Relativistic Heavy Ion Physics
experimental perspective

today

ideas

Lecture 1: overview and historical perspective

- Introduction
- Basic Kinematics
- Hagedorn bootstrap model

Lecture 2: ideas

- Glauber Model
- Bjorken energy density
- The Bag Model

A biased mind can not grasp reality

Dalai Lama

tomorrow

facts

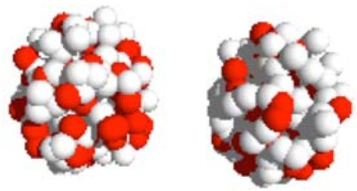
Lecture 3: Experimental probes and signatures

Experimental probes and signatures

Lecture 4: Experimental aspects

Accelerator
Detectors

Introduction



High Energy Heavy Ion collisions is an emerging field of research

LARGE amount of energy involved

RHIC at Brookhaven National Laboratory

100 GeV/nucleon → Gold nucleus

each nucleus 100×197 GeV i.e. 19.7 TeV

$\sqrt{s} = 39.4$ TeV

LHC at CERN

$\sqrt{s} = 1262$ TeV Lead – Lead collisions

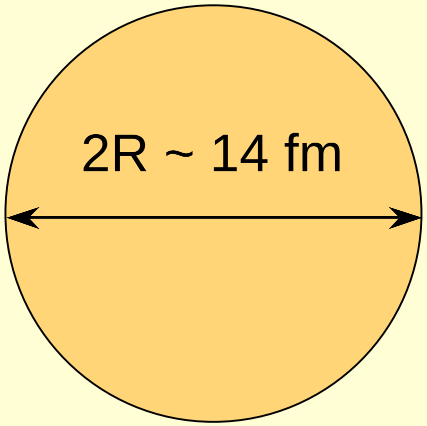
A large amount of energy is deposited in a small region of space in a very short time

In this region the density of energy is very large … this may favor the appearance of new forms of matter

matter in extreme conditions

The search for new forms of matter under extreme conditions of high energy densities is an important objective of high energy heavy ion collisions

$$energy - density = \frac{energy}{volume}$$

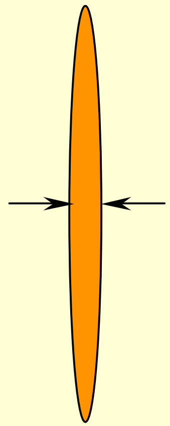


Rest Frame

$$\varepsilon = E/V = M/V_0$$

$$\varepsilon \sim 0.14 \text{ GeV/fm}^3 = \varepsilon_0$$

2R/γ ~ 0.13 fm



Boosted Frame

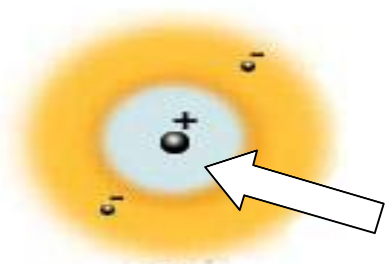
$$\varepsilon = E/V = \gamma M/(V_0/\gamma) = \varepsilon_0\gamma^2$$

$$\gamma = \frac{E}{m} = \frac{100 \times N}{N \times m_N} \qquad \gamma_{RHIC} \approx 106$$

$$\gamma = \frac{574 TeV}{207 \times 0.938 GeV} \qquad \gamma_{LHC} \approx 3000$$

$$\varepsilon_{RHIC} \approx 1570 \quad \varepsilon_{LHC} \approx 1260000 \quad GeV/fm^3$$

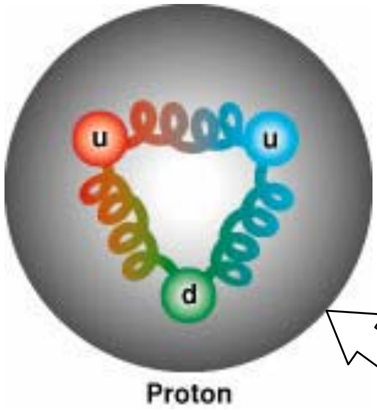
energy density of the nucleus

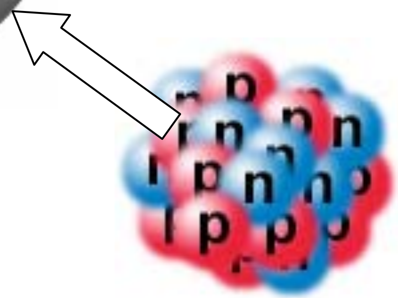


0.14 GeV/fm$^{\mathbf{3}}$

$$\varepsilon = \frac{A \times nucleon_{mass}}{Volume_{nuclear}}$$

$$Volume_{nuclear} = \frac{4}{3}\pi (r_0 A^{1/3})^3$$





energy density of the proton

0.44 GeV/fm$^{\mathbf{3}}$

mass proton= 940 MeV

$$Volume_{proton} = \frac{4}{3}\pi (0.8)^3 \quad fm^3$$

temperature

$$k_B = 8.63 \times 10^{-11} MeV / K$$

1 MeV = 1.2 × 10^{10} ^{o}K

200 MeV = 240 × 10^{10} ^{o}K

100 000 times the temperature in the center of the sun

a brief history of relativistic heavy-ion facilities

Lawrence Berkeley National Laboratory – Bevalac (1974)

EOS - TPC : Equation of State Time Projection Chamber Experiment

DLS: DiLepton Spectrometer

Brookhaven National Laboratory – Alternating Gradient Synchrotron (1986-1995)

E802/866/917; E810/891; E877; E878; E864; E895; E896

CERN – SPS (1986-present) (1994-2000)

HELIOS(NA34); NA35/NA49/NA61(Shine); NA36; NA38/NA50/NA60; NA44;
CERES(NA45); NA52, WA85/WA94/WA97/NA57; WA80/WA9898

Brookhaven National Laboratory – Relativistic Heavy Iion Collider (2000-present)

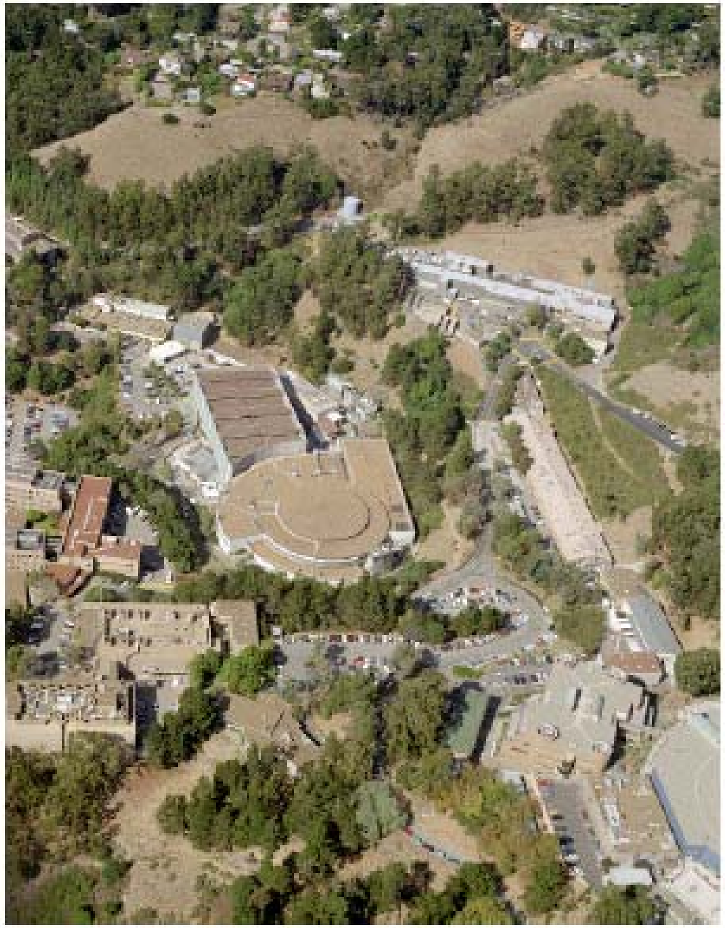
STAR, PHENIX, PHOBOS BRAHMS

Centre Europeen pour la Recherche Nucleair – Large Hadron Collider (2009)

ALICE
CMS
ATLAS

Lawrence Berkeley National Laboratory
Bevalac

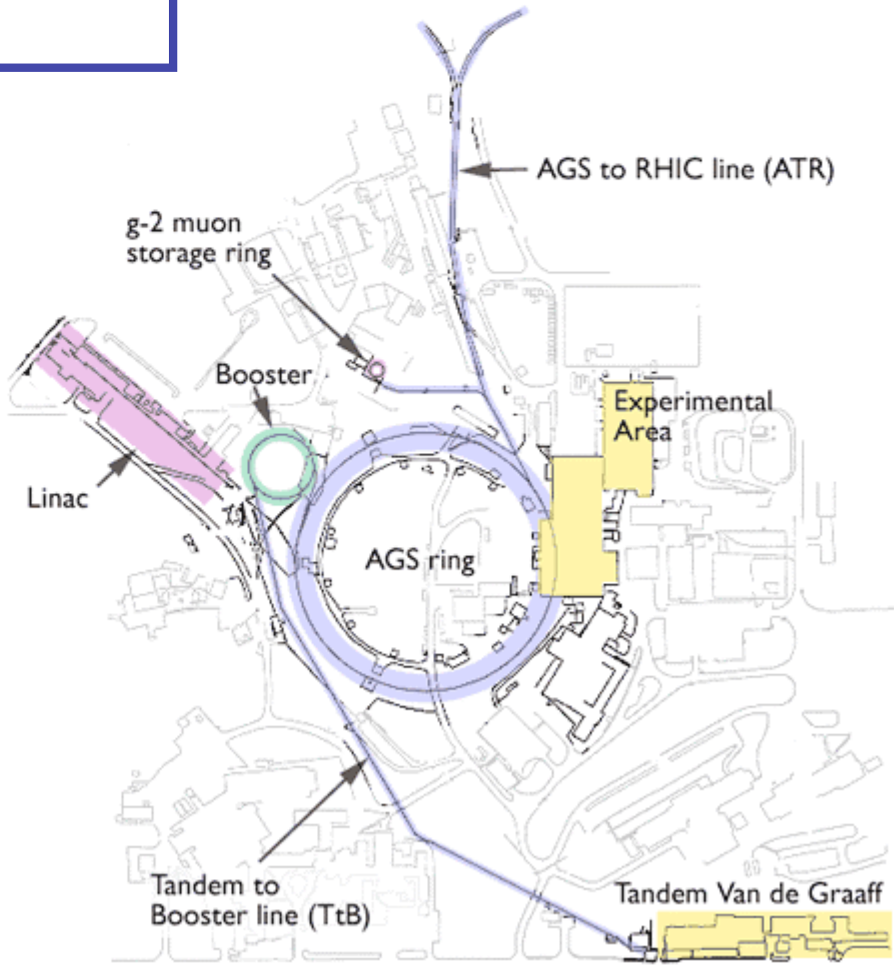
Ions of heavy elements created at SUPERHILAC



experimental work

further accelerated at the Bevatron

Brookhaven National Laboratory
AGS

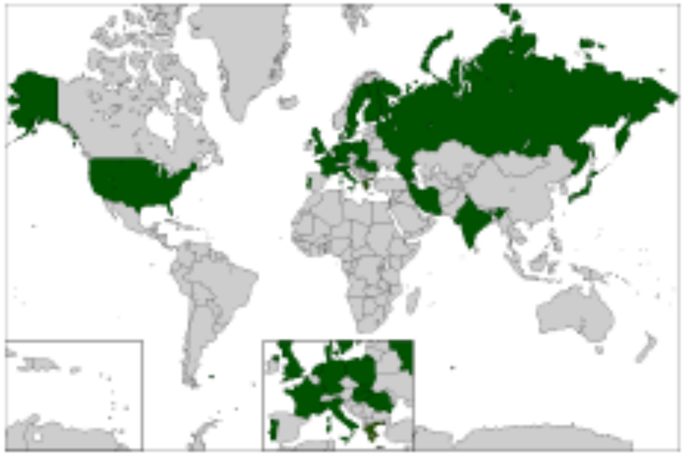


Protons, Gold, Iron

40 years of Heavy Ion research 1974 - 2014

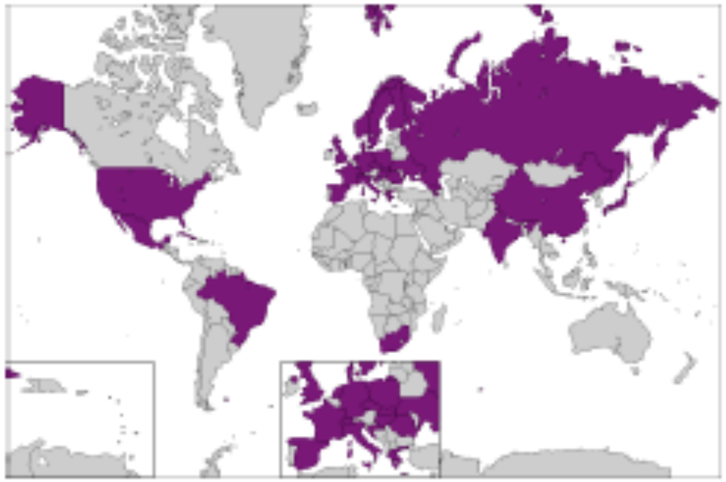
… from Bevalac to LHC

AGS



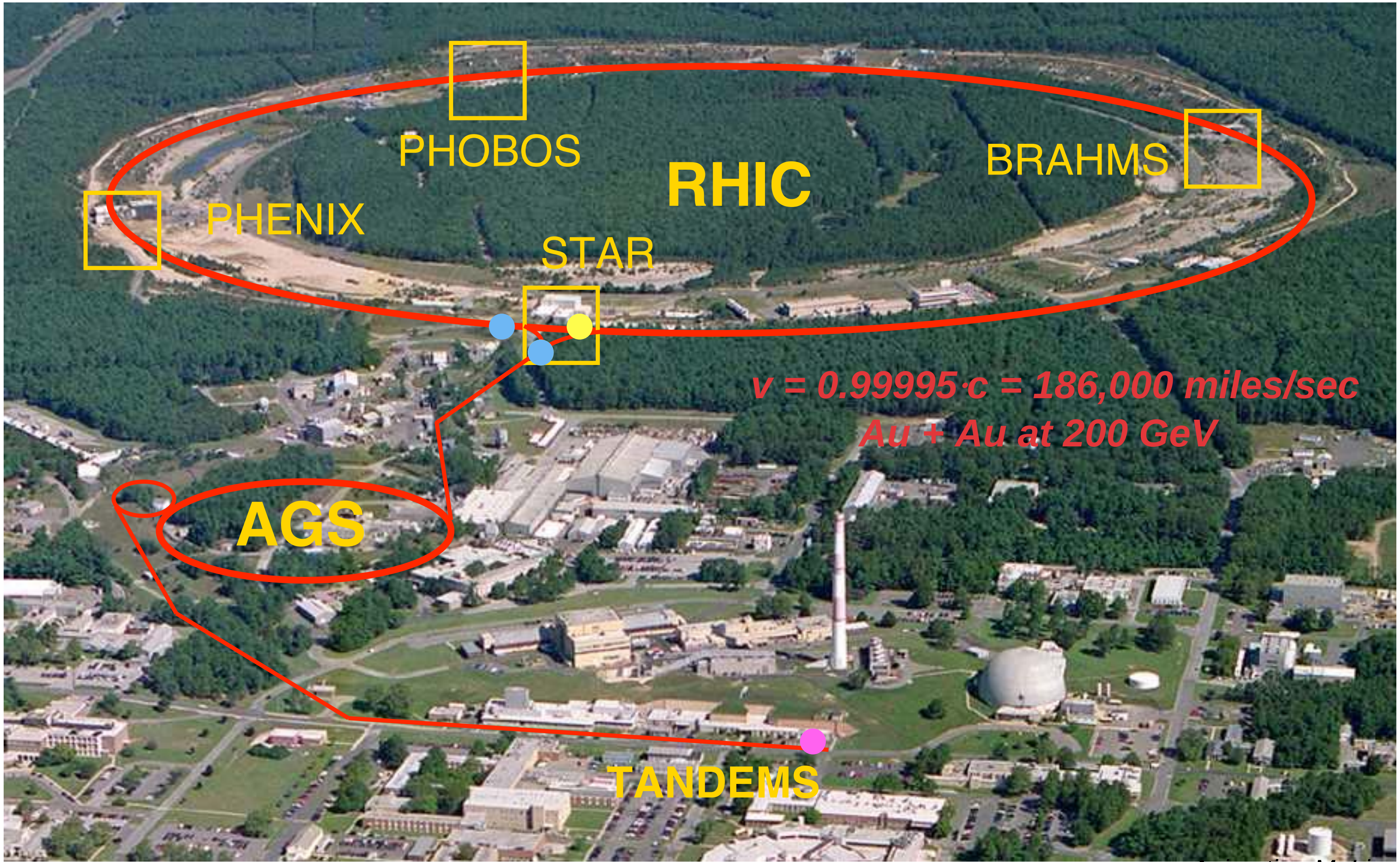
SPS

RHIC



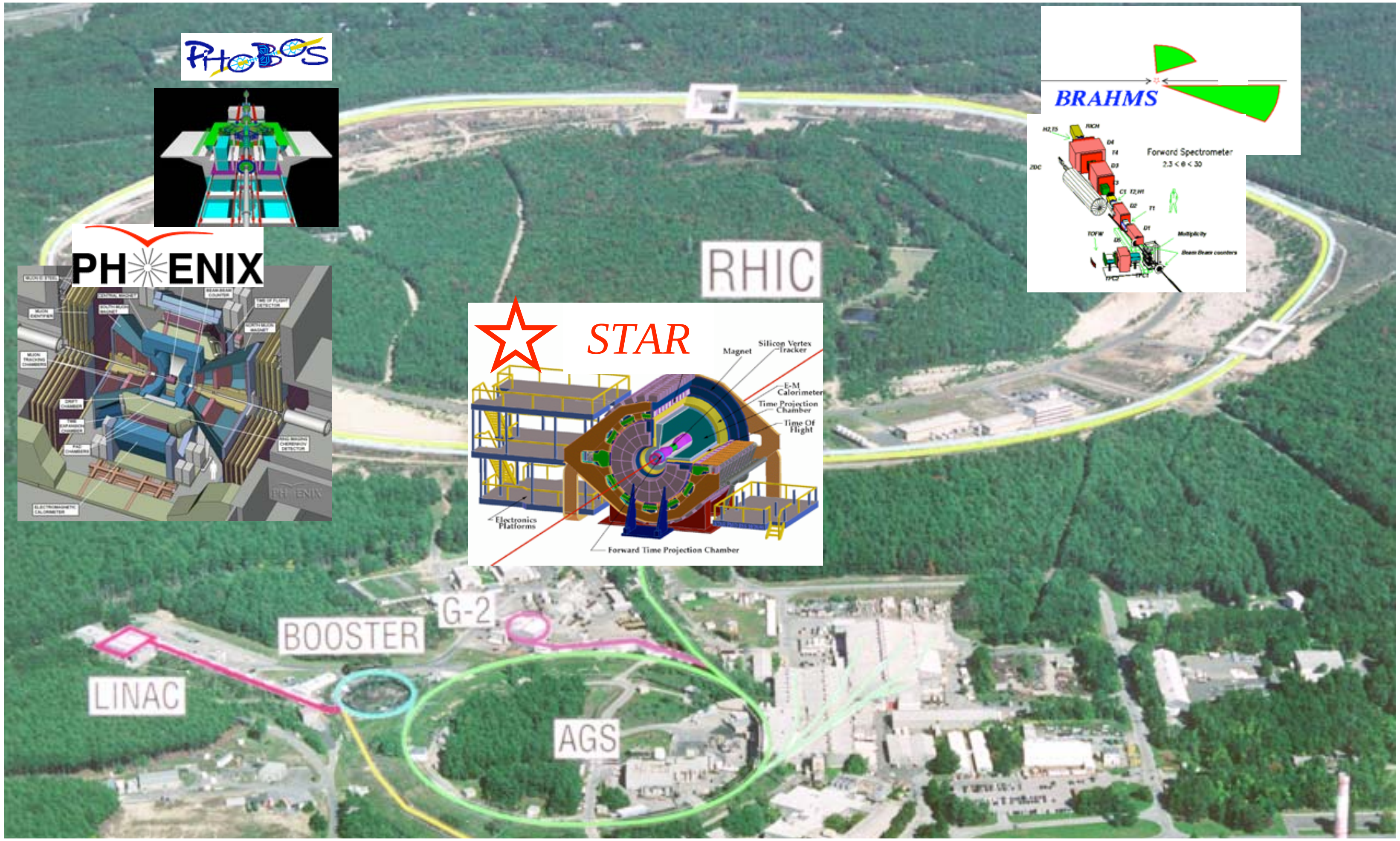
Heavy Ions at LHC

Relativistic **H**eavy **I**on **C**ollider (RHIC)
Brookhaven National Laboratory (BNL), Upton, NY



Animation M. Lisa

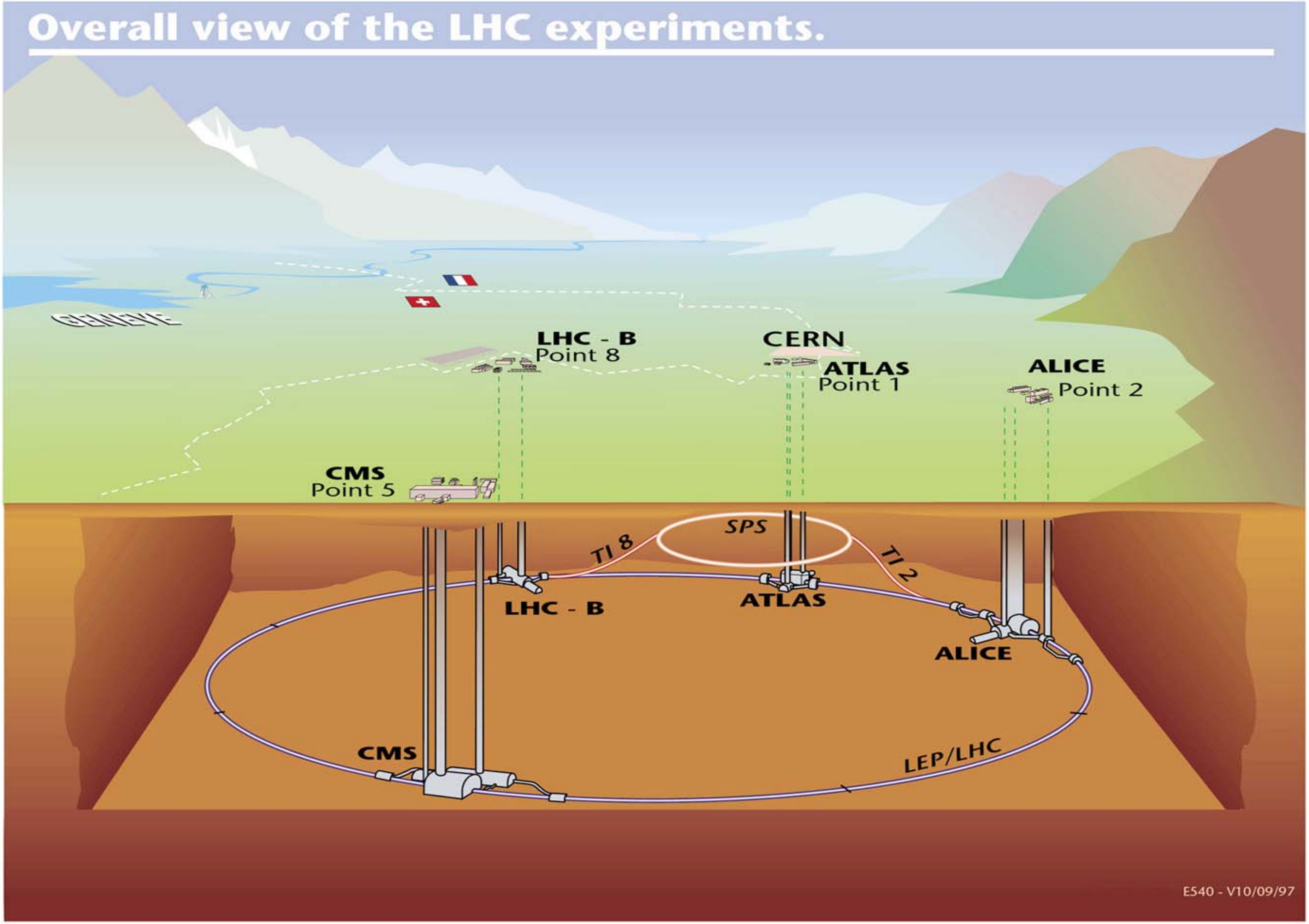
Relativistic Heavy Ion Collider --- RHIC



Au+Au 200 GeV N-N CM energy

Polarized p+p up to 500 GeV CM energy

CERN

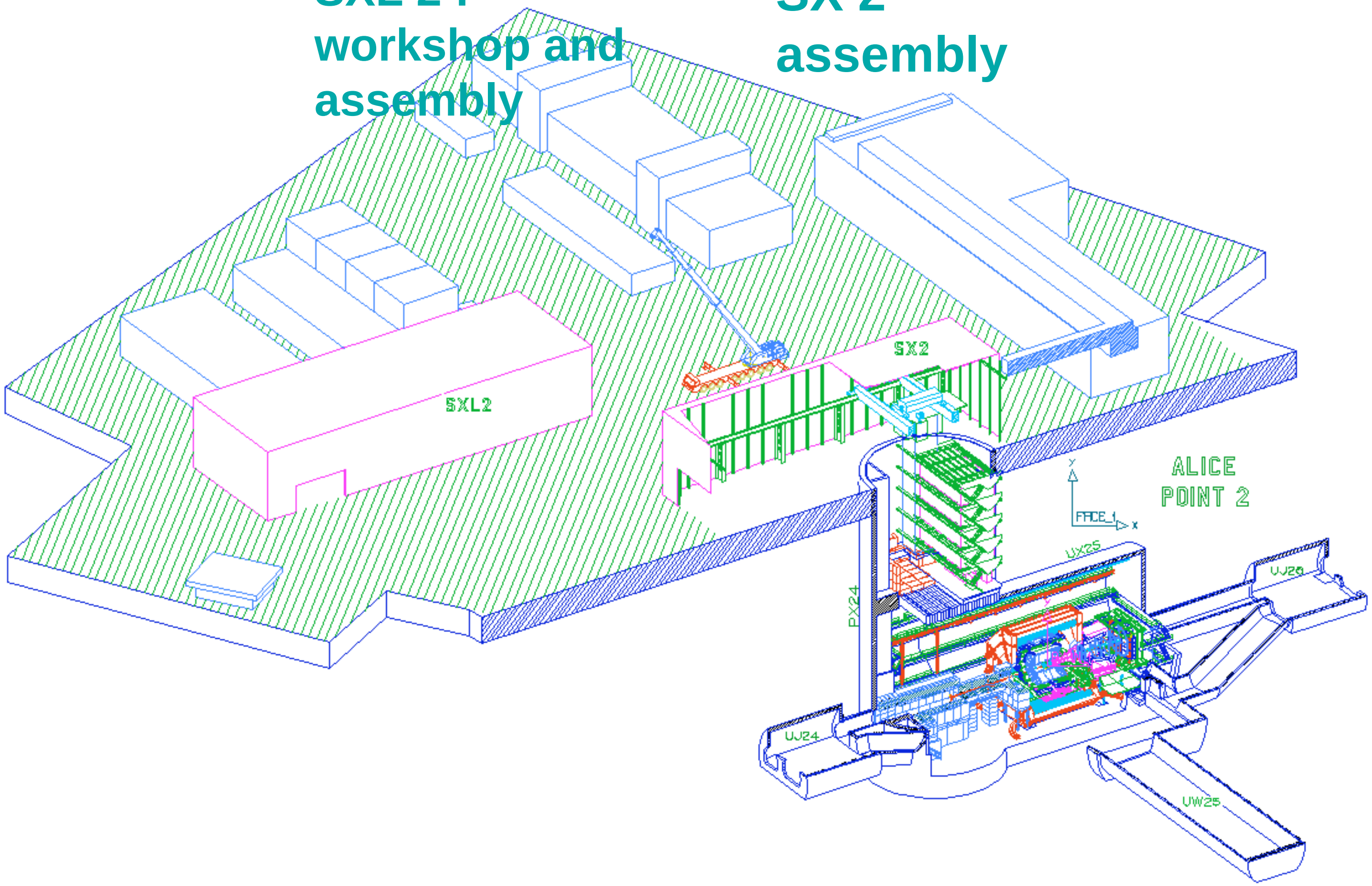

Overall view of the LHC experiments.
GENEVE
LHC - B
Point 8
CERN
ATLAS
Point 1
ALICE
Point 2
CMS
Point 5
SPS
TI 8
TI 2
LHC - B
ATLAS
ALICE
CMS
LEP/LHC
E540 - V10/09/97

Point 2

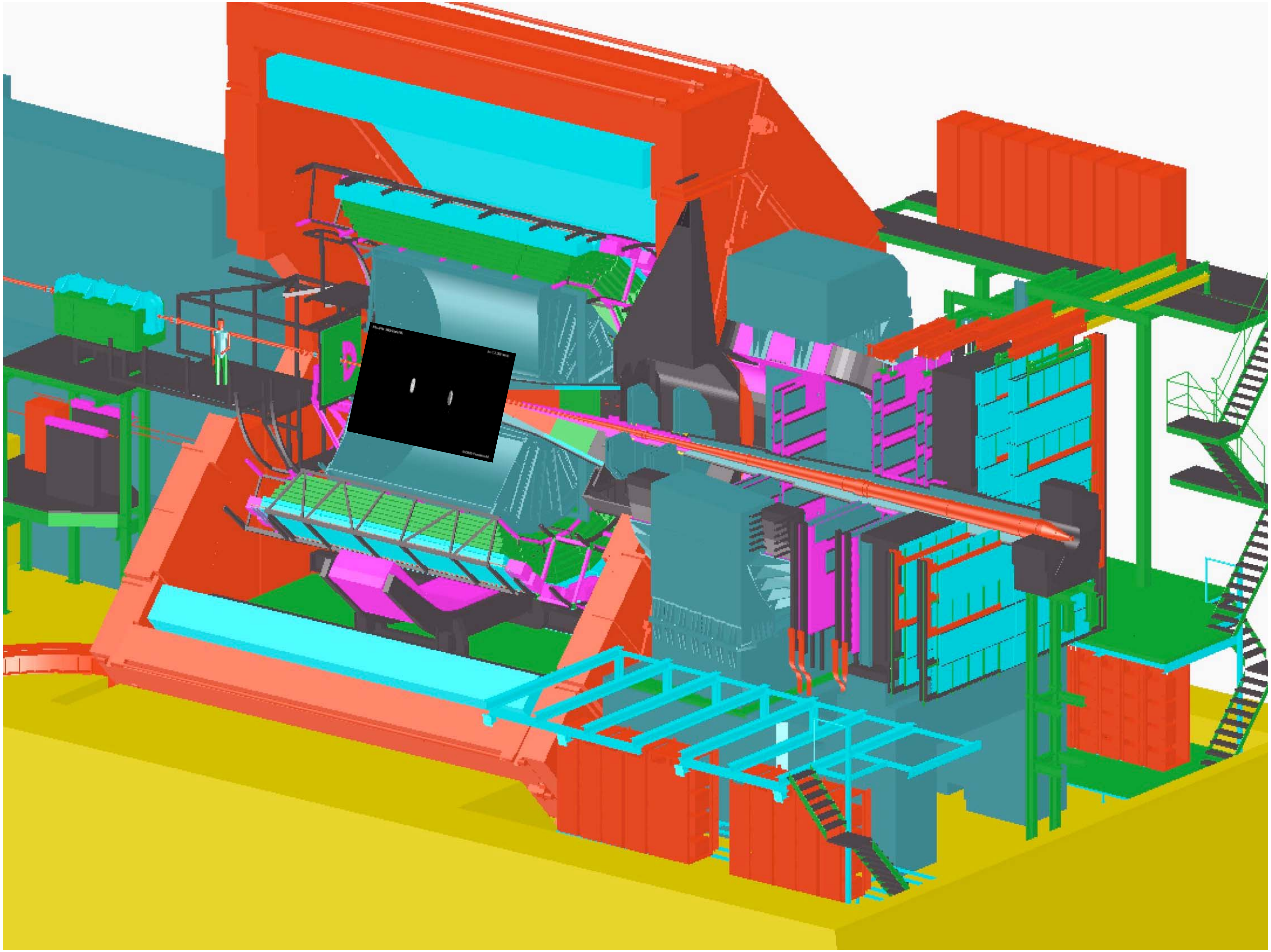
Alice at Point 2

SXL 2 : workshop and assembly

SX 2 assembly

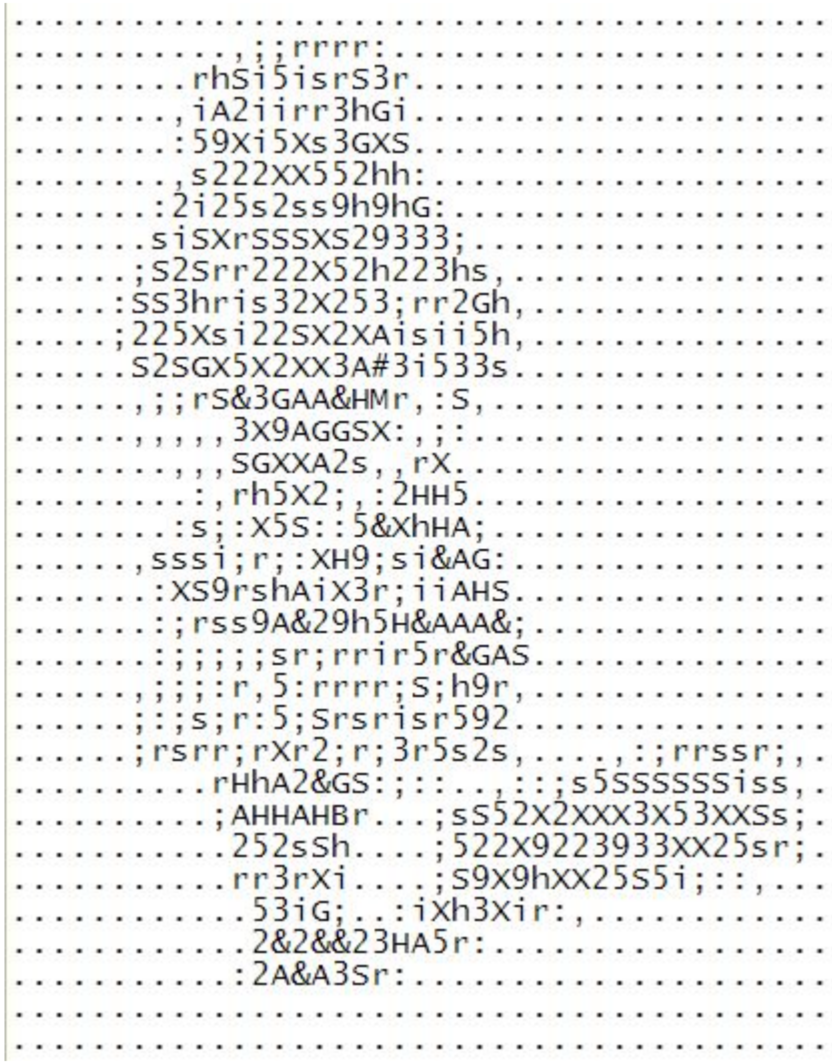


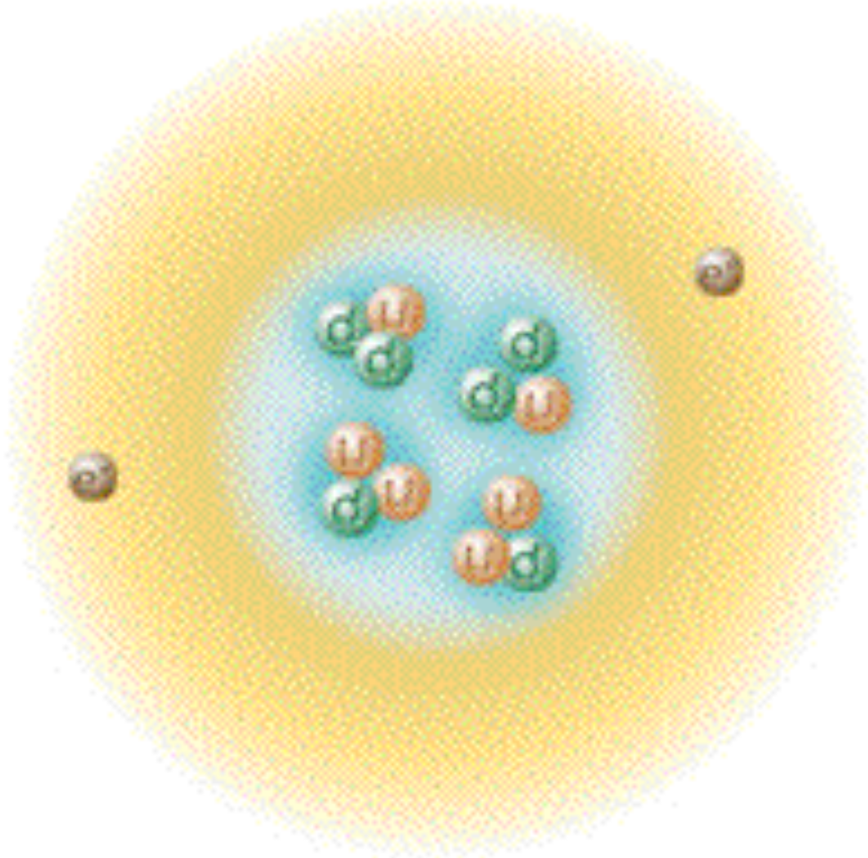
Experimental cavern UX25



A Large Ion Collider Experiment

ALICE in woderland

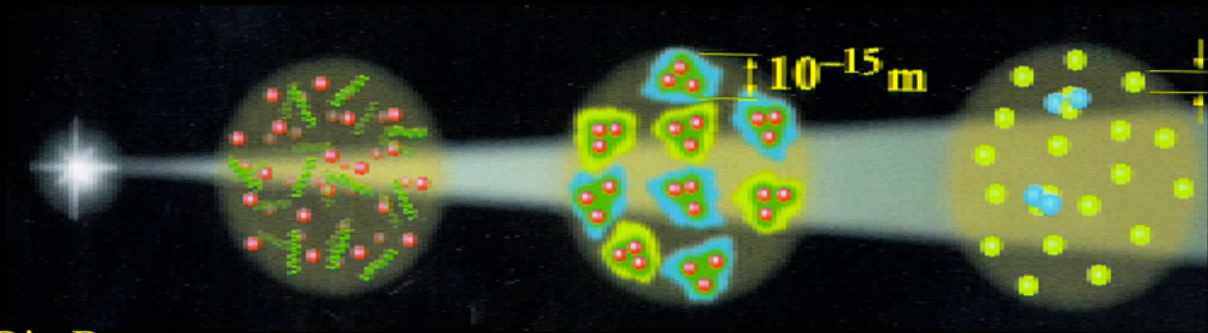




The LHC will be the last look into Relativistic Heavy Ion Collisions for a long time…

... may be forever.

History of the Universe



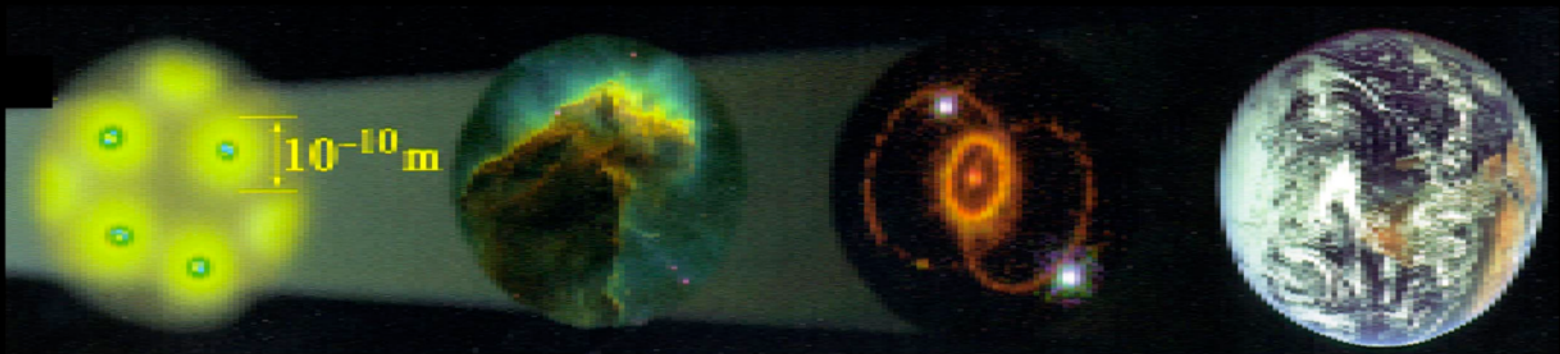
10^{-15} m

Big Bang

Quark-Gluon Plasma
10^{13}K, 10^{-6}s

Protons& Neutrons
10^{12}K, 10^{-4}s

Low-mass Nuclei
10^{9}K, 3 min



Neutral Atoms
4000K, 10^{5}y

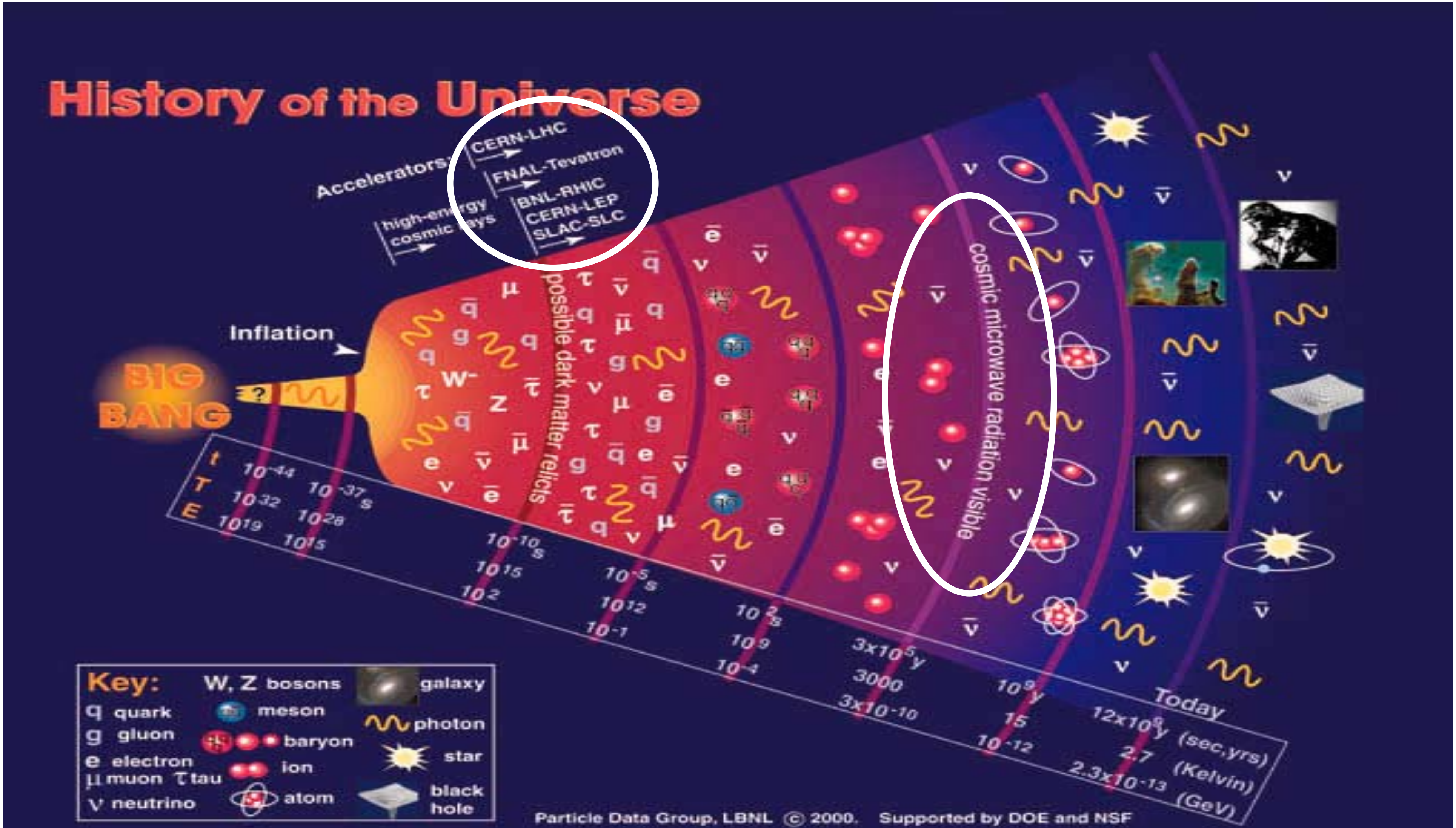
Star Formation
10^{9}y

Heavy Elements
$>10^{9}$y

Today

Source: Nuclear Science Wall Chart

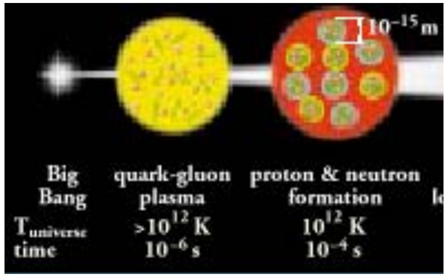
Early Universe

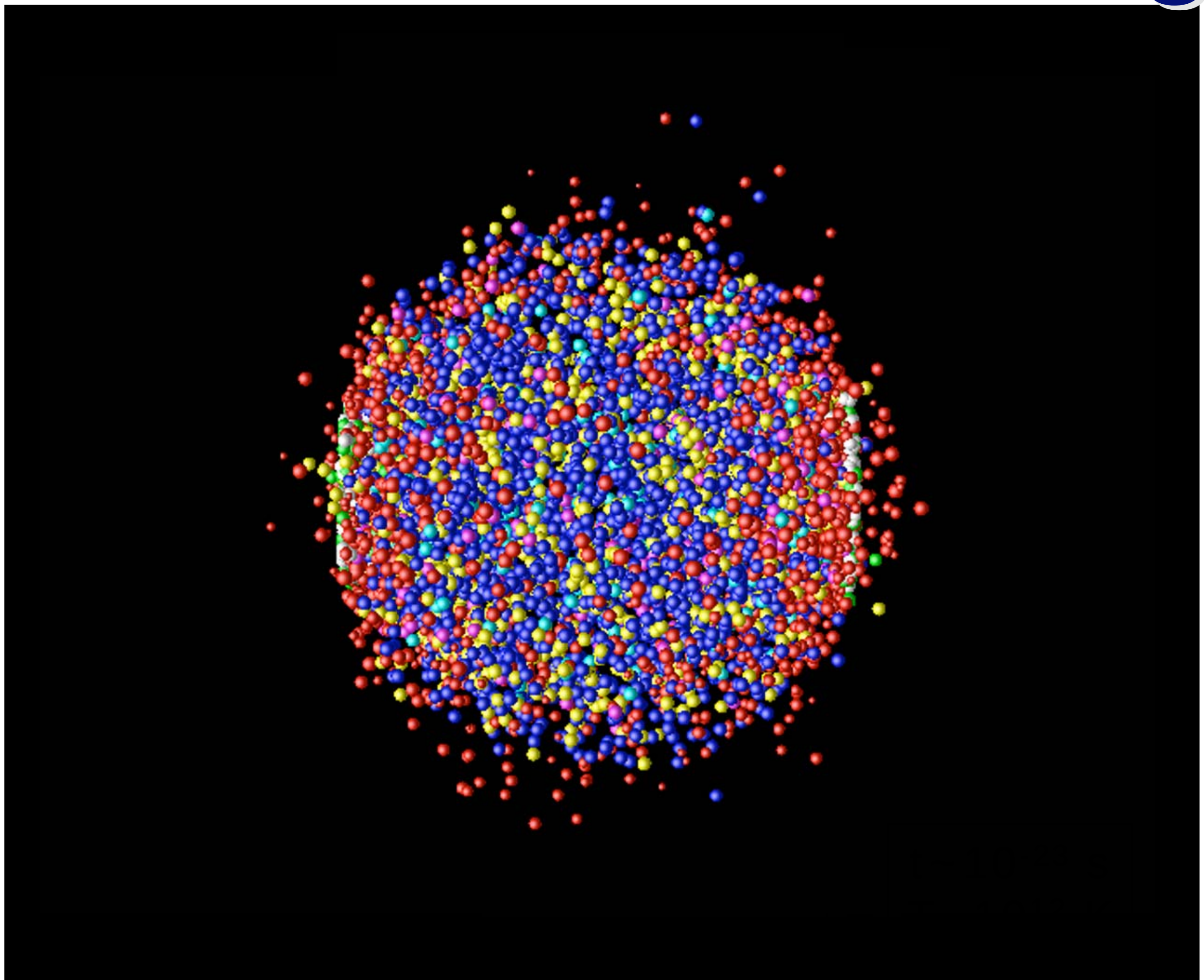


COBE and WMAP: universe is highly equilibrated

⇒ relics of early phase transitions are difficult to see

The mini Big Bang





v/c = 0,99999993
Lorentz Contraction : 7 fm → 0,003 fm

1. Accelerated ions will collide head on

2. The energy of collision is materialized into quarks and gluons

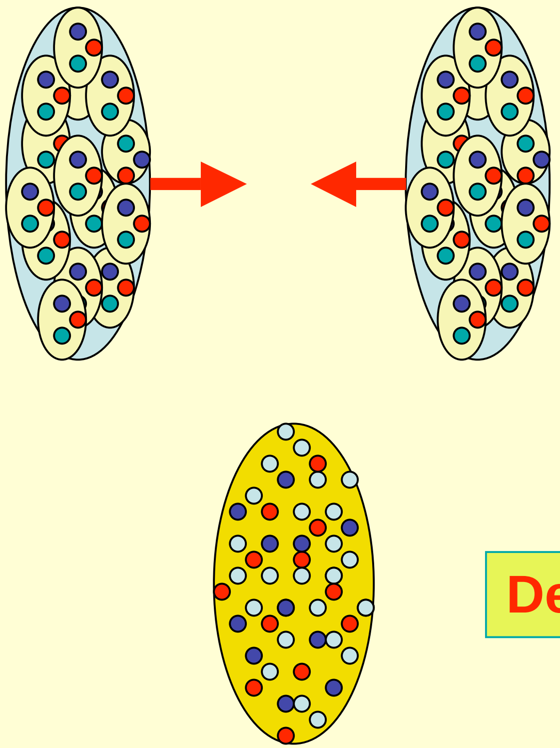
3. Quarks and gluons interact via the strong interaction: matter equilibrates

4. The system expands and cools down

5. Quarks and gluons condensate into hadrons

The Melting of Quarks and Gluons
Quark-Gluon Plasma

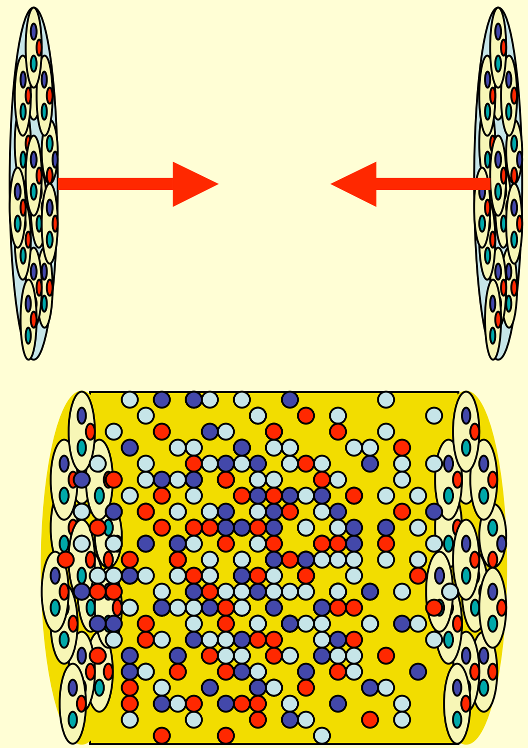
Matter Compression:



Deconfinement

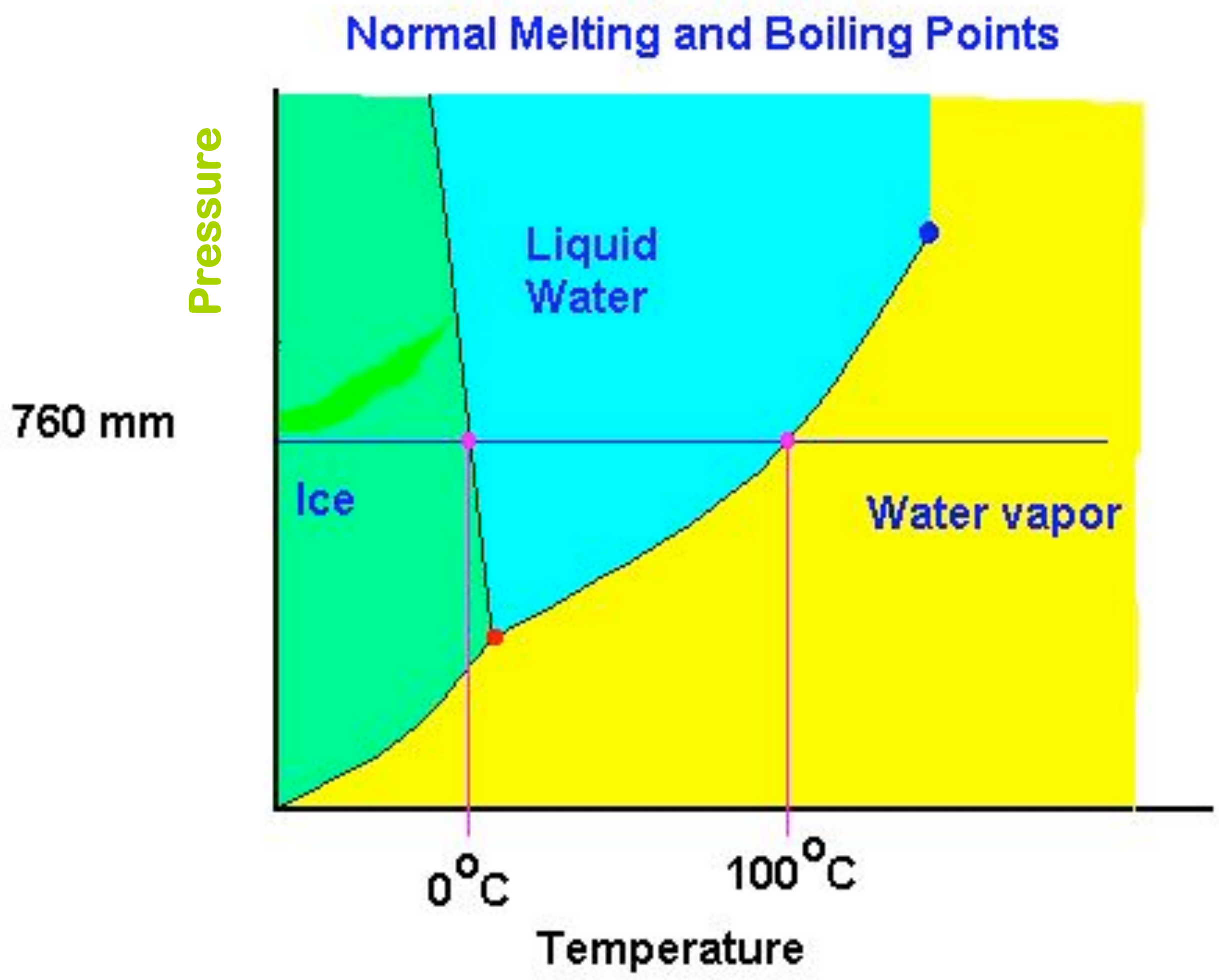
High Baryon Density
- low energy heavy ion collisions
- neutron star◊quark star

Vacuum Heating:



High Temperature Vacuum
- high energy heavy ion collisions

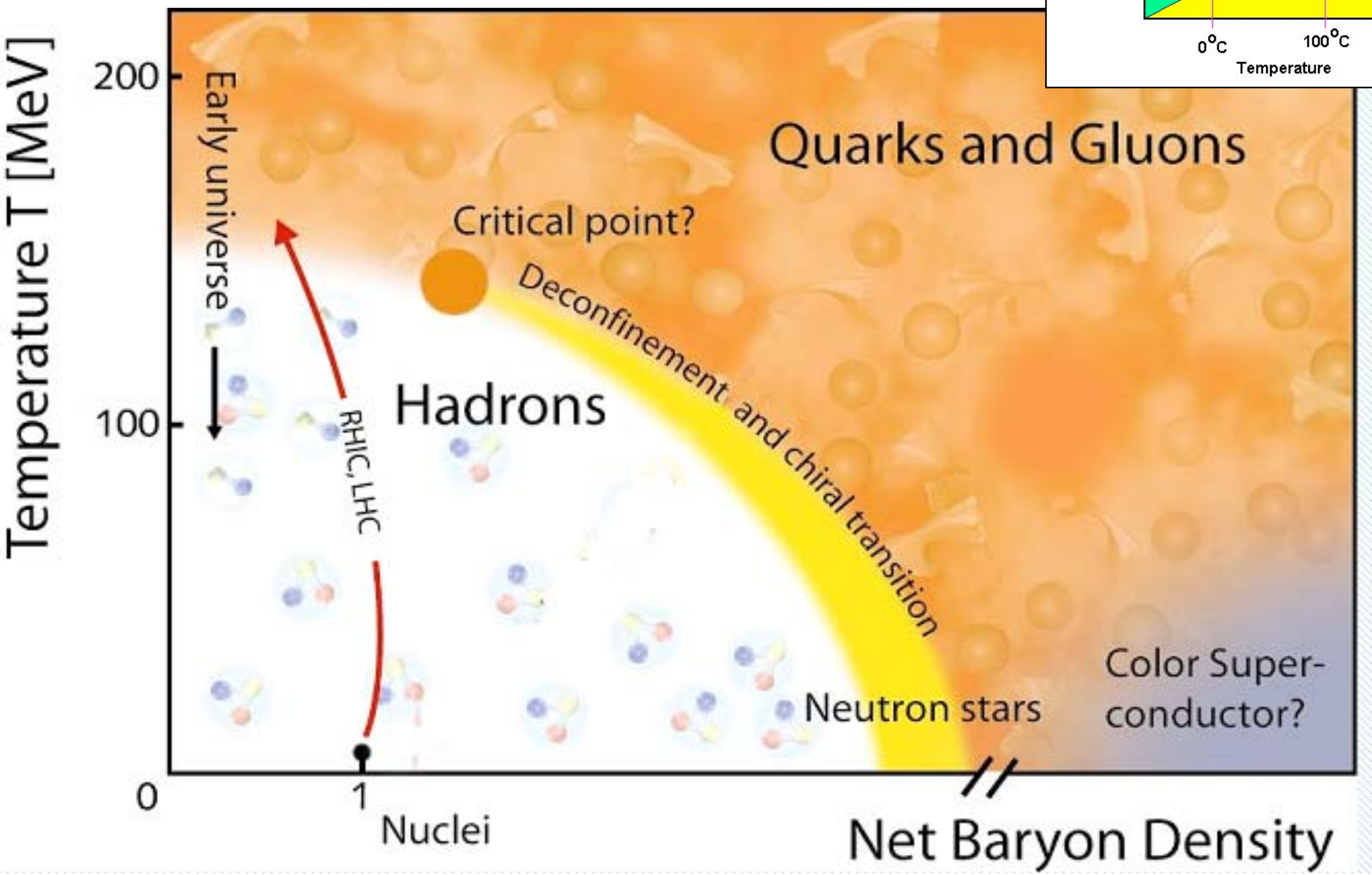
Phase Diagram of Water

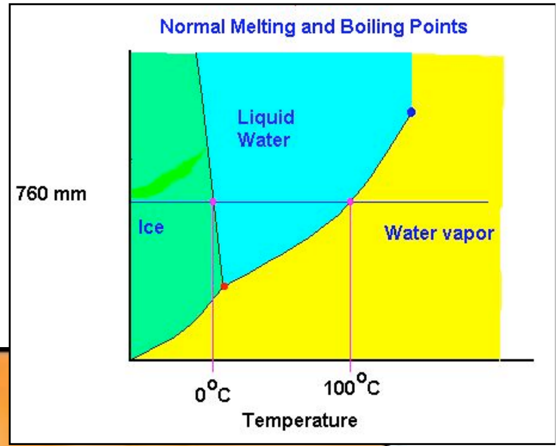


Not directly calculable from the QED Lagrangian: complex *emergent* features

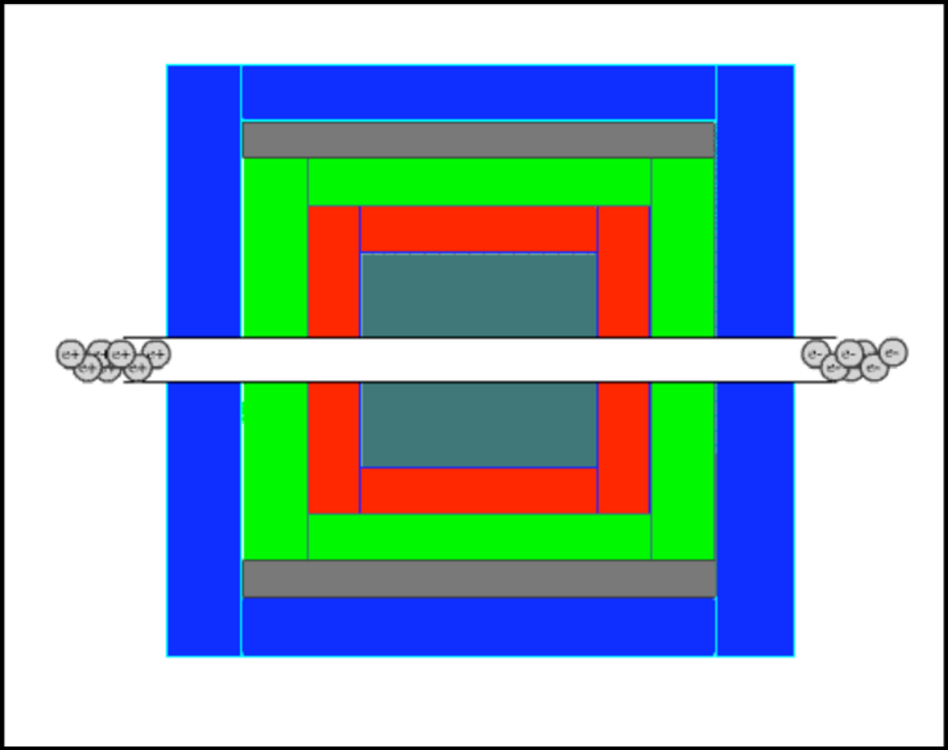
Phase Diagram of QCD Matter

high T ~ high Q^2 ~ deconfinement





Basic Kinematics



Kinematics: What is Rapidity?

In non-relativistic physics the Galileo law of summation of velocities is valid:

$$\boxed{v_2 = v_1 + v \quad \text{(non-rel),}}$$

where v_1 and v_2 are the velocities measured in reference frames one of which moves at a velocity v with respect to the other.

In relativistic physics instead of the above, the Einstein law of summation of velocities is valid:

$$v_2 = (v_1 + v) / (1+v_1 v/c^2) \quad \text{(relativistic)}$$

This is non-additive one. This is inconvenient as difference in velocities of two particles depends on the choice of the moving reference frame.

To retain the property of additivity a new kinematic quantity – the rapidity (y) is introduced in relativistic kinematics . By definition:

$$y = ½ \ ln\ (c+v)/(c-v)$$

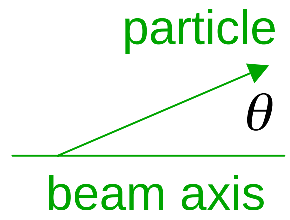
And with this, one can show that: $\boxed{y_2 = y_1 + y \quad \text{(relativistic)}}$

Thus the difference $y_A - y_B$ in rapidities of two particles in same in all moving reference frame.

Kinematic Variables

Rapidity: $$y = \frac{1}{2}\ln\left(\frac{E + P_Z}{E - P_Z}\right)$$ dimensionless

Pseudo-rapidity: $$\eta = \frac{1}{2}\ln\left(\frac{|P| + P_Z}{|P| - P_Z}\right) = -\ln\left(\tan\frac{\theta}{2}\right)$$



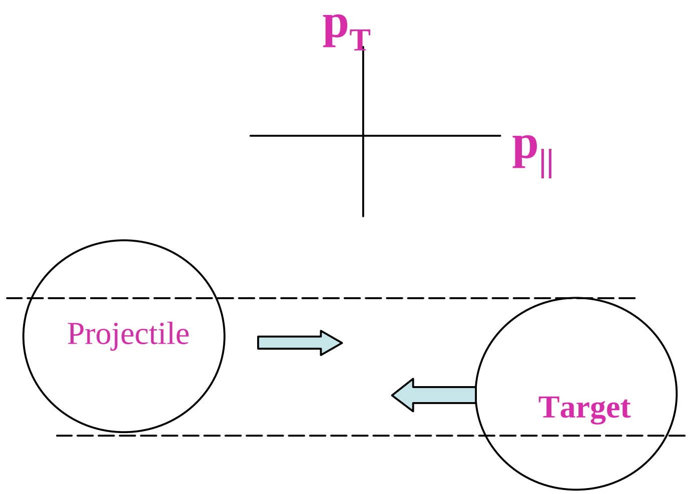
$\eta \to y$ large momentum i.e. $|P| \to E$

Transverse Momentum: $$p_T = \sqrt{p_X^2 + p_Y^2}$$

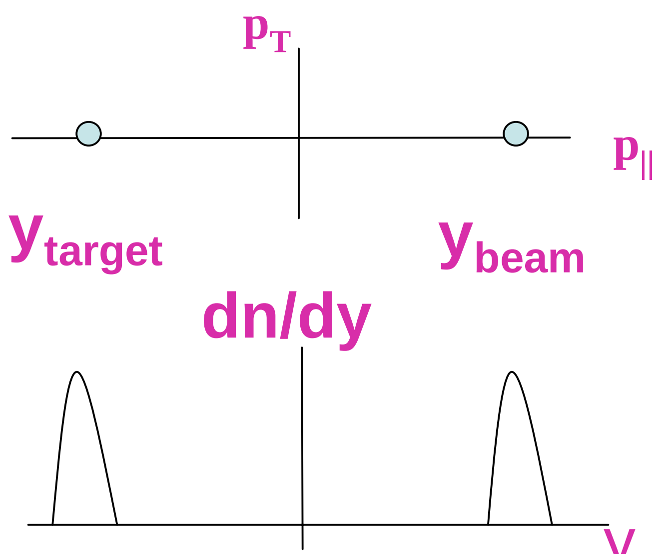
Transverse Mass: $$m_T = \sqrt{p_T^2 + m_0^2}$$

Kinematics: y, η

Heavy-ion Collision



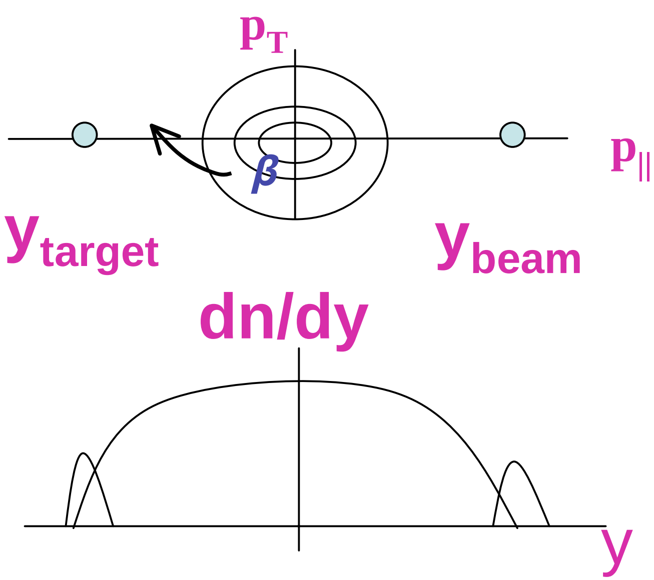
Before



e.g. lead-lead at 158 AGeV

$$y_a = y_b = 2.92$$

After



$\Rightarrow$ $+$ β

Pseudorapidity:

$$\eta = -\ln \tan(\theta / 2)$$

Kinematic Variables

In high energy reactions $a + b \rightarrow c + X$

four-momentum $c = (c_0, c_T, c_z)$

Forward light cone momentum $c_+ = c_0 + c_z$

Backward light cone momentum $c_+ = c_0 - c_z$

Forward light cone variable: $x_+ = \frac{(c_0 + c_z)}{(b_0 + b_z)}$ Lorenz invariant

Backward light cone variable: $x_- = \frac{(c_0 - c_z)}{(b_0 - b_z)}$ Lorenz invariant

Light-cone x_+: $x_+ = \frac{(E + p_z)}{(E + p_z)_{beam}}$

Feymann x_F: $x_F = \frac{p_L^*}{p_{\max}^*} = \frac{p_L^*}{\sqrt{s}/2}$

Useful Expressions

$$E = m_T \cosh y$$

$$p_Z = m_T \sinh y$$

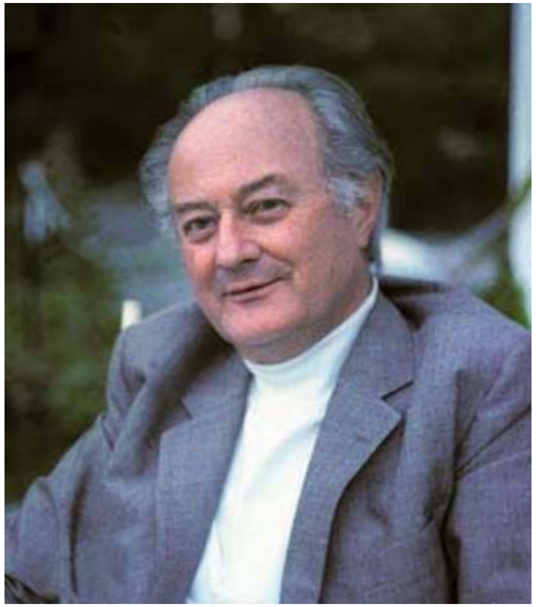
$$\beta_z = \tanh y$$

$$dp_z = E dy$$

Light cone variables, rapidity and all that.
John C. Collins (Penn State U.) . May 1997. 14pp.
e-Print: **hep-ph/9705393**

Download pdf file
from the SPIRES

Hagedorn Model



Hagedorn observed that the measured density of hadrons states grows exponentially

$$\frac{d\rho}{dm} \approx m^a e^{m/m_0}$$

and that this implies a limiting temperature

$$dn(E) \approx dE \int_0^E pE\,dm \frac{d\rho}{dm} e^{-E/kT}$$

$$p = \sqrt{E^2 - m^2}$$

$$dn(E) \approx dE \int_0^E m^a e^{m/m_0} e^{-E/kT} \sqrt{E^2 - m^2}\, E\,dm$$

$$= E^{a+3} dE \int_0^1 z^a e^{zE/m_0} e^{-E/kT} \sqrt{1-z^2}\, dz$$

here $m = zE$

and now with $z = \cos(\varphi)$

$$= E^{a+3} e^{-E/kT} dE \boxed{\int_0^{\pi/2} \cos^a(\varphi) \sin^2(\varphi) e^{E\cos(\varphi)/m_0} d\varphi}$$

assuming $\frac{E}{m_0} >> 1$ we can approximate

$$\approx \int_0^{\pi} \cos^a(\varphi) \sin^2(\varphi) e^{E\cos(\varphi)/m_0} d\varphi$$

$$\boxed{\approx \int_0^{\pi} \sin^2(\varphi) e^{E\cos(\varphi)/m_0} d\varphi = \sqrt{\pi}\,\frac{2m_0}{E}\,\Gamma\left(\frac{3}{2}\right) I_1\left(\frac{E}{m_0}\right)}$$

$$= E^{a+3} e^{-E/kT} dE \sqrt{\pi}\,\frac{2m_0}{E}\,\frac{\sqrt{\pi}}{2}\,\frac{e^{E/m_0}}{\sqrt{2\pi E/m_0}}$$

$$= E^{a+3} e^{-E/kT} dE \sqrt{\frac{\pi m_0^3}{2E^3}}\, e^{E/m_0}$$

$$= E^{a+3} dE \sqrt{\frac{\pi m_0^3}{2E^3}}\, e^{(E/m_0 - E/kT)}$$

… the total energy density $\int_0^\infty E\,dn(E)$ diverges for $kT > kT_0 = m_0$

$$= E^{a+3}dE\sqrt{\frac{\pi m_0^3}{2E^3}}e^{(E/m_0 - E/kT)}$$

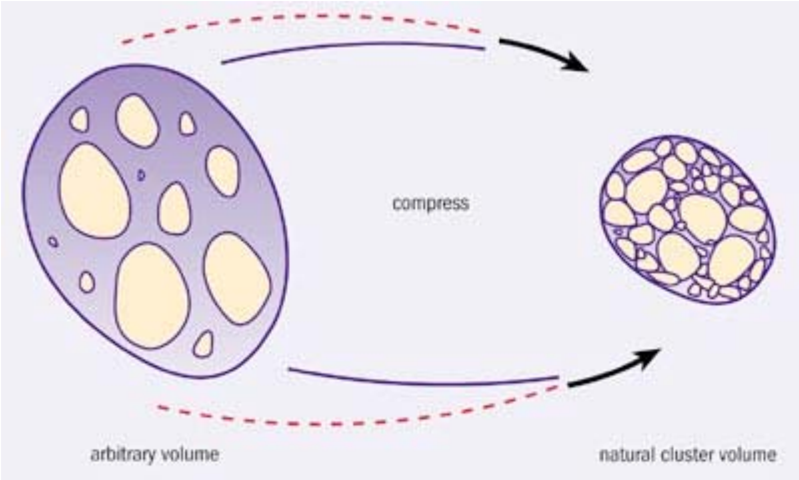
grows with energy

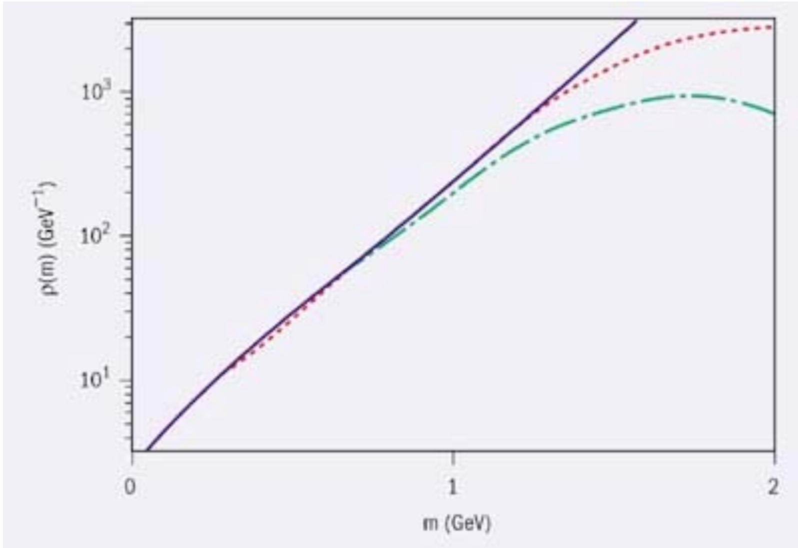
No higher temperatures are possible

or

some new physics must become relevant

Statistical Bootstrap Model





The smoothed mass spectrum of hadronic states as a function of mass.

Experimental data: long-dashed green line with the 1411 states known in 1967; short-dashed red line with the 4627 states of 1996.

The solid blue line represents the exponential fit yielding TH=158 MeV.

http://cerncourier.com/cws/article/cern/28919

R. Hagedorn, *Supplemento al Nuovo Cimento, 1965*

Statistical Thermodynamics of Strong Interactions at High Energies

Thermodynamical model of the mass spectrum of hadrons

$$\rho(m) = \frac{a}{m^{5/2}} e^{m/T_0}$$

$$\rho_{\exp}(m) = \sum \nu_i \delta(m - m_i)$$

with $\nu_i = (2J_i + 1)(2I_i + 1)2^{\lambda_i}$

J	**spin**
I	**isospin**

$$\lambda_i = \begin{matrix} 1 \, \text{part} \neq \text{anti} \\ 0 \, \text{part} = \text{anti} \end{matrix}$$

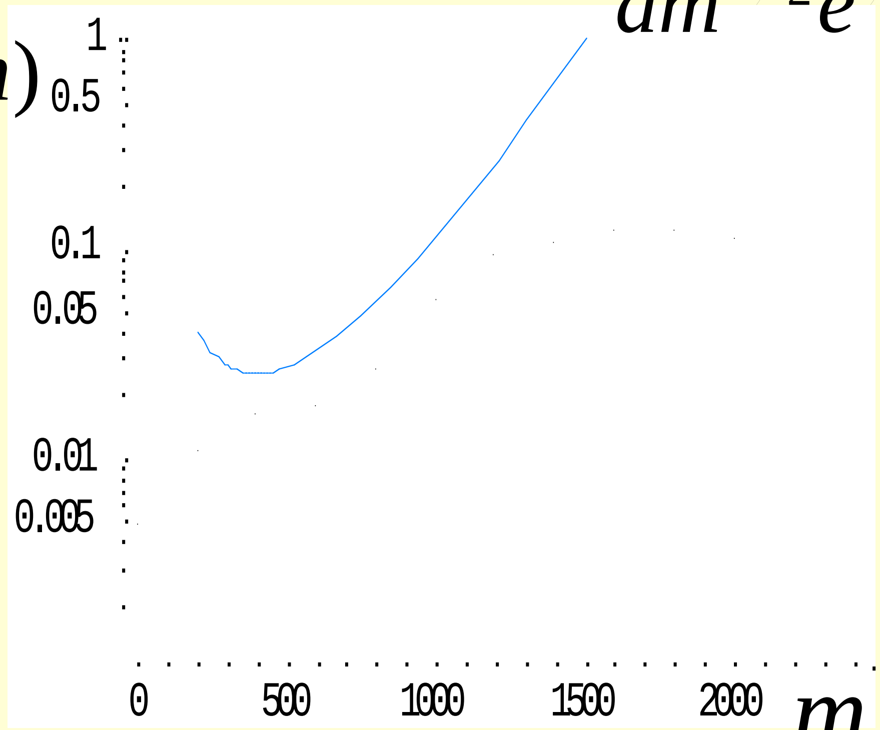
smooth function for *m=0,200,400,… 2000 MeV*

$$\bar{\rho}_{\exp}(m) = \frac{1}{\sqrt{2\pi\tau^2}} \sum \nu_i e^{-\frac{(m-m_i)^2}{2\tau^2}} \qquad \tau = 200 \text{ MeV}$$

asymptotic formula $m \to \infty$

$$a m^{-5/2} e^{m/158}$$

$\rho(m)$



m

Hagedorn Limiting Temperature

Ordinary statistical mechanics

$$E \sim \sum_{\text{states } i} E_i \, g_i \exp(-E_i / T) \sim \int E \frac{dN}{dE} \exp(-E/T)\, dE$$

For thermal hadron gas (somewhat crudely):

$$E \sim \int M \frac{dN}{dM} \exp(-M/T)\, dM \qquad \text{now add in } \frac{dN}{dM} \sim \exp(M/T_{\mathrm{H}})$$

$$\sim \int M \exp\left(-M\left[\frac{1}{T} - \frac{1}{T_{\mathrm{H}}}\right]\right) dM$$

Energy *diverges* as T --> T_{H}

Maximum achievable temperature?

"…a veil, obscuring our view of the very beginning." Steven Weinberg, *The First Three Minutes* (1977)

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Suppl. Al Nuovo Cimento, 3(1965)147
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Exponential hadronic spectrum and quark liberation
N. Cabibbo and G. Parisi
Phys. Lett. B59(1975)67

The long way to the statistical bootstrap model (SBM)
R. Hagedorn
Invited lecture at the “Advanced NATO Workshop: Hot Hadronic Matter: Theory and Experiment”, Divonne-les-Bains, France, June 27-July 27, 1994

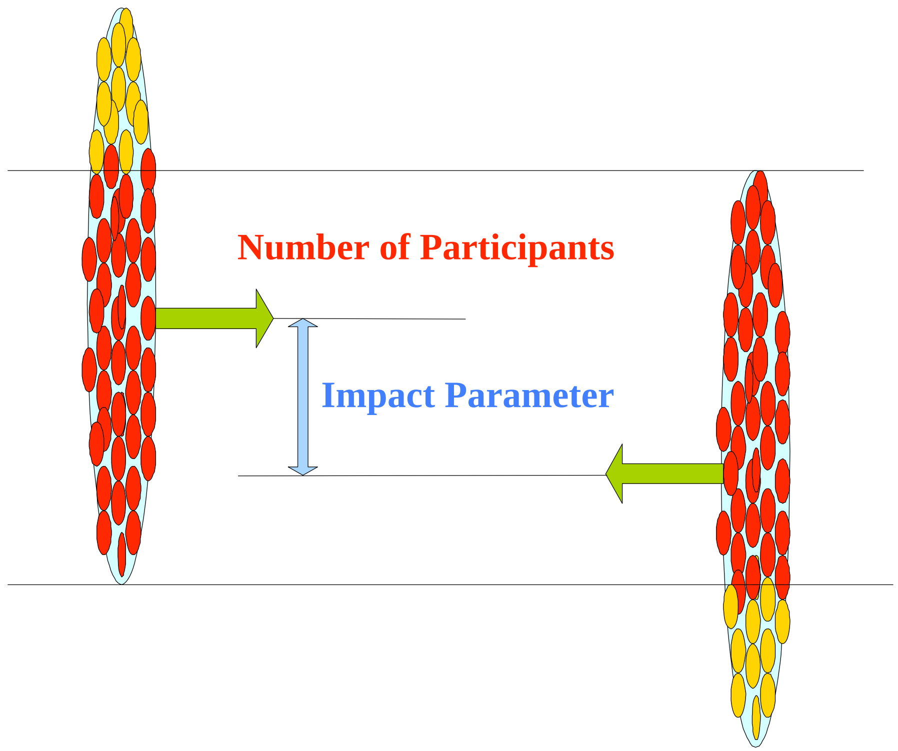
Global properties of Nucleus Nucleus collisions
M. Kliemant, R. Sahoo, T. Schuster, R. Stock
arXiv:0809.2482[nucl-ex]

activity

```
{
//
//       Program to generate the Hagedorn Plot
//       The 5th CERN - Latin American School of
//                High-Energy Physics
//                 march 15-28, 2009
//        Recinto Quirama, Antioquia Colombia
//
//                         Gerardo Herrera Corral
//                            CERN-CINVESTAV

Int_t i,j,k,N=221,M=12;                        // N,  PDG registered in the PDG
                                               // M, mass bins for the Hagedorn Plot
Double_t sigma = 200.0;                        //  tau parameter (Hagedorn s paper)
Double_t Mass[N],SMass[N],J[N],SJ[N],I[N],lambda[N];
// mass, spin, isospin, lambda
Double_t factor,sumafactor,rho[M],masa_bin[M],tau;

TCanvas *c1 = new TCanvas("c1","a simple start",200,10,700,500);

//spectrum = new TH1F("masses","mass spectrum",100,0,12000); // the masses
//spectrum ->SetFillColor(42);

fun1 = new TF1("fun1","6450*(1/x**(2.5))*exp(x/158)",0,2400);
```

Glauber Model

Nuclear Collision Geometry



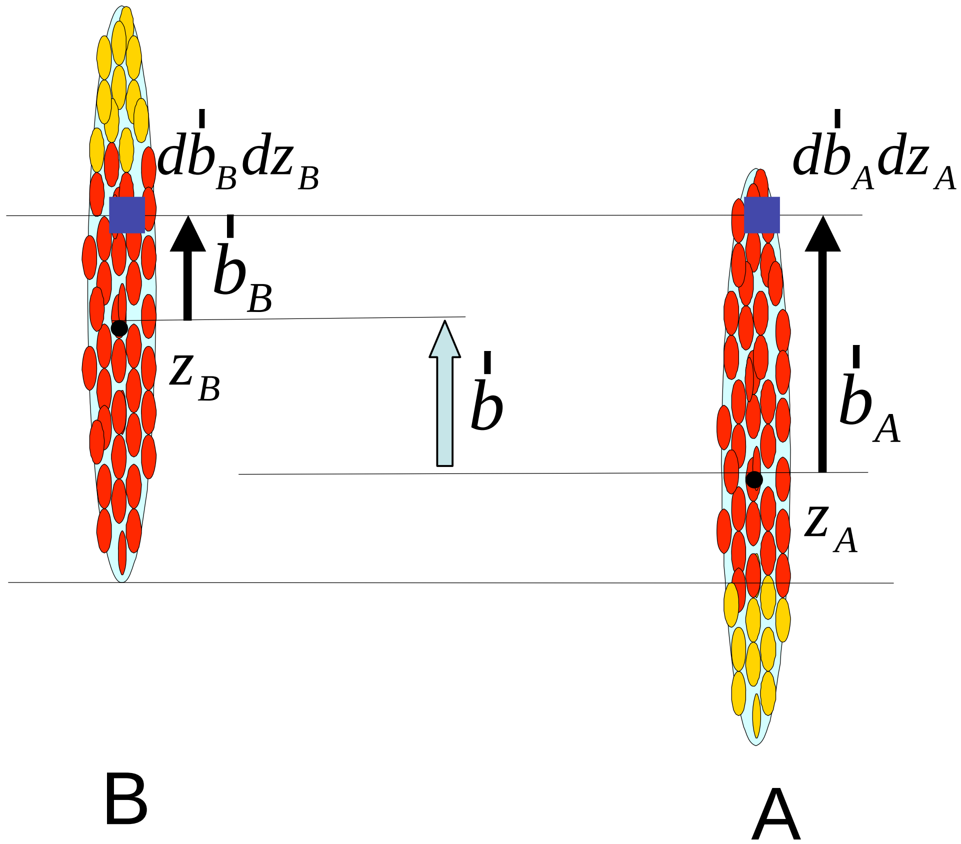
Particle Production is assumed to be directly related to the impact parameter or number of participant nucleons.

thickness function

$$t(\vec{b})d\vec{b}$$

the probability of a baryon-baryon collision in a transverse area element $d\vec{b}$ when a baryon is located at $\vec{b}$

$$\int t(\vec{b})d\vec{b} = 1$$



Probability of finding a baryon in the volume element $d\vec{b}_B dz_B$ of B at position $(\vec{b}_B, z_B)$:

$$\rho(\vec{b}_B, z_B) d\vec{b}_B dz_B$$

Probability element of a baryon baryon collision:

$$\boxed{dP = \rho_A(\vec{b}_A, z_A) d\vec{b}_A dz_A \rho_B(\vec{b}_B, z_B) d\vec{b}_B dz_B t(\vec{b} - \vec{b}_A - \vec{b}_B) \sigma_{in}}$$

$$T(\vec{b}) = \int d\vec{b}_A d\vec{b}_B T_A(\vec{b}_A) T_B(\vec{b}_B) t(\vec{b} - \vec{b}_A - \vec{b}_B)$$

with

$$T_A(\vec{b}_A) = \int dz_A \rho_A(\vec{b}_A, z_A)$$

$$T_B(\vec{b}_B) = \int dz_B \rho_B(\vec{b}_B, z_B)$$

probability of ***n*** inelastic baryon-baryon collisions:

$$P(n,b) = \binom{AB}{n} \left[T(b)\sigma_{in}\right]^{n} \left[1 - T(b)\sigma_{in}\right]^{AB-n}$$

n out of AB possible

probability of *n* collisions

probability of *AB-n* misses

Total probability of an inelastic event

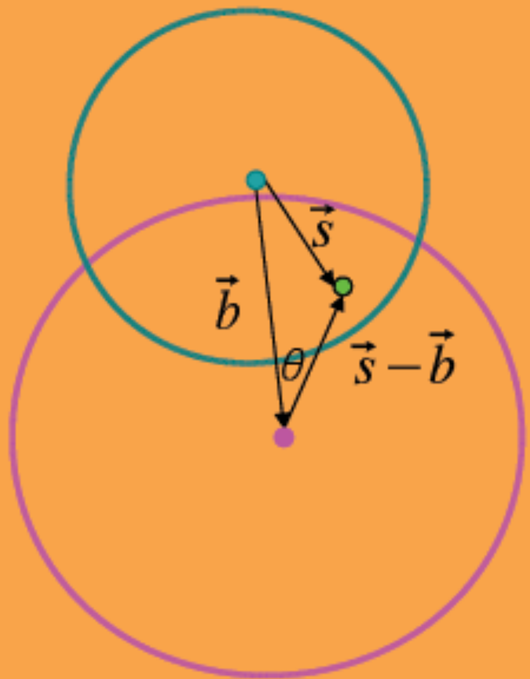
$$\frac{d\sigma_{inel}^{AB}}{db} = \sum_{n=1}^{AB} P(n,b) = 1 - \left[1 - T(b)\sigma_{in}\right]^{AB}$$

total inelastic cross section

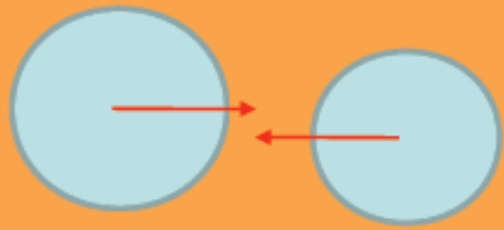
$$\sigma_{inel}^{AB} = \int db \left\{1 - \left[1 - T(b)\sigma_{in}\right]^{AB}\right\}$$

Overlap function

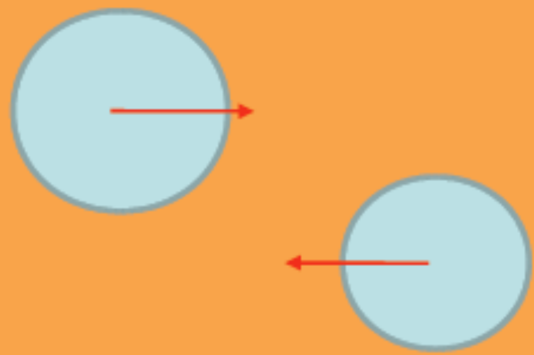
$$T_{AB}(b) = \int_{-\infty}^{\infty} d^2s \, T_A(s) \, T_B(s-b)$$



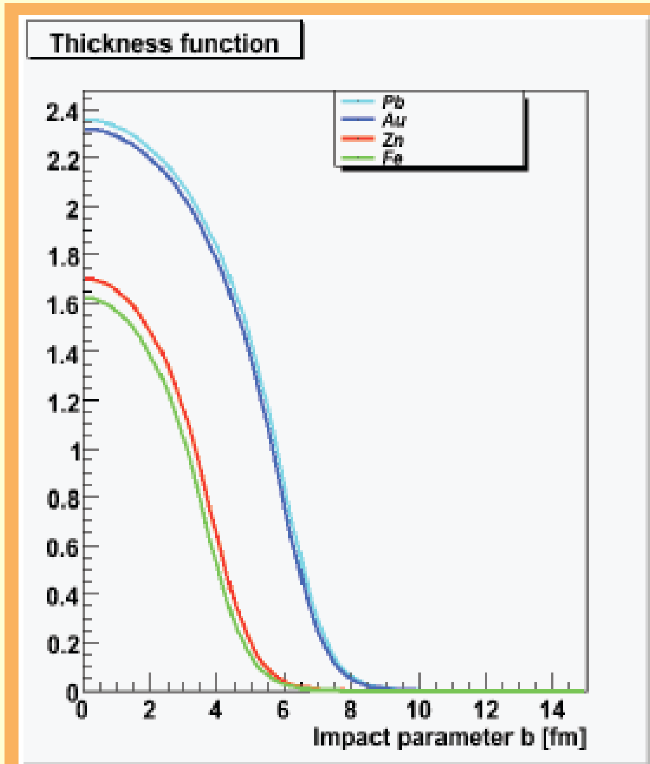
Colisiones centrales.

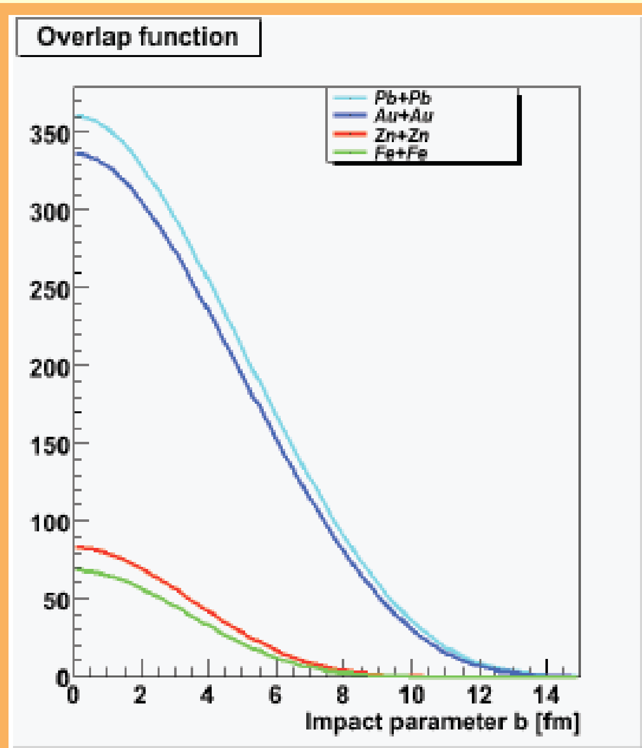


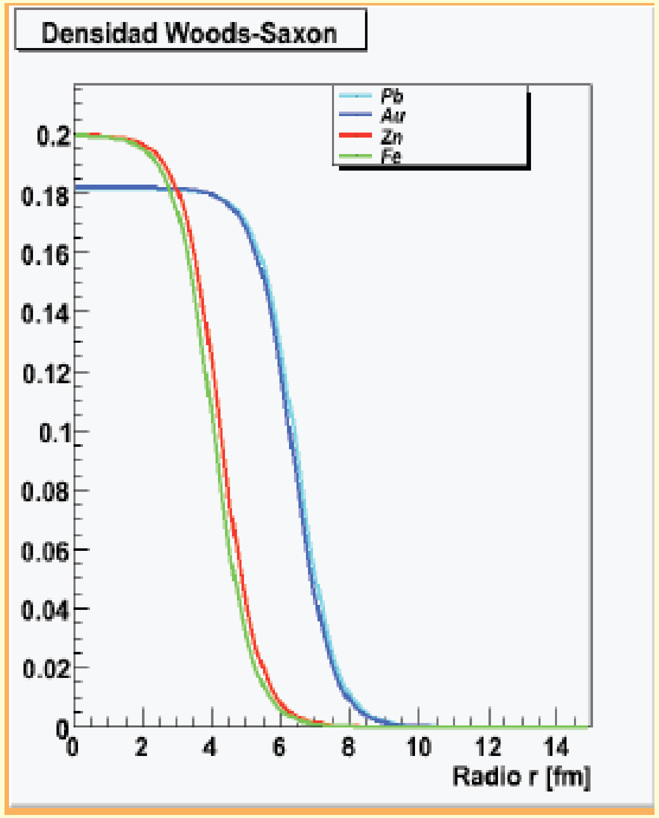
Colisiones periféricas.



$T_A(b)$ $T_{AA}(b)$


Thickness function
Pb
Au
Zn
Fe
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14
Impact parameter b [fm]


Overlap function
Pb+Pb
Au+Au
Zn+Zn
Fe+Fe
350
300
250
200
150
100
50
0
0 2 4 6 8 10 12 14
Impact parameter b [fm]

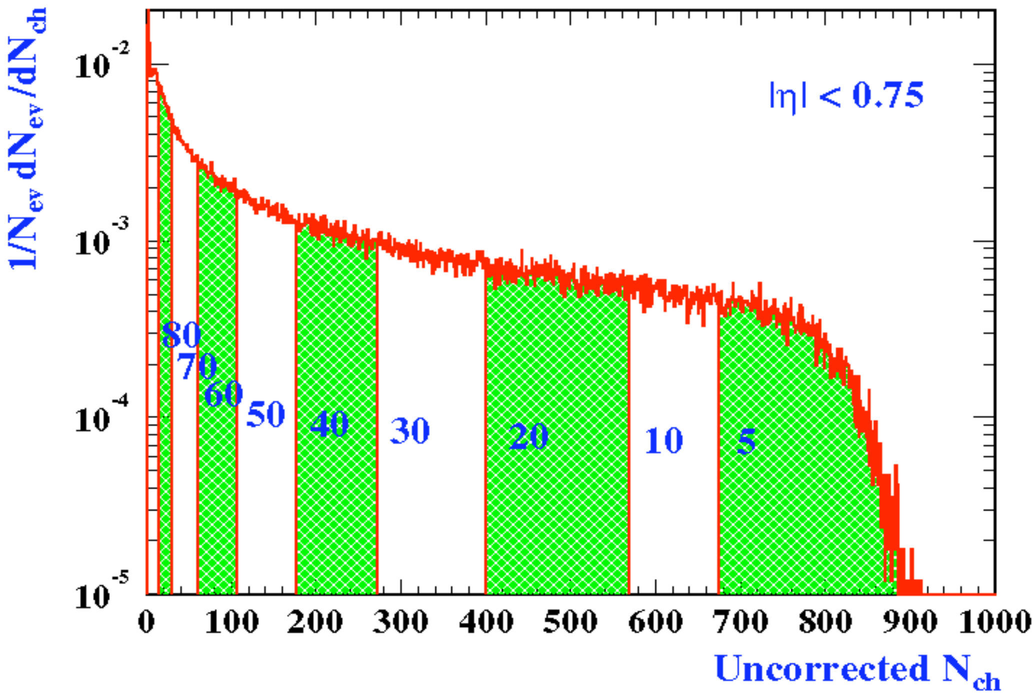


PARAMETRO DE IMPACTO	TAA			
	Pb	Au	Fe	Zn
0.08	360.98	335.95	68.36	82.84
0.83	355.24	331.05	66.19	80.39
1.58	340.7	317.11	60.82	74.29
2.33	319.01	296.33	53.04	65.5
3.08	292.14	270.61	44.09	55.13
3.83	261.85	241.71	34.72	44.22
4.58	229.66	211.05	25.78	33.64
5.33	196.91	179.98	17.87	24.07
6.08	164.7	149.55	11.43	16.02
6.83	133.96	120.65	6.64	9.77
7.58	105.46	94.05	3.46	5.39
8.33	79.93	70.37	1.61	2.65
9.08	57.8	49.97	0.67	1.17
9.83	39.6	33.66	0.26	0.47
10.58	25.27	20.95	0.09	0.17
11.33	14.81	11.92	0.03	0.06

Number of Participant Nucleons

a) Geometrical Interpretation of Observables

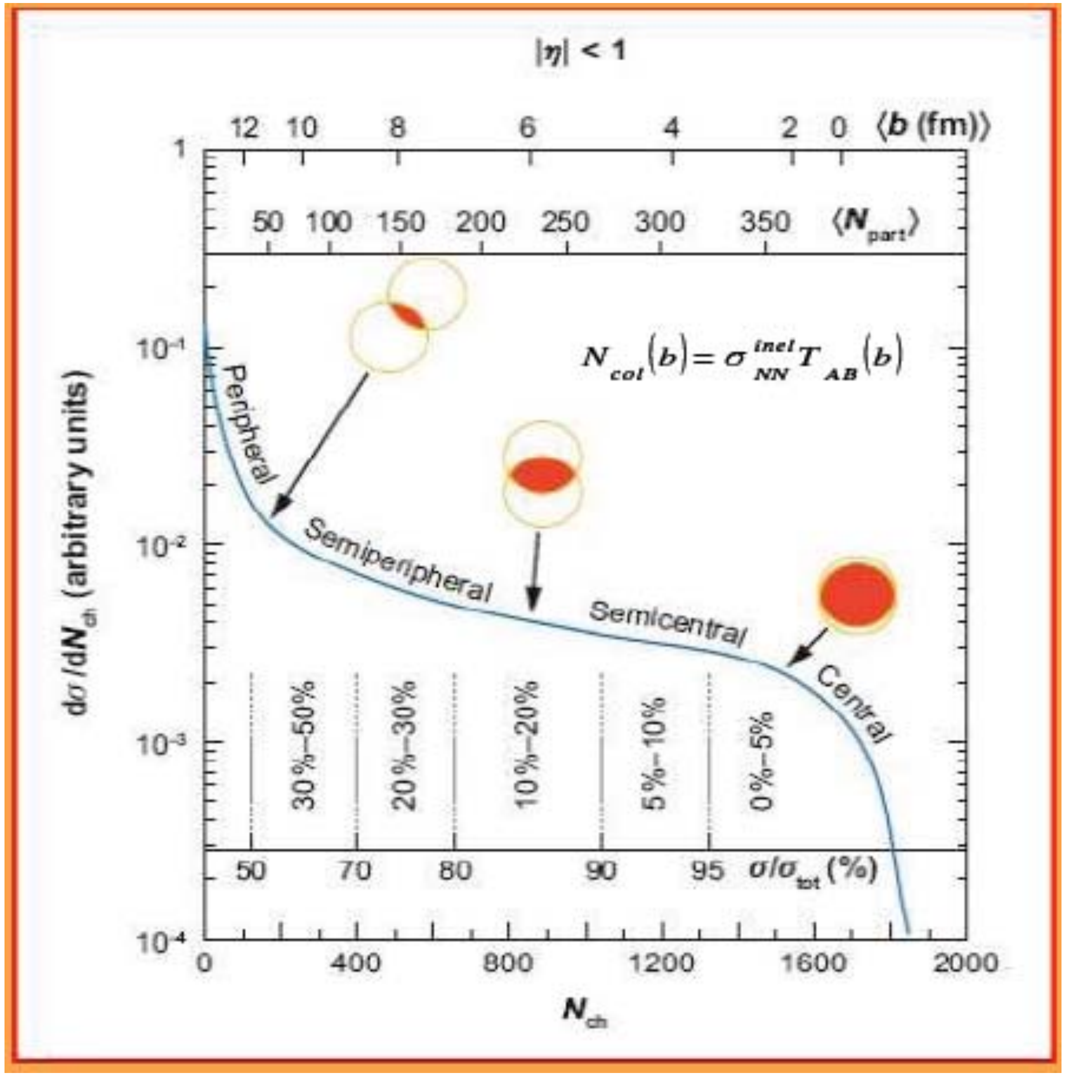
A monotonic relation between the observable and collision centrality is assumed



b) Estimate from direct measurement

missing energy from Zero-degree calorimeter

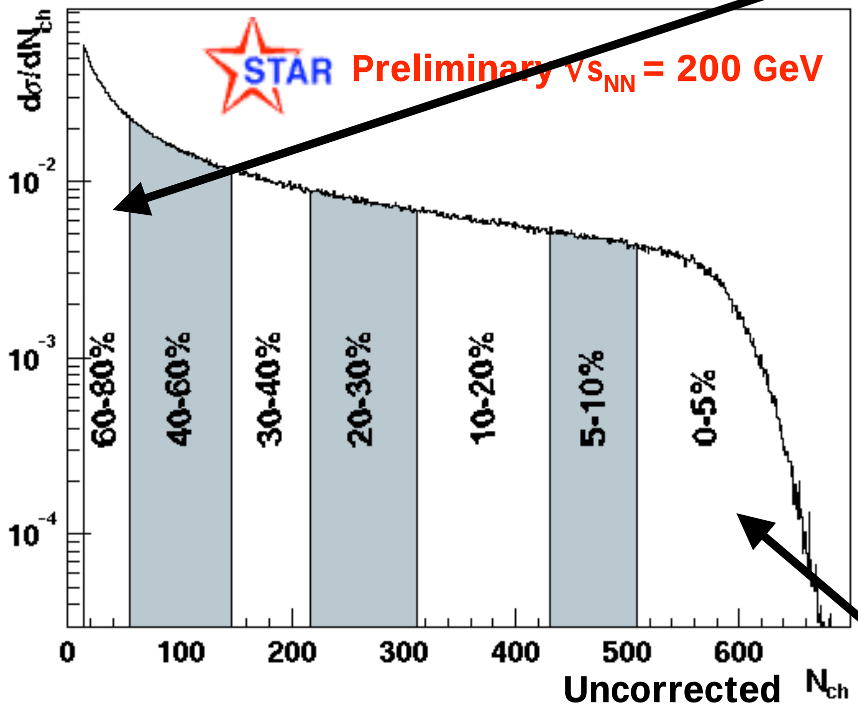
from dn/dy of protons….



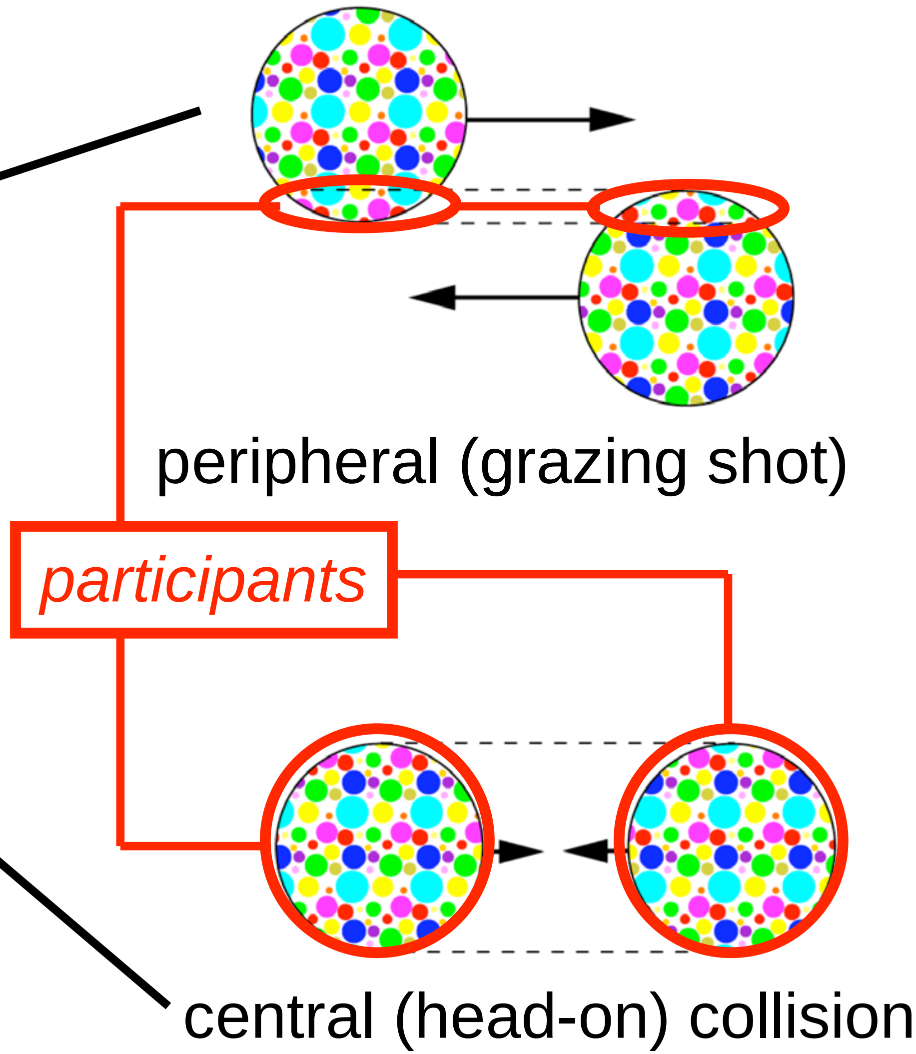
|η| < 1
12 10 8 6 4 2 0 ⟨b (fm)⟩
50 100 150 200 250 300 350 ⟨N_part⟩
N_col(b) = σ_NN^inel T_AB(b)
Peripheral
Semiperipheral
Semicentral
Central
30%–50%
20%–30%
10%–20%
5%–10%
0%–5%
50 70 80 90 95 σ/σ_tot (%)
dσ/dN_ch (arbitrary units)
1
10^-1
10^-2
10^-3
10^-4
0 400 800 1200 1600 2000
N_ch

Centrality: Experimental Control

Particle production scales with increasing centrality



Centrality classes based on mid-rapidity multiplicity



Procesos duros.

•Colisión A+B : Número de procesos duros entre las partículas puntuales contituyentes de nucleones es proporcional a $T_{AB}(b)$.

$$N_{hard}^{AB,enc} = T_{AB}(b)\ \sigma_{hard}^{pp}$$

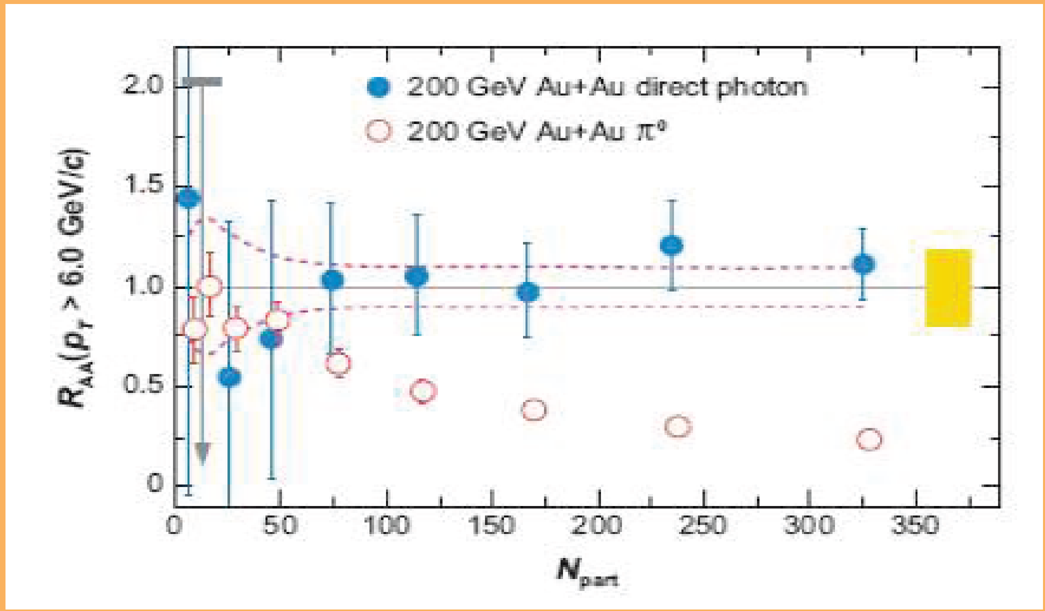
•En RHIC, la producción de partículas es usualmente medido en función de p_T. Si la sección eficaz invariante de un proceso duro que produce la partícula *x* ha sido medida para *p+p*, entonces la multiplicidad de *x* para una colisión inelástica *A+B* es:

$$\frac{1}{N_{inel}^{AB}} \frac{dN_x^{AB}}{dp_T} = \frac{T_{AB}(b)}{p_{inel}^{AB}(b)} \frac{d\sigma_x^{pp}}{dp_T}$$

Esto es al considerar que no hay efectos nucleares.

Si hay efectos nucleares, se introduce el factor de modificación nuclear $\boldsymbol{R_{AB}}$.

$R_{AB}(p_T)$=1 en ausencia de efectos nucleares.



Order of Magnitude

Geometrical CS:

pp $\pi r^2 = \pi(1\text{fm})^2 = 32$ mb

Au+Au Collisions:

$R_{au} = 1.2\, A^{1/3} = 6.98$ fm

$\pi b_{max} = \pi(2R)^2 = 6$ barn

1 barn = 10^{-24} cm^2

38. Plots of cross sections and related quantities 9

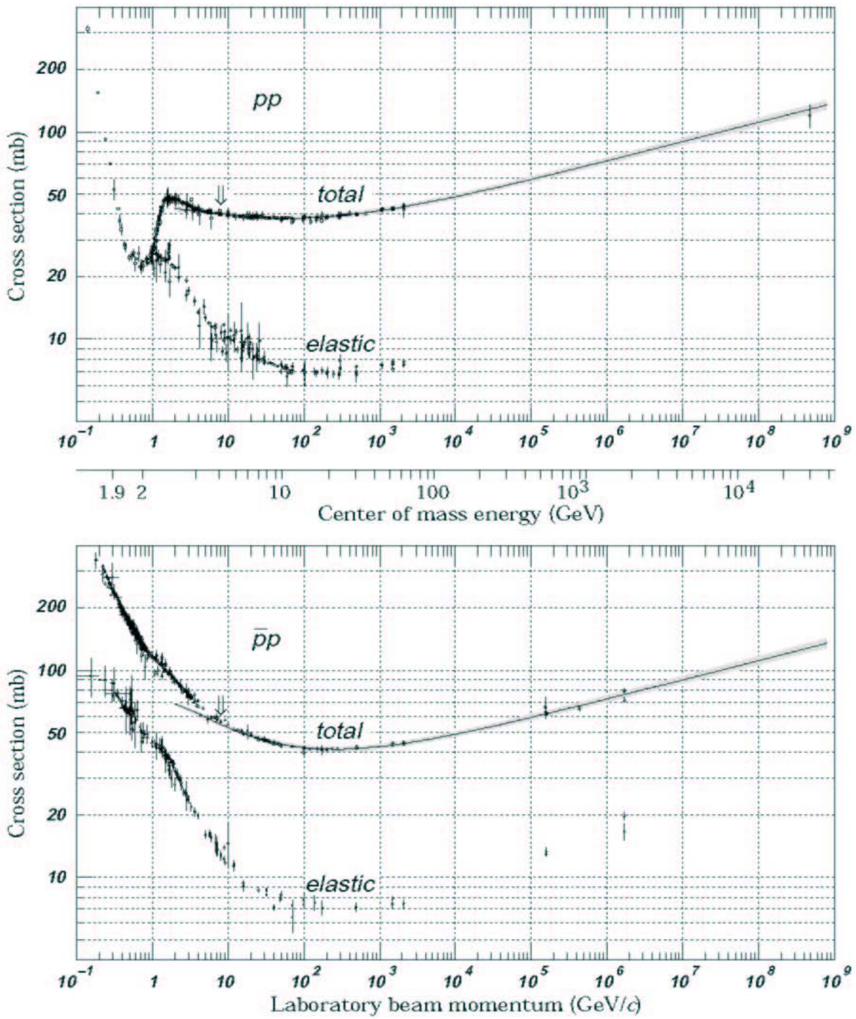
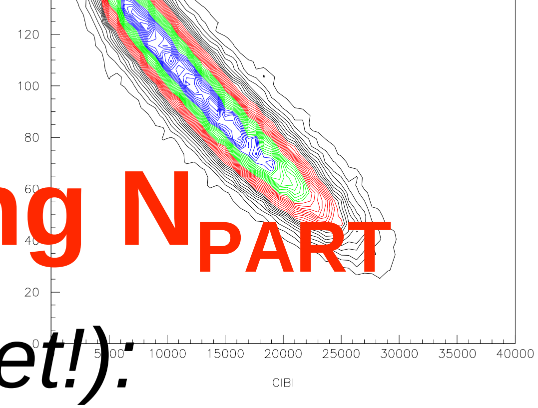


Figure 38.9: Total and elastic cross sections for pp and $\bar{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html (Courtesy of the COMPAS Group, IHEP, Protvino, Russia, August 1999.)

Directly Determining N_{PART}

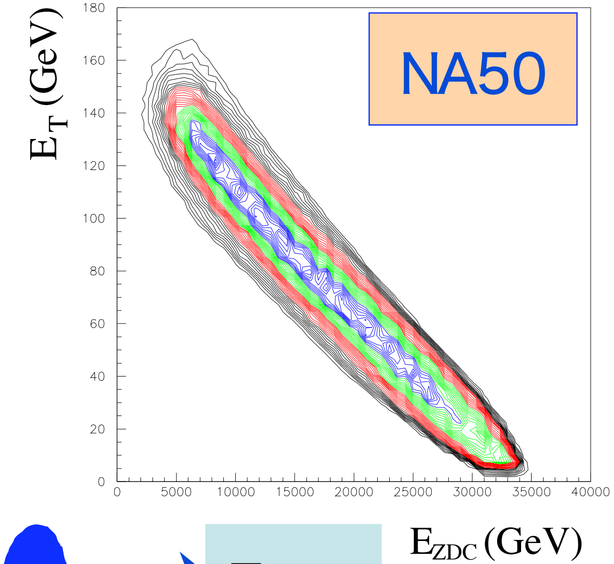


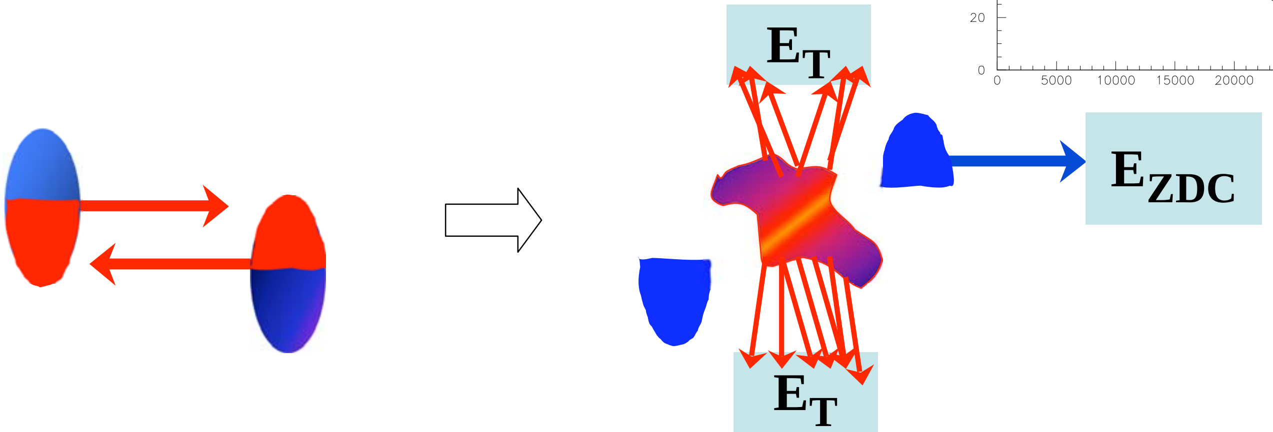
Best approach *(for fixed target!)*:

– Directly measure in a "zero degree calorimeter"

– [illegible] (for A+A collisions)

– Strongly (anti)-correlated with produced transverse energy:





Number of Participant Nucleons

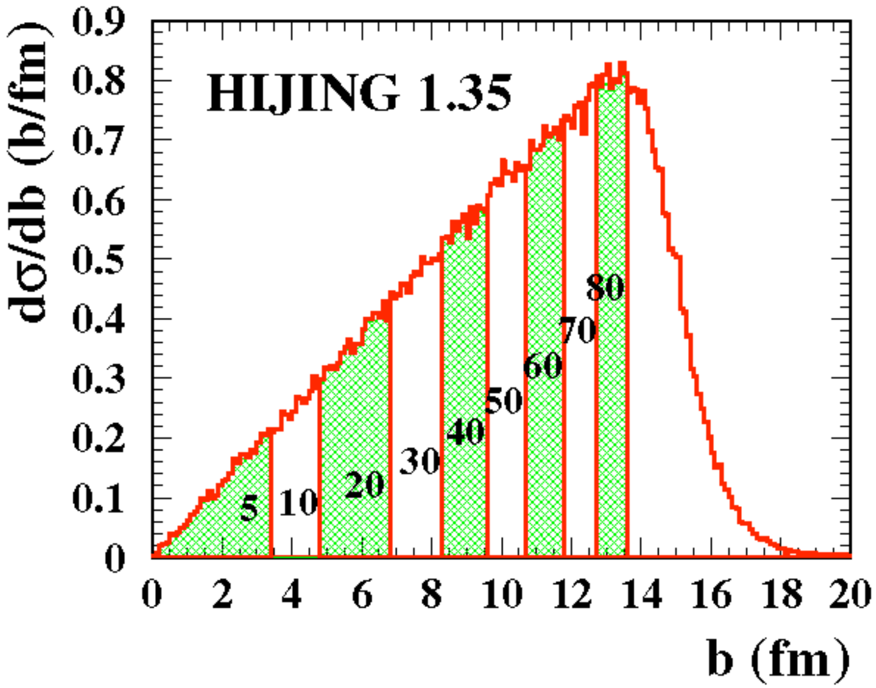
c) Dynamical Model

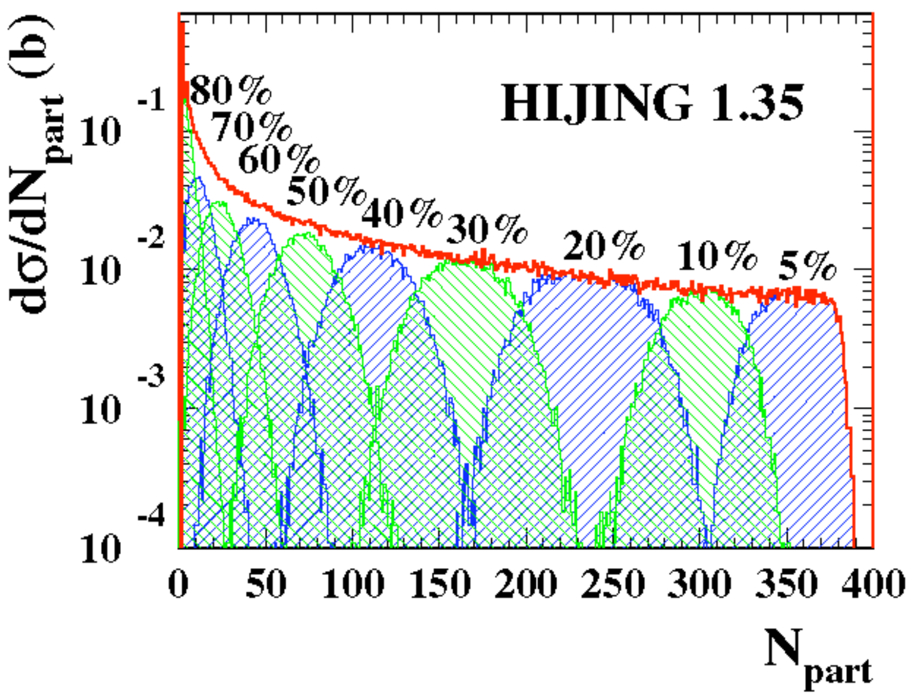
Tune to fit experimental measurement

From model to convert measurement to impact parameter and number of participant nucleons

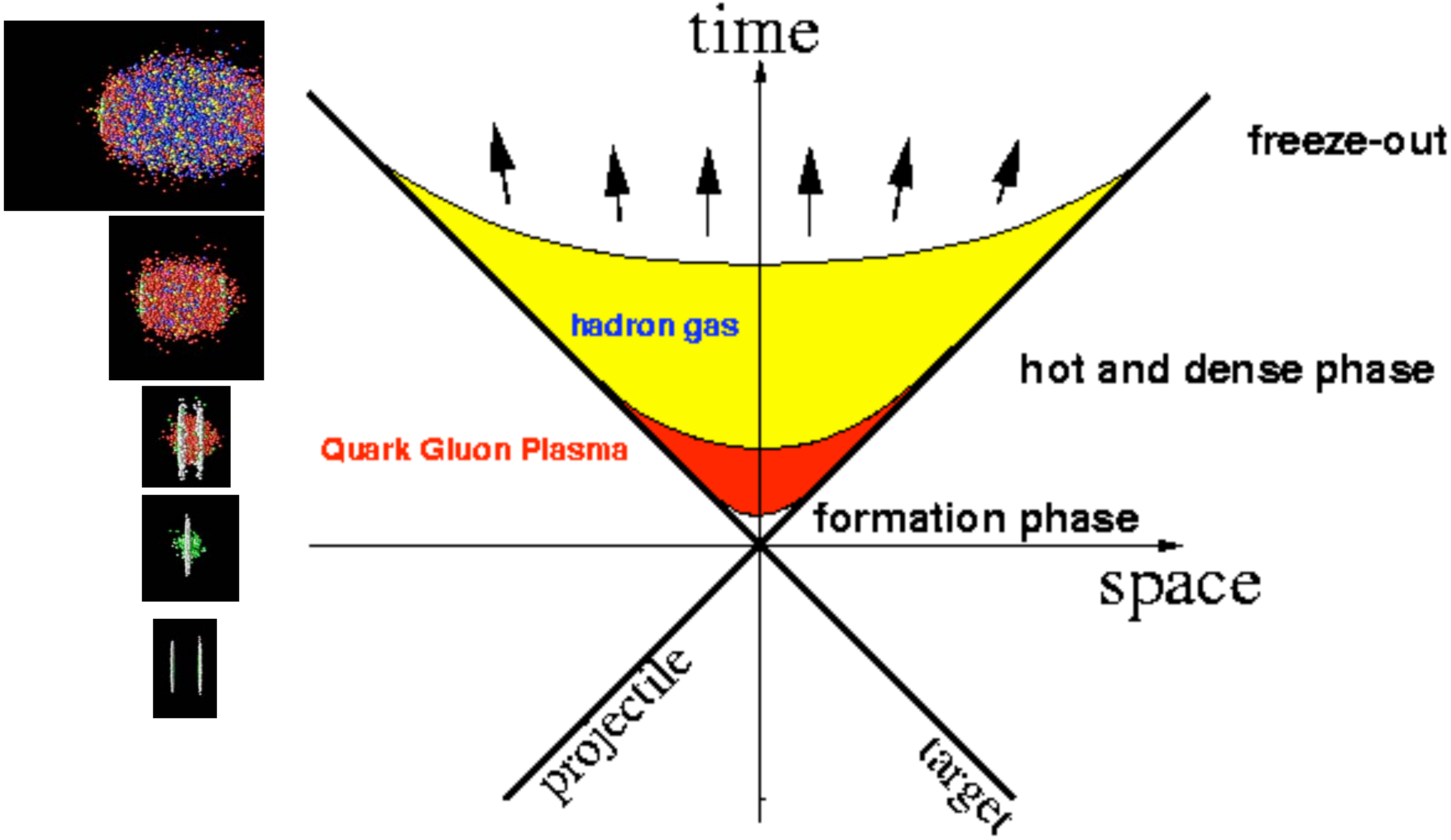
++ Fluctuations are included

- - Physical picture is biased to begin with

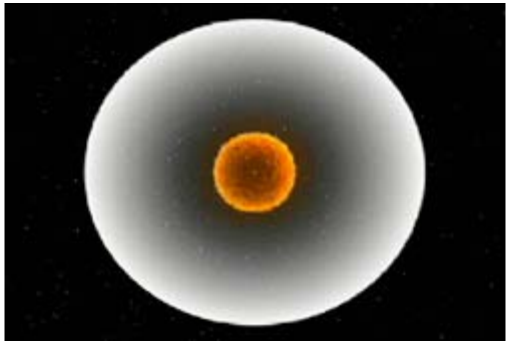




Collision Dynamics



Energy density



Bjorken Energy Density

Energy density $$\varepsilon = \frac{E\Delta N}{A\Delta z}$$

E ◊ average energy per particle
A ◊ transverse area of the collision volume
Δz ◊ longitudinal interval
ΔN ◊ number of particles in Δz interval

$v_z = z/t = \tanh y; \quad z = \tau \sinh y$
$\Delta z = \tau \cosh y \, \Delta y$
$E = m_T \cosh y$

$$\varepsilon = \frac{m_T \cosh y \, \Delta N}{A \, \tau \cosh y \, \Delta y} \rightarrow \frac{m_T \, dn/dy}{A\tau}$$

count energy in *produced* particles/matter

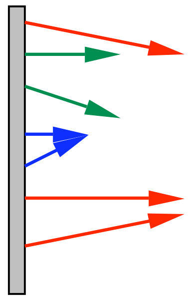
where "produced" is everything at rapidities intermediate between those of the original incoming nuclei.

And then assume

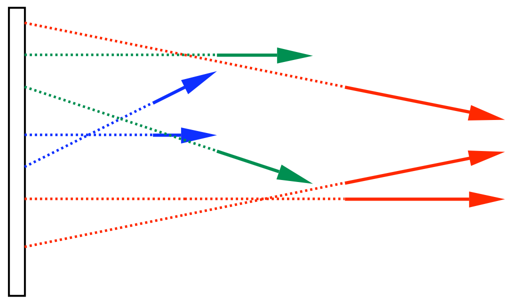
- thin radiator
- classical trajectories
- finite formation time

Highly relativistic nucleus-nucleus collisions: the central rapidity region

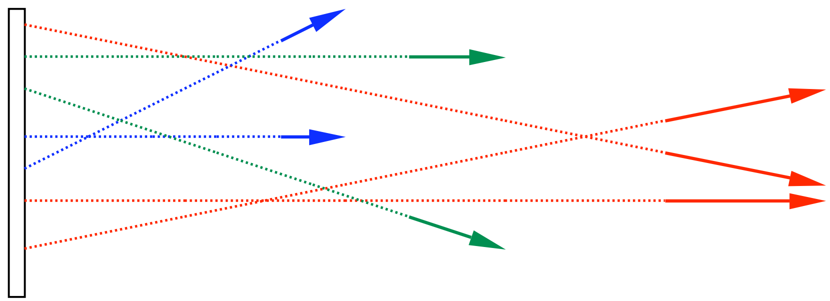
J.D. Bjorken, Phys. Rev. D 27(1983)140



Particles in a thin box with
random velocities



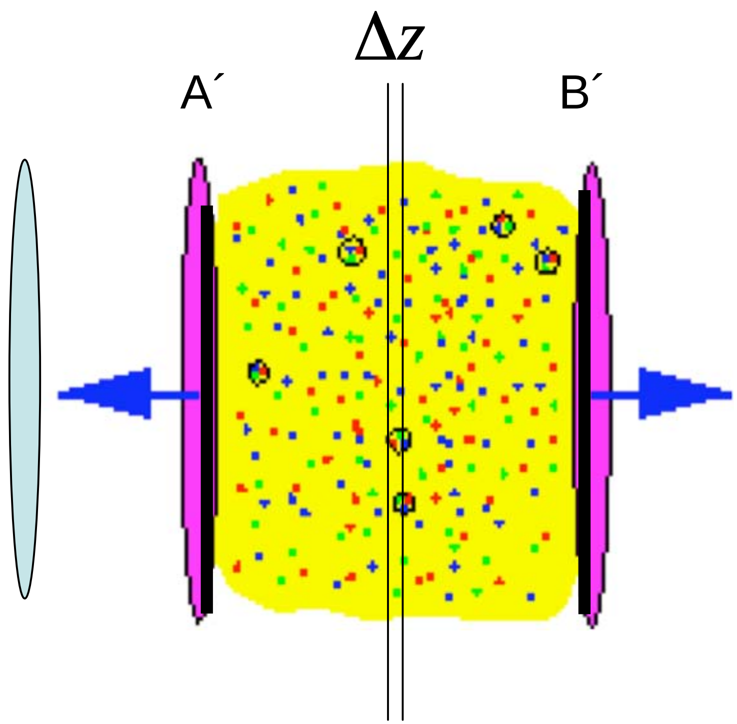
Release them suddenly,
and let them follow
classical trajectories
without interactions



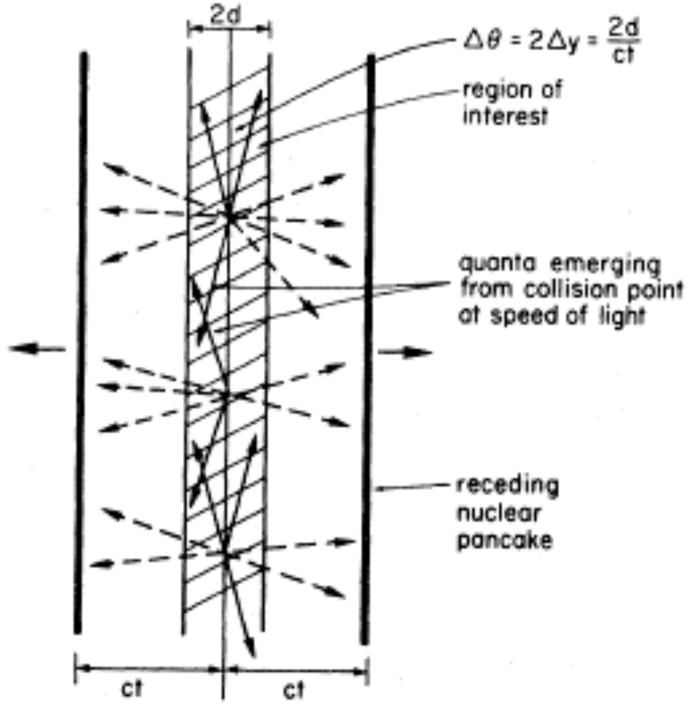
Strong position-
momentum
correlations!

after collision

overlapping area
A



at extremely high energy we can neglect the thickness

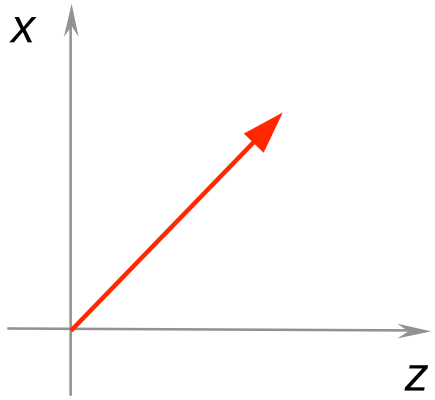


Volume = ***A*** Δz

The number density in this volume at $z = 0$ and $\tau = \tau_0$

$$\frac{\Delta N}{A\Delta z} = \frac{1}{A}\frac{dN}{dy}\frac{dy}{dz}\bigg|_{y=0}$$

$$\frac{\Delta N}{A\Delta z} = \frac{1}{A}\frac{dN}{dy}\frac{1}{\tau_0 \cosh y}\bigg|_{y=0}$$



$$p_T = p_x$$

$$p_z = m_T \sinh y$$

$$\beta_z = \frac{p_z}{E} = \tanh y$$

$$y = \tanh^{-1} \beta_z$$

for $\beta_z << 1$, $\quad y \approx \beta_z$

The energy ***E*** of a particle with rapidity ***y*** is: $E = m_T \cosh y$

Therefore the initial energy density is:

$$\varepsilon_0 = E \frac{\Delta N}{A \Delta z} = m_T \cosh y \frac{\Delta N}{A \Delta z}$$

since

$$\frac{\Delta N}{A \Delta z} = \frac{1}{A} \frac{dN}{dy} \frac{1}{\tau_0 \cosh y} \bigg|_{y=0}$$

the initial energy density averaged over the transverse area A at the proper time τ_0

$$\varepsilon_0 = \frac{m_T}{\tau_0 A} \frac{dN}{dy} \bigg|_{y=0}$$

energy density $\Longleftrightarrow$ rapidity in his paper, Bjorken says $\frac{N}{A} = \frac{A}{\pi(1.2A^{1/3} fm)} = \frac{A^{1/3}}{4.5 fm^2}$

so that for $\tau_0 \approx 1 fm/c$

one gets an initial energy density:

$$\varepsilon_0 \approx 1 - 10 GeV/fm^3$$

It is not clear at this energy density what the produced quanta which carry this energy really are

J.D. Bjorken

For many years people used $\tau_0 \approx 1 fm/c$

here some estimates of the formation time …

$$\langle \varepsilon(t) \rangle = \frac{1}{t\,A} \frac{dE_{\mathrm{T}}(t)}{dy}$$

How low can t go? Two basic limits:

$t >> R\left(1/\gamma_{\mathrm{A}} + 1/\gamma_{\mathrm{B}}\right)$	Crossing time
$t \geq \tau_{\mathrm{Form}}$	Formation time

$$\langle \varepsilon(\tau_{\mathrm{Form}}) \rangle = \frac{1}{\tau_{\mathrm{Form}}\,A} \frac{dE_{\mathrm{T}}(\tau_{\mathrm{Form}})}{dy} \approx \underbrace{\frac{1}{\tau_{\mathrm{Form}}\,A} \frac{dE_{\mathrm{T}}^{\mathrm{Final\ State}}}{dy}}_{\varepsilon_{\mathrm{Bjorken}}}$$

$2R/\gamma = 5.3$ fm/c for AGS Au+Au, 1.6 fm/c for SPS Pb+Pb.

formation time estimates

Generic quantum mechanics:

a particle can't be considered *formed* in a frame faster than

$$\hbar/E$$

Translation: $\tau_{\text{Form}} \geq 1/m_{\text{T}} \sim 1/\langle m_{\text{T}}\rangle$

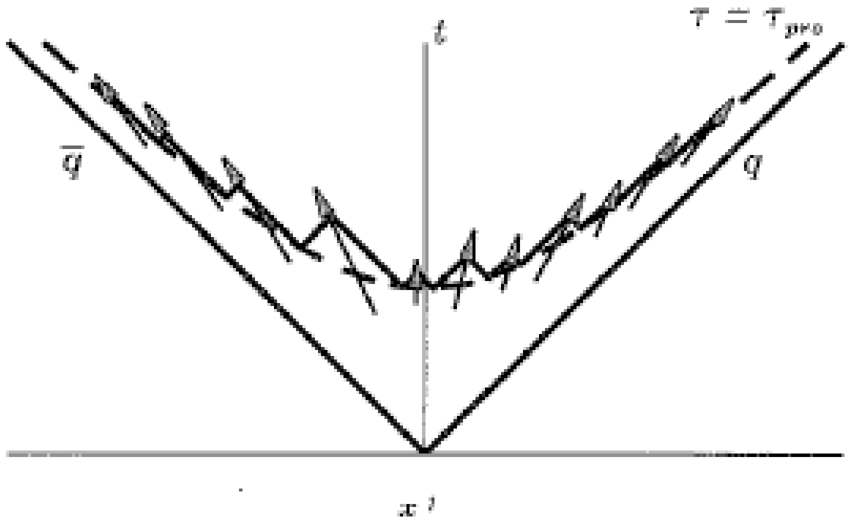
$$\langle m_{\text{T}}\rangle = \frac{dE_{\text{T}}(\tau_{\text{Form}})/dy}{dN(\tau_{\text{Form}})/dy} \approx \frac{dE_{\text{T}}/d\eta}{dN/d\eta} \quad \text{(Final State)}$$

PHENIX Data: $(dE_T/d\eta)/(dN^{ch}/d\eta) \sim 0.85$ GeV

Assuming 2/3 of particles are charged, this implies $\tau_{\text{Form}} \sim 0.35$ fm/c

formation time estimate

Show that, if all the vertices fall on the proper time curve



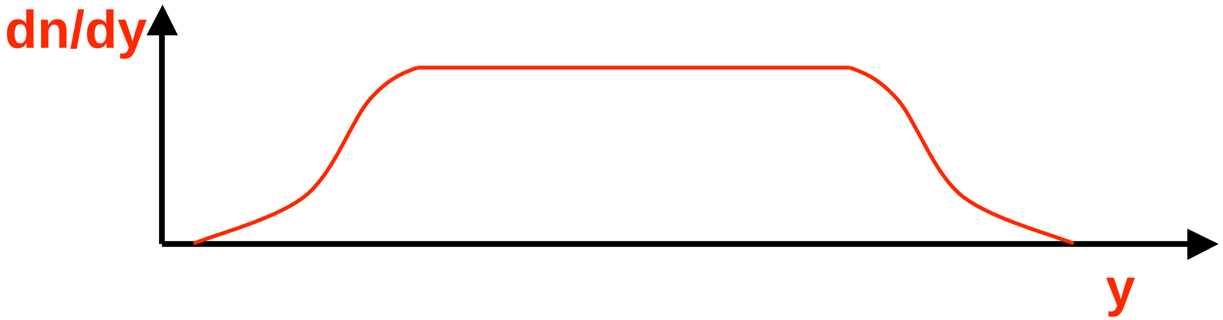
the rapidity distribution of the produced particles is a constant given by

$$\frac{dN}{dy} = \frac{\kappa \tau_{production}}{m_T}$$

For $\sqrt{s} = 62.8$ GeV $\frac{dN}{dy} \approx 3$ including neutral particles $\tau_{production} \approx 0.6\, fm/c$

Bjorken Scaling

Bjorken Ansatz: "…… at sufficient high energy there is a 'central-plateau' structure for the particle production as a function of the rapidity variable."



Physics must be invariant under Lorentz-boost:

1) Local thermodynamic quantity must be a function of

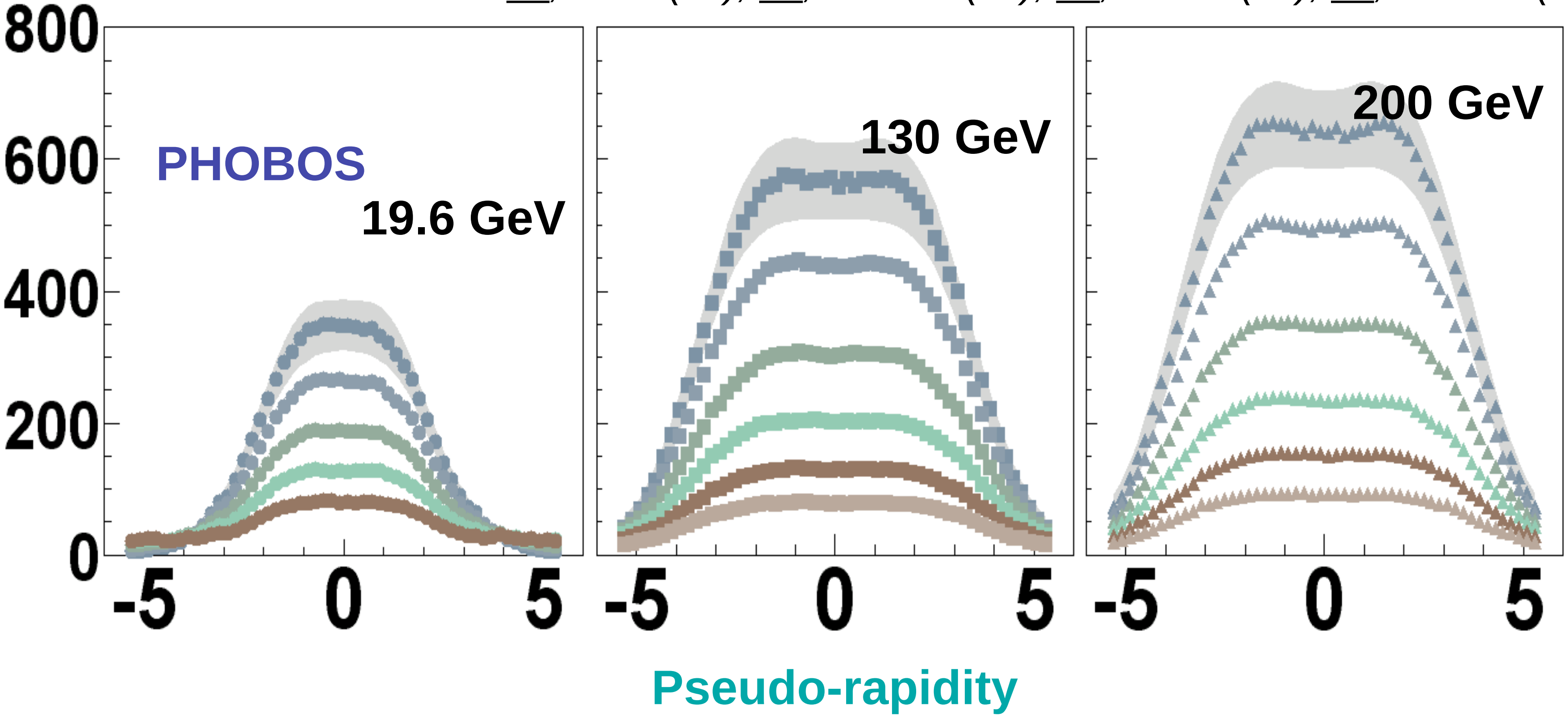
proper time $\tau = \sqrt{t^2 - z^2}$ **only.**

2) Longitudinal velocity

$v_z = z/t$ **or** $y = 0.5 \ln((t+z)/(t-z))$

Initial Energy Density Estimate

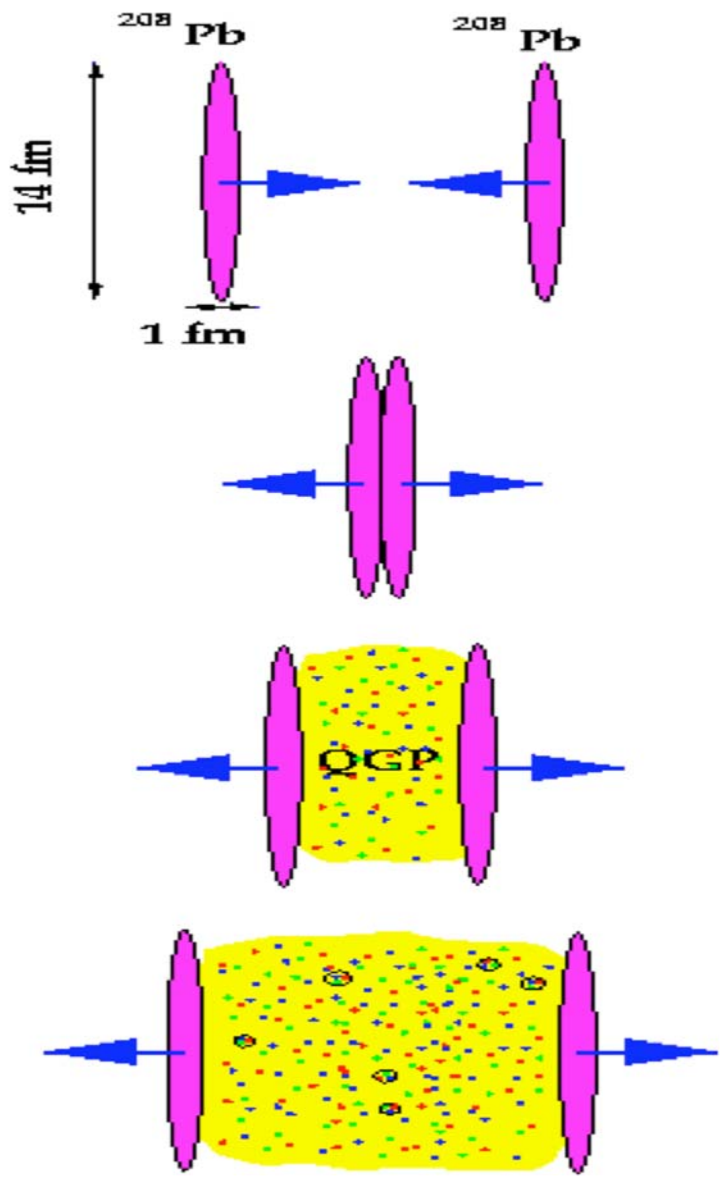
*PRL **85**, 3100 (00); **91**, 052303 (03); **88**, 22302 (02), **91**, 052303 (03)*



hminus:
Central Au+Au <pT>=0.508GeV/c
pp: 0.390GeV/c

Within |η|<0.5 the total transverse momentum created is 1.5x650x0.508 ~ 500 GeV from an initial transverse overlap area of $\pi R^2 \sim 153$ fm² !

Energy density
$\varepsilon \sim$ 5-30 ε_0 at early time τ=0.2-1 fm/c !



Bjorken formula -PRD(1983)-

$$\frac{dE}{dy} \approx \frac{3}{2}\frac{dN_{ch}}{dy}\langle E \rangle$$

$$N = \frac{3}{2} N_{ch}$$

density of energy:

$$\varepsilon \approx \frac{3}{2}\frac{1}{\pi R_N^{2} A^{2/3}}\langle E \rangle \frac{dN_{ch}}{dy}$$

$2-3\ {GeV}/{fm^3}$ CERN SPS $5-6\ {GeV}/{fm^3}$ RHIC

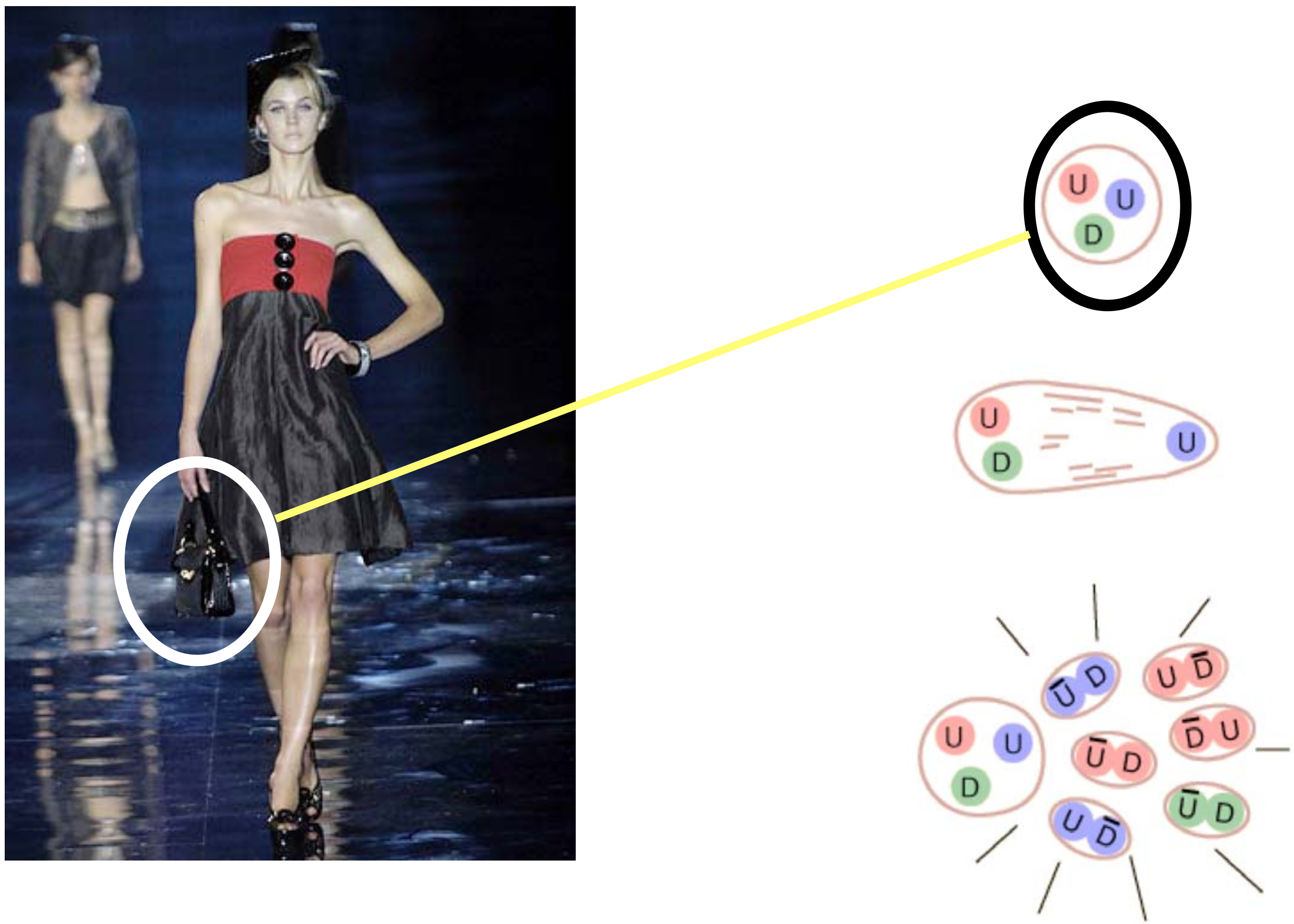
Pb-Pb at LHC:

$$10-38\ {GeV}/{fm^3}$$

Bag Model

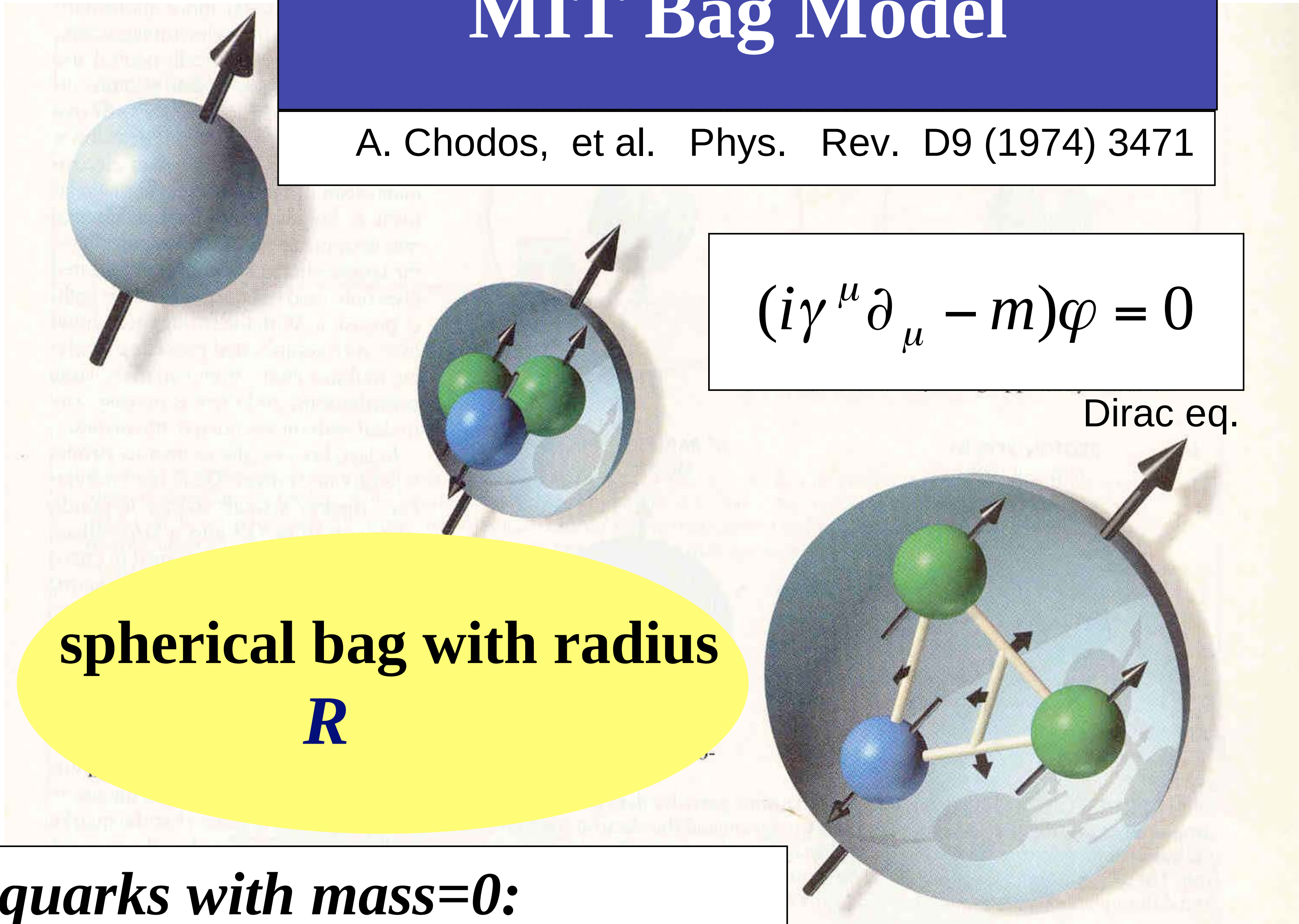
What´s in model´s bag

What´s in a bag´s model



MIT Bag Model

A. Chodos, et al. Phys. Rev. D9 (1974) 3471



$$(i\gamma^{\mu}\partial_{\mu} - m)\varphi = 0$$

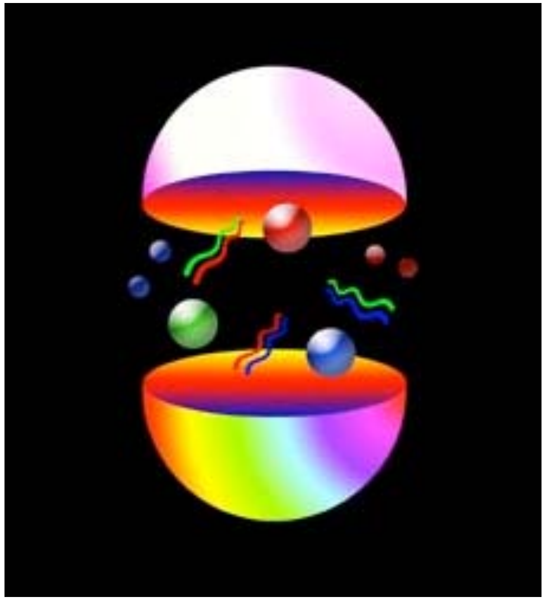
Dirac eq.

spherical bag with radius

R

quarks with mass=0:

$$\gamma p \varphi = 0$$



Using the Dirac representation of the γ matrices

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad \gamma = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}$$

2x2 unit matrix

Pauli matrices

$$\varphi = \begin{pmatrix} \varphi_+ \\ \varphi_- \end{pmatrix}$$

four component wave function

MIT Bag Model

$$(\gamma^0 p^0 - \vec{\gamma} \cdot \vec{p})\varphi = 0$$

$$\begin{pmatrix} p^0 & -\vec{\sigma} \cdot \vec{p} \\ \vec{\sigma} \cdot \vec{p} & -p^0 \end{pmatrix} \begin{pmatrix} \varphi_+ \\ \varphi_- \end{pmatrix} = 0$$

$$p^0 \varphi_+ - \vec{\sigma} \cdot \vec{p} \varphi_- = 0 \qquad \vec{\sigma} \cdot \vec{p} \varphi_+ - p_0 \varphi_- = 0$$

$$(\vec{p}^2 - (p^0)^2)\varphi_+ = 0$$

$$\boxed{\begin{aligned} \varphi_+(\vec{r}, t) &= N e^{-ip^0 t} j_0(p^0 r)\chi_+ \\ \varphi_-(\vec{r}, t) &= N e^{-ip^0 t} \vec{\sigma} \cdot \hat{r} j_1(p^0 r)\chi_- \end{aligned}}$$

lowest energy solution

Spherical Bessel function

$j_0 \quad j_1$

Confinement $\Rightarrow$ ***current flux through the surface of the sphere*** = 0

i.e. the normal component of $J_\mu = \bar{\varphi}\gamma_\mu\varphi$

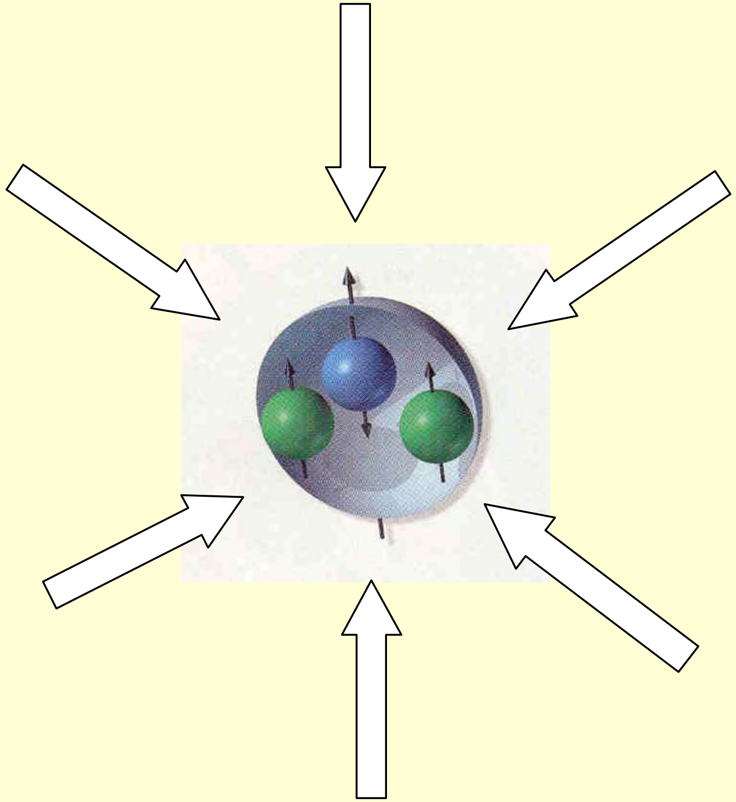
$$n^\mu \bar{\varphi}\gamma_\mu \varphi = 0 \quad \Rightarrow \quad \bar{\varphi}\varphi = 0$$

$$\bar{\varphi}\varphi\big|_{r=R} = \left[j_0(p^0R)\right]^2 - \vec{\sigma}\cdot\hat{r}\,\sigma\cdot\hat{r}\left[j_1(p^0R)\right]^2 = 0$$

$$\left[j_0(p^0R)\right]^2 - \left[j_1(p^0R)\right]^2 = 0$$

$$p^0R = 2.04$$

energy of N quarks: $E = \dfrac{2.04N}{R}$



Bag pressure

B

energy of N quarks in the bag with presure B:

$$E = \frac{2.04N}{R} + \frac{4\pi}{3} R^3 B$$

Equilibrium

$$\frac{\partial E}{\partial R} = 0 \Longrightarrow 4\pi R^2 B - \frac{2.04N}{R^2} = 0$$

$$\therefore R = \left(\frac{2.04N}{4\pi B} \right)^{1/4}$$

for a proton $R = 0.8\, fm$ ***with three quarks***

$$B^{1/4} = \left(\frac{2.04N}{4\pi} \right)^{1/4} \frac{1}{R} \qquad \hbar c = 197 MeV - fm$$

$$B^{1/4} = 1.044 \times 197.3$$

$$B^{1/4} = 206 MeV$$

QGP High Temperature

Consider a quark gluon system in thermal equilibrium
Quarks and Gluons are non interacting and massles

$$P = [g_g + \frac{7}{8}(g_q + g_{\bar{q}})]\frac{\pi^2}{90}T^4$$

degeneracy number of gluons quarks anti-quarks

$$g_g = 8 \times 2$$

gluons polarization

$$g_q = g_{\bar{q}} = N_{colors} \times N_{spin} \times N_{flavor}$$

3 2 2 or 3

$$\boxed{P = 37\frac{\pi^2}{90}T^4} \quad \boxed{\varepsilon = 37\frac{\pi^2}{30}T^4}$$

The phase space volume of quarks in a spatial volume ***V*** with momentum ***p*** in the interval ***dp*** is

$$4\pi p^2 V dp$$

since each state occupies a phase space volume of $(2\pi h)^3$ the number of states characterized by a momentum ***p*** in the interval ***dp*** is $4\pi p^2 dp V/(2\pi)^3$

The ocupation probability is given by the Fermi –Dirac distribution

The number of quarks in a volume ***V*** with momentum ***p*** within the interval ***dp*** is

$$dN_q = \frac{g_q V\, 4\pi p^2 dp}{(2\pi)^3}\left[\frac{1}{1+e^{(p-\mu_q)/T}}\right]$$

chemical potential

quark degeneracy

Fermi-Dirac distribution

The presence of anti-quarks corresponds to the absence of quarks in the negative energy states

$$n_q = \frac{g_q}{(2\pi)^3}\int_{-\infty}^{0}\left[1-\frac{1}{1+e^{(p_0-\mu_q)/T}}\right]4\pi p_0^2\, dp_0$$

$$n_q = \frac{g_q}{(2\pi)^3}\int_0^\infty \frac{1}{1+e^{(p_0+\mu_q)/T}} 4\pi p_0^2 dp_0$$

In our case the number of quarks is the same as the antiquarks, $\mu_q = 0$

Relativistic massless quark gas ➡ $E^2 = p^2 + \cancel{m^2}$

The energy of a quark in a system of Volume **V** and temperature **T**

valor esperado

$$E_q = \frac{g_q V}{2\pi^2}\int_0^\infty \frac{p^3 dp}{1+e^{p/T}}$$

$$E_q = \frac{g_q V}{2\pi^2} T^4 \int_0^\infty \frac{z^3 dz}{1+e^z}$$

$$E_q = \frac{g_q V}{2\pi^2} T^4 \Gamma(4) \sum_{n=0}^\infty (-1)^n \frac{1}{(n+1)^4}$$

The energy of the system due to quarks is therefore

$$E_q = \frac{7}{8} g_q V \frac{\pi^2}{30} T^4$$

for massles fermions and bosons pressure and energy are related:

$$P = \frac{1}{3}\frac{E}{V}$$

so that, pressure due to quarks

$$P_q = \frac{7}{8} g_q V \frac{\pi^2}{90} T^4$$

anti-quarks

$$P_{\bar{q}} = \frac{7}{8} g_{\bar{q}} V \frac{\pi^2}{90} T^4$$

the pressure due to gluons

$$E_g = \frac{Vg_g}{2\pi^2}\int_0^\infty p^3 dp\left[\frac{1}{e^{p/T}-1}\right]$$

gluon degeneracy

Bose-Einstein distribution

$$E_g = g_g V \frac{\pi^2}{30} T^4$$

and

$$P_g = g_g V \frac{\pi^2}{90} T^4$$

so that, the total pressure is $P = P_q + P_{\bar{q}} + P_g$

We have therefore an energy density of $2.54 \; {}^{GeV}\!/\!_{fm^3}$

at $T = 200 \; MeV$

$$T_c = \left(\frac{90}{37\pi^2} \right)^{1/4} B^{1/4}$$

$$T_c = \left(\frac{90}{37\pi^2} \right)^{1/4} 206$$

$$T_c = 145 \quad MeV$$

If quark matter is heated to a

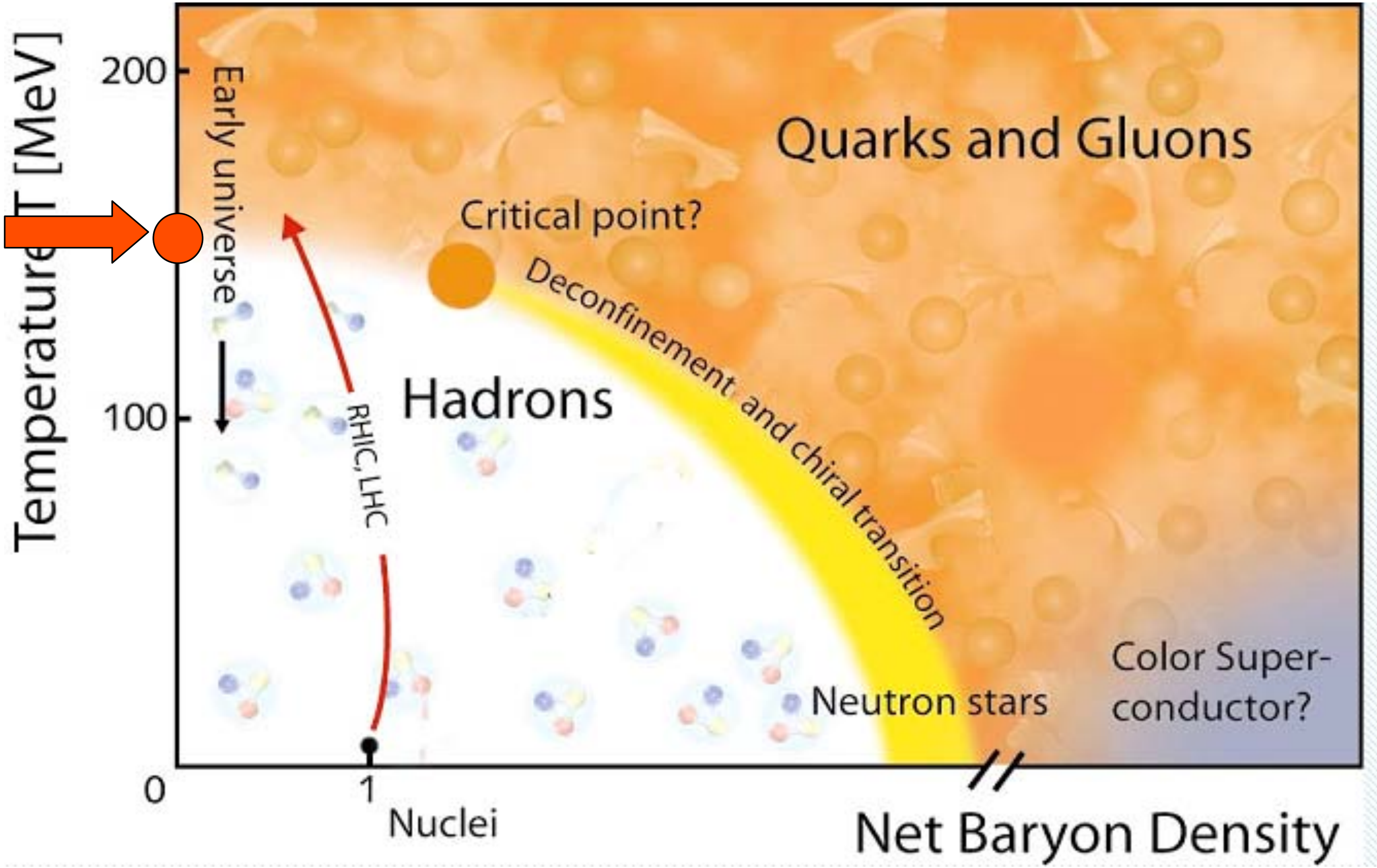
$$T > T_c$$

the quark gluon matter in the bag will have pressure greater that the bag-pressure

$\Rightarrow$

bag breaks deconfined QGP

Phase Diagram of QCD Matter



QGP High Baryon Density

deconfinement may happen even at T = 0 → at which baryon density ?

The number of states in a volume ***V*** with momentum ***p*** in the interval ***dp***

$$\frac{g_q V}{(2\pi)^3} 4\pi p^2 dp$$

The number of quarks is

$$N_q = \frac{g_q V}{(2\pi)^3} \int_0^{\mu_q} 4\pi p^2 dp = \frac{g_q V}{6\pi^2} \mu_q^3$$

The number density of quarks is

$$\boxed{n_q = \frac{N_q}{V} = \frac{g_q}{6\pi^2} \mu_q^3}$$

The energy of the quark gas in a volume ***V***

$$E_q = \frac{g_q V}{(2\pi)^3}\int_0^{\mu_q} 4\pi p^3 dp = \frac{V g_q}{8\pi^2}\mu_q^4$$

energy density

$$\varepsilon = \frac{E_q}{V} = \frac{g_q}{8\pi^2}\mu_q^4$$

from the relation between the pressure and the energy density

$$P_q = \frac{1}{3}\frac{E}{V} = \frac{g_q}{24\pi^2}\mu_q^4$$

Change of state $\rightarrow$ $P_q = B$ $\Rightarrow$ $\mu_q = \left(\frac{24\pi^2}{g_q}B\right)^{1/4}$

this corresponds to a CRITICAL quark number density

$$n_q(QGP) = 4\left(\frac{g_q}{24\pi^2}\right)^{1/4} B^{3/4}$$

i.e. **Critical Baryon Number**

$$n_q(QGP) = \frac{4}{3}\left(\frac{g_q}{24\pi^2}\right)^{1/4} B^{3/4}$$

Let's take ordinary nuclear matter as composed only of ***u*** and ***d*** quarks

$$g_q = 3_{colors} \times 2_{spin} \times 2_{flavor}$$

For a Bag pressure $B^{1/4} = 206\ MeV$

The critical baryon number density $= 0.72 / fm^3$

at T=0

Phase Diagram of QCD Matter