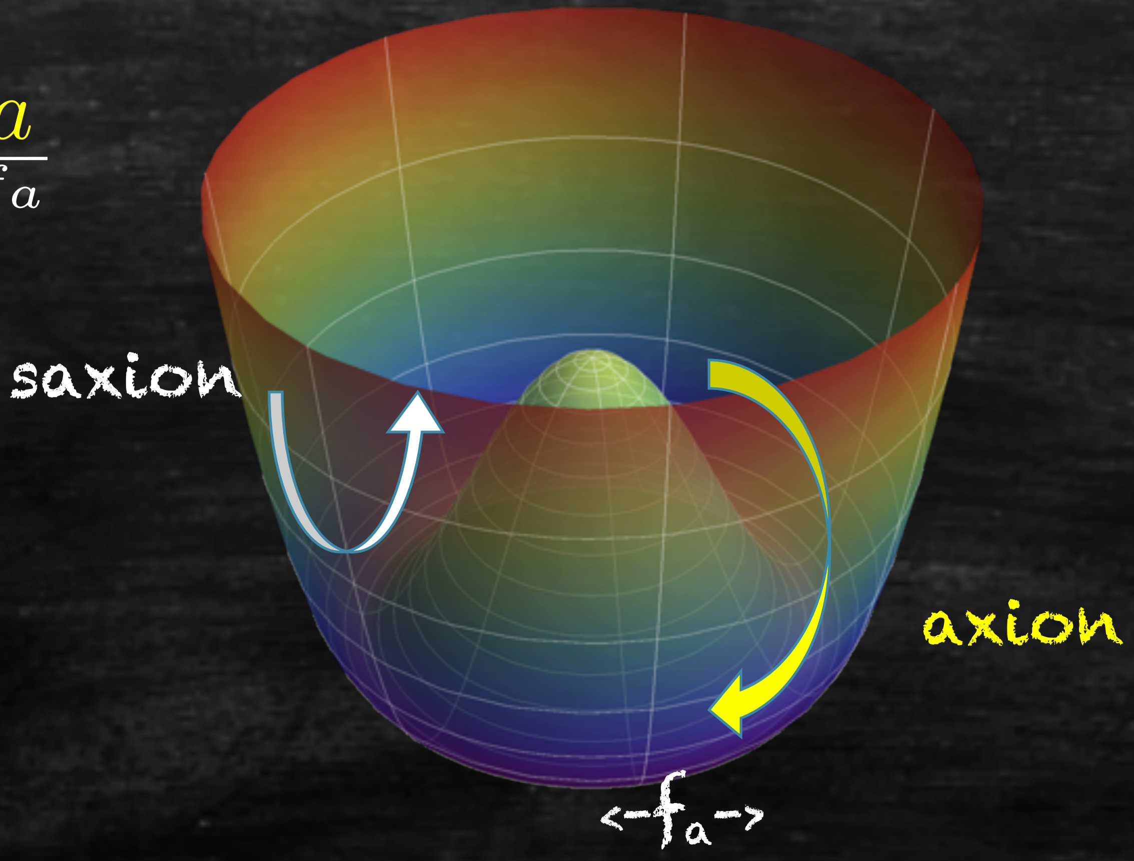
QCD axion

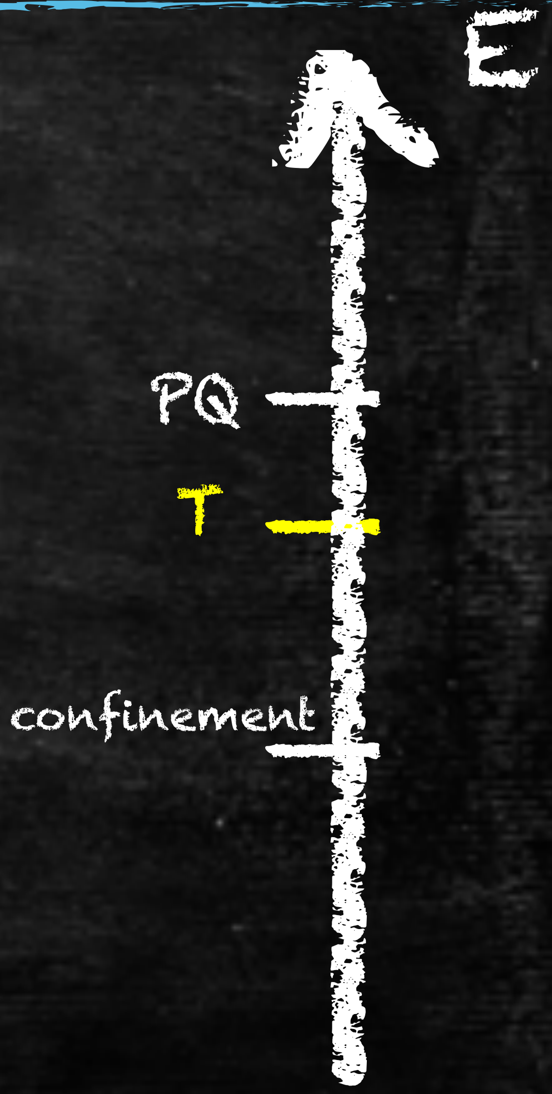
See 2006.04809 RC, L. Hall, K. Harigaya for axion-like particles.

ALP cogenesis

Axions

$$P = \frac{S + f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$$





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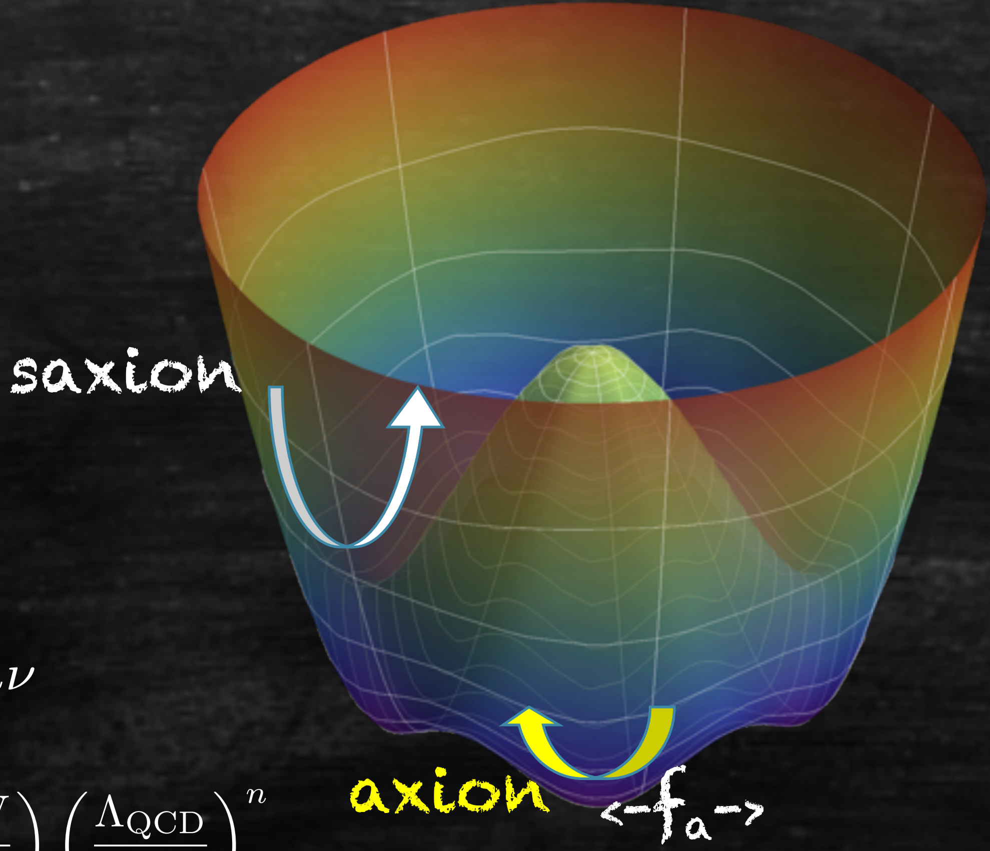
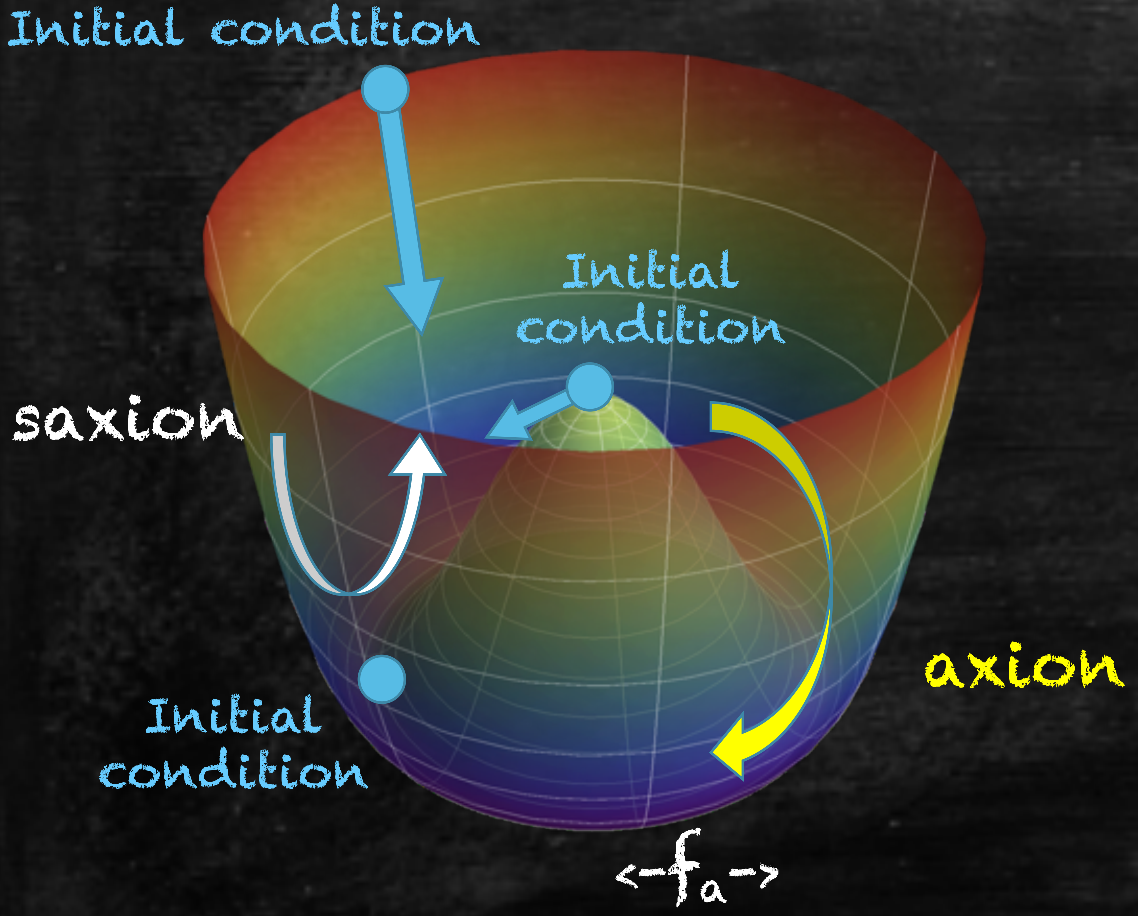
Axions

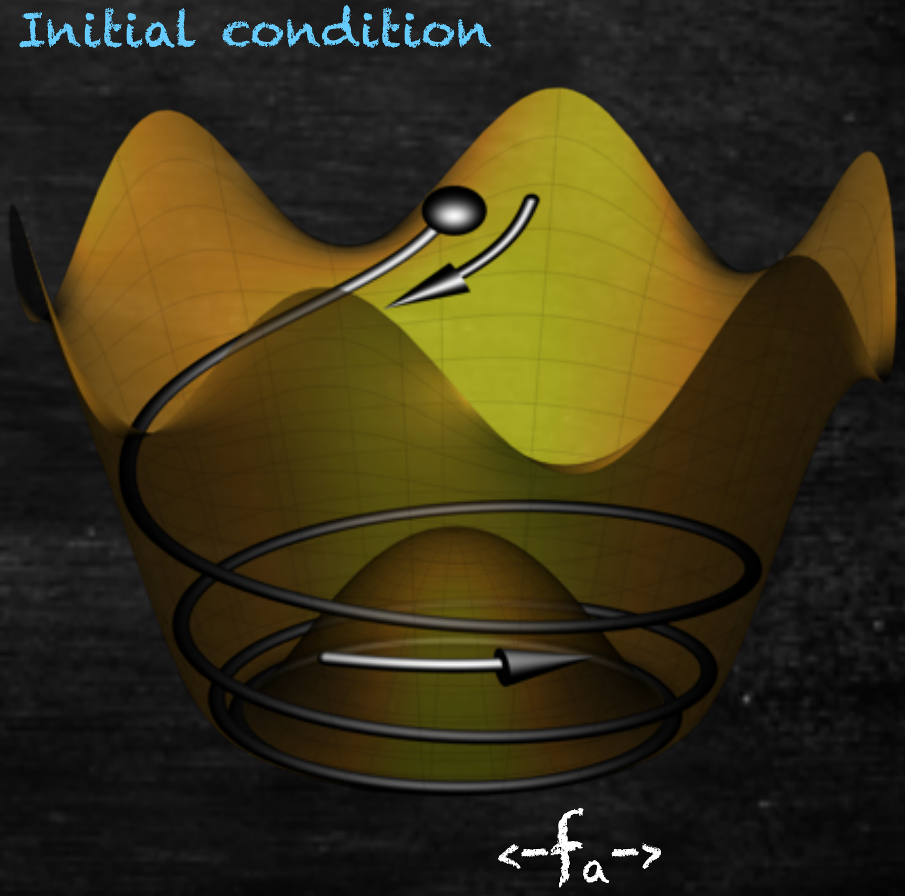


$$\mathcal{L} \supset \frac{\alpha}{8\pi}\frac{a}{f_a}G^{\mu\nu}\tilde{G}_{\mu\nu}$$

$$m_a(T \geq \Lambda_{\rm QCD}) = 6\,{\rm eV}\left(\frac{10^6\ {\rm GeV}}{f_a}\right)\left(\frac{\Lambda_{\rm QCD}}{T}\right)^n$$

Rotation





PRL 124, 111602 (2020) RC and K. Harigaya

Why Rotation?

$$P = \frac{S + f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$$

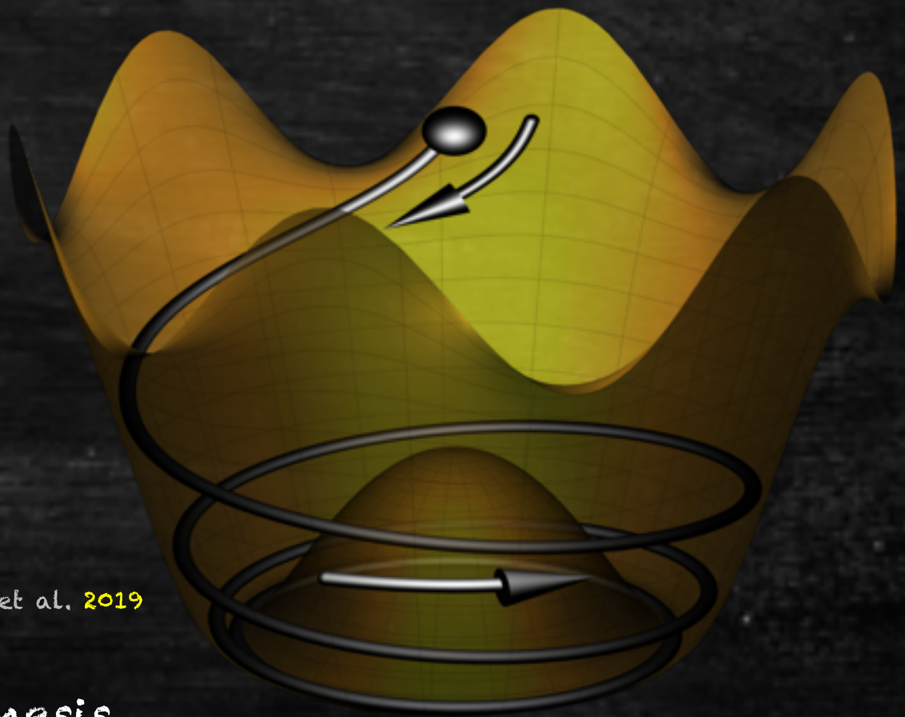
Large field value : Inflaton coupling

Initial condition

$$V(|P|) \sim -H_I^2 |P|^2 + \frac{|P|^{2d}}{M^{2d-4}}$$

Angular motion : Explicit PQ breaking

$$V(P) \sim \frac{P^n}{M^{n-4}} + \text{h.c.}$$



expected from quantum gravity
or PQ as an accidental symmetry

S. Giddings et al. 1988, S. Coleman 1988, G. Gilbert 1988, D. Harlow et al. 2019
R. Holman 1992, S. Barr 1992, M. Kamiokowski 1992, D. Dine 1992

Dynamics analogous to that in Affleck-Dine baryogenesis

I. Affleck and M. Dine 1991

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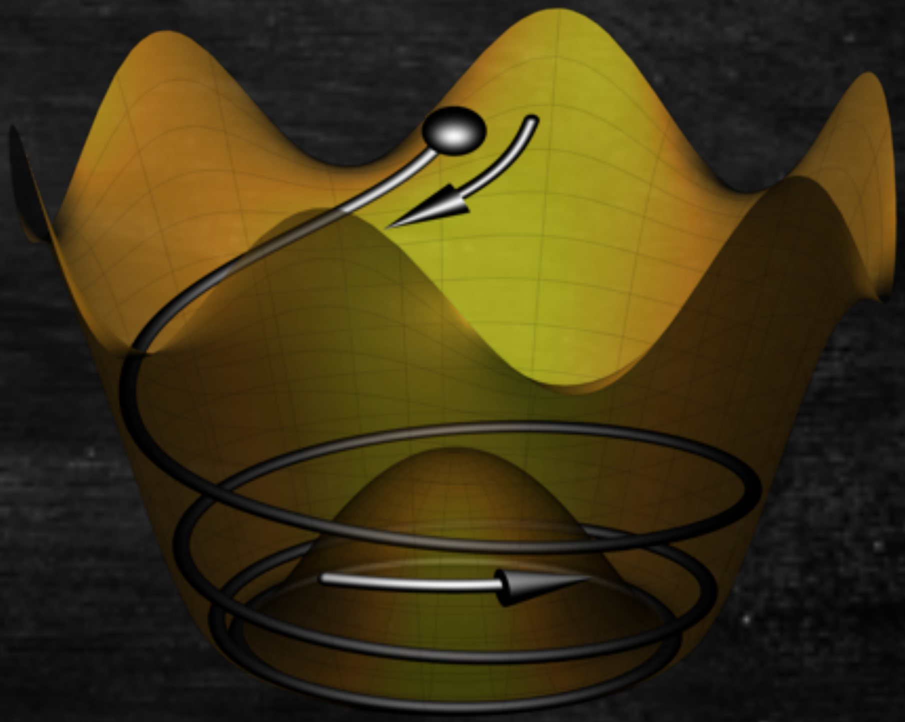
Asymmetry of PQ Charge

$$P = \frac{S + f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$$

Noether charge associated with the shift symmetry

$$n_{\rm PQ} = iP\dot{P}^* - iP^*\dot{P}$$

$$n_{\rm PQ} = S^2\dot{\theta}$$



PQ asymmetry
PQ charge density = Rotation of PQ field

PQ charge is conserved soon after the onset.

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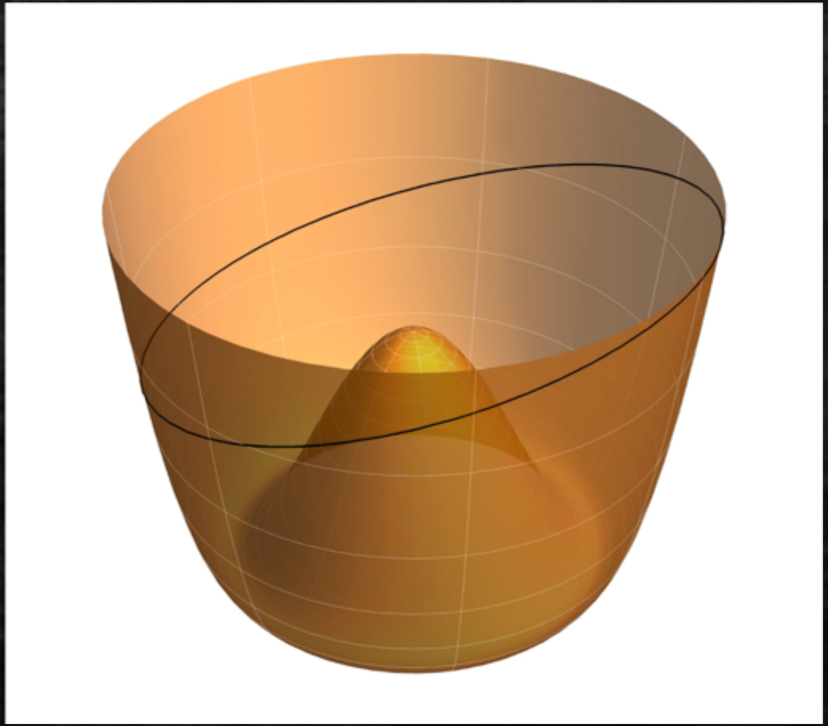
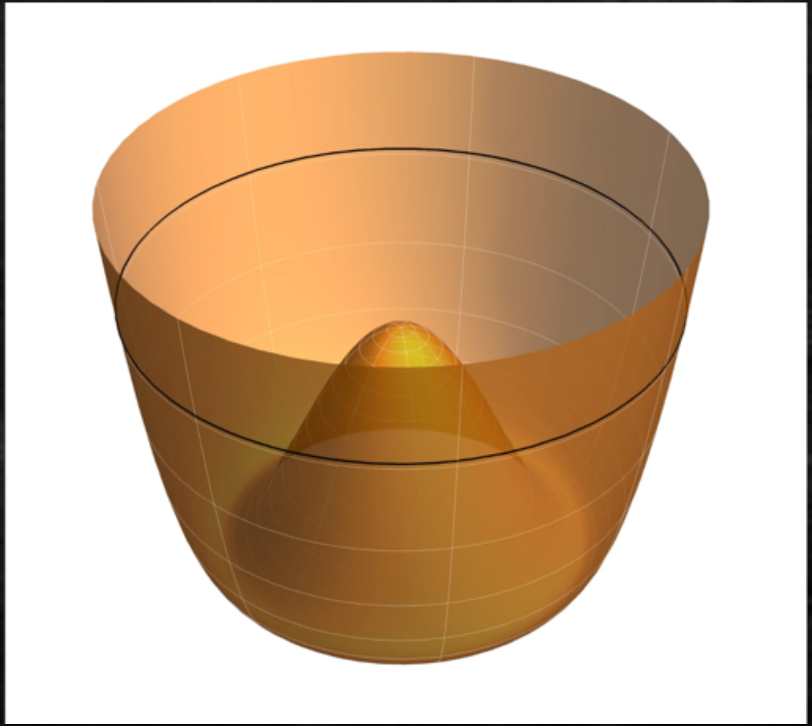
PQ Field Evolution

Thermalization

$$n_{\mathrm{PQ}} = S^2\dot{\theta}$$

Redshift



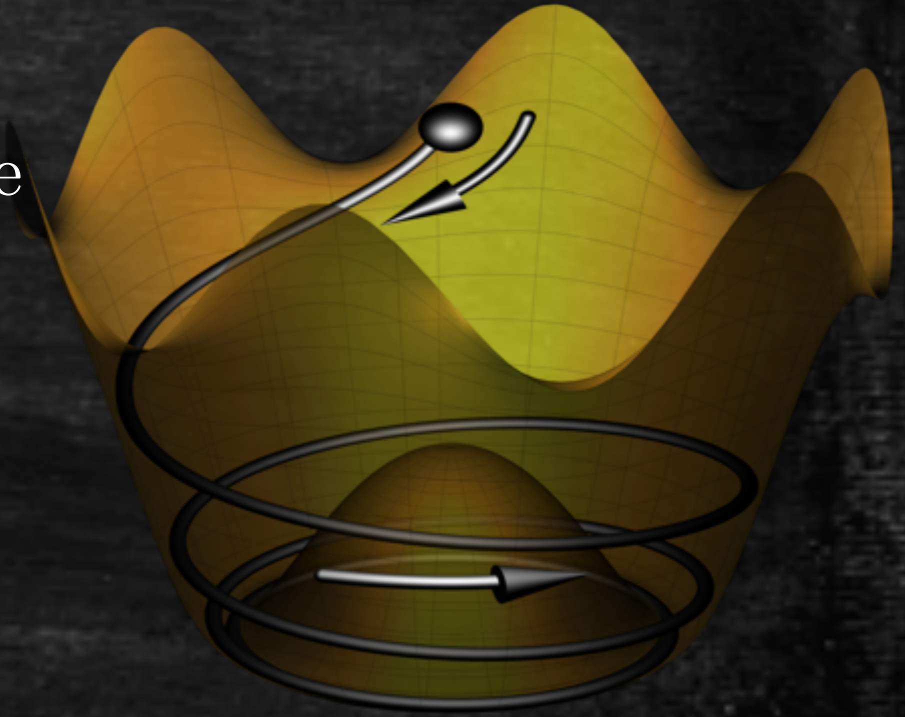


PRL 124, 111602 (2020) RC and K. Harigaya

Why a large angular speed?

Reason:

$$n_{\rm PQ} = S^2\dot{\theta} \qquad n_{\rm PQ}R^3 = \text{conserved charge}$$



Conventional:

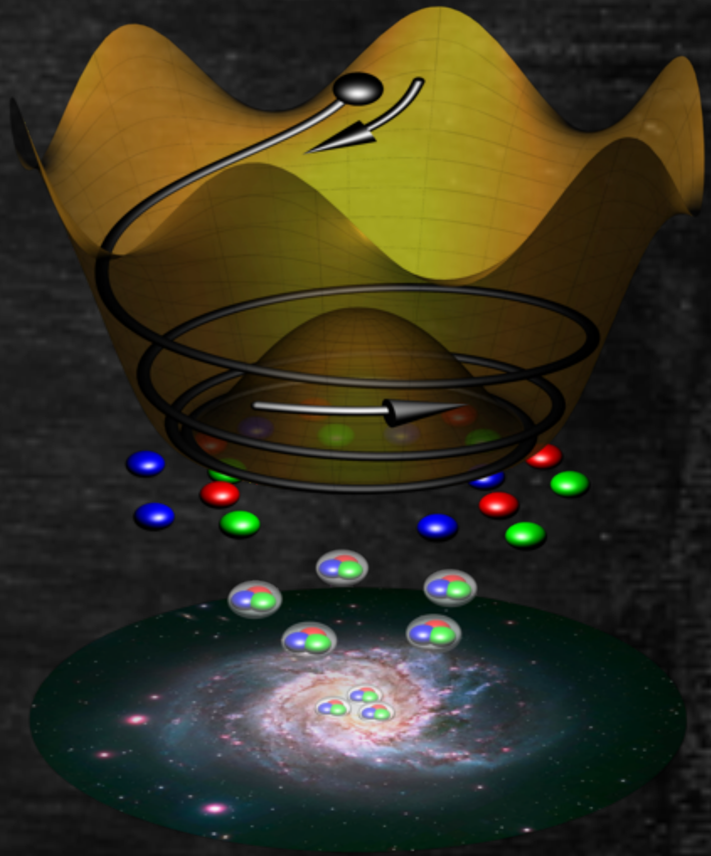
$$S^2 = f_a^2 \qquad \dot{\theta} \propto R^{-3}$$

Our scenario ($S \gg f_a$):

quartic	$S^2 \propto R^{-2}$	$\dot{\theta} \propto R^{-1}$	Slower redshift!
quadratic	$S^2 \propto R^{-3}$	$\dot{\theta} = \text{constant}$	

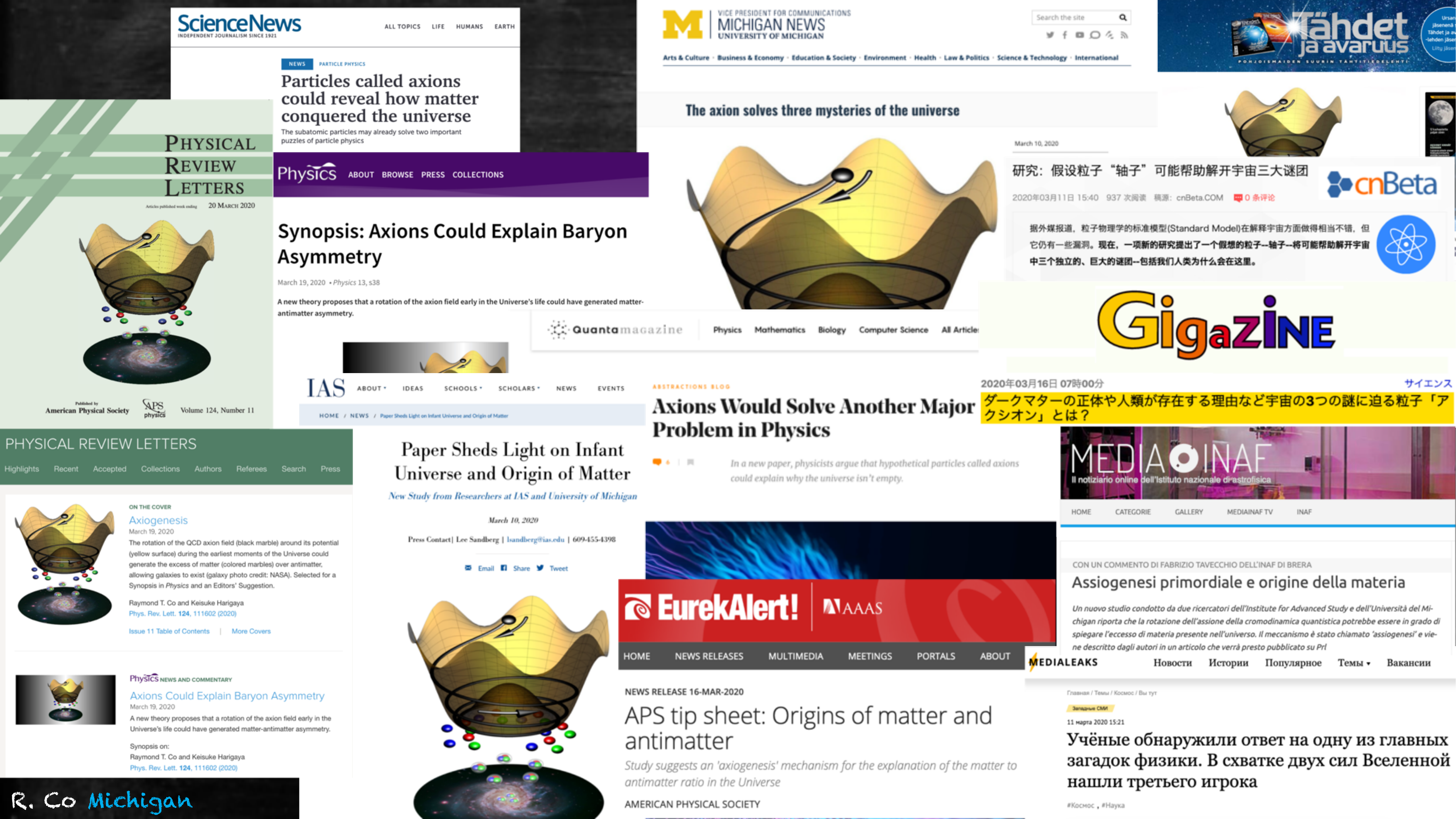
arXiv:1910.14152 RC, L. Hall, and K. Harigaya

Axiogenesis



(QCD axion + baryogenesis)

PRL 124, 111602 (2020) RC and K. Harigaya



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NEWS PARTICLE PHYSICS
Particles called axions could reveal how matter conquered the universe
The subatomic particles may already solve two important puzzles of particle physics
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The axion solves three mysteries of the universe
Tähdet ja avaruus
PHYSICAL REVIEW LETTERS
20 March 2020
Published by American Physical Society
Volume 124, Number 11
Physics ABOUT BROWSE PRESS COLLECTIONS
Synopsis: Axions Could Explain Baryon Asymmetry
March 19, 2020 • Physics 13, s38
A new theory proposes that a rotation of the axion field early in the Universe's life could have generated matter-antimatter asymmetry.
March 10, 2020
研究：假设粒子“轴子”可能帮助解开宇宙三大谜团
2020年03月11日 15:40 937 次阅读 稿源：cnBeta.COM 0 条评论
据外媒报道，粒子物理学的标准模型(Standard Model)在解释宇宙方面做得相当不错，但它仍有一些漏洞。现在，一项新的研究提出了一个假想的粒子--轴子--将可能帮助解开宇宙中三个独立的、巨大的谜团--包括我们人类为什么会在这里。
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Axions Would Solve Another Major Problem in Physics
In a new paper, physicists argue that hypothetical particles called axions could explain why the universe isn't empty.
2020年03月16日 07時00分
サイエンス
ダークマターの正体や人類が存在する理由など宇宙の3つの謎に迫る粒子「アクシオン」とは？
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ON THE COVER
Axiogenesis
March 19, 2020
The rotation of the QCD axion field (black marble) around its potential (yellow surface) during the earliest moments of the Universe could generate the excess of matter (colored marbles) over antimatter, allowing galaxies to exist (galaxy photo credit: NASA). Selected for a Synopsis in Physics and an Editors' Suggestion.
Raymond T. Co and Keisuke Harigaya
Phys. Rev. Lett. 124, 111602 (2020)
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Axions Could Explain Baryon Asymmetry
March 19, 2020
A new theory proposes that a rotation of the axion field early in the Universe's life could have generated matter-antimatter asymmetry.
Synopsis on:
Raymond T. Co and Keisuke Harigaya
Phys. Rev. Lett. 124, 111602 (2020)
Paper Sheds Light on Infant Universe and Origin of Matter
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March 10, 2020
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CON UN COMMENTO DI FABRIZIO TAVECCHIO DELL'INAF DI BRERA
Assiogenesi primordiale e origine della materia
Un nuovo studio condotto da due ricercatori dell'Institute for Advanced Study e dell'Università del Michigan riporta che la rotazione dell'assione della cromodinamica quantistica potrebbe essere in grado di spiegare l'eccesso di materia presente nell'universo. Il meccanismo è stato chiamato 'assiogenesi' e viene descritto dagli autori in un articolo che verrà presto pubblicato su Prl
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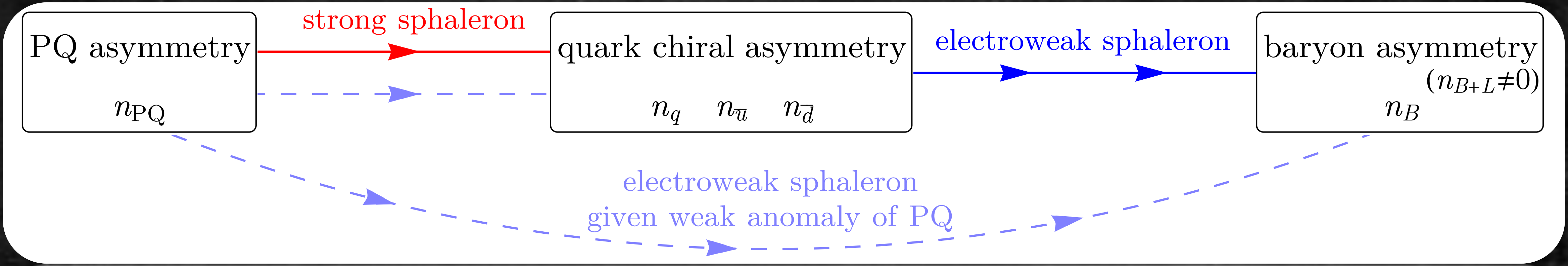
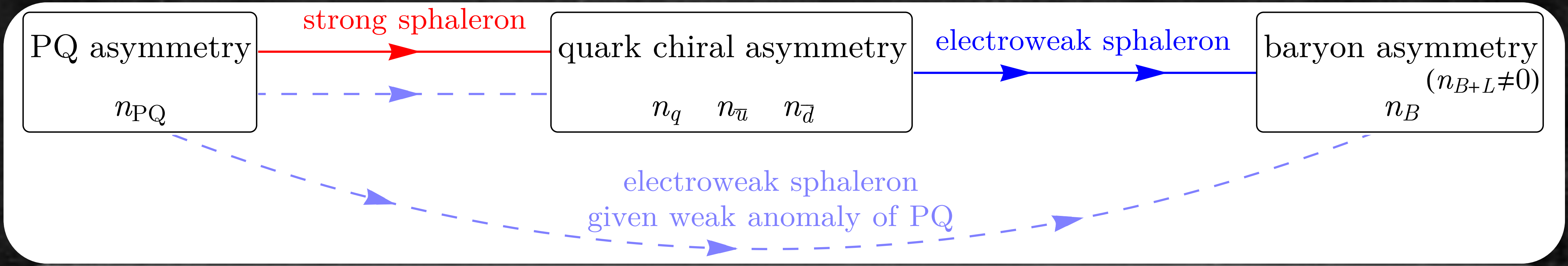
NEWS RELEASE 16-MAR-2020
APS tip sheet: Origins of matter and antimatter
Study suggests an 'axiogenesis' mechanism for the explanation of the matter to antimatter ratio in the Universe
AMERICAN PHYSICAL SOCIETY
MEDIALEAKS Новости Истории Популярное Темы Вакансии
Главная / Темы / Космос / Вы тут
11 марта 2020 15:21
Учёные обнаружили ответ на одну из главных загадок физики. В схватке двух сил Вселенной нашли третьего игрока
#Космос, #Наука
R. Co Michigan

Axiogenesis



PRL 124, 111602 (2020) RC and K. Harigaya

Axiogenesis



$$n_B = c_B \dot{\theta} T^2$$

$$c_B \simeq 0.1 - 0.15\, c_W$$

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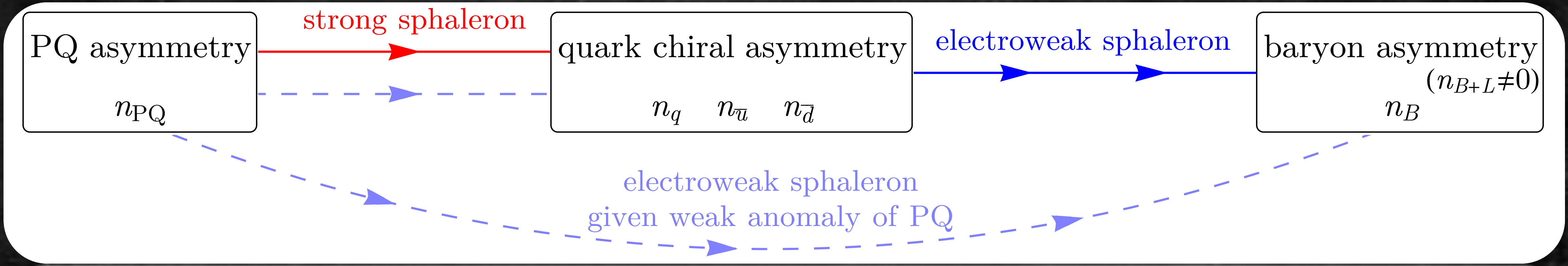
Axiogenesis



$$n_B = c_B \dot{\theta} T^2$$

$$c_B \simeq 0.1 - 0.15\, c_W$$

$$Y_B \equiv \frac{n_B}{s} = \left.\frac{c_B \dot{\theta} T^2}{s}\right|_{T=T_{\rm EW}} = c_B Y_{\rm PQ} \left(\frac{T_{\rm EW}}{f_a}\right)^2$$

Baryon asymmetry fixes rotational speed, equivalently $Y_{\rm PQ}$.

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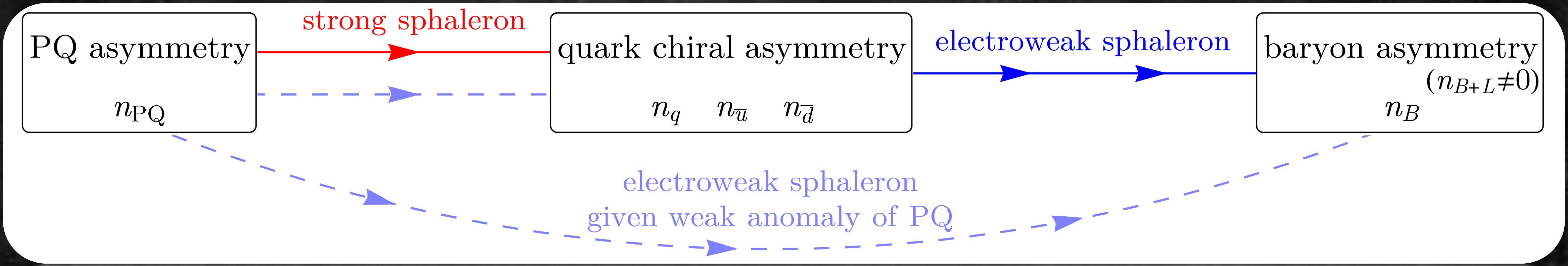
Axiogenesis



$$Y_B \simeq 10^{-10}\left(\frac{c_B}{0.1}\right)\left(\frac{T_{\rm EW}}{130\ {\rm GeV}}\right)^2\left(\frac{10^8\ {\rm GeV}}{f_a}\right)^2\left(\frac{Y_{\rm PQ}}{500}\right)^2$$

Baryon asymmetry fixes rotational speed, equivalently $Y_{\rm PQ}$.

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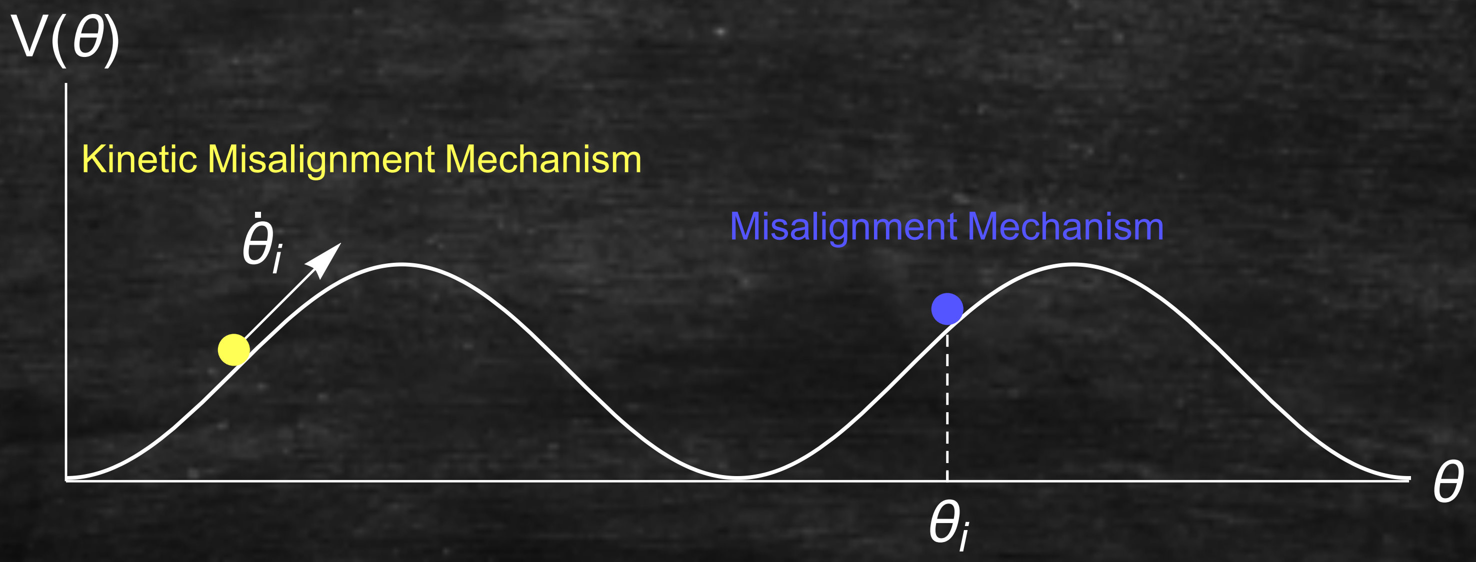


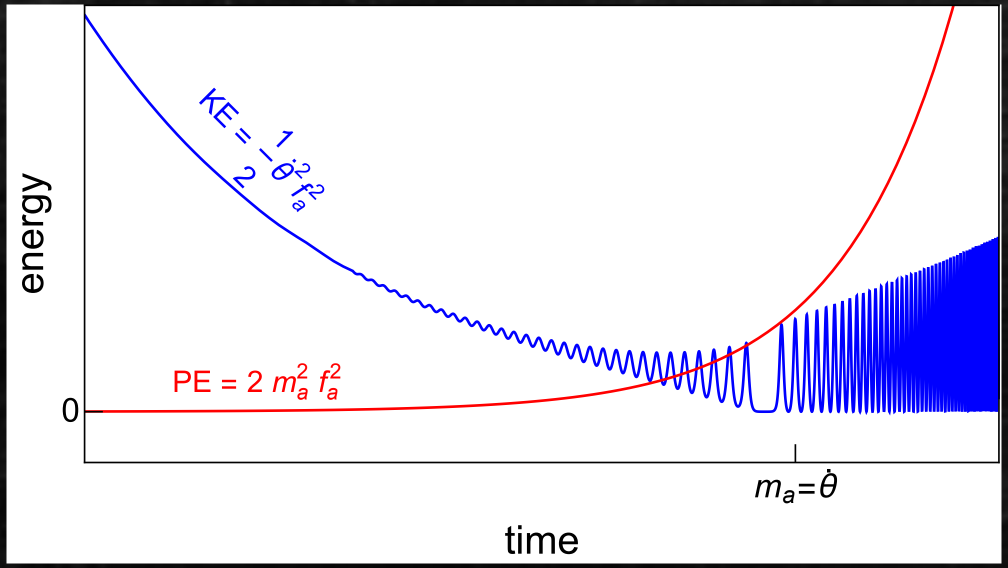
Kinetic Misalignment Mechanism

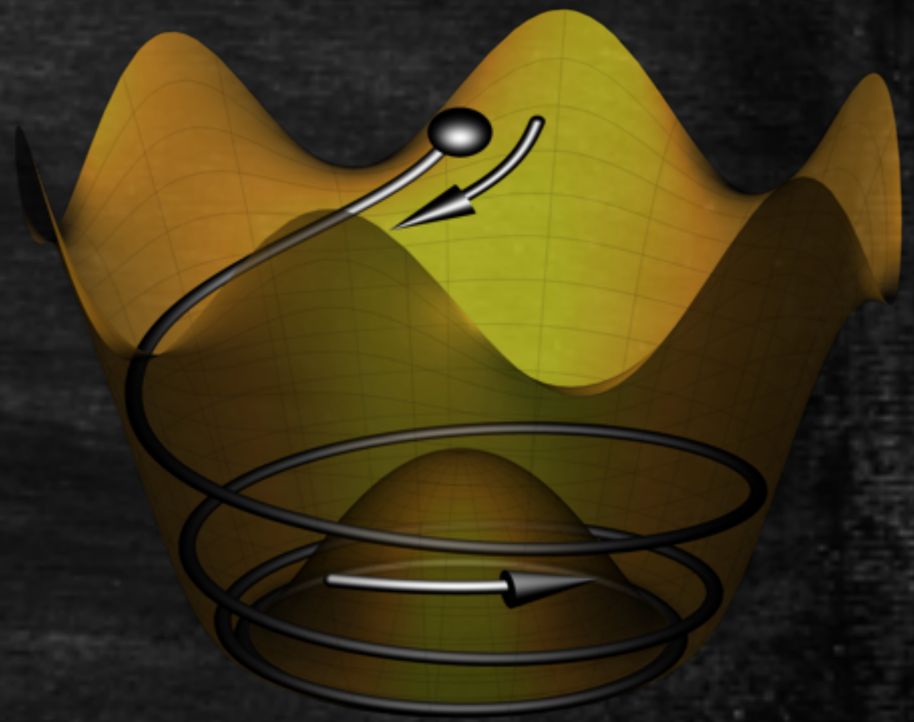
(Misalignment + non-zero kinetic energy)

arXiv:1910.14152 RC, L. Hall, and K. Harigaya

Kinetic Misalignment Mechanism

Consequence: delaying usual T_{osc} until KE = PE





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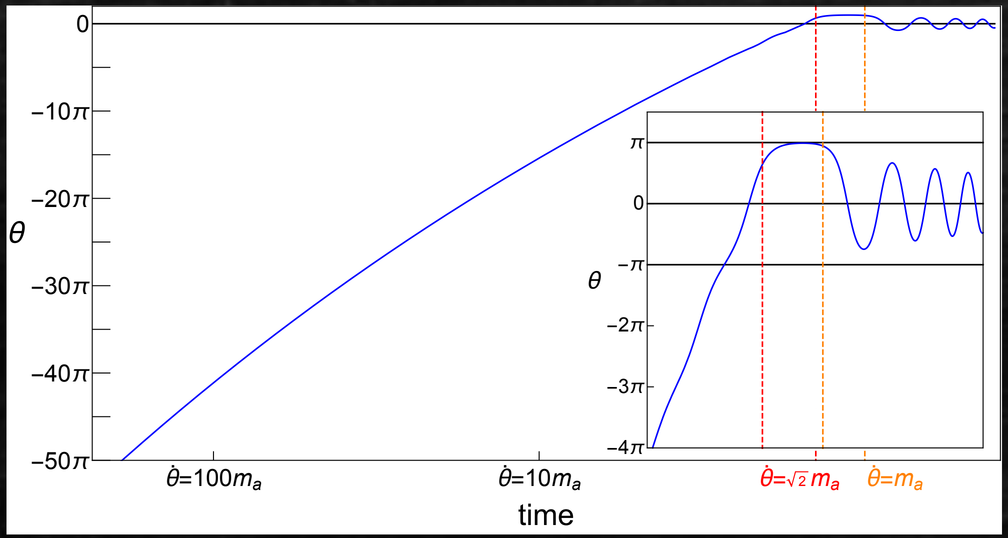
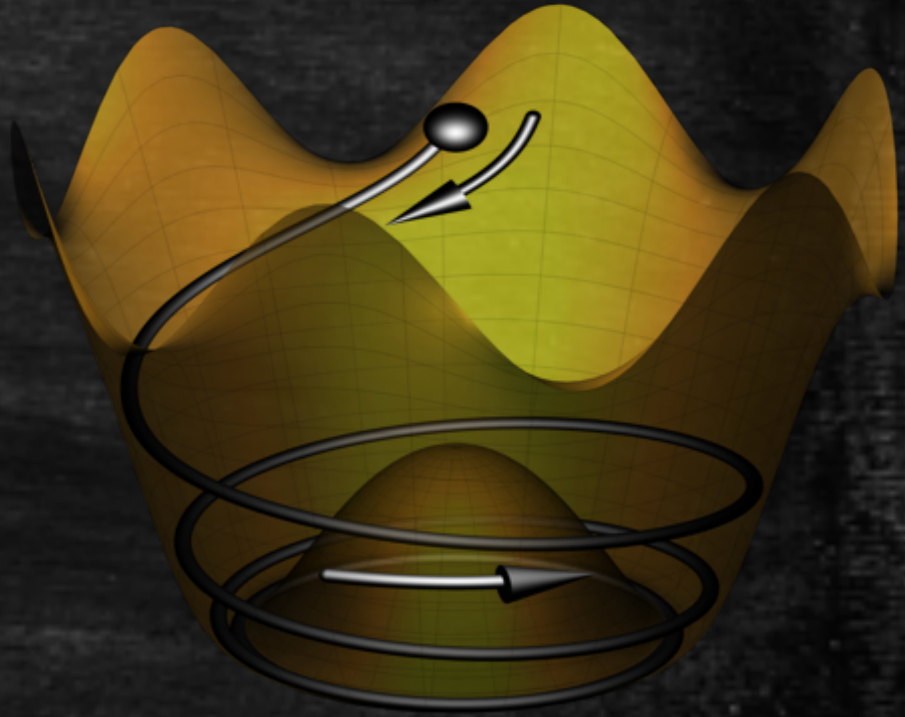
arXiv:1910.14152 RC, L. Hall, and K. Harigaya

Kinetic Misalignment Mechanism

Consequence: delaying usual T_{osc} until KE = PE





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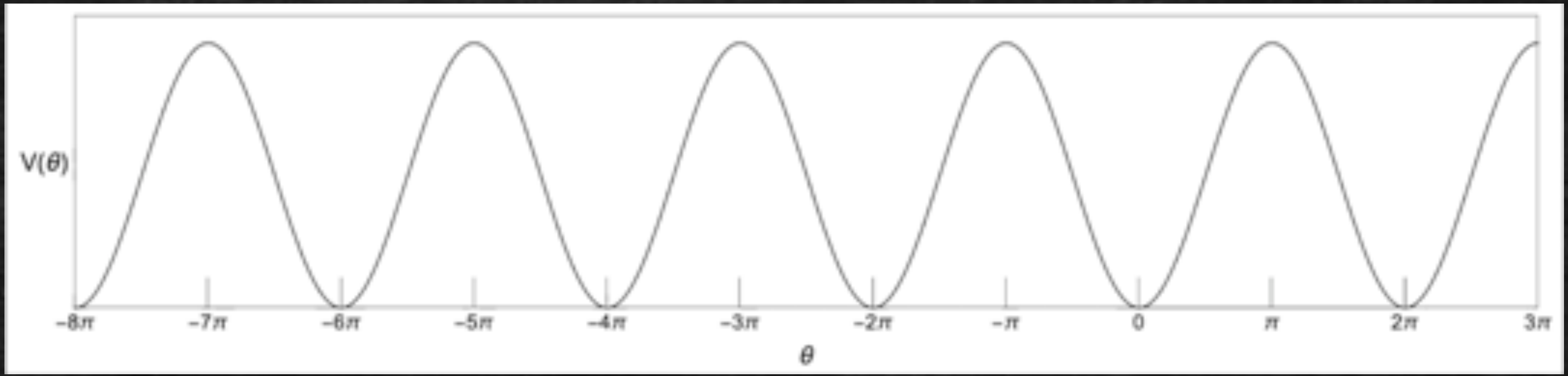
arXiv:1910.14152 RC, L. Hall, and K. Harigaya



Kinetic Misalignment Mechanism

Abundance:

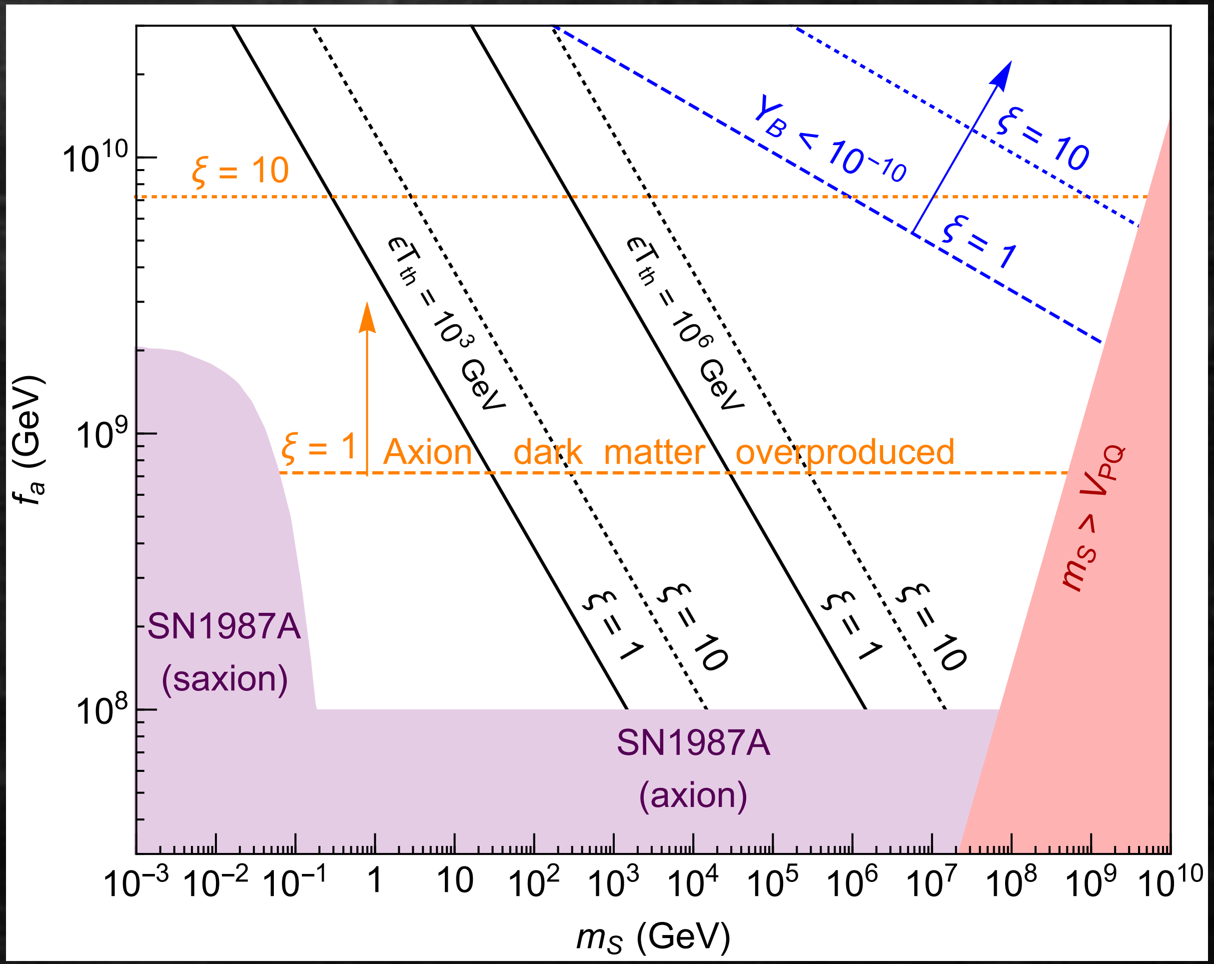
$$\Omega_a h^2 \simeq \Omega_{\rm DM} h^2 \left(\frac{10^8 \mathrm{GeV}}{f_a}\right)\left(\frac{Y_{\rm PQ}}{4}\right)$$

arXiv:1910.14152 RC, L. Hall, and K. Harigaya

Axiogenesis + Kinetic Misalignment

$$\frac{\Omega_a h^2}{\Omega_{\rm DM} h^2} \simeq 140 \left(\frac{f_a}{10^8\ \mathrm{GeV}}\right)\left(\frac{130\ \mathrm{GeV}}{T_{\rm EW}}\right)^2\left(\frac{0.1}{c_B}\right)$$

$$\xi \equiv 10^{-2} \times \left(\frac{T_{\rm EW}}{130\mathrm{GeV}}\right)^2\left(\frac{c_B}{0.1}\right)$$



T_{th} = thermalization temperature of saxions

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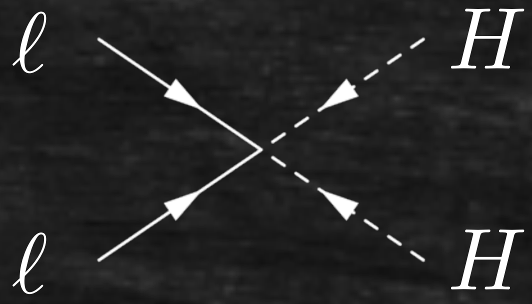
Axion with Majorana neutrinos

$$\dot{\theta} \neq 0$$



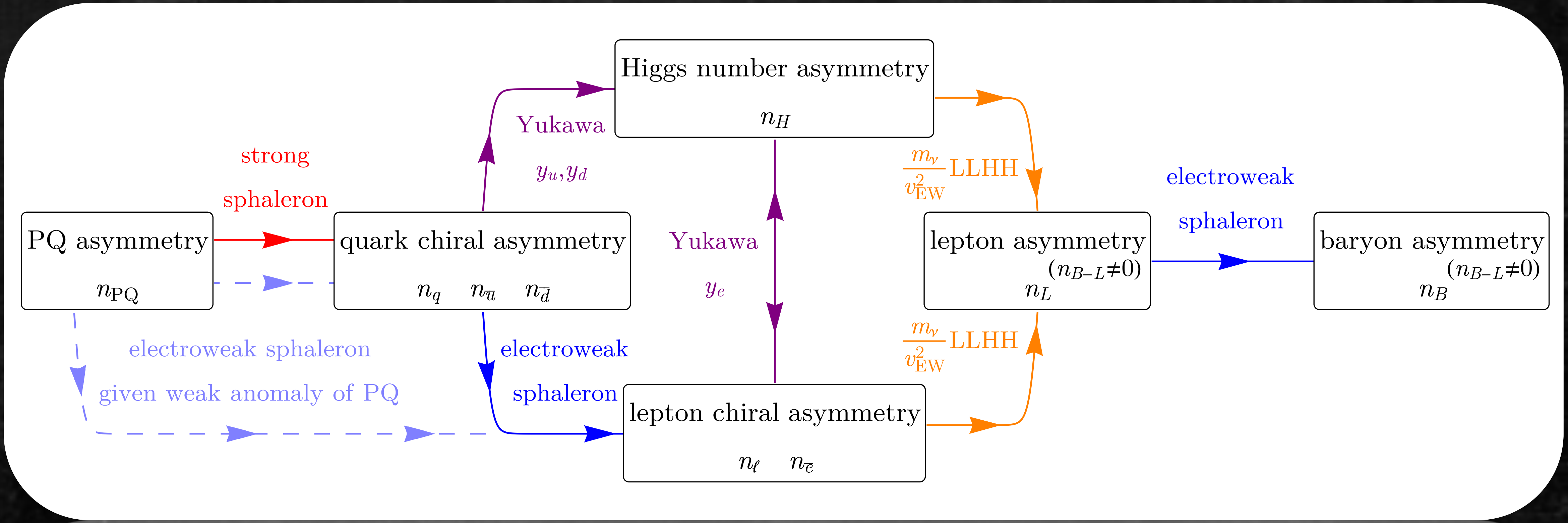
$$n_B \propto \dot{\theta} T^2$$

$$\mathcal{L} = \frac{m_\nu}{2 v_{\rm EW}^2} \ell \ell \, H^\dagger H^\dagger$$

2006.03148 V. Domcke, Y. Ema, K. Mukaida, M. Yamada

2006.05687 RC, N. Fernandez, A. Ghalsasi, L. Hall, K. Harigaya

Lepto-Axiogenesis

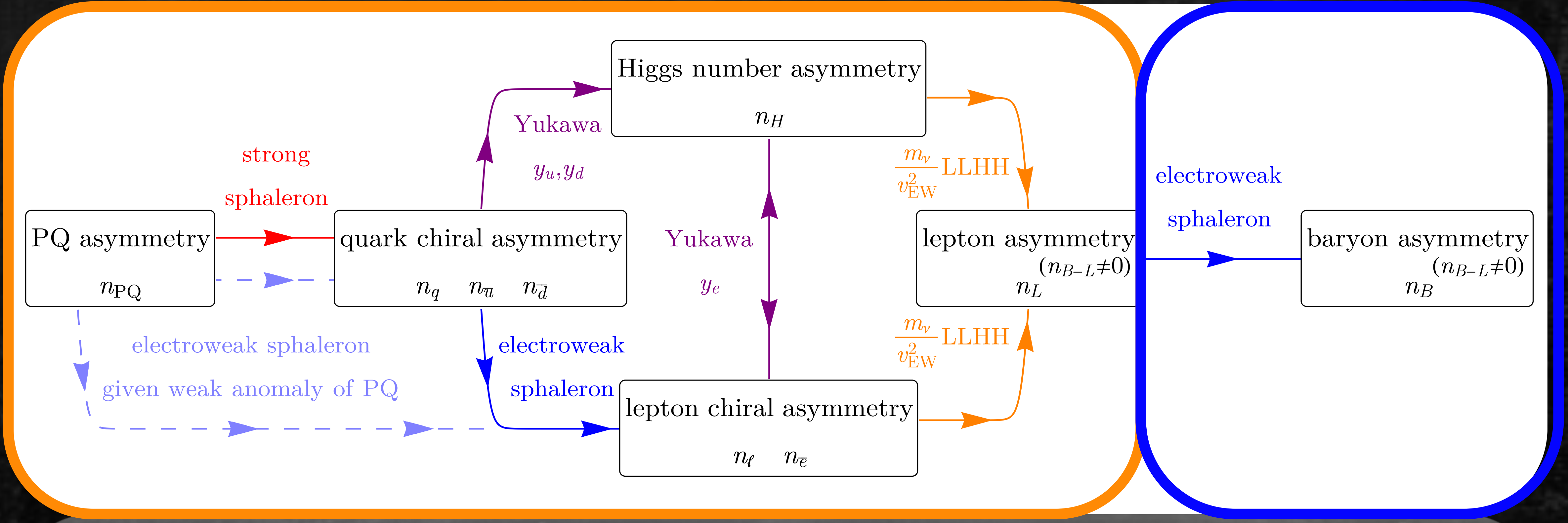


2006.05687 RC, N. Fernandez, A. Ghalsasi, L. Hall, K. Harigaya

Lepto-Axiogenesis

Producing L at high temperatures

Converting to B at T_{EW}



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Lepto-Axiogenesis

Producing L at high temperatures

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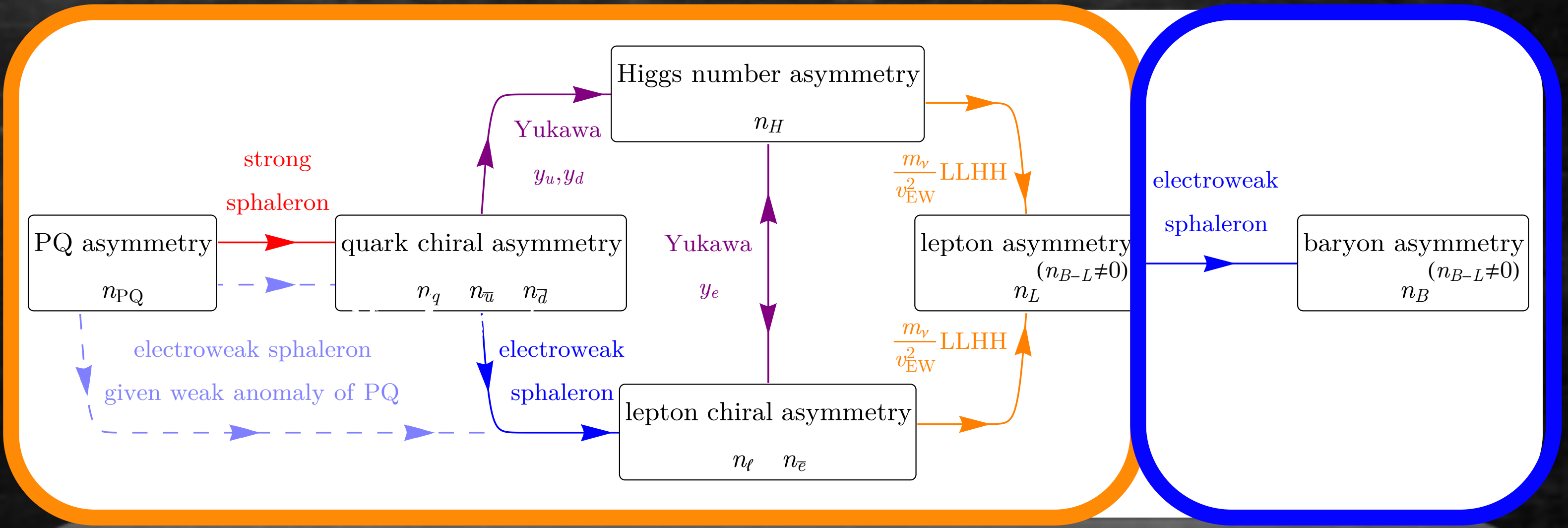


Lepton number violating interaction rate

$$\Gamma_L \simeq \frac{1}{4\pi^3}\frac{\bar{m}^2}{v_{\rm EW}^4}T^3$$

$$\bar{m}^2 \equiv \sum m_\nu^2$$

Out of equilibrium when

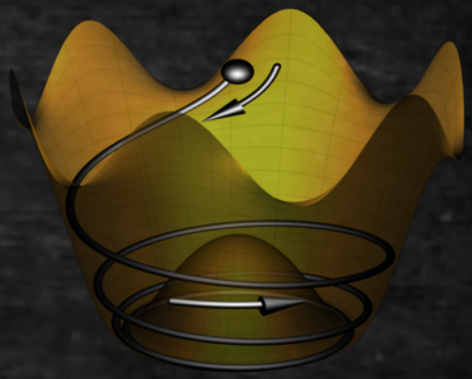
$$T \lesssim 5 \times 10^{12}\,{\rm GeV}\left(\frac{0.03{\rm eV}^2}{\bar{m}^2}\right)$$

$$Y_B = \frac{28}{79} Y_{B-L}$$

$$Y_{B-L} \equiv \frac{n_{B-L}}{s} = \frac{\Gamma_L}{H}\frac{c_{B-L}\dot{\theta}T^2}{s}$$

2006.05687 RC, N. Fernandez, A. Ghalsasi, L. Hall, K. Harigaya

Lepto-Axiogenesis



$$Y_{B-L} \equiv \frac{n_{B-L}}{s} = \frac{\Gamma_L}{H}\frac{c_{B-L}\dot{\theta}T^2}{s}$$

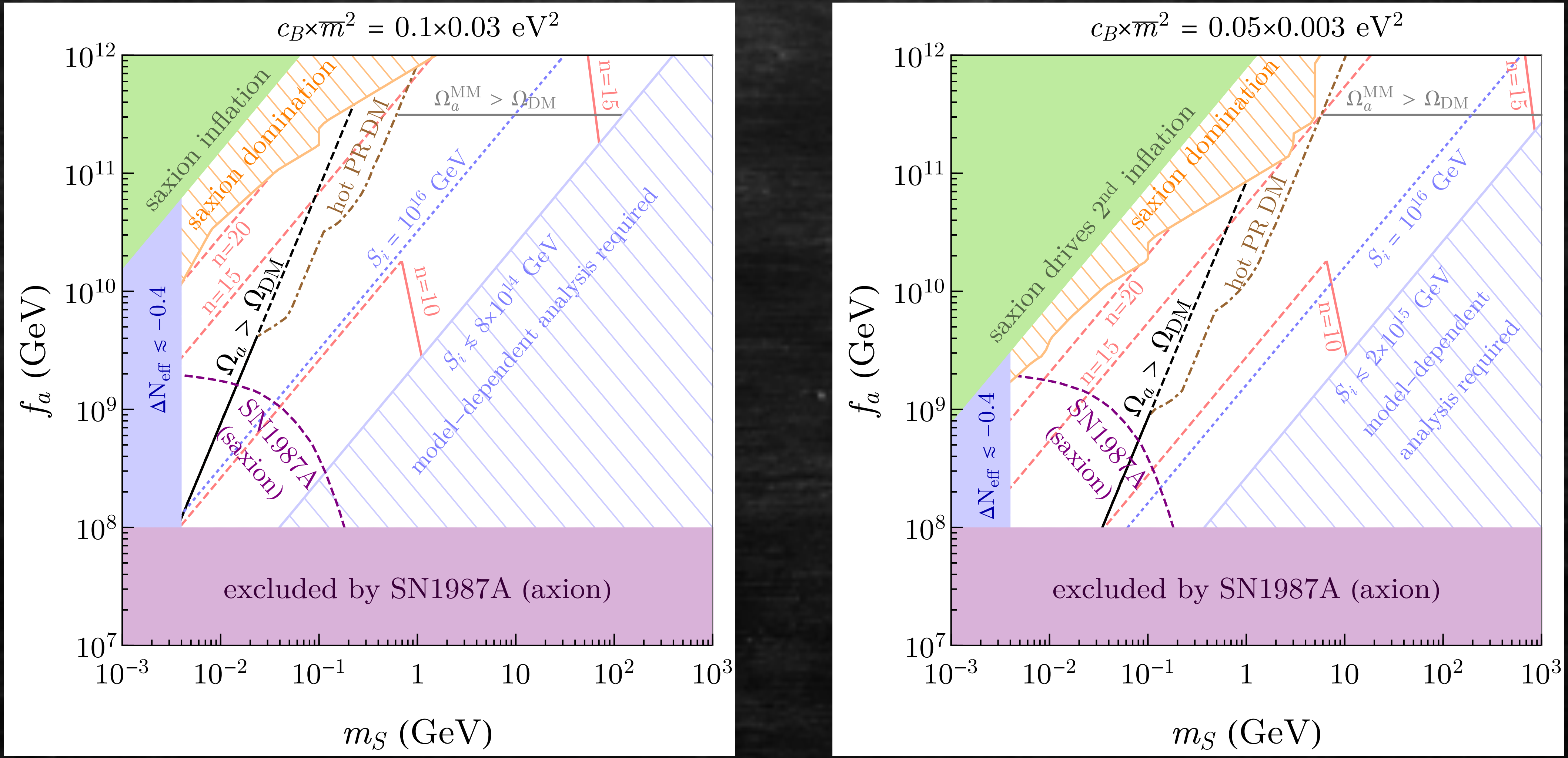
In a radiation-dominated epoch $T \times \frac{\dot{\theta}}{T}$ → { quartic / quadratic }

	$S \gg f_a$	$S = f_a$
quartic	$\dot{\theta} \propto T$	$\dot{\theta} \propto T^3$
quadratic	$\dot{\theta} = \text{constant}$	

2006.05687 RC, N. Fernandez, A. Ghalsasi, L. Hall, K. Harigaya

Radiation-dominated universe

Quartic potential



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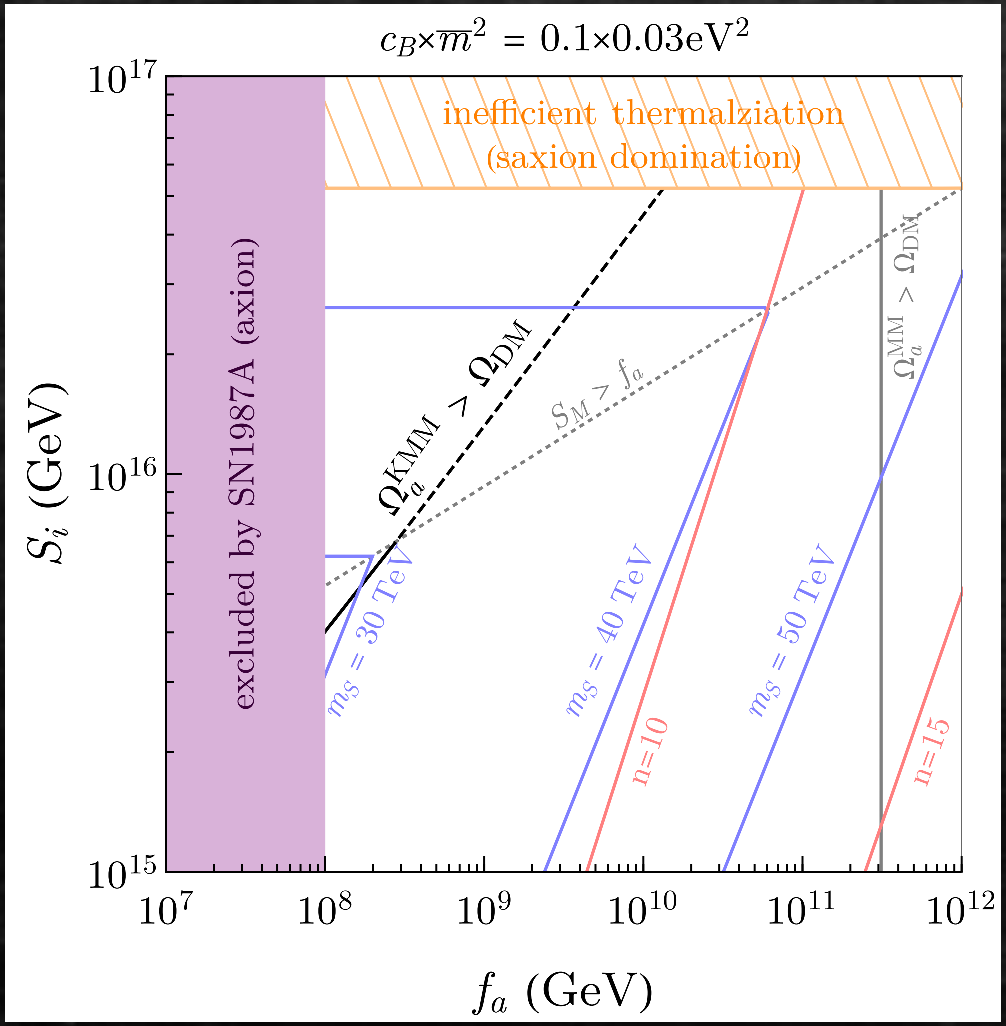
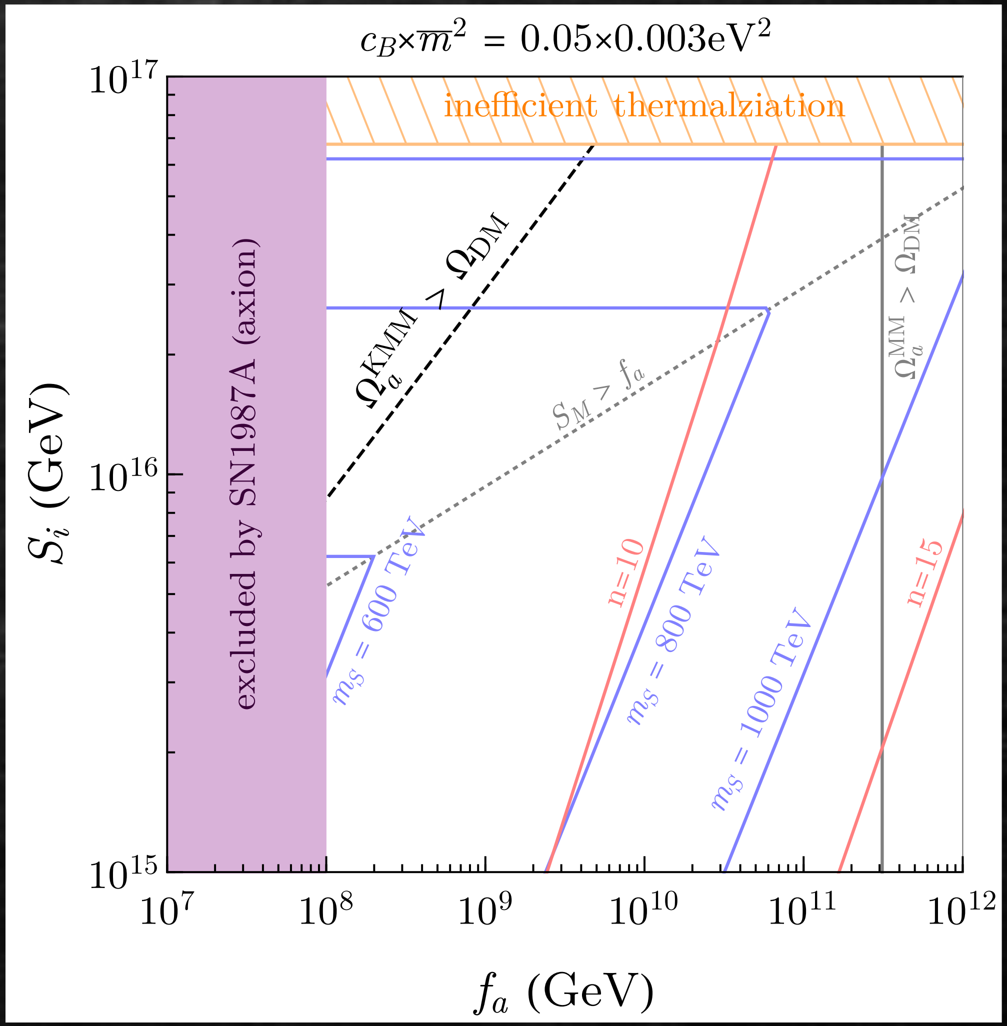
Radiation-dominated universe

Quadratic potential

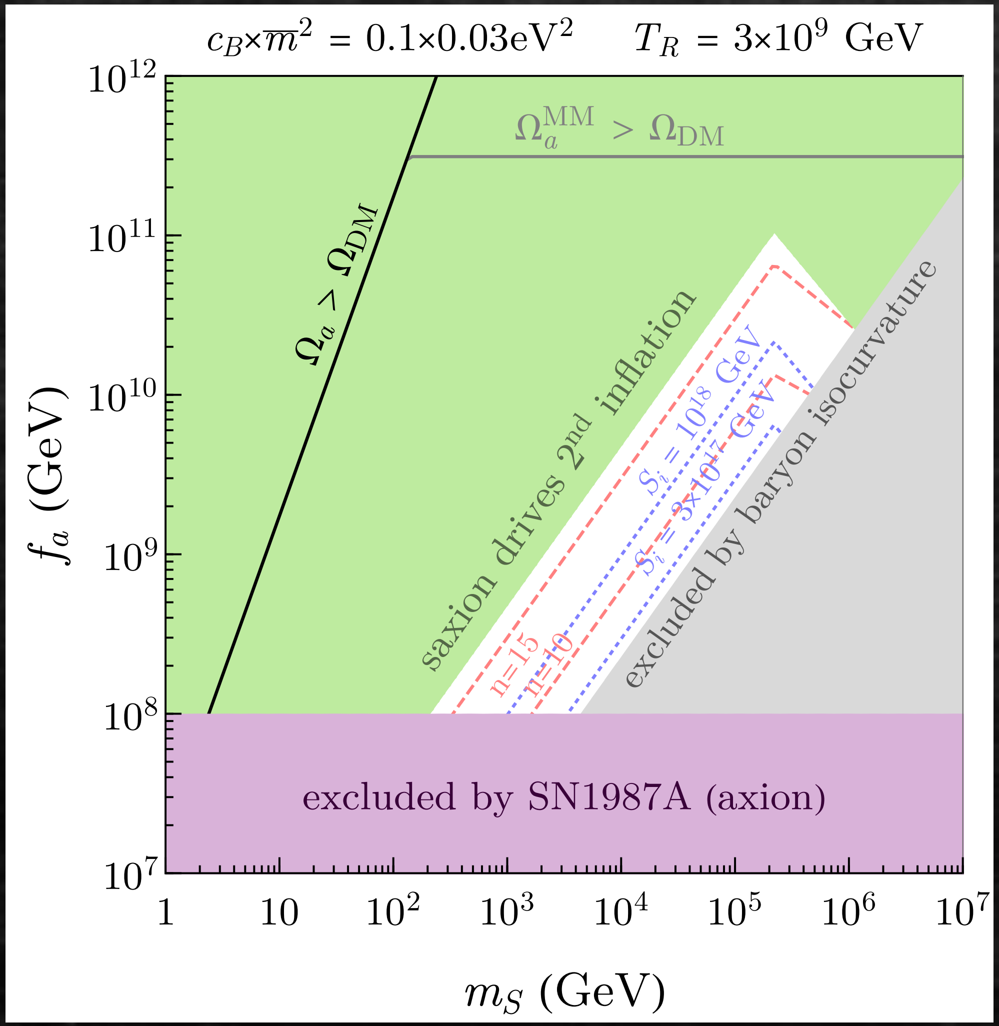


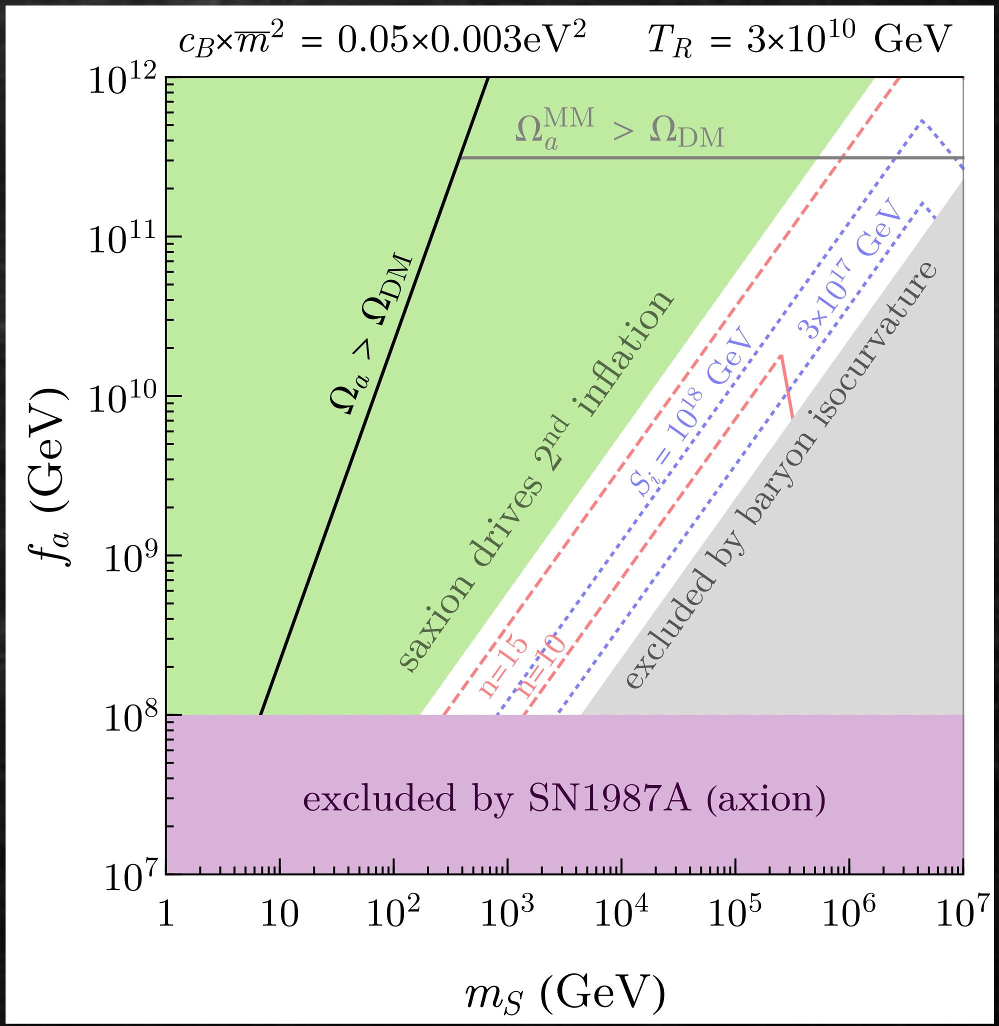


2006.05687 RC, N. Fernandez, A. Ghalsasi, L. Hall, K. Harigaya

Low reheat temperature

Quartic potential





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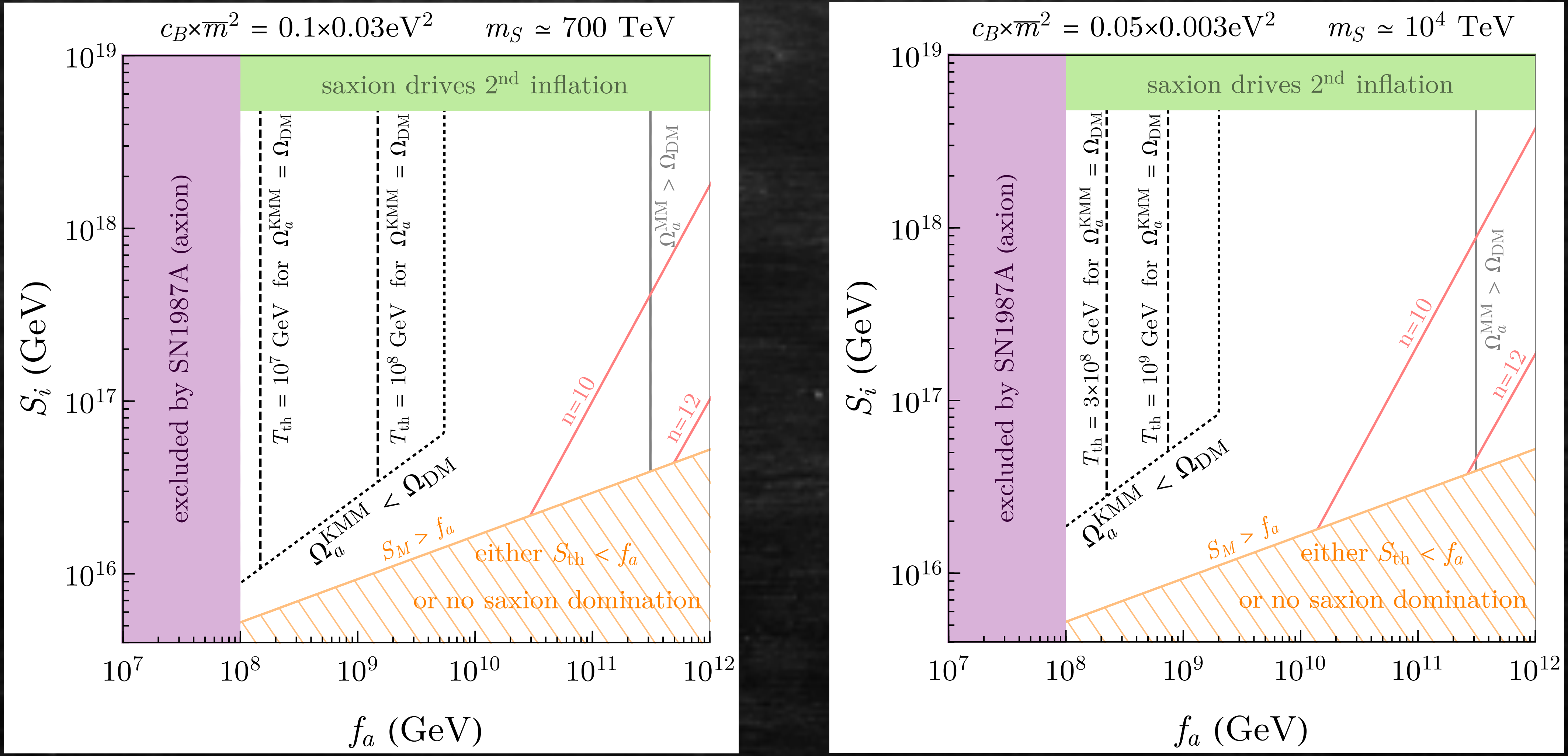
2006.05687 RC, N. Fernandez, A. Ghalsasi, L. Hall, K. Harigaya

Matter-dominated universe

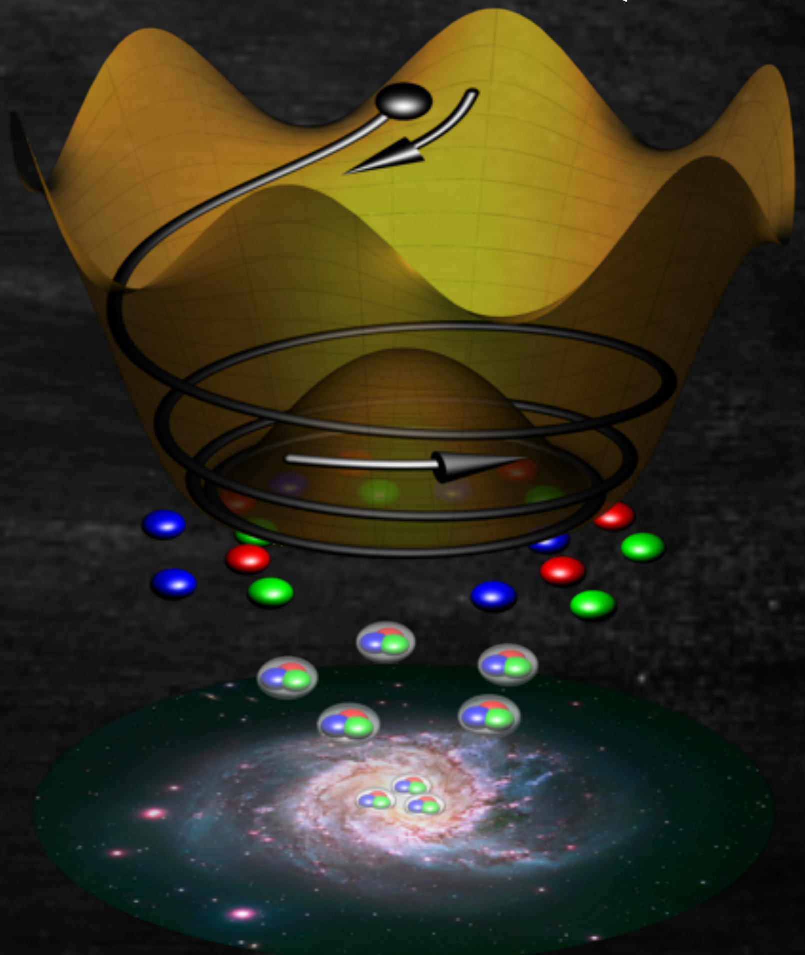
Quadratic potential



2006.05687 RC, N. Fernandez, A. Ghalsasi, L. Hall, K. Harigaya

Conclusions

- ✓ Lepto-axiogenesis allows the QCD axion to simultaneously explain
  - ✓ the Strong CP problem
  - ✓ the dark matter abundance
  - ✓ the baryon asymmetry
- ✓ Possible signatures:
  - ✓ QCD axion searches
  - ✓ Saxion-Higgs mixing
  - ✓ dark matter ultracompact minihalos
- ✓ New model building opportunities

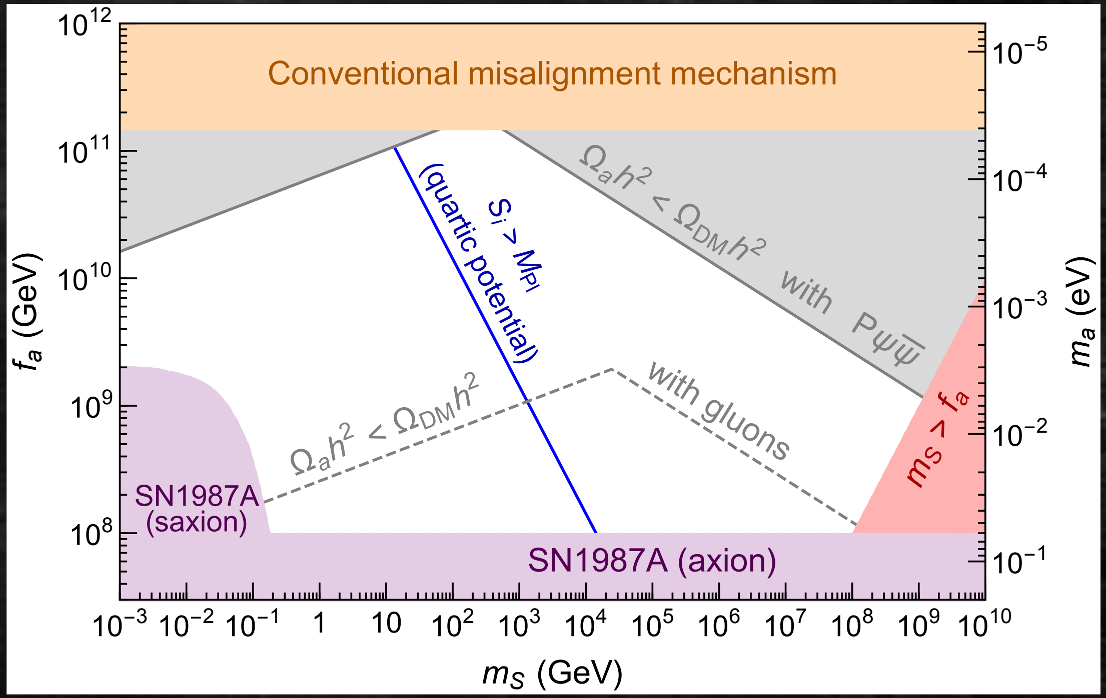


Thank you!

Happy birthday to Martin Perl!

Backup

Kinetic Misalignment Mechanism



Abundance:

$$\frac{\rho_a}{s} \simeq 2 m_a(0) Y_{\rm PQ}$$

$$\Omega_a h^2 \simeq \Omega_{\rm DM} h^2 \left(\frac{10^9\,{\rm GeV}}{f_a}\right)\left(\frac{Y_{\rm PQ}}{40}\right)$$

arXiv:1910.14152 RC, L. Hall, and K. Harigaya

Axiogenesis + Kinetic Misalignment

$$\frac{\Omega_a h^2}{\Omega_{\rm DM} h^2} \simeq 140 \left(\frac{f_a}{10^8 \text{ GeV}} \right) \left(\frac{130 \text{ GeV}}{T_{\rm EW}} \right)^2 \left(\frac{0.1}{c_B} \right)$$

$f_a \sim 10^6$ GeV - hadronic axion window?

It seems to have been closed.

M.S. Raffelt 1988

J. Engel, D. Seckel, and A. C. Hayes 1990

S. Chang and K. Choi 1993

J. H. Chang, R. Essig, and S. D. McDermott 2018

P. Carenza, T. Fischer, M. Giannotti, G. Guo, G. MartinezPinedo, and A. Mirizzi 2019

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Axiogenesis + Kinetic Misalignment

$$\frac{\Omega_a h^2}{\Omega_{\rm DM} h^2} \simeq 140 \left(\frac{f_a}{10^8 \text{ GeV}}\right) \left(\frac{130 \text{ GeV}}{T_{\rm EW}}\right)^2 \left(\frac{0.1}{c_B}\right)$$

T_{EW} ~ TeV ?

New physics at colliders!

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Axiogenesis + Kinetic Misalignment

$$\frac{\Omega_a h^2}{\Omega_{\rm DM} h^2} \simeq 140 \left(\frac{f_a}{10^8~\mathrm{GeV}}\right)\left(\frac{130~\mathrm{GeV}}{T_{\rm EW}}\right)^2\left(\frac{0.1}{c_B}\right)$$

$$c_B \simeq 0.1 - 0.15\, c_W$$

C_B ~ 100 so C_W ~ 1000 ?

Kim-Nilles-Peloso mechanism (clockwork axions)!

Kim, Nilles and Peloso 2004
Harigaya and Ibe 2014
Choi, Kim and Yun 2014
Choi and Im 2015
Kaplan and Rattazzi 2015
Farina, Pappadopulo, Rompineve and Tesi 2016

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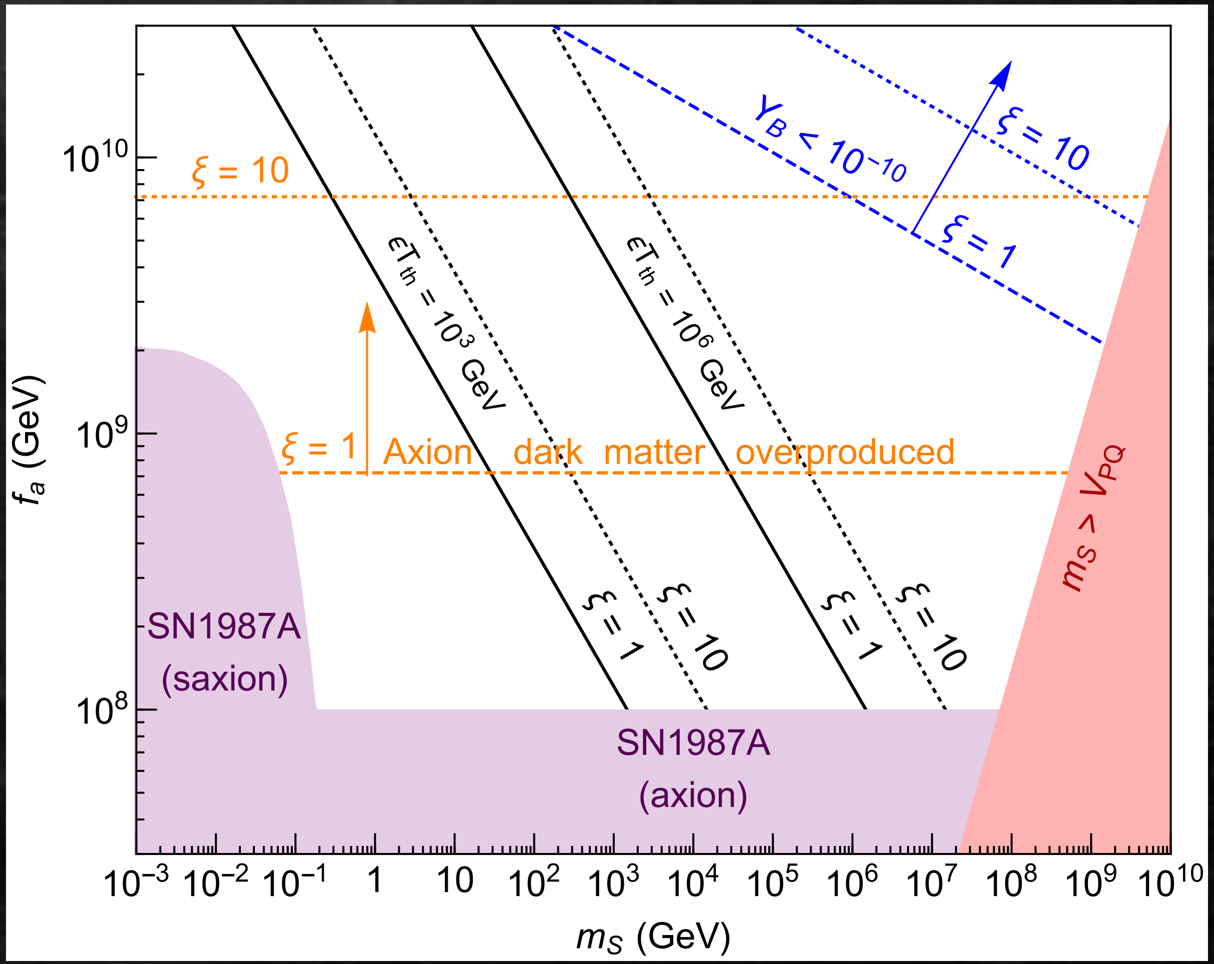
Axiogenesis + Kinetic Misalignment

$$\frac{\Omega_a h^2}{\Omega_{\rm DM} h^2} \simeq 140 \left(\frac{f_a}{10^8\ \text{GeV}}\right)\left(\frac{130\ \text{GeV}}{T_{\rm EW}}\right)^2\left(\frac{0.1}{c_B}\right)$$

$$\xi \equiv 10^{-2} \times \left(\frac{T_{\rm EW}}{130\text{GeV}}\right)^2\left(\frac{c_B}{0.1}\right)$$



T_{th} = thermalization temperature of saxions

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Baryon Asymmetry

Epoch	H	T	Γ_L	$\Gamma_{\rm ss}$	ρ_P	$\dot{\theta}$	$T < T_{\rm ss}$		$T_{\rm ss} < T < T_L$	
							$\frac{\Delta n_B}{s}$	$\frac{\Delta n_B}{\rho_P}$	$\frac{\Delta n_B}{s}$	$\frac{\Delta n_B}{\rho_P}$
$\mathrm{MD}_{\rm NA}^{\rm inf}$	$R^{-\frac{3}{2}}$	$R^{-\frac{3}{8}}$	$R^{-\frac{9}{8}}$	$R^{-\frac{3}{8}}$	R^{-3}	R^0	–	$R^{\frac{21}{8}}$	–	$R^{\frac{15}{4}}$
RD_i	R^{-2}	R^{-1}	R^{-3}	R^{-1}	R^{-3}	R^0	–	R^0	–	R^1
$\mathrm{MD}_{\rm A}^{\rm osc}$	$R^{-\frac{3}{2}}$	R^{-1}	R^{-3}	R^{-1}	R^{-3}	R^0	–	$R^{-\frac{1}{2}}$	–	R^0
$\mathrm{MD}_{\rm NA}^{\rm osc}$ gauge bosons	$R^{-\frac{3}{2}}$	$R^{\frac{3}{2}}$	$R^{\frac{9}{2}}$	$R^{\frac{3}{2}}$	R^{-3}	R^0	–	R^{12}	–	R^{15}
$\mathrm{MD}_{\rm NA}^{\rm osc}$ fermion	$R^{-\frac{3}{2}}$	$R^{-\frac{1}{2}}$	$R^{-\frac{3}{2}}$	$R^{-\frac{1}{2}}$	R^{-3}	R^0	–	R^2	–	R^3
$\mathrm{MD}_{\rm A}^{\rm rot}$	$R^{-\frac{3}{2}}$	R^{-1}	R^{-3}	R^{-1}	R^{-3}	R^0	$R^{-\frac{1}{2}}$	–	R^0	–
KD	R^{-3}	R^{-1}	R^{-3}	R^{-1}	R^{-6}	R^{-3}	R^{-2}	–	R^0	–
RD_f	R^{-2}	R^{-1}	R^{-3}	R^{-1}	R^{-6}	R^{-3}	R^{-3}	–	R^{-2}	–

TABLE I. Scalings of physical quantities relevant for the estimation of the baryon asymmetry.

2006.05687 RC, N. Fernandez, A. Ghalsasi, L. Hall, K. Harigaya

Baryon Asymmetry

$$\begin{aligned}
\dot{n}_{q_i} =& \sum_j \gamma^u_{ij}\left(-\frac{n_{q_i}}{6} - \frac{n_{\bar{u}_j}}{3} + \frac{n_H}{4} + \frac{c_{q_i}+c_{\bar{u}_j}}{6}\dot{\theta}T^2\right) + \sum_j \gamma^d_{ij}\left(-\frac{n_{q_i}}{6} - \frac{n_{\bar{d}_j}}{3} - \frac{n_H}{4} + \frac{c_{q_i}+c_{\bar{d}_j}}{6}\dot{\theta}T^2\right) \\
&+ 2\Gamma_{\rm ss}\sum_j \left(-n_{q_j} - n_{\bar{u}_j} - n_{\bar{d}_j} + \frac{2c_{q_j}+c_{\bar{u}_i}+c_{\bar{d}_j}-c_g/N_g}{2}\dot{\theta}T^2\right), \\
&+ 3\Gamma_{\rm ws}\sum_j \left(-n_{q_j} - n_{\ell_j} + \frac{3c_{q_j}+c_{\ell_j}-c_W/N_g}{3}\dot{\theta}T^2\right) \qquad \text{(A.3)}
\end{aligned}$$

$$\begin{aligned}
\dot{n}_{\bar{u}_i} =& \sum_j \gamma^u_{ji}\left(-\frac{n_{q_j}}{6} - \frac{n_{\bar{u}_i}}{3} + \frac{n_H}{4} + \frac{c_{q_j}+c_{\bar{u}_i}}{6}\dot{\theta}T^2\right) \\
&+ \Gamma_{\rm ss}\sum_j \left(-n_{q_j} - n_{\bar{u}_j} - n_{\bar{d}_j} + \frac{2c_{q_j}+c_{\bar{u}_i}+c_{\bar{d}_j}-c_g/N_g}{2}\dot{\theta}T^2\right), \qquad \text{(A.4)}
\end{aligned}$$

$$\begin{aligned}
\dot{n}_{\bar{d}_i} =& \sum_j \gamma^d_{ji}\left(-\frac{n_{q_j}}{6} - \frac{n_{\bar{d}_i}}{3} - \frac{n_H}{4} + \frac{c_{q_j}+c_{\bar{d}_i}}{6}\dot{\theta}T^2\right) \\
&+ \Gamma_{\rm ss}\sum_j \left(-n_{q_j} - n_{\bar{u}_j} - n_{\bar{d}_j} + \frac{2c_{q_j}+c_{\bar{u}_i}+c_{\bar{d}_j}-c_g/N_g}{2}\dot{\theta}T^2\right), \qquad \text{(A.5)}
\end{aligned}$$

$$\begin{aligned}
\dot{n}_{\ell_i} =& \sum_j \gamma^e_{ij}\left(-\frac{n_{\ell_i}}{2} - n_{\bar{e}_j} - \frac{n_H}{4} + \frac{c_{\ell_i}+c_{\bar{e}_j}}{6}\dot{\theta}T^2\right) \\
&+ \Gamma_{\rm ws}\sum_j \left(-n_{q_j} - n_{\ell_j} + \frac{3c_{q_j}+c_{\ell_j}-c_W/N_g}{3}\dot{\theta}T^2\right), \qquad \text{(A.6)}
\end{aligned}$$

$$\dot{n}_{\bar{e}_i} = \sum_j \gamma^e_{ji}\left(-\frac{n_{\ell_j}}{2} - n_{\bar{e}_i} - \frac{n_H}{4} + \frac{c_{\ell_j}+c_{\bar{e}_i}}{6}\dot{\theta}T^2\right), \qquad \text{(A.7)}$$

$$\begin{aligned}
\dot{n}_H =& -\sum_{ij} \gamma^u_{ij}\left(-\frac{n_{q_i}}{6} - \frac{n_{\bar{u}_j}}{3} + \frac{n_H}{4} + \frac{c_{q_i}+c_{\bar{u}_j}}{6}\dot{\theta}T^2\right) + \sum_{ij} \gamma^d_{ij}\left(-\frac{n_{q_i}}{6} - \frac{n_{\bar{d}_j}}{3} - \frac{n_H}{4} + \frac{c_{q_i}+c_{\bar{d}_j}}{6}\dot{\theta}T^2\right) \\
&+ \sum_{ij} \gamma^e_{ij}\left(-\frac{n_{\ell_i}}{2} - n_{\bar{e}_j} - \frac{n_H}{4} + \frac{c_{\ell_i}+c_{\bar{e}_j}}{6}\dot{\theta}T^2\right), \qquad \text{(A.8)}
\end{aligned}$$

$$\begin{aligned}
\dot{n}_\theta =& -\sum_{ij}(c_{q_i}+c_{\bar{u}_j})\gamma^u_{ij}\left(-\frac{n_{q_i}}{6} - \frac{n_{\bar{u}_j}}{3} + \frac{n_H}{4} + \frac{c_{q_i}+c_{\bar{u}_j}}{6}\dot{\theta}T^2\right) \\
&- \sum_{ij}(c_{q_i}+c_{\bar{d}_j})\gamma^d_{ij}\left(-\frac{n_{q_i}}{6} - \frac{n_{\bar{d}_j}}{3} - \frac{n_H}{4} + \frac{c_{q_i}+c_{\bar{d}_j}}{6}\dot{\theta}T^2\right) \\
&- \sum_{ij}(c_{\ell_i}+c_{\bar{e}_j})\gamma^e_{ij}\left(-\frac{n_{\ell_i}}{2} - n_{\bar{e}_j} - \frac{n_H}{4} + \frac{c_{\ell_i}+c_{\bar{e}_j}}{6}\dot{\theta}T^2\right) \\
&- \sum_{ij}(2c_{q_i}+c_{\bar{u}_i}+c_{\bar{d}_i}-c_g/N_g)\Gamma_{\rm ss}\left(-n_{q_j} - n_{\bar{u}_j} - n_{\bar{d}_j} + \frac{2c_{q_j}+c_{\bar{u}_i}+c_{\bar{d}_j}-c_g/N_g}{2}\dot{\theta}T^2\right), \\
&- \sum_{ij}(3c_{q_i}+c_{\ell_i}-c_W/N_g)\Gamma_{\rm ws}\left(-n_{q_j} - n_{\ell_j} + \frac{3c_{q_j}+c_{\ell_j}-c_W/N_g}{3}\dot{\theta}T^2\right), \qquad \text{(A.9)}
\end{aligned}$$

$$c_B = \left(\frac{21}{158} - \delta\right)c_g - \frac{12}{79}c_W + \sum_i\left(\frac{18}{79}c_{q_i} - \frac{21}{158}c_{\bar{u}_i} - \frac{15}{158}c_{\bar{d}_i} + \frac{25}{237}c_{\ell_i} - \frac{11}{237}c_{\bar{e}_i}\right)$$

2006.04809 RC, L. Hall, K. Harigaya