Linearized method for e-cloud instabilities

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Many thanks to:
H. Bartosik, N. Biancacci, K. Li, E. Métral, G. Rumolo, M. Schenk, C. Zannini

Outline

- **Introduction**
- **A Vlasov solver for e-cloud instabilities**
  - The linearized Vlasov equation
  - Description of the e-cloud detuning forces
  - Description of the e-cloud dipolar forces
  - Solving the equation
- **Benchmark against DELPHI (dipolar impedance)**
- **Application to LHC e-cloud at 450 GeV**
  - Benchmark against PyHEADTAIL simulations (linearized model)
  - Comparison against PIC simulations
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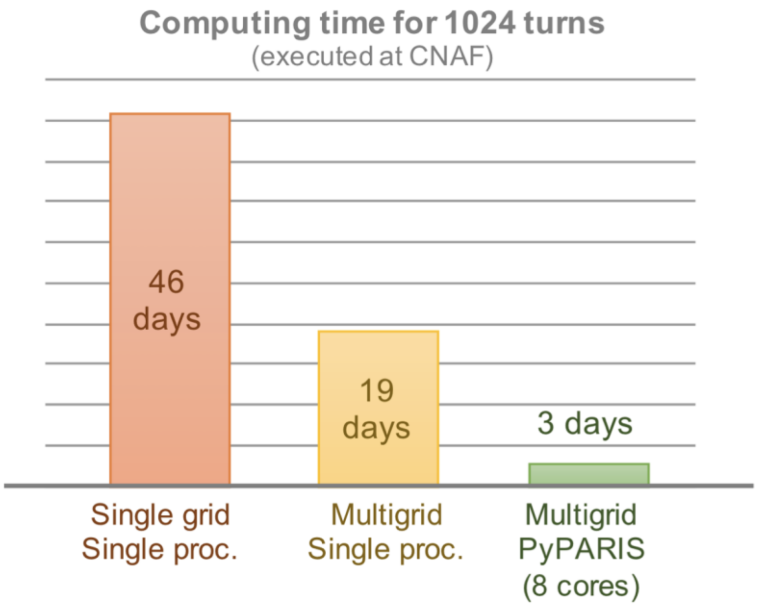
- **Summary and conclusions**

Macroparticle simulations: joys and sorrows

The study of **single-bunch e-cloud instabilities** heavily rely on **macroparticle simulations**:

- Simulate **turn after turn** the **coupled dynamics** of the beam and of the cloud distributions using the **Particle-In-Cell** (PIC) method.
- Allow for **detailed modelling of complex features** of e-cloud and beam dynamics

- Need **very small timestep** (~10 ps) to follow the fast electron motion → **extremely demanding** in terms of computing resources and calculation time
  - **Parallelization** available for PyECLOUD-PyHEADTAIL and regularly exploited
  - Still For LHC/HL-LHC simulations for **more than 20k turns remain unaffordable** (while observed instabilities have longer time scales)



- Large **multiparametric scans strongly limited** by available computing resources
- **Difficult to get an insight** on **underlying mechanisms** driving the instability (e.g. mode-coupling vs weak head-tail)

Is there an alternative to this "brute-force" approach?

The impedance case

In the study of **conventional impedance-driven instabilities we do not simulate at each turn the electromagnetic interaction** of the beam with the accelerator components

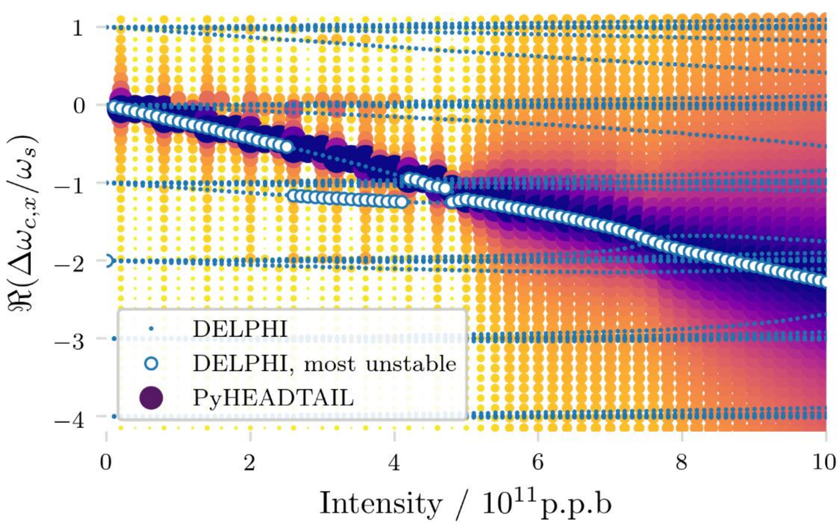
- Instead we **characterize the short-term response** of the environment through **effective quantities**, namely wakefields and impedances

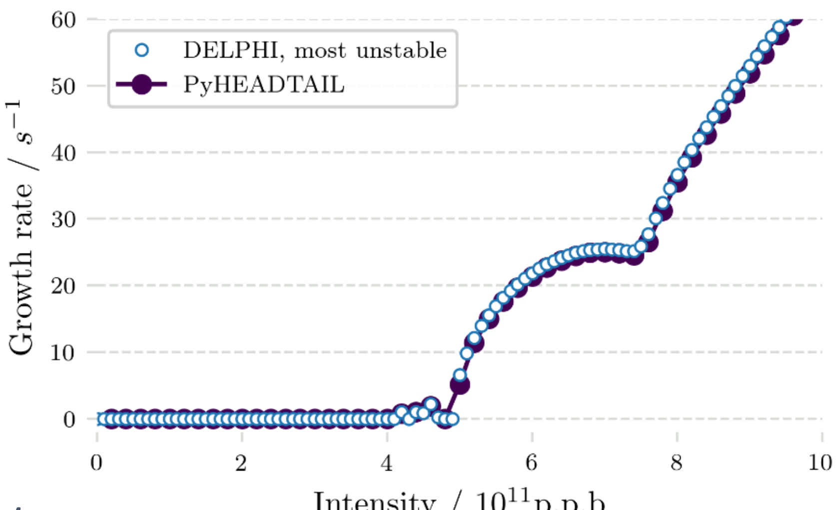
Then, we **use these effective quantities for predicting the long term behavior** of the beam using **fast macroparticle simulations** or **analytical approaches**

- Analytical methods **(Vlasov)** provide predictions on beam stability an insight on **underlying mechanisms** (e.g. transverse mode vs weak head-tail)

Can we do something similar for the e-cloud?

Vlasov solver vs macroparticle simulations (impedance)





D. Amorim

The e-cloud is not just another impedance

Attempts of using **Vlasov methods on e-cloud instabilities** made in early 2000s (Ohmi, Perevedentsev, Zimmermann), but **limited success** due to **features specific of the e-cloud**:

- e-cloud **dipolar forces cannot be modeled by conventional wakefields** (the system is not time-invariant due to the electron pinch)
- e-cloud introduces **tune modulation as a function of the z coordinate** which plays an important role on beam stability

Both features are not included in available Vlasov solvers (MOSES, NHTVS, DELPHI, GALACTIC)

e-cloud pseudo wake-fields

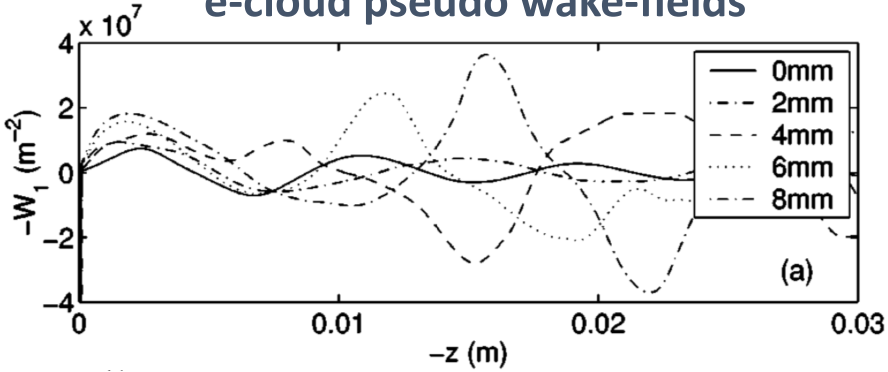
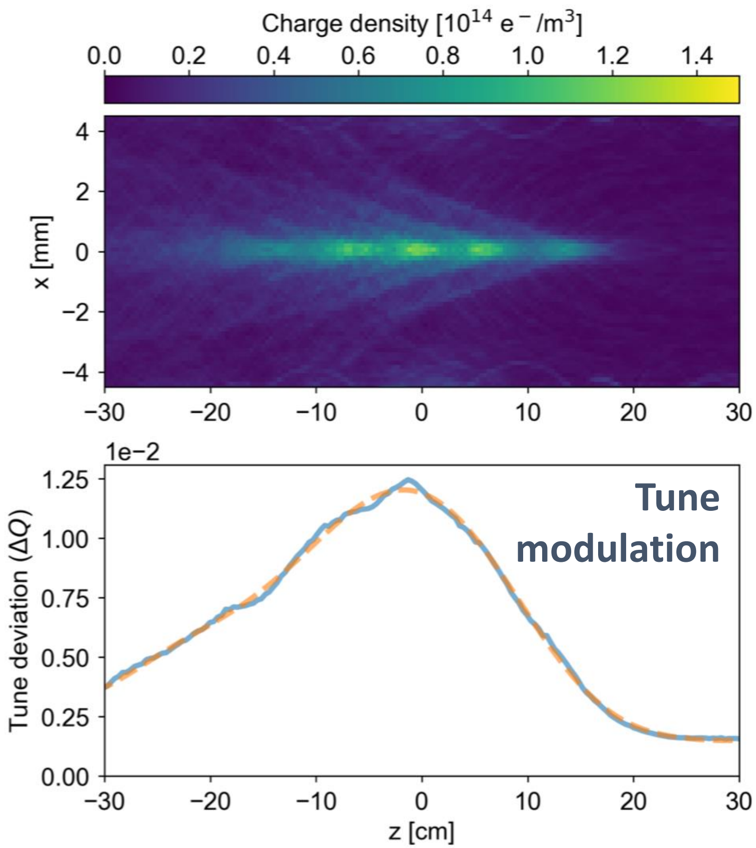


FIG. 6. Vertical and horizontal wake field computed by displacing microbunches at various longitudinal positions along the bunch, for electron-cloud sizes of (10,10) and (10,50), respectively.

K. Ohmi, F. Zimmermann, and E. Perevedentsev, Phys. Rev. E 65, 016502 (2001)



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A generalized DELPHI solver

- We investigated the possibility of **building a Vlasov solver including**:
  - **Detuning forces** dependent on the z coordinate
  - **A description of the dipolar forces** that is general enough to be applicable to an e-cloud

- We used as a **starting point** the approach used in **DELPHI**. **Advantages**:
  - It's known to have good **convergence properties** for typical LHC cases (impedance)
  - **Document very clearly** in [1] and [2] + very responsive support from Nicolas

- Most of the work consisted in the **analytical derivation** (only main steps will be described in this presentation)
  - **Documentation:** a paper has been submitted to PRAB and an extended note with all the steps is in preparation

[1] N. Mounet, ``Direct Vlasov solvers'', CAS proceedings, Thessaloniki, Greece, 2018.
[2] N. Mounet, "Vlasov solvers and macroparticle simulations'", proceedings of the ICFA Mini-Workshop on Impedances and Beam Instabilities in Particle Accelerators, Benevento, Italy, 2018.

In the Vlasov approach we don't simulate the dynamics of the single particles but the evolution of the **bunch phase-space distribution** $\psi(x, x', y, y', z, \delta)$.

We search for its (first-order) **deviation $\Delta\psi$ from the unperturbed stationary distribution** by solving the linearized Vlasov equation:

$$\frac{\partial \Delta\psi}{\partial t} - \omega_0 \left(Q_{x0} + \Delta Q(r,\phi)\right) \frac{\partial \Delta\psi}{\partial \theta_x} + \omega_s \frac{\partial \Delta\psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma} \frac{d f_0}{d J_x} \sqrt{\frac{2 J_x R}{Q_{x0}}} e^{j\theta_x} F_x^{coh}(z, t, \Delta\psi)$$

Tune modulation from the e-cloud

unperturbed long. distribution

unperturbed transv. distribution

Dipolar forces from the e-cloud

r, ϕ are the polar coordinate in the **longitudinal plane** $z = r\cos\phi$ $\delta = \frac{\omega_S}{v\eta} r \sin\phi$

J_x, θ_x are the action and angle in the **transverse plane** $x = \sqrt{\frac{2J_x R}{Q_{x0}}}\cos\theta_x$ $x' = \sqrt{\frac{2J_x Q_{x0}}{R}}\sin\theta_x$

The rest are constants:

ω_0	revolution frequency	R	machine radius
ω_s	synchrotron frequency	m_0	particle mass,
Q_{y0}	betatron tune	γ	relativistic gamma
		v	and velocity

Equation to be solved:

$$\frac{\partial \Delta\psi}{\partial t} - \omega_0 \left(Q_{x0} + \Delta Q(r,\phi)\right) \frac{\partial \Delta\psi}{\partial \theta_x} + \omega_s \frac{\partial \Delta\psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma} \frac{df_0}{dJ_x} \sqrt{\frac{2 J_x R}{Q_{x0}}} e^{j\theta_x} F_x^{coh}(z,t,\Delta\psi)$$

We search for **solutions in the form of dipolar coherent mode**:

Phase-shift term to be chosen later

$$\Delta\psi\left(J_x,\theta_x,r,\phi,t\right) = e^{j(\Omega t - \Delta\Phi(r,\phi))} f_1\left(J_x\right) e^{j\theta_x} \sum_{l=-\infty}^{+\infty} R_l(r) e^{-jl\phi}$$

Unknowns:

Complex tune shift

Known for dipolar oscillation

Longitudinal distribution of the oscillation

Replacing in the above we obtain a **new equation in the unknowns Ω and $R_l(z)$:**

$$e^{j(\Omega t - \Delta\Phi(r,\phi))} \sum_{l=-\infty}^{+\infty} R_l^p(r) e^{-jl\phi} \left(j\Omega - j\omega_s \frac{\partial \Delta\Phi}{\partial \phi} - jl\omega_s - j\omega_0\left(Q_{x0} + \Delta Q\right)\right)$$

$$= -\frac{\eta g_0(r)}{\omega_s m_0 \gamma} F_x^{coh}(z,t,\Delta\psi)$$

Decomposition of the detuning

To handle the detuning from the e-cloud we **decompose it** in two terms (as done in [1]):

$$\Delta Q(r,\phi) = \Delta Q_R(r) + \Delta Q_\Phi(r,\phi)$$

Average **detuning with longitudinal amplitude**

zero-average term

$$\Delta Q_R(r) = \frac{1}{2\pi}\int_0^{2\pi} \Delta Q(r,\phi)d\phi$$

Independent on ϕ

$$\frac{1}{2\pi}\int_0^{2\pi} \Delta Q_\Phi(r,\phi)d\phi = 0$$

No net change over the synchrotron period but **head-tail phase shift** (like for linear chromaticity)

[1] M. Schenk et al., "Vlasov description of the effects of nonlinear chromaticity on transverse coherent beam instabilities", Phys. Rev. Accel. Beams ***21****, 084402*

Choice of the phase-shift term

Replacing in Vlasov equation we find:

$$e^{j(\Omega t-\Delta\Phi(r,\phi))}\sum_{l=-\infty}^{+\infty}R_l^p(r)e^{-jl\phi}\left(j\Omega-j\omega_s\cancel{\frac{\partial\Delta\Phi}{\partial\phi}}-jl\omega_s-j\omega_0\left(Q_{x0}+\Delta Q_R(r)+\cancel{\Delta Q_\Phi(r,\phi)}\right)\right)$$

$$=-\frac{\eta g_0(r)}{\omega_s m_0\gamma}F_x^{coh}(z,t,\Delta\psi)$$

We can simplify the equation by **choosing $\Delta\Phi$** such that (generalization of what is done for linear chromaticity):

$$\frac{\partial\Delta\Phi}{\partial\phi}=-\frac{\omega_0}{\omega_s}\Delta Q_\Phi(r,\phi)$$

The equation becomes:

$$e^{j(\Omega t-\Delta\Phi(r,\phi))}\sum_{l=-\infty}^{+\infty}R_l^p(r)e^{-jl\phi}\left[j\Omega-j\omega_0\left(Q_{x0}+\Delta Q_R(r)-jl\omega_s\right)\right]=-\frac{\eta g_0(r)}{\omega_s m_0\gamma}F_x^{coh}(z,t,\Delta\psi)$$

The e-cloud defines these three terms
(we need to describe e-cloud detuning and dipolar forces in a way that can be plugged in this equation and allows to solve it)

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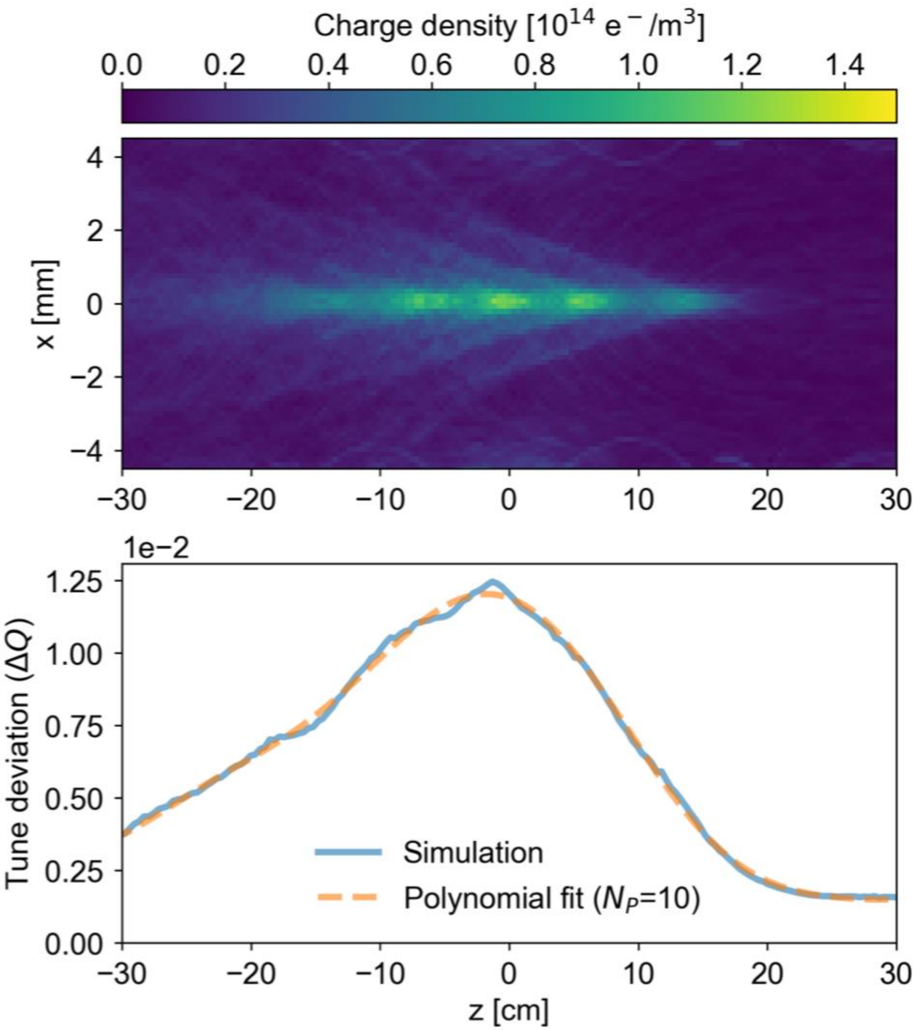
Modeling e-cloud detuning forces

We look at the detuning forces from a **realistic e-cloud** (LHC quadrupole, 450 GeV)



They can be **well approximated by a polynomial**

$$\Delta Q(z) = \sum_{n=0}^{N_P} A_n z^n$$

where the coefficients A_n can be found by a simple polynominal fit

We generalize, adding linear and non-linear chromaticity from the lattice:

$$\Delta Q(z, \delta) = \sum_{n=1}^{N} A_n z^n + B_n \delta^n$$

Modeling e-cloud detuning forces

For a detuning in this form:

$$\Delta Q(z,\delta) = \sum_{n=1}^{N} A_n z^n + B_n \delta^n$$

we could derive **explicit expressions for $\Delta\Phi$** and **ΔQ_R**:

$$\Delta\Phi(r,\phi) = -\frac{\omega_0}{\omega_s}\sum_{n=1}^{N_P} r^n \left[A_n\left(C_n(\phi) - \overline{C}_n\frac{\phi}{2\pi}\right) + \left(\frac{\omega_s}{v\eta}\right)^n B_n\left(S_n(\phi) - \overline{S}_n\frac{\phi}{2\pi}\right)\right]$$

$$\Delta Q_R(r) = \sum_{n=0}^{N_P} r^n \left[A_n\frac{\overline{C}_n}{2\pi} + B_n\left(\frac{\omega_s}{v\eta}\right)^n\frac{\overline{S}_n}{2\pi}\right]$$

where:

$$C_0(\phi) = \phi\,, \quad C_1(\phi) = \sin\phi\,,$$

$$C_n(\phi) = \frac{\cos^{n-1}\phi\sin\phi}{n} + \frac{n-1}{n}C_{n-2}(\phi)\,,$$

$$S_0(\phi) = \phi\,, \quad S_1(\phi) = -\cos\phi\,,$$

$$S_n(\phi) = -\frac{\sin^{n-1}\phi\cos\phi}{n} + \frac{n-1}{n}S_{n-2}(\phi)\,.$$

$$\overline{C}_n = C_n(2\pi) - C_n(0)$$

$$\overline{S}_n = S_n(2\pi) - S_n(0)$$

These expressions can be directly plugged in our equation:

$$e^{j(\Omega t - \Delta\Phi(r,\phi))}\sum_{l=-\infty}^{+\infty} R_l^p(r)e^{-jl\phi}\left[j\Omega - j\omega_0\left(Q_{x0} + \Delta Q_R(r) - jl\omega_s\right)\right] = -\frac{\eta g_0(r)}{\omega_s m_0\gamma}F_x^{coh}(z,t,\Delta\psi)$$

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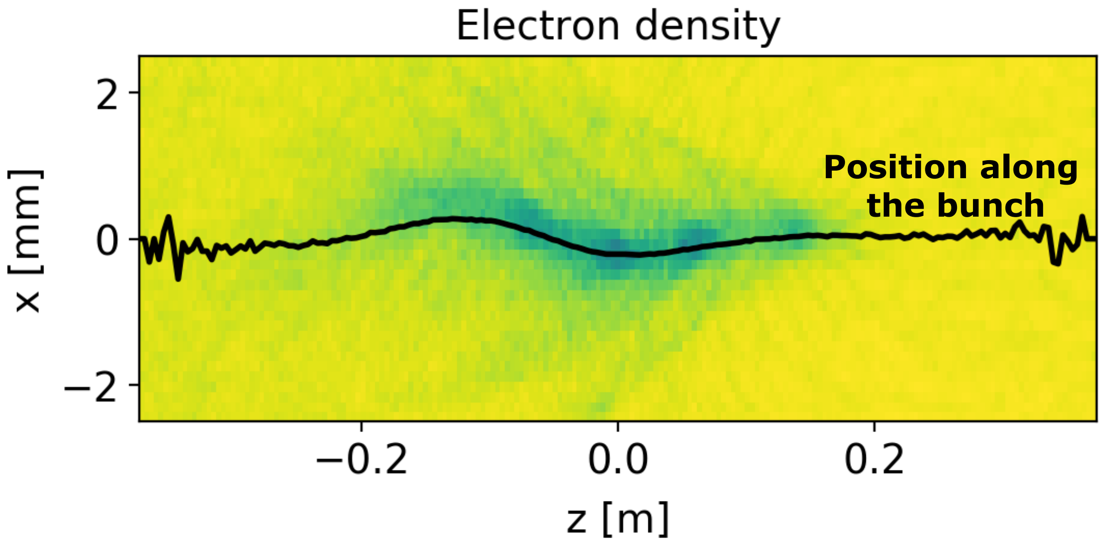
Handling the e-cloud dipolar forces

$$e^{j(\Omega t-\Delta\Phi(r,\phi))}\sum_{l=-\infty}^{+\infty}R_l^p(r)e^{-jl\phi}\left[j\Omega-j\omega_0\left(Q_{x0}+\Delta Q_R(r)-jl\omega_s\right)\right]=-\frac{\eta g_0(r)}{\omega_s m_0\gamma}F_x^{coh}(z,t,\Delta\psi)$$

- This is more tricky because the **dipolar forces depend on the beam oscillation itself**
  → We need to find a way to **characterize how a realistic e-cloud responds to intra-bunch distortions**
- An approach based on **generalized wakefields was proposed by Perevedentsev** in 2002 but, to our knowledge, never implemented
- We developed a **different approach**, mathematically equivalent to Perevedentev's but **easier to implement numerically** also in the presence of detuning forces



Test and response functions

We use a **set of sinusoidal test functions**, $h_n(z)$

$$h_n(z) = \begin{cases} \mathcal{A}_n \cos(2\pi f_n^z z), & \text{if } n \text{ is even} \\ \mathcal{A}_n \sin(2\pi f_n^z z), & \text{if } n \text{ is odd} \end{cases} \qquad f_n^z = \begin{cases} \dfrac{n}{2}\dfrac{1}{L_{\mathrm{bkt}}} & \text{if } n \text{ is even} \\ \dfrac{n+1}{2}\dfrac{1}{L_{\mathrm{bkt}}} & \text{if } n \text{ is odd} \end{cases}$$

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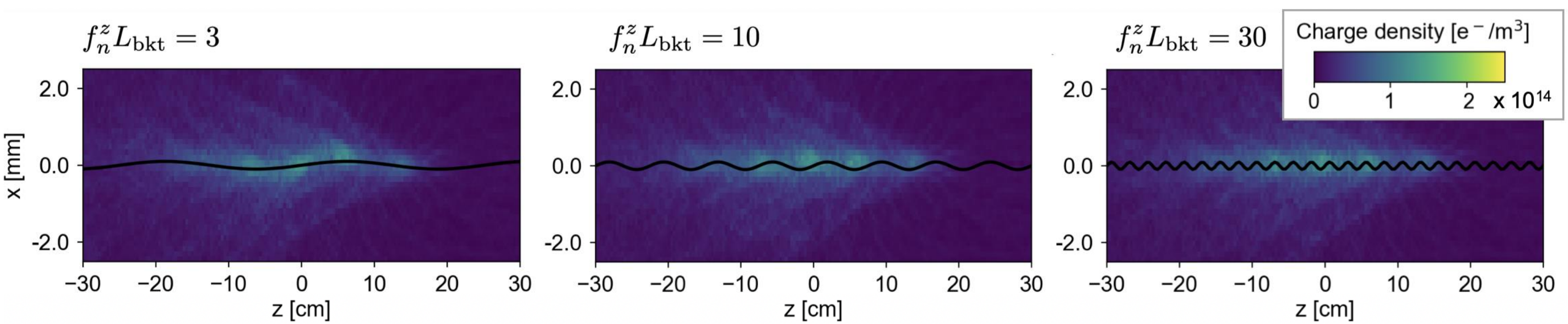
We probe the response of the e-cloud with **short (single-passage) simulations** where:

- we apply a **distortion equal to the test-function** on the position along the bunch



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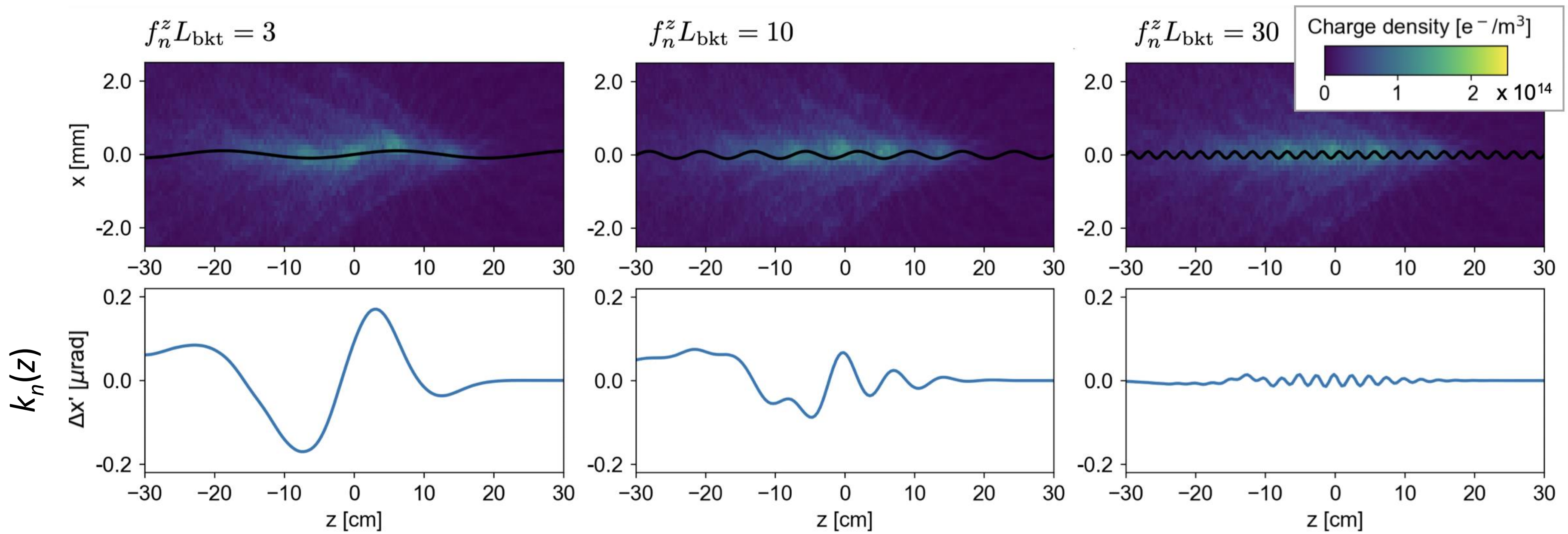
We probe the response of the e-cloud with **short (single-passage) simulations** where:

- we apply a **distortion equal to the test-function** on the position along the bunch
- we **measure the resulting kick** $k_n(z)$



Superposition principle

We test whether the **superposition principle** can be applied to get the **response to an arbitrary transverse distortion** (small in amplitude)

We **decompose the average transverse position along the bunch** using our test functions:

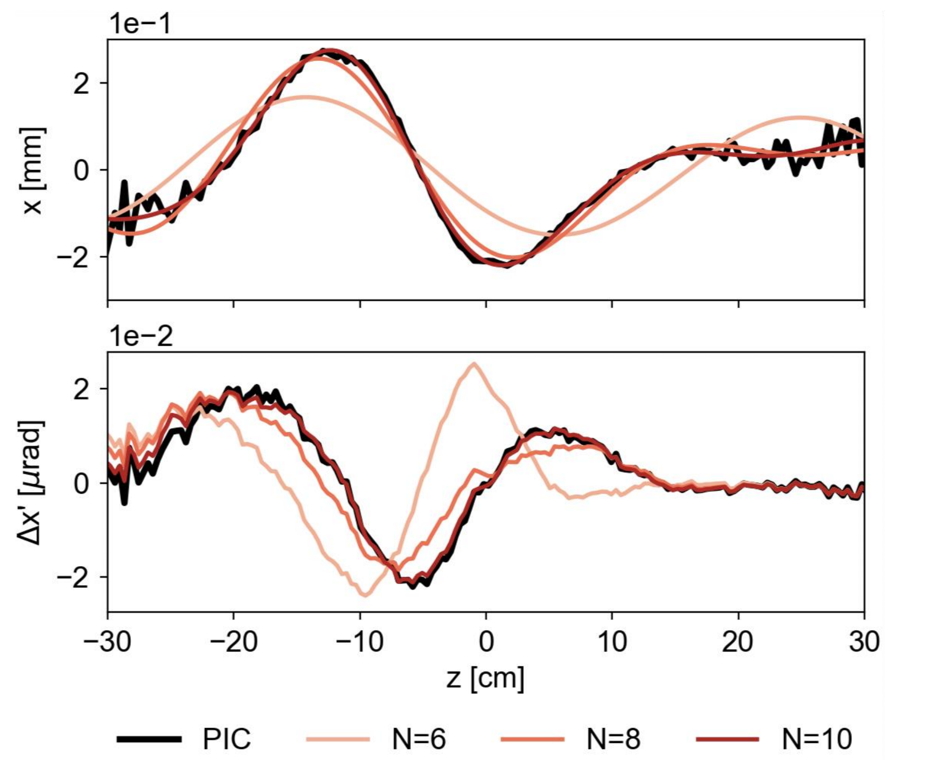
$$\bar{x}(z) = \sum_{n=0}^{\infty} a_n h_n(z) = \sum_{n=0}^{\infty} h_n(z) \int \bar{x}(\tilde{z}) \frac{h_n(\tilde{z})}{H_n^2} d\tilde{z}$$

And we try to **predict the kick** using our response function set:

$$\Delta x'(z) = \sum_{n=0}^{\infty} a_n k_n(z) = \sum_{n=0}^{\infty} k_n(z) \int \bar{x}(\tilde{z}) \frac{h_n(\tilde{z})}{H_n^2} d\tilde{z}$$

We **tested** it on oscillations from a simulated e-cloud instability

- It **works remarkably well**!
- **Convergence is fast**!



Superposition principle

From the previous slide:

$$\Delta x'(z)=\sum_{n=0}^{\infty}a_n k_n(z)=\sum_{n=0}^{\infty}k_n(z)\int \bar{x}(\tilde{z})\frac{h_n(\tilde{z})}{H_n^2}d\tilde{z}$$

We need to **write it in terms of phase-space distribution** (the unknown of the Vlasov problem):

$$\bar{x}(z,t)=\frac{1}{\lambda_0(z)}\iint d\tilde{x}d\tilde{x}'\int d\tilde{\delta}\,\tilde{x}\Delta\psi\left(\tilde{x},\tilde{x}',z,\tilde{\delta},t\right)$$

We replace the **expression of the coherent mode**

$$\Delta\psi(J_x,\theta_x,r,\phi,t)=e^{j(\Omega t-\Delta\Phi(r,\phi))}f_1(J_x)\,e^{j\theta_x}\sum_{l=-\infty}^{+\infty}R_l(r)e^{-jl\phi}$$

We get an **expression relating the coherent force to the unknowns** (Ω, $R_l(z)$), which can be plugged in the Vlasov equation:

$$F_x^{coh}(r,\phi,t)=-\frac{N_b m_0\gamma v\omega_s}{2\pi\eta Q_{x0}}e^{j\Omega t}\iint \tilde{r}d\tilde{r}d\tilde{\phi}\,e^{-j\Delta\Phi(\tilde{r},\tilde{\phi})}\sum_{l'=-\infty}^{+\infty}R_{l'}(\tilde{r})e^{-jl'\tilde{\phi}}\sum_{n=0}^{N}k_n(r\cos\phi)\frac{h_n\left(\tilde{r}\cos\tilde{\phi}\right)}{H_n^2\lambda_0(\tilde{r}\cos\tilde{\phi})}$$

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Putting it all together, with some standard manipulation, we get the **(integral) equation to be solved**:

$$R_l(r)\left(\Omega - Q_{x0}\omega_0 - \omega_0\Delta Q_R - l\omega_s\right) = -\frac{Nv}{8\pi^2 Q_{x0}} g_0(r) \int_0^{2\pi} d\phi e^{jl\phi} e^{j\Delta\Phi(r,\phi)} \iint \tilde{r} d\tilde{r} d\tilde{\phi}\, e^{-j\Delta\Phi(\tilde{r},\tilde{\phi})}$$

$$\times \sum_{l'=-\infty}^{+\infty} R_{l'}(\tilde{r}) e^{-jl'\tilde{\phi}} \sum_{n=0}^{N} k_n\left(r\cos\phi\right) \frac{h_n\left(\tilde{r}\cos\tilde{\phi}\right)}{H_n^2 \lambda_0(\tilde{r}\cos\tilde{\phi})}$$

In red: **unknowns**

In green: **terms defined by the e-cloud (built as described before)**

Solving the equation

We want to go **from continuous unknowns** $R_l(z)$ to **discrete unknowns**. For this purpose we expand $R_l(z)$ using orthogonal polynomials:

$$R_l(r) = W_l(r) \sum_{m=0}^{+\infty} b_{lm} f_{lm}(r)$$

This transforms our integral equation into a **discrete eigenvalue problem**:

$$b_{lm}(\Omega - Q_{x0}\omega_0 - l\omega_s) = \sum_{l'm'} \left(M_{lm,l'm'} + \tilde{M}_{lm,l'm'} \right) b_{l'm'}$$

In red: unknowns

In green: terms defined by the e-cloud

The effect of the e-cloud is encoded in two matrices:

e-cloud phase shift

e-cloud dipolar response

$$M_{lm,l'm'} = -\frac{Nv}{8\pi^2 Q_{x0} F_{lm}} \sum_{n=0}^{N} \iint dr\, d\phi\, e^{jl\phi} e^{j\Delta\Phi(r,\phi)} w_l(r) f_{lm}(r) \frac{g_0(r)}{W_l(r)} k_n(r\cos\phi)$$
$$\times \iint \tilde{r} d\tilde{r} d\tilde{\phi}\, e^{-jl'\tilde{\phi}} e^{-j\Delta\Phi(\tilde{r},\tilde{\phi})} f_{l'm'}(\tilde{r}) \frac{W_{l'}(\tilde{r}) h_n(\tilde{r}\cos\tilde{\phi})}{\lambda_0(\tilde{r}\cos\tilde{\phi}) H_n^2}$$

$$\tilde{M}_{lm,l'm'} = \delta_{l,l'} \frac{\omega_0}{F_{lm}} \int dr\, w_l(r) \Delta Q_R(r) f_{lm}(r) f_{lm'}(r)$$

e-cloud detuning with long. amplitude

The **implementation** consists in a (python) code that:

- Uses **single-pass PyECLOUD simulations** to compute the e-cloud detuning coefficients (A_n, defining ΔQ_R and $\Delta\Phi$) and response functions ($k_n(z)$)
- Computes the matrices by **numerical integration** (speed-up obtained exploiting matrix properties and optionally "easy" parallelization)
- Computes **eigenvalues** (Ω) and **eigenvectors** (b_{lm}) using a standard linear algebra package

We want to g

expand $R_l(z)$ u

This transforms our integral equation into a discrete eigenvalue problem:

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$$\times \iint \tilde{r} d\tilde{r} d\tilde{\phi}\, e^{-jl'\tilde{\phi}} e^{-j\Delta\Phi(\tilde{r},\tilde{\phi})} f_{l'm'}(\tilde{r}) \frac{W_{l'}(\tilde{r}) h_n(\tilde{r}\cos\tilde{\phi})}{\lambda_0(\tilde{r}\cos\tilde{\phi}) H_n^2}$$

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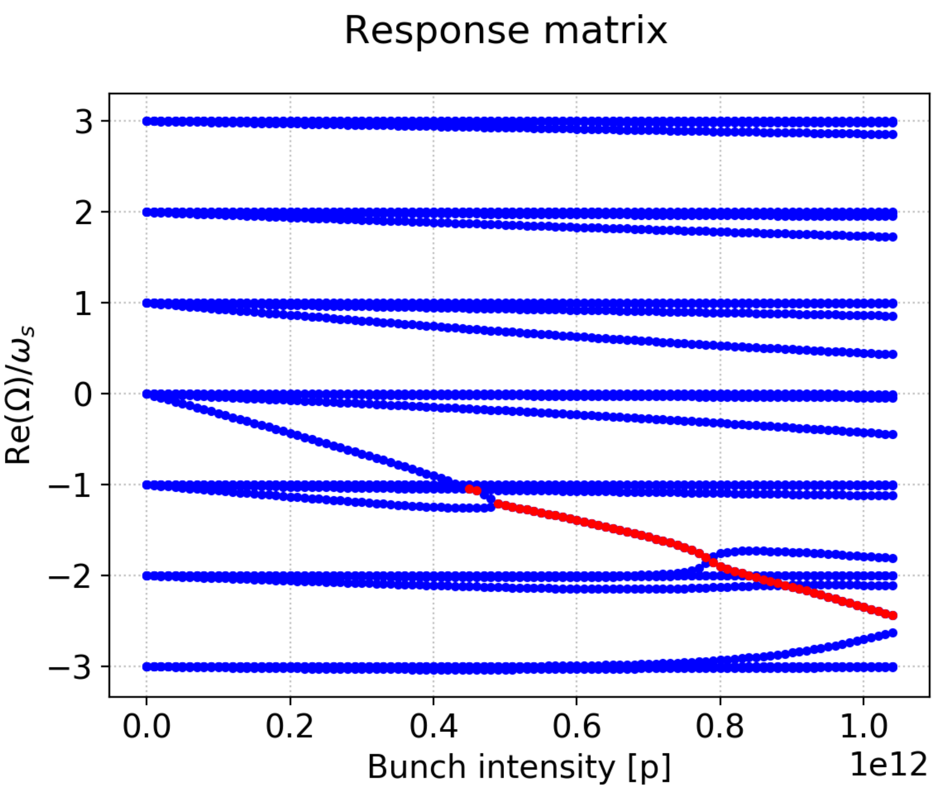
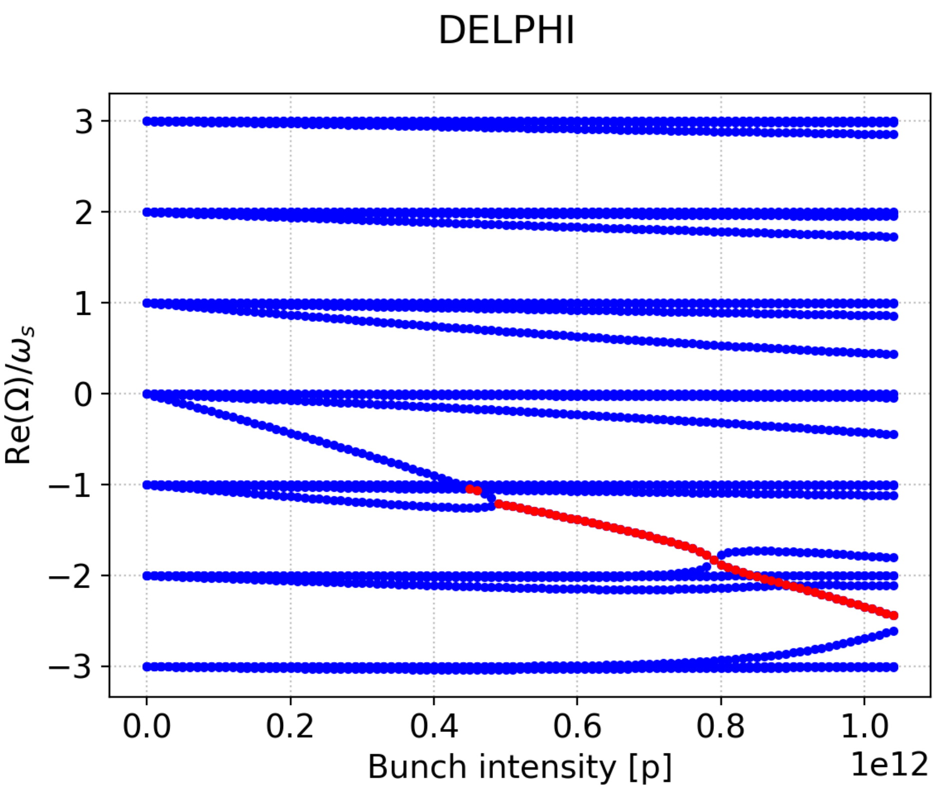
The method is not specific for e-cloud effects, and **can in fact be applied to a conventional impedance**:

- This was done in order to **benchmark the new tool against the DELPHI Vlasov solver**

  → **extremely good agreement found**, in spite of very different method to compute the coupling matrix





Used study case from D. Amorim's (https://indico.cern.ch/event/824835/contributions/3449637/)

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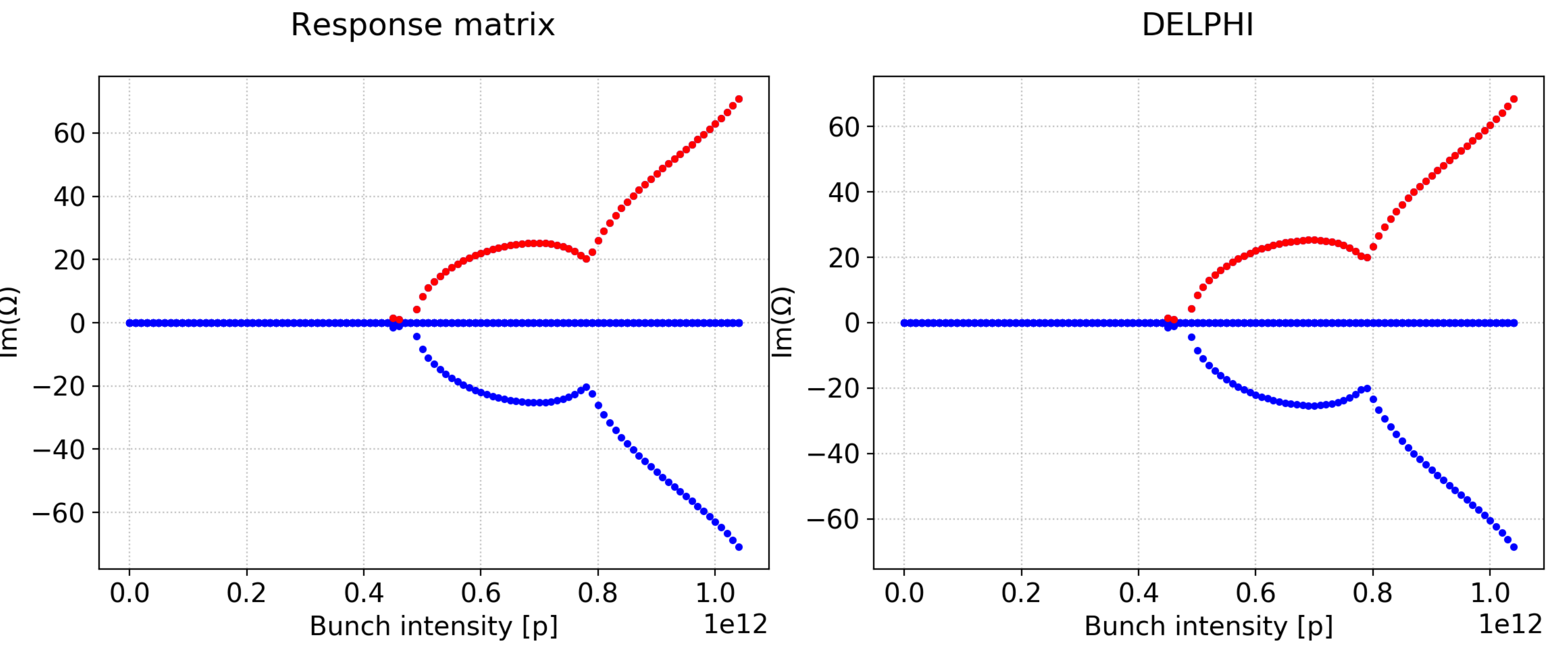
  → **extremely good agreement found**, in spite of very different method to compute the coupling matrix
- The test **covers only the dipolar forces**



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e-cloud - applications and benchmark

- We applied the new method to the case of **e-cloud instabilities at LHC injection**
  - Solid results for this case are available from macroparticle simulation studies for comparison (see L. Sabato, [WP2 meeting 10 Dec 2019]())

- To fully **validate the Vlasov method** and its **implementation** we **simulated in PyHEADTAIL the same linearized model for the e-cloud:**
  - Simulation approach **in itself interesting for applications** as it allows including **effects not handled by the Vlasov solver** (external transverse non-linearities, complex feedback systems, or non-linear longitudinal motion)
  - **Much faster** (x10) compared to brute-force PIC simulations
- For comparison we **needed to reconstruct the mode spectrum from the turn-by-turn data saved by PyHEADTAIL** → used advanced spectral analysis based on **intra-bunch motion**

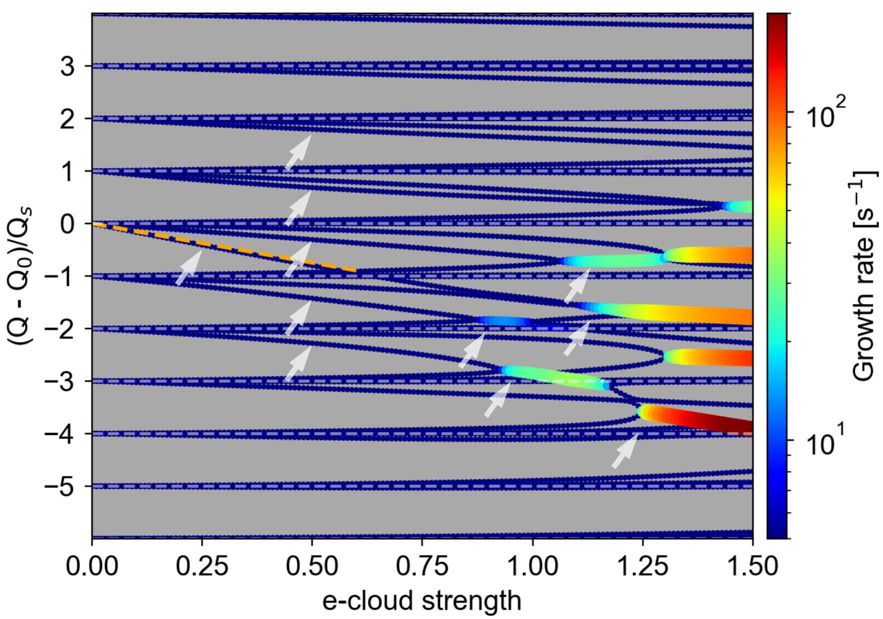
- The different terms from the e-cloud were introduced in steps to identify their effect

Benchmarks with linearized model

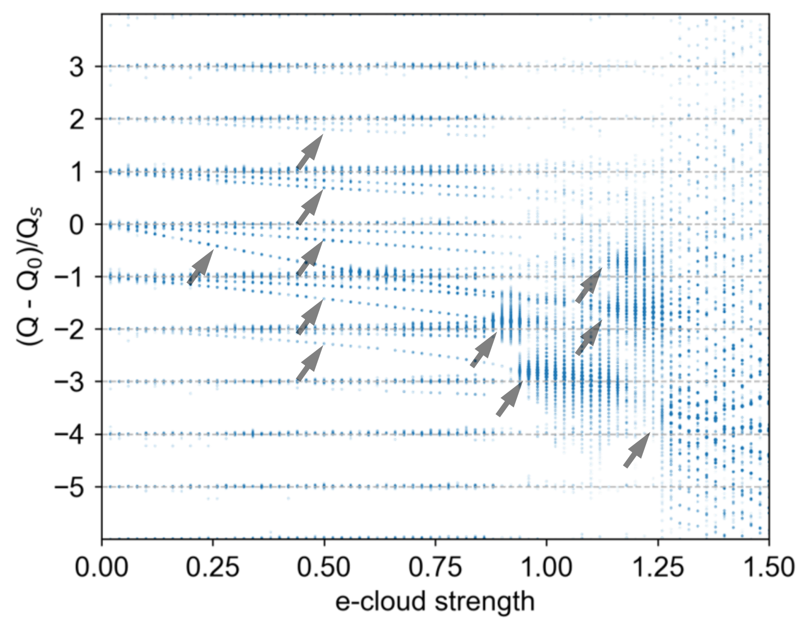
- With the e-cloud **dipolar force alone** (response functions)
  - **Negative tune shifts** are observed when increasing the cloud strength
  - Instabilities are triggered by **transverse mode coupling**
- **Very good agreement** between Vlasov method and macroparticle simulations

Dipolar	Quadrupolar	
☑ F_{coh}	☒ $\Delta\Phi(r, \phi)$	☒ $\Delta Q(r)$

(a) Eigenvalues from linearized Vlasov equation

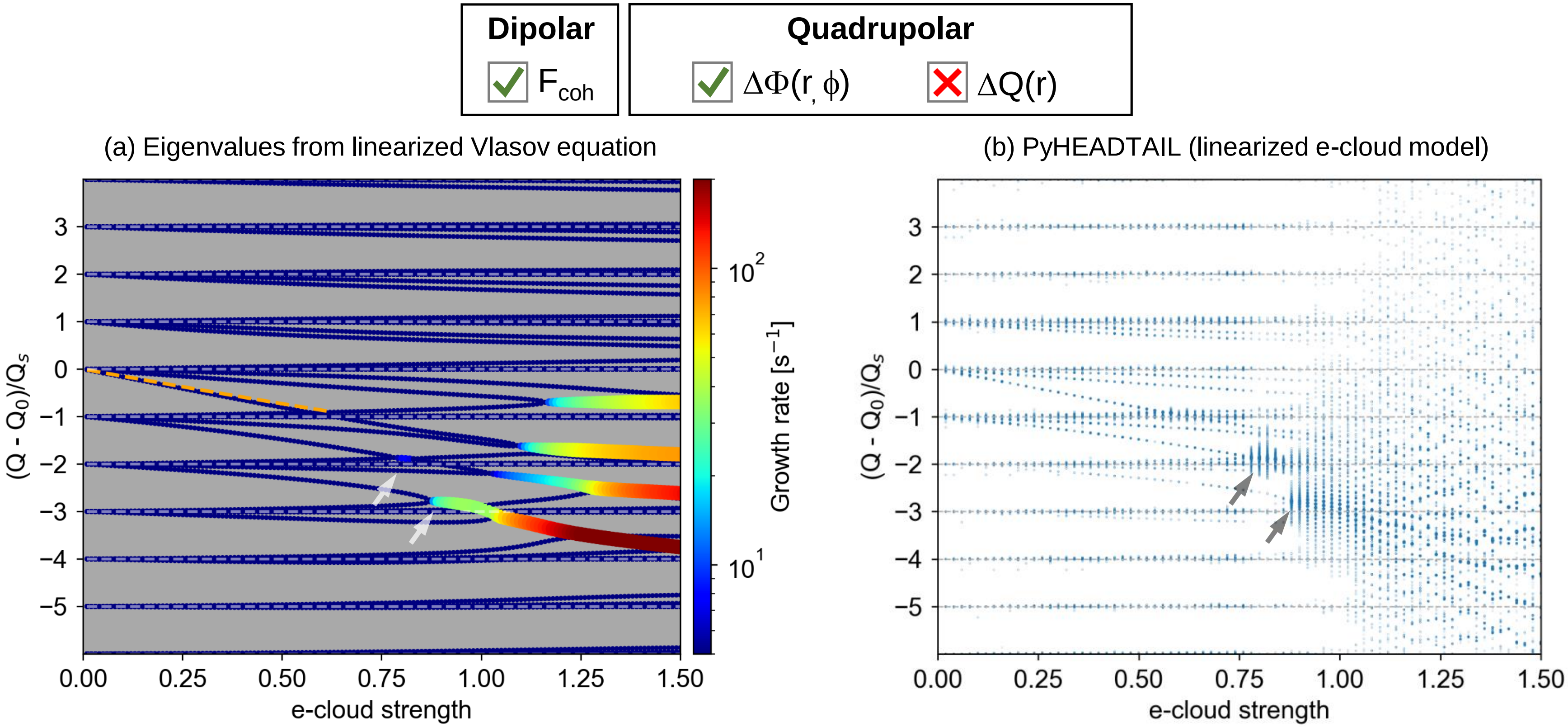


(b) PyHEADTAIL (linearized e-cloud model)



Benchmarks with linearized model

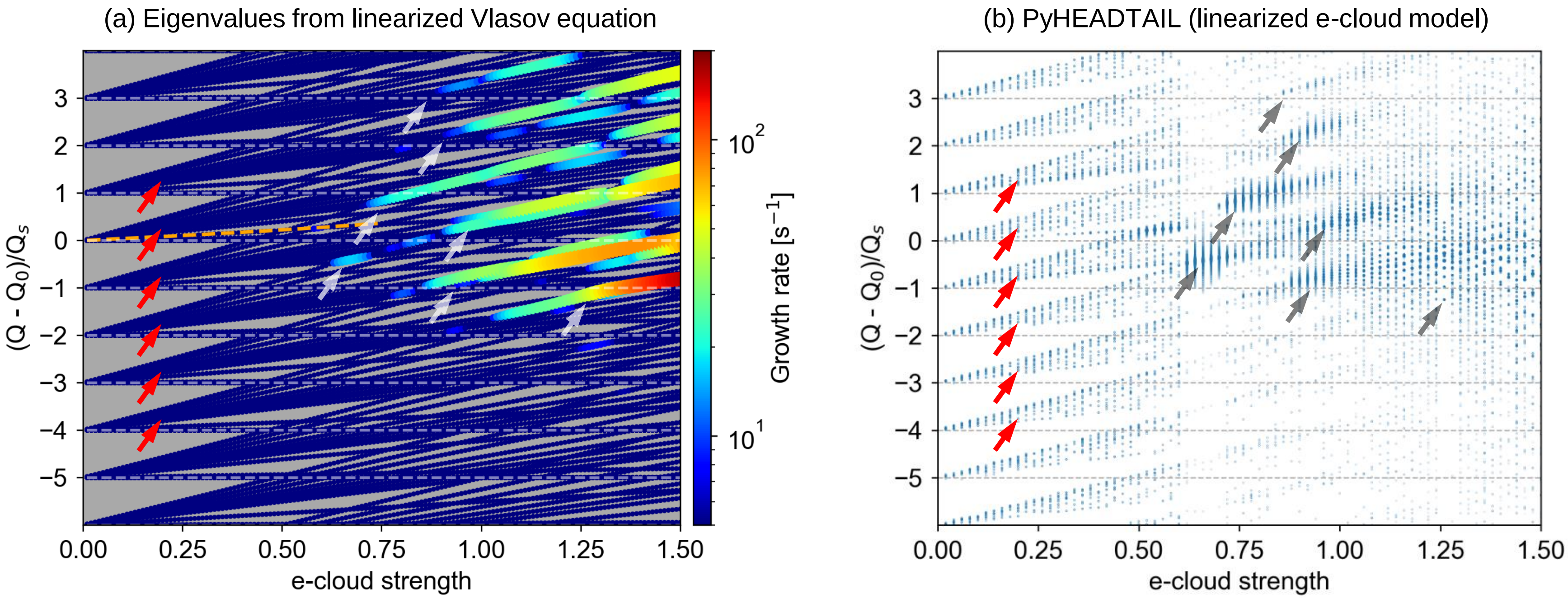
- When introducing the **head-tail phase shift** due to the e-cloud quadrupolar forces
  - Modes **tune shifts** at low intensity **remain very similar**
  - The **mode coupling behavior is affected** (threshold decreases)
- **Very good agreement** between Vlasov method and macroparticle simulations



Benchmarks with linearized model

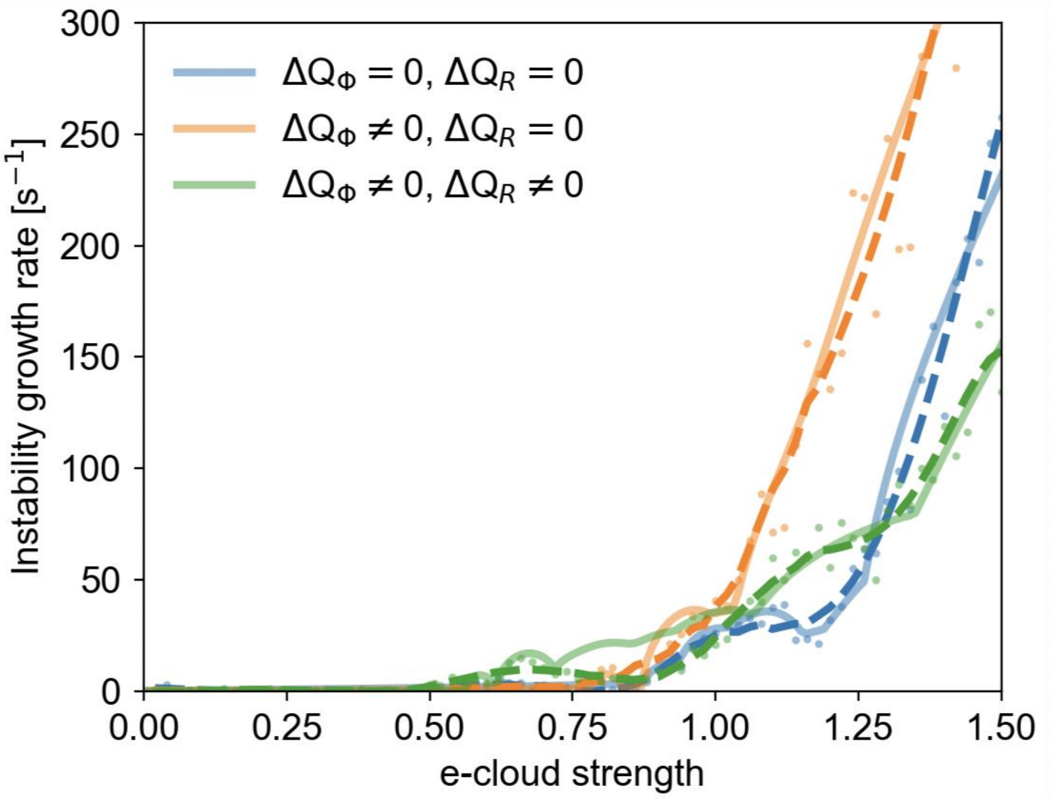
- When introducing the **detuning with longitudinal amplitude** due to e-cloud
  - Characteristic **"fans of modes"** appear at each synchrotron sideband
  - The **most unstable mode becomes less strong**
  - **Other weaker instabilities** appear
- **Very good agreement** between Vlasov method and macroparticle simulations

Dipolar	Quadrupolar	
☑ F_{coh}	☑ $\Delta\Phi(r,\phi)$	☑ $\Delta Q(r)$



Benchmarks with linearized model

In all cases **good agreement is found also on the growth rate** between the macroparticle simulations and the most unstable mode identified by the Vlasov solver



Continuous lines: Vlasov method

Dots: PyHT simulation (raw)

Dashed lines: PyHT simulations (smoothed)

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Comparison against PIC simulations

- So far we have compared Vlasov method with MP simulation using the **same linearized e-cloud model**, defined by ΔQ_R and $\Delta\Phi$ and response functions $k_n(z)$
- We now want to **check** whether such a **linearized model is appropriate** to describe the e-cloud instability
- For this purpose we **compare against conventiola PyEC-PyHT (PIC) simulations**

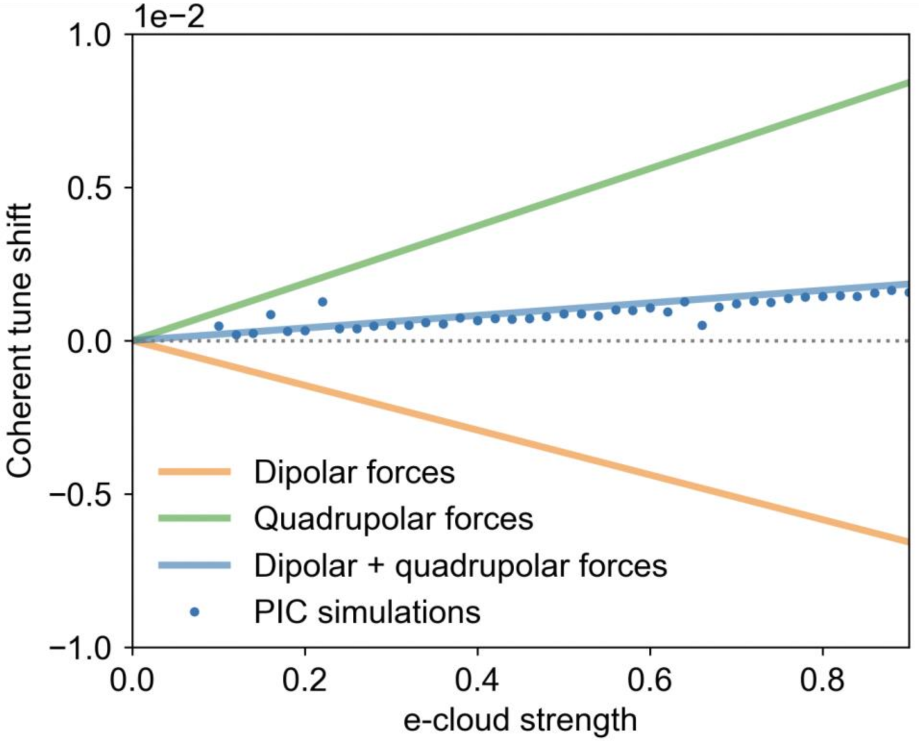
  → PIC simulations include **transverse non-linearities** from the e-cloud which are neglected in the linearized model

Comparison against PIC simulations

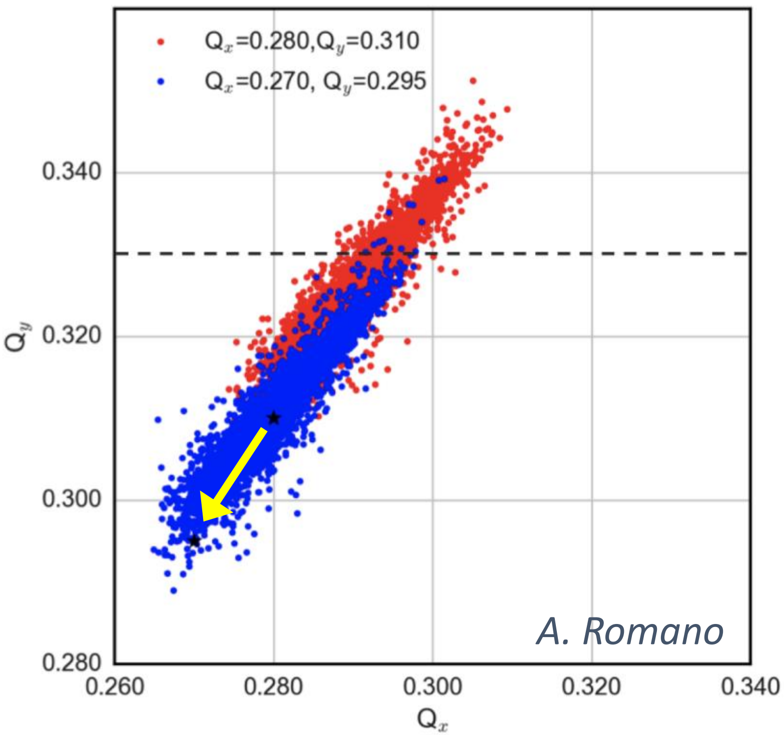
As a first check we compared the **tune-shift on the "rigid-bunch mode"** for low e-cloud strength (effect of phase shift negligible)

- **Excellent agreement** between Vlasov method and PIC simulation
- Coherent **tune shift very small** (especially when compared to tune spread)
  - Consistent with experimental observations
  - Effect or cancellation between **dipolar** and **quadrupolar** forces (important to take into account both!)

Coherent tune shift



Incoherent tune spread

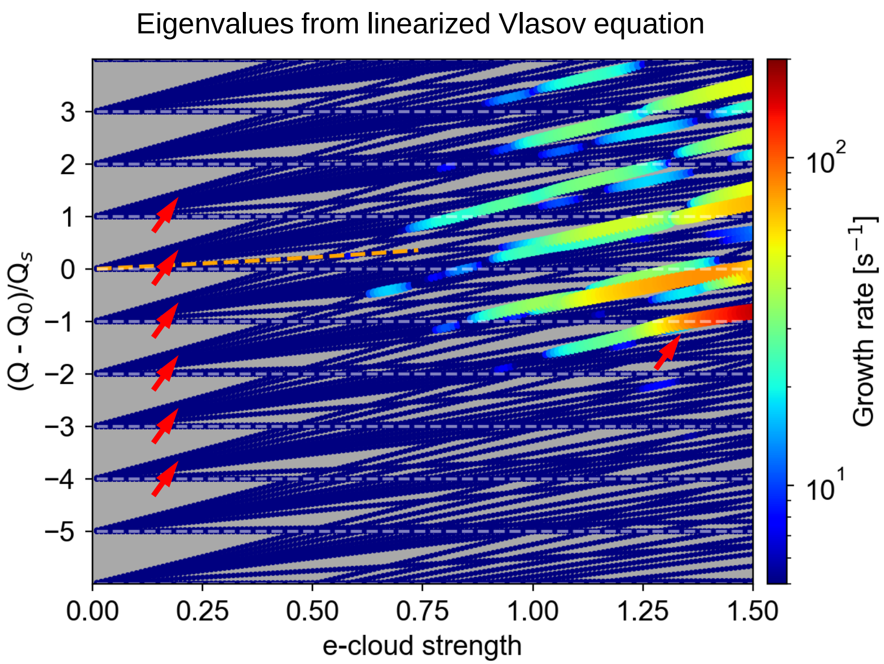


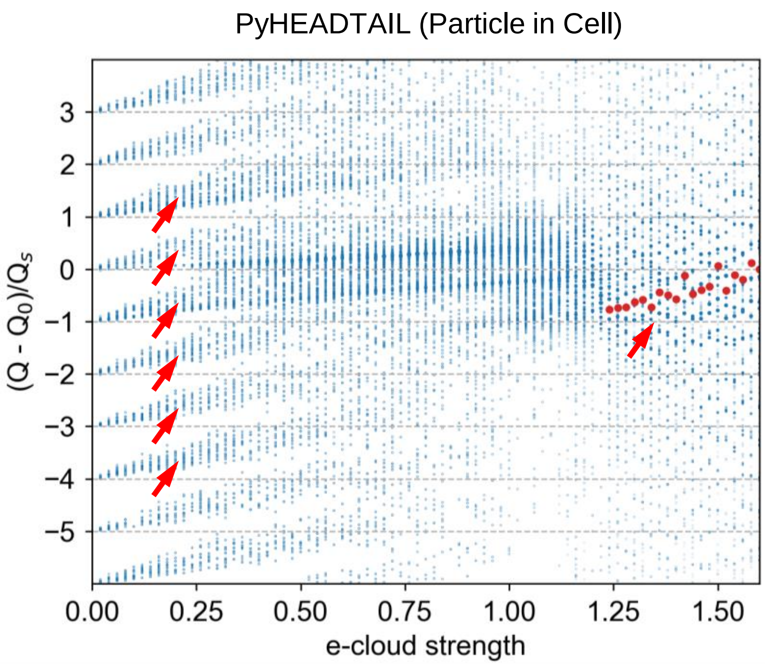
A. Romano

Comparison against PIC simulations

Comparing the spectra we observe:

- The **existence of "fans of modes"** associated to each synchrotron sideband (effect of quadrupolar forces) is **confirmed** by PIC simulations
- The **frequency of the most unstable** mode is **quite well reproduced** (again the quadrupolar terms are crucial for this purpose)

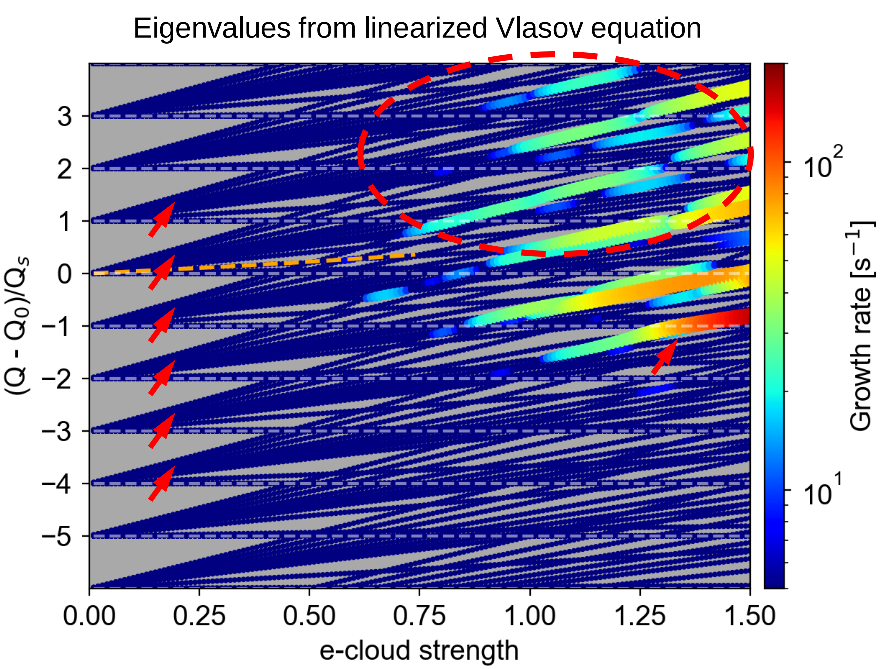


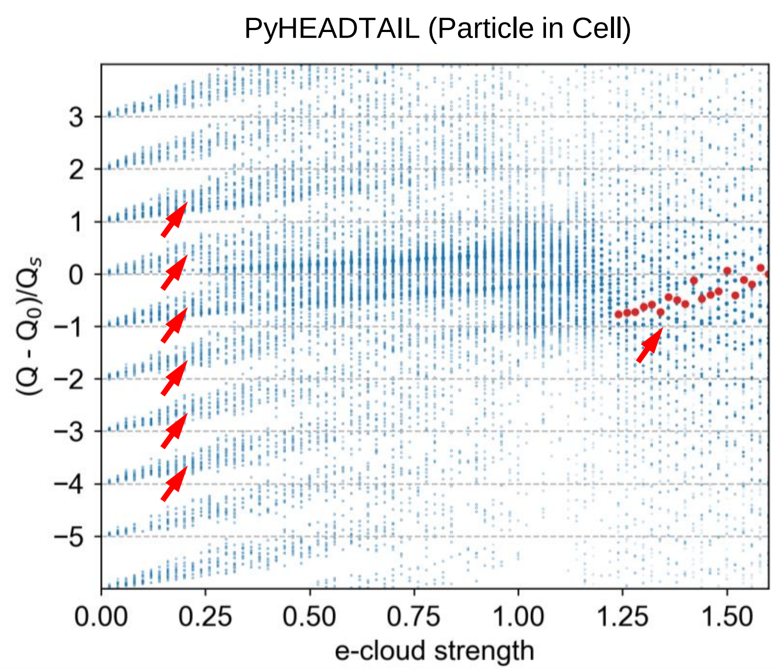


Comparison against PIC simulations

Comparing the spectra we observe:

- **Unstable modes with $\Delta Q/Q_s$ >0 are not visible with the PIC simulation** but only with linearized model (both Vlasov and macroparticles)





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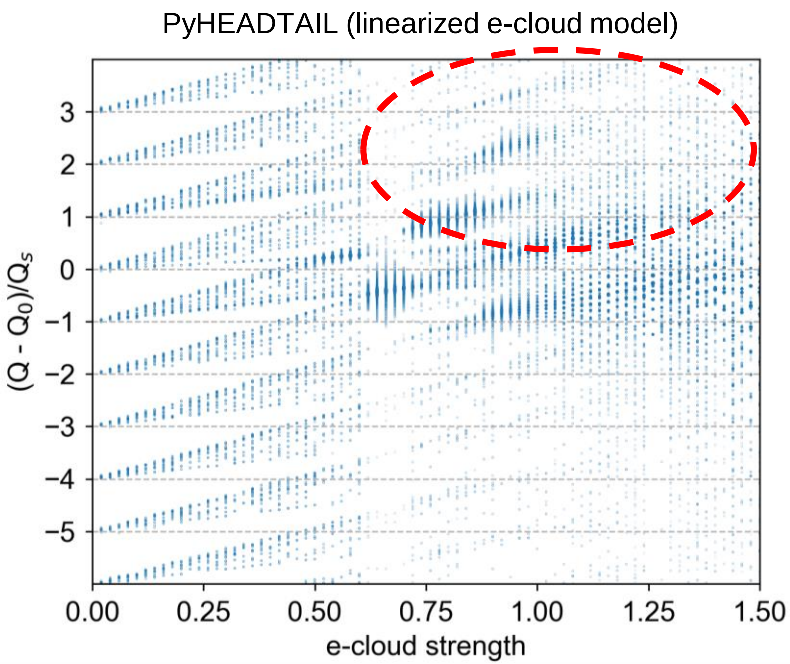
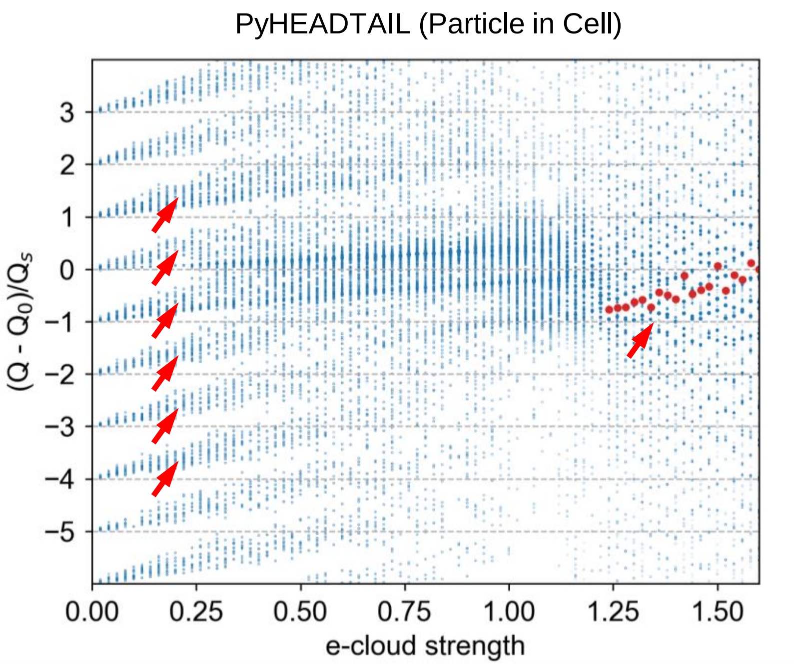
- Our hypothesis: that **these modes are stabilized by the e-cloud transverse non-linearities**, which are not included in the linearized model
  - To check this hypothesis we **added** to the MP simulations with the **linearized model** a **simplified (z-independent) non-linear map**





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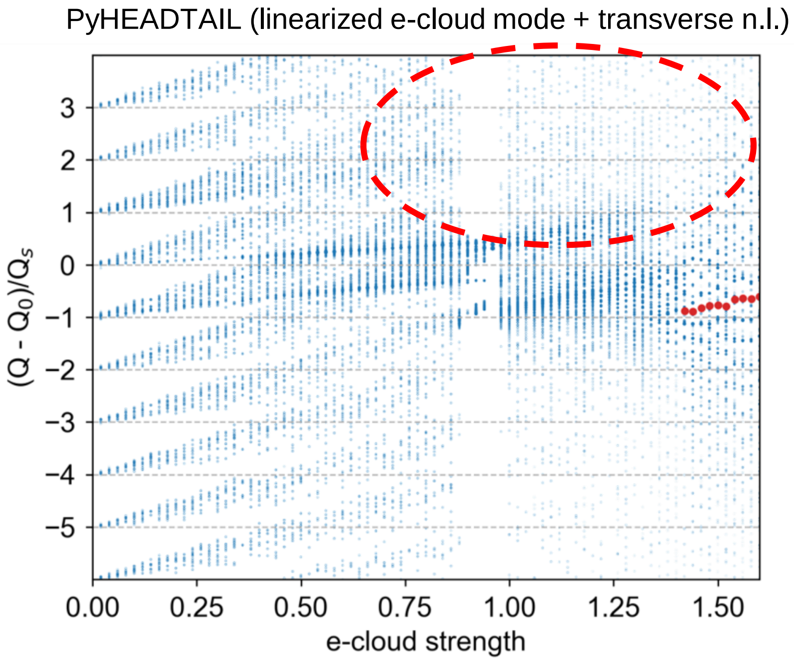
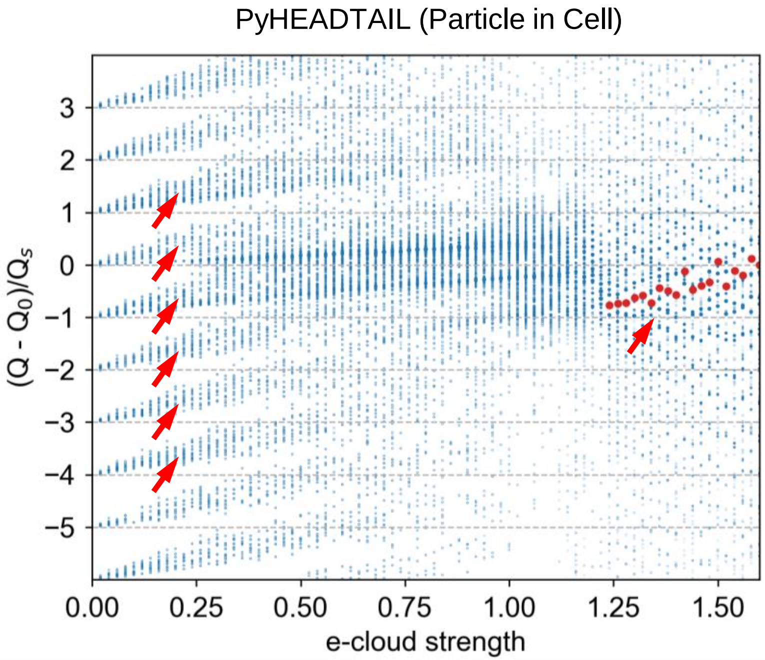
    → Modes with $\Delta Q/Q_s$ >0 are **indeed stabilized!**





Summary and conclusions

- We have developed a **linearized description** of the **dipolar** and **quadrupolar** forces introduced by and **e-cloud** and a **Vlasov method** including these effects
    - **Benchmarked** against **DELPHI** for the special case a dipolar impedance and against **MP simulations with the same linearized model** for the case of LHC e-cloud instabilities at 450 GeV
    - Comparing against **PIC simulations**, the linearized approach predicts well the **coherent tune shifts** as well as the **frequency** and **risetime** the most unstable mode

- This approach is **much faster** (x100) compared to PIC simulations:

    → Provides a way of **rapidly obtaining a general picture** on the beam stability and its dependence on different parameters

    → This is **complementary to PIC simulations**, which can then be used to refine the picture (including the effect of transverse non-linearities)

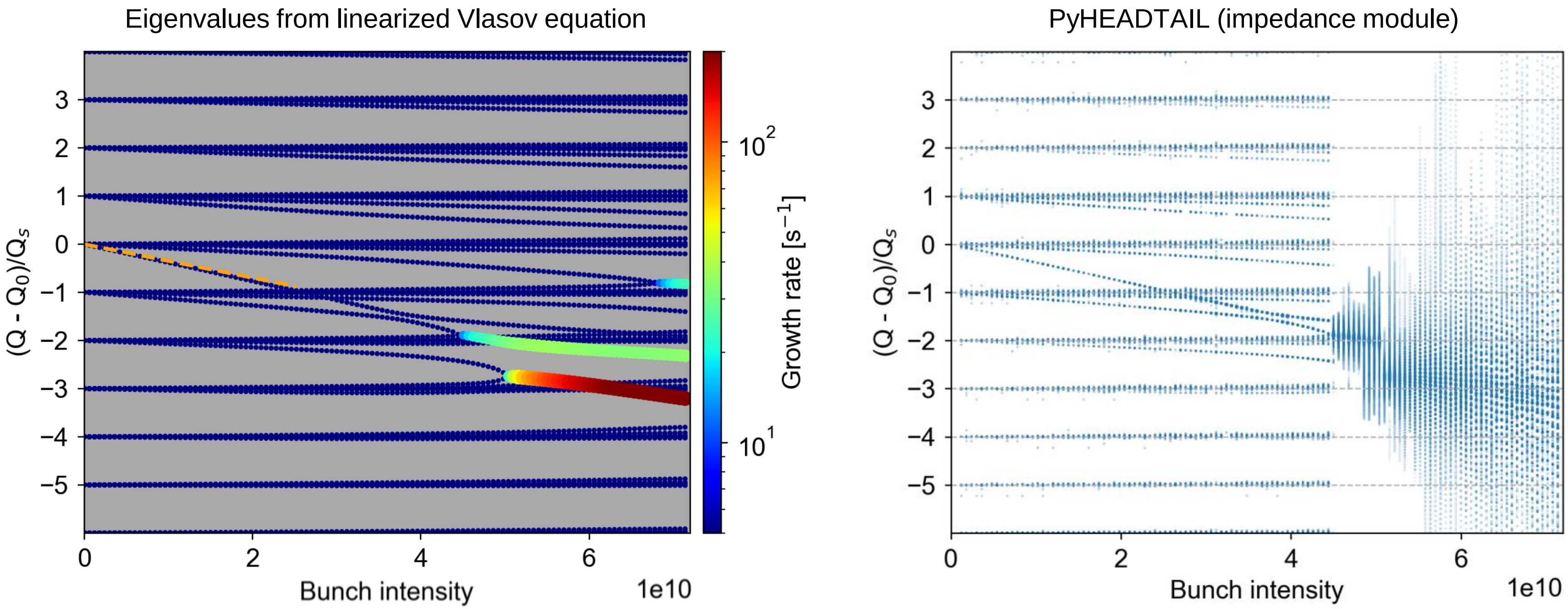
Next steps:

- **Code development**: from “demonstrator” to “proper tool” (including integration with DELPHI)
- **Application** to a wider range of scenarios (scan intensity, damper, Q’, etc.)

A useful byproduct: impedance applications

- The new method turned out to be useful also to study **conventional impedance-driven instabilities** as it capable to handle **quadrupolar impedances**, which are not available in conventional Vlasov solvers (MOSES, DELPHI, NHTVS)
  → Successfully checked against MP simulations

Only dipolar impedance



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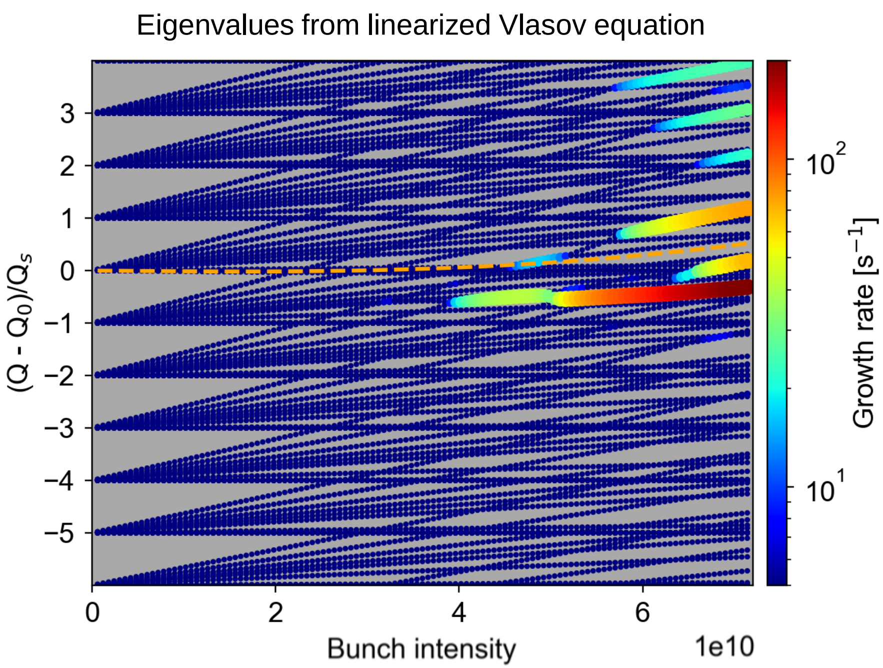
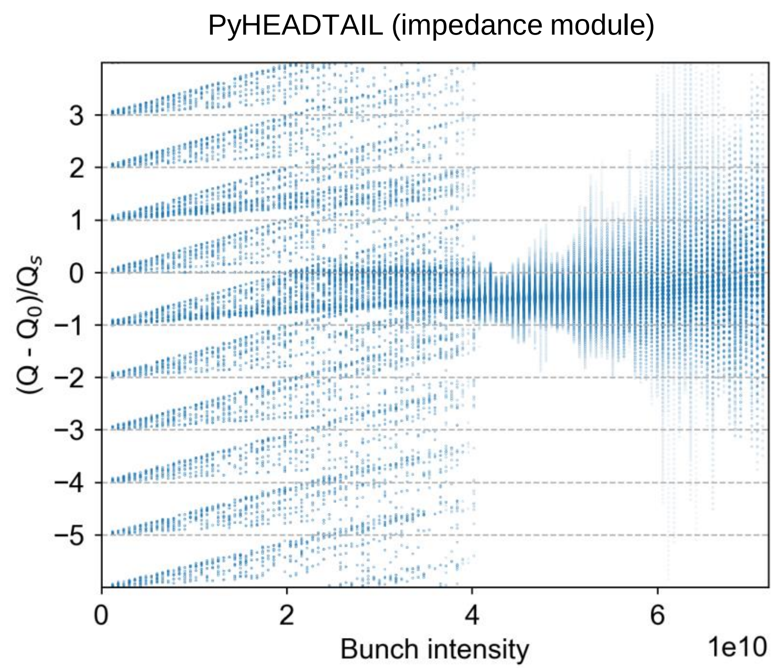
Dipolar and quadrupolar impedance





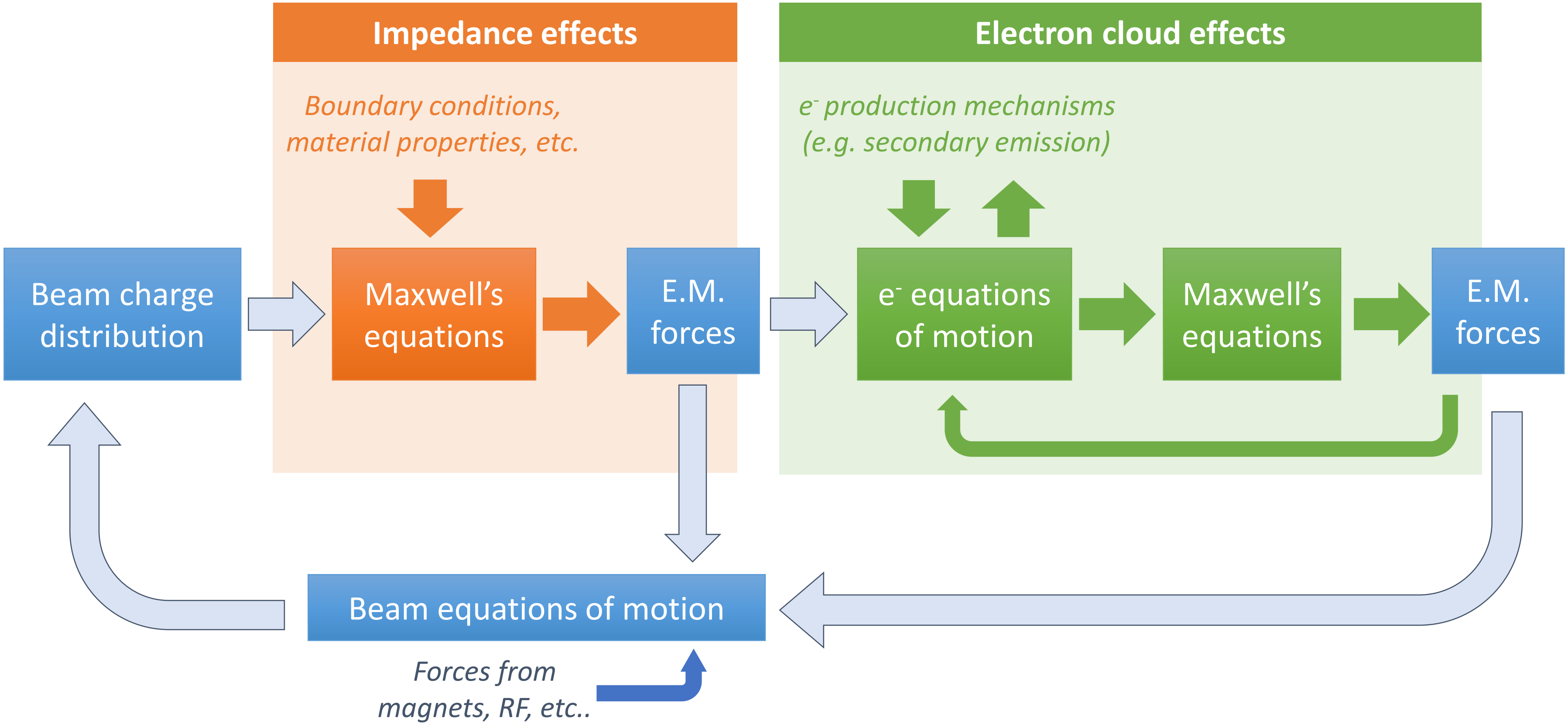
Thanks for your attention!

Electron cloud driven instabilities

This system is **linear and time-invariant:**

- The system is **fully described by its pulse response** (wakefield)
- **Superposition principle** (convolution) is applicable

This system is **....**



In the Vlasov approach we don't simulate the dynamics of the single particles but the evolution of the bunch phase space distribution ψ(x, x', y, y', z, δ).

Its (first-order) deviation from the unperturbed stationary distribution is solution of the linearized Vlasov equation (for a "well-behaved" accelerator):

$$\frac{\partial \Delta\psi}{\partial t} - \omega_0 \left(Q_{x0} + \Delta Q(r,\phi)\right) \frac{\partial \Delta\psi}{\partial \theta_x} + \omega_s \frac{\partial \Delta\psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma} \frac{df_0}{dJ_x} \sqrt{\frac{2J_x R}{Q_{x0}}} e^{j\theta_x} F_x^{coh}(z,t,\Delta\psi)$$

Tune modulation from the e-cloud

unperturbed long. distribution

unperturbed transv. distribution

Dipolar forces from the e-cloud

We search for solutions in the form of dipolar coherent mode:

Phase shift to be chosen to ease the solution

$$\Delta\psi\left(J_x, \theta_x, r, \phi, t\right) = e^{j(\Omega t - \Delta\Phi(r,\phi))} f_1\left(J_x\right) e^{j\theta_x} \sum_{l=-\infty}^{+\infty} R_l(r) e^{-jl\phi}$$

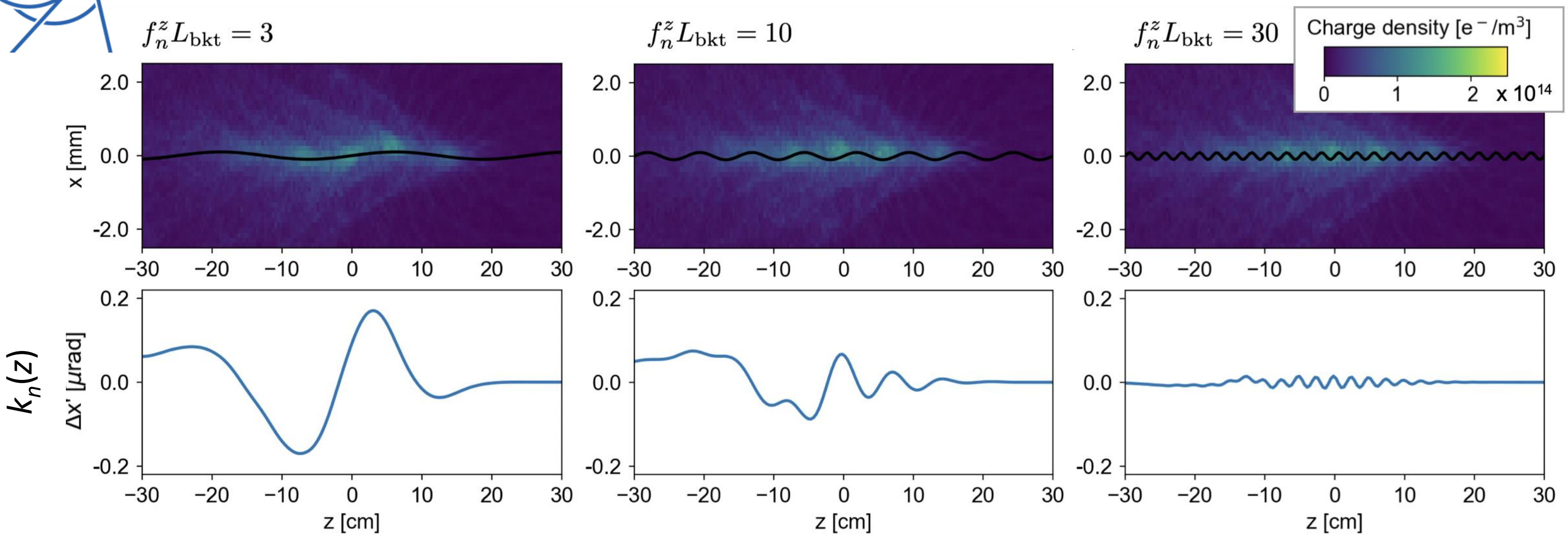
Unknowns:

Complex tune shift

Known for dipolar oscillation

Longitudinal distribution of the oscillation

Test and response functions



The strength of the resulting kick **reduces for larger excitation frequency**:

- Effect of the **inertia of the electrons**, preventing them from responding to very fast oscillations within the bunch
- The number of relevant terms is small