Quantum ChromoDynamics

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Outline

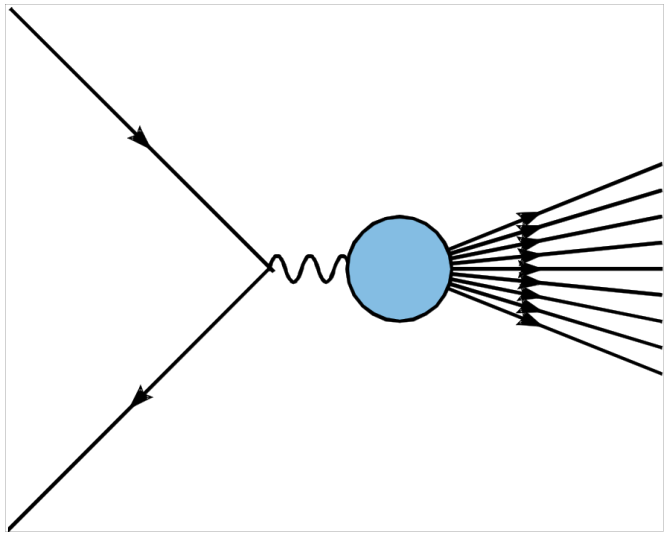
1. Basics
2. QCD Phenomenology at Tree Level
3. Higher Order corrections
4. Monte Carlo techniques

QCD Phenomenology – Tree Level

- The e^+e^- annihilation cross section
- Jets in e^+e^- annihilation
- Deep Inelastic Scattering
  - the parton model
- Hadron collisions
  - the Drell—Yan process
  - jet production

e^+e^- Annihilation to ???

- Most final states contain many hadrons
- Feynman rules don't tell us anything about hadrons!
- How to calculate:



- How to sum over all hadronic final states?
- Symmetries can give us a clue…

- Matrix element to produce n hadrons:

$$\mathcal{M} = \{\bar{v}(q_2) e \gamma_\mu u(q_1)\} \frac{-g^{\mu\nu}}{q^2} T_\nu(n, q, \{p_1 \dots p_n\})$$

- *T* is a guess (parametrization) of the unknown part.
- Gives total cross section:

$$\sigma = \frac{1}{2s}\frac{1}{4}\frac{e^2}{s^2}\mathrm{Tr}(\not{q}_2\gamma^\mu \not{q}_1 \gamma^\nu) \times \sum_n \int dPS_n \, T_\mu(n, q, \{p_1 \dots p_n\}) \, T^*_\nu(n, q, \{p_1 \dots p_n\})$$

- Define:

$$H_{\mu\nu}(q) \equiv \sum_n \int dPS_n \, T_\mu \, T^*_\nu$$

1) Lorentz covariance:

$$H_{\mu\nu} = A(q^2) g_{\mu\nu} + B(q^2) q_\mu q_\nu$$

2) Gauge invariance:

$$q^\mu H_{\mu\nu} = q^\nu H_{\mu\nu} = 0 \Rightarrow A = -q^2 B$$

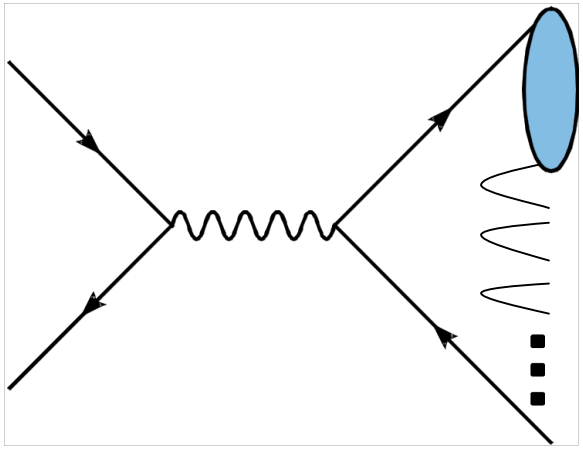
- Dimensions: B(s) dimensionless.
- Gives fundamental prediction:

$$R(e^+e^-) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \text{constant}$$

without knowing anything about interaction of hadrons!

Quark Parton Model

- Space time picture of e^+e^- → hadrons:



- Annihilation takes time $\sim 1/\sqrt{s}$
- Confinement takes time $\sim 1/m_{\text{hadron}}$

→ No time for confinement to affect cross section.

$$\Rightarrow \sigma(e^+e^- \to \text{hadrons}) \approx \sigma(e^+e^- \to \text{quarks}) \times \left(1+\mathcal{O}\left(\frac{m_{\text{hadron}}}{\sqrt{s}}\right)^n\right)$$

- Feynman rules do tell us how to calculate that.

$\sigma(e^+e^- \rightarrow \text{hadrons})$

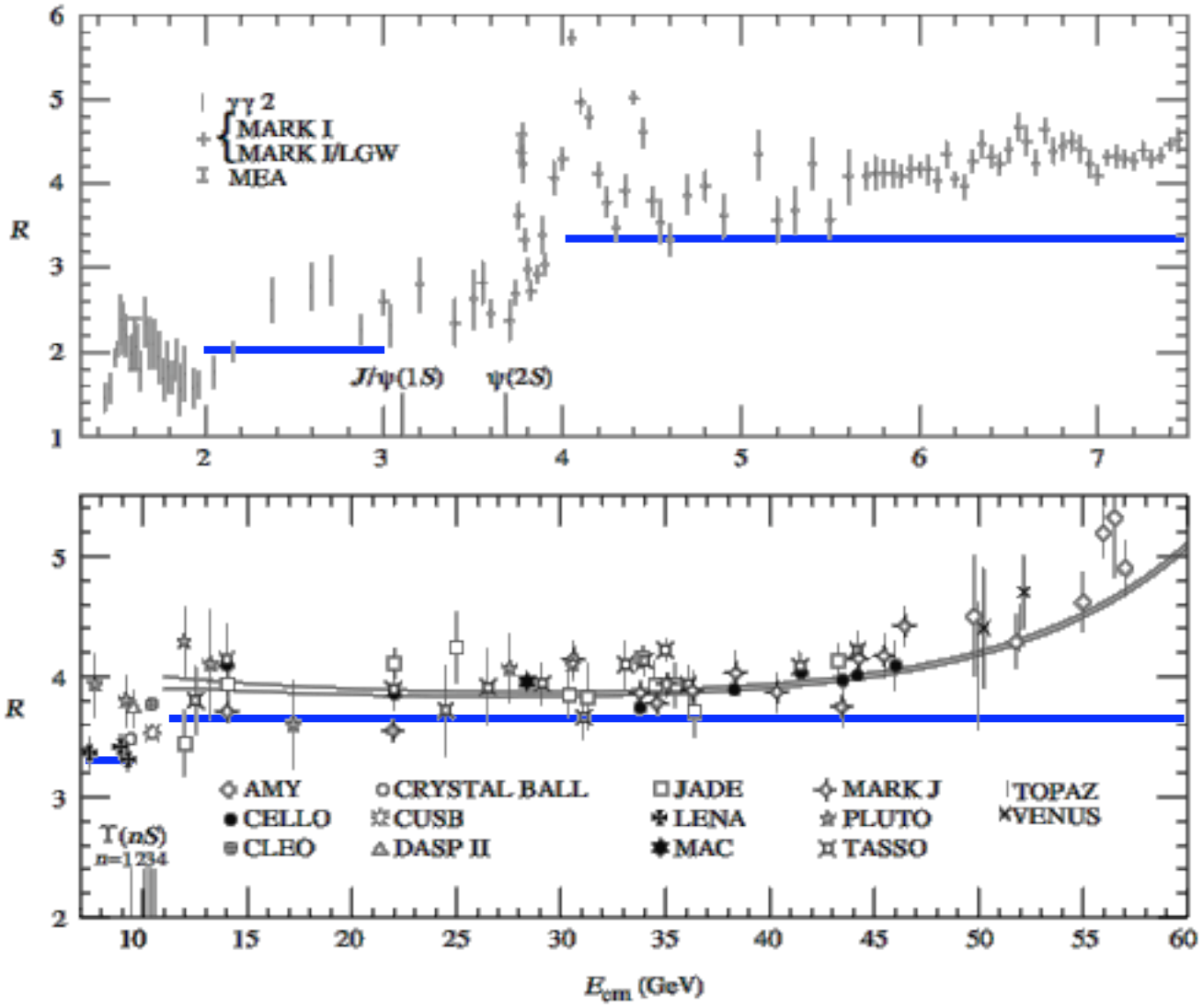
- Start with photon exchange only:

$$\rightarrow \sigma(\text{hadrons}) = \frac{4\pi\alpha^2}{3s}\sum_q e_q^2$$

  where sum is over all quarks that can appear in the final state. → 3 (N_c) colours per flavour
- From here on explicitly extract this factor:

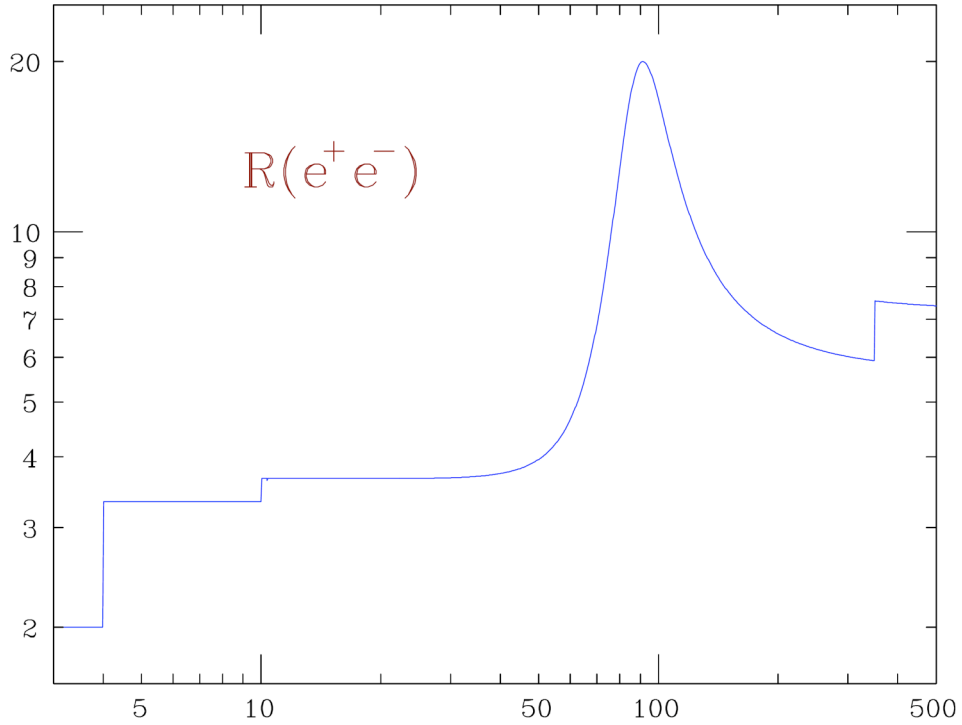
$$R \equiv \frac{\sigma(\text{hadrons})}{\sigma(\text{muons})} = N_c \sum_q e_q^2$$

- Kinematically allowed if $s > 2m_q$ → series of steps at $2m_c \sim 4$ GeV, $2m_b \sim 10$ GeV, …


R
γγ 2
MARK I
MARK I/LGW
MEA
J/ψ(1S)
ψ(2S)
AMY
CELLO
CLEO
CRYSTAL BALL
CUSB
DASP II
JADE
LENA
MAC
MARK J
PLUTO
TASSO
TOPAZ
VENUS
Υ(nS)
n=1234
E_cm (GeV)

R(e^+e^-) at the Z^0

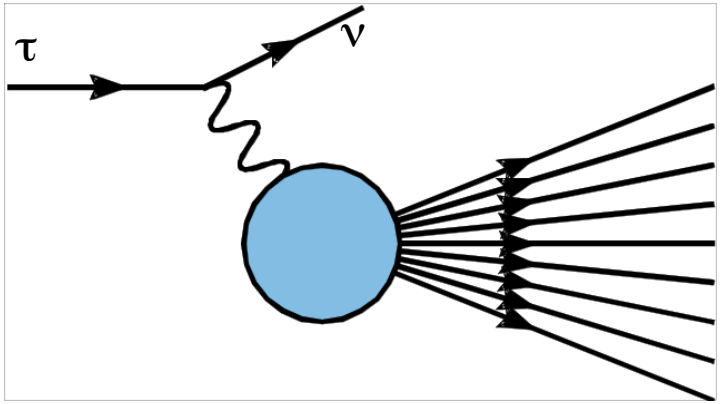
- $R = N_c \frac{\sum_q v_q^2 + a_q^2}{v_\mu^2 + a_\mu^2} = 20.095$ cf LEP average: 20.775±0.027
- In general, sensitive to γ–Z interference:



- (actually → 19.984 on Z peak)

τ decays

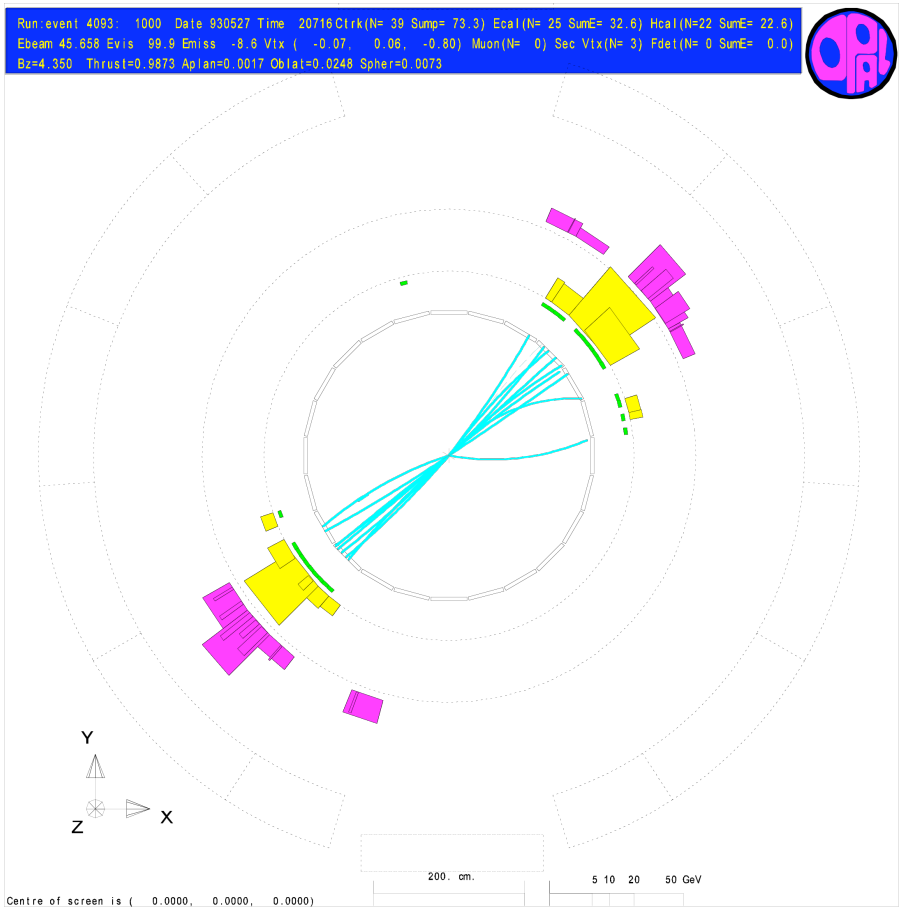
- Very closely-related process

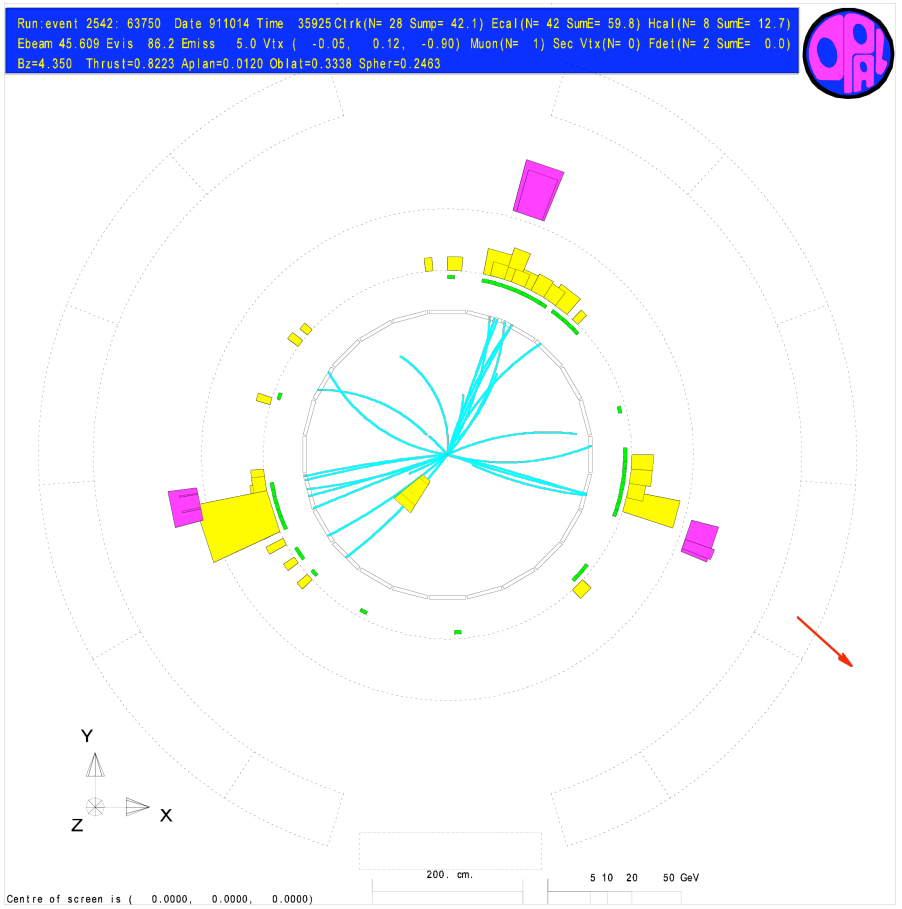


- $R_\tau \equiv \mathcal{B}(\tau \to \text{hadrons})/\mathcal{B}(\tau \to \mu)$

Jet production in e^+e^- → hadrons

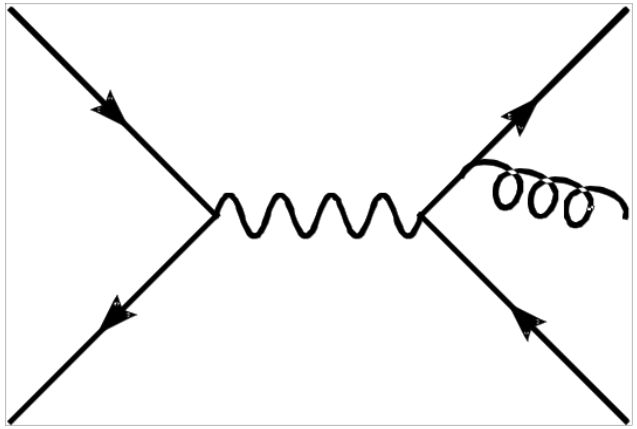
- Most e^+e^- events consist of two back-to-back jets
- But some consist of three (or more) jets → gluons

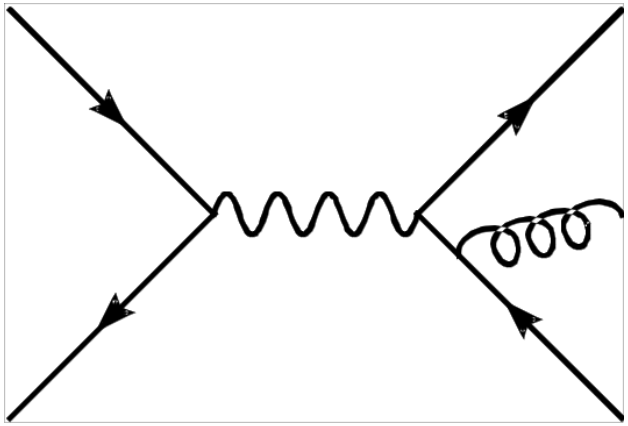




Three-jet cross section

- Two Feynman diagrams





- After some algebra…

$$\sum |\mathcal{M}|^2 = \frac{16 C_F N_c e^4 e_q^2 g_s^2}{s\, p_q\cdot p_g\, p_{\bar{q}}\cdot p_g} \left((p_q\cdot p_{e^+})^2 + (p_{\bar{q}}\cdot p_{e^+})^2 + (p_q\cdot p_{e^-})^2 + (p_{\bar{q}}\cdot p_{e^-})^2 \right)$$

Aside on Colour factor for e^+e^- → qqbarg

$$\sum_{a,b,A} t^A_{ba} \left(t^A_{ba}\right)^* = \sum_{a,b,A} t^A_{ba} t^A_{ab} = \sum_A \mathsf{Tr}(t^A t^A) = C_F N_c$$

Crash course in Three-Body Phase Space

$$dPS = \frac{d^3\underline{p_1}}{(2\pi)^3 2E_1}\frac{d^3\underline{p_2}}{(2\pi)^3 2E_2}\frac{d^3\underline{p_3}}{(2\pi)^3 2E_3}(2\pi)^4\delta^{(4)}(q_1+q_2-p_1-p_2-p_3)$$

- All massless:

$$dPS = \frac{1}{8(2\pi)^5}\, p_1 dp_1\, d\cos\theta\, d\phi\, p_2 dp_2\, d\cos\beta\, d\alpha\, \frac{1}{p_3}\delta(\sqrt{s}-p_1-p_2-p_3)$$

- Momentum conservation:

$$p_3 = |\underline{p_1}+\underline{p_2}| = \sqrt{p_1^2+p_2^2+2p_1p_2\cos\beta}$$

$$\Rightarrow \int d\cos\beta\ \delta(\sqrt{s}-p_1-p_2-p_3) = \frac{p_3}{p_1p_2}$$

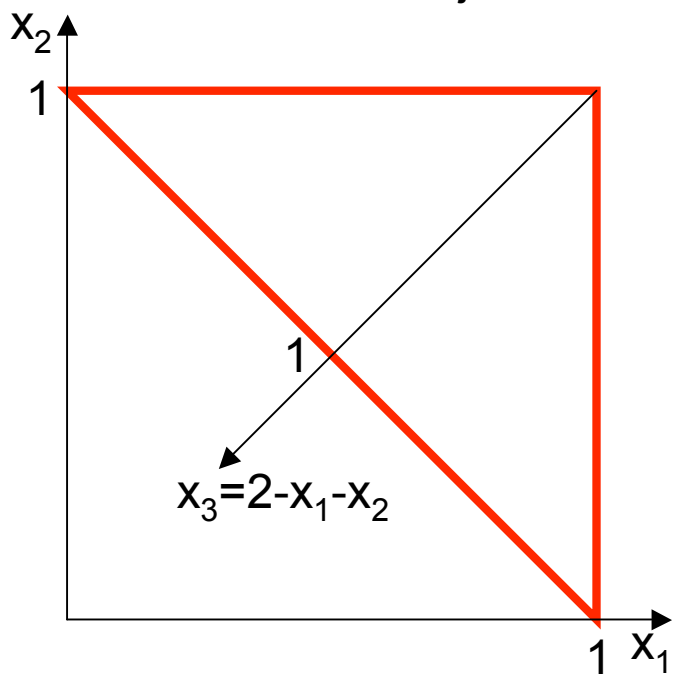
$$dPS = \frac{1}{8(2\pi)^5}\, dp_1\, d\cos\theta\, d\phi\, dp_2\, d\alpha$$

- Define energy fractions $x_i \equiv 2p_i/\sqrt{s}$

$$dPS = \frac{s}{16(2\pi)^3}\, dx_1\, dx_2\, \frac{d\cos\theta\, d\phi\, d\alpha}{2(2\pi)^2}$$

- θ, ϕ, α Euler angles: generally uninteresting for QCD

- phase space limits: $0 \leq (p_i+p_j)^2 = 2p_i \cdot p_j = s(1-x_k) \Rightarrow x_i \leq 1$

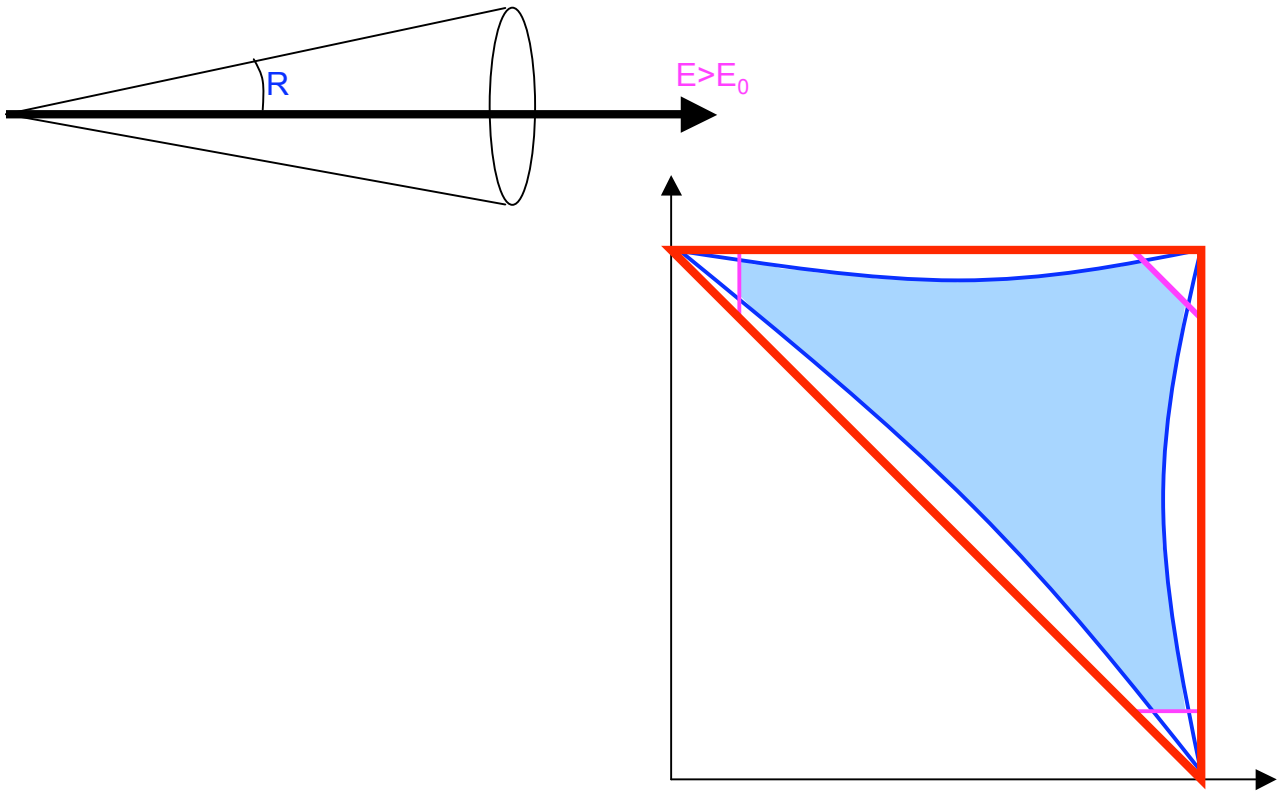


Three-jet cross section

- $\sigma = \sigma_0 \, C_F \frac{\alpha_s}{2\pi} \int dx_1 \, dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$
- Note:
  1. Total cross section infinite (see later…)
  2. Cross section depends on **jet definition**

Jet Definitions: 1) cone ("Sterman—Weinberg", "Snowmass")

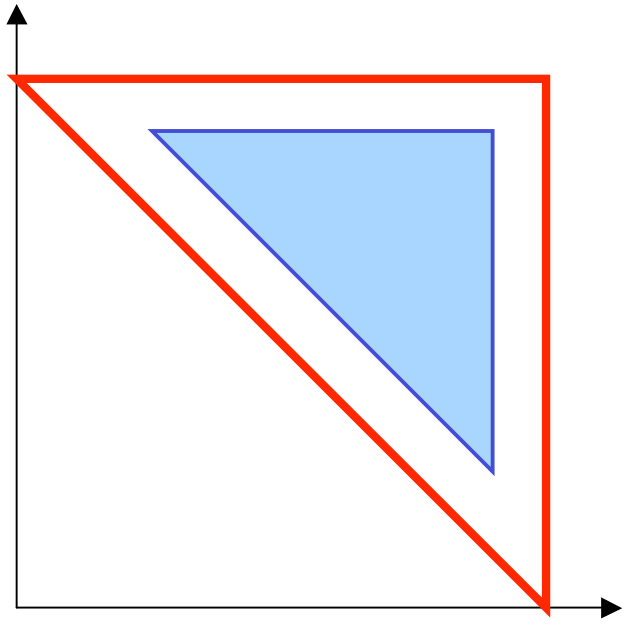
- Simple concept



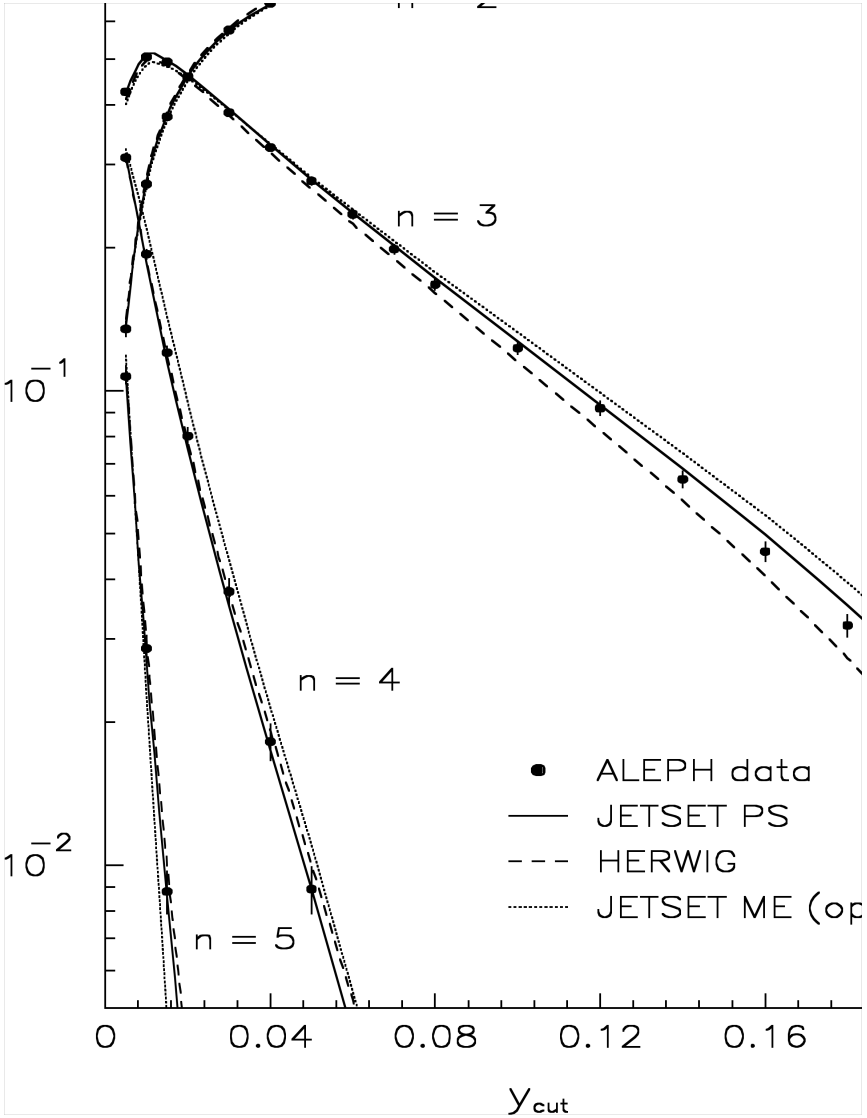
- Complicated in practice → rarely used in e^+e^-

Jet definitions: 2) cluster ("Jade", "Durham", "k_t")

- Define
  - 'closeness measure' y_{ij}
  - eg s_{ij}/s (JADE) or $k_t^2{}_{ij}/s$ (Durham)
  - 'combination scheme' $p_{ij} = p_i \otimes p_j$
  - eg p_i+p_j
- Iterate
  - find closest pair
  - combine them until all pairs are above y_{cut}
- Many theoretical advantages



- Note: jet cross section is (strongly) a function of jet definition
- Compare like with like in theory and experiment
- JADE n-jet rate as a function of y_{cut}



Deep Inelastic Lepton–Hadron Scattering

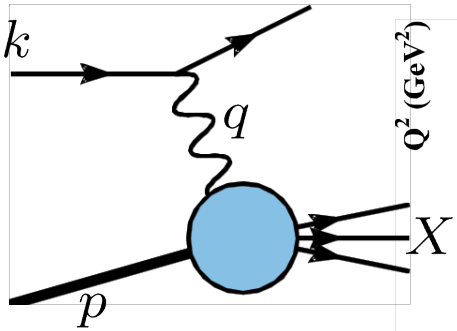
- Electron proton inelastic scattering = electron parton elastic scattering

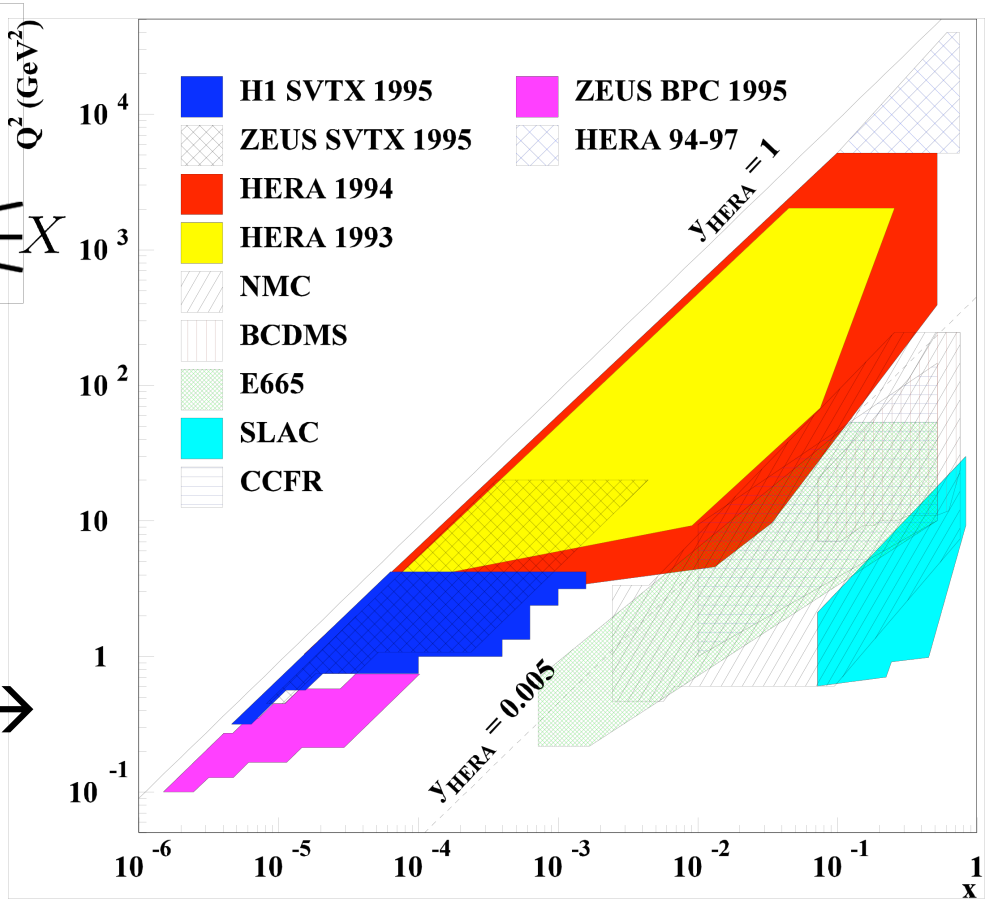
→ Inelastic scattering tells us about internal structure

- Q^2 gives resolving power

Lorentz invariant variables

$$
\begin{aligned}
Q^2 &= -q^2 \\
x &= \frac{Q^2}{2p\cdot q} \\
s &= (k+p)^2 \\
W^2 &= (p+q)^2 = Q^2\frac{1-x}{x} \\
y &= \frac{p\cdot q}{p\cdot k} = \frac{Q^2}{xs}
\end{aligned}
$$



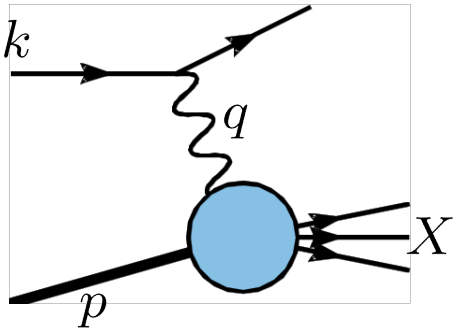


- Deep inelastic scattering → neglect proton mass M
- Kinematic limits:
  - $x>Q^2/s$
  - $Q^2<s$

- HERA: $s\sim 10^5$ GeV

General Form of Cross Section

$$d\sigma = \frac{1}{s}\sum_X \int dPS\, \frac{1}{4}\sum |\mathcal{M}|^2$$



- Phase space: $dPS = \dfrac{\pi Q^2}{2sx^2} dQ^2\, dx\, dPS_X$
- Matrix element: like e^+e^-, don't know about hadrons

$$\sum_X \int dPS_X \frac{1}{4}\sum |\mathcal{M}|^2 = \frac{e^4}{Q^4} L^{\mu\nu} H_{\mu\nu}$$

$\Rightarrow$ use Lorentz covariance, gauge invariance, etc:

$$H_{\mu\nu} = -H_1 g_{\mu\nu} + H_2 \frac{p_\mu p_\nu}{Q^2} + H_4 \frac{q_\mu q_\nu}{Q^2} + H_5 \frac{p_\mu q_\nu + q_\mu p_\nu}{Q^2}$$

- H_i: scalar fns of $q\cdot q = -Q^2$ and $p\cdot q = Q^2/2x$ only

$$\Rightarrow L^{\mu\nu} H_{\mu\nu} = 4k\cdot k'\, H_1 + 4\frac{p\cdot k\, p\cdot k'}{Q^2} H_2$$

- Define: $H_1 = 4\pi F_1$ and $H_2 = 8\pi x F_2$ to give:

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[y^2 x F_1(x,Q^2) + (1-y) F_2(x,Q^2)\right]$$

Other Forms

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[y^2 x F_1(x,Q^2) + (1-y)F_2(x,Q^2)\right]$$

- Parameterizes cross section in terms of two unknown functions: **structure functions**
- Note: only *s*-dependence comes via $y = Q^2/sx$
- Often see other linear combinations:

$$\begin{aligned} F_T(x,Q^2) &\equiv 2xF_1(x,Q^2) \\ F_L(x,Q^2) &\equiv F_2(x,Q^2) - 2xF_1(x,Q^2) \end{aligned}$$

  where $F_{T,L}$ correspond to scattering of transverse and longitudinally polarized photons

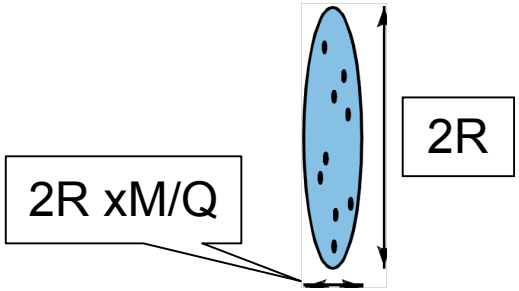
$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[(1+(1-y)^2)F_T(x,Q^2) + 2(1-y)F_L(x,Q^2)\right]$$

- Especially:

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[(1+(1-y)^2)F_2(x,Q^2) - y^2 F_L(x,Q^2)\right]$$

Parton Model

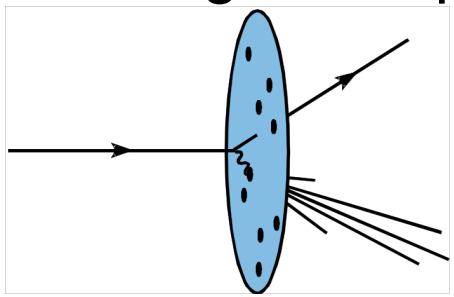
- Suppose that proton is a bound state of partons
- Look at proton in the Breit frame:



- Virtual photon sees tiny fraction of thin disk:

→ partons have no time to interact with each other

→ photon collides with single free parton



- Elastic electron—parton scattering

Parton Distribution Functions

- Suppose that parton type q carries a fraction η of proton's momentum a fraction $f_q(\eta)$ of the time
- Provided partons are
  - pointlike ($r^2 \ll 1/Q^2$)
  - dilute ($f(\eta) \ll Q^2 R^2_{\text{proton}}$)

  photons scatter incoherently off them:

$$\frac{d^2\sigma}{dx\,dQ^2} = \sum_q \int_0^1 d\eta\; f_q(\eta)\; \frac{d^2\sigma(e+q(\eta p))}{dx\,dQ^2}$$

- Outgoing parton on shell $\Rightarrow \eta = x$
- Therefore: $\frac{d^2\sigma}{dx\,dQ^2} = \sum_q f_q(x)\; \frac{d^2\sigma(e+q(xp))}{dQ^2}$

Callan Gross Relation

- Suppose partons = fermions:

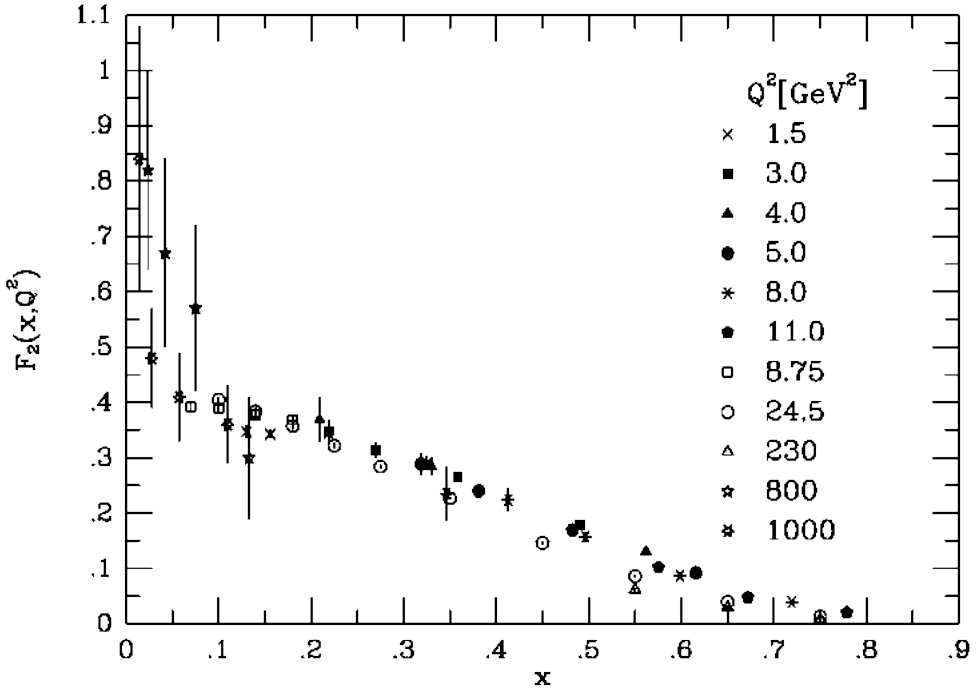
$$F_2 = \sum_q e_q^2 \, x \, f_q(x)$$
$$F_L = 0$$

→ partons = quarks!

- (partons = scalars $\Rightarrow F_T = 0$)

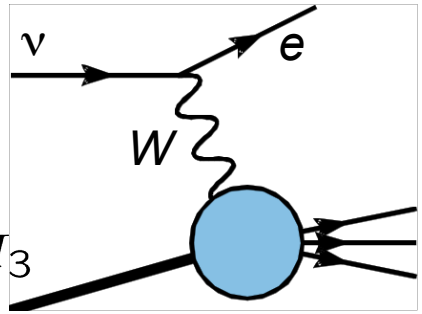
Bjorken Scaling

- Non-interacting pointlike partons → structure functions Q^2-independent…



Neutrino DIS (i.e. Charged Current)

- Extra parity-violating terms…



$$L^{\nu}_{\bar{\nu}\,\mu\nu} = L^{e}_{\mu\nu} \pm 2i\epsilon_{\mu\nu\rho\sigma}k^{\rho}k'^{\sigma}$$

$$H^{\mu\nu} = -H_1 g^{\mu\nu} + H_2\frac{p^{\mu}p^{\nu}}{Q^2} - \frac{i}{Q^2}\epsilon^{\mu\nu\rho\sigma}p_{\rho}q_{\sigma}H_3$$

$$\Rightarrow L^{\nu}_{\bar{\nu}\,\mu\nu}H^{\mu\nu} = 2Q^2H_1 + Q^2\frac{1-y}{x^2y^2}H_2 \pm \frac{Q^2}{xy}H_3\,(1-y/2)$$

- $H_3 = 8\pi x F_3$

$$\Rightarrow \frac{d^2\sigma^{\nu}_{\bar{\nu}}}{dx\,dQ^2} = \frac{G_F^2}{2\pi\,x}\overbrace{\left(\frac{M_w^2}{Q^2+M_w^2}\right)^2}^{\sim 1,\ Q^2 \ll M_w^2}$$
$$\left\{(1+(1-y)^2)F_2^{\nu} - y^2F_L^{\nu} \pm y(2-y)xF_3^{\nu}(x,Q^2)\right\}$$

- (Also get non-zero F_3 for neutral current for $Q^2 \gtrsim M_z^2$)

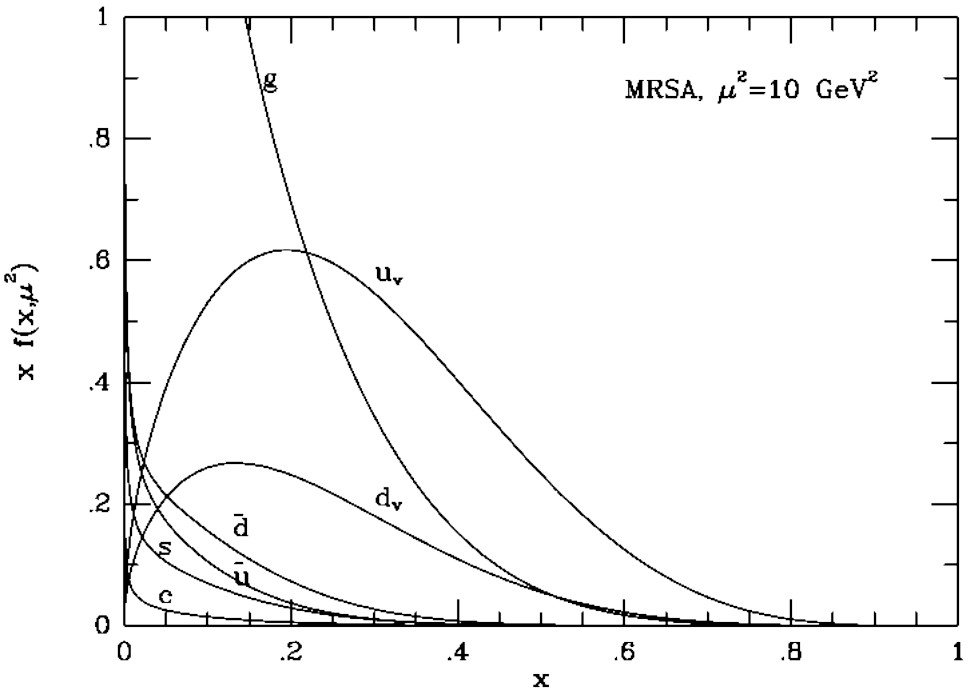
Parton Distribution Functions

- Constrain pdfs using many different beams/targets:

$$
\begin{aligned}
F_2^{ep}(x,Q^2) &= \tfrac{1}{9}xf_d + \tfrac{4}{9}xf_u + \tfrac{1}{9}xf_{\bar{d}} + \tfrac{4}{9}xf_{\bar{u}} + \tfrac{1}{9}xf_s + \ldots \\
F_2^{en}(x,Q^2) &= \tfrac{4}{9}xf_d + \tfrac{1}{9}xf_u + \tfrac{4}{9}xf_{\bar{d}} + \tfrac{1}{9}xf_{\bar{u}} + \tfrac{1}{9}xf_s + \ldots \\
F_2^{\nu p}(x,Q^2) &= 2xf_d + 2xf_{\bar{u}} + 2xf_s + 2xf_{\bar{c}} + \ldots \\
F_3^{\nu p}(x,Q^2) &= 2xf_d - 2xf_{\bar{u}} + 2xf_s - 2xf_{\bar{c}} + \ldots \\
F_2^{\bar{\nu} p}(x,Q^2) &= 2xf_u + 2xf_{\bar{d}} + 2xf_c + 2xf_{\bar{s}} + \ldots \\
F_3^{\bar{\nu} p}(x,Q^2) &= 2xf_u - 2xf_{\bar{d}} + 2xf_c - 2xf_{\bar{s}} + \ldots
\end{aligned}
$$

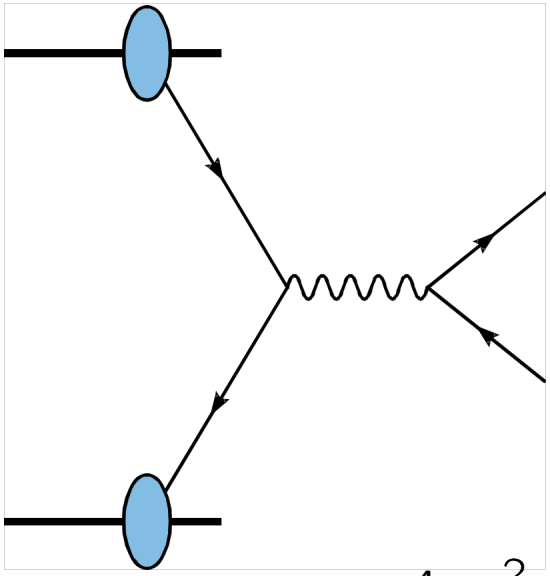
  etc…

→ Global fits…

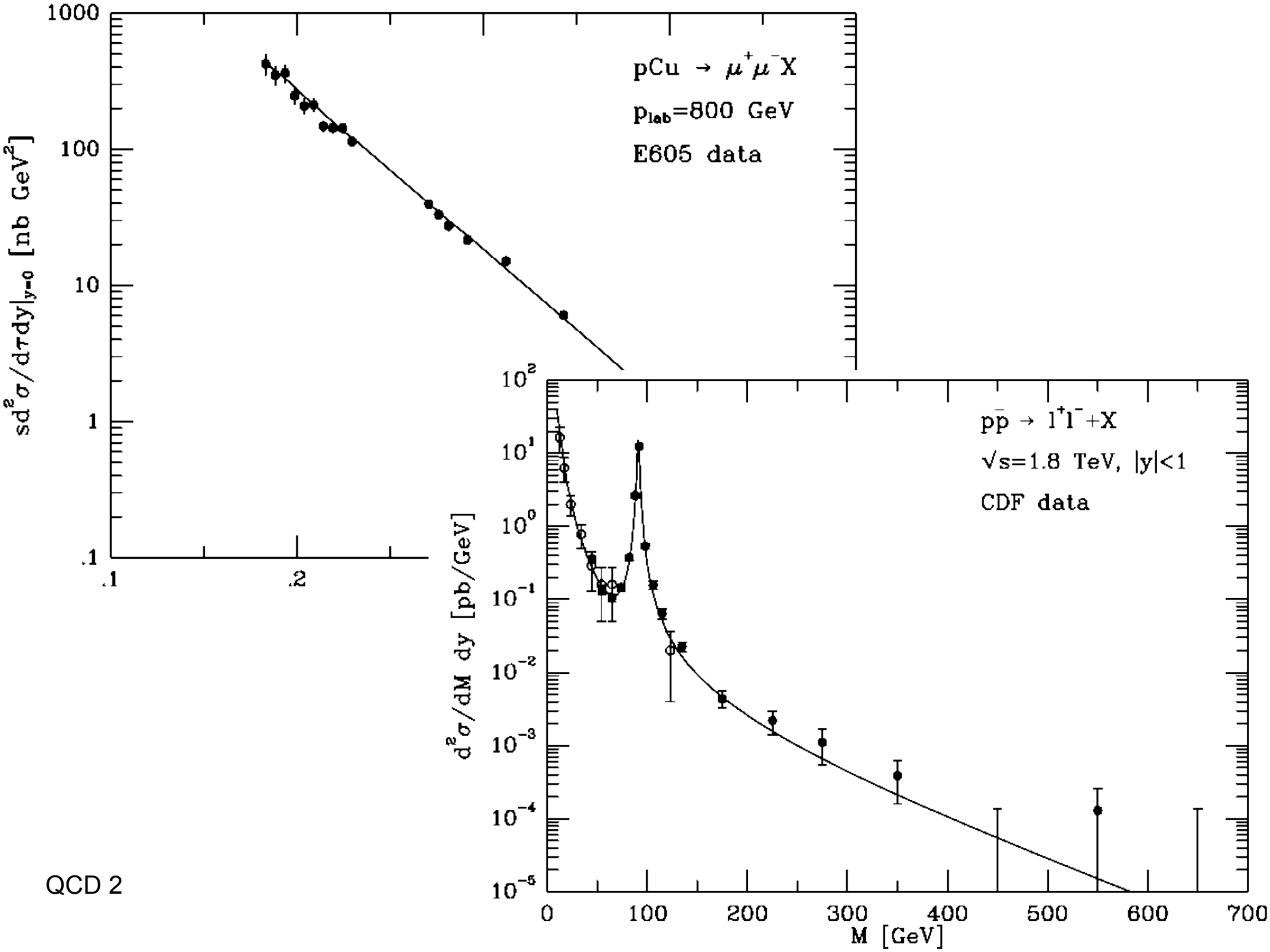

MRSA, μ^2=10 GeV^2
g
u_v
d_v
$\bar{d}$
s
$\bar{u}$
c
x f(x,μ^2)
x
0
.2
.4
.6
.8
1

The Drell—Yan Process

- If parton model is correct, pdfs measured in DIS should be applicable to hadron collisions,
- eg qqbar → $\mu^+\mu^-$, "Drell—Yan pair production"

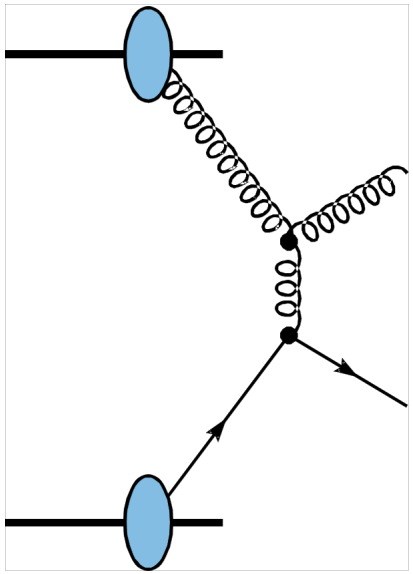


$$\sigma(q\bar{q} \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3M^2}\,\frac{1}{N_c}\,e_q^2$$


pCu → μ⁺μ⁻X
p_lab=800 GeV
E605 data
sd²σ/dτdy|y=0 [nb GeV²]
1000
100
10
1
.1
.1
2
p̄p → l⁺l⁻+X
√s=1.8 TeV, |y|<1
CDF data
d²σ/dM dy [pb/GeV]
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0
100
200
300
400
500
600
700
M [GeV]

Jets in Hadron Collisions

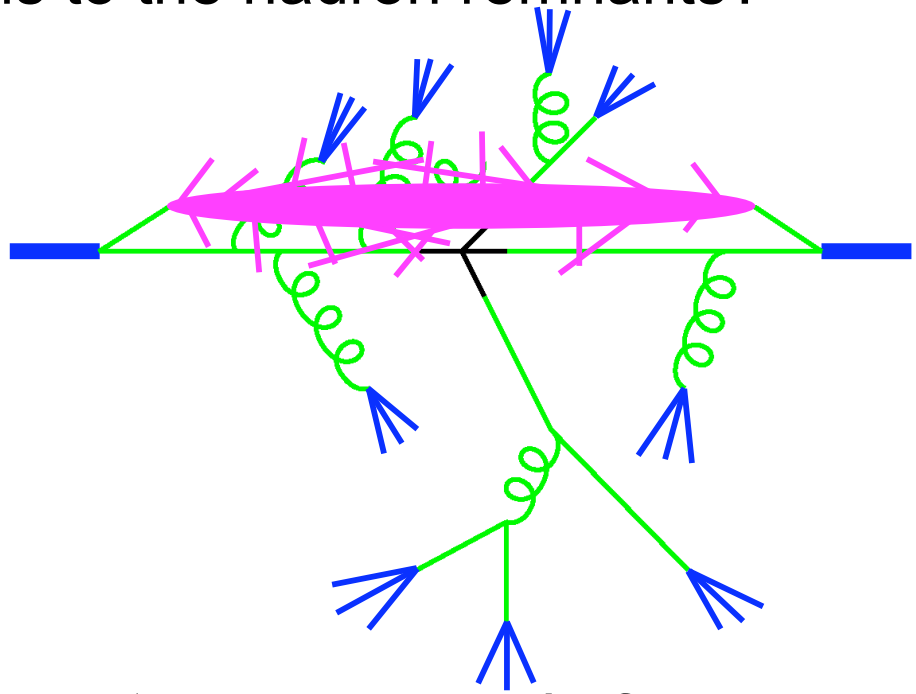
- Sensitive to gluon pdf
- Proportional to α_s^2



- Fits data over 6 orders of magnitude in rate!

The ‘Underlying’ Event

- Incoming hadrons are colour singlets
- Annihilation/scattering removes coloured parton
- What happens to the hadron remnants?



- Non-perturbative ‘contamination’ of all jet cross sections

Summary

- Most QCD phenomenology can be understood from tree level calculations, but…

- α_s is not so small → higher order corrections?
- can we derive parton model from QCD?