Constraints on the opacity profile of the sun from helioseismic and solar neutrino data

F. L. Villante – Università dell' Aquila and LNGS-INFN

Work done in collaboration with B. Ricci

Outline

- Solar abundances: the solar composition problem and the SSM
- Linear Solar Models: a tool to investigate the solar interior
- Application: what we know about opacity (and metals) in the sun
- Summary and conclusions

The solar composition problem

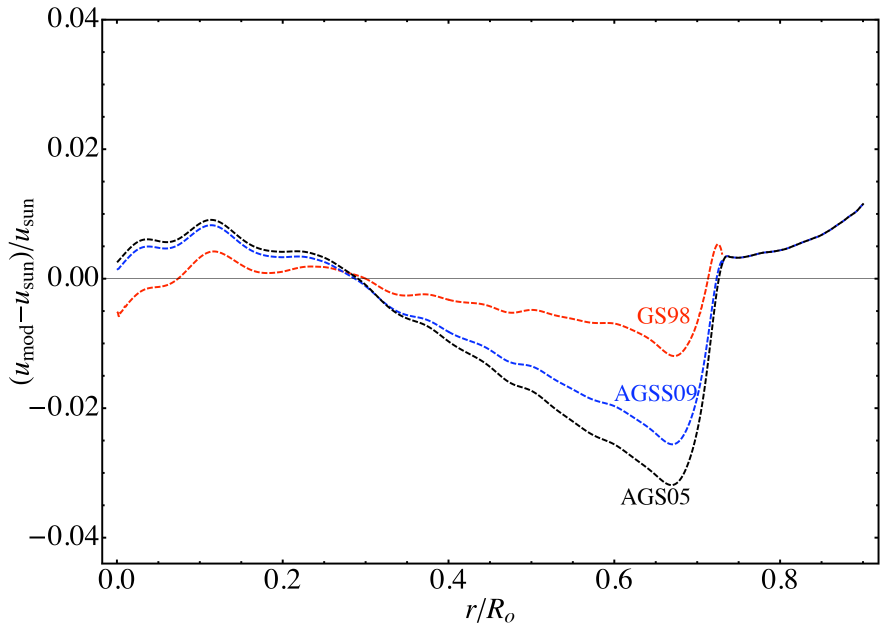
The latest solar photospheric abundances leads to SSMs which do not correctly reproduce helioseismic observables

squared isothermal sound speed

$$u = P/\rho$$

Note that: $c^2 = \gamma\, u$

$$\begin{cases} c^2 = \partial P/\partial \rho|_{\rm ad} \\ \gamma = \partial \ln P/\partial \ln \rho|_{\rm ad} \end{cases}$$



See e.g. Basu & Antia 07

	FRANEC(*)-GS98	FRANEC-AGS05	FRANEC(*)-AGSS09
Photospheric abund. $(Z/X)_b$ Y_b	 0.0231 0.245	 0.0165 0.229	 0.0181 0.232
Convective Zone R_b/R_o	 0.716	 0.730	 0.725

Helioseismic Values
0.2485 ± 0.0034
0.715 ± 0.001

(*) Estimated by LSM approach. See Later

Standard Solar Models

Stellar structure equations are solved, starting from a ZAMS model to present solar age (we neglect rotation, magnetic fields, etc.):

$$
\begin{aligned}
\frac{\partial m}{\partial r} &= 4\pi r^2 \rho \\
\frac{\partial P}{\partial r} &= -\frac{G_{\rm N} m}{r^2}\rho \\
P &= P(\rho, T, X_i) \\
\frac{\partial l}{\partial r} &= 4\pi r^2 \rho\, \epsilon(\rho, T, X_i) \\
\frac{\partial T}{\partial r} &= -\frac{G_{\rm N} m T \rho}{r^2 P}\nabla
\end{aligned}
$$

$$
\nabla = {\rm Min}(\nabla_{\rm rad}, \nabla_{\rm ad}) \longrightarrow \begin{cases} \nabla_{\rm rad} = \dfrac{3}{16\pi a c\, G_{\rm N}} \dfrac{\kappa(\rho, T, X_i)\, l\, P}{m\, T^4} \\ \nabla_{\rm ad} = (d \ln T / d \ln P)_{\rm s} \simeq 0.4 \end{cases}
$$

Chemical evolution driven by nuclear reaction, diffusion and gravitational settling, convection

Standard input physics for equation of states, nuclear reaction rates, opacity, etc.

Free-parameters (mixing length, Y_{ini}, Z_{ini}) adjusted to match the observed properties of the Sun (radius, luminosity, Z/X).

Note that equations are non-linear → Iterative method to determine mixing length, Y_{ini}, Z_{ini}

Metals in the Sun

- Z_{CNO} control the efficiency of CNO cycle

- Metals give a substantial contribution to opacity:

Energy producing region ($R < 0.3\ R_o$)

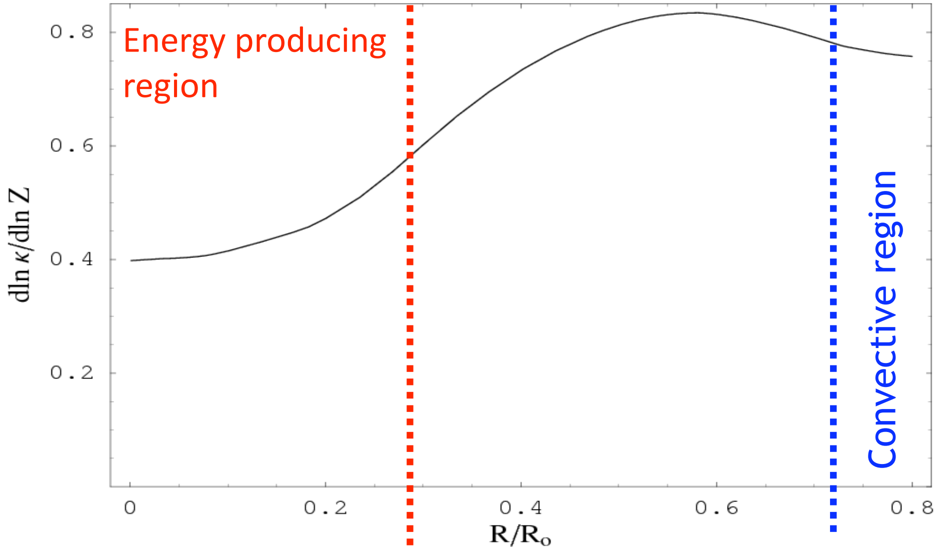
$$\kappa_Z \approx \frac{1}{2}\kappa_{tot}$$

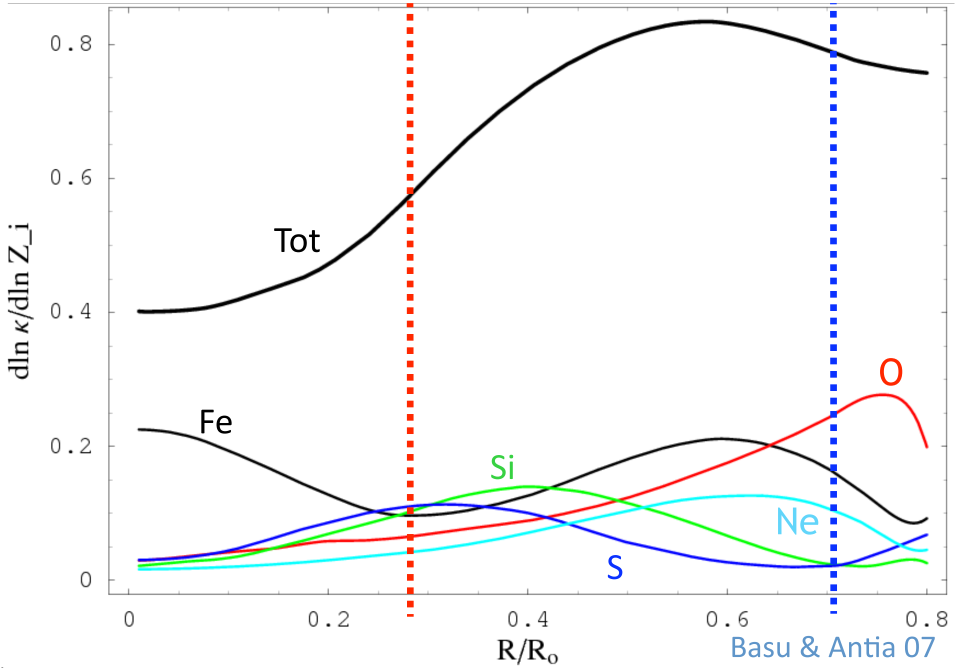
Fe gives the largest contribution.

Outer radiative region ($0.3 < R < 0.73\ R_o$)

$$\kappa_Z \sim 0.8\ \kappa_{tot}$$

Relevant contributions from several diff. elements (O,Fe,Si,Ne,...)





To understand metals: → What we know about opacity in the sun?

Linear Solar Models

F.L. Villante and B. Ricci - ***Astrophys.J.714:944-959,2010***
F.L. Villante - ***J.Phys.Conf.Ser.203:012084,2010***

Linear Solar Models: the basic idea

- The starting point:

SSMs provide a good approximation of the real sun. Small modifications are likely to explain disagreement with helioseismology.

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- The method:

We write:

$$\begin{aligned} h(r) &= \overline{h}(r)[1 + \delta h(r)] \\ X_i(r) &= \overline{X}_i(r)[1 + \delta X_i(r)] \\ Y(r) &= \overline{Y}(r) + \Delta Y(r) \end{aligned} \qquad h = l,\ m,\ \rho,\ P,\ T$$

where $\overline{h}(r)$, $\overline{X}_i(r)$ are the SSMs predicted values, and we expand linearly in $\begin{cases} \delta h(r) \\ \delta X_{\rm i}(r) \\ \Delta Y(r) \end{cases}$

Assumption: the variation of the *present* solar composition (i.e. the $\delta X_{\rm i}(r),\ \ \Delta Y(r)$) can be deduced with sufficient accuracy from the variation of the nuclear reaction efficiency and diffusion velocities in the *present* sun (i.e. the $\delta h(r)$)

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- The result:

A linear system of ordinary differential equations that can be used to study the response of the sun to an arbitrary modification input parameters.

Linear Solar Models – Final set of equations equations

$$\frac{d\delta m}{dr} = \frac{1}{l_m}\left[\gamma_P\,\delta P + \gamma_T\,\delta T - \delta m + \gamma_Y\,\Delta Y_{\rm ini} + \gamma_\epsilon\,\delta\epsilon\right]$$

$$\frac{d\delta P}{dr} = \frac{1}{l_P}\left[(\gamma_P - 1)\,\delta P + \gamma_T\,\delta T + \delta m + \gamma_Y\,\Delta Y_{\rm ini} + \gamma_\epsilon\,\delta\epsilon\right]$$

$$\frac{d\delta l}{dr} = \frac{1}{l_l}\left[\beta'_P\,\delta P + \beta'_T\,\delta T - \delta l + \beta'_Y\,\Delta Y_{\rm ini} + \beta'_C\,\delta C + \beta'_\epsilon\,\delta\epsilon\right]$$

$$\frac{d\delta T}{dr} = \frac{1}{l_T}\left[\alpha'_P\,\delta P + \alpha'_T\,\delta T + \delta l + \alpha'_Y\,\Delta Y_{\rm ini} + \alpha'_C\,\delta C + \delta\kappa + \alpha'_\epsilon\,\delta\epsilon\right]$$

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→ System of four linear differential equations in *δm, δP, δl, δT*

Note that:

EOS: we assumed perfect gas scaling and neglect the role of metals

$$\delta\rho(r) = \delta P(r) - \delta T(r) - P_Y \Delta Y(r)$$

$$P_Y(r) = -\frac{\partial \ln\mu}{\partial Y} = -\frac{5}{8 - 5Y(r) - 6Z(r)}$$

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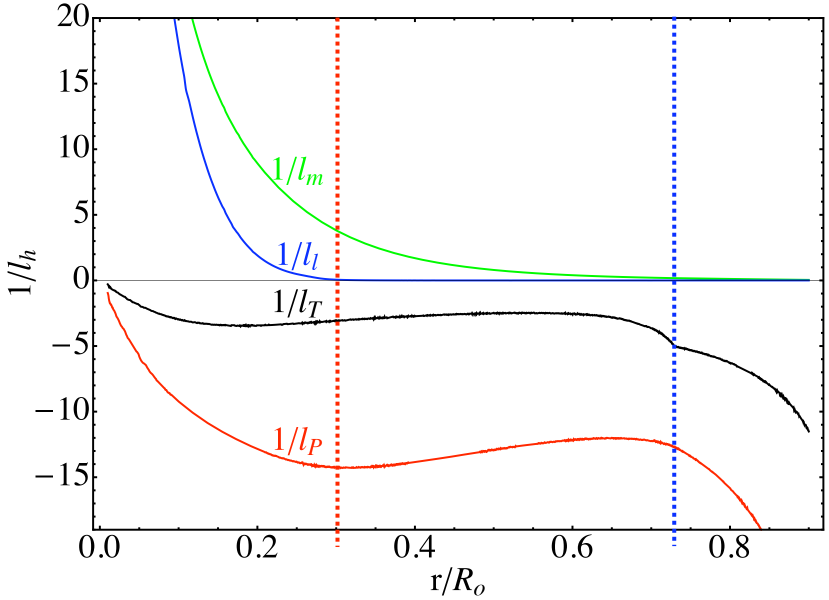
$\delta\varepsilon$, $\delta\kappa$ = fractional variations of the input params → provide the source terms

Inverse scale height of *h* in SSM

$$l_h = \left[\frac{d\ln(\overline{h})}{dr}\right]^{-1}$$



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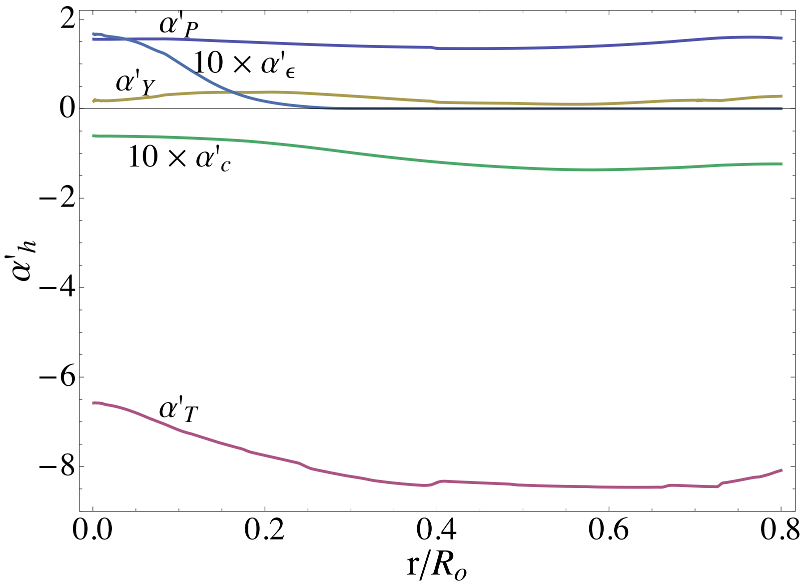
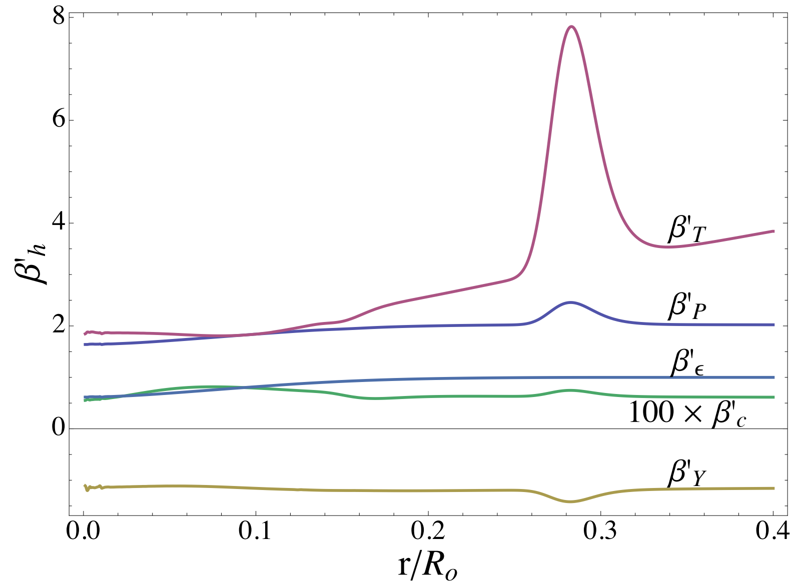
The coefficients γ_h, β'_h and α'_h describes the response of the plasma (EOS, energy generations and radiative transfer) to variation of structural (δm, δP, δL, δT) and chemical properties.





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To be solved between with the boundary conditions

At the center of the sun (r = 0)

$$\delta m = \gamma_{P,0}\,\delta P_0 + \gamma_{T,0}\,\delta T_0 + \gamma_{Y,0}\,\Delta Y_{\rm ini} + \gamma_{\epsilon,0}\,\delta\epsilon_0$$
$$\delta P = \delta P_0$$
$$\delta T = \delta T_0$$
$$\delta l = \beta'_{P,0}\,\delta P_0 + \beta'_{T,0}\,\delta T_0 + \beta'_{Y,0}\,\Delta Y_{\rm ini} + \beta'_{C,0}\,\delta C + \beta'_{\epsilon,0}\,\delta\epsilon_0$$

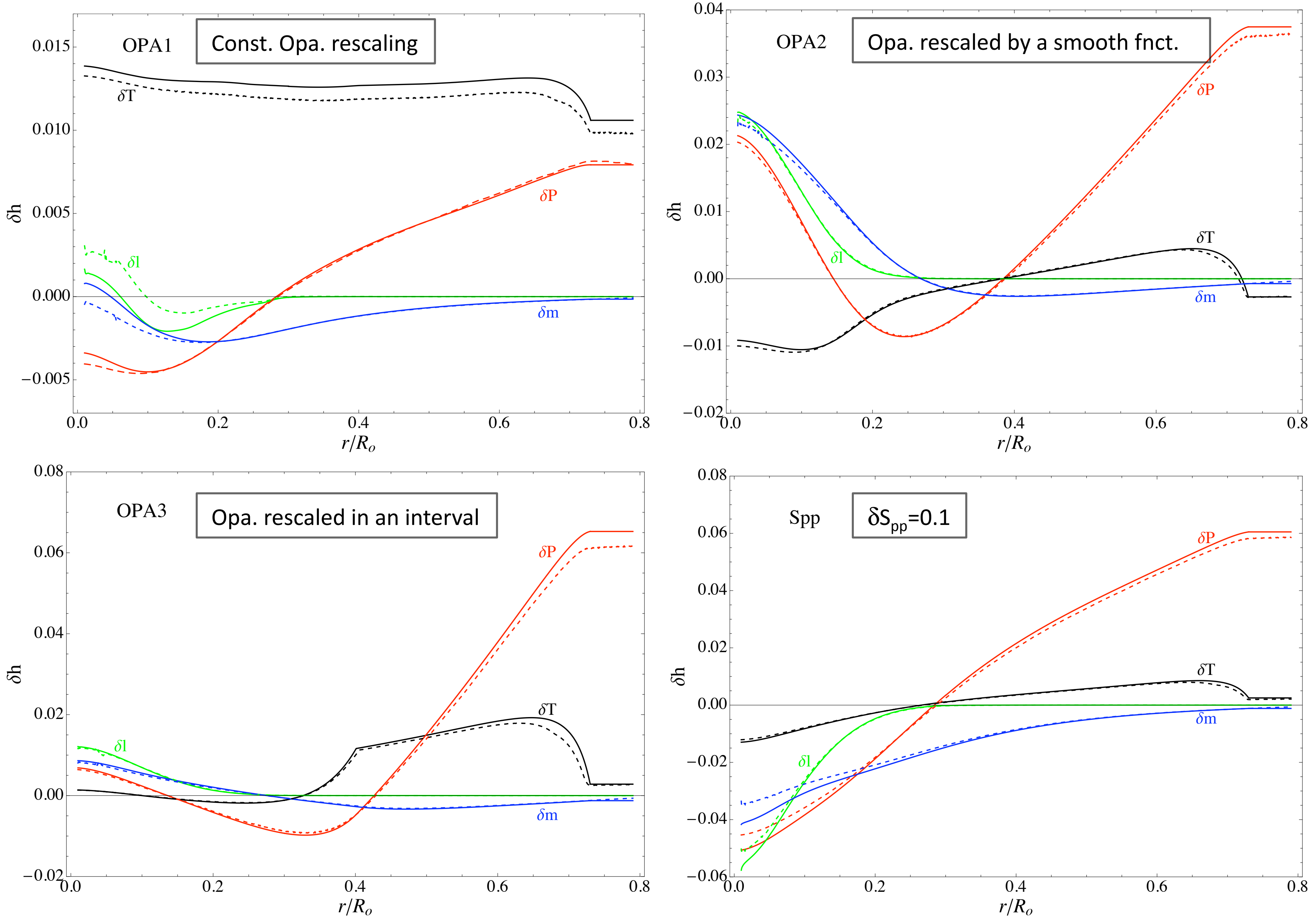
At the convective boundary ($r = \overline{R}_b$)

$$\delta m = -\overline{m}_{\rm conv}\,\delta C$$
$$\delta P = \delta C$$
$$\delta T = A'_Y\,\Delta Y_{\rm ini} + A'_C\,\delta C$$
$$\delta l = 0$$

Univocally determine the parameters $\delta P_0, \delta T_0, \Delta Y_{ini}, \delta C$

Linear Solar Models – Validation

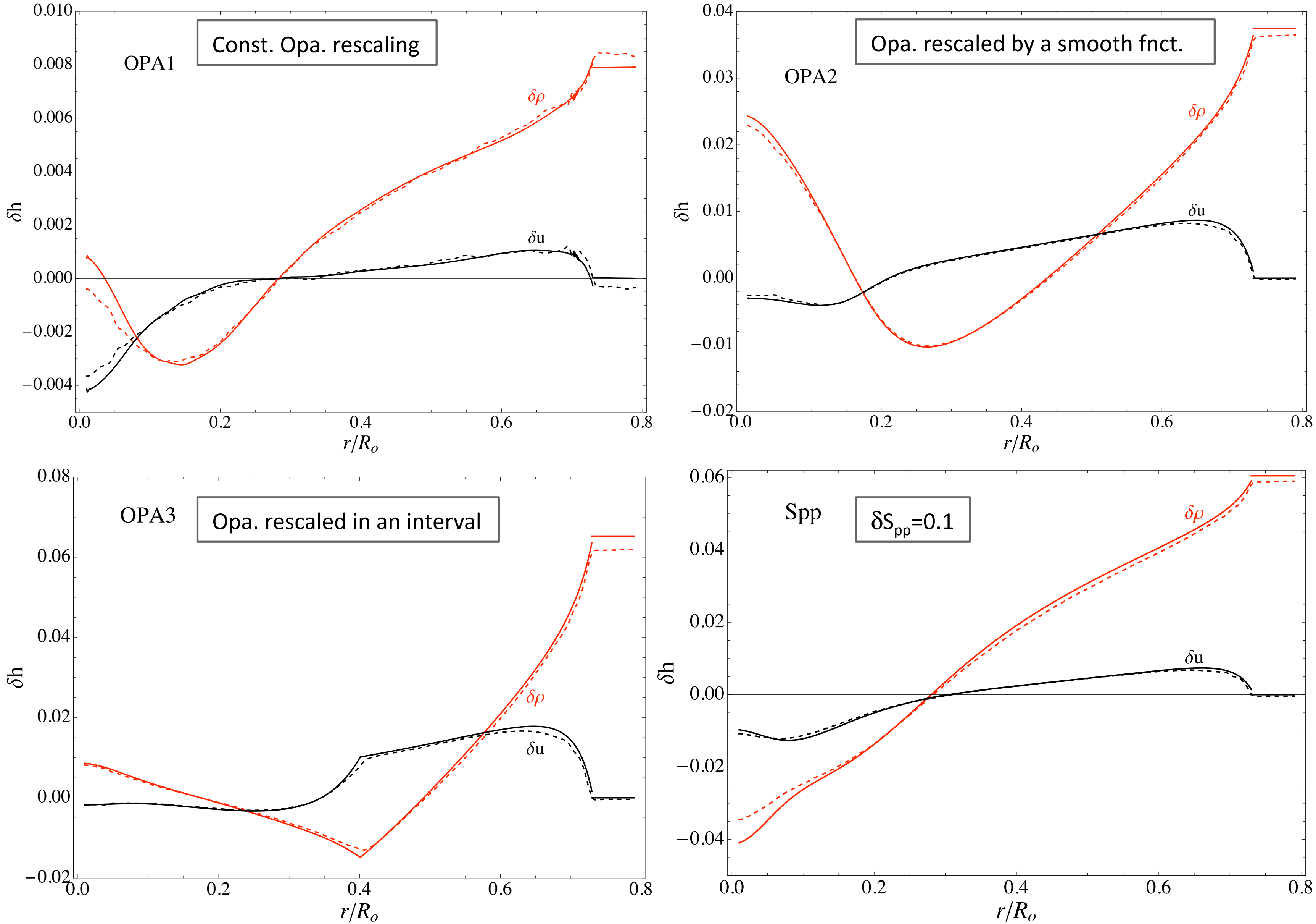
Solid - Linear solar models
Dotted - SSMs



Linear Solar Models – Validation

Solid - Linear solar models
Dotted - SSMs

$$\delta u(r) = \delta P(r) - \delta\rho(r)$$



Linear Solar Models – Validation

	OPA1		OPA2		OPA3		Spp	
	SM	LSM	SM	LSM	SM	LSM	SM	LSM
$\Delta Y_{\rm b}$	0.014	0.014	-0.0037	-0.0036	0.0038	0.0038	0.0031	0.0034
$\delta R_{\rm b}$	-0.0020	-0.0020	-0.0067	-0.0070	-0.014	-0.015	-0.0058	-0.0064
$\delta\Phi_{\rm pp}$	-0.011	-0.010	0.0045	0.0052	-0.0020	-0.0011	0.0090	0.0092
$\delta\Phi_{\rm Be}$	0.13	0.13	-0.067	-0.064	0.017	0.016	-0.11	-0.11
$\delta\Phi_{\rm B}$	0.27	0.27	-0.17	-0.17	0.029	0.028	-0.27	-0.28
$\delta\Phi_{\rm N}$	0.14	0.14	-0.10	-0.094	0.003	0.004	-0.21	-0.22
$\delta\Phi_{\rm O}$	0.21	0.22	-0.14	-0.14	0.012	0.012	-0.29	-0.31

Surface helium:

$$\Delta Y_{\rm b} = A_{\rm Y}\,\Delta Y_{\rm ini} + A_{\rm C}\,\delta C$$

$$\begin{cases} A_Y = 0.838 \\ A_C = 0.033 \end{cases}$$

Convective radius:

$$\delta R_{\rm b} = \Gamma_Y\,\Delta Y_{\rm ini} + \Gamma_C\,\delta C + \Gamma_\kappa\,\delta\kappa_{\rm b}$$

$$\begin{cases} \Gamma_Y = 0.449 \\ \Gamma_C = -0.117 \\ \Gamma_\kappa = -0.085 \end{cases}$$

Neutrino fluxes:

$$\delta\Phi_\nu = \int dr\;[\phi_{\nu,\rho}(r)\,\delta\rho(r) + \phi_{\nu,T}(r)\,\delta T(r) + \phi_{\nu,Y}(r)\,\Delta Y(r) + \phi_{\nu,Z}(r)\,\delta Z(r) + \phi_{\nu,{\rm Spp}}(r)\,\delta S_{\rm pp}]$$

where:

$$\phi_{\nu,j}(r) = \frac{r^2\,\overline{\rho}(r)\,\overline{n}_\nu(r)\,n_{\nu,j}(r)}{\int dr\;r^2\,\overline{\rho}(r)\,\overline{n}_\nu(r)}$$

Opacity (and metals) in the sun

F.L. Villante – **ApJ, in press**

The relation between opacity and metals

$$\frac{d\delta m}{dr} = \frac{1}{l_m}\left[\gamma_P\,\delta P + \gamma_T\,\delta T - \delta m + \gamma_Y\,\Delta Y_{\rm ini}\right]$$
$$\frac{d\delta P}{dr} = \frac{1}{l_P}\left[(\gamma_P - 1)\,\delta P + \gamma_T\,\delta T + \delta m + \gamma_Y\,\Delta Y_{\rm ini}\right]$$
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$$\frac{d\delta T}{dr} = \frac{1}{l_T}\left[\alpha'_P\,\delta P + \alpha'_T\,\delta T + \delta l + \alpha'_Y\,\Delta Y_{\rm ini} + \alpha'_C\,\delta C + \delta\kappa\right]$$

The source term that is responsibile for the modification of the sun (and that can be bounded from obs. data) is :

$$\delta\kappa(r) = \delta\kappa_{\rm I}(r) + \delta\kappa_{\rm Z}(r)$$

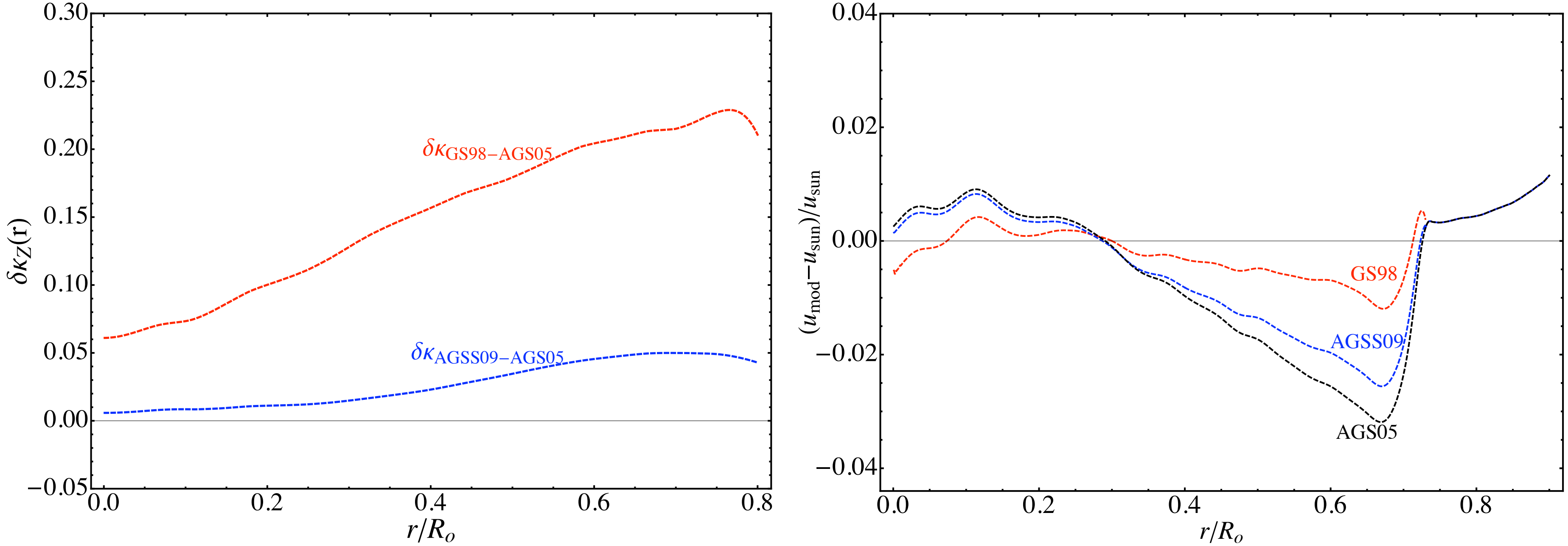
Intrinsic opacity change

$$\delta\kappa_{\rm I}(r) = \frac{\kappa(\overline{\rho}(r), \overline{T}(r), \overline{Y}(r), \overline{Z}_i(r))}{\overline{\kappa}(\overline{\rho}(r), \overline{T}(r), \overline{Y}(r), \overline{Z}_i(r))} - 1$$

Composition opacity change

$$\delta\kappa_{\rm Z}(r) = \frac{\overline{\kappa}(\overline{\rho}(r), \overline{T}(r), \overline{Y}(r), Z_i(r))}{\overline{\kappa}(\overline{\rho}(r), \overline{T}(r), \overline{Y}(r), \overline{Z}_i(r))} - 1$$

The relation between opacity and metals



$$\delta\kappa_{\rm Z}(r) = \frac{\overline{\kappa}(\overline{\rho}(r), \overline{T}(r), \overline{Y}(r), Z_i(r))}{\overline{\kappa}(\overline{\rho}(r), \overline{T}(r), \overline{Y}(r), \overline{Z}_i(r))} - 1 \simeq \sum_i \frac{\partial \ln \overline{\kappa}}{\partial \ln Z_i}\ \delta z_{\rm i}$$

$$\delta z_{\rm i} = \frac{(Z_{\rm i,b}/X_{\rm b}) - (\overline{Z}_{\rm i,b}/\overline{X}_{\rm b})}{(\overline{Z}_{\rm i,b}/\overline{X}_{\rm b})}$$

The opacity kernels

We study the response of the sun to arbitrary opacity variations:

$$\delta\kappa(r) = \delta\kappa_{\rm I}(r) + \delta\kappa_{\rm Z}(r)$$

If we consider a small variation of the opacity, the sun respond linearly. The variation of a generic quantity Q is then given by:

$$\delta Q = \int dr \, \boxed{K_Q(r)} \, \delta\kappa(r)$$

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$$\delta Q = \int dr\, \boxed{K_Q(r)}\, \delta\kappa(r)$$

We calculate numerically the kernel $K_Q(r)$ by considering localised increase of opacity in LSM:

$$\delta\kappa(r) = G(r - r_0) \quad = \quad \frac{1}{\sqrt{2\pi}\delta r} \exp\left[-\frac{(r - r_0)^2}{2\delta r^2} \right]$$

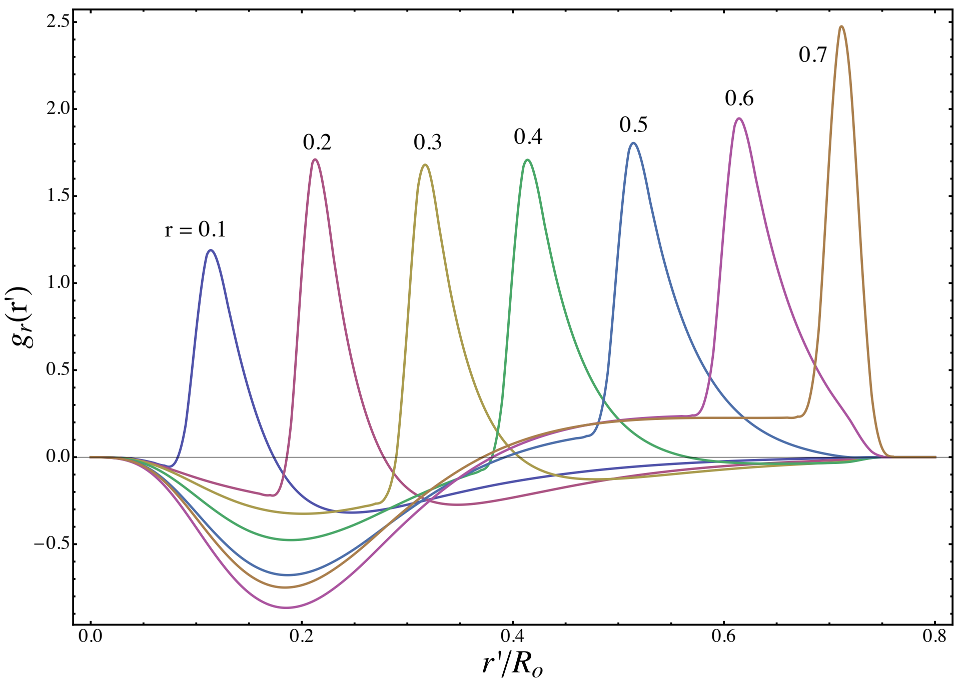
$$(\delta r = 0.01 R_\odot)$$

The obtained results are adequate to describe all the situations in which opacity varies on scales larger the $\delta r = 0.01\ R_o$

$$\delta Q(r_0) = \int dr\ K_Q(r)\ G(r - r_0) \simeq K_Q(r_0)$$

The sound speed kernels

$$\delta u(r) = \int dr' \; K_u(r, r') \; \delta\kappa(r')$$



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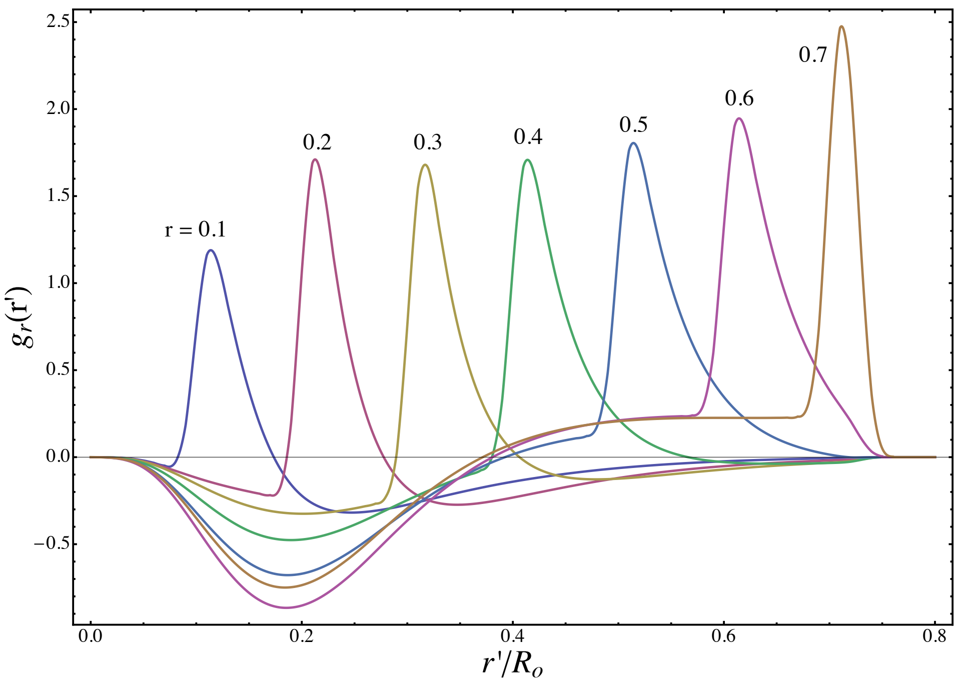
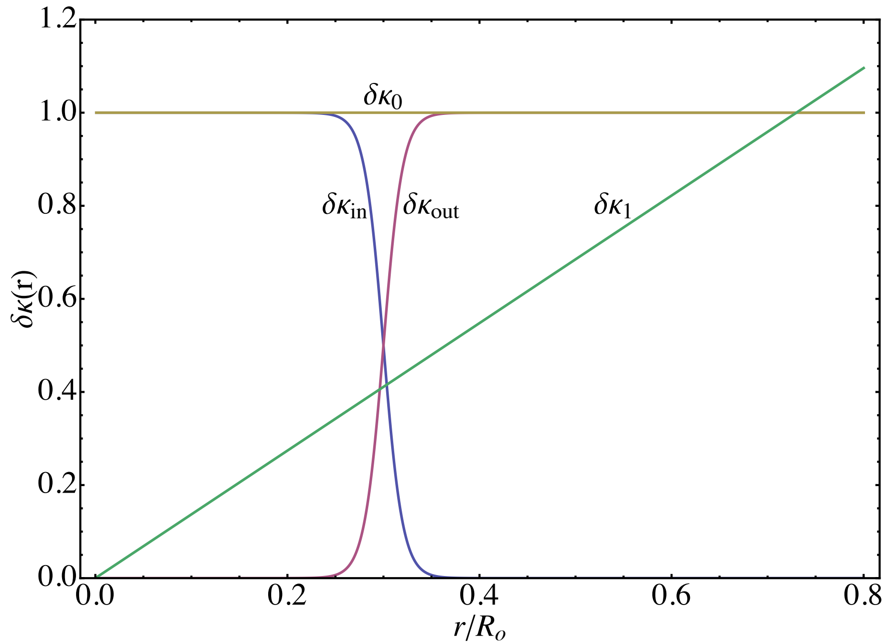


The kernels are not positive definite → compensating effects can occur …

$$\delta u_0(r) = \int dr' K_u(r, r') \simeq 0$$

The sound speed is *insensitive to a global rescaling of opacity*

Useful parameterizations for δκ(r)



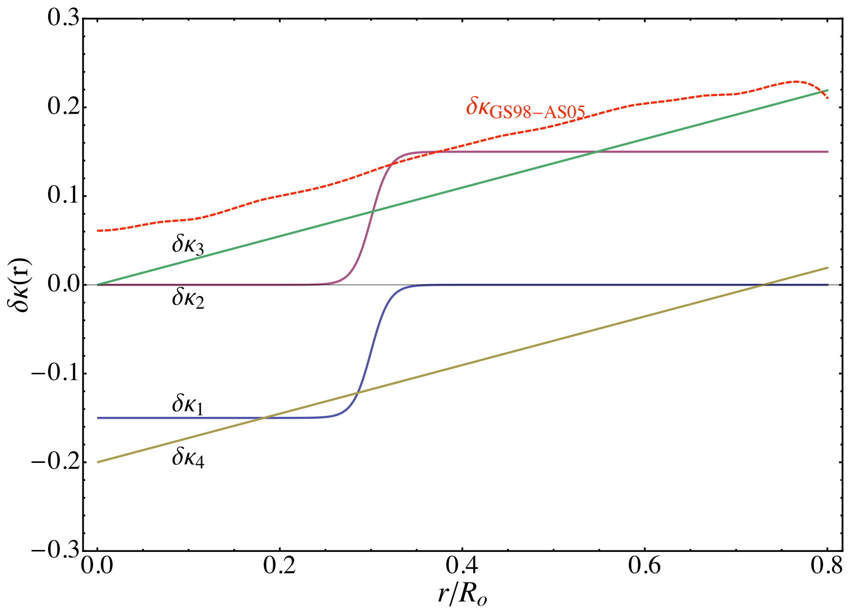
Two zones:

$$\delta\kappa(r) \quad = \quad A_{\rm in}\ \delta\kappa_{\rm in}(r) + A_{\rm out}\ \delta\kappa_{\rm out}(r)$$

Linear tilt:

$$\delta\kappa(r) \quad = \quad A_0\ \delta\kappa_0(r) + A_1\ \delta\kappa_1(r) = A_0 + A_1\ (r/\overline{R}_{\rm b})$$

The sound speed



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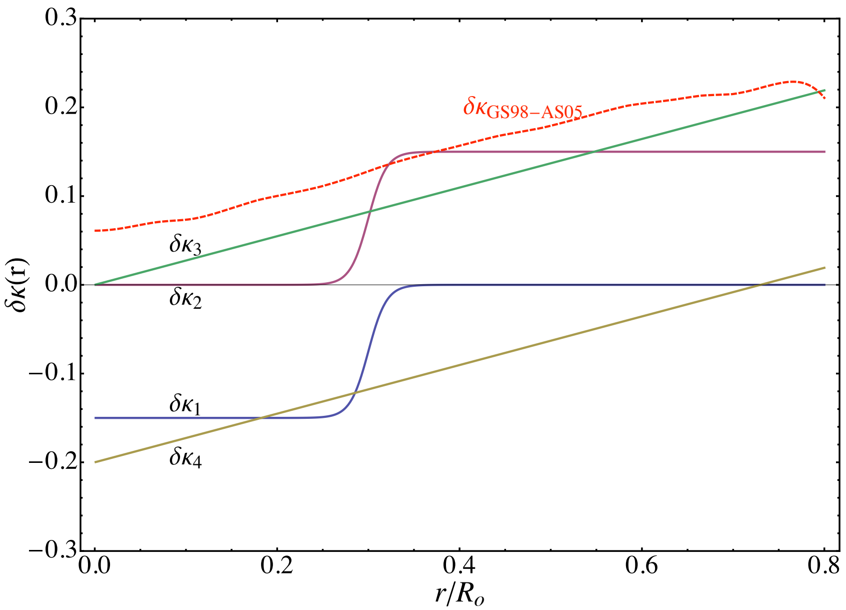

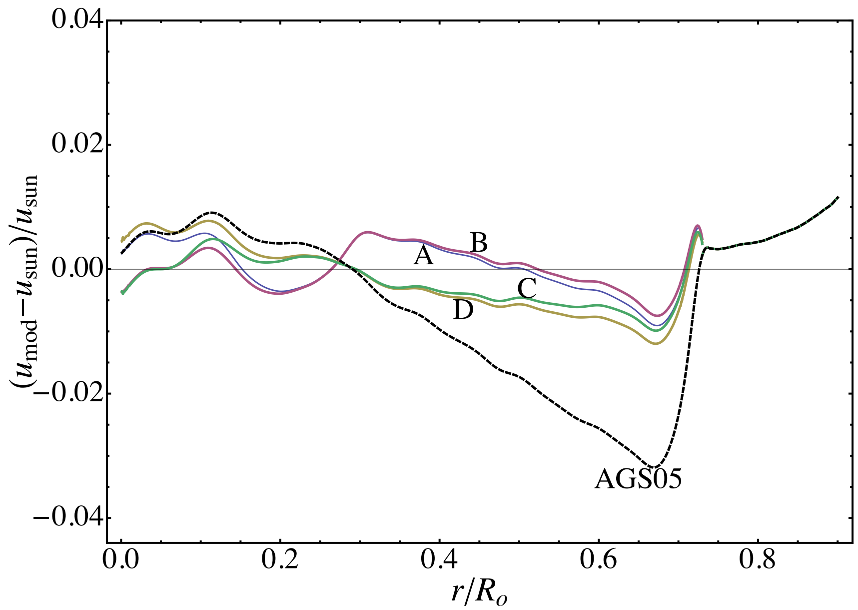

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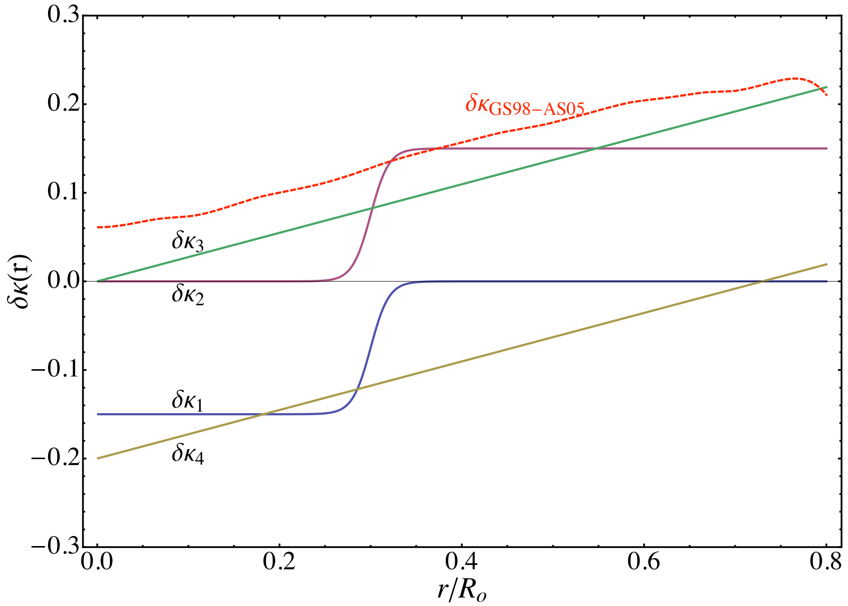
$$\delta u(r) \simeq (A_{\rm out} - A_{\rm in})\ \delta u_{\rm out}(r)$$
$$\delta u(r) \simeq A_1\ \delta u_1(r)$$

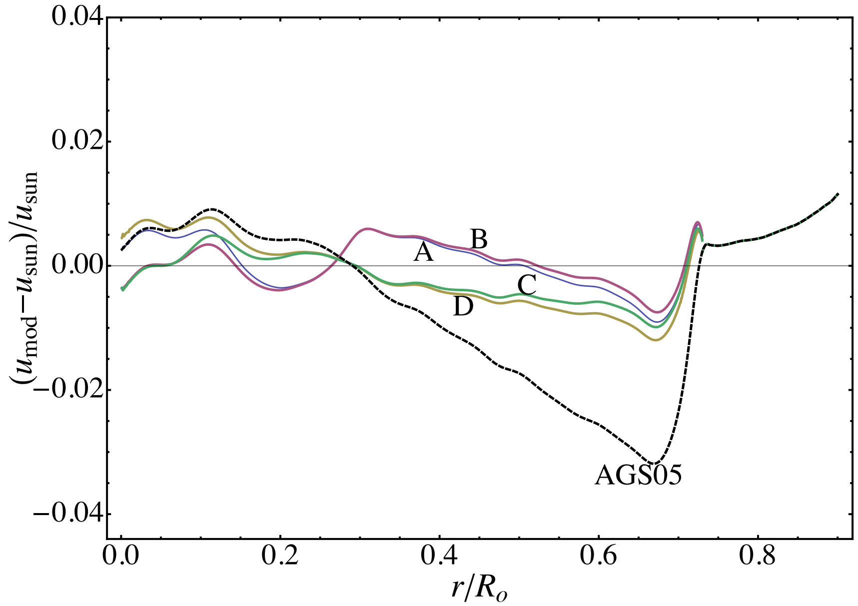
The sound speed provide a bound on the differential opacity increase (A_{out}-A_{in}) or on the tilt (A_1):

$$A_{\rm out} - A_{\rm in} \simeq 0.15$$
$$A_1 \simeq 0.2$$

No relevant bound on the opacity scale A_0 (or A_{in} + A_{out}).

The sound speed





$$\delta\kappa(r) = A_{in}\ \delta\kappa_{in}(r) + A_{out}\ \delta\kappa_{out}(r)$$
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?? Few GeV WIMPs in the core??

Villante@Physun08 and TAUP09
Frandsen talk later

The “stability” of sound speed …

Schematically, we can note that:

$$\frac{GMm_u}{R} \sim \frac{k_B T}{\mu} = \frac{P}{\rho} = u$$

Virial theorem

↓

This quantity is fixed for the Sun

In a “normal star”, opacity determine luminosity:

$$L \sim \frac{E_\gamma}{t_{diff}} = \frac{M^3 \mu^4}{\kappa}$$

In the sun:

To keep L constant, we have to vary helium abundace.

An increase of Y implies a decrease of κ and an increase of μ).

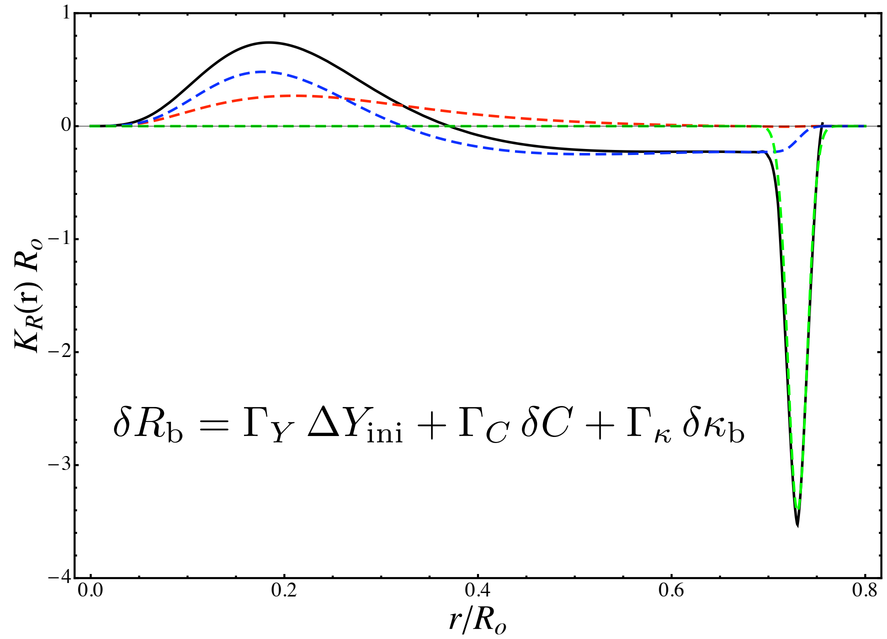
The convective radius and the surface helium abundance

Convective radius:

$$\delta R_{\rm b} = \int dr\ K_{\rm R}(r)\ \delta\kappa(r)$$

$$\begin{aligned}\delta R_{\rm b} &= 0.12\ A_{\rm in} - 0.14\ A_{\rm out} \\ &\simeq 0.13\ (A_{\rm in} -\ A_{\rm out})\end{aligned}$$

$$\delta R_{\rm b} = -0.02\ A_0 - 0.10\ A_1$$



The convective radius and the surface helium abundance

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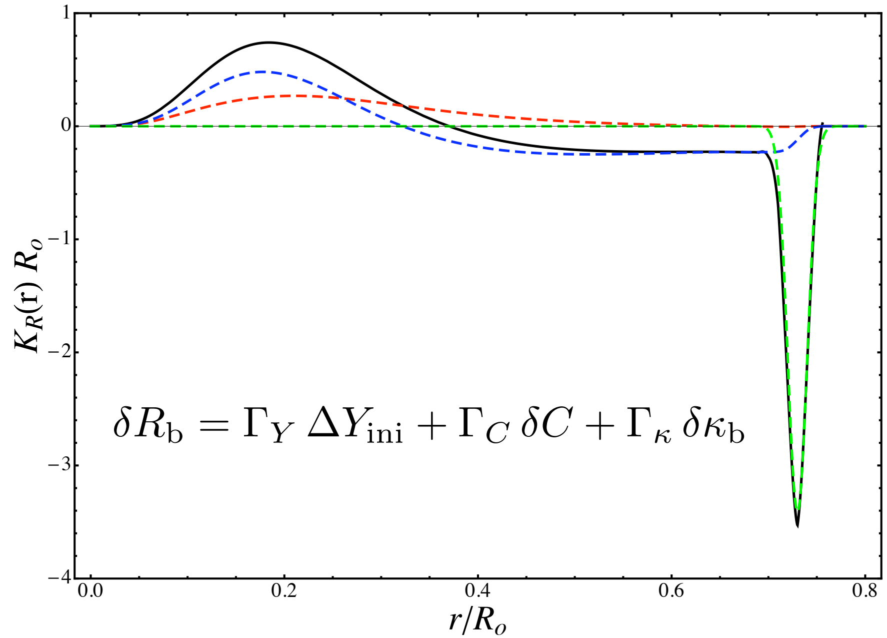
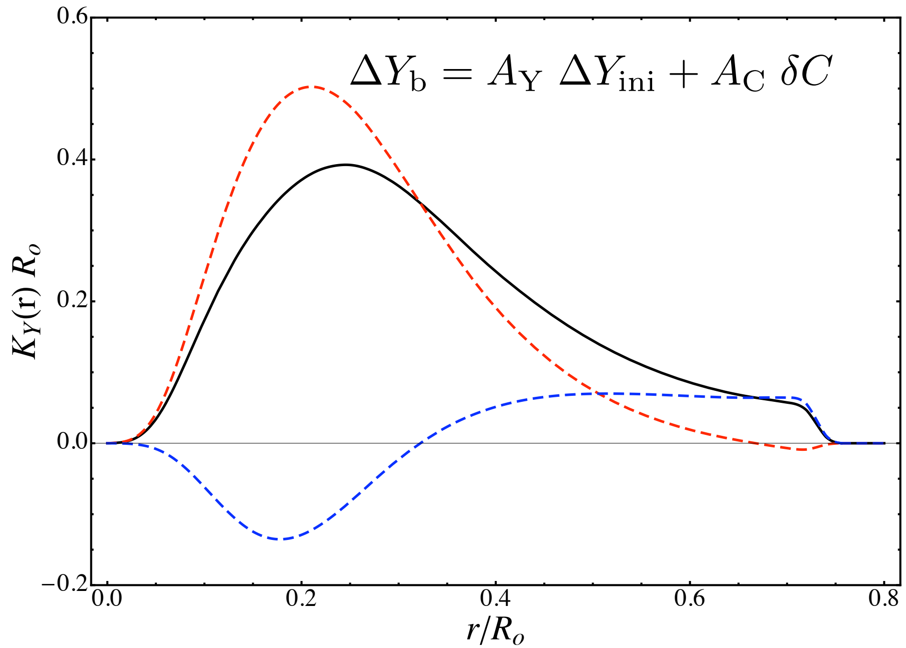


Surface helium:

$$\Delta Y_{\rm b} = \int dr\; K_{\rm Y}(r)\; \delta\kappa(r)$$

$$\begin{aligned} \Delta Y_{\rm b} &= 0.073\; A_{\rm in} + 0.069\; A_{\rm out} \\ &\simeq 0.07\; (A_{\rm in} + A_{\rm out}) \end{aligned}$$

$$\Delta Y_{\rm b} = 0.142\; A_0 + 0.062\; A_1$$



To reproduce helioseismic results:

$A_{\rm in} = 0.07 \pm 0.04 \qquad A_{\rm out} = 0.21 \pm 0.04$

A model independent relation for $\delta\kappa_b$

We have that:

$$\Delta Y_{\rm b} = A_{\rm Y}\,\Delta Y_{\rm ini} + A_{\rm C}\,\delta C$$

$$\left\{\begin{array}{lcl} A_Y & = & 0.838 \\ A_C & = & 0.033 \end{array}\right.$$

$$\delta R_{\rm b} = \Gamma_Y\,\Delta Y_{\rm ini} + \Gamma_C\,\delta C + \Gamma_\kappa\,\delta\kappa_{\rm b}$$

$$\left\{\begin{array}{lcl} \Gamma_Y & = & 0.449 \\ \Gamma_C & = & -0.117 \\ \Gamma_\kappa & = & -0.085 \end{array}\right.$$

Remenber that: $\delta C = \delta P_{\rm b} = \delta\rho_{\rm b}$

We eliminate $\Delta Y_{\rm ini}$ from equations and obtain:

$$\delta\kappa_{\rm b} = C_Y\,\Delta Y_{\rm b} + C_{\rm R}\,\delta R_{\rm b} + C_\rho\,\delta\rho_{\rm b}$$

$$\left\{\begin{array}{lcl} C_Y & = & -\dfrac{\Gamma_Y}{A_Y\,\Gamma_\kappa} = 6.27 \\ C_{\rm R} & = & \dfrac{1}{\Gamma_\kappa} = -11.71 \\ C_\rho & = & \dfrac{1}{\Gamma_\kappa}\left[\dfrac{A_C\,\Gamma_Y}{A_Y} - \Gamma_C\right] = -1.58 \end{array}\right.$$

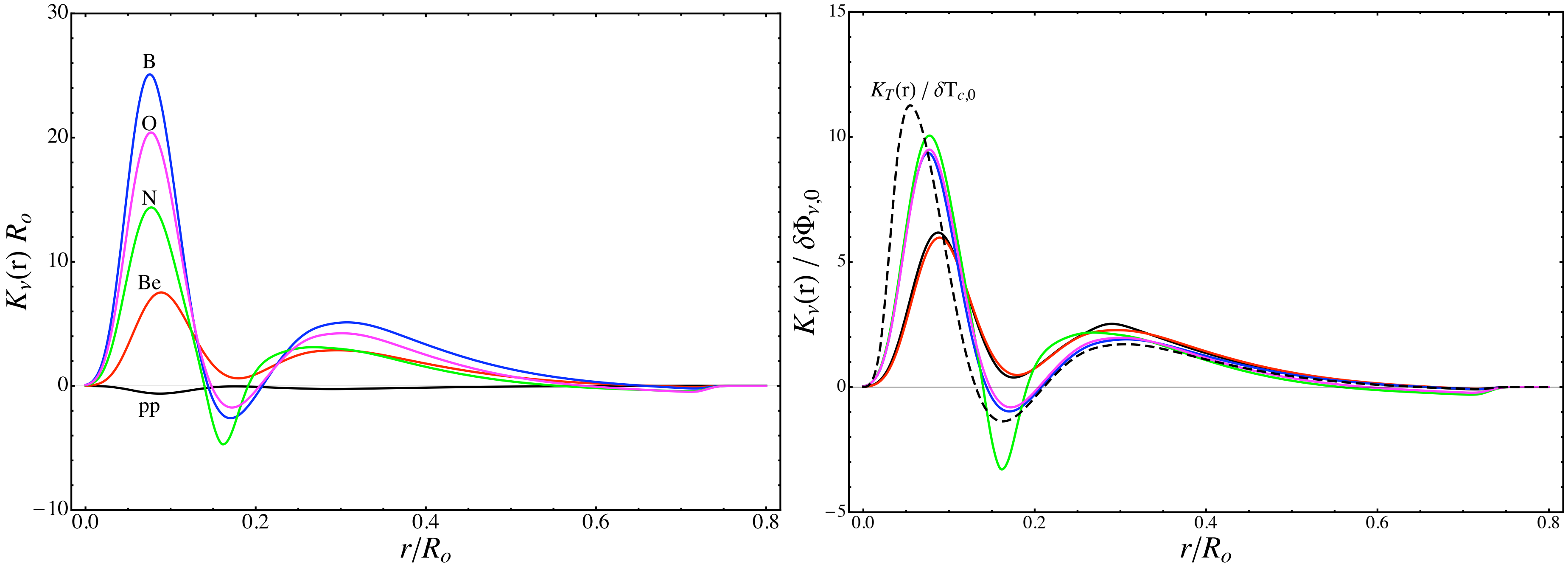
Model independent:
no parametrization for δκ(r) is needed

By using $\delta R_{\rm b} = -0.0205 \pm 0.0015$, $\Delta Y_{\rm b} = 0.0195 \pm 0.0034$ and $\delta\rho_{\rm b} = 0.08$:

$$\delta\kappa_{\rm b} \simeq 0.24 \pm 0.03$$

Neutrino Fluxes

See also Tripathy and Christensen-Dalsgaard 98



$$\delta\Phi_\nu = \int dr\ K_\nu(r)\ \delta\kappa(r)$$

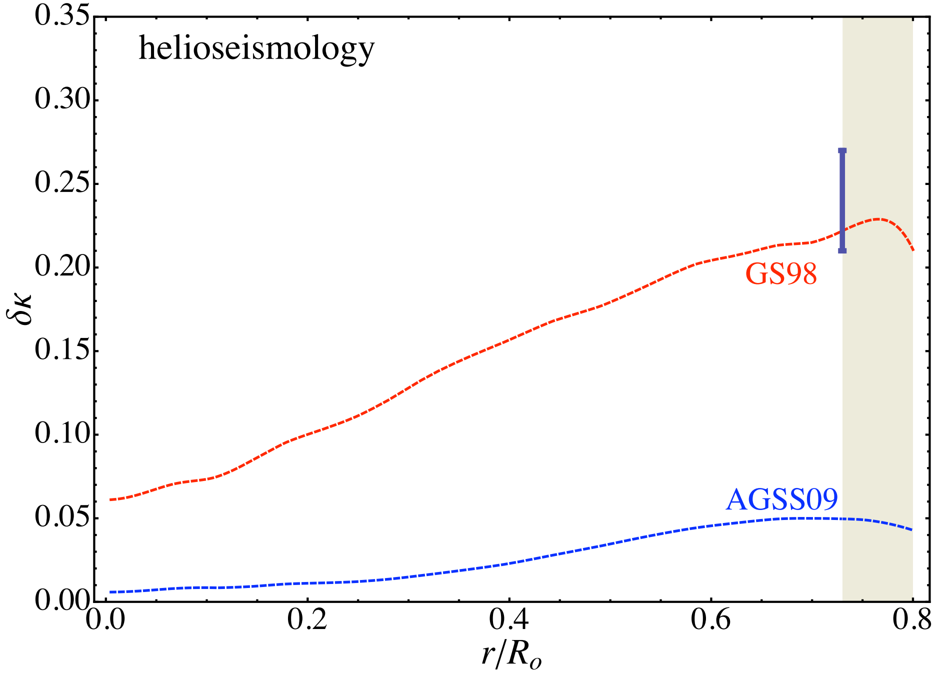
$$\begin{aligned}\delta\Phi_\nu &= A_{\rm in}\ \delta\Phi_{\nu,{\rm in}} + A_{\rm out}\ \delta\Phi_{\nu,{\rm out}} \\ &\simeq (\xi\ A_{\rm in} + A_{\rm out})\,\delta\Phi_{\nu,{\rm in}}\end{aligned}$$

$[\xi \sim 2 \to 3]$

$$\delta\Phi_\nu = A_0\ \delta\Phi_{\nu,0} + A_1\ \delta\Phi_{\nu,1}$$

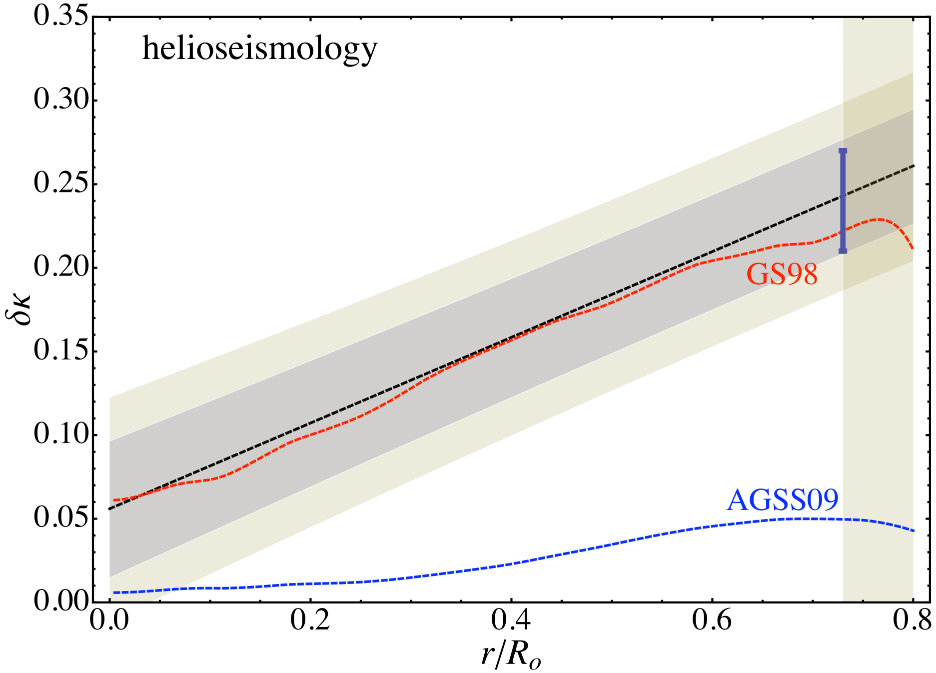
$\delta\Phi_\nu$	$\delta\Phi_{\nu,\rm in}$	$\delta\Phi_{\nu,\rm out}$	$\delta\Phi_{\nu,0}$	$\delta\Phi_{\nu,1}$
$\delta\Phi_{\rm pp}$	-0.069	-0.031	-0.100	-0.030
$\delta\Phi_{\rm Be}$	0.85	0.41	1.26	0.38
$\delta\Phi_{\rm B}$	1.93	0.75	2.68	0.68
$\delta\Phi_{\rm O}$	1.65	0.50	2.15	0.48
$\delta\Phi_{\rm N}$	1.14	0.28	1.43	0.30

A final look ...



helioseismology
GS98
AGSS09
δκ
r/R_o
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8

A final look …



$$\delta\kappa(r) = A_0\ \delta\kappa_0(r) + A_1\ \delta\kappa_1(r) = A_0 + A_1\ (r/\overline{R}_{\rm b})$$

$$Y_{\rm b} = 0.2485 \pm 0.0034$$

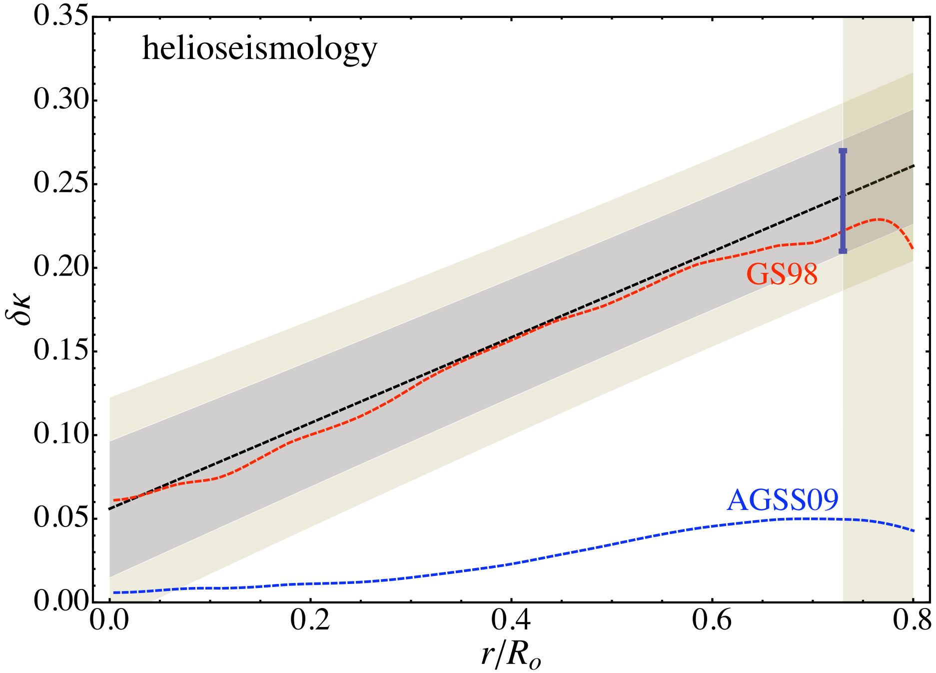
$$R_{\rm b} = (0.715 \pm 0.001) R_\odot$$

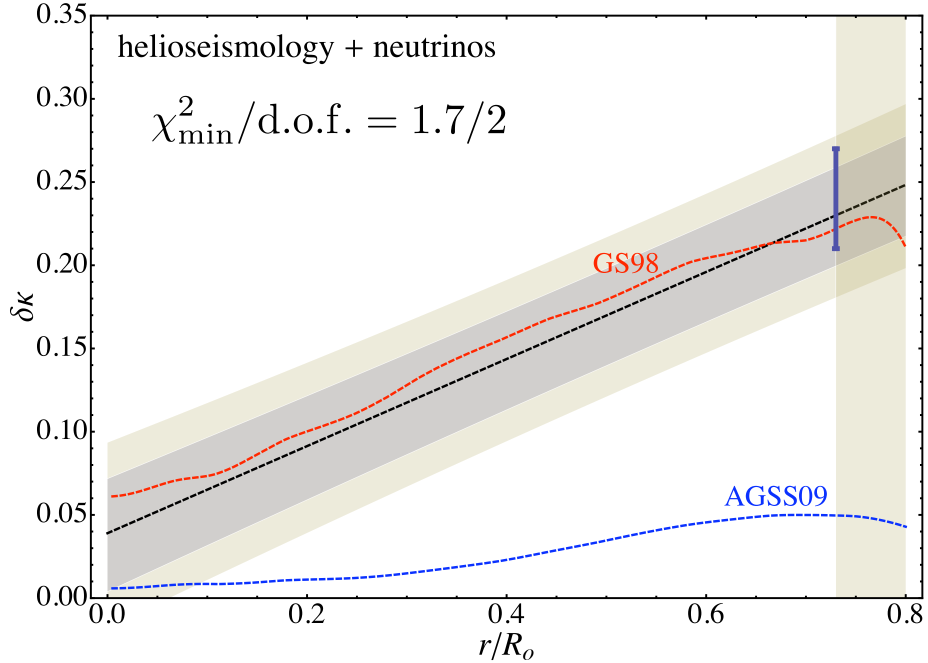
AGS05 predictions

$$\overline{Y}_{\rm b} = 0.229$$

$$\overline{R}_{\rm b} = 0.730 R_\odot$$

A final look ...





$$\delta\kappa(r) = A_0\ \delta\kappa_0(r) + A_1\ \delta\kappa_1(r) = A_0 + A_1\ (r/\overline{R}_{\rm b})$$

$$Y_{\rm b} = 0.2485 \pm 0.0034$$

$$R_{\rm b} = (0.715 \pm 0.001) R_\odot$$

$$\Phi_{\rm B} = (5.18 \pm 0.29) \times 10^6\ {\rm cm^{-2}\ s^{-1}}$$

SNO 2002, 2005, 2008

$$\Phi_{\rm Be} = (5.18 \pm 0.51) \times 10^9\ {\rm cm^{-2}\ s^{-1}}$$

Borexino 2008

AGS05 predictions

$$\overline{Y}_{\rm b} = 0.229$$

$$\overline{R}_{\rm b} = 0.730 R_\odot$$

$$\overline{\Phi}_{\rm B} = (4.66 \pm 0.42) \times 10^6\ {\rm cm^{-2}\ s^{-1}}$$

$$\overline{\Phi}_{\rm Be} = (4.54 \pm 0.23) \times 10^9\ {\rm cm^{-2}\ s^{-1}}$$

ν-fluxes from Serenelli 09 – (error estimate sdo not include opa and metals)

Summary and conclusions

- Linear Solar Models: a simple and accurate tool to investigate the sun interior
- Application: analysis of the role of opacity (and metals) in the sun
- Sound speed and convective radius: sensitive to differential opacity variations
- Helium and neutrinos: sensitive to overall opacity rescaling and, in particular, to opacity of the inner radiative region.
- Nice complementarity between different observational data.
- The opacity changes required to fit helioseismic and solar neutrino data seem large with respect to current opacity (and composition) uncertainties.
- Hopefully, future neutrino flux measurements (Be, CNO, pep) will provide additional clues for the solar composition puzzle.

Additional Slides

Linear Solar Models

• SSMs provide a good approximation of the real sun. Small modifications are likely to explain disagreement with helioseismology.

• We can expand linearly the solar models around the SSM and calculate:

$$\delta(\mathrm{output}) = \mathrm{L}\left[\delta(\mathrm{input})\right]$$

$$\delta(\mathrm{input}) = \delta\kappa(\mathrm{r}),\ \delta\epsilon(\mathrm{r}),\ \delta(\mathrm{composition}),\ \ldots$$

$$\delta(\mathrm{output}) = \delta\mathrm{u}(\mathrm{r}),\ \delta\mathrm{R_b}, \Delta\mathrm{Y_b},\ \delta\Phi_\nu,\ \ldots$$

• A tool to investigate the solar interior (accessible also to non SSM builders)

• It can help to understand the origin of discrepancy with helioseismic data or to describe new effects.

Linear Solar Models – Structure equations

We write:

$$\begin{aligned} h(r) &= \overline{h}(r)[1+\delta h(r)] \\ X_i(r) &= \overline{X}_i(r)[1+\delta X_i(r)] \\ Y(r) &= \overline{Y}(r)+\Delta Y(r) \end{aligned} \qquad h = l,\ m,\ \rho,\ P,\ T$$

where $\overline{h}(r)$, $\overline{X}_i(r)$ are the SSMs predicted values.

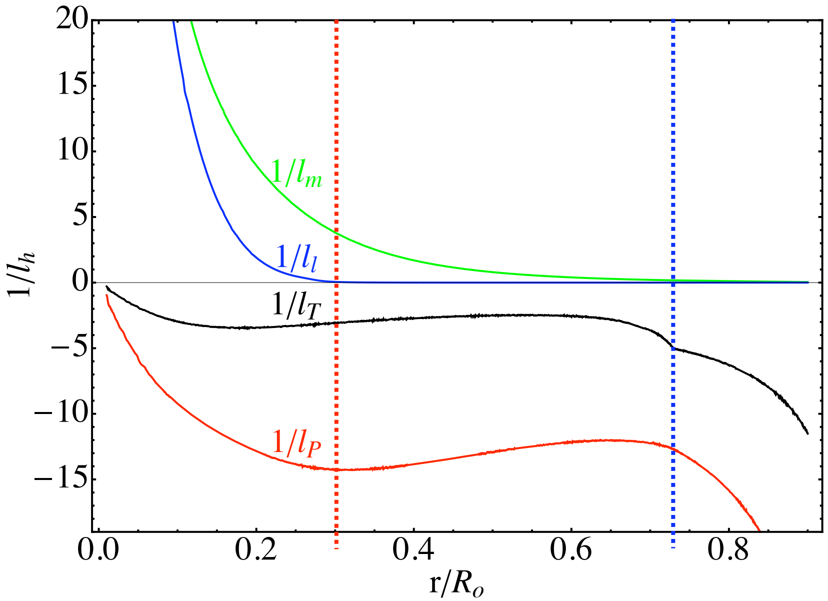
We then expand structure eqs. to first order in perturbations:

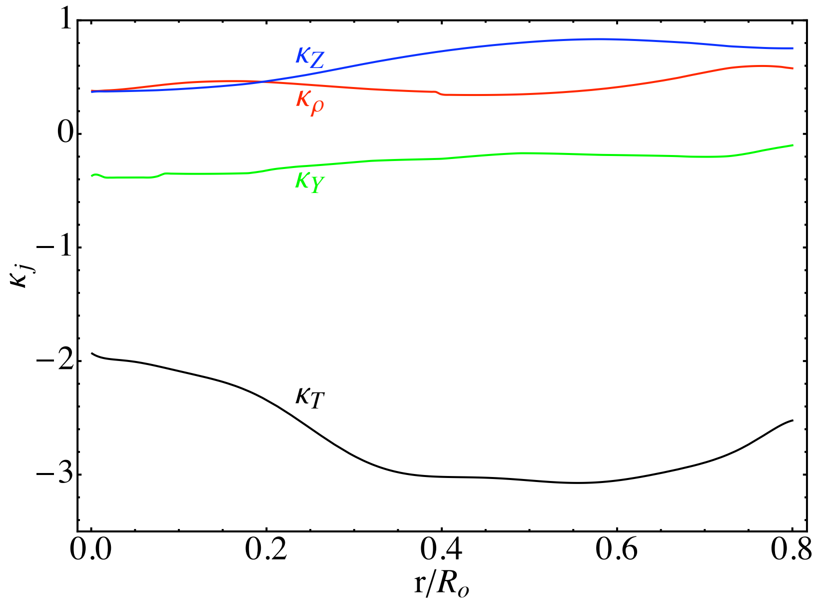
$$\begin{aligned} \frac{\partial\,\delta m}{\partial r} &= \frac{1}{l_m}\,[\delta\rho - \delta m] \\ \frac{\partial\,\delta P}{\partial r} &= \frac{1}{l_P}\,[\delta m + \delta\rho - \delta P] \\ \delta P &= [P_\rho\,\delta\rho + P_T\,\delta T + P_Y \Delta Y + \sum_i P_i\,\delta X_i] \\ \frac{\partial\,\delta l}{\partial r} &= \frac{1}{l_l}\,[(1+\epsilon_\rho)\delta\rho + \epsilon_T\,\delta T + \epsilon_Y \Delta Y + \sum_i \epsilon_i\,\delta X_i - \delta l + \delta\epsilon] \end{aligned}$$

$$\begin{cases} \dfrac{\partial\,\delta T}{\partial r} = \dfrac{1}{l_T}\,[\delta l + (\kappa_T - 4)\delta T + (\kappa_\rho + 1)\delta\rho + \kappa_Y \Delta Y + \sum_i \kappa_i\,\delta X_i + \delta\kappa] & \text{Rad.} \\ \dfrac{\partial\,\delta T}{\partial r} = \dfrac{1}{l_T}\,[\delta m + \delta\rho - \delta P] & \text{Conv.} \end{cases}$$

where $l_h = \left[\dfrac{d\ln(\overline{h})}{dr}\right]^{-1}$ and $P_h = \left[\dfrac{d\ln(P)}{d\ln(h)}\right]$, $\epsilon_h = \left[\dfrac{d\ln(\epsilon)}{d\ln(h)}\right]$, $\kappa_h = \left[\dfrac{d\ln(\kappa)}{d\ln(h)}\right]$

Linear Solar Models – Structure equations





$$\frac{\partial\,\delta m}{\partial r} = \frac{1}{l_m}\,[\delta\rho - \delta m]$$

$$\frac{\partial\,\delta P}{\partial r} = \frac{1}{l_P}\,[\delta m + \delta\rho - \delta P]$$

$$\delta P = [P_\rho\,\delta\rho + P_T\,\delta T + P_Y\,\Delta Y + \sum_i P_i\,\delta X_i]$$

$$\frac{\partial\,\delta l}{\partial r} = \frac{1}{l_l}\,[(1+\epsilon_\rho)\delta\rho + \epsilon_T\,\delta T + \epsilon_Y\,\Delta Y + \sum_i \epsilon_i\,\delta X_i - \delta l + \delta\epsilon]$$

$$\begin{cases} \dfrac{\partial\,\delta T}{\partial r} = \dfrac{1}{l_T}\,[\delta l + (\kappa_T - 4)\delta T + (\kappa_\rho + 1)\delta\rho + \kappa_Y\,\Delta Y + \sum_i \kappa_i\,\delta X_i + \delta\kappa] & \text{Rad.} \\ \dfrac{\partial\,\delta T}{\partial r} = \dfrac{1}{l_T}\,[\delta m + \delta\rho - \delta P] & \text{Conv.} \end{cases}$$

Linear Solar Models – Simplifications

EOS: we assume perfect gas scaling and neglect the role of metals

$$\delta\rho(r) = \delta P(r) - \delta T(r) - P_Y \Delta Y(r) \qquad P_Y(r) = -\frac{\partial \ln \mu}{\partial Y} = -\frac{5}{8 - 5Y(r) - 6Z(r)}$$

We eliminate density from equations, obtaining:

$$\begin{aligned}
\frac{d\delta m}{dr} &= \frac{1}{l_m}\left[\delta P - \delta T - \delta m - P_Y\,\Delta Y\right] \\
\frac{d\delta P}{dr} &= \frac{1}{l_P}\left[-\delta T + \delta m - P_Y\,\Delta Y\right] \\
\frac{d\delta l}{dr} &= \frac{1}{l_l}\left[\beta_P\,\delta P + \beta_T\,\delta T - \delta l + \beta_Y\,\Delta Y + \sum_i \beta_i\,\delta Z_i + \delta\epsilon\right] \\
\frac{d\delta T}{dr} &= \frac{1}{l_T}\left[\alpha_P\,\delta P + \alpha_T\,\delta T + \delta l + \alpha_Y\,\Delta Y + \sum_i \alpha_i\,\delta Z_i + \delta\kappa\right] && \text{Rad.} \\
\frac{d\delta T}{dr} &= \frac{1}{l_T}\left[-\delta T + \delta m - P_Y\,\Delta Y\right] && \text{Conv.}
\end{aligned}$$

with:

$$\begin{aligned}
\alpha_P &= \kappa_\rho + 1 \qquad \alpha_T = \kappa_T - \kappa_\rho - 5 \qquad \alpha_Y = -(\kappa_\rho + 1)P_Y + \kappa_Y \qquad \alpha_i = \partial \ln \kappa / \partial \ln Z_i \\
\beta_P &= \epsilon_\rho + 1 \qquad \beta_T = \epsilon_T - \epsilon_\rho - 1 \qquad \beta_Y = -(\epsilon_\rho + 1)P_Y + \epsilon_Y \qquad \beta_i = \partial \ln \epsilon / \partial \ln Z_i
\end{aligned}$$

Linear Solar Models – Boundary conditions

Central conditions (r=0):

$$\begin{aligned} m(0) &= 0 \\ l(0) &= 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} \delta m(0) &= \delta P_0 - \delta T_0 - P_{Y,0}\,\Delta Y_0 \\ \delta P(0) &= \delta P_0 \\ \delta T(0) &= \delta T_0 \\ \delta l(0) &= \beta_{P,0}\,\delta P_0 + \beta_{T,0}\,\delta T_0 + \beta_{Y,0}\,\Delta Y_0 + \sum_i \beta_{i,0}\,\delta Z_{i,0} + \delta\epsilon_0 \end{aligned}$$

Surface conditions implemented at the convective boundary:

$$\begin{aligned} m(\overline{R}_{\rm b}) &\simeq M_\odot \\ l(\overline{R}_{\rm b}) &= L_\odot \\ Y(r \geq \overline{R}_{\rm b}) &= {\rm const} \end{aligned} \quad \longrightarrow \quad \frac{\partial\,\delta u}{\partial r} = -\frac{\delta u}{l_T} \quad \longrightarrow \quad \begin{aligned} \delta m(\overline{R}_{\rm b}) &= -\overline{m}_{\rm conv}\,\delta C \\ \delta P(\overline{R}_{\rm b}) &= \delta C = \delta\rho(\overline{R}_{\rm b}) \\ \delta T(\overline{R}_{\rm b}) &= \delta\mu_{\rm b} = -P_{Y,{\rm b}}\,\Delta Y_{\rm b} \\ \delta l(\overline{R}_{\rm b}) &= 0 \end{aligned}$$

$$\boxed{\delta u(r \geq \overline{R}_{\rm b}) = 0}$$

Remind that:

$$\delta u(r) = \delta P(r) - \delta\rho(r) = \delta T(r) - \delta\mu(r)$$

Linear Solar Models – Chemical composition

The elemental abundaces in the present Sun are determined by:

- initial abundances;
- nuclear burnings;
- diffusion and gravitational settling.

$$
\begin{aligned}
Y(r) &= Y_{\rm ini}\,[1 + D_Y(r)] + Y_{\rm nuc}(r) \\
Z_{\rm i}(r) &= Z_{\rm ini,i}\,[1 + D_Z(r)]
\end{aligned}
$$

We assume that:

- helium nuclear production Y_{nuc} scales proportionally to the nuclear reaction efficiency in the present sun

$$\Delta Y_{\rm nuc}(r) = \overline{Y}_{\rm nuc}(r)\delta\epsilon^{\rm tot}(r)$$

- Diffusion effects in the convective envelope scales proportionally to diffusion velocity of the present sun

$$
\begin{aligned}
\Delta Y_{\rm b} &= A_{\rm Y}\,\Delta Y_{\rm ini} + A_{\rm C}\,\delta C \\
\Delta Z_{\rm b,i} &= B_{\rm Y}\,\Delta Y_{\rm ini} + B_{\rm C}\,\delta C + \delta z_{\rm i}
\end{aligned}
$$

$$\delta z_{\rm i} = \frac{(Z_{\rm i,b}/X_{\rm b}) - (\overline{Z}_{\rm i,b}/\overline{X}_{\rm b})}{(\overline{Z}_{\rm i,b}/\overline{X}_{\rm b})}$$

- Variations of diffusion efficiency in the radiative core can be neglected:

$$\Delta Y(r) = \xi_Y(r)\,\Delta Y_{\rm ini} + \xi_T(r)\,\delta T(r) + \xi_P(r)\,\delta P(r)$$

$$\delta Z_{\rm i}(r) = \delta Z_{\rm i,ini} = Q_Y\,\Delta Y_{\rm ini} + Q_C\,\delta C + \delta z_{\rm i}$$

Linear Solar Models – Final set of equations equations

$$
\begin{aligned}
\frac{d\delta m}{dr} &= \frac{1}{l_m}\left[\gamma_P\,\delta P + \gamma_T\,\delta T - \delta m + \gamma_Y\,\Delta Y_{\rm ini} + \gamma_\epsilon\,\delta\epsilon\right] \\
\frac{d\delta P}{dr} &= \frac{1}{l_P}\left[(\gamma_P - 1)\,\delta P + \gamma_T\,\delta T + \delta m + \gamma_Y\,\Delta Y_{\rm ini} + \gamma_\epsilon\,\delta\epsilon\right] \\
\frac{d\delta l}{dr} &= \frac{1}{l_l}\left[\beta'_P\,\delta P + \beta'_T\,\delta T - \delta l + \beta'_Y\,\Delta Y_{\rm ini} + \beta'_C\,\delta C + \beta'_\epsilon\,\delta\epsilon\right] \\
\frac{d\delta T}{dr} &= \frac{1}{l_T}\left[\alpha'_P\,\delta P + \alpha'_T\,\delta T + \delta l + \alpha'_Y\,\Delta Y_{\rm ini} + \alpha'_C\,\delta C + \delta\kappa + \alpha'_\epsilon\,\delta\epsilon\right]
\end{aligned}
$$

To be solved between with the boundary conditions

<u>*r = 0*</u>

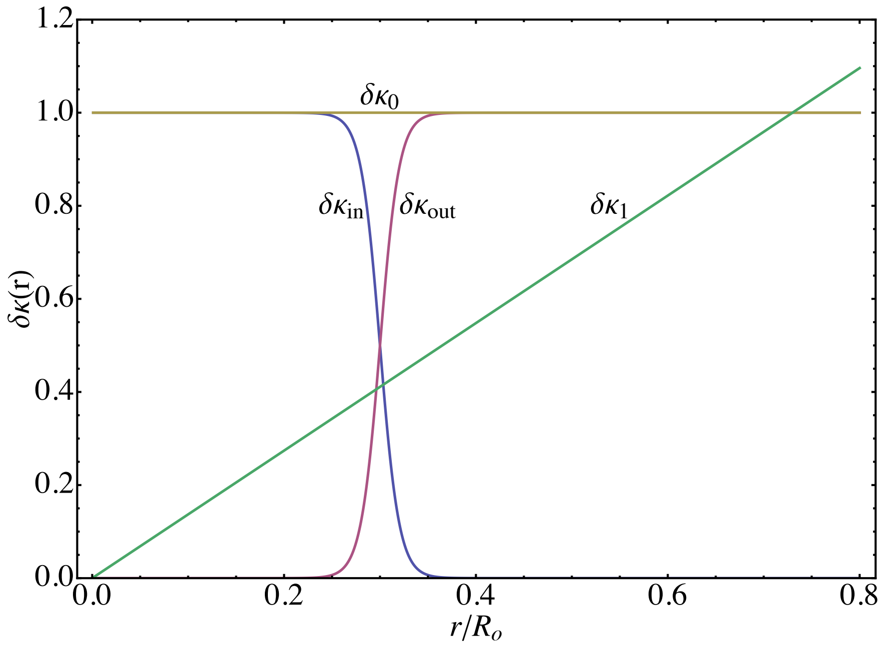
$$
\begin{aligned}
\delta m &= \gamma_{P,0}\,\delta P_0 + \gamma_{T,0}\,\delta T_0 + \gamma_{Y,0}\,\Delta Y_{\rm ini} + \gamma_{\epsilon,0}\,\delta\epsilon_0 \\
\delta P &= \delta P_0 \\
\delta T &= \delta T_0 \\
\delta l &= \beta'_{P,0}\,\delta P_0 + \beta'_{T,0}\,\delta T_0 + \beta'_{Y,0}\,\Delta Y_{\rm ini} + \beta'_{C,0}\,\delta C + \beta'_{\epsilon,0}\,\delta\epsilon_0
\end{aligned}
$$

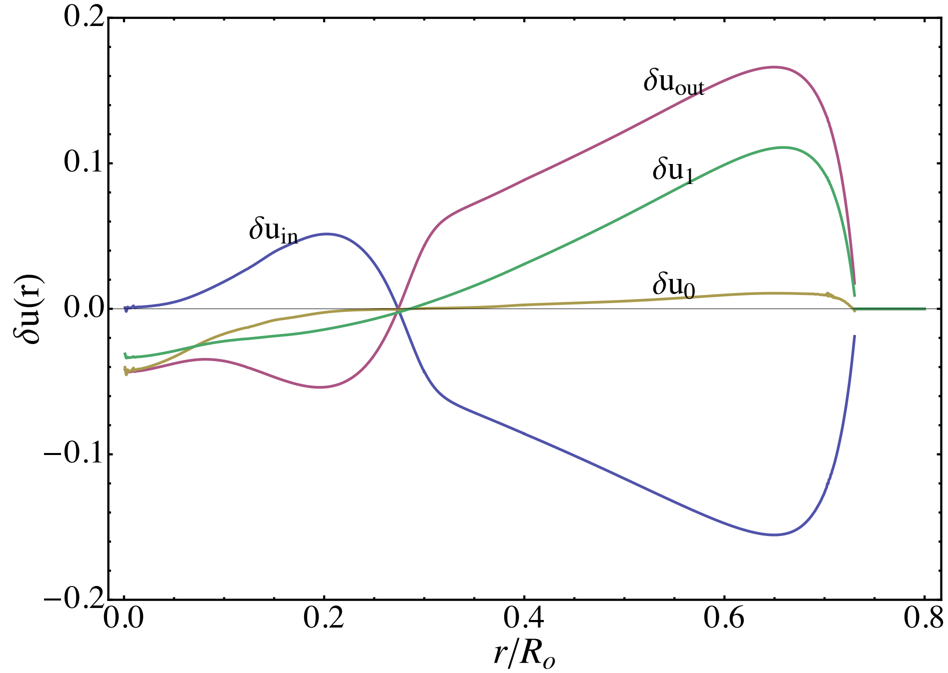
r = $\overline{R}_b$

$$
\begin{aligned}
\delta m &= -\overline{m}_{\rm conv}\,\delta C \\
\delta P &= \delta C \\
\delta T &= A'_Y\,\Delta Y_{\rm ini} + A'_C\,\delta C \\
\delta l &= 0
\end{aligned}
$$

Univocally determine the parameters $\delta P_0, \delta T_0, \Delta Y_{ini}, \delta C$

The sound speed





$$\delta\kappa(r) = A_{\rm in}\ \delta\kappa_{\rm in}(r) + A_{\rm out}\ \delta\kappa_{\rm out}(r)$$
$$\delta\kappa(r) = A_0\ \delta\kappa_0(r) + A_1\ \delta\kappa_1(r)$$

$$\delta u(r) = A_{\rm in}\ \delta u_{\rm in}(r) + A_{\rm out}\ \delta u_{\rm out}(r)$$
$$\delta u(r) = A_0\ \delta u_0(r) + A_1\ \delta u_1(r)$$

$$\delta u_{\rm in}(r) \simeq -\delta u_{\rm out}(r)$$
$$\delta u_0(r) \ll \delta u_1(r)$$