Flavor Physics and CP Violation*

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Abstract

This is a written version of a series of lectures aimed at graduate students and postdoctoral fellows in particle theory/string theory/particle experiment familiar with the basics of the Standard Model. We explain the many reasons for the interest in flavor physics. We describe flavor physics and the related CP violation within the Standard Model, and explain how the B-factories proved that the Kobayashi-Maskawa mechanism dominates the CP violation that is observed in meson decays. We explain the implications of flavor physics for new physics. We emphasize the "new physics flavor puzzle". As an explicit example, we explain how the recent measurements of $D^0 - \overline{D}^0$ mixing constrain the supersymmetric flavor structure. We explain how the ATLAS and CMS experiments can solve the new physics flavor puzzle and perhaps shed light on the standard model flavor puzzle. Finally, we describe various interpretations of the neutrino flavor data and their impact on flavor models.

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I. WHAT IS FLAVOR?

The term "**flavors**" is used, in the jargon of particle physics, to describe several copies of the same gauge representation, namely several fields that are assigned the same quantum charges. Within the Standard Model, when thinking of its unbroken $SU(3)_{\rm C}\times U(1)_{\rm EM}$ gauge group, there are four different types of particles, each coming in three flavors:

- Up-type quarks in the $(3)_{+2/3}$ representation: u, c, t;
- Down-type quarks in the $(3)_{-1/3}$ representation: d, s, b;
- Charged leptons in the $(1)_{-1}$ representation: e, μ, τ;
- Neutrinos in the $(1)_0$ representation: ν_1, ν_2, ν_3.

The term "**flavor physics**" refers to interactions that distinguish between flavors. By definition, gauge interactions, namely interactions that are related to unbroken symmetries and mediated therefore by massless gauge bosons, do not distinguish among the flavors and do not constitute part of flavor physics. Within the Standard Model, flavor-physics refers to the weak and Yukawa interactions.

The term "**flavor parameters**" refers to parameters that carry flavor indices. Within the Standard Model, these are the nine masses of the charged fermions and the four "mixing parameters" (three angles and one phase) that describe the interactions of the charged weak-force carriers ($W^\pm$) with quark-antiquark pairs. If one augments the Standard Model with Majorana mass terms for the neutrinos, one should add to the list three neutrino masses and six mixing parameters (three angles and three phases) for the $W^\pm$ interactions for lepton-antilepton pairs.

The term "**flavor universal**" refers to interactions with couplings (or to flavor parameters) that are proportional to the unit matrix in flavor space. Thus, the strong and electromagnetic interactions are flavor-universal.[1] An alternative term for "flavor-universal" is "**flavor-blind**".

The term "**flavor diagonal**" refers to interactions with couplings (or to flavor parameters) that are diagonal, but not necessarily universal, in the flavor space. Within the

[1] In the interaction basis, the weak interactions are also flavor-universal, and one can identify the source of all flavor physics in the Yukawa interactions among the gauge-interaction eigenstates.

Standard Model, the Yukawa interactions of the Higgs particle are flavor diagonal in the mass basis.

The term "**flavor changing**" refers to processes where the initial and final flavor-numbers (that is, the number of particles of a certain flavor minus the number of anti-particles of the same flavor) are different. In "flavor changing charged current" processes, both up-type and down-type flavors, and/or both charged lepton and neutrino flavors are involved. Examples are (i) muon decay via $\mu \to e\bar{\nu}_i\nu_j$, and (ii) $K^- \to \mu^-\bar{\nu}_j$ (which corresponds, at the quark level, to $s\bar{u} \to \mu^-\bar{\nu}_j$). Within the Standard Model, these processes are mediated by the W-bosons and occur at tree level. In "**flavor changing neutral current**" (FCNC) processes, either up-type or down-type flavors but not both, and/or either charged lepton or neutrino flavors but not both, are involved. Example are (i) muon decay via $\mu \to e\gamma$ and (ii) $K_L \to \mu^+\mu^-$ (which corresponds, at the quark level, to $s\bar{d} \to \mu^+\mu^-$). Within the Standard Model, these processes do not occur at tree level, and are often highly suppressed.

Another useful term is "**flavor violation**". We will explain it later in these lectures.

II. WHY IS FLAVOR PHYSICS INTERESTING?

- Flavor physics can discover new physics or probe it before it is directly observed in experiments. Here are some examples from the past:
  - The smallness of $\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)}$ led to predicting a fourth (the charm) quark;
  - The size of Δm_K led to a successful prediction of the charm mass;
  - The size of Δm_B led to a successful prediction of the top mass;
  - The measurement of ε_K led to predicting the third generation.
- CP violation is closely related to flavor physics. Within the Standard Model, there is a single CP violating parameter, the Kobayashi-Maskawa phase $\delta_{\rm KM}$ [1]. Baryogenesis tells us, however, that there must exist new sources of CP violation. Measurements of CP violation in flavor changing processes might provide evidence for such sources.
- The fine-tuning problem of the Higgs mass, and the puzzle of the dark matter imply that there exists new physics at, or below, the TeV scale. If such new physics had a generic flavor structure, it would contribute to flavor changing neutral current (FCNC)

processes orders of magnitude above the observed rates. The question of why this does not happen constitutes the *new physics flavor puzzle*.

- Most of the charged fermion flavor parameters are small and hierarchical. The Standard Model does not provide any explanation of these features. This is the *Standard Model flavor puzzle*. The puzzle became even deeper after neutrino masses and mixings were measured because, so far, neither smallness nor hierarchy in these parameters have been established.

III. FLAVOR IN THE STANDARD MODEL

A model of elementary particles and their interactions is defined by the following ingredients: (i) The symmetries of the Lagrangian and the pattern of spontaneous symmetry breaking; (ii) The representations of fermions and scalars. The Standard Model (SM) is defined as follows:

(i) The gauge symmetry is

$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}. \tag{1}$$

It is spontaneously broken by the VEV of a single Higgs scalar, $\phi(1,2)_{1/2}$ ($\langle\phi^0\rangle = v/\sqrt{2}$):

$$G_{\rm SM} \to SU(3)_{\rm C} \times U(1)_{\rm EM}. \tag{2}$$

(ii) There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q_{Li}(3,2)_{+1/6}, \quad U_{Ri}(3,1)_{+2/3}, \quad D_{Ri}(3,1)_{-1/3}, \quad L_{Li}(1,2)_{-1/2}, \quad E_{Ri}(1,1)_{-1}. \tag{3}$$

A. The interactions basis

The Standard Model Lagrangian, $\mathcal{L}_{\rm SM}$, is the most general renormalizable Lagrangian that is consistent with the gauge symmetry (1), the particle content (3) and the pattern of spontaneous symmetry breaking (2). It can be divided to three parts:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}. \tag{4}$$

As concerns the kinetic terms, to maintain gauge invariance, one has to replace the derivative with a covariant derivative:

$$D^\mu = \partial^\mu + ig_s G^\mu_a L_a + igW^\mu_b T_b + ig' B^\mu Y. \tag{5}$$

Here G^μ_a are the eight gluon fields, W^μ_b the three weak interaction bosons and B^μ the single hypercharge boson. The L_a's are $SU(3)_{\rm C}$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_{\rm L}$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the Y's are the $U(1)_{\rm Y}$ charges. For example, for the quark doublets Q_L, we have

$$\mathcal{L}_{\rm kinetic}(Q_L) = i\overline{Q_{Li}}\gamma_\mu\left(\partial^\mu + \frac{i}{2}g_s G^\mu_a\lambda_a + \frac{i}{2}gW^\mu_b\tau_b + \frac{i}{6}g'B^\mu\right)\delta_{ij}Q_{Lj}, \tag{6}$$

while for the lepton doublets L^I_L, we have

$$\mathcal{L}_{\rm kinetic}(L_L) = i\overline{L_{Li}}\gamma_\mu\left(\partial^\mu + \frac{i}{2}gW^\mu_b\tau_b - \frac{i}{2}g'B^\mu\right)\delta_{ij}L_{Lj}. \tag{7}$$

The unit matrix in flavor space, δ_{ij}, signifies that these parts of the interaction Lagrangian are flavor-universal. In addition, they conserve CP.

The Higgs potential, which describes the scalar self interactions, is given by:

$$\mathcal{L}_{\rm Higgs} = \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2. \tag{8}$$

For the Standard Model scalar sector, where there is a single doublet, this part of the Lagrangian is also CP conserving.

The quark Yukawa interactions are given by

$$-\mathcal{L}^q_{\rm Y} = Y^d_{ij}\overline{Q_{Li}}\phi D_{Rj} + Y^u_{ij}\overline{Q_{Li}}\tilde{\phi}U_{Rj} + {\rm h.c.}, \tag{9}$$

(where $\tilde{\phi} = i\tau_2\phi^\dagger$) while the lepton Yukawa interactions are given by

$$-\mathcal{L}^\ell_{\rm Y} = Y^e_{ij}\overline{L_{Li}}\phi E_{Rj} + {\rm h.c.}. \tag{10}$$

This part of the Lagrangian is, in general, flavor-dependent (that is, $Y^f \not\propto \mathbf{1}$) and CP violating.

B. Global symmetries

In the absence of the Yukawa matrices Y^d, Y^u and Y^e, the SM has a large $U(3)^5$ global symmetry:

$$G_{\rm global}(Y^{u,d,e}=0) = SU(3)^3_q \times SU(3)^2_\ell \times U(1)^5, \tag{11}$$

where

$$\begin{aligned} SU(3)_q^3 &= SU(3)_Q \times SU(3)_U \times SU(3)_D, \\ SU(3)_\ell^2 &= SU(3)_L \times SU(3)_E, \\ U(1)^5 &= U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\rm PQ} \times U(1)_E. \end{aligned} \tag{12}$$

Out of the five $U(1)$ charges, three can be identified with baryon number (B), lepton number (L) and hypercharge (Y), which are respected by the Yukawa interactions. The two remaining $U(1)$ groups can be identified with the PQ symmetry whereby the Higgs and D_R, E_R fields have opposite charges, and with a global rotation of E_R only.

The point that is important for our purposes is that $\mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Higgs}$ respect the non-Abelian flavor symmetry $S(3)_q^3 \times SU(3)_\ell^2$, under which

$$Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R, \quad L_L \to V_L L_L, \quad E_R \to V_E E_R, \tag{13}$$

where the V_i are unitary matrices. The Yukawa interactions (9) and (10) break the global symmetry,

$$G_{\rm global}(Y^{u,d,e} \neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau. \tag{14}$$

(Of course, the gauged $U(1)_Y$ also remains a good symmetry.) Thus, the transformations of Eq. (13) are not a symmetry of $\mathcal{L}_{\rm SM}$. Instead, they correspond to a change of the interaction basis. These observations also offer an alternative way of defining flavor physics: it refers to interactions that break the $SU(3)^5$ symmetry (13). Thus, the term "**flavor violation**" is often used to describe processes or parameters that break the symmetry.

One can think of the quark Yukawa couplings as spurions that break the global $SU(3)_q^3$ symmetry (but are neutral under $U(1)_B$),

$$Y^u \sim (3, \bar{3}, 1)_{SU(3)_q^3}, \quad Y^d \sim (3, 1, \bar{3})_{SU(3)_q^3}, \tag{15}$$

and of the lepton Yukawa couplings as spurions that break the global $SU(3)_\ell^2$ symmetry (but are neutral under $U(1)_e \times U(1)_\mu \times U(1)_\tau$),

$$Y^e \sim (3, \bar{3})_{SU(3)_\ell^2}. \tag{16}$$

The spurion formalism is convenient for several purposes: parameter counting (see below), identification of flavor suppression factors (see Section V), and the idea of minimal flavor violation (see Section VII).

C. Counting parameters

How many independent parameters are there in $\mathcal{L}_{\rm Y}^q$? The two Yukawa matrices, Y^u and Y^d, are 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. The pattern of $G_{\rm global}$ breaking means that there is freedom to remove 9 real and 17 imaginary parameters (the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$). For example, we can use the unitay transformations $Q_L \to V_Q Q_L$, $U_R \to V_U U_R$ and $D_R \to V_D D_R$, to lead to the following interaction basis:

$$Y^d = \lambda_d, \quad Y^u = V^\dagger \lambda_u, \tag{17}$$

where $\lambda_{d,u}$ are diagonal,

$$\lambda_d = {\rm diag}(y_d, y_s, y_b), \quad \lambda_u = {\rm diag}(y_u, y_c, y_t), \tag{18}$$

while V is a unitary matrix that depends on three real angles and one complex phase. We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. In the mass basis, we will identify the nine real parameters as six quark masses and three mixing angles, while the single phase is $\delta_{\rm KM}$.

How many independent parameters are there in $\mathcal{L}_{\rm Y}^\ell$? The Yukawa matrix Y^e is 3×3 and complex. Consequently, there are 9 real and 9 imaginary parameters in this matrix. There is, however, freedom to remove 6 real and 9 imaginary parameters (the number of parameters in two 3×3 unitary matrices minus the phases related to $U(1)^3$). For example, we can use the unitay transformations $L_L \to V_L L_L$ and $E_R \to V_E E_R$, to lead to the following interaction basis:

$$Y^e = \lambda_e = {\rm diag}(y_e, y_\mu, y_\tau). \tag{19}$$

We conclude that there are 3 real lepton flavor parameters. In the mass basis, we will identify these parameters as the three charged lepton masses. We must, however, modify the model when we take into account the evidence for neutrino masses.

D. The mass basis

Upon the replacement $\mathcal{R}e(\phi^0) \to \frac{v+H^0}{\sqrt{2}}$, the Yukawa interactions (9) give rise to the mass matricess

$$M_q = \frac{v}{\sqrt{2}} Y^q. \tag{20}$$

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{qL} and V_{qR} such that

$$V_{qL} M_q V_{qR}^\dagger = M_q^{\rm diag} \equiv \frac{v}{\sqrt{2}} \lambda_q. \tag{21}$$

The four matrices V_{dL}, V_{dR}, V_{uL} and V_{uR} are then the ones required to transform to the mass basis. For example, if we start from the special basis (17), we have $V_{dL} = V_{dR} = V_{uR} = \mathbf{1}$ and $V_{uL} = V$. The combination $V_{uL} V_{dL}^\dagger$ is independent of the interaction basis from which we start this procedure.

We denote the left-handed quark mass eigenstates as U_L and D_L. The charged current interactions for quarks [that is the interactions of the charged $SU(2)_{\rm L}$ gauge bosons $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$], which in the interaction basis are described by (6), have a complicated form in the mass basis:

$$-\mathcal{L}_{W^\pm}^q = \frac{g}{\sqrt{2}} \overline{U_{Li}} \gamma^\mu V_{ij} D_{Lj} W_\mu^+ + \text{h.c.}. \tag{22}$$

where V is the 3×3 unitary matrix ($V V^\dagger = V^\dagger V = \mathbf{1}$) that appeared in Eq. (17). For a general interaction basis,

$$V = V_{uL} V_{dL}^\dagger. \tag{23}$$

V is the Cabibbo-Kobayashi-Maskawa (CKM) *mixing matrix* for quarks [1, 2]. As a result of the fact that V is not diagonal, the $W^\pm$ gauge bosons couple to quark mass eigenstates of different generations. Within the Standard Model, this is the only source of *flavor changing* quark interactions.

Exercise 1: *Prove that, in the absence of neutrino masses, there is no mixing in the lepton sector.*

Exercise 2: *Prove that there is no mixing in the Z couplings. (In the physics jargon, there are no flavor changing neutral currents at tree level.)*

The detailed structure of the CKM matrix, its parametrization, and the constraints on its elements are described in Appendix A.

IV. TESTING CKM

Measurements of rates, mixing, and CP asymmetries in B decays in the two B factories, BaBar abd Belle, and in the two Tevatron detectors, CDF and D0, signified a new era in our understanding of CP violation. The progress is both qualitative and quantitative. Various basic questions concerning CP and flavor violation have received, for the first time, answers based on experimental information. These questions include, for example,

- Is the Kobayashi-Maskawa mechanism at work (namely, is $\delta_{\rm KM} \neq 0$)?
- Does the KM phase dominate the observed CP violation?

As a first step, one may assume the SM and test the overall consistency of the various measurements. However, the richness of data from the B factories allow us to go a step further and answer these questions model independently, namely allowing new physics to contribute to the relevant processes. We here explain the way in which this analysis proceeds.

A. $S_{\psi K_S}$

The CP asymmetry in $B \to \psi K_S$ decays plays a major role in testing the KM mechanism. Before we explain the test itself, we should understand why the theoretical interpretation of the asymmetry is exceptionally clean, and what are the theoretical parameters on which it depends, within and beyond the Standard Model.

The CP asymmetry in neutral meson decays into final CP eigenstates f_{CP} is defined as follows:

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\,] - d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}\,]}{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\,] + d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}\,]}\,. \tag{24}$$

A detailed evaluation of this asymmetry is given in Appendix B. It leads to the following form:

$$\begin{aligned} \mathcal{A}_{f_{CP}}(t) &= S_{f_{CP}}\,\sin(\Delta m t) - C_{f_{CP}}\,\cos(\Delta m t), \\ S_{f_{CP}} &\equiv \frac{2\,\mathcal{I}m(\lambda_{f_{CP}})}{1+|\lambda_{f_{CP}}|^2}, \quad C_{f_{CP}} \equiv \frac{1-|\lambda_{f_{CP}}|^2}{1+|\lambda_{f_{CP}}|^2}\,, \end{aligned} \tag{25}$$

where

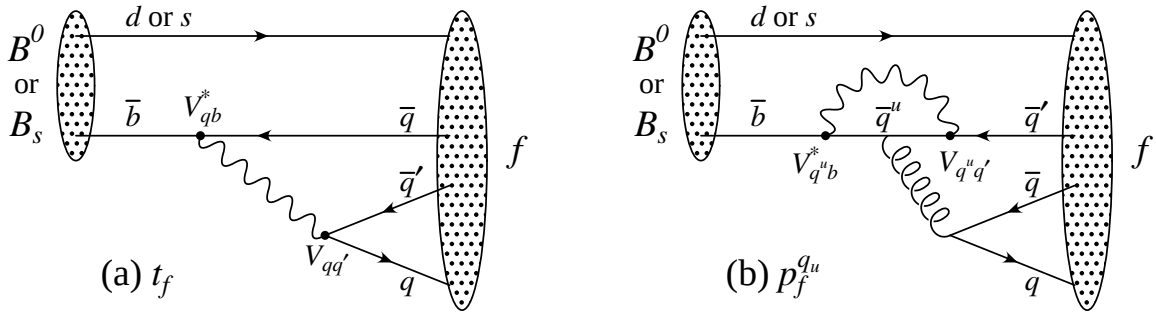
$$\lambda_{f_{CP}} = e^{-i\phi_B}(\overline{A}_{f_{CP}}/A_{f_{CP}})\,. \tag{26}$$

Here ϕ_B refers to the phase of M_{12} [see Eq. (B23)]. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_B} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*) \; . \tag{27}$$

The decay amplitudes A_f and $\overline{A}_f$ are defined in Eq. (B1).

FIG. 1: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s \to f$ via a $\bar{b} \to \bar{q}q\bar{q}'$ quark-level process.



The $B^0 \to J/\psi K^0$ decay [3, 4] proceeds via the quark transition $\bar{b} \to \bar{c}c\bar{s}$. There are contributions from both tree (t) and penguin (p^{q_u}, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 1) which carry different weak phases:

$$A_f = (V_{cb}^* V_{cs})\, t_f + \sum_{q_u = u,c,t} \left(V_{q_u b}^* V_{q_u s}\right) p_f^{q_u} \; . \tag{28}$$

(The distinction between tree and penguin contributions is a heuristic one, the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, ref. [5].) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations:

$$A_{\psi K} = (V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u, \tag{29}$$

where $T_{\psi K} = t_{\psi K} + p_{\psi K}^c - p_{\psi K}^t$ and $P_{\psi K}^u = p_{\psi K}^u - p_{\psi K}^t$. A subtlety arises in this decay that is related to the fact that $B^0 \to J/\psi K^0$ and $\overline{B}^0 \to J/\psi \overline{K}^0$. A common final state, e.g. $J/\psi K_S$, can be reached via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor corresponding to neutral K mixing, $e^{-i\phi_K} = (V_{cd}^* V_{cs})/(V_{cd} V_{cs}^*)$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb} V_{cs}^*)\, T_{\psi K} + (V_{ub} V_{us}^*)\, P_{\psi K}^u}{(V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}. \tag{30}$$

The crucial point is that, for $B \to J/\psi K_S$ and other $\bar{b} \to \bar{c}c\bar{s}$ processes, we can neglect the P^u contribution to $A_{\psi K}$, in the SM, to an approximation that is better than one percent:

$$|P^u_{\psi K}/T_{\psi K}| \times |V_{ub}/V_{cb}| \times |V_{us}/V_{cs}| \sim (\text{loop factor}) \times 0.1 \times 0.23 \lesssim 0.005. \tag{31}$$

Thus, to an accuracy of better than one percent,

$$\lambda_{\psi K_S} = \left(\frac{V^*_{tb}V_{td}}{V_{tb}V^*_{td}}\right)\left(\frac{V_{cb}V^*_{cd}}{V^*_{cb}V_{cd}}\right) = -e^{-2i\beta}, \tag{32}$$

where β is defined in Eq. (A9), and consequently

$$S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0\ . \tag{33}$$

(Below the percent level, several effects modify this equation [6–9].)

Exercise 3: *Show that, if the $B \to \pi\pi$ decays were dominated by tree diagrams, then $S_{\pi\pi} = \sin 2\alpha$.*

Exercise 4: *Estimate the accuracy of the predictions $S_{\phi K_S} = \sin 2\beta$ and $C_{\phi K_S} = 0$.*

When we consider extensions of the SM, we still do not expect any significant new contribution to the tree level decay, $b \to c\bar{c}s$, beyond the SM W-mediated diagram. Thus, the expression $\bar{A}_{\psi K_S}/A_{\psi K_S} = (V_{cb}V^*_{cd})/(V^*_{cb}V_{cd})$ remains valid, though the approximation of neglecting sub-dominant phases can be somewhat less accurate than Eq. (31). On the other hand, M_{12}, the $B^0 - \overline{B}^0$ mixing amplitude, can in principle get large and even dominant contributions from new physics. We can parametrize the modification to the SM in terms of two parameters, r_d^2 signifying the change in magnitude, and $2\theta_d$ signifying the change in phase:

$$M_{12} = r_d^2\ e^{2i\theta_d}\ M_{12}^{\rm SM}(\rho,\eta). \tag{34}$$

This leads to the following generalization of Eq. (33):

$$S_{\psi K_S} = \sin(2\beta + 2\theta_d), \quad C_{\psi K_S} = 0\ . \tag{35}$$

The experimental measurements give the following ranges [10]:

$$S_{\psi K_S} = +0.671 \pm 0.024, \quad C_{\psi K_S} = +0.005 \pm 0.019\ . \tag{36}$$

B. Self-consistency of the CKM assumption

The three generation standard model has room for CP violation, through the KM phase in the quark mixing matrix. Yet, one would like to make sure that indeed CP is violated by the SM interactions, namely that $\sin\delta_{\rm KM} \neq 0$. If we establish that this is the case, we would further like to know whether the SM contributions to CP violating observables are dominant. More quantitatively, we would like to put an upper bound on the ratio between the new physics and the SM contriubtions.

As a first step, one can assume that flavor changing processes are fully described by the SM, and check the consistency of the various measurements with this assumption. There are four relevant mixing parameters, which can be taken to be the Wolfenstein parameters λ, A, ρ and η defined in Eq. (A4). The values of λ and A are known rather accurately [11] from, respectively, $K \to \pi\ell\nu$ and $b \to c\ell\nu$ decays:

$$\lambda = 0.2257 \pm 0.0010, \quad A = 0.814 \pm 0.022. \tag{37}$$

Then, one can express all the relevant observables as a function of the two remaining parameters, ρ and η, and check whether there is a range in the $\rho - \eta$ plane that is consistent with all measurements. The list of observables includes the following:

- The rates of inclusive and exclusive charmless semileptonic B decays depend on $|V_{ub}|^2 \propto \rho^2 + \eta^2$;
- The CP asymmetry in $B \to \psi K_S$, $S_{\psi K_S} = \sin 2\beta = \frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2}$;
- The rates of various $B \to DK$ decays depend on the phase γ, where $e^{i\gamma} = \frac{\rho+i\eta}{\rho^2+\eta^2}$;
- The rates of various $B \to \pi\pi, \rho\pi, \rho\rho$ decays depend on the phase $\alpha = \pi - \beta - \gamma$;
- The ratio between the mass splittings in the neutral B and B_s systems is sensitive to $|V_{td}/V_{ts}|^2 = \lambda^2[(1-\rho)^2 + \eta^2]$;
- The CP violation in $K \to \pi\pi$ decays, ϵ_K, depends in a complicated way on ρ and η.

The resulting constraints are shown in Fig. 2.

The consistency of the various constraints is impressive. In particular, the following ranges for ρ and η can account for all the measurements [11]:

$$\rho = +0.135^{+0.031}_{-0.016}, \quad \eta = +0.349 \pm 0.017. \tag{38}$$

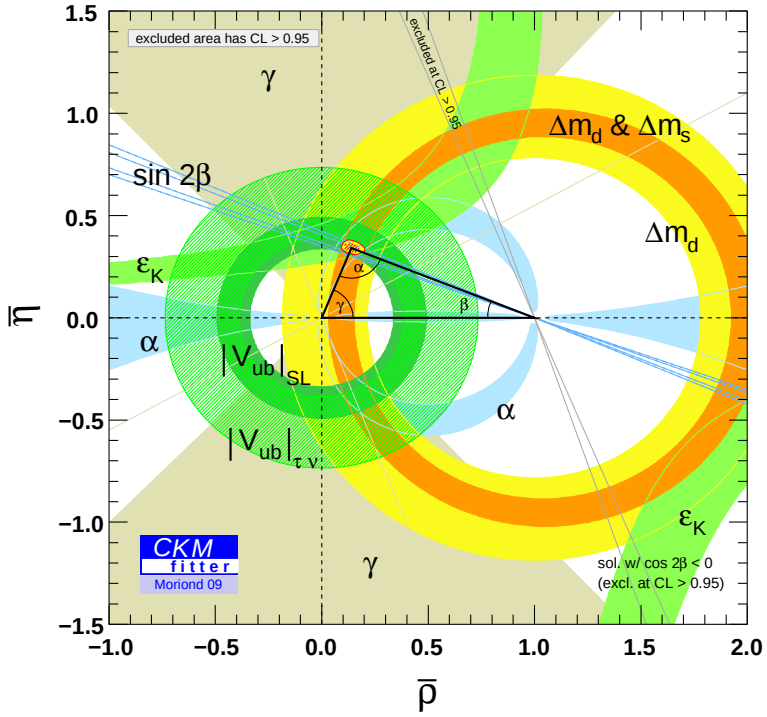


FIG. 2: Allowed region in the ρ, η plane. Superimposed are the individual constraints from charmless semileptonic B decays ($|V_{ub}/V_{cb}|$), mass differences in the B^0 (Δm_d) and B_s (Δm_s) neutral meson systems, and CP violation in $K \to \pi\pi$ (ε_K), $B \to \psi K$ ($\sin 2\beta$), $B \to \pi\pi, \rho\pi, \rho\rho$ (α), and $B \to DK$ (γ). Taken from [12].

One can make then the following statement [13]:

Very likely, CP violation in flavor changing processes is dominated by the Kobayashi-Maskawa phase.

In the next two subsections, we explain how we can remove the phrase "very likely" from this statement, and how we can quantify the KM-dominance.

C. Is the KM mechanism at work?

In proving that the KM mechanism is at work, we assume that charged-current tree-level processes are dominated by the W-mediated SM diagrams (see, for example, [14]). This is a very plausible assumption. I am not aware of any viable well-motivated model where this assumption is not valid. Thus we can use all tree level processes and fit them to ρ and η, as

we did before. The list of such processes includes the following:

1. Charmless semileptonic B-decays, $b \to u\ell\nu$, measure R_u [see Eq. (A8)].

2. $B \to DK$ decays, which go through the quark transitions $b \to c\bar{u}s$ and $b \to u\bar{c}s$, measure the angle γ [see Eq. (A9)].

3. $B \to \rho\rho$ decays (and, similarly, $B \to \pi\pi$ and $B \to \rho\pi$ decays) go through the quark transition $b \to u\bar{u}d$. With an isospin analysis, one can determine the relative phase between the tree decay amplitude and the mixing amplitude. By incorporating the measurement of $S_{\psi K_S}$, one can subtract the phase from the mixing amplitude, finally providing a measurement of the angle γ [see Eq. (A9)].

In addition, we can use loop processes, but then we must allow for new physics contributions, in addition to the (ρ, η)-dependent SM contributions. Of course, if each such measurement adds a separate mode-dependent parameter, then we do not gain anything by using this information. However, there is a number of observables where the only relevant loop process is $B^0 - \overline{B}^0$ mixing. The list includes $S_{\psi K_S}$, Δm_B and the CP asymmetry in semileptonic B decays:

$$\begin{aligned} S_{\psi K_S} &= \sin(2\beta + 2\theta_d), \\ \Delta m_B &= r_d^2 (\Delta m_B)^{\rm SM}, \\ \mathcal{A}_{\rm SL} &= -\mathcal{R}e\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\sin 2\theta_d}{r_d^2} + \mathcal{I}m\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\cos 2\theta_d}{r_d^2}. \end{aligned} \tag{39}$$

As explained above, such process involve two new parameters [see Eq. (34)]. Since there are three relevant observables, we can further tighten the constraints in the (ρ, η)-plane. Similarly, one can use measurements related to $B_s - \overline{B}_s$ mixing. One gains three new observables at the cost of two new parameters (see, for example, [15]).

The results of such fit, projected on the $\rho - \eta$ plane, can be seen in Fig. 3. It gives [12]

$$\eta = 0.44^{+0.05}_{-0.23} \ \ (3\sigma). \tag{40}$$

[A similar analysis in Ref. [16] obtains the 3σ range $(0.31 - 0.46)$.] It is clear that $\eta \neq 0$ is well established:

The Kobayashi-Maskawa mechanism of CP violation is at work.

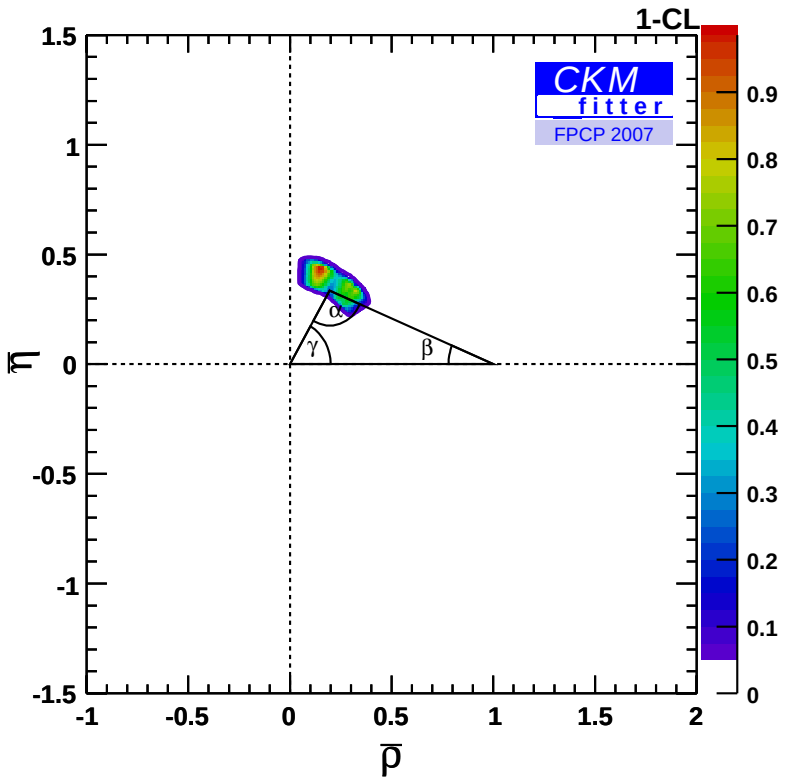


FIG. 3: The allowed region in the $\rho - \eta$ plane, assuming that tree diagrams are dominated by the Standard Model [12].

Another way to establish that CP is violated by the CKM matrix is to find, within the same procedure, the allowed range for $\sin 2\beta$ [16]:

$$\sin 2\beta^{\rm tree} = 0.76 \pm 0.04. \tag{41}$$

(Ref. [12] finds $0.82^{+0.02}_{-0.13}$.) Thus, $\beta \neq 0$ is well established.

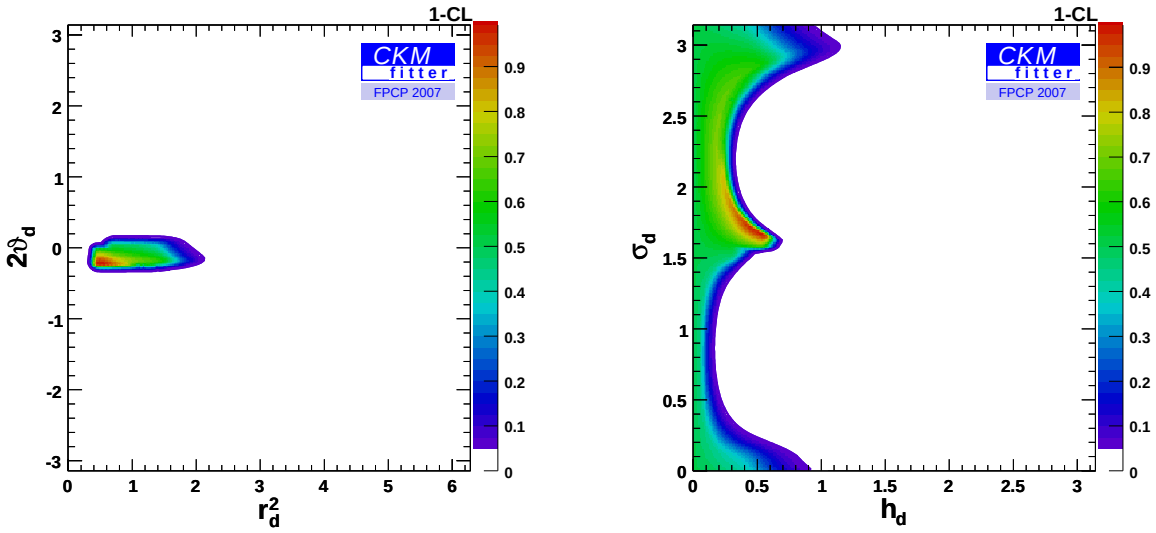
The consistency of the experimental results (36) with the SM predictions (33,41) means that the KM mechanism of CP violation dominates the observed CP violation. In the next subsection, we make this statement more quantitative.

D. How much can new physics contribute to $B^0 - \overline{B}^0$ mixing?

All that we need to do in order to establish whether the SM dominates the observed CP violation, and to put an upper bound on the new physics contribution to $B^0 - \overline{B}^0$ mixing, is to project the results of the fit performed in the previous subsection on the $r_d^2 - 2\theta_d$

plane. If we find that $\theta_d \ll \beta$, then the SM dominance in the observed CP violation will be established. The constraints are shown in Fig. 4(a). Indeed, $\theta_d \ll \beta$.

FIG. 4: Constraints in the (a) $r_d^2 - 2\theta_d$ plane, and (b) $h_d - \sigma_d$ plane, assuming that NP contributions to tree level processes are negligible [12].



An alternative way to present the data is to use the h_d, σ_d parametrization,

$$r_d^2 e^{2i\theta_d} = 1 + h_d e^{2i\sigma_d}. \tag{42}$$

While the r_d, θ_d parameters give the relation between the full mixing amplide and the SM one, and are convenient to apply to the measurements, the h_d, σ_d parameters give the relation between the new physics and SM contributions, and are more convenient in testing theoretical models:

$$h_d e^{i\sigma_d} = \frac{M_{12}^{\rm NP}}{M_{12}^{\rm SM}}. \tag{43}$$

The constraints in the $h_d - \sigma_d$ plane are shown in Fig. 4(b). We can make the following two statements:

1. A new physics contribution to $B^0 - \overline{B}^0$ mixing amplitude that carries a phase that is significantly different from the KM phase is constrained to lie below the 20-30% level.

2. A new physics contribution to the $B^0 - \overline{B}^0$ mixing amplitude which is aligned with the KM phase is constrained to be at most comparable to the CKM contribution.

One can reformulate these statements as follows:

1. The KM mechanism dominates CP violation in $B^0 - \overline{B}^0$ mixing.

2. The CKM mechanism is a major player in $B^0 - \overline{B}^0$ mixing.

V. THE NEW PHYSICS FLAVOR PUZZLE

It is clear that the Standard Model is not a complete theory of Nature:

1. It does not include gravity, and therefore it cannot be valid at energy scales above $m_{\rm Planck} \sim 10^{19}$ GeV:

2. It does not allow for neutrino masses, and therefore it cannot be valid at energy scales above $m_{\rm seesaw} \sim 10^{15}$ GeV;

3. The fine-tuning problem of the Higgs mass and the puzzle of the dark matter suggest that the scale where the SM is replaced with a more fundamental theory is actually much lower, $\Lambda_{\rm NP} \lesssim 1$ TeV.

Given that the SM is only an effective low energy theory, non-renormalizable terms must be added to $\mathcal{L}_{\rm SM}$ of Eq. (4). These are terms of dimension higher than four in the fields which, therefore, have couplings that are inversely proportional to the scale of new physics $\Lambda_{\rm NP}$. For example, the lowest dimension non-renormalizable terms are dimension five:

$$-\mathcal{L}^{\rm dim-5}_{\rm Yukawa} = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} L^I_{Li} L^I_{Lj} \phi\phi + {\rm h.c.}. \tag{44}$$

These are the seesaw terms, leading to neutrino masses. We will return to the topic of neutrino masses in section VIII.

Exercise 5: *How does the global symmetry breaking pattern (14) change when (44) is taken into account?*

Exercise 6: *What is the number of physical lepton flavor parameters in this case? Identify these parameters in the mass basis.*

As concerns quark flavor physics, consider, for example, the following dimension-six, four-fermion, flavor changing operators:

$$\mathcal{L}_{\Delta F=2} = \frac{z_{sd}}{\Lambda^2_{\rm NP}} (\overline{d_L} \gamma_\mu s_L)^2 + \frac{z_{cu}}{\Lambda^2_{\rm NP}} (\overline{c_L} \gamma_\mu u_L)^2 + \frac{z_{bd}}{\Lambda^2_{\rm NP}} (\overline{d_L} \gamma_\mu b_L)^2 + \frac{z_{bs}}{\Lambda^2_{\rm NP}} (\overline{s_L} \gamma_\mu b_L)^2. \tag{45}$$

Each of these terms contributes to the mass splitting between the corresponding two neutral mesons. For example, the term $\mathcal{L}_{\Delta B=2} \propto (\overline{d_L}\gamma_\mu b_L)^2$ contributes to Δm_B, the mass difference between the two neutral B-mesons. We use $M_{12}^B = \frac{1}{2m_B}\langle B^0|\mathcal{L}_{\Delta F=2}|\overline{B}^0\rangle$ and

$$\langle B^0|(\overline{d_{La}}\gamma^\mu b_{La})(\overline{d_{Lb}}\gamma_\mu b_{Lb})|\overline{B}^0\rangle = -\frac{1}{3}m_B^2 f_B^2 B_B. \tag{46}$$

Analogous expressions hold for the other neutral mesons.[2] This leads to $\Delta m_B/m_B = 2|M_{12}^B|/m_B \sim (z_{bd}/3)(f_B/\Lambda_{\rm NP})^2$. Experiments give, for CP conserving observables (the experimental evidence for Δm_D is at the 3σ level):

$$\begin{aligned}
\Delta m_K/m_K &\sim 7.0\times 10^{-15},\\
\Delta m_D/m_D &\sim 8.6\times 10^{-15},\\
\Delta m_B/m_B &\sim 6.3\times 10^{-14},\\
\Delta m_{B_s}/m_{B_s} &\sim 2.1\times 10^{-12},
\end{aligned} \tag{47}$$

and for CP violating ones

$$\begin{aligned}
\epsilon_K &\sim 2.3\times 10^{-3},\\
A_\Gamma &\lesssim 0.2,\\
S_{\psi K_S} &= 0.67\pm 0.02,\\
S_{\psi\phi} &\lesssim 1.
\end{aligned} \tag{48}$$

These measurements give then the following constraints:

$$\Lambda_{\rm NP} \gtrsim \begin{cases}
\sqrt{z_{sd}}\ 1\times 10^3\ TeV & \Delta m_K\\
\sqrt{z_{cu}}\ 1\times 10^3\ TeV & \Delta m_D\\
\sqrt{z_{bd}}\ 4\times 10^2\ TeV & \Delta m_B\\
\sqrt{z_{bs}}\ 7\times 10^1\ TeV & \Delta m_{B_s}
\end{cases} \tag{49}$$

and, for maximal phases,

$$\Lambda_{\rm NP} \gtrsim \begin{cases}
\sqrt{z_{sd}}\ 2\times 10^4\ TeV & \epsilon_K\\
\sqrt{z_{cu}}\ 3\times 10^3\ TeV & A_\Gamma\\
\sqrt{z_{bd}}\ 8\times 10^2\ TeV & S_{\psi K}\\
\sqrt{z_{bs}}\ 7\times 10^1\ TeV & S_{\psi\phi}
\end{cases} \tag{50}$$

[2] The PDG [11] quotes the following values, extracted from leptonic charged meson decays: $f_K \approx 0.16\ GeV$, $f_D \approx 0.23\ GeV$, $f_B \approx 0.18\ GeV$. We further use $f_{B_s} \approx 0.20\ GeV$.

If the new physics has a generic flavor structure, that is $z_{ij} = \mathcal{O}(1)$, then its scale must be above $10^3 - 10^4$ TeV (or, if the leading contributions involve electroweak loops, above $10^2 - 10^3$ TeV).[3]

If indeed $\Lambda_{\rm NP} \gg TeV$, it means that we have misinterpreted the hints from the fine-tuning problem and the dark matter puzzle. There is, however, another way to look at these constraints:

$$\begin{aligned} z_{sd} &\lesssim 9\times 10^{-7}\ (\Lambda_{\rm NP}/TeV)^2, \\ z_{cu} &\lesssim 6\times 10^{-7}\ (\Lambda_{\rm NP}/TeV)^2, \\ z_{bd} &\lesssim 5\times 10^{-6}\ (\Lambda_{\rm NP}/TeV)^2, \\ z_{bs} &\lesssim 2\times 10^{-4}\ (\Lambda_{\rm NP}/TeV)^2, \end{aligned} \tag{51}$$

$$\begin{aligned} z^I_{sd} &\lesssim 4\times 10^{-9}\ (\Lambda_{\rm NP}/TeV)^2, \\ z^I_{cu} &\lesssim 1\times 10^{-7}\ (\Lambda_{\rm NP}/TeV)^2, \\ z^I_{bd} &\lesssim 1\times 10^{-6}\ (\Lambda_{\rm NP}/TeV)^2, \\ z^I_{bs} &\lesssim 2\times 10^{-4}\ (\Lambda_{\rm NP}/TeV)^2. \end{aligned} \tag{52}$$

It could be that the scale of new physics is of order TeV, but its flavor structure is far from generic.

One can use that language of effective operators also for the SM, integrating out all particles significantly heavier than the neutral mesons (that is, the top, the Higgs and the weak gauge bosons). Thus, the scale is $\Lambda_{\rm SM} \sim m_W$. Since the leading contributions to neutral meson mixings come from box diagrams, the z_{ij} coefficients are suppressed by α_2^2. To identify the relevant flavor suppression factor, one can employ the spurion formalism. For example, the flavor transition that is relevant to $B^0 - \overline{B}^0$ mixing involves $\overline{d_L} b_L$ which transforms as $(8,1,1)_{SU(3)^3_q}$. The leading contribution must then be proportional to $(Y^u Y^{u\dagger})_{13} \propto y_t^2 V_{tb} V_{td}^*$. Indeed, an explicit calculation (using VIA for the matrix element and neglecting QCD corrections) gives[4]

$$\frac{2M_{12}^B}{m_B} \approx -\frac{\alpha_2^2}{12}\frac{f_B^2}{m_W^2} S_0(x_t)(V_{tb}V_{td}^*)^2, \tag{53}$$

[3] The bounds from the corresponding four-fermi terms with LR structure, instead of the LL structure of Eq. (45), are even stronger.

[4] A detailed derivation can be found in Appendix B of [17].

where $x_i = m_i^2/m_W^2$ and

$$S_0(x) = \frac{x}{(1-x)^2}\left[1 - \frac{11x}{4} + \frac{x^2}{4} - \frac{3x^2\ln x}{2(1-x)}\right]. \tag{54}$$

Similar spurion analyses, or explicit calculations, allow us to extract the weak and flavor suppression factors that apply in the SM:

$$\begin{aligned}
\mathcal{I}m(z_{sd}^{\rm SM}) &\sim \alpha_2^2 y_t^2 |V_{td}V_{ts}|^2 \sim 1\times 10^{-10},\\
z_{sd}^{\rm SM} &\sim \alpha_2^2 y_c^2 |V_{cd}V_{cs}|^2 \sim 5\times 10^{-9},\\
z_{bd}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{td}V_{tb}|^2 \sim 7\times 10^{-8},\\
z_{bs}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{ts}V_{tb}|^2 \sim 2\times 10^{-6}.
\end{aligned} \tag{55}$$

(We did not include $z_{cu}^{\rm SM}$ in the list because it requires a more detailed consideration. The naively leading short distance contribution is $\propto \alpha_2^2(y_s^4/y_c^2)|V_{cs}V_{us}|^2 \sim 5\times 10^{-13}$. However, higher dimension terms can replace a y_s^2 factor with $(\Lambda/m_D)^2$ [18]. Moreover, long distance contributions are expected to dominate. In particular, peculiar phase space effects [19, 20] have been identified which are expected to enhance Δm_D to within an order of magnitude of the its measured value.)

It is clear than that contributions from new physics at $\Lambda_{\rm NP} \sim 1\,TeV$ should be suppressed by factors that are comparable or smaller than the SM ones. Why does that happen? This is the new physics flavor puzzle.

The fact that the flavor structure of new physics at the TeV scale must be non-generic means that flavor measurements are a good probe of the new physics. Perhaps the best-studied example is that of supersymmetry. Here, the spectrum of the superpartners and the structure of their couplings to the SM fermions will allow us to probe the mechanism of dynamical supersymmetry breaking.

VI. LESSONS FOR SUPERSYMMETRY FROM $D^0 - \overline{D}^0$ MIXING

Interesting experimental results concerning $D^0 - \overline{D}^0$ mixing have been recently achieved by the BELLE and BABAR experiments. For the first time, there is evidence for width splitting [21, 22] and mass splitting (of order one percent) between the two neutral D-mesons. Allowing for indirect CP violation, the world averages of the mixing parameters

are [10]

$$x = (1.00 \pm 0.25) \times 10^{-2},$$
$$y = (0.77 \pm 0.18) \times 10^{-2}. \quad (56)$$

It is important to note, however, that there is no evidence for CP violation in this mixing [10]:

$$1 - |q/p| = +0.06 \pm 0.14,$$
$$\phi_D = -0.04 \pm 0.09. \quad (57)$$

We use this recent experimental information to draw important lessons on supersymmetry. This demonstrates how flavor physics – at the GeV scale – provides a significant probe of supersymmetry – at the TeV scale.

A. Neutral meson mixing with supersymmetry

We consider the contributions from the box diagrams involving the squark doublets of the first two generations, $\tilde{Q}_{L1,2}$, to the $D^0 - \overline{D}^0$ and $K^0 - \overline{K}^0$ mixing amplitudes. The contributions that are relevant to the neutral D system are proportional to $K^u_{2i} K^{u*}_{1i} K^u_{2j} K^{u*}_{1j}$, where K^u is the mixing matrix of the gluino couplings to a left-handed up quark and their supersymmetric squark partners. (In the language of the mass insertion approximation, we calculate here the contribution that is $\propto [(\delta^u_{LL})_{12}]^2$.) The contributions that are relevant to the neutral K system are proportional to $K^{d*}_{2i} K^d_{1i} K^{d*}_{2j} K^d_{1j}$, where K^d is the mixing matrix of the gluino couplings to a left-handed down quark and their supersymmetric squark partners ($\propto [(\delta^d_{LL})_{12}]^2$ in the mass insertion approximation). We work in the mass basis for both quarks and squarks. A detailed derivation [23] is given in Appendix C. It gives:

$$M^D_{12} = \frac{\alpha_s^2 m_D f_D^2 B_D \eta_{\rm QCD}}{108 m^2_{\tilde{u}}} [11 \tilde{f}_6(x_u) + 4 x_u f_6(x_u)] \frac{(\Delta m^2_{\tilde{u}})^2}{m^4_{\tilde{u}}} (K^u_{21} K^{u*}_{11})^2, \quad (58)$$

$$M^K_{12} = \frac{\alpha_s^2 m_K f_K^2 B_K \eta_{\rm QCD}}{108 m^2_{\tilde{d}}} [11 \tilde{f}_6(x_d) + 4 x_d f_6(x_d)] \frac{(\Delta \tilde{m}^2_{\tilde{d}})^2}{\tilde{m}^4_d} (K^{d*}_{21} K^d_{11})^2. \quad (59)$$

Here $m_{\tilde{u},\tilde{d}}$ is the average mass of the corresponding two squark generations, $\Delta m^2_{\tilde{u},\tilde{d}}$ is the mass-squared difference, and $x_{u,d} = m^2_{\tilde{g}} / m^2_{\tilde{u},\tilde{d}}$.

One can immediately identify three generic ways in which supersymmetric contributions to neutral meson mixing can be suppressed:

1. Heaviness: $m_{\tilde{q}} \gg 1\ TeV$;

2. Degeneracy: $\Delta m^2_{\tilde{q}} \ll m^2_{\tilde{q}}$;

3. Alignment: $K^{d,u}_{21} \ll 1$.

When heaviness is the only suppression mechanism, as in split supersymmetry [24], the squarks are very heavy and supersymmetry no longer solves the fine tuning problem.[5] If we want to maintain supersymmetry as a solution to the fine tuning problem, either degeneracy or alignment or a combination of both is needed. This means that the flavor structure of supersymmetry is not generic, as argued in the previous section.

The 2×2 mass-squared matrices for the relevant squarks have the following form:

$$\begin{aligned} \tilde{M}^2_{U_L} &= \tilde{m}^2_{Q_L} + \left(\frac{1}{2} - \frac{2}{3}s^2_W\right) m^2_Z \cos 2\beta + M_u M_u^\dagger, \\ \tilde{M}^2_{D_L} &= \tilde{m}^2_{Q_L} - \left(\frac{1}{2} - \frac{1}{3}s^2_W\right) m^2_Z \cos 2\beta + M_d M_d^\dagger. \end{aligned} \tag{60}$$

We note the following features of the various terms:

- $\tilde{m}^2_{Q_L}$ is a 2×2 hermitian matrix of soft supersymmetry breaking terms. It does not break $SU(2)_{\rm L}$ and consequently it is common to $\tilde{M}^2_{U_L}$ and $\tilde{M}^2_{D_L}$. On the other hand, it breaks in general the $SU(2)_Q$ flavor symmetry.

- The terms proportional to m^2_Z are the D-terms. They break supersymmetry (since they involve $D_{T_3} \neq 0$ and $D_Y \neq 0$) and $SU(2)_{\rm L}$ but conserve $SU(2)_Q$.

- The terms proportional to M^2_q come from the F_{U_R}- and F_{D_R}-terms. They break the gauge $SU(2)_{\rm L}$ and the global $SU(2)_Q$ but, since $F_{U_R} = F_{D_R} = 0$, conserve supersymmetry.

Given that we are interested in squark masses close to the TeV scale (and the experimental lower bounds are of order 300 GeV), the scale of the eigenvalues of $\tilde{m}^2_{Q_L}$ is much higher than m^2_Z which, in turn, is much higher than m^2_c, the largest eigenvalue in $M_q M_q^\dagger$. We can draw the following conclusions:

[5] When the first two squark generations are mildly heavy and the third generation is light, as in effective supersymmetry [25], the fine tuning problem is still solved, but additional suppression mechanisms are needed.

1. $m^2_{\tilde{u}} = m^2_{\tilde{d}} \equiv m^2_{\tilde{q}}$ up to effects of order m^2_Z, namely to an accuracy of $\mathcal{O}(10^{-2})$.

2. $\Delta m^2_{\tilde{u}} = \Delta m^2_{\tilde{d}} \equiv \Delta m^2_{\tilde{q}}$ up to effects of order m^2_c, namely to an accuracy of $\mathcal{O}(10^{-5})$.

3. Since $K_u \simeq V_{uL}\tilde{V}_L^\dagger$ and $K_d \simeq V_{dL}\tilde{V}_L^\dagger$ (the matrices V_{qL} are defined in Eq. (21), while $\tilde{V}_L$ diagonalizes $\tilde{m}^2_{Q_L}$), the mixing matrices K^u and K^d are different from each other, but the following relation to the CKM matrix holds to an accuracy of $\mathcal{O}(10^{-5})$:

$$K^u K^{d\dagger} = V. \tag{61}$$

B. Non-degenerate squarks at the LHC?

Eqs. (58) and (59) can be translated into our generic language:

$$\begin{aligned} \Lambda_{\rm NP} &= m_{\tilde{q}}, & (62)\\ z_{cu} &= z_{12}\sin^2\theta_u, \\ z_{sd} &= z_{12}\sin^2\theta_d, \\ z_{12} &= \frac{11\tilde{f}_6(x) + 4x f_6(x)}{18}\alpha_s^2\left(\frac{\Delta\tilde{m}^2_{\tilde{q}}}{m^2_{\tilde{q}}}\right)^2, & (63) \end{aligned}$$

with Eq. (61) giving

$$\sin\theta_u - \sin\theta_d \approx \sin\theta_c = 0.23. \tag{64}$$

We now ask the following question: Is it possible that the first two generation squarks, $\tilde{Q}_{L1,2}$, are accessible to the LHC ($m_{\tilde{q}} \lesssim 1\ TeV$), and are not degenerate ($\Delta m^2_{\tilde{q}}/m^2_{\tilde{q}} = \mathcal{O}(1)$)?

To answer this question, we use Eqs. (52). For $\Lambda_{\rm NP} \lesssim 1\ TeV$, we have $z_{cu} \lesssim 5\times10^{-7}$ and, for a phase that is $\not\ll 0.1$, $z_{sd} \lesssim 6\times10^{-8}$. On the other hand, for non-degenerate squarks, and, for example, $11\tilde{f}_6(1) + 4f_6(1) = 1/6$, we have $z_{12} = 8\times10^{-5}$. Then we need, simultaneously, $\sin\theta_u \lesssim 0.08$ and $\sin\theta_d \lesssim 0.03$, but this is inconsistent with Eq. (64).

There are three ways out of this situation:

1. The first two generation squarks are quasi-degenerate. The minimal level of degeneracy is $(\tilde{m}_2-\tilde{m}_1)/(\tilde{m}_2+\tilde{m}_1) \lesssim 0.1$. It could be the result of RGE [26]. However, for maximal phases, the bound is even stronger, of order 0.04 [27], which is difficult to achieve with just RGE effects.

2. The first two generation squarks are heavy. Putting $\sin\theta_u = 0.23$ and $\sin\theta_d \approx 0$, as in models of alignment [28, 29], Eq. (50) leads to

$$m_{\tilde{q}} \gtrsim 3\ TeV. \tag{65}$$

3. The ratio $x = \tilde{m}_g^2/\tilde{m}_q^2$ is in a fine-tuned region of parameter space where there are accidental cancelations in $11\tilde{f}_6(x) + 4x f_6(x)$. For example, for $x = 2.33$, this combination is ~ 0.003 and the bound (65) is relaxed by a factor of 7.

Barring accidental cancelations, the *model independent* conclusion is that, if the first two generations of squark doublets are within the reach of the LHC, they must be quasi-degenerate [30, 31]. Analogous conclusions can be drawn for many TeV-scale new physics scenarios: a strong level of degeneracy is required (for definitions and detailed analysis, see [27]).

Exercise 7: *Does $K_{31}^d \sim |V_{ub}|$ suffice to satisfy the Δm_B constraint with neither degeneracy nor heaviness? (Use the two generation approximation and ignore the second generation.)*

Is there a natural way to make the squarks degenerate? Examining Eqs. (60) we learn that degeneracy requires $\tilde{m}_{Q_L}^2 \simeq \tilde{m}_{\tilde{q}}^2 \mathbf{1}$. We have mentioned already that flavor universality is a generic feature of gauge interactions. Thus, the requirement of degeneracy is perhaps a hint that supersymmetry breaking is *gauge mediated* to the MSSM fields.

VII. FLAVOR AT THE LHC

The LHC will study the physics of electroweak symmetry breaking. There are high hopes that it will discover not only the Higgs, but also shed light on the fine-tuning problem that is related to the Higgs mass. Here, we focus on the issue of how, through the study of new physics, the LHC can shed light on the new physics flavor puzzle.

A. Minimal flavor violation (MFV)

If supersymmetry breaking is gauge mediated, the squark mass matrices of Eq. (60), and those for the SU(2)-singlet squarks, have the following form at the scale of mediation m_M:

$$\tilde{M}_{U_L}^2(m_M) \;=\; \left(m_{\tilde{Q}_L}^2 + D_{U_L}\right)\mathbf{1} + M_u M_u^\dagger,$$

$$
\begin{aligned}
\tilde{M}^2_{D_L}(m_M) &= \left(m^2_{\tilde{Q}_L} + D_{D_L}\right)\mathbf{1} + M_d M_d^\dagger, \\
\tilde{M}^2_{U_R}(m_M) &= \left(m^2_{\tilde{U}_R} + D_{U_R}\right)\mathbf{1} + M_u^\dagger M_u, \\
\tilde{M}^2_{D_R}(m_M) &= \left(m^2_{\tilde{D}_R} + D_{D_R}\right)\mathbf{1} + M_d^\dagger M_d,
\end{aligned} \tag{66}
$$

where $D_{q_A} = (T_3)_{q_A} - (Q_{\rm EM})_{q_A} s^2_W m^2_Z \cos 2\beta$ are the D-term contributions. Here, the only source of the $SU(3)^3_q$ breaking are the SM Yukawa matrices.

This statement holds also when the renormalization group evolution is applied to find the form of these matrices at the weak scale. Taking the scale of the soft breaking terms $m_{\tilde{q}_A}$ to be somewhat higher than the electroweak breaking scale m_Z allows us to neglect the D_{q_A} and M_q terms in (66). Then we obtain

$$
\begin{aligned}
\tilde{M}^2_{Q_L}(m_Z) &\sim m^2_{\tilde{Q}_L}\left(r_3\mathbf{1} + c_u Y_u Y_u^\dagger + c_d Y_d Y_d^\dagger\right), \\
\tilde{M}^2_{U_R}(m_Z) &\sim m^2_{\tilde{U}_R}\left(r_3\mathbf{1} + c_{uR} Y_u^\dagger Y_u\right), \\
\tilde{M}^2_{D_R}(m_Z) &\sim m^2_{\tilde{D}_R}\left(r_3\mathbf{1} + c_{dR} Y_d^\dagger Y_d\right).
\end{aligned} \tag{67}
$$

Here r_3 represent the universal RGE contribution that is proportional to the gluino mass ($r_3 = \mathcal{O}(6) \times (M_3(m_M)/m_{\tilde{q}}(m_M))$) and the c-coefficients depend logarithmically on m_M/m_Z and can be of $\mathcal{O}(1)$ when m_M is not far below the GUT scale.

Models of gauge mediated supersymmetry breaking (GMSB) provide a concrete example of a large class of models that obey a simple principle called *minimal flavor violation* (MFV) [32]. This principle guarantees that low energy flavor changing processes deviate only very little from the SM predictions. The basic idea can be described as follows. The gauge interactions of the SM are universal in flavor space. The only breaking of this flavor universality comes from the three Yukawa matrices, Y_U, Y_D and Y_E. If this remains true in the presence of the new physics, namely Y_U, Y_D and Y_E are the only flavor non-universal parameters, then the model belongs to the MFV class.

Let us now formulate this principle in a more formal way, using the language of spurions that we presented in section III B. The Standard Model with vanishing Yukawa couplings has a large global symmetry (11,12). In this section we concentrate only on the quarks. The non-Abelian part of the flavor symmetry for the quarks is $SU(3)^3_q$ of Eq. (12) with the three generations of quark fields transforming as follows:

$$
Q_L(3,1,1), \quad U_R(1,3,1), \quad D_R(1,1,3). \tag{68}
$$

The Yukawa interactions,

$$\mathcal{L}_Y = \overline{Q_L} Y_D D_R H + \overline{Q_L} Y_U U_R H_c, \tag{69}$$

($H_c = i\tau_2 H^*$) break this symmetry. The Yukawa couplings can thus be thought of as spurions with the following transformation properties under $SU(3)_q^3$ [see Eq. (15)]:

$$Y_U \sim (3, \bar{3}, 1), \qquad Y_D \sim (3, 1, \bar{3}). \tag{70}$$

When we say "spurions", we mean that we pretend that the Yukawa matrices are fields which transform under the flavor symmetry, and then require that all the Lagrangian terms, constructed from the SM fields, Y_D and Y_U, must be (formally) invariant under the flavor group $SU(3)_q^3$. Of course, in reality, $\mathcal{L}_Y$ breaks $SU(3)_q^3$ precisely because $Y_{D,U}$ are *not* fields and do not transform under the symmetry.

The idea of minimal flavor violation is relevant to extensions of the SM, and can be applied in two ways:

1. If we consider the SM as a low energy effective theory, then all higher-dimension operators, constructed from SM-fields and Y-spurions, are formally invariant under $G_{\rm global}$.

2. If we consider a full high-energy theory that extends the SM, then all operators, constructed from SM and the new fields, and from Y-spurions, are formally invariant under $G_{\rm global}$.

Exercise 8: *Use the spurion formalism to argue that, in MFV models, the $K_L \to \pi^0 \nu\bar{\nu}$ decay amplitude is proportional to $y_t^2 V_{td} V_{ts}^*$.*

Examples of MFV models include models of supersymmetry with gauge-mediation or with anomaly-mediation of its breaking. If the LHC discovers new particles that couple to the SM fermions, then it will be able to test solutions to the new physics flavor puzzle such as MFV [33]. Much of its power to test such frameworks is based on identifying top and bottom quarks.

To understand this statement, we notice that the spurions Y_U and Y_D can always be written in terms of the two diagonal Yukawa matrices λ_u and λ_d and the CKM matrix V, see Eqs. (17,18). Thus, the only source of quark flavor changing transitions in MFV models

is the CKM matrix. Next, note that to an accuracy that is better than $\mathcal{O}(0.05)$, we can write the CKM matrix as follows:

$$V = \begin{pmatrix} 1 & 0.23 & 0 \\ -0.23 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{71}$$

Exercise 9: *The approximation (71) should be intuitively obvious to top-physicists, but definitely counter-intuitive to bottom-physicists. (Some of them have dedicated a large part of their careers to experimental or theoretical efforts to determine V_{cb} and V_{ub}.) What does the approximation imply for the bottom quark? When we take into account that it is only good to $\mathcal{O}(0.05)$, what would the implications be?*

We learn that the third generation of quarks is decoupled, to a good approximation, from the first two. This, in turn, means that any new particle that couples to the SM quarks (think, for example, of heavy quarks in vector-like representations of $G_{\rm SM}$), decay into either third generation quark, or to non-third generation quark, but not to both. For example, in Ref. [33], MFV models with additional charge $-1/3$, $SU(2)_{\rm L}$-singlet quarks – B' – were considered. A concrete test of MFV was proposed, based on the fact that the largest mixing effect involving the third generation is of order $|V_{cb}|^2 \sim 0.002$: Is the following prediction, concerning events of B' pair production, fulfilled:

$$\frac{\Gamma(B'\overline{B'} \to X q_{1,2} q_3)}{\Gamma(B'\overline{B'} \to X q_{1,2} q_{1,2}) + \Gamma(B'\overline{B'} \to X q_3 q_3)} \lesssim 10^{-3}. \tag{72}$$

If not, then MFV is excluded.

B. Supersymmetric flavor at the LHC

One can think of analogous tests in the supersymmetric framework [34–36]. Here, there is also a generic prediction that, in each of the three sectors (Q_L, U_R, D_R), squarks of the first two generations are quasi-degenerate, and do not decay into third generation quarks. Squarks of the third generation can be separated in mass (though, for small $\tan\beta$, the degeneracy in the $\tilde{D}_R$ sector is threefold), and decay only to third generation quarks.

It is not necessary, however, that the mediation of supersymmetry breaking is MFV. Examples of natural and viable solutions to the supersymmetric flavor problem that are not MFV include the following:

1. The leading contribution to the soft supersymmetry breaking terms is gauge mediated, and therefore MFV, but there are subleading contributions that are gravity mediated and provide new sources of flavor and CP violation [34, 36]. The gravity mediated contributions could either have some structure (dictated, for example, by a Froggatt-Nielsen symmetry [34] or by localization in extra dimensions [37]) or be anarchical [38].

2. The first two sfermion generations are heavy, and their mixing with the third generation is suppressed (for a recent analysis, see [39]). These features can come, for example, from conformal dynamics [40].

Such framework have different predictions concerning the mass splitting between sfermion generations and the flavor decomposition of the sfermion mass eigenstates. Note that measurements of flavor changing neutral current processes are only sensitive to the products of the form

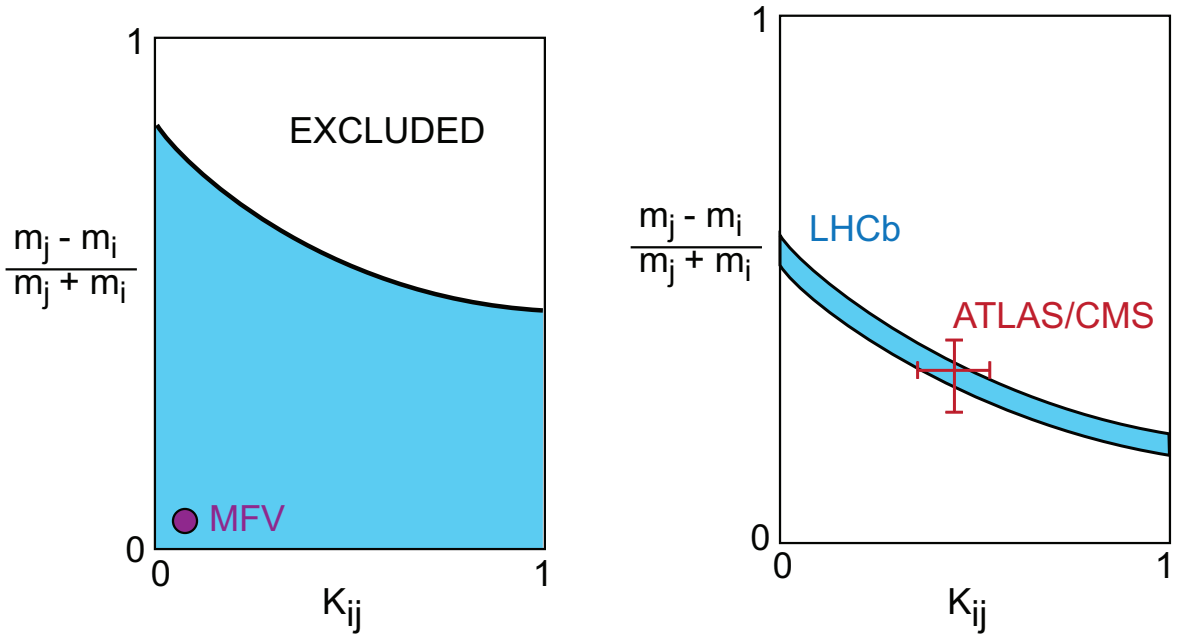
$$\delta_{ij} = \frac{\Delta\tilde{m}_{ij}^2}{\tilde{m}^2}\, K_{ij}K_{jj}^*, \tag{73}$$

where $\Delta\tilde{m}_{ij}^2$ is the mass-squared splitting between the sfermion generations i and j, $\tilde{m}^2$ is their average mass-squared, and K is the mixing matrix of gaugino couplings to these sfermions. On the other hand, the LHC experiments – ATLAS and CMS – can, at least in principle, measure the mass splitting and the mixing separately.

The present situation is depicted schematically in Fig. 5(a). Flavor factories have provided only upper bounds on deviations of FCNC processes, such as $\mu \to e\gamma$ or $D^0 - \overline{D}^0$ mixing, from the standard model predictions. In the supersymmetric framework, such bounds translate into an upper bound on a δ_{ij} parameter of Eq. (73), corresponding to the blue region in the figure. The supersymmetric flavor puzzle can be stated as the question of why the region in the upper right corner – where the flavor parameters are of order one – is excluded. MFV often puts us in the lower left corner of the plot, far from the experimental constraints. (This is particularly true for δ_{12} parameters.)

The optimal future situation is depicted schematically in Fig. 5(b). Imagine that a flavor factory does provide evidence for new physics, such as observation of $\Gamma(\mu \to e\gamma) \neq 0$ or CP violation in $D^0 - \overline{D}^0$ mixing. This will constrain the corresponding δ parameter, which is shown as the blue region in the Figure. If ATLAS/CMS measure the corresponding sfermion mass splitting and/or mixing, we will get a small allowed region in this flavor plane.

FIG. 5: Schematic description of the constraints in the plane of sfermion mass-squared splitting, $\Delta\tilde{m}^2_{ij}/\tilde{m}^2$, and mixing, $K_{ij}K^*_{jj}$: (a) Upper bounds from not observing any deviation from the SM predictions in present experiments; (b) Hypothetical future situation, where deviations have been observed in flavor factories (such as LHCb, a super-B factory, a $\mu \to e\gamma$ measurement, etc.) and the mass splitting and flavor decomposition have been measured by ATLAS/CMS.



If we have at our disposal such three consistent measurements (rate of FCNC process, spectrum and splitting), then we will understand the mechanism by which supersymmetry has its flavor violation suppressed. This will provide strong hints about the mechanism of supersymmetry breaking mediation.

If the sfermions are quasi-degenerate, then the mixing is determined by the small corrections to the unit mass-squared matrix. As mentioned above, the structure of such corrections may be dictated by the same symmetry or dynamics that gives the structure of the Yukawa couplings. If that is the case, then the measurement of the flavor decomposition might shed light on the Standard Model flavor puzzle.

We conclude that measurements at the LHC related to new particles that couple to the SM fermions are likely to teach us much more about flavor physics.

VIII. NEUTRINO ANARCHY VERSUS QUARK HIERARCHY

A detailed presentation of the physics and the formalism of neutrino flavor transitions is given in Appendix D for both vacuum oscillations (D 1) and the matter transitions (D 2). It follows Ref. [41].

Exercise 10: *For atmospheric ν_μ's with $E \sim 1\ GeV$, the flux coming from above has $P_{\mu\mu}(L \sim 10\ \text{km}) \approx 1$, while the flux from below has $P_{\mu\mu}(L \sim 10^4\ \text{km}) \approx 0.5$. Assuming that for the flux coming from below the oscillations are averaged out, estimate Δm^2 and $\sin^2 2\theta$.*

Exercise 11: *For solar ν_e's, the transition between matter ($\beta_{\text{MSW}} > 1$) and vacuum ($\beta_{\text{MSW}} < \cos 2\theta$) flavor transitions occurs around $E \sim 2\ MeV$. The transition probability is measured to be roughly $P_{ee} \sim 0.30$ for $\beta_{\text{MSW}} > 1$. Estimate Δm^2 and θ and predict P_{ee} for $\beta_{\text{MSW}} \ll 1$.*

The derived ranges for the three mixing angles and two mass-squared differences at 1σ are [42]:

$$\begin{aligned} \Delta m^2_{21} &= (7.9 \pm 0.3) \times 10^{-5}\ eV^2, \quad |\Delta m^2_{32}| = (2.6 \pm 0.2) \times 10^{-3}\ eV^2, \\ \sin^2\theta_{12} &= 0.31 \pm 0.02, \quad \sin^2\theta_{23} = 0.47 \pm 0.07, \quad \sin^2\theta_{13} = 0^{+0.008}_{-0.0}. \end{aligned} \tag{74}$$

The 3σ range for the matrix elements of U are the following [42]:

$$|U| = \begin{pmatrix} 0.79 \to 0.86 & 0.50 \to 0.61 & 0.00 \to 0.20 \\ 0.25 \to 0.53 & 0.47 \to 0.73 & 0.56 \to 0.79 \\ 0.21 \to 0.51 & 0.42 \to 0.69 & 0.61 \to 0.83 \end{pmatrix}. \tag{75}$$

A. New physics

The simplest and most straightforward lesson of the evidence for neutrino masses is also the most striking one: there is new physics beyond the Standard Model. This is the first experimental result that is inconsistent with the SM.

Most likely, the new physics is related to the existence of G_{SM}-singlet fermions at some high energy scale that induce, at low energies, the effective terms of Eq. (44) through the seesaw mechanism. The existence of heavy singlet fermions is predicted by many extensions of the SM, especially by GUTs [beyond $SU(5)$] and left-right-symmetric theories. The seesaw mechanism could also be driven by an $SU(2)_{\text{L}}$-triplet fermion.

There are other possibilities. In particular, neutrino masses can be generated without introducing any new fermions beyond those of the SM. Instead, the existence of a scalar $\Delta_L(1,3)_{+1}$, that is, an $SU(2)_{\rm L}$-triplet, is required. The smallness of the neutrino masses is related here to the smallness of the vacuum expectation value $\langle\Delta_L^0\rangle$ (required also by the success of the $\rho = 1$ relation) and does not have a generic natural explanation.

In left-right-symmetric models, however, where the breaking of $SU(2)_{\rm R} \times U(1)_{\rm B-L} \to U(1)_{\rm Y}$ is induced by the VEV of an $SU(2)_{\rm R}$-triplet, Δ_R, there must exist also an $SU(2)_{\rm L}$-triplet scalar. Furthermore, the Higgs potential leads to an order of magnitude relation between the various VEVs, $\langle\Delta_L^0\rangle\langle\Delta_R^0\rangle \sim v^2$, and the smallness of $\langle\Delta_L^0\rangle$ is correlated with the high scale of $SU(2)_{\rm R}$ breaking. This situation can be thought of as a seesaw of VEVs. In this model there are, however, also SM-singlet fermions. The light neutrino masses arise from both the seesaw mechanism ("type I") and the triplet VEV ("type II").

Neutrino masses could also be of the Dirac type. Here, again, singlet fermions are introduced, but lepton number is imposed by hand. This possibility is disfavored by theorists since it is likely that global symmetries are violated by gravitational effects. Furthermore, the lightness of the neutrinos (compared to charged fermions) is unexplained.

Another possibility is that neutrino masses are generated by mixing with singlet fermions but the mass scale of these fermions is not high. Here again the lightness of neutrino masses remains a puzzle. The best known example of such a scenario is the framework of supersymmetry without R parity.

Let us emphasize that the seesaw mechanism or, more generally, the extension of the SM with non-renormalizable terms, is the simplest explanation of neutrino masses. Models in which neutrino masses are generated by new physics at low energy imply a much more dramatic departure from the SM. Furthermore, the existence of seesaw masses is an unavoidable prediction of various extensions of the SM. In contrast, many (but not all) of the low energy mechanisms are introduced for the specific purpose of generating neutrino masses.

B. The scale of new physics

Eq. (44) gives a light neutrino mass matrix:

$$(M_\nu)_{ij} = Z^\nu_{ij}\frac{v^2}{\Lambda_{\rm NP}}. \tag{76}$$

It is straightforward to use the measured neutrino masses of Eq. (74) in combination with Eq. (76) to estimate the scale of new physics that is relevant to their generation. In particular, if there is no quasi-degeneracy in the neutrino masses, the heaviest of the active neutrino masses can be estimated:

$$m_h = m_3 \sim \sqrt{\Delta m^2_{32}} \approx 0.05\ eV. \tag{77}$$

(In the case of inverted hierarchy, the implied scale is $m_h = m_2 \sim \sqrt{\Delta m^2_{32}} \approx 0.05\ eV$.) It follows that the scale in the nonrenormalizable terms (44) is given by

$$\Lambda_{\rm NP} \sim v^2/m_h \approx 10^{15}\ GeV. \tag{78}$$

We should clarify two points regarding Eq. (78):

1. There could be some level of degeneracy between the neutrino masses. In such a case, Eq. (77) is modified into a lower bound on m_3 and, consequently, Eq. (78) becomes an upper bound on $\Lambda_{\rm NP}$.

2. It could be that the Z_{ij} of Eq. (44) are much smaller than 1. In such a case, again, Eq. (78) becomes an upper bound on the scale of new physics.

On the other hand, in models of approximate flavor symmetries, there are relations between the structures of the charged lepton and neutrino mass matrices that give, quite generically, $Z_{33} \gtrsim m^2_\tau/v^2 \sim 10^{-4}$. We conclude that the likely range for $\Lambda_{\rm NP}$ is given by

$$10^{11}\ GeV \lesssim \Lambda_{\rm NP} \lesssim 10^{15}\ GeV. \tag{79}$$

The estimates (78) and (79) are very exciting. First, the upper bound on the scale of new physics is well below the Planck scale. This means that there is new physics in Nature which is intermediate between the two known scales, the Planck scale, $m_{\rm Pl} \sim 10^{19}\ GeV$, and the electroweak breaking scale, $v \sim 10^2\ GeV$.

Second, the scale $\Lambda_{\rm NP} \sim 10^{15}\ GeV$ is intriguingly close to the scale of gauge coupling unification.

Third, the range (79) for the scale of lepton number breaking is optimal for leptogenesis [43] (for a recent review, see [44]). If (i) leptogenesis is generated by the decays of the lightest singlet neutrino N_1, and (ii) the masses of the singlet neutrinos are hierarchical, $M_1/M_{2,3...} \ll 1$, and (iii) the temperature when leptogenesis occurs is high enough, $T_{\rm LG} >$

$10^{12}\ GeV$, so that flavor effects are unimportant, then there is an upper bound on the CP asymmetry in N_1 decays [45]:

$$|\epsilon_{N_1}| \leq \frac{3}{16\pi}\frac{M_1(m_3 - m_2)}{v^2}. \tag{80}$$

Given that $Y_B^{\rm obs} \sim 9 \times 10^{-11}$, and that $Y_B \sim 10^{-3}\eta\epsilon_{N_1}$, where $\eta \lesssim 1$ is a washout factor, we must require $|\epsilon_{N_1}| \gtrsim 10^{-7}$. Moreover, we have $m_3 - m_2 \leq \sqrt{\Delta m^2_{32}} \sim 0.05\ eV$ and therefore obtain $M_1 \gtrsim 10^9\ GeV$. Violating any of the three conditions will relax this bound, but typically not by more than about an order of magnitude.

C. The flavor puzzle

In the absence of neutrino masses, there are 13 flavor parameters in the SM:

$$\begin{aligned}
y_t &\sim 1, \quad y_c \sim 10^{-2}, \quad y_u \sim 10^{-5}, \\
y_b &\sim 10^{-2}, \quad y_s \sim 10^{-3}, \quad y_d \sim 10^{-4}, \\
y_\tau &\sim 10^{-2}, \quad y_\mu \sim 10^{-3}, \quad y_e \sim 10^{-6}, \\
|V_{us}| &\sim 0.2, \quad |V_{cb}| \sim 0.04, \quad |V_{ub}| \sim 0.004, \quad \sin\delta_{\rm KM} \sim 1.
\end{aligned} \tag{81}$$

These flavor parameters are hierarchical (their magnitudes span six orders of magnitude), and all but two or three (the top Yukawa, the CP violating phase, and perhaps the Cabibbo angle) are small. The unexplained smallness and hierarchy pose the SM *flavor puzzle*. Its solution may direct us to physics beyond the Standard Model.

Several mechanisms have been proposed in response to this puzzle. For example, approximate horizontal symmetries, broken by a small parameter, can lead to selection rules that explain the hierarchy of the Yukawa couplings.

In the extension of the SM with three active neutrinos that have Majorana masses, there are nine new flavor parameters in addition to those of Eq. (81). These are three neutrino masses, three lepton mixing angles, and three phases in the mixing matrix. Of the nine new parameters, four have been measured: two mass-squared differences and two mixing angles [see Eq. (74)]. This adds significantly to the input data on flavor physics and provides an opportunity to test and refine flavor models.

If neutrino masses arise from effective terms of the form of Eq. (44), then the overall scale of neutrino masses is related to the scale $\Lambda_{\rm NP}$ and, in most cases, does not tell us anything

about flavor physics. More significant information for flavors models can be written in terms of three dimensionless parameters whose values can be read from Eq. (74), that is $\sin\theta_{12}$, $\sin\theta_{23}$ and

$$\Delta m^2_{21}/|\Delta m^2_{32}| = 0.030 \pm 0.003. \tag{82}$$

In addition, the upper bound on $\sin\theta_{13}$ often plays a significant role in flavor model building.

There are several features in the numerical estimates (74,82) that have drawn much attention and have driven numerous investigations:

(i) Large mixing and strong hierarchy: The mixing angle that is relevant to the $2-3$ sector is large, $\sin\theta_{23} \sim 0.7$. On the other hand, if there is no quasi-degeneracy in the neutrino masses, the corresponding mass ratio is small, $m_2/m_3 \sim 0.17$. It is difficult to explain in a natural way a situation where there is an $\mathcal{O}(1)$ mixing but the corresponding masses are hierarchical.

(ii) Two large and one small mixing angles: The mixing angles relevant to the $2-3$ sector ($\sin\theta_{23} \sim 0.7$) and $1-2$ sector ($\sin\theta_{12} \sim 0.55$) are large, yet the $1-3$ mixing angle is small ($\sin\theta_{13} \lesssim 0.20$). Such a situation is, again, difficult – though not impossible – to explain from approximate symmetries. An example of a symmetry that does predict such a pattern is that of $L_e - L_\mu - L_\tau$. This symmetry predicts, however, $\theta_{12} \simeq \pi/4$, which is experimentally excluded.

(iii) Maximal mixing: The value of θ_{23} is intriguingly close to maximal mixing ($\sin^2 2\theta_{23} = 1$). It is interesting to understand whether a symmetry could explain this special value.

(iv) Tribimaximal mixing: The mixing matrix (75) has a structure that is consistent with the following unitary matrix [46]:

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \tag{83}$$

It is interesting to understand whether a symmetry could explain this special structure.

All four features enumerated above are difficult to explain in a large class of flavor models that do very well in explaining the flavor features of the quark sector. In particular, models with Abelian horizontal symmetries (Froggatt-Nielsen type [47]) predict that, in general, $|V_{ub}| \sim |V_{us}V_{cb}|$, $|V_{ij}| \gtrsim m_i/m_j$ $(i < j)$ and $V \sim \mathbf{1}$ [29, 48]. All of these are successful predictions. At the same time, however, these models predict [49] that for the neutrinos, in general, $|U_{ij}|^2 \sim m_i/m_j$ and $|U_{e3}| \sim |U_{e2}U_{\mu 3}|$, in contradiction to, respectively, points (i)

and (ii) above (and there is no way to make θ_{23} parametrically close to $\pi/4$). On the other hand, there exist very specific models where these features are related to a symmetry.

It is possible, however, that the above interpretation of the results is wrong. Indeed, the data can be interpreted in a very different way:

(v) No small parameters. The two measured mixing angles are larger than any of the quark mixing angles. Indeed, they are both of order one. The measured mass ratio, $m_2/m_3 \gtrsim 0.16$ is larger than any of the quark and charged lepton mass ratios, and could be interpreted as an $\mathcal{O}(1)$ parameter (namely, it is accidentally small, without any parametric suppression). If this is the correct way of reading the data, the measured neutrino parameters may actually reflect the absence of any hierarchical structure in the neutrino mass matrices [50]. The possibility that there is no structure – neither hierarchy, nor degeneracy – in the neutrino sector has been called "neutrino mass anarchy". An important test of this idea will be provided by the measurement of $|U_{e3}|$. If indeed the entries in M_ν have random values of the same order, all three mixing angles are expected to be of order one. If experiments measure $|U_{e3}| \sim 0.1$, that is, close to the present bound, it can be argued that its smallness is accidental. The stronger the upper bound on this angle becomes, the more difficult it will be to maintain this view.

Neutrino mass anarchy can be accommodated within models of Abelian flavor symmetries, if the three lepton doublets carry the same charge. Indeed, consider a supersymmetric model with a $U(1)_H$ symmetry that is broken by a single small spurion ϵ_H of charge -1. Let us assume that the three fermion generations contained in the 10-representation of $SU(5)$ carry charges $(2, 1, 0)$, while the three $\bar{5}$-representations carry charges $(0, 0, 0)$. (The Higgs fields carry no H charges.) Such a model predicts ϵ_H^2 hierarchy in the up sector, ϵ_H hierarchy in the down and charged lepton sectors, and anarchy in the neutrino sector.

Exercise 12: *The selection rule for this model is that a term in the superpotential that carries H charge $n \geq 0$ is suppressed by ϵ_H^n. Find the parametric suppression of the various entries in M_u, M_d, M_ℓ and M_ν. Find the parametric suppression of the mixing angles.*

It would be nice if the features of quark mass hierarchy and neutrino mass anarchy can be traced back to some fundamental principle or to a stringy origin (see, for example, [51]).

IX. CONCLUSIONS

(i) Measurements of CP violating B-meson decays have established that the Kobayashi-Maskawa mechanism is the dominant source of the observed CP violation.

(ii) Measurements of flavor changing B-meson decays have established the the Cabibbo-Kobayashi-Maskawa mechanism is a major player in flavor violation.

(iii) The consistency of all these measurements with the CKM predictions sharpens the new physics flavor puzzle: If there is new physics at, or below, the TeV scale, then its flavor structure must be highly non-generic.

(iv) Measurements of $D^0 - \overline{D}^0$ mixing imply that alignment by itself cannot solve the supersymmetric flavor problem. The first two squark generations must be quasi-degenerate.

(v) Measurements of neutrino flavor parameters have not only not clarified the standard model flavor puzzle, but actually deepened it. Whether they imply an anarchical structure, or a tribimaximal mixing, it seems that the neutrino flavor structure is very different from that of quarks.

(vi) If the LHC experiments, ATLAS and CMS, discover new particles that couple to the Standard Model fermions, then, in principle, they will be able to measure new flavor parameters. Consequently, the new physics flavor puzzle is likely to be understood.

(vii) If the flavor structure of such new particles is affected by the same physics that sets the flavor structure of the Yukawa couplings, then the LHC experiments (and future flavor factories) may be able to shed light also on the standard model flavor puzzle.

The huge progress in flavor physics in recent years has provided answers to many questions. At the same time, new questions arise. We look forward to the LHC era for more answers and more questions.

Acknowledgments

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APPENDIX A: THE CKM MATRIX

The CKM matrix V is a 3×3 unitary matrix. Its form, however, is not unique:

(i) There is freedom in defining V in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The elements of V are written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{A1}$$

(ii) There is further freedom in the phase structure of V. This means that the number of physical parameters in V is smaller than the number of parameters in a general unitary 3×3 matrix which is nine (three real angles and six phases). Let us define P_q $(q = u, d)$ to be diagonal unitary (phase) matrices. Then, if instead of using V_{qL} and V_{qR} for the rotation (21) to the mass basis we use $\tilde{V}_{qL}$ and $\tilde{V}_{qR}$, defined by $\tilde{V}_{qL} = P_q V_{qL}$ and $\tilde{V}_{qR} = P_q V_{qR}$, we still maintain a legitimate mass basis since $M_q^{\rm diag}$ remains unchanged by such transformations. However, V does change:

$$V \to P_u V P_d^*. \tag{A2}$$

This freedom is fixed by demanding that V has the minimal number of phases. In the three generation case V has a single phase. (There are five phase differences between the elements of P_u and P_d and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of CP violation in the quark sector of the Standard Model [1].

The fact that V is unitary and depends on only four independent physical parameters can be made manifest by choosing a specific parametrization. The standard choice is [52]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{A3}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The θ_{ij}'s are the three real mixing parameters while δ is the Kobayashi-Maskawa phase. It is known experimentally that $s_{13} \ll s_{23} \ll s_{12} \ll 1$. It is convenient to choose an approximate expression where this hierarchy is manifest. This is the Wolfenstein parametrization, where the four mixing parameters are (λ, A, ρ, η) with

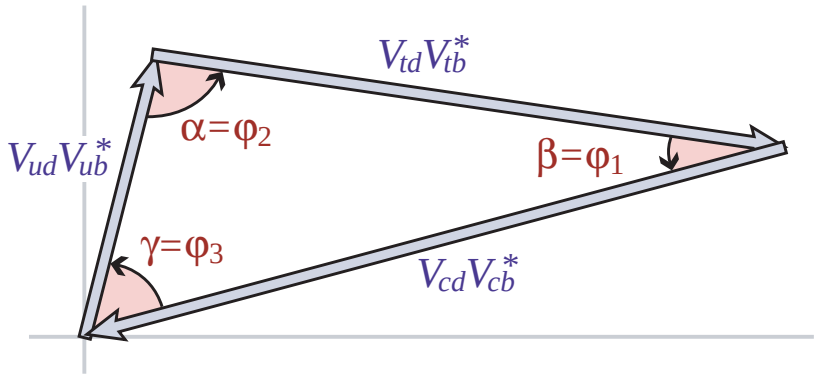

FIG. 6: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

$\lambda = |V_{us}| = 0.23$ playing the role of an expansion parameter and η representing the CP violating phase [53, 54]:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} . \quad \text{(A4)}$$

A very useful concept is that of the *unitarity triangles*. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \quad \text{(A5)}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \quad \text{(A6)}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad \text{(A7)}$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (A7) only. The unitarity triangle related to Eq. (A7) is depicted in Fig. 6.

The rescaled unitarity triangle is derived from (A7) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ, η). The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the (ρ, η) plane, the lengths of the two complex sides are

$$R_u \equiv \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2+\eta^2}, \quad R_t \equiv \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right| = \sqrt{(1-\rho)^2+\eta^2}. \tag{A8}$$

The three angles of the unitarity triangle are defined as follows [55, 56]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{A9}$$

They are physical quantities and can be independently measured by CP asymmetries in B decays. It is also useful to define the two small angles of the unitarity triangles (A6,A5):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{A10}$$

The λ and A parameters are very well determined at present, see Eq. (37). The main effort in CKM measurements is thus aimed at improving our knowledge of ρ and η:

$$\rho = 0.14^{+0.03}_{-0.02}, \quad \eta = 0.35 \pm 0.02. \tag{A11}$$

The present status of our knowledge is best seen in a plot of the various constraints and the final allowed region in the $\rho - \eta$ plane. This is shown in Fig. 2.

APPENDIX B: CP VIOLATION IN NEUTRAL B DECAYS TO FINAL CP EIGENSTATES

We define decay amplitudes of B (which could be charged or neutral) and its CP conjugate $\overline{B}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$A_f = \langle f|\mathcal{H}|B\rangle \quad , \quad \overline{A}_f = \langle f|\mathcal{H}|\overline{B}\rangle \quad , \quad A_{\overline{f}} = \langle \overline{f}|\mathcal{H}|B\rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{B}\rangle \ , \tag{B1}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases ξ_B and ξ_f according to

$$\begin{aligned} CP\,|B\rangle &= e^{+i\xi_B}\,|\overline{B}\rangle \quad , \quad CP\,|f\rangle = e^{+i\xi_f}\,|\overline{f}\rangle \ , \\ CP\,|\overline{B}\rangle &= e^{-i\xi_B}\,|B\rangle \quad , \quad CP\,|\overline{f}\rangle = e^{-i\xi_f}\,|f\rangle \ , \end{aligned} \tag{B2}$$

so that $(CP\,)^2 = 1$. The phases ξ_B and ξ_f are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP\,, \mathcal{H}] = 0$, then A_f and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_B)}\, A_f \ . \tag{B3}$$

A state that is initially a superposition of B^0 and $\overline{B}^0$, say

$$|\psi(0)\rangle = a(0)|B^0\rangle + b(0)|\overline{B}^0\rangle \ , \tag{B4}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\overline{B}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots \ . \tag{B5}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times t in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [57]. The simplified time evolution is determined by a 2×2 effective Hamiltonian $\mathcal{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathcal{H}$, can be written in terms of Hermitian matrices M and Γ as

$$\mathcal{H} = M - \frac{i}{2}\,\Gamma \ . \tag{B6}$$

M and Γ are associated with $(B^0, \overline{B}^0) \leftrightarrow (B^0, \overline{B}^0)$ transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. Diagonal elements of M and Γ are associated with the flavor-conserving transitions $B^0 \to B^0$ and $\overline{B}^0 \to \overline{B}^0$ while off-diagonal elements are associated with flavor-changing transitions $B^0 \leftrightarrow \overline{B}^0$.

The eigenvectors of $\mathcal{H}$ have well defined masses and decay widths. We introduce complex parameters $p_{L,H}$ and $q_{L,H}$ to specify the components of the strong interaction eigenstates, B^0 and $\overline{B}^0$, in the light (B_L) and heavy (B_H) mass eigenstates:

$$|B_{L,H}\rangle = p_{L,H}|B^0\rangle \pm q_{L,H}|\overline{B}^0\rangle \tag{B7}$$

with the normalization $|p_{L,H}|^2 + |q_{L,H}|^2 = 1$. If either CP or CPT is a symmetry of $\mathcal{H}$ (independently of whether T is conserved or violated) then $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, and solving the eigenvalue problem for $\mathcal{H}$ yields $p_L = p_H \equiv p$ and $q_L = q_H \equiv q$ with

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}} \ . \tag{B8}$$

From now on we assume that CPT is conserved. If either CP or T is a symmetry of $\mathcal{H}$ (independently of whether CPT is conserved or violated), then M_{12} and Γ_{12} are relatively real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_B} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1 \ , \tag{B9}$$

where ξ_B is the arbitrary unphysical phase introduced in Eq. (B2).

The real and imaginary parts of the eigenvalues of $\mathcal{H}$ corresponding to $|B_{L,H}\rangle$ represent their masses and decay-widths, respectively. The mass difference Δm_B and the width difference $\Delta\Gamma_B$ are defined as follows:

$$\Delta m_B \equiv M_H - M_L, \quad \Delta\Gamma_B \equiv \Gamma_H - \Gamma_L. \tag{B10}$$

Note that here Δm_B is positive by definition, while the sign of $\Delta\Gamma_B$ is to be experimentally determined. The average mass and width are given by

$$m_B \equiv \frac{M_H + M_L}{2}, \quad \Gamma_B \equiv \frac{\Gamma_H + \Gamma_L}{2}. \tag{B11}$$

It is useful to define dimensionless ratios x and y:

$$x \equiv \frac{\Delta m_B}{\Gamma_B}, \quad y \equiv \frac{\Delta\Gamma_B}{2\Gamma_B}. \tag{B12}$$

Solving the eigenvalue equation gives

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \qquad \Delta m_B \Delta\Gamma_B = 4\mathcal{R}e(M_{12}\Gamma_{12}^*). \tag{B13}$$

All CP-violating observables in B and $\overline{B}$ decays to final states f and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of A_f, $\overline{A}_f$, $A_{\overline{f}}$ and $\overline{A}_{\overline{f}}$, together with, for neutral-meson decays only, q/p. CP violation in charged-meson decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in neutral-meson decays is complicated by $B^0 \leftrightarrow \overline{B}^0$ oscillations and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

For neutral D, B, and B_s mesons, $\Delta\Gamma/\Gamma \ll 1$ and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|B^0\rangle$ or $|\overline{B}^0\rangle$ after an elapsed proper time t as $|B^0{}_{\rm phys}(t)\rangle$ or $|\overline{B}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, we obtain

$$\begin{aligned} |B^0{}_{\rm phys}(t)\rangle &= g_+(t)\,|B^0\rangle - \frac{q}{p}\,g_-(t)|\overline{B}^0\rangle, \\ |\overline{B}^0_{\rm phys}(t)\rangle &= g_+(t)\,|\overline{B}^0\rangle - \frac{p}{q}\,g_-(t)|B^0\rangle\,, \end{aligned} \tag{B14}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \tag{B15}$$

One obtains the following time-dependent decay rates:

$$\frac{d\Gamma[{B^0}_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} = \left(|A_f|^2+|(q/p)\overline{A}_f|^2\right)\cosh(y\Gamma t)+\left(|A_f|^2-|(q/p)\overline{A}_f|^2\right)\cos(x\Gamma t) + 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t)-2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t)\ , \quad \text{(B16)}$$

$$\frac{d\Gamma[{\overline{B}}^0_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} = \left(|(p/q)A_f|^2+|\overline{A}_f|^2\right)\cosh(y\Gamma t)-\left(|(p/q)A_f|^2-|\overline{A}_f|^2\right)\cos(x\Gamma t) + 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t)-2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t)\ , \quad \text{(B17)}$$

where $\mathcal{N}_f$ is a common normalization factor. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f=\mathcal{N}_{\overline{f}}$ and the substitutions $A_f\to A_{\overline{f}}$ and $\overline{A}_f\to\overline{A}_{\overline{f}}$ in Eqs. (B16,B17). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $B\leftrightarrow\overline{B}$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$ are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (B16,B17) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

One possible manifestation of CP-violating effects in meson decays [58] is in the interference between a decay without mixing, $B^0\to f$, and a decay with mixing, $B^0\to\overline{B}^0\to f$ (such an effect occurs only in decays to final states that are common to B^0 and $\overline{B}^0$, including all CP eigenstates). It is defined by

$$\mathcal{I}m(\lambda_f)\neq 0\ , \quad \text{(B18)}$$

with

$$\lambda_f\equiv\frac{q}{p}\frac{\overline{A}_f}{A_f}\ . \quad \text{(B19)}$$

This form of CP violation can be observed, for example, using the asymmetry of neutral meson decays into final CP eigenstates f_{CP}

$$\mathcal{A}_{f_{CP}}(t)\equiv\frac{d\Gamma/dt[{\overline{B}}^0_{\rm phys}(t)\to f_{CP}\,]-d\Gamma/dt[{B^0}_{\rm phys}(t)\to f_{CP}\,]}{d\Gamma/dt[{\overline{B}}^0_{\rm phys}(t)\to f_{CP}\,]+d\Gamma/dt[{B^0}_{\rm phys}(t)\to f_{CP}\,]}\ . \quad \text{(B20)}$$

For $\Delta\Gamma=0$ and $|q/p|=1$ (which is a good approximation for B mesons), $\mathcal{A}_{f_{CP}}$ has a particularly simple form [59–61]:

$$\mathcal{A}_f(t) = S_f\sin(\Delta mt)-C_f\cos(\Delta mt), \qquad S_f\equiv\frac{2\,\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2},\quad C_f\equiv\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\ , \quad \text{(B21)}$$

Consider the $B \to f$ decay amplitude A_f, and the CP conjugate process, $\overline{B} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in A_f and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^\pm$ bosons and hence are often called "weak phases". The weak phase of any single term is convention dependent. However, the difference between the weak phases in two different terms in A_f is convention independent. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Their origin is the possible contribution from intermediate on-shell states in the decay process. Since these phases are generated by CP-invariant interactions, they are the same in A_f and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The 'weak' and 'strong' phases discussed here appear in addition to the 'spurious' CP-transformation phases of Eq. (B3). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution a_i to A_f in three parts: its magnitude $|a_i|$, its weak phase ϕ_i, and its strong phase δ_i. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$\begin{aligned} A_f &= |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}, \\ \overline{A}_{\overline{f}} &= |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}. \end{aligned} \tag{B22}$$

Similarly, for neutral meson decays, it is useful to write

$$M_{12} = |M_{12}| e^{i\phi_M} \quad , \quad \Gamma_{12} = |\Gamma_{12}| e^{i\phi_\Gamma} \ . \tag{B23}$$

Each of the phases appearing in Eqs. (B22,B23) is convention dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$ and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

In the approximations that only a single weak phase contributes to decay, $A_f = |a_f| e^{i(\delta_f+\phi_f)}$, and that $|\Gamma_{12}/M_{12}| = 0$, we obtain $|\lambda_f| = 1$ and the CP asymmetries in decays

to a final CP eigenstate f [Eq. (B20)] with eigenvalue $\eta_f = \pm 1$ are given by

$$\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f)\ \sin(\Delta m t) \ \text{ with } \ \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f). \tag{B24}$$

Note that the phase so measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$.

APPENDIX C: SUPERSYMMETRIC CONTRIBUTIONS TO NEUTRAL MESON MIXING

We consider the squark-gluino box diagram contribution to $D^0 - \overline{D}^0$ mixing amplitude that is proportional to $K^u_{2i}K^{u*}_{1i}K^u_{2j}K^{u*}_{1j}$, where K^u is the mixing matrix of the gluino couplings to left-handed up quarks and their up squark partners. (In the language of the mass insertion approximation, we calculate here the contribution that is $\propto [(\delta^u_{LL})_{12}]^2$.) We work in the mass basis for both quarks and squarks.

The contribution is given by

$$M_{12}^D = -i\frac{4\pi^2}{27}\alpha_s^2 m_D f_D^2 B_D \eta_{\rm QCD} \sum_{i,j}(K^u_{2i}K^{u*}_{1i}K^u_{2j}K^{u*}_{1j})(11\tilde{I}_{4ij} + 4\tilde{m}_g^2 I_{4ij}). \tag{C1}$$

where

$$\begin{aligned}
\tilde{I}_{4ij} &\equiv \int \frac{d^4p}{(2\pi)^4}\frac{p^2}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_i^2)(p^2-\tilde{m}_j^2)} \\
&= \frac{i}{(4\pi)^2}\left[\frac{\tilde{m}_g^2}{(\tilde{m}_i^2-\tilde{m}_g^2)(\tilde{m}_j^2-\tilde{m}_g^2)}\right. \\
&\quad \left. + \frac{\tilde{m}_i^4}{(\tilde{m}_i^2-\tilde{m}_j^2)(\tilde{m}_i^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^4}{(\tilde{m}_j^2-\tilde{m}_i^2)(\tilde{m}_j^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_j^2}{\tilde{m}_g^2}\right],
\end{aligned} \tag{C2}$$

$$\begin{aligned}
I_{4ij} &\equiv \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_i^2)(p^2-\tilde{m}_j^2)} \\
&= \frac{i}{(4\pi)^2}\left[\frac{1}{(\tilde{m}_i^2-\tilde{m}_g^2)(\tilde{m}_j^2-\tilde{m}_g^2)}\right. \\
&\quad \left. + \frac{\tilde{m}_i^2}{(\tilde{m}_i^2-\tilde{m}_j^2)(\tilde{m}_i^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^2}{(\tilde{m}_j^2-\tilde{m}_i^2)(\tilde{m}_j^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_j^2}{\tilde{m}_g^2}\right].
\end{aligned} \tag{C3}$$

We now follow the discussion in refs. [23, 26]. To see the consequences of the super-GIM mechanism, let us expand the expression for the box integral around some value $\tilde{m}_q^2$ for the

squark masses-squared:

$$\begin{aligned} I_4(\tilde{m}_g^2,\tilde{m}_i^2,\tilde{m}_j^2) &= I_4(\tilde{m}_g^2,\tilde{m}_q^2+\delta\tilde{m}_i^2,\tilde{m}_q^2+\delta\tilde{m}_j^2) \\ &= I_4(\tilde{m}_g^2,\tilde{m}_q^2,\tilde{m}_q^2)+(\delta\tilde{m}_i^2+\delta\tilde{m}_j^2)I_5(\tilde{m}_g^2,\tilde{m}_q^2,\tilde{m}_q^2,\tilde{m}_q^2) \\ &+ \frac{1}{2}\left[(\delta\tilde{m}_i^2)^2+(\delta\tilde{m}_j^2)^2+2(\delta\tilde{m}_i^2)(\delta\tilde{m}_j^2)\right] I_6(\tilde{m}_g^2,\tilde{m}_q^2,\tilde{m}_q^2,\tilde{m}_q^2,\tilde{m}_q^2)+\cdots, \end{aligned} \tag{C4}$$

where

$$I_n(\tilde{m}_g^2,\tilde{m}_q^2,\ldots,\tilde{m}_q^2) \equiv \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_q^2)^{n-2}}, \tag{C5}$$

and similarly for $\tilde{I}_{4ij}$. Note that $I_n \propto (\tilde{m}_q^2)^{n-2}$ and $\tilde{I}_n \propto (\tilde{m}_q^2)^{n-3}$. Thus, using $x \equiv \tilde{m}_g^2/\tilde{m}_q^2$, it is customary to define

$$I_n \equiv \frac{i}{(4\pi)^2(\tilde{m}_q^2)^{n-2}} f_n(x), \qquad \tilde{I}_n \equiv \frac{i}{(4\pi)^2(\tilde{m}_q^2)^{n-3}} \tilde{f}_n(x). \tag{C6}$$

The unitarity of the mixing matrix implies that

$$\sum_i (K^u_{2i}K^{u*}_{1i}K^u_{2j}K^{u*}_{1j}) = \sum_j (K^u_{2i}K^{u*}_{1i}K^u_{2j}K^{u*}_{1j}) = 0. \tag{C7}$$

We learn that the terms that are proportional $f_4, \tilde{f}_4, f_5$ and $\tilde{f}_5$ vanish in their contribution to M_{12}. When $\delta\tilde{m}_i^2 \ll \tilde{m}_q^2$ for all i, the leading contributions to M_{12} come from f_6 and $\tilde{f}_6$. We learn that for quasi-degenerate squarks, the leading contribution is quadratic in the small mass-squared difference. The functions $f_6(x)$ and $\tilde{f}_6(x)$ are given by

$$\begin{aligned} f_6(x) &= \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(1-x)^5}, \\ \tilde{f}_6(x) &= \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(1-x)^5}. \end{aligned} \tag{C8}$$

For example, with $x = 1$, $f_6(1) = -1/20$ and $\tilde{f}_6 = +1/30$; with $x = 2.33$, $f_6(2.33) = -0.015$ and $\tilde{f}_6 = +0.013$.

To further simplify things, let us consider a two generation case. Then

$$\begin{aligned} M_{12}^D &\propto 2(K^u_{21}K^{u*}_{11})^2(\delta\tilde{m}_1^2)^2 + 2(K^u_{22}K^{u*}_{12})^2(\delta\tilde{m}_2^2)^2 + (K^u_{21}K^{u*}_{11}K^u_{22}K^{u*}_{12})(\delta\tilde{m}_1^2+\delta\tilde{m}_2^2)^2 \\ &= (K^u_{21}K^{u*}_{11})^2(\tilde{m}_2^2-\tilde{m}_1^2)^2. \end{aligned} \tag{C9}$$

We thus rewrite Eq. (C1) for the case of quasi-degenerate squarks:

$$M_{12}^D = \frac{\alpha_s^2 m_D f_D^2 B_D \eta_{\rm QCD}}{108\tilde{m}_q^2}[11\tilde{f}_6(x)+4xf_6(x)]\frac{(\Delta\tilde{m}_{21}^2)^2}{\tilde{m}_q^4}(K^u_{21}K^{u*}_{11})^2. \tag{C10}$$

For example, for $x = 1$, $11\tilde{f}_6(x) + 4xf_6(x) = +0.17$. For $x = 2.33$, $11\tilde{f}_6(x) + 4xf_6(x) = +0.003$.

APPENDIX D: NEUTRINO FLAVOR TRANSITIONS

1. Neutrinos in vacuum

Neutrino oscillations in vacuum [62] arise since neutrinos are massive and mix. In other words, the neutrino state that is produced by electroweak interactions is not a mass eigenstate. The weak eigenstates ν_α ($\alpha = e, \mu, \tau$ denotes the charged lepton mass eigenstates and their neutrino doublet-partners) are linear combinations of the mass eigenstates ν_i ($i = 1, 2, 3$):

$$|\nu_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle. \tag{D1}$$

After traveling a distance L (or, equivalently for relativistic neutrinos, time t), a neutrino originally produced with a flavor α evolves as follows:

$$|\nu_\alpha(t)\rangle = U^*_{\alpha i}|\nu_i(t)\rangle. \tag{D2}$$

It can be detected in the charged-current interaction $\nu_\alpha(t)N' \to \ell_\beta N$ with a probability

$$P_{\alpha\beta} = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = \left|\sum_{i=1}^{3}\sum_{j=1}^{3} U^*_{\alpha i}U_{\beta j}\langle\nu_j(0)|\nu_i(t)\rangle\right|^2. \tag{D3}$$

We follow the analysis of ref. [41]. We use the standard approximation that $|\nu\rangle$ is a plane wave, $|\nu_i(t)\rangle = e^{-iE_i t}|\nu_i(0)\rangle$. In all cases of interest to us, the neutrinos are relativistic:

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}, \tag{D4}$$

where E_i and m_i are, respectively, the energy and the mass of the neutrino mass eigenstate. Furthermore, we can assume that $p_i \simeq p_j \equiv p \simeq E$. Then, we obtain the following transition probability:

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i=1}^{2}\sum_{j=i+1}^{3} \mathcal{R}e\left(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right)\sin^2 x_{ij}, \tag{D5}$$

where $x_{ij} \equiv \Delta m^2_{ij}L/(4E)$, $\Delta m^2_{ij} = m^2_i - m^2_j$ and $L = t$ is the distance between the source (that is, the production point of ν_α) and the detector (that is, the detection point of ν_β). In deriving Eq. (D5) we used the orthogonality relation $\langle\nu_j(0)|\nu_i(0)\rangle = \delta_{ij}$. It is convenient to use the following units:

$$x_{ij} = 1.27\,\frac{\Delta m^2_{ij}}{eV^2}\,\frac{L/E}{m/MeV}. \tag{D6}$$

TABLE I: Characteristic values of L and E for various neutrino sources and experiments.

Experiment	L (m)	E (MeV)	Δm^2 (eV^2)
Solar	10^{10}	1	10^{-10}
Atmospheric	$10^4 - 10^7$	$10^2 - 10^5$	$10^{-1} - 10^{-4}$
Reactor	$10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
Kamland	10^5	1	10^{-5}
Accelerator	10^2	$10^3 - 10^4$	$\gtrsim 10^{-1}$
Long-baseline Accelerator	$10^5 - 10^6$	10^4	$10^{-2} - 10^{-3}$

The transition probability [Eq. (D5)] has an oscillatory behavior, with oscillation lengths

$$L^{\rm osc}_{0,ij} = \frac{4\pi E}{\Delta m^2_{ij}} \tag{D7}$$

and amplitude that is proportional to elements of the mixing matrix. Thus, in order to have oscillations, neutrinos must have different masses ($\Delta m^2_{ij} \neq 0$) and they must mix ($U_{\alpha i} U_{\beta i} \neq 0$).

An experiment is characterized by the typical neutrino energy E and by the source-detector distance L. In order to be sensitive to a given value of Δm^2_{ij}, the experiment has to be set up with $E/L \approx \Delta m^2_{ij}$ ($L \sim L^{\rm osc}_{0,ij}$). The typical values of L/E for different types of neutrino sources and experiments are summarized in Table I.

If $(E/L) \gg \Delta m^2_{ij}$ ($L \ll L^{\rm osc}_{0,ij}$), the oscillation does not have time to give an appreciable effect because $\sin^2 x_{ij} \ll 1$. The case of $(E/L) \ll \Delta m^2_{ij}$ ($L \gg L^{\rm osc}_{0,ij}$) requires more careful consideration. One must take into account that, in general, neutrino beams are not monochromatic. Thus, rather than measuring $P_{\alpha\beta}$, the experiments are sensitive to the average probability

$$\langle P_{\alpha\beta} \rangle = \delta_{\alpha\beta} - 4 \sum_{i=1}^{2} \sum_{j=i+1}^{3} \mathcal{R}e \left(U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} \right) \langle \sin^2 x_{ij} \rangle . \tag{D8}$$

For $L \gg L^{\rm osc}_{0,ij}$, the oscillation phase goes through many cycles before the detection and is averaged to $\langle \sin^2 x_{ij} \rangle = 1/2$.

For a two neutrino case,

$$P_{\alpha\beta} = \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1) \sin^2 2\theta \sin^2 x. \tag{D9}$$

For averaged oscillations we get, for example,

$$P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta. \tag{D10}$$

For a recent careful derivation of the oscillation formulae, see [63].

2. Neutrinos in matter

When neutrinos propagate in dense matter, the interactions with the medium affect their properties. These effects are either coherent or incoherent. For purely incoherent $\nu - p$ scattering, the characteristic cross section is very small,

$$\sigma \sim \frac{G_F^2 s}{\pi} \sim 10^{-43}\ \mathrm{cm}^2 \left(\frac{E}{1\ MeV}\right)^2. \tag{D11}$$

The smallness of this cross section is demonstrated by the fact that if a beam of 10^{10} neutrinos with $E \sim 1\ MeV$ was aimed at Earth, only one would be deflected by the Earth's matter. It may seem then that for neutrinos matter is irrelevant. However, one must take into account that Eq. (D11) does not contain the contribution from forward elastic coherent interactions. In coherent interactions, the medium remains unchanged and it is possible to have interference of scattered and unscattered neutrino waves which enhances the effect. Coherence further allows one to decouple the evolution equation of neutrinos from the equations of the medium. In this approximation, the effect of the medium is described by an effective potential which depends on the density and composition of the matter [64].

Consider, for example, the effective potential for ν_e induced by its charged-current interactions with electrons in matter:

$$V_C = \langle \nu_e | \int d^3x H_C^{(e)} | \nu_e \rangle = \sqrt{2} G_F N_e. \tag{D12}$$

For $\overline{\nu_e}$ the sign of V is reversed. The potential can also be expressed in terms of the matter density ρ:

$$V_C = 7.6\ \frac{N_e}{N_p + N_n}\ \frac{\rho}{10^{14}\ \mathrm{g/cm}^3}\ eV. \tag{D13}$$

Two examples that are relevant to observations are the following:

- At the Earth's core $\rho \sim 10\ \mathrm{g/cm}^3$ and $V \sim 10^{-13}\ eV$;
- At the solar core $\rho \sim 100\ \mathrm{g/cm}^3$ and $V \sim 10^{-12}\ eV$.

Consider a state that is an admixture of two neutrino species, $|\nu_e\rangle$ and $|\nu_a\rangle$ or, equivalently, $|\nu_1\rangle$ and $|\nu_2\rangle$. With some approximations, the time evolution can be written in the following matrix form [64]:

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_e\\ \nu_a\end{pmatrix} = -\frac{1}{2E}M_w^2\begin{pmatrix}\nu_e\\ \nu_a\end{pmatrix}, \tag{D14}$$

where we have defined an effective mass matrix in matter,

$$M_w^2 = \frac{1}{2}\begin{pmatrix} m_1^2+m_2^2+4EV_e-\Delta m^2\cos 2\theta & \Delta m^2\sin 2\theta \\ \Delta m^2\sin 2\theta & m_1^2+m_2^2+4EV_a+\Delta m^2\cos 2\theta\end{pmatrix}, \tag{D15}$$

with $\Delta m^2 = m_2^2 - m_1^2$.

We define the instantaneous mass eigenstates in matter, ν_i^m, as the eigenstates of M_w for a fixed value of x. They are related to the interaction eigenstates by a unitary transformation,

$$\begin{pmatrix}\nu_e\\ \nu_a\end{pmatrix} = U(\theta_m)\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix} = \begin{pmatrix}\cos\theta_m & \sin\theta_m\\ -\sin\theta_m & \cos\theta_m\end{pmatrix}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}. \tag{D16}$$

The eigenvalues of M_w, that is, the effective masses in matter, are given by [64, 65]

$$\mu_{1,2}^2 = \frac{m_1^2+m_2^2}{2} + E(V_e+V_a) \mp \frac{1}{2}\sqrt{(\Delta m^2\cos 2\theta - A)^2 + (\Delta m^2\sin 2\theta)^2}, \tag{D17}$$

while the mixing angle in matter is given by

$$\tan 2\theta_m = \frac{\Delta m^2\sin 2\theta}{\Delta m^2\cos 2\theta - A}, \tag{D18}$$

where

$$A \equiv 2E(V_e - V_a). \tag{D19}$$

The instantaneous mass eigenstates ν_i^m are, in general, not energy eigenstates: they mix in the evolution. The importance of this effect is controlled by the relative size of $4E\dot{\theta}_m(t)$ with respect to $\mu_2^2(t) - \mu_1^2(t)$. When the latter is much larger than the first, ν_i^m behave approximately as energy eigenstates and do not mix during the evolution. This is the adiabatic transition approximation. The adiabaticity condition reads

$$\mu_2^2(t) - \mu_1^2(t) \gg 2EA\Delta m^2\sin 2\theta\left|\dot{A}/A\right|. \tag{D20}$$

The transition probability for the adiabatic case is given by

$$P_{ee}(t) = \left|\sum_i U_{ei}(\theta)U_{ei}^*(\theta_p)\exp\left(-\frac{i}{2E}\int_{t_0}^t \mu_i^2(t')dt'\right)\right|^2, \tag{D21}$$

where θ_p is the mixing angle at the production point. For the case of two-neutrino mixing, Eq. (D21) takes the form

$$P_{ee}(t) = \cos^2\theta_p \cos^2\theta + \sin^2\theta_p \sin^2\theta + \frac{1}{2}\sin 2\theta_p \sin 2\theta \cos\left(\frac{\delta(t)}{2E}\right), \tag{D22}$$

where

$$\delta(t) = \int_{t_p}^{t} [\mu_2^2(t') - \mu_1^2(t')] dt'. \tag{D23}$$

For $\mu_2^2(t) - \mu_1^2(t) \gg E$, the last term in Eq. (D22) is averaged out and the survival probability takes the form

$$P_{ee} = \frac{1}{2}[1 + \cos 2\theta_p \cos 2\theta]. \tag{D24}$$

The relative importance of the MSW matter term [A of Eq. (D19)] and the kinematic vacuum oscillation term in the Hamiltonian [the off-diagonal term in Eq. (D15)] can be parametrized by the quantity $\beta_{\rm MSW}$, which represents the ratio of matter to vacuum effects (see, for example [66]). From Eq. (D15) we see that the appropriate ratio is

$$\beta_{\rm MSW} = \frac{2\sqrt{2} G_F n_e E_\nu}{\Delta m^2}. \tag{D25}$$

The quantity $\beta_{\rm MSW}$ is the ratio between the oscillation length in matter and the oscillation length in vacuum. In convenient units, $\beta_{\rm MSW}$ can be written as

$$\beta_{\rm MSW} = 0.19 \left(\frac{E_\nu}{1\ MeV}\right) \left(\frac{\mu_e \rho}{100\ {\rm g\ cm^{-3}}}\right) \left(\frac{8 \times 10^{-5}\ eV^2}{\Delta m^2}\right). \tag{D26}$$

Here μ_e is the electron mean molecular weight ($\mu_e \approx 0.5(1 + X)$, where X is the mass fraction of hydrogen) and ρ is the total density. If $\beta_{\rm MSW} \lesssim \cos 2\theta$, the survival probability corresponds to vacuum averaged oscillations [see Eq. (D9)],

$$P_{ee} = \left(1 - \frac{1}{2}\sin^2 2\theta\right) \quad (\beta_{\rm MSW} < \cos 2\theta,\ \text{vacuum}). \tag{D27}$$

If $\beta_{\rm MSW} > 1$, the survival probability corresponds to matter dominated oscillations [see Eq. (D24)],

$$P_{ee} = \sin^2\theta \quad (\beta_{\rm MSW} > 1,\ \text{MSW}). \tag{D28}$$

The survival probability is approximately constant in either of the two limiting regimes, $\beta_{\rm MSW} < \cos 2\theta$ and $\beta_{\rm MSW} > 1$. There is a strong energy dependence only in the transition region between the limiting regimes.

For the Sun, $N_e(R) = N_e(0)\exp(-R/r_0)$, with $r_0 \equiv R_\odot/10.54 = 6.6 \times 10^7$ m $= 3.3 \times 10^{14}\ eV^{-1}$. Then, the adiabaticity condition for the Sun reads

$$\frac{(\Delta m^2/eV^2)\sin^2 2\theta}{(E/MeV)\cos 2\theta} \gg 3 \times 10^{-9}. \tag{D29}$$

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