7. Quantum computers: physical realization

7.1~7.2
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7.1 Guiding principles

An overview of the tradeoffs in selecting a physical realization of a quantum computer.

7.2 Conditions for quantum computation

1. Robustly represent quantum information
2. Perform a universal family of unitary transformations
3. Prepare a fiducial initial state
4. Measure the output result
The requirement of experimental realization is often only partially met.

eg.1, Coin: has exactly two states, however don’t have superposition state

eg.2, nuclear spin: A very good qubit because it has good superposition state, however it is difficult to measure the state.

The most difficult: a quantum computer need to be well isolated to retain the states,

However, on the other hand, its qubits have to be accessible.
What are the experimental requirements for building a quantum computer?

- **decoherence times**: The time for which a system remains quantum-mechanically coherent
- **Operation times**: The time it takes to perform elementary unitary transformations

These two times are actually related by the formula: \( n_{op} = \lambda^{-1} = \tau_Q / \tau_{op} \)

<table>
<thead>
<tr>
<th>System</th>
<th>( \tau_Q )</th>
<th>( \tau_{op} )</th>
<th>( n_{op} )</th>
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</thead>
<tbody>
<tr>
<td>Nuclear spin</td>
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7.2 Conditions for quantum computation

Four basic requirements for quantum computation

1. Robustly represent quantum information

1. Qubit with two state

2. Accessible state need to be finite

Choice of representation is important

2. Perform a universal family of unitary transformations

3. Prepare a fiducial initial state

4. Measure the output result
Square well

\[ |\psi_n\rangle = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi}{L} x \right) \quad \rightarrow \quad |\psi_n(t)\rangle = e^{-iE_n t} |\psi_n\rangle. \]

Suppose that we arrange matters such that only the two lowest energy levels

\[ |\psi\rangle = a |\psi_1\rangle + b |\psi_2\rangle. \]

\[ |\psi(t)\rangle = e^{-i(E_1+E_2)/2t} \left[ a e^{-i\omega t} |\psi_1\rangle + b e^{i\omega t} |\psi_2\rangle \right], \quad \omega = (E_1 - E_2)/2, \]

For performing operation, we introduce a perturbation

\[ \delta V(x) = -V_0(t) \frac{9\pi^2}{16L} \left( \frac{x}{L} - \frac{1}{2} \right) \]

\[ V_{nm} = \langle \psi_n | \delta V(x) | \psi_m \rangle \quad \text{giving} \quad V_{11} = V_{22} = 0, \quad \text{and} \quad V_{12} = V_{21} = V_0, \]

\[ H \quad \text{is} \quad H_1 = V_0(t)X. \quad \rightarrow \quad \text{This generates rotations about the} \, \hat{x} \, \text{axis} \]
Last page shows the simple operation on the Square well qubit.

However, the other term in \[ |\psi_n\rangle = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right) \] would be affected.

1. Perturbations introduce higher order effects
2. Square well boxes are not infinitely deep

Our two-level qubit states lead to decoherence
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1. Robustly represent quantum information

2. Perform a universal family of unitary transformations

Section 4.5
Any unitary transform can be composed from single spin operations and controlled-Not gates

eg.
Ion trap can be selectively excited, but we need to separate ions by a wavelength or more

3. Prepare a fiducial initial state

4. Measure the output result
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Qubits may not stay there for very long due to thermal heating.

Because the energy difference between the $|0\rangle$ and $|1\rangle$ states is much smaller than $k_B T$

$$\hbar = 10^{-34} \quad \text{and} \quad k_B = 1.38 \times 10^{-23}$$

For example, ions can be prepared in good input states by physically cooling them into their ground state

4. Measure the output result
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Let us think of measurement as a process of coupling one or more qubits to a classical system

a qubit state $a|0> + b|1>$

Wavefunction collapse

The output from a good quantum algorithm is a superposition state which gives a useful answer with high probability when measured
Projective measurements (strong measurements) are often difficult to implement.

Like $\alpha|0> + \beta|1>$, when we measure it, it collapse.

Weak measurements are usable for quantum computation

1. Giving little information for us
2. Disturb the state little

It is possible by completing the computation in time short compared with the measurement coupling, and by using large ensembles of quantum computers.

Other problem: In factoring algorithm, its output $q<c>/r$ will be fail

Not necessarily a integral
7.1 Guiding principles

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