GWs from axion preheating and SGWB recostruction at LISA

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CERN workshop:
Zooming in on axions in the early Universe

June 26, 2020

Outline

Slow-roll inflation

Homogeneous scalar field ϕ in a homogeneous and isotropic universe:

$$\mathcal{S} = \int \mathrm{d}^x \sqrt{-g} \left[\frac{m_p^2 R}{2} + \frac{\dot{\phi}^2}{2} - V(\phi) \right]$$

The evolution is fixed by:

$$3 \left(\frac{\dot{a}}{a} \right)^2 m_p^2 \equiv 3H^2 m_p^2 = \frac{\dot{\phi}^2}{2} + V, \qquad -2\dot{H} m_p^2 = \dot{\phi}^2, \qquad \ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \, .$$

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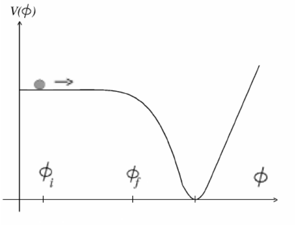
$\dot{\phi}^2/2 \ll V(\phi)$

The e.o.ms. simplify to:

$$3H^2 \simeq V\kappa^2, \qquad -2\dot{H} = \dot{\phi}^2 \kappa^2 \, ,$$

$$3H\dot{\phi} \simeq -\frac{\partial V}{\partial \phi} \, .$$



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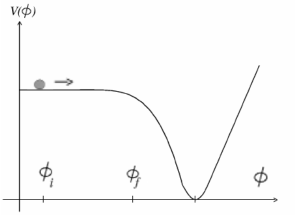


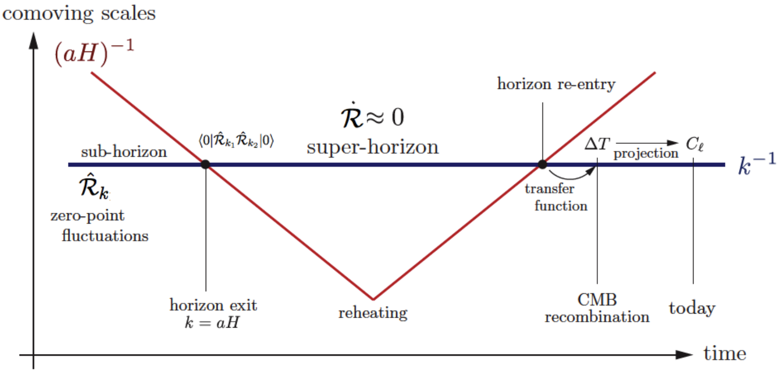
As usual we introduce the slow-roll parameters and the number of e-foldings:

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2}\,, \qquad \epsilon_2 \equiv \frac{\mathrm{d}\ln(\epsilon_1)}{\mathrm{d}\ln a}\,, \qquad N(t) \equiv -\int_{a_f}^{a} \mathrm{d}\ln\hat{a}\,.$$

Cosmological perturbations

Perturbations around the homogeneous background:

$$\Phi(t,\vec{x}) = \phi(t) + \delta\phi(t,\vec{x}) \qquad \mathbf{g}_{\mu\nu}(t,\vec{x}) = g_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\vec{x})$$



J. M. Bardeen 1980, P. J. Steinhardt, and M. S. Turner 1983, H. Kodama and M. Sasaki 1984, V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger 1992

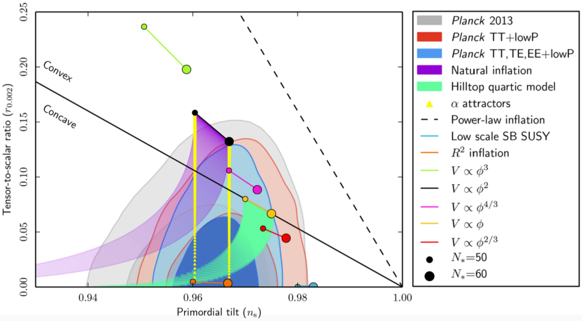
Power spectra and CMB Observables

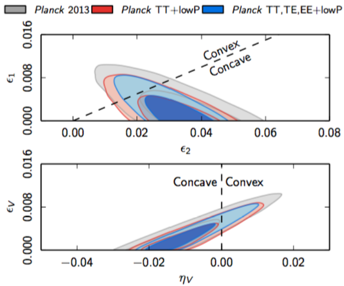
Scalar and tensor power spectra:

$$\Delta_s^2(k,\tau)\Big|_{\tau=k^{-1}} = \frac{1}{8\pi^2}\frac{H^2\kappa^2}{\epsilon_V}\,, \qquad \Delta_t^2(k,\tau)\Big|_{\tau=k^{-1}} = 2\left(\frac{\kappa H}{\pi}\right)^2 .$$

Tensor-to-scalar ratio (r) and the scalar spectral index (n_s):

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2}\Big|_{k=aH} \simeq 16\epsilon_V\,, \qquad n_s \equiv 1 + \frac{\mathrm{d}\ln\Delta_s^2(k)}{\mathrm{d}\ln k}\Big|_{k=aH} \simeq 1 + 2\eta_V - 6\epsilon_V\,.$$





Planck 2015 results. XX. Constraints on inflation (1502.02114)

Detection of Primordial GWs

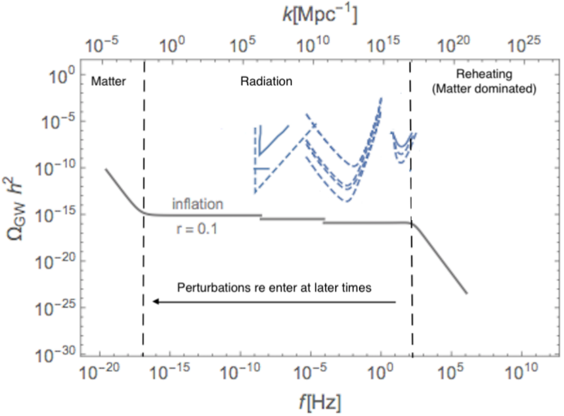
The normalized energy density of the GW background is:

$$\Omega_{GW}(f,\tau_0)h_0^2 \equiv \frac{h_0^2}{\rho_c}\frac{\mathrm{d}\rho_{GW}(f,\tau_0)}{\mathrm{d}\ln f} = \frac{h_0^2(2\pi f)^2}{4\kappa^2\rho_c}\, T_f(f,\tau_0)\,\Delta_t^2(k(f),\tau=k)\ .$$

The transfer function scales as:

Radiation
$T_f(f,\tau_0) \propto (2\pi f)^{-2}$,

Matter
$T_f(f,\tau_0) \propto (2\pi f)^{-4}$.



Axion inflation with gauge fields

Inflaton non-minimally coupled to some Abelian gauge fields:

$$\mathcal{L} = \frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Turner, Widrow '88,
Garretson, Field, Caroll '92,
Anber, Sorbo '06./'10/'12,
Barnaby, Namba, Peloso '11,
Barnaby, Pajer, Peloso '12 ,
.....

The equations of motion for the fields are:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = \frac{\alpha}{\Lambda}\langle \vec{E}\cdot\vec{B}\rangle$$

$$\frac{\mathrm{d}^2\vec{A}^a(\tau,\vec{k})}{\mathrm{d}\tau^2} - \vec{\nabla}^2\vec{A}^a = \frac{\alpha}{\Lambda}\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\vec{\nabla}\times\vec{A}^a$$

$$\mathrm{d}t \equiv a\,\mathrm{d}\tau$$

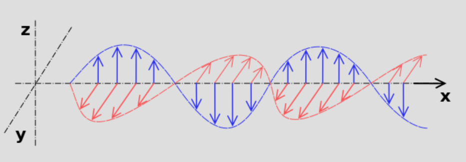
$$N \equiv -\int H\,\mathrm{d}t$$

Friedman equation reads:

$$3H^2\kappa^{-2} = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle$$

Gauge field amplification

Assuming
$\vec{k}$ parallel to $\hat{x}$



$$\vec{e}_\pm \equiv (\hat{y} \pm i\hat{z})/\sqrt{2}$$
$$\vec{A}^a \equiv \vec{e}_\pm A^a_\pm$$

The equations of motion for the gauge fields (in Fourier transform) read:

$$\frac{\mathrm{d}^2\, A^a_\pm(\tau,\vec{k})}{\mathrm{d}\tau^2} + \left[k^2 \pm 2k\frac{\xi}{\tau}\right] A^a_\pm(\tau,\vec{k}) = 0 \qquad \boxed{\xi \equiv \frac{\alpha}{2\Lambda}\left|\frac{\dot{\phi}}{H}\right| \propto \sqrt{\epsilon_1}}$$

If ξ is nearly constant one mode (A^a_+) is exponentially growing with ξ.

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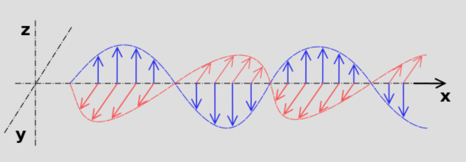

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Substituting $\langle \vec{E}\cdot\vec{B}\rangle$ into the equation of motion for ϕ we get:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial\phi} = \frac{\alpha}{\Lambda}\langle \vec{E}\cdot\vec{B}\rangle \simeq \frac{\alpha}{\Lambda} 2.4\cdot 10^{-4}\mathcal{N}\left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

Friction term that dominates the last part of the evolution.

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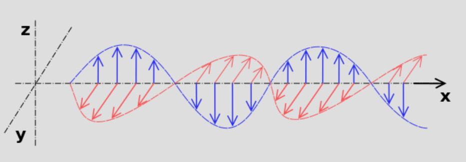

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Friction term that dominates the last part of the evolution.

Modified dynamics also affects the scalar and tensor power spectra!

Modified tensor spectrum

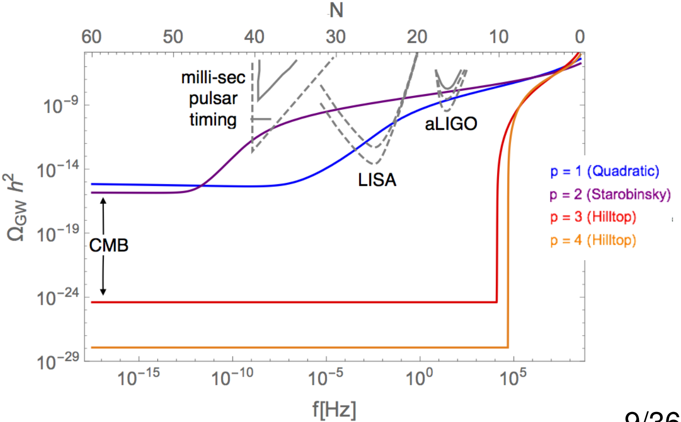
GW spectrum $\longrightarrow$

$$\Delta_t^2(k) = \frac{1}{12}\left(\frac{\kappa H}{\pi}\right)^2\left(1 + 4.3\cdot 10^{-7}\frac{\kappa^2 H^2}{\xi^6}e^{4\pi\xi}\right)$$

N-frequency relation $\longrightarrow$

$$N = N_{\rm CMB} + \ln\frac{k_{\rm CMB}}{0.002\ {\rm Mpc}^{-1}} - 44.9 - \ln\frac{f}{10^2\ {\rm Hz}}$$

- Parametrizing $\epsilon_1 \simeq \beta/(1+N)^p$
- GWs in the range of direct GW detectors
- Stronger production at smaller scales
- Steeper increase for larger p



V. Domcke, **M. P.** and P. Binetruy 2016 (1603.01287)

Modified scalar spectrum

Scalar spectrum $\longrightarrow$

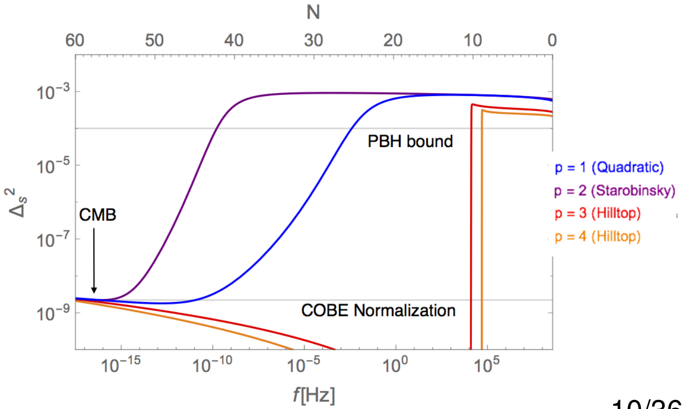
$$\Delta_s^2(k) = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 + \left(\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3bH\dot{\phi}}\right)^2$$

where:

$$b \equiv 1 - 2\pi\xi\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3\Lambda H\dot{\phi}}$$

- COBE normalization fixes V_0
- Nearly universal behavior at large scales
- $\Delta_s^2(k) \simeq \frac{1}{\mathcal{N}(2\pi\xi)^2}$ at small scales

  A. D. Linde, S. Mooij, and E. Pajer 2012 (1212.1693)
- Strong increase at small scales $\rightarrow$ PBHs



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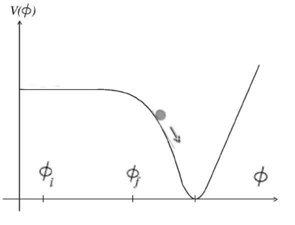
Preheating: a slightly different problem

At the end of inflation the Universe is cold and empty (*i.e.* $\rho_{TOT} \simeq \rho_{inflaton}$)

The inflaton must (explosively) decay into other particles to repopulate the Universe!

This typically occurs while the inflaton oscillates around the minimum of its potential



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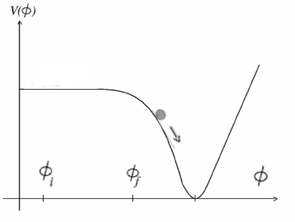

Some crucial differences with slow-roll inflation are:

- $\dot{\phi}^2 \ll V(\phi)$ is not satisfied anymore
- Inhomogeneities may be important (*i.e.* $\vec{\nabla}\phi \neq 0$)
- Interactions with other fields have to be taken into account.

Analytical approximations for $\phi F\tilde{F}$ models break down!

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Discrete gauge symmetry
is a symmetry of the system

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J. R. C. Cuissa and D. G. Figueroa 2018 (1812.03132)

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J. R. C. Cuissa and D. G. Figueroa 2018 (1812.03132)

We follow the second strategy and evolve on a periodic $N^3 = 384^3$ lattice

(also tested on $N = 512$ cubed lattice to check stability)

- Initial conditions are set by solving the linear system
- E.o.m. are evolved via a fourth-order Runge-Kutta method
- Spacetime expansion by averaging energy density and pressure

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GWs are generated during preheating!
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The total energy density in gravitational waves is given by

$$\Omega_{\rm gw}^{TOT} h^2 = \int \Omega_{\rm gw}(k) h^2 \, \mathrm{d} \ln k \equiv \int \frac{h^2}{\rho} \frac{\mathrm{d}\rho_{\rm gw}}{\mathrm{d} \ln k} \, \mathrm{d} \ln k .$$

CMB experiment set (via $N_{\rm eff}$) constraints on $\Omega_{{\rm gw},0} h^2$ $\longrightarrow$

$$\frac{\Omega_{{\rm gw},0} h^2}{\Omega_{\gamma,0} h^2} = \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \Delta N_{\rm eff},$$

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For different experiments we get:

- Planck: $|\Delta N_{\rm eff}| \lesssim 0.33$, implying $\Omega_{{\rm gw},0}^{TOT} h^2 \lesssim 1.85 \times 10^{-6}$
  Planck Collaboration 2015, 2018 (1502.01589,1807.06209)
- CMB-S4: $|\Delta N_{\rm eff}| \leq .03$ at $1\sigma (\leq .06$ at $2\sigma)$, implying $\Omega_{{\rm gw},0}^{TOT} h^2 \lesssim 1.68 - 3.36 \times 10^{-7}$
  K. Abazajian et al. 2019 (1907.04473)
- COrE and Euclid: $|\Delta N_{\rm eff}| < .013$ at 2σ implying $\Omega_{{\rm gw},0} h^2 \lesssim 7.6 \times 10^{-8}$
  COrE Collaboration 2011 (1102.2181), EUCLID Collaboration 2011 (1110.3193)

Some estimates

Given the fraction of the Universe's energy residing in the gauge fields,

$$f_{\text{gauge}} \equiv \frac{\rho_{\text{gauge}}}{\rho} \approx \frac{1.4 \cdot 10^{-4}}{3} \frac{(H/M_{pl})^2}{\xi^3} e^{2\pi\xi} \,,$$

for different models we roughly have

$$\frac{f_{\text{gauge},1}}{f_{\text{gauge},2}} = \left(\frac{H_1}{H_2}\right)^2 \left(\frac{\xi_2}{\xi_1}\right)^3 e^{2\pi(\xi_1 - \xi_2)} \,,$$

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GWs produced in a cosmological process satisfy $\Omega_{\rm gw,0} \propto (H_*/k_*)^2$
P. Binétruy, A. Bohé, C. Caprini, and J.-F. Dufaux 1201.0983, J. T. Giblin and E. Thrane 1410.4779

i.e. if the source is deeper inside the horizon GWs redshift and lose energy!

Recalling $k/H \simeq \xi \simeq \alpha/f \;\longrightarrow$ GWs are suppressed in models with larger ξ!
P. Adshead, J. T. Giblin, **M. P.** and Z. J. Weiner 2019 (1909.12842,1909.12843)

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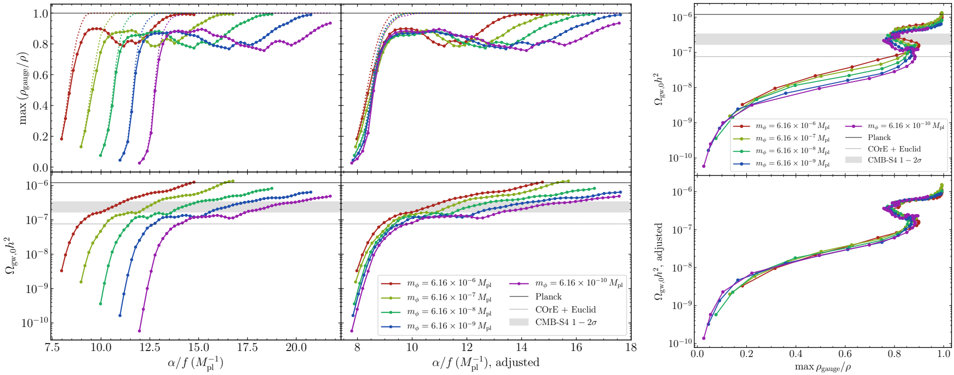
Vary the inflation scale for a fixed potential $\Longleftrightarrow$ test the estimates
Ultimately this can be used to check the shape dependency of the results!
(*i.e.* how the results change for different potentials)

Scale dependence

Assuming $V(\phi) = m_\phi^2 \phi^2/2$



Where we 'adjusted' α/f using the first estimate and Ω_{GW} using the second.

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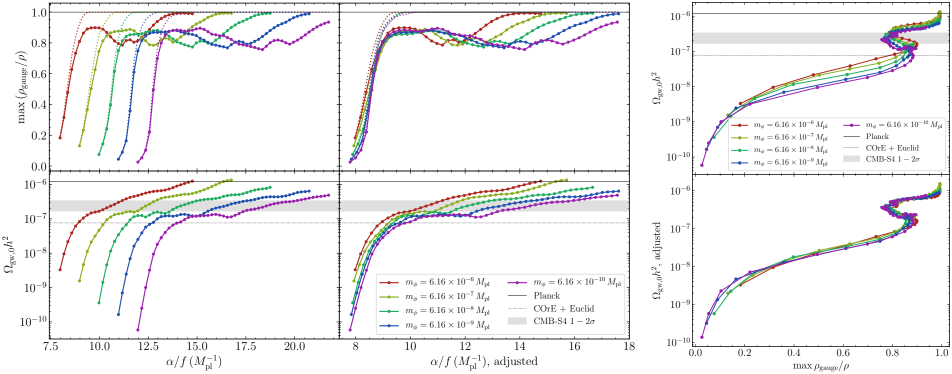

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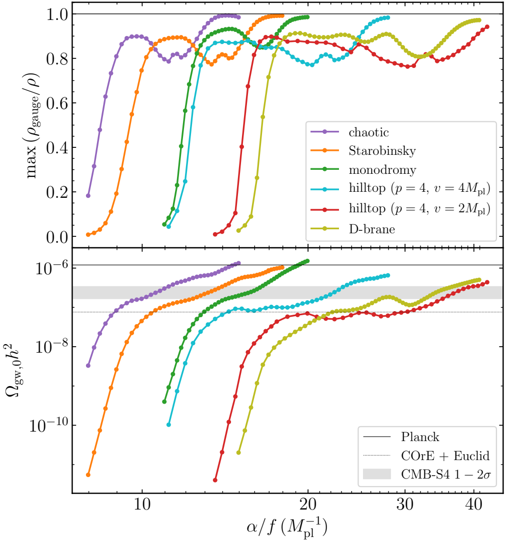
Planck constraint: $\alpha/f \lesssim 15 M_{pl}$ ($\alpha/f \lesssim 9 M_{pl}$ for CMB-S4),

P. Adshead, J. T. Giblin, **M. P.** and Z. J. Weiner 2019 (1909.12842,1909.12843)

Stronger than the PBH bound ($\xi \lesssim 1.3$) $\alpha/f \lesssim 20 M_{pl}$!

A. D. Linde, S. Mooij, and E. Pajer 2012 (1212.1693)

Shape dependence



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Large scale models giving successful reheating, leave signatures which are (will soon be) observable!

First clear preheating observable !?

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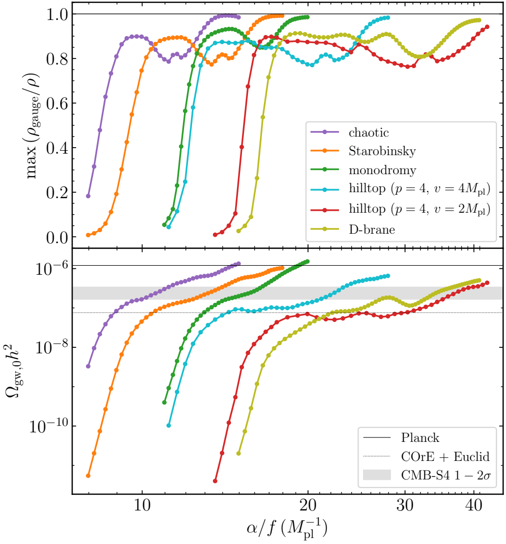

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Large scale modes produce more GWs also at CMB scales! *i.e.* larger value of *r* (observable by future experiments)

Joint detection of ΔN_{eff} and r may be a hint for $\phi F\tilde{F}$ (p)reheating!

P. Adshead, J. T. Giblin, **M. P.** and Z. J. Weiner 2019 (1909.12842,1909.12843)

Conclusions and future perspectives

Conclusions:

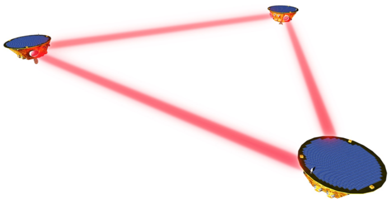
- $\phi F\tilde{F}$ models feature an extremely rich phenomenology (GWs, PBHs, ...)
- Any detection would give informations on the microphysics of inflation
- Very natural mechanism for (a successful) preheating
- Possible correlation between CMB observables and preheating!

Future perspectives:

- New models? Embedding in high energy theories?
- More studies on lattice? (Non-Abelian gauge fields, fermions...)

Laser Interferometer Space Antenna

Few details on LISA:



- First direct GW detector in space
- Constellation of three satellites
- 2.5 million km arm lengths
- Peak sensitivity $10^{-2} \div 10^{-3}$Hz
- $\sim$ two independent detectors
- Expected launch in 2034
- Operating for 4yrs (nominal)

Very interesting for cosmology since we can:

- Measure H_0 (see 1601.07112)
- Test modified gravity (see1906.01593)
- (Hopefully) detect and characterize SGWBs! (see 1906.09244, 2006.03313)

Response function and sensitivity

The data $\tilde{d}$ (in frequency space) can be expressed as

$$\tilde{d} = \tilde{s} + \tilde{n}$$

For an isotropic SGWB $\longrightarrow$ $\langle h_\lambda(\vec{k})\, h^*_{\lambda'}(\vec{k}')\rangle = P_h^\lambda(k)(2\pi)^3\delta_{\lambda\lambda'}\delta(\vec{k}-\vec{k}')$

Assuming $\langle \tilde{s}\tilde{n}\rangle = 0$ and Gaussian signal

$$\left\langle \tilde{d}^2 \right\rangle = \left\langle \tilde{s}^2 \right\rangle + \left\langle \tilde{n}^2 \right\rangle = \mathcal{R}\, P_h^\lambda + N \equiv \mathcal{R}\left[P_h^\lambda + S_n\right]$$

where we have introduced

- The response function of the instrument $\mathcal{R}$
- The noise power spectrum N
- The (square of the) Strain sensitivity S_n (in 1/Hz)

The noise model

Two analytical approximations for acceleration and interferometric noise:

$$P_{acc}(f,A) = A^2 \cdot 10^{-30} \cdot \left[1 + \left(\frac{4\cdot 10^{-4}}{f}\right)^2\right]\left[1 + \left(\frac{f}{8\cdot 10^{-3}}\right)^4\right]\left(\frac{1}{2\pi f}\right)^4\left(\frac{2\pi f}{c}\right)^2 ,$$

$$P_{IMS}(f,P) = P^2 \cdot 10^{-24} \cdot \left[1 + \left(\frac{2\cdot 10^{-3}}{f}\right)^4\right]\left(\frac{2\pi f}{c}\right)^2 .$$

The power spectral density is:

$$P_{PSD}(f) = 16\sin^2\left(\frac{2\pi f L}{c}\right)\left\{P_{IMS}(f,P) + \left[3 + \cos\left(\frac{4\pi f L}{c}\right)\right]P_{acc}(f,A)\right\} ,$$

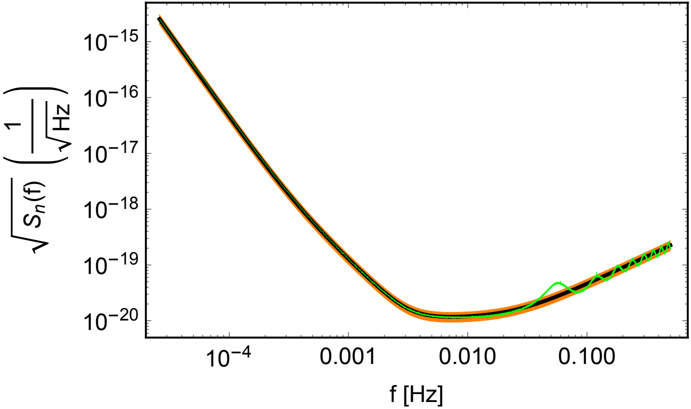
where $L = 2.5 \times 10^9$m is the arm length.

The response function and the strain sensitivity are ((See 1906.09244)):

$$\tilde{R}(f) = \frac{0.3}{1 + 0.6\left(\frac{2\pi f L}{c}\right)^2} , \qquad S_n(f,P,A) = \frac{P_{PSD}(f,P,A)}{\tilde{R}(f) \times 16\sin^2\left(\frac{2\pi f L}{c}\right) \times \left(\frac{2\pi f L}{c}\right)^2} .$$

For the techniques to compute $\tilde{R}(f)$, see for example 1910.08052

Analytical vs numerical



The LISA strain is well approximated by a two parameter analytical model:

- Acceleration noise at small frequencies
- Interferometric noise at large frequencies

Central values $\pm 20\%$ in orange

SGWBs and SNR

In order to compare with cosmological predictions it's customary to introduce

$$\Omega_n(f) = \frac{4\pi^2}{3H_0^2} f^3 S_n(f) \,, \qquad \text{and} \qquad \Omega_{\rm GW} \equiv \frac{1}{3H_0^2 M_p^2} \frac{\partial \rho_{\rm GW}}{\partial \ln f} = \frac{4\pi^2}{3H_0^2} f^3 \sum_\lambda P_h^\lambda$$

where $H_0 \simeq 3.24 \times 10^{-18}\, h_0\, \mathrm{Hz}$ is the Hubble constant today.

We can also introduce the signal-to-noise ratio (SNR)

$$\mathrm{SNR} = \sqrt{T \int_{f_{\rm min}}^{f_{\rm max}} df \left(\frac{\Omega_{\rm GW}(f)}{\Omega_n(f)} \right)^2}$$

where T denotes the observation time.

Power Law Sensitivity definition

The simplest SGWB we can aim at detecting is a Power Law (PL)

$$h^2\Omega_{GW}(f) = 10^\alpha \left(\frac{f}{f_*}\right)^p$$

for which we can easily compute the SNR

$$\mathrm{SNR} = \sqrt{T\int_{f_{\rm min}}^{f_{\rm max}} df \left(\frac{10^\alpha \ (f/f_*)^p}{\Omega_n(f)}\right)^2}$$

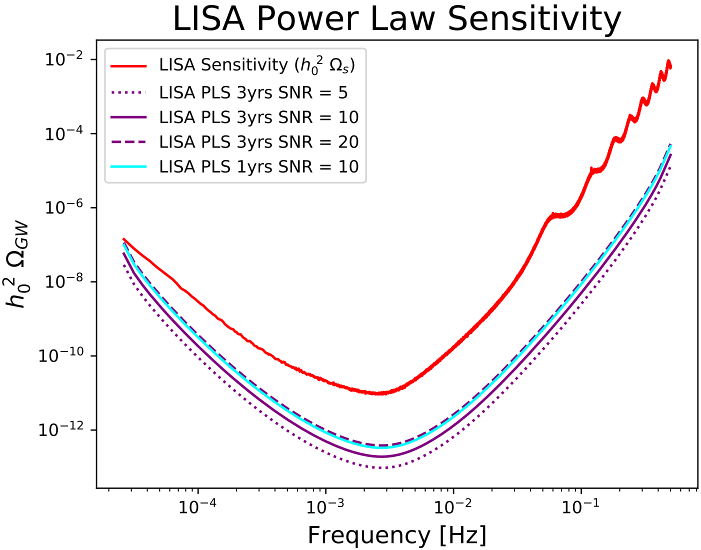
where α is $\log_{10}$ of the amplitude, p is the tilt and f_* is some (irrelevant) pivot.

For every fixed value of $\mathrm{SNR}_{\rm th}$ and of observation time T we introduce the Power Law Sensitivity (PLS) as

$$h^2\Omega_{\rm PLS}(f) \equiv \max_{\alpha,\,p} 10^\alpha \left(\frac{f}{f_*}\right)^p$$

where the parameters α and p are chosen in order to satisfy $\mathrm{SNR} = \mathrm{SNR}_{th}$

LISA Power Law Sensitivity



Towards "realistic" data

The spectra ($\Omega_{\rm GW}$ and Ω_n) quantify the variance of fluctuations
For each chunk of data (independent observation) get a random realization:

$$\tilde{s}_c(f_i) = \left| \frac{G(0, \sqrt{\Omega_{\rm GW}(f_i)}) + i\, G(0, \sqrt{\Omega_{\rm GW}(f_i)})}{\sqrt{2}} \right|$$

$$\tilde{n}_c(f_i) = \left| \frac{G(0, \sqrt{\Omega_n(f_i)}) + i\, G(0, \sqrt{\Omega_n(f_i)})}{\sqrt{2}} \right|$$

where signal and noise (from now on in units of Ω) are assumed to be gaussianly distributed.

We can assume:

- Each chunk consists of roughly 12 days
- LISA will be operating for 4yrs (75% efficiency)

we conclude that:

- The resolution of the detector is roughly 10^{-6}Hz
- Roughly 94 independent measurements at each frequency.

Data pre-processing and likelihood

Starting from $D_c(f_i)$ (our data), defined as:

$$D_c(f_i) \equiv \langle \tilde{d}_c^2(f_i) \rangle = \langle (\tilde{s}_c(f_i) + \tilde{n}_c(f_i))^2 \rangle = \langle \tilde{s}_c^2(f_i) \rangle + \langle \tilde{n}_c^2(f_i) \rangle \ .$$

we can reduce the complexity of the problem by performing two operations:

- We average over the (94) chunks:
  This leaves us with some $D(f_i)$ (the averaged data)
  and $\sigma(f_i)$ (the corresponding standard deviation).
- We coarse grain the data:
  i.e. from the initial linear 10^{-6}Hz spacing ($\sim 5 \times 10^5$ points)
  $\longrightarrow$ we go to some log spacing ($\sim 2 \times 10^3$ points).

Finally we assume the data to be described by the likelihood:

$$\mathcal{L}\left(\vec{\theta}, \vec{n}\right) \propto \exp\left[-\frac{N_{chunks}}{2} \sum_i \left(\frac{D\left(f_i\right) - h^2\Omega_{\rm GW}\left(f_i, \vec{\theta}\right) - h^2\Omega_n\left(f_i, \vec{n}\right)}{\sigma\left(f_i\right)}\right)^2\right]$$

with i labeling the data points and $\Omega_{\rm GW}$,Ω_n models for signal and noise.

An exact solution for the parameters

Based on 2004.01135, in collaboration with Enrico Barausse

If the model (for both signal and noise) is linear in the $\vec{\theta}$:

- The log likelihood is quadratic in the parameters
- The Fisher matrix does not depend on the parameters
- Finding the best fit reduces to solving a linear equation

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- Finding the best fit reduces to solving a linear equation

Starting from:

$$-\ln\mathcal{L}\left(\vec{\theta},\vec{n}\right)\propto\frac{1}{2}\sum_i\left(\frac{D\left(f_i\right)-M\left(f_i,\,\vec{\theta}\right)}{\sigma\left(f_i\right)}\right)^2\,,$$

for a linear model we get:

$$F_{lk}\equiv\sum_i\frac{1}{\sigma^2(f_i)}\frac{\partial M\left(f_i,\,\vec{\theta}\right)}{\partial\theta_l}\frac{\partial M\left(f_i,\,\vec{\theta}\right)}{\partial\theta_k}\,,\qquad\bar{\theta}_l=F_{lk}^{-1}\sum_i\frac{1}{\sigma^2(f_i)}D_i\frac{\partial M\left(f_i,\,\vec{\theta}\right)}{\partial\theta_k}$$

where $\bar{\theta}_l$ is the MLE for the parameters.

A simple model for the signal

Assume the signal can be expressed as:

$$S(f) = \sum_{j=1}^{n} a_j \, \delta_w(f - f_j) \, ,$$

where:

- a_j are the parameters
- w is some correlation length
- $\delta_w(f - f_j)$ are some functions

The choice of $\delta_w(f - f_j)$ defines a basis to express the signal.

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The choice of $\delta_w(f - f_j)$ defines a basis to express the signal.

Depending on the choice of w we have two regimes:

- Small w: the measurements in f_j are not correlated
- Large w: the measurements in f_j are correlated

Properly choosing w we can smooth the signal!

Principal component analysis

It is interesting to notice that:

- In general the parameters a_j are correlated
- Eigenvectors $e_j^{(i)}$ of F_{lk} are uncorrelated combinations of a_j.
- Eigenvalues $\lambda^{(i)}$ of F_{lk} give the information on the $e_j^{(i)}$.

Principal component analysis

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- In general the parameters a_j are correlated
- Eigenvectors $e_j^{(i)}$ of F_{lk} are uncorrelated combinations of a_j.
- Eigenvalues $\lambda^{(i)}$ of F_{lk} give the information on the $e_j^{(i)}$.

Principal Component Analysis (PCA):

1. Compute the eigensystem of F_{lk}
2. Cut $e_j^{(i)}$ corresponding to $\lambda^{(i)}$ smaller than some threshold
3. Project $\delta_w(f - f_j)$ and a_j on this subset of $e_j^{(i)}$ (say $\eta_k(f)$, b_k)
4. Reconstruct the signal as: $S(f) = \sum_k b_k \eta_k(f)$

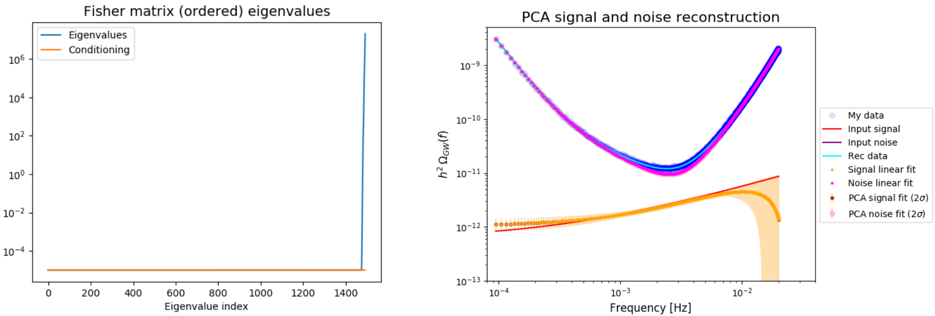
Corresponds to reconstructing the signal in terms of the components which can be well determined!

In the following plots all parameters are normalized to 1!

An explicit example

Let us consider:

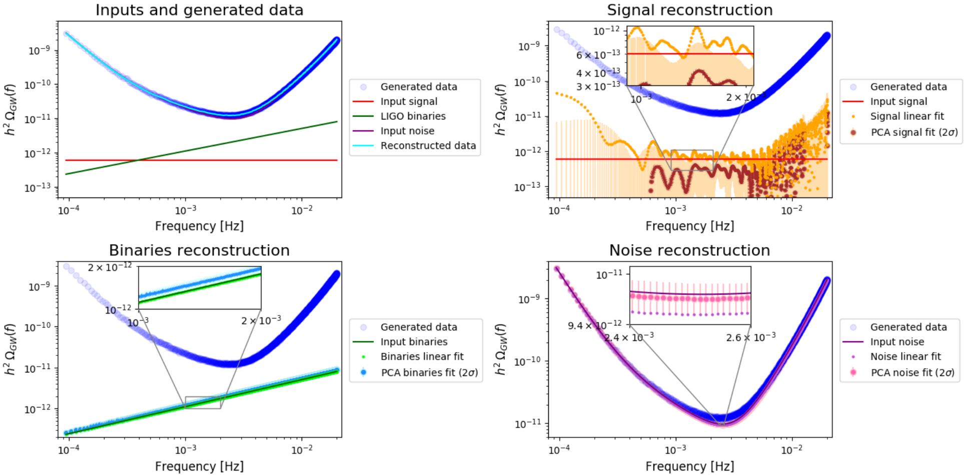
$$h^2\Omega_{\rm GW}(f) = h^2\Omega_{\rm GW,const}(f)+h^2\Omega_{\rm GW,BHB+NSB}(f) = 6\times10^{-13}+8.9\times10^{-10}\left(\frac{f}{25}\right)^{2/3}$$



After cutting the low information components we recover the input signal!

Subtracting the foreground 1

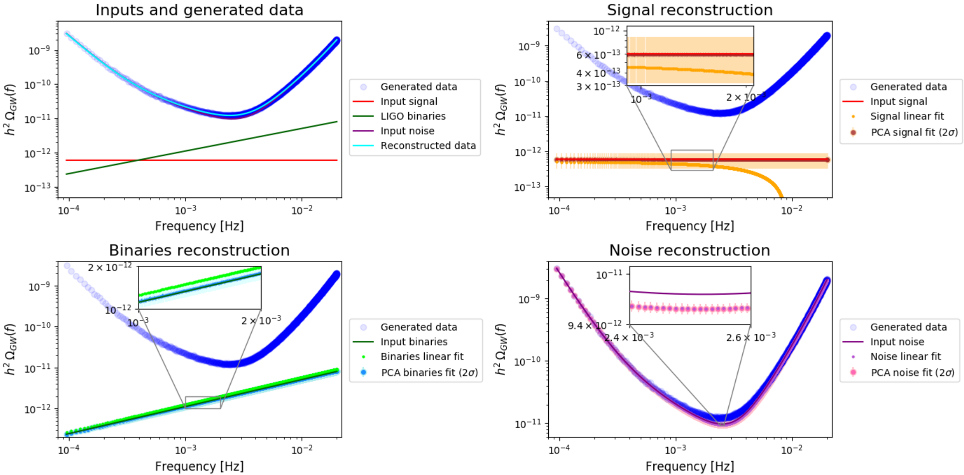
Again flat signal + LIGO binaries (gaussian prior $\sigma = 0.5$) + $w = 10^{-5}$Hz



$L = 1.096 \pm 0.037$, $A = 0.998 \pm 0.014$, $O = 0.987 \pm 0.002$

Subtracting the foreground 2

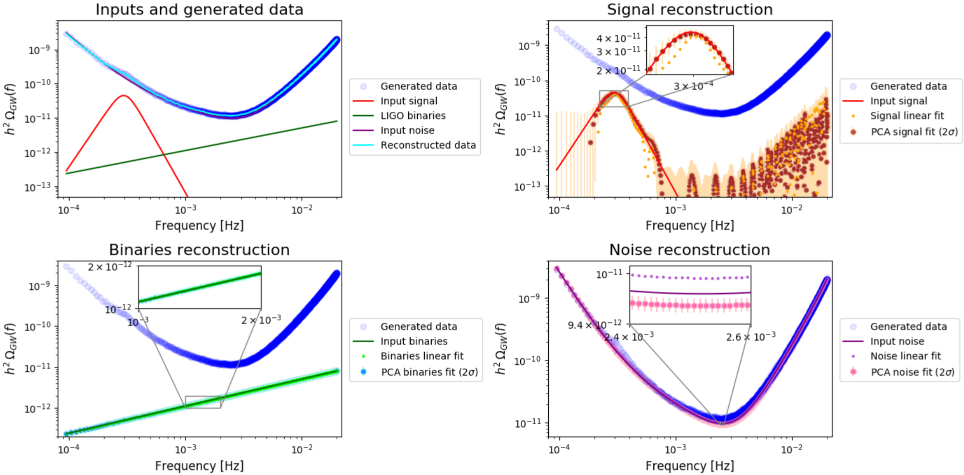
Again flat signal + LIGO binaries (gaussian prior $\sigma = 0.5$) + $w = 1$Hz



$L = 1.014 \pm 0.052$, $A \simeq 0.983 \pm 0.005$ and $O \simeq 0.980 \pm 0.001$

Broken PL 1

Broken PL (SNR $\sim$ 30) + LIGO binaries (gaussian prior $\sigma = 0.5$) + $w = 5 \times 10^{-5}$Hz

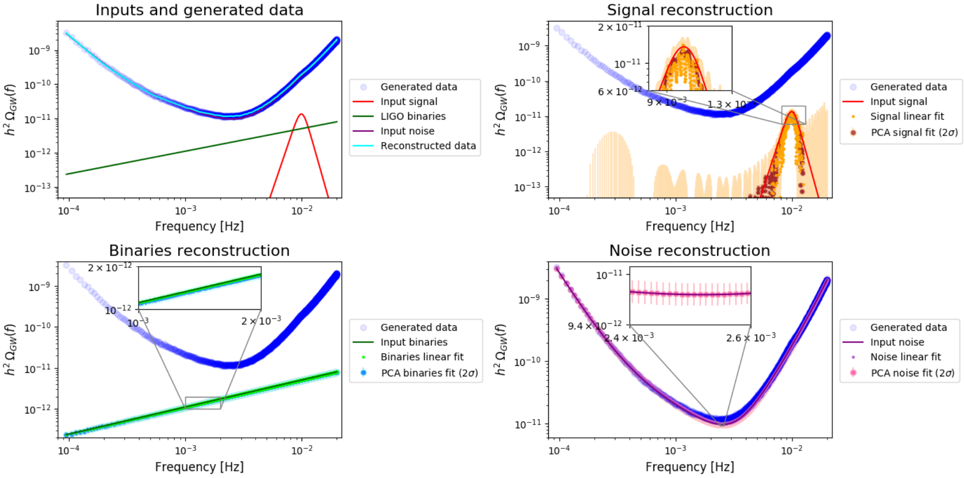


$L \simeq 1.007 \pm 0.021, A \simeq 0.988 \pm 0.006, O \simeq 0.981 \pm 0.001$

Some examples

Broken PL 2

Bump (SNR $\sim$ 30) + LIGO binaries (gaussian prior $\sigma = 0.5$) + $w = 10^{-4}$Hz



$L \simeq 0.981 \pm 0.032$, $A \simeq 1.001 \pm 0.010$, $O \simeq 0.998 \pm 0.002$

Conclusions and future perspective

Conclusions

- LISA is a wonderful experiment for cosmology
- PLS is (qualitatively) useful but not the end of the story
- Template-free approaches may be very useful
- PCA reconstruction is a quite simple but robust method

Future perspectives

- More realistic models for the response and for the noise
- Application to concrete case (inflation, phase transitions, ...)
- New techniques?

Last Slide

The End

Thank you