Neutrino Physics

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Plan of Lectures

Lecture I:

- Introduction
- Neutrino masses and mixings: Dirac versus Majorana
- Neutrino oscillations in vacuum and in matter

Lecture II:

- Evidence for ν masses
- The standard scenario of 3-ν mixing

Lecture III:

- Propects in neutrino physics
- Leptogenesis
- Theory outlook

Introduction

LEP era has established the validity of the Standard Model (SM) based on the gauge group:

$$SU(3) \times SU(2) \times U(1) \overset{SSB}{\rightarrow} \quad SU(3) \times U(1)_{EM}$$

below the per cent level!

$\nu's$ are the most elusive particles of the MSM:

$$m_\nu \simeq 0, \;\; Q_{em} = 0, \;\; Q_{color} = 0, \;\; Q_{SU(2)\times U(1)/U(1)_{em}} \neq 0$$

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"I have done a terrible thing, I have postulated a particle that cannot be detected"

W. Pauli

Not only have they been detected, but have provided essential information on the two most striking features of the fermionic content of the SM:

- Left-handedness of the weak interactions
- The family structure

$(1,2)_{-\frac{1}{2}}$	$(3,2)_{-\frac{1}{6}}$	$(1,1)_{-1}$	$(3,1)_{-\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

Only left-handed fields carry $SU(2)$ charge:

$$\Psi = \Psi_R + \Psi_L = \underbrace{\left(\frac{1+\gamma_5}{2}\right)}_{P_R}\Psi + \underbrace{\left(\frac{1-\gamma_5}{2}\right)}_{P_L}\Psi$$

For $v \longrightarrow c$:

$$P_{R,L} = \frac{1}{2}\left(1 \pm 2\frac{\vec{s}\cdot\vec{p}}{|p|}\right) + O\left(\frac{m_i}{E}\right)$$

$Q_{SU(2)} \neq 0$		$Q_{SU(2)} = 0$	
L		R	
$\longrightarrow$		$\longrightarrow$	$\vec{p}$
$\Leftarrow$		$\Rightarrow$	$\vec{s}$

$\mathcal{CPT} \leftrightarrow$ Lorentz invariance $\oplus$ unitarity: is an essential building block of field theory

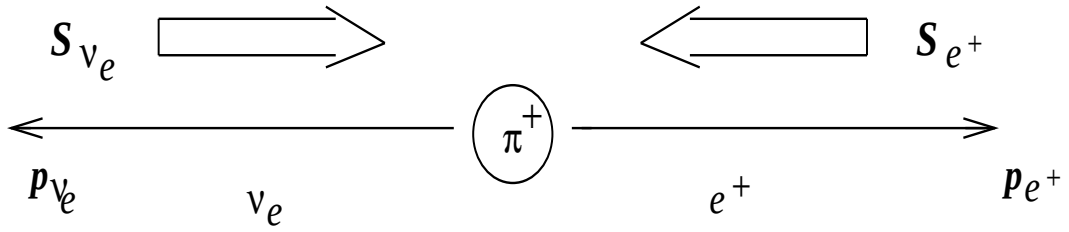
$\mathcal{CPT}$: L particle $\leftrightarrow$ R antiparticle

Neutrinos in the MSM are massless and exist only in two states: particle with negative helicity and antiparticle with positive one: Weyl fermion

$\mathcal{P}$: L particle $\leftrightarrow$ R particle

Parity violation is nowhere more obvious than in the neutrino sector: the reflection of a left-handed neutrino in a mirror is nothing !

The weak interactions are $V - A$:

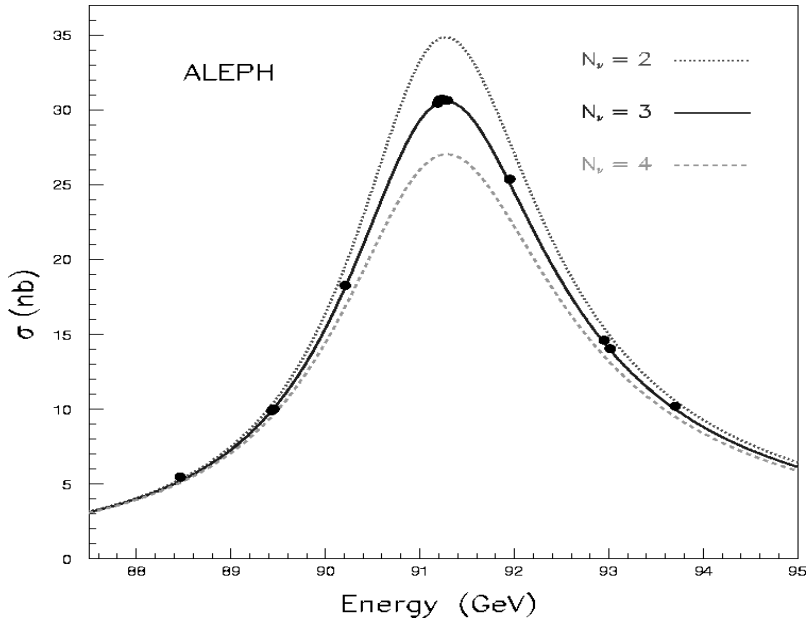


$$R_\pi \equiv \frac{\Gamma(\pi^+ \to e^+ + \nu_e)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.28 \times 10^{-4}$$

to be compared the measured: $R_\pi = (1.230 \pm 0.004) \times 10^{-4}$

The Z^0 decays invisibly into exactly three ν:

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{\nu\bar{\nu}}} = 2.984 \pm 0.008$$

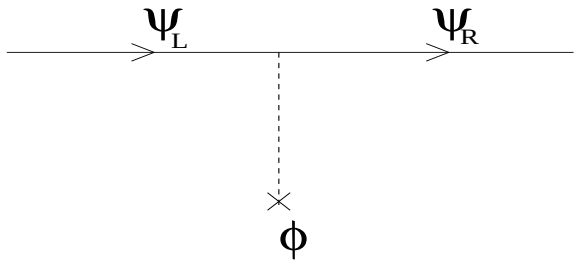


Fermion masses

A mass can be thought of as a $L \leftrightarrow R$ transition:

$$m\ \overline{\psi_L}\psi_R + h.c.$$

In the SM fermion masses originate in the interaction with the Higgs field:

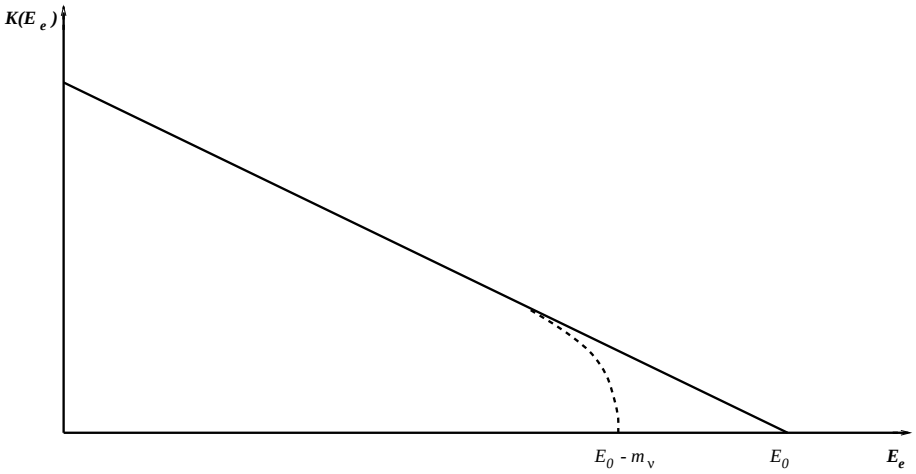


$$\lambda_f\ \overline{\psi_L}\ \Phi\ \psi_R + h.c. \rightarrow m_f = \lambda_f\ v$$

When the SM was invented there were only upper bounds on the ν masses from weak decay kinematics and these were conjectured to be zero.

Most sensitive direct measurement of ν mass is from Tritium β decay:

$$H^3 \rightarrow^3 He + e^- + \bar{\nu}_e$$



$m_{\nu_e} < 2.2$eV (Mainz-Troitsk)

$m_{\nu_\mu} < 170$keV (PSI: $\pi^+ \rightarrow \mu^+ \nu_\mu$)

$m_{\nu_\tau} < 18.2$MeV (LEP: $\tau^- \rightarrow 5\pi\nu_\tau$)

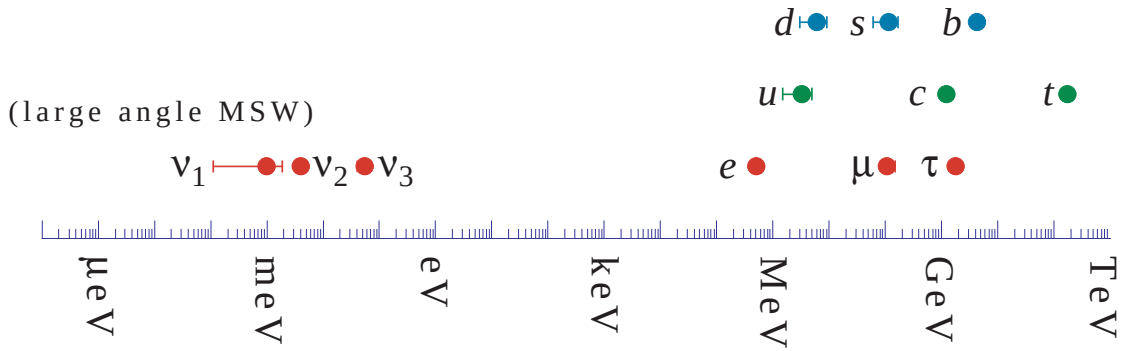
$$K(E_e) \propto \sqrt{(E_0 - E_e)((E_0 - E_e)^2 - m_\nu^2)^{1/2}}$$

ν masses: I. Dirac way

ν masses can be easily added to the MSM in the same way as for the remaining leptons: add three singlets,$(\nu_\alpha)_R, \alpha = e, \mu, \tau$ with $Q_{SU(3)} = Q_{SU(2)} = Q_{U(1)} = 0$ and proceed like for the quarks and charged leptons:

$$m_\nu = \lambda_\nu v \qquad \lambda_\nu \ll \lambda_l$$

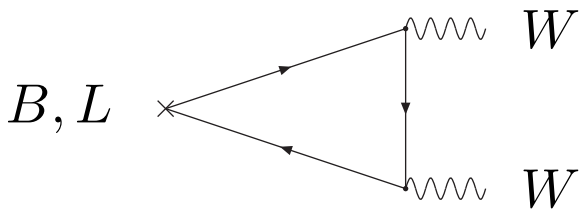
- New hierarchy problem: why $m_{\nu_i} \ll m_{l_i}$?



- $B - L$ is an exactly conserved global symmetry:

$$B : \Psi_q \to e^{i\alpha/3}\, \Psi_q \quad L : \Psi_l \to e^{i\alpha} \Psi_l$$

B, L are conserved at the classical level, but $B + L$ is broken by the anomaly:



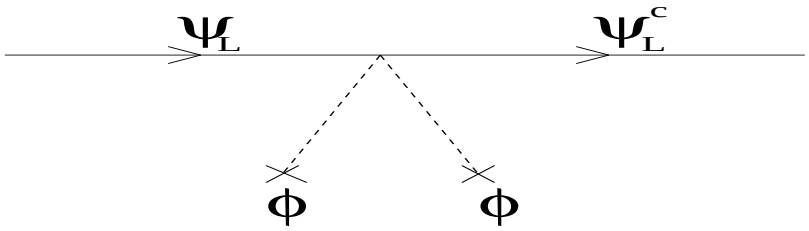
$$\partial_\mu J^B_\mu = \partial_\mu J^L_\mu = \frac{3g_W^2}{32\pi^2}\,\epsilon^{\mu\nu\alpha\beta} W^a_{\mu\nu} W^a_{\alpha\beta}$$

ν masses: II. Majorana way

CPT : L particle $\leftrightarrow$ R antiparticle

For neutral particles, Majorana realized that one can get rid of half of the degrees of freedom in a massive Dirac spinor by identifying:

$$\psi = \psi_L + \psi_R \qquad \psi_R \to (\psi_L)^c = C\bar{\psi}_L^T = C\gamma_0\psi_L^*$$



Weinberg

In the SM one can indeed write a $L \leftrightarrow R$ coupling of this form in a gauge invariant way:

$$\frac{1}{\Lambda}\nu_L^T C\ \alpha_\nu \tilde{\Phi}^T\ \tilde{\Phi}\ \nu_L + h.c. \to m_\nu = \alpha_\nu \frac{v^2}{\Lambda}$$

Only as a d=5 operator: a new scale Λ is needed! . If $\Lambda \gg v$ neutrinos will be naturally much lighter than the remaining fermions!

The consequences of this alternative are profound:

- Physics beyond the SM at a scale Λ!

- Majorana fermions carry no conserved charge: L is violated !

$$\nu_L \to e^{i\alpha}\nu_L$$

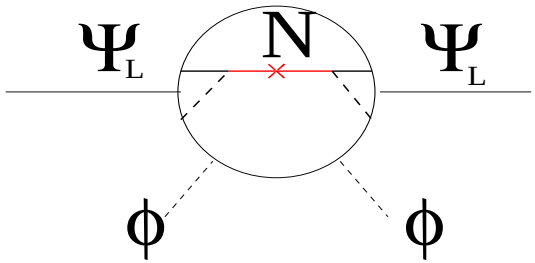
does not leave the Majorana mass term invariant.

$\longrightarrow$ Most welcome for baryogenesis: a mechanism to understand the matter-antimatter asymmetry in the Universe emerges naturally

See-saw Model

Minkowski, Gell-Mann, Ramond, Slansky, Yanagida '79

Add to the SM heavy Majorana neutrinos:



$$\begin{aligned}\delta\mathcal{L}_Y^\nu &= \bar{L}_L\tilde{\lambda}_\nu\,\tilde{\Phi}\,N_R + \frac{1}{2}N_R^T C\,M_R\,N_R + h.c.\\ &\to \frac{1}{2}\begin{pmatrix}\nu_L^T & N_R^{cT}\end{pmatrix} C \begin{pmatrix}0 & \tilde{\lambda}_\nu v\\ \tilde{\lambda}_\nu^T v & M_R\end{pmatrix}\begin{pmatrix}\nu_L\\ N_R^c\end{pmatrix}\end{aligned}$$

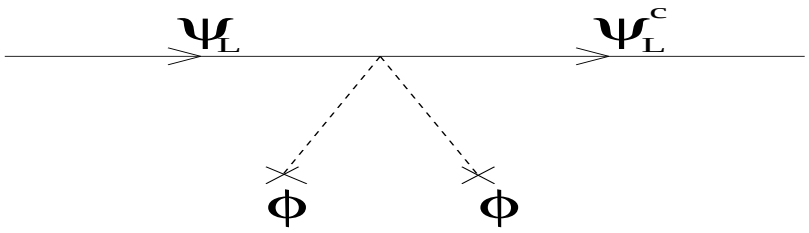
When $v \ll M_R$, the diagonalization of the mass matrix can be done in perturbation theory:

$$\mathcal{M} = \mathcal{M}^{(0)} + \delta\mathcal{M} \equiv \begin{pmatrix}0 & 0\\ 0 & M_R\end{pmatrix} + \begin{pmatrix}0 & \tilde{\lambda}_\nu v\\ \tilde{\lambda}_\nu^T v & 0\end{pmatrix}$$

To second order we find: $\nu_L' = \nu_L + \tilde{\lambda}_\nu \frac{v}{M_R} N_R$

$$U^T\mathcal{M}U = \begin{pmatrix}-v^2\tilde{\lambda}_\nu\frac{1}{M_R}\tilde{\lambda}_\nu^T & 0\\ 0 & M_R\end{pmatrix} \quad U = \begin{pmatrix}1 & \tilde{\lambda}_\nu\frac{v}{M_R}\\ -\frac{v}{M_R}\tilde{\lambda}_\nu^T & 1\end{pmatrix}$$

Equivalently we say that the heavy Majoranas can be integrated out leaving a trace of higher dimensional operators:



$$\mathcal{L}_{eff}^{d=5} = \frac{1}{2} L_L^T C \; \tilde{\Phi}^T \left(\tilde{\lambda}_\nu \frac{1}{M_R} \tilde{\lambda}_\nu^T \right) \; \tilde{\Phi} \; L_L$$

$$\mathcal{L}_{eff}^{d=6} = \mathcal{O}\left(\frac{1}{M_R^2} \right) ....$$

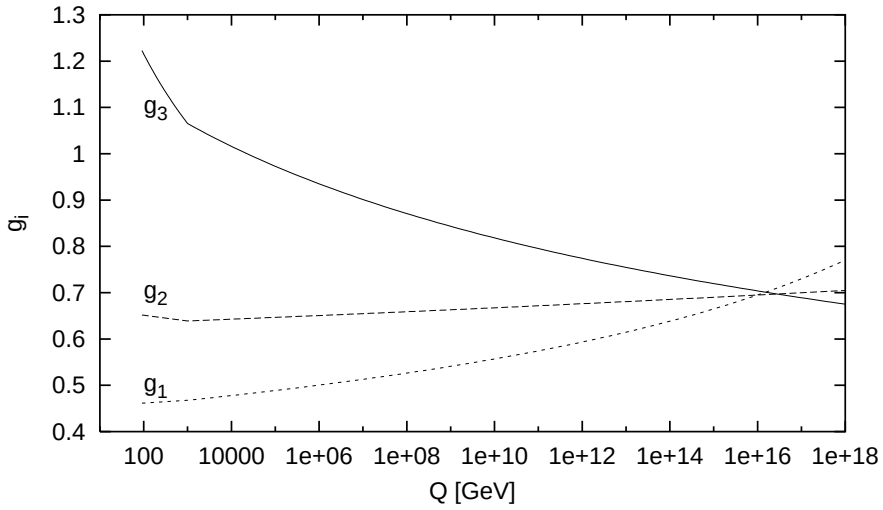
- The new physics scale in this case is simply the masses of these heavy Majorana neutrinos:

$$\frac{\alpha_\nu}{\Lambda} = \tilde{\lambda}_\nu \frac{1}{M_R} \tilde{\lambda}_\nu^T$$

Data imply there is at least one $m_\nu \geq 0.05\text{eV}$. If $\tilde{\lambda}_\nu \sim O(1)$:

$$v \; < \; M_R \sim 10^{15}\text{GeV} \; < \; M_{Planck}$$

close to the typical Grand Unification (GUT) scale !



- In order to give masses to all the three left-handed neutrinos $n_R \geq n_L = 3$

  The matrix $\underbrace{\tilde{\lambda}_\nu}_{n_L \times n_R} \underbrace{\frac{1}{M_R}}_{n_R \times n_R} \underbrace{\tilde{\lambda}_\nu^T}_{n_R \times n_L}$ has $n_L - n_R$ zero modes

Dirac or Majorana ?

All differences between Dirac and Majorana at low-energies $E, v \ll \Lambda$ are in the mass term:

$$\mathcal{L}_m^{\text{Dirac}} = \overline{\nu_L}\lambda_\nu v \nu_R + h.c. \qquad \mathcal{L}_m^{\text{Majorana}} = \frac{1}{2}\nu_L^T C \alpha_\nu \frac{v^2}{\Lambda}\nu_L + h.c.$$

- If $\lambda_\nu = \alpha_\nu = 0$, both cases are indistinguishable
- Mixing in the lepton sector

$$\mathcal{L}^{\text{Dirac}} = \bar{l}_L^i \gamma_\mu W_\mu^+ V_{MNS}^{ij} \nu_L^j + \frac{1}{2}\overline{\nu_L^i}\gamma_\mu Z_\mu \nu_L^i + \overline{\nu_L^i} m_i \nu_R^i + h.c.$$

$$\mathcal{L}^{\text{Majorana}} = \bar{l}_L^i \gamma_\mu W_\mu^+ \tilde{V}_{MNS}^{ij} \nu_L^j + \frac{1}{2}\overline{\nu_L^i}\gamma_\mu Z_\mu \nu_L^i + \frac{1}{2}{\nu_L^i}^T C m_i \nu_L^i + h.c.$$

In both cases, there is neutrino mixing as in the quark sector.

Number of physical parameters in the lepton mixing matrix is different:

	Yukawas	Field redefinitons	$\#m$	$\#\theta$	$\#\phi$
Dirac	$\lambda_l,\ \lambda_\nu$ $2n^2 + 2n^2$	$U(n)^3/U(1)_L$ $\frac{3(n^2-n)}{2}, \frac{3(n^2+n)}{2} - 1$	$2n$	$\frac{n^2-n}{2}$	$\frac{(n-2)(n-1)}{2}$
Majorana	$\lambda_l,\ \alpha_\nu^T = \alpha_\nu$ $3n^2 + n$	$U(n)^2$ $n^2 - n,\ n^2 + n$	$2n$	$\frac{n^2-n}{2}$	$\frac{n^2-n}{2}$

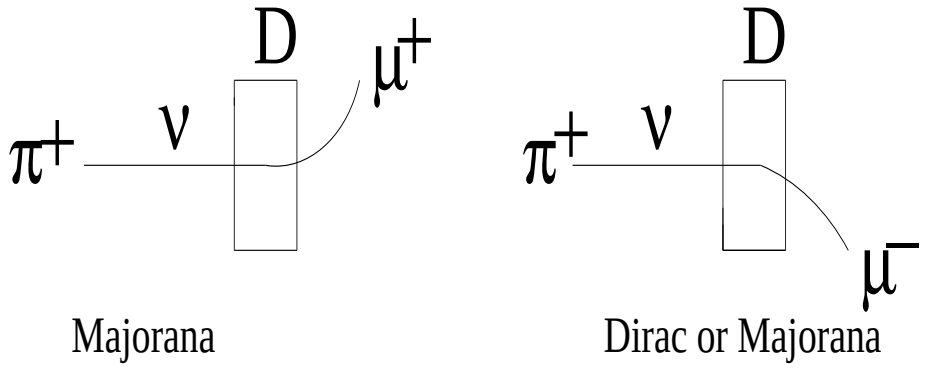
$$v\lambda_l = V_l^\dagger m_l \tilde{V}_l \quad v\lambda_\nu = V_\nu^\dagger m_\nu \tilde{V}_\nu \quad \frac{v^2}{M}\alpha_\nu = O_\nu^T m_\nu O_\nu$$

$$V_{MNS} = V_l^+ V_\nu \equiv$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{V}_{MNS} = V_l^\dagger O_\nu \equiv V_{MNS}(\theta_{12}, \theta_{13}, \theta_{23}, \delta) \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}$$

In principle very clear signatures:



In practice, these processes are suppressed by $(m_\nu/E)^2$: very rare!

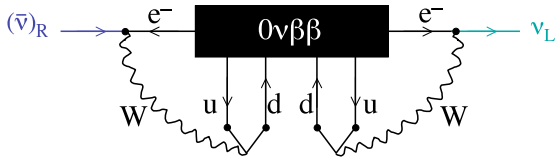
e.g: $E_\nu = O(1)$MeV, $m_\nu \sim O(1)$eV $\rightarrow$ the cross section for this process will be roughly 10^{-12} times the usual CC neutrino cross-section !

Best hope neutrinoless double-beta decay

$T_{2\beta 2\nu} > 10^{19} - 10^{21}$years $\quad T^{-1}_{2\beta 0\nu} \sim \left(\frac{m_\nu}{E}\right)^2 10^9 \, T^{-1}_{2\beta 2\nu}$

If ν are Majorana particles, this process must be there, and if this process is there ν are Majorana particles...

Schetcher,Valle



If the source of L violation is just the ν mass

$$T^{-1}_{2\beta 0\nu} \simeq \underbrace{G^{0\nu}}_{\text{Phase}} \underbrace{\left|M^{0\nu}\right|^2}_{\text{NuclearM.E.}} \underbrace{\left|\sum_i \left(V^{ei}_{MNS}\right)^2 m_i\right|^2}_{|m_{ee}|^2}$$

Best present bounds and a controversial positive signal...

Experiment	Nucleus	$\lvert m_{ee}\rvert$
Heidelberg-Moscow I	^{76}Ge	$< 0.34 - 1.1eV(90\%CL)$
Heidelberg-Moscow II	^{76}Ge	$0.2 - 0.6eV$
CUORICINO	^{120}Te	$< 0.2 - 1.1eV(90\%CL)$
NEMO-3	^{100}Mo	$< 0.6 - 2eV(90\%CL)$

Neutrino oscillations

The fact that ν are so weakly interacting particles allows them to have coherence over very long distances

E.g: $E_\nu = 1$ MeV moving in lead $\rho = 7.9 gr/cm^3$ will have a mean free path $l \sim \frac{1}{\sigma\rho} \sim 4 \times 10^{16} m \sim 4$ light-years !!

Neutrinos produced in a flavour eigenstate can change flavour as they propagate, because they are not mass eigenstates

Pontecorvo

Many ways to derive the oscillation formula, all require:

- there must be uncertainty in momentum and/or energy in the production of the neutrino (we know they are better localized than L)
- propagating neutrinos are on-shell (macroscopic distances)
- there must be coherence of the mass eigenstates

Neutrino source at $\vec{x} = (0, 0, 0)$

$$|\nu_\alpha^S\rangle = \sum_i V_{\alpha i} \int d^3k \; f_i(\vec{k}) \; |i; \vec{k}\rangle \; e^{-iE_i t}, \quad {E_i}^2 = \vec{k}^2 + m_i^2$$

Neutrino detector at $\vec{x}_D = (L, 0, 0)$

$$|\nu_\beta^D\rangle = \sum_j V_{\beta j} |j; \vec{x}_D\rangle$$

Oscillation probability:

$$P(\nu_\alpha \to \nu_\beta) = \frac{\int dt |\langle \nu_\beta^D | \nu_\alpha^S \rangle|^2}{\sum_\beta \int dt |\langle \nu_\beta^D | \nu_\alpha^S \rangle|^2}$$

The wave packets $f_i(k)$ depend on the production/detection process (uncertainty in momentum of the inicial states, kinematics), but oscillation formula does not depend on this, e.g. a simple choice:

$$f_i(\vec{k}) \sim e^{-(k_x-q)^2/(2\sigma^2)}\delta(k_y)\delta(k_z)$$

$$\int dt|\langle\nu_\beta^D|\nu_\alpha^S\rangle|^2 = \sum_{i,j} V_{\alpha i}V_{\beta i}^* V_{\alpha j}^* V_{\beta j} \int_{k_x,k_x'} \delta\left(\sqrt{k_x^2+m_i^2}-\sqrt{k_x'^2+m_j^2}\right) \times$$

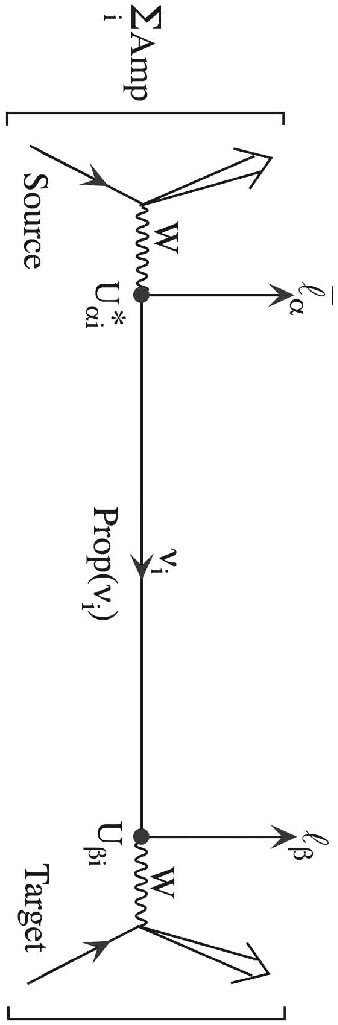
$$e^{-(k_x-q)^2/(2\sigma^2)} e^{-(k_x'-q)^2/(2\sigma^2)}\ e^{i(k_x-k_x')L}$$

$$\simeq_{m_i\ll q} \sum_{i,j} V_{\alpha i}V_{\beta i}^* V_{\alpha j}^* V_{\beta j} \int dk_x\ e^{-(k_x-q)^2/(2\sigma^2)}\ e^{-\left(\sqrt{k_x^2+\Delta_{ij}}-q\right)^2/(2\sigma^2)}\ e^{i\frac{\Delta_{ij}}{2k_x}L}$$

$$\simeq_{\text{coherence}} \sum_{i,j} V_{\alpha i}V_{\beta i}^* V_{\alpha j}^* V_{\beta j}\ e^{i\frac{\Delta_{ij}}{2q}L} \int dk_x e^{-(k_x-q)^2/\sigma^2}$$

$$P(\nu_\alpha \to \nu_\beta) \simeq \sum_{i,j} V_{\alpha i}V_{\beta i}^* V_{\alpha j}^* V_{\beta j} e^{-i\frac{\Delta_{ij}L}{2E}} \quad \Delta_{ij} \equiv m_i^2 - m_j^2$$

A more systematic treatment requires looking at the ν a virtual state



Using the unitarity of the mixing matrix: ($W^{jk}_{\alpha\beta} \equiv [V_{\alpha j} V^*_{\beta j} V^*_{\alpha k} V_{\beta k}]$)

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{k>j} \mathrm{Re}[\mathrm{W}^{\mathrm{jk}}_{\alpha\beta}] \sin^2\left(\frac{\Delta \mathrm{m}^2_{\mathrm{jk}}\, \mathrm{L}}{4\mathrm{E}_\nu}\right)$$

$$\pm \; 2 \sum_{k>j} \mathrm{Im}[\mathrm{W}^{\mathrm{jk}}_{\alpha\beta}] \sin\left(\frac{\Delta \mathrm{m}^2_{\mathrm{jk}}\, \mathrm{L}}{2\mathrm{E}_\nu}\right)$$

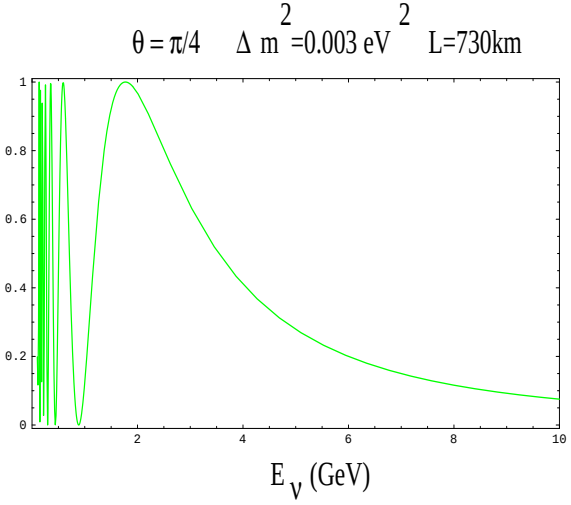
Oscillation probabilities show the expected GIM suppression of any flavour changing process: they vanish if the neutrinos are degenerate

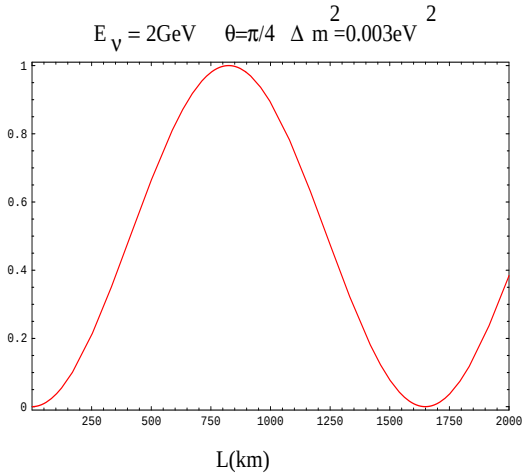
For 2 families: $V_{MNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$$P_{\alpha\beta} = \sin^2 2\theta \ \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right) \rightarrow \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance}$$

$$L_{\text{osc}}(km) = 2\pi \frac{E_\nu(GeV)}{1.27\Delta m^2(eV^2)}$$





Observations:

- There is an intrinsic limit to coherence, because $\sigma \neq 0$!

$$\left| \frac{\Delta_{ij} L_D}{2} \left(\frac{1}{q} - \frac{1}{q+\sigma} \right) \right| \sim 2\pi \Rightarrow L_D \sim L_{osc} \frac{|\vec{q}|}{\sigma}$$

If $L \gg L_D$:

$$P\left(\nu_\alpha \to \nu_\beta\right) = \sum_i \left|V_{\alpha i} V_{\beta i}\right|^2 = 2\cos^2\theta \sin^2\theta = \frac{1}{2}\sin^2 2\theta$$

- In practice, the smearing in L and E_ν produces the same effect when $L \gg L_{osc}$: averaged oscillations

$$\left\langle P\left(\nu_\alpha \to \nu_\beta\right)\right\rangle = \frac{1}{2}\sin^2 2\theta$$

- Smoking guns for neutrino oscillations are:

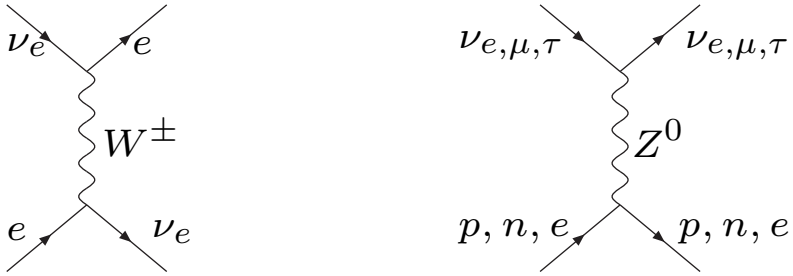
1. flavour transitions $\longrightarrow$ need experiments that can tell flavour
2. the L/E_ν dependence $\longrightarrow$ need experiments with $L \sim L_{\rm osc}(E_\nu)$

Measuring only flavour transitions imply neutrino mixing but not oscillations

Neutrino oscillations in matter

When neutrinos propagate in matter (Earth, Sun...), the amplitude for this propagation is modified due to coherent forward scattering on electrons and nucleons:

Wolfenstein 1978



$$\mathcal{H}_{CC} = 2\sqrt{2}G_F\,[\bar{e}\gamma_\mu P_L \nu_e][\bar{\nu}_e\gamma^\mu P_L e] = 2\sqrt{2}G_F\,[\bar{e}\gamma_\mu P_L e][\bar{\nu}_e\gamma^\mu P_L \nu_e]$$

$$\langle \bar{e}\gamma_\mu P_L e\rangle_{\text{unpol.medium}} \quad = \quad \delta_{\mu 0}\frac{N_e}{2}$$

$$\mathcal{H}^{eff} = \mathcal{H}^{vac} + \bar{\nu} V_m \gamma^0 (1 - \gamma_5) \nu$$

$$V_m = \begin{pmatrix} \frac{G_F}{\sqrt{2}} \left(N_e - \frac{N_n}{2} \right) & 0 & 0 \\ 0 & \frac{G_F}{\sqrt{2}} \left(-\frac{N_n}{2} \right) & 0 \\ 0 & 0 & \frac{G_F}{\sqrt{2}} \left(-\frac{N_n}{2} \right) \end{pmatrix}$$

E.g.: In the center of the sun $V_e \sim 10^{-12} eV$, Earth $V_e \sim 10^{-13} eV$

There is a new dispersion relation is complicated because M_ν, V_m are not diagonal on the same basis:

$$E - V_m - M_\nu = (\pm|\vec{p}| + V_m) \frac{1}{E + M_\nu - V_m} (\pm|\vec{p}| + V_m) \quad h = \pm$$

Approximations: neglet effects of $O(M_\nu^3), O(V^2), O(V M_\nu)$ which should be ok for $m_\nu \gg V_m$

$$E > 0 \;\; E^2 = |p|^2 + M_\nu^2 + 4EV_m \quad h = - \quad E^2 = |p|^2 + M_\nu^2, \; h = +$$

For antineutrinos $V_m \to -V_m$.

We can then describe neutrino oscillations in a medium as in vacuum, but with and effective mass matrix given by

$$\tilde{M}^2_\nu = M^2_\nu \pm 4EV_m$$

The mixing matrix is the one that takes us from the original flavour basis to that which diagonalizes the effective Hamiltonian:

$$\begin{pmatrix} \tilde{m}_1^2 & 0 & 0 \\ 0 & \tilde{m}_2^2 & 0 \\ 0 & 0 & \tilde{m}_3^2 \end{pmatrix} = \tilde{U}_{MNS} \left(M^2_\nu \pm 4E \begin{pmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau \end{pmatrix} \right) \tilde{U}^+_{MNS}$$

- The matter potential does not change the number of physical parameters in the mixing matrix: $\tilde{\theta}_{12}, \tilde{\theta}_{13}, \tilde{\theta}_{23}, \tilde{\delta}$
- The mixing angles and effective masses depend on the energy !

Neutrino oscillations in constant density

$N_\nu = 2$ simple case: ($- \to$ neutrinos, $+ \to$ antineutrinos)

$$\sin^2 2\tilde{\theta} = \frac{\left(\Delta m^2 \sin 2\theta\right)^2}{\left(\Delta m^2 \cos 2\theta \pm 2\sqrt{2}\, G_F E\, N_e\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}$$

$$\Delta\tilde{m}^2 = \sqrt{\left(\Delta m^2 \cos 2\theta \pm 2\sqrt{2} E\, G_F\, N_e\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}$$

The oscillation amplitude has a resonance: MSW effect

Mikheyev, Smirnov 1985

$$\sqrt{2}\, G_F\, N_e = \frac{\Delta m^2}{2E} \cos 2\theta \quad \Rightarrow \sin^2 2\tilde{\theta} = 1 \quad \Delta\tilde{m}^2 = \Delta m^2 \sin 2\theta$$

The oscillation amplitude can be maximal even with a very small vacuum mixing angle!

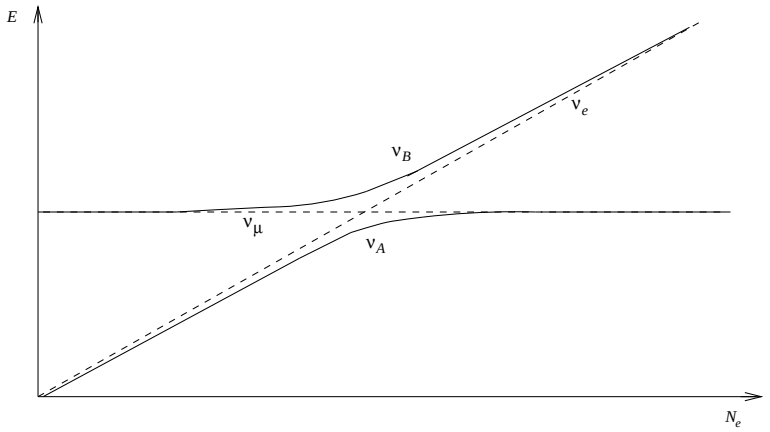
Observations:

- Oscillations vanish at $\theta = 0$, because the oscillation length becomes infinite for $\theta = 0$
- The resonance is only there for ν or $\bar{\nu}$ but not both
- The resonance condition depends on the sign$(\Delta m^2 \cos 2\theta) \Rightarrow$ we can measure the hierarchy!

Resonance in $\nu \rightarrow \Delta m^2 \cos 2\theta > 0$ Resonance in $\bar{\nu} \rightarrow \Delta m^2 \cos 2\theta < 0$

Varying density: adiabatic approximation

In the sun the density of electrons is not constant. However, if the variation is sufficiently slow, the eigenstates of H_{eff} change slowly with the density and we can assume that the neutrino remains an eigenstate along the trayectory: adiabatic approximation



$$\left.\begin{aligned}\tilde{\nu}_1 &= \nu_e \cos\tilde{\theta} + \nu_\mu \sin\tilde{\theta} \\ \tilde{\nu}_2 &= -\nu_e \sin\tilde{\theta} + \nu_\mu \cos\tilde{\theta}\end{aligned}\right\}$$

$$\begin{aligned} x = 0 \qquad & 2\sqrt{2}G_F N_e \gg \Delta m^2 \cos 2\theta \Rightarrow \tilde{\theta} \to \tfrac{\pi}{2} \quad \nu_e \simeq \tilde{\nu}_2 \\ x = R_\odot \qquad & N_e = 0 \Rightarrow \quad \tilde{\theta} \to \theta \quad \nu_\mu \simeq \nu_2 \end{aligned}$$

If θ is small, an electron neutrino produced at $x = 0$ is the eigenstate $\tilde{\nu}_2$, but this eigenstate outside the sun is mostly ν_μ. There is maximum $\nu_e \to \nu_\mu$ conversion!

The conditions for this:

- The density at the production is above the critical one (resonant condition)

$$N_e(x=0) > \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2} E G_F}$$

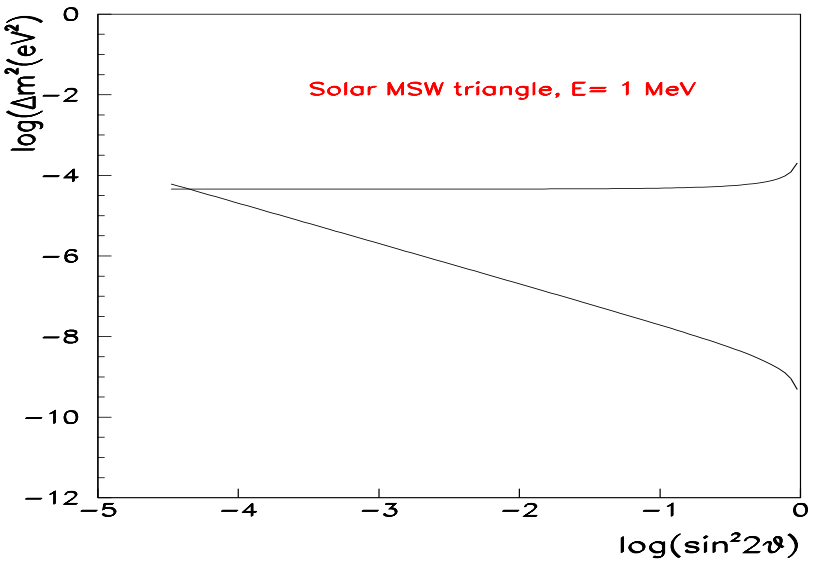
- Adiabaticity: the splitting of the levels is large compared to energy injected in the system by the variation if N_e

$$\gamma = \frac{\sin^2 2\theta}{\cos 2\theta} \frac{\Delta m^2}{2E} \frac{1}{|\nabla \log N_e|} \gg 1$$

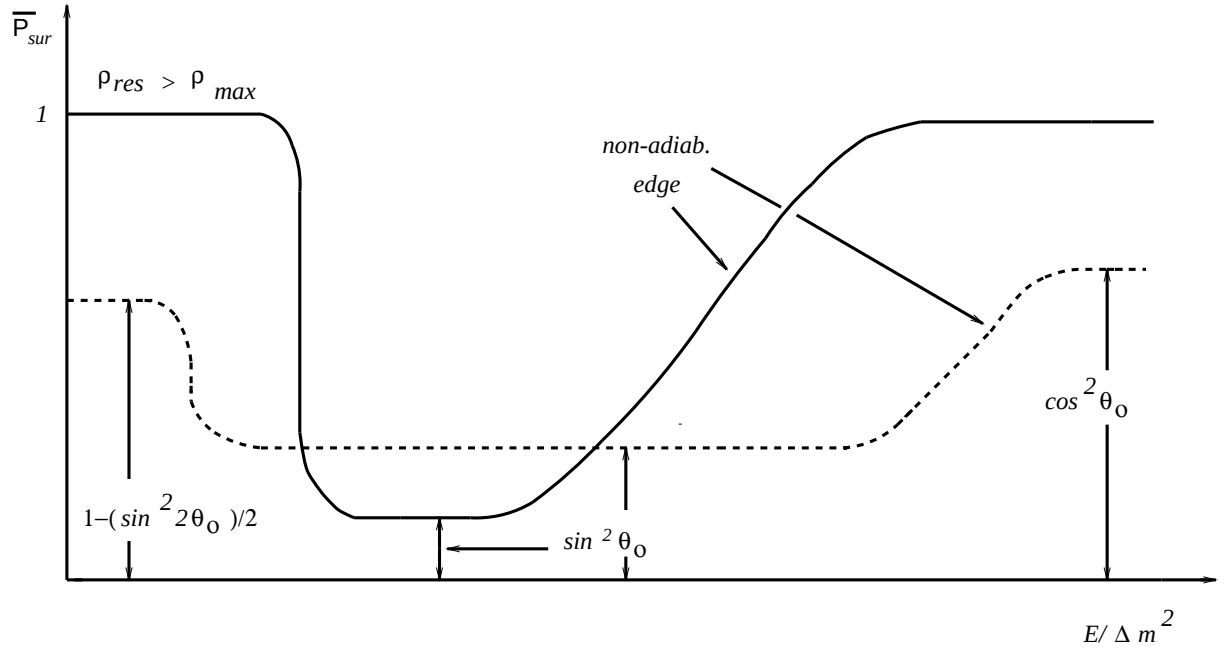
At fixed energy both conditions give the famous MSW triangles, if plotted on the plane $(\log(\sin^2 2\theta), \log(\Delta m^2))$:

$$\log\left(\Delta m^2\right) < \log\left(\frac{2\sqrt{2} G_F N_e(0) E}{\cos 2\theta}\right) \qquad \log\left(\Delta m^2\right) > \log\left(\gamma_{\min} 2E \nabla \log N_e \frac{\cos 2\theta}{\sin^2 2\theta}\right)$$

E.g.: $N_e = N_c exp(-r/R_0) \quad R_0 = R_\odot/10.54, N_c = 1.6 \times 10^{26} cm^{-3}, E = 1MeV$

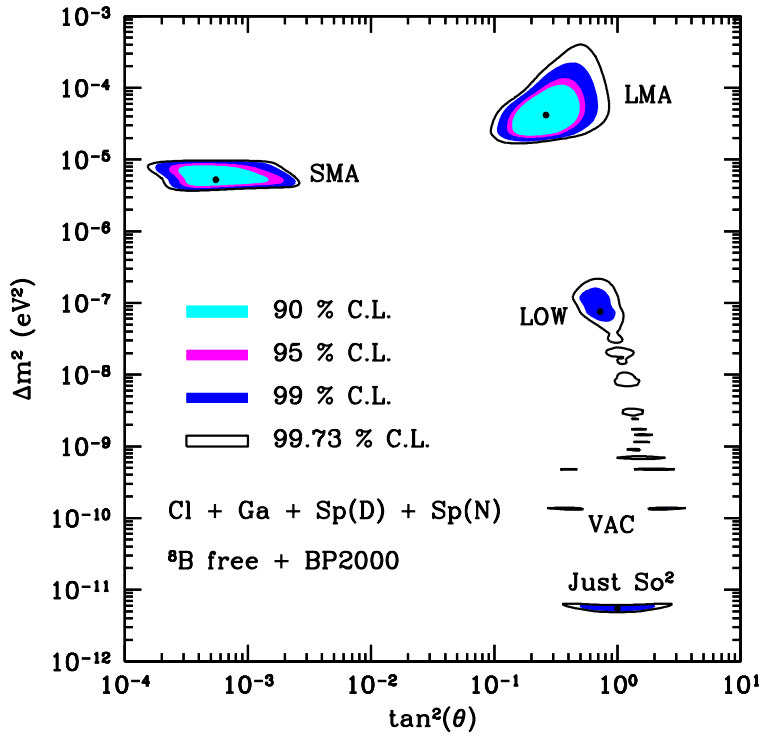


The triangle and the survival probability for ν_e depends on the energy:



The energy dependence is larger for smaller θ

Solar neutrino puzzle before 2002...



Bahcall, *et al* 2001