D-instanton Perturbation Theory

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Plan:

1. The problem

2. The solution and some results

A.S., work in progress

Also, arXiv:1908.02782, 2002.04043, 2003.12076

More details can be found at

https://www.ictp-saifr.org/workshop-on-fundamental-aspects-of-string-theory/

The problem

String theory began with Veneziano amplitude

– tree level scattering amplitude of four tachyons in open string theory

World-sheet expression for the amplitude (in $\alpha' = 1$ unit)

$$\int_0^1 dy\, y^{2p_1.p_2} (1-y)^{2p_2.p_3}$$

– diverges for $2p_1.p_2 \le -1$ or $2p_2.p_3 \le -1$.

Conventional viewpoint: Define the amplitude for $2p_1.p_2 > -1$, $2p_2.p_3 > -1$ and then analytically continue to the other kinematic regions.

However, analytic continuation does not always work

It may not be possible to move away from the singularity by changing the external momenta

Examples: Mass renormalization, Vacuum shift

– discussed earlier

Today we shall discuss another situation where analytic continuation fails

– D-instanton contribution to string amplitudes

D-instanton: A D-brane with Dirichlet boundary condition on all non-compact directions including (euclidean) time.

D-instantons give non-perturbative contribution to string amplitudes that may be important in many situations

Problem: Open strings living on the D-instanton do not carry any continuous momenta

$\Rightarrow$ we cannot move away from the singularities by varying the external momenta

Polchinski; Green, Gutperle; ...

Even if the divergent parts cancel after suitable choice of regulators, the finite parts that remain after the cancellation become ambiguous.

Fischler, Susskind

We shall study this in the context of a particular example – bosonic string theory in two dimensions

World-sheet theory: A free scalar X describing time coordinate and a Liouville field theory with central charge 25

Total central charge adds up to 26, cancelling anomalies on the world-sheet

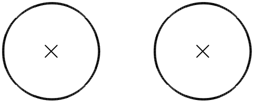
In this case the closed string ‘tachyon’ is actually a massless state of the theory

In arXiv:1907.07688 Balthazar, Rodriguez and Yin (BRY) computed the single D-instanton contribution to the two point amplitude of closed string tachyons

This model is interesting because there is a dual matrix model description that gives the exact results.

Das, Jevicki; Gross, Klebanov; Sengupta, Wadia; Moore, Plesser, Ramgoolam; · · ·

The leading contribution comes from the product of two disk one point functions.



BRY result:

$$8\,\pi\,N\,e^{-1/g_s}\,\delta(\omega_1+\omega_2)\,\sinh(\pi|\omega_1|)\,\sinh(\pi|\omega_2|)$$

N: An overall normalization constant

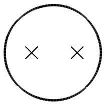
g_s: string coupling constant

$-\omega_1, \omega_2$: energies of incoming / outgoing 'tachyons'

Note: Individual disk amplitudes do not conserve energy

At the next order, there are three contributions.

1. Two point function on the disk.



BRY result:

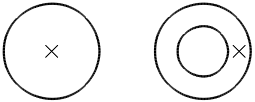
$$8\,\pi\,N\,e^{-1/g_s}\,g_s\,\delta(\omega_1+\omega_2)\,\sinh(\pi|\omega_1|)\,\sinh(\pi|\omega_2|)\times f(\omega_1,\omega_2)$$

$$f(\omega_1,\omega_2)=\frac{1}{2}\int_0^1 dy\,y^{-2}(1+2\omega_1\omega_2 y)+\text{ finite terms}$$

Note the divergences from the $y\to 0$ limit that cannot be tamed by deforming the ω_i's.

Write $f(\omega_1,\omega_2)=A_f+B_f\,\omega_1\,\omega_2+$ finite terms

2. Product of disk one point function and annulus one point function.



Annulus: UHP with identification $z \equiv z/v$
Position of vertex operator: $e^{2\pi i x}$

BRY result:

$$8\pi N e^{-1/g_s} g_s \delta(\omega_1 + \omega_2) \sinh(\pi|\omega_1|) \sinh(\pi|\omega_2|) \{g(\omega_1) + g(\omega_2)\}$$

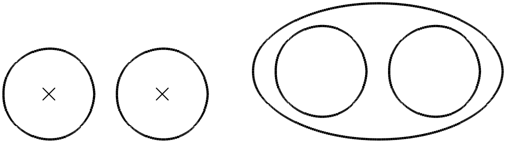
$$g(\omega) = \int_0^1 dv \int_0^{1/4} dx \left\{ \frac{v^{-2} - v^{-1}}{\sin^2(2\pi x)} + 2\,\omega^2\, v^{-1} \right\} + \text{ finite terms}$$

Note the divergences from $x \to 0$ and $v \to 0$ that cannot be tamed by adjusting the ω_i's.

Write $g(\omega) = A_g + B_g\, \omega^2 + $ finite terms

3. Product of two disk one point functions and the zero point function on a surface of Euler number -1

– disk with two holes or torus with one hole.



Result:

$$8\,\pi\,\mathbf{N}\,\mathbf{e}^{-1/g_s}\,\mathbf{g_s}\,\delta(\omega_1+\omega_2)\,\mathbf{sinh}(\pi|\omega_1|)\,\mathbf{sinh}(\pi|\omega_2|)\,\mathbf{C}$$

C: a real constant that can in principle be computed but also has divergences.

After setting $\omega_1 = -\omega_2$, the total unknown factor is:

$$(A_f + 2\, A_g + C) + (2B_g - B_f)\, \omega_2^2$$

with unknown constants A_f, B_f, A_g, B_g, C.

BRY numerically compared the result with matrix model results as function of ω_2.

Leading order result $\Rightarrow$ $N = -1/(8\pi^2)$

They found excellent fit at the subleading order if we choose:

$$A_f + 2\, A_g + C \simeq -0.496, \qquad 2B_g - B_f \simeq -1.399$$

Question: Can we get these results from string theory without invoking the matrix model?

Actually the same functions f, g, C can be used to compute $1 \to n$ tachyon scattering for which the matrix model result is known.

$$A_{n+1} = N\, e^{-1/g_s}\, 2\pi\delta(\omega_1 + \omega_2 + \cdots \omega_n + \omega_{n+1}) \left[\prod_{i=1}^{n+1} \{2\sinh(\pi|\omega_i|)\}\right]$$
$$\left[1 + g_s \sum_{\substack{i,j=1 \\ i<j}}^{n+1} f(\omega_i, \omega_j) + g_s \sum_{i=1}^{n+1} g(\omega_i) + C\, g_s + O(g_s^2)\right].$$

'Divergent part' of the amplitude at order g_s:

$$\frac{n(n+1)}{2} A_f + (n+1) A_g + C + \frac{1}{2}(2B_g - B_f) \sum_i \omega_i^2$$

Comparing this with matrix model results for different n, we can fix

$$A_f, \quad A_g, \quad C, \quad (2B_g - B_f)$$

‘Divergent’ constants in two dimensional string theory:

$$\mathbf{A_f}, \quad \mathbf{A_g}, \quad \mathbf{C}, \quad (\mathbf{2B_g} - \mathbf{B_f})$$

Since these constants appear in the S-matrix, string theory should be able to determine these constants unambiguously without invoking duality with the matrix model

How?

The solution

We use the general framework of open-closed string field theory to fix these constants

Zwiebach, · · ·

Our results so far:

$$A_f = -\frac{1}{2}, \quad 2B_g - B_f = -\ln[4] \simeq -1.386...$$

A_g: in progress

C: computable in principle but more difficult than others.

If we assume equality of quantum corrected actions of the D-instanton and the matrix model instanton, then one can argue that

$$C = 0$$

Numerical values of these constants from fitting the marix model results (BRY, private communications)

$$A_f = -.50, \quad A_g = .00, \quad C = .00, \quad 2B_g - B_f = -1.40, \quad \text{error} \sim \pm .01$$

String field theory (SFT)

SFT is a regular quantum field theory (QFT) with infinite number of fields

Perturbative amplitudes: sum of Feynman diagrams

Contribution from a given diagram gives us back the world-sheet result, but the integral runs over only part of the full region of integration.

Sum of the diagrams covers the full integration region.

String field theory is not unique but comes in a whole family which are all related to each other by field redefibition. Hata, Zwiebach

How do we get integral over world-sheet variables from a Feynman diagram?

Express internal propagator as

$$(k^2 + m^2)^{-1} = \int_0^\infty ds\, e^{-s(k^2+m^2)} = \int_0^1 dq\, q^{k^2+m^2-1}, \quad q \equiv e^{-s}$$

The integration over q gives integration over world-sheet variables after a change of variable.

Divergences come from the $q \to 0$ region for $k^2 + m^2 \leq 0$.

All divergences in string theory are of this kind.

For D-instantons k=0, and we cannot analytically continue in momenta to make $k^2 + m^2 > 0$.

$$(m^2)^{-1} = \int_0^1 dq\, q^{m^2-1}$$

This equation is:

1. An identity for $m^2 > 0$.

2. For $m^2 < 0$ the lhs is finite but the rhs is divergent

$\Rightarrow$ use lhs to define the integral.

– Change variables from the moduli of Riemann surfaces to the variables $q_1, q_2, \cdots$ associated with the propagators

– Replace $\int_0^1 dq\, q^{\beta-1}$ by $1/\beta$ for $\beta \neq 0$

– can be used to deal with power law divergences like $\int_0^1 dy\, y^{-2}$ in the earlier formulæ

Comment: Making the correct change of variables is important for getting the correct result.

Replacement rule: $\int_0^1 \mathrm{d}q\, q^{-2} = -1$

Suppose we change variable to

$$q' = \frac{q}{(1 - c\,q)} \quad \Leftrightarrow \quad \frac{1}{q'} = \frac{1}{q} - c, \qquad c = \text{constant}$$

Then $\mathrm{d}q\, q^{-2} = \mathrm{d}q'\, q'^{-2}$

$$\Rightarrow \int_0^1 \mathrm{d}q\, q^{-2} = \int_0^{1/(1-c)} \mathrm{d}q' q'^{-2} = \int_0^1 \mathrm{d}q' q'^{-2} + \int_1^{1/(1-c)} \mathrm{d}q' q'^{-2}$$

If we now replace the first term on the rhs by -1 using the replacement rule, we get

$$-1 + 1 - (1 - c) = -1 + c$$

– a different answer!

$$(m^2)^{-1} = \int_0^1 dq\, q^{m^2-1},$$

For $m = 0$ both sides are divergent.

– associated with zero modes on the D-instanton

– produces logarithmic divergence in the world-sheet description

Strategy: Understand the physical origin of the zero modes and then find suitable remedy by drawing insights from QFT.

D-instantons have zero modes associated with translation of the instanton position along transverse directions

– known as collective coordinates χ

$\Rightarrow$ massless open string states

Treatment of these zero modes in QFT:

1. Carry out path integral over all modes of the instanton other than χ, in the background of χ

$\Rightarrow$ while evaluating Feynman diagrams we remove the χ contribution from the internal propagators but keep the D-instanton position χ arbitrary

After summing over Feynman diagrams we get a given amplitude as a function F(χ).

2. Then we compute $\int$ dχF(χ)

Strategy: Follow the same procedure for D-instantons

a. Drop terms of the from $\int_0^1 dq\, q^{-1}$ coming from the open string zero mode ϕ describing collective coordinates

– corresponds to removing ϕ propagators from internal lines.

b. Allow external states to be both closed strings and the open string zero mode ϕ, leading to some function F(ϕ) for given closed string amplitude.

c. Carry out the integration over ϕ

ϕ is related to the collective coordinate χ via a field redefinition.

After field redefinition, χ dependence of F is expected to be of the form $e^{ip\cdot\chi}$

p: total momenta carried by the external closed strings

χ integration will generate the $\delta(p)$ factor.

The field redefinition relating ϕ to χ may give rise to Jacobian in the integration measure that needs to be taken into account in the analysis.

Note: Integration measure over ϕ is fixed by Batalin-Vilkovisky formalism and cannot be tampered with.

Open strings have other zero modes from the ghost sector.

Consider the $L_0 = 0$ sector of the expansion of a general open string state:

$$|\chi\rangle = \psi^1\, c_0|0\rangle + \psi^2|0\rangle + \xi_1\, c_1 c_{-1}|0\rangle + \xi_2\, c_1 c_0 c_{-1}|0\rangle + \cdots$$

$\psi^1, \psi^2, \xi_1, \xi_2$ are open string 'fields'

Gauge fixing requires setting two of the four fields to zero.

(ψ^1 or ξ_1) and (ψ^2 or ξ_2).

The world-sheet description emerges in the Siegel gauge:

$$\psi^1 = 0,\ \xi_2 = 0 \quad \Rightarrow \quad |\chi\rangle = \psi^2|0\rangle + \xi_1\, c_1 c_{-1}|0\rangle + \cdots$$

Problem: In this gauge, the quadratic term of SFT, $\frac{1}{2}\langle\chi|Q_B|\chi\rangle$, becomes independent of the remaining fields ξ_1 and ψ^2.

$\frac{1}{2}\langle\chi|Q_B|\chi\rangle$ is independent of ξ_1 and ψ^2

$\Rightarrow$ ξ_1 and ψ^2 are additional zero modes of the open string

– give additional logarithmic divergence in the world-sheet integrals that is not removed by removing the collective modes from the propagators.

Remedy: Choose a different gauge $\xi_1 = 0$, $\xi_2 = 0$

$$|\chi\rangle = \psi^1 c_0|0\rangle + \psi^2|0\rangle + \cdots$$

$$S = \frac{1}{2}\langle\chi|Q_B|\chi\rangle = (\psi^1)^2 + \cdots$$

ψ^1 has a non-vanishing kinetic term and therefore a finite propagator.

ψ^2 has no kinetic term $\Rightarrow$ a zero mode

However it decouples from the action completely by ghost number conservation

Physically, integration over ψ^2 corresponds to division by the volume of the rigid U(1) gauge group.

Therefore integration over ψ^2 factorizes and can be dropped (possibly after a field redefinition).

Algorithm on the world-sheet:

1. Remove all logarithmically divergent integrals of the form $\int_0^1 dq/q$, including those that come from the Siegel gauge ghost zero mode pairs ξ_1, ψ^2.

2. Explicitly add the contribution from ψ^1 propagators

3. Take into account the Jacobians that arise form change of variable from string fields to collective modes and ghost associated with rigid U(1) symmetry.

This leads to the results in two dimensional string theory described earlier.

Example: Contributions to A_f in a two parameter family of SFT

Three types of contributions:

(a) (b) (c)

▬ : closed string C ── : open string O •: interaction vertex

Contributions:

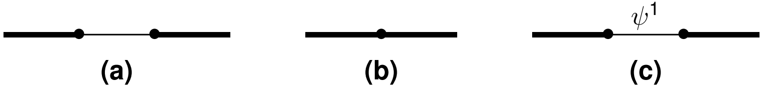
$$(\mathbf{a}) : -\frac{1}{2}\lambda^2, \qquad (\mathbf{b}) : \frac{1}{2}\lambda^2 + \gamma^2 - \frac{1}{2}, \qquad (\mathbf{c}) : -\gamma^2$$

λ, γ: parameters labelling choice of SFT

Sum gives $-1/2$

– independent of SFT parameters

Computation of A_f (for $\gamma = 0$)



We define the C-O interaction vertex by two point function on UHP with C at i and O at 0

Insert the off-shell open string in coordinate $w = \lambda z$

Diagram (a) now describes the Riemann surface obtained by sewing two copies of UHP via

$$\lambda z \lambda z' = -q, \qquad 0 \leq q \leq 1$$

The C's are placed at z=i and at z'=i $\rightarrow z = iq/\lambda^2 \equiv iy$

Therefore diagram (a) covers $0 \leq q \leq 1 \Rightarrow 0 \leq y \leq 1/\lambda^2$

Diagram (b) covers $1/\lambda^2 \leq y \leq 1$

Net contribution to A_f:

$$\frac{1}{2}\int_0^1 dy\, y^{-2} = \frac{1}{2}\int_0^{1/\lambda^2} dy\, y^{-2} + \frac{1}{2}\int_{1/\lambda^2}^1 dy\, y^{-2}$$

$$= \frac{\lambda^2}{2}\int_0^1 dq\, q^{-2} + \frac{1}{2}\int_{1/\lambda^2}^1 dy\, y^{-2}$$

$$\to -\frac{\lambda^2}{2} + \frac{1}{2}(\lambda^2 - 1) = -\frac{1}{2}$$

In this case the ψ^1 exchange contribution vanishes due to vanishing of the C-O veretx

Open questions:

1. The same matrix model with a different vacuum is expected to be dual to type 0B string theory.

Takayanagi, Toumbas; Douglas, Klebanov, Kutasov, Maldacena, Martinec, Seiberg

– same perturbative S-matrix but different instanton contribution.

Can we verify this explicitly by string theory computation?

2. The method described here is applicable to any string theory, including type IIB string theory in D=10

Can we extract instanton contributions to the effective action and compare the results with those expected from S-duality?

Green, Gutperle;...

3. The overall normalization constant N has not been fixed in string theory

– represents the ratio of the path integral measure in the D-instanton and the vacuum sector.

Can we fix this by starting with an unstable D-brane system and by regarding the D-instanton and the vacuum as different classical solutions in the same theory?

4. Can we combine the D-instanton contribution and perturbative contribution to string amplitudes with the idea of resurgence to give a full non-perturbative definition of string amplitudes?

Some references:

1. Computation of $2B_g - B_f$ can be found in arXiv:1908.02782

2. Computation of A_f and the argument for vanishing of C can be found at the lectures at ICTP-SAIFR school

https://www.ictp-saifr.org/workshop-on-fundamental-aspects-of-string-theory/