Simple Models of Black Hole Evaporation

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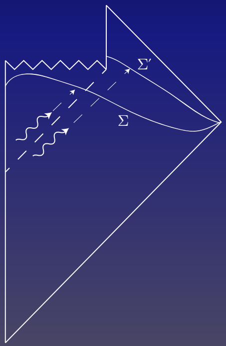
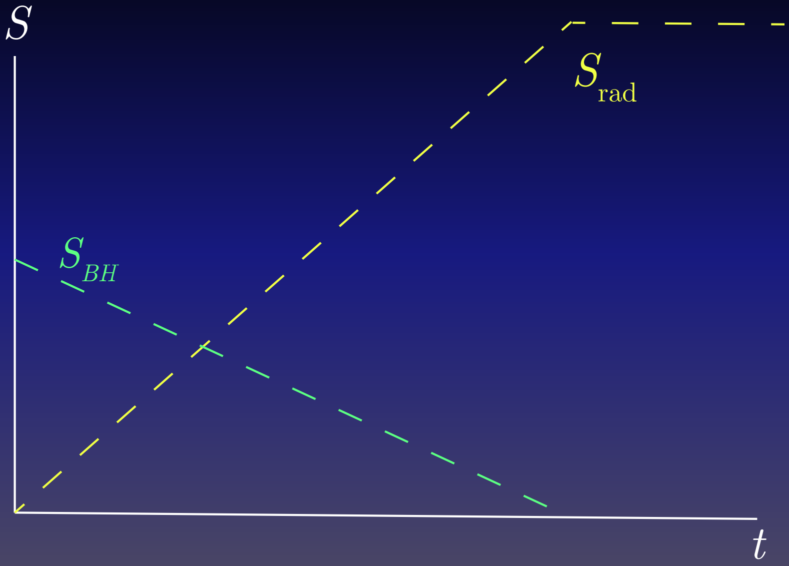
The Information Paradox

- Most of us will probably agree that quantum gravity is a unitary theory, but resolving the black hole information paradox requires an understanding of the detailed dynamics via which information gets out in the radiation.

- The von Neumann entropy of the radiation, $-\mathrm{tr}\rho_{rad} \log \rho_{rad}$ has traditionally served as a diagnostic for information loss/conservation via the Page curve.



The Page Curve



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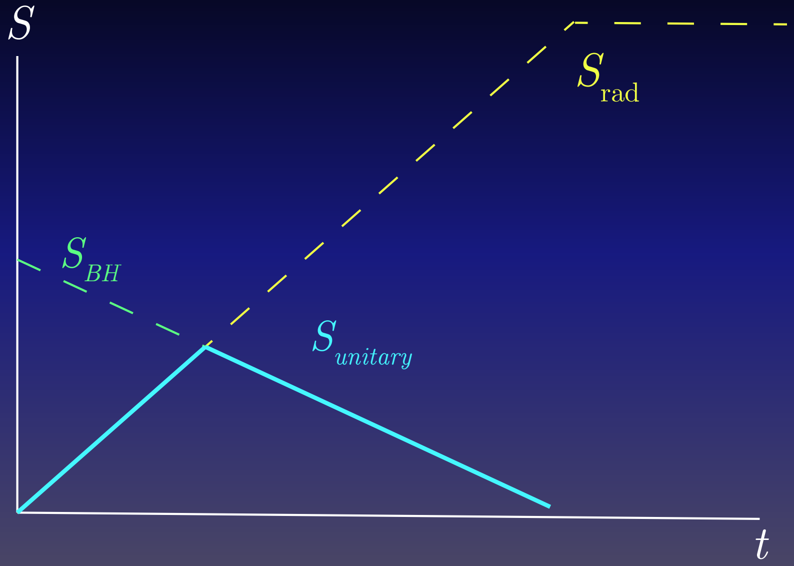

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- Nonetheless, a direct calculation of the Page curve is a large step forward, and until recently it was not considered feasible by the broad community.

- This was partly because of an expectation that such a calculation would require extensive knowledge of nonperturbative physics.

The New Developments

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- Today I'll discuss the original setup Penington; Almheiri, NE, Marolf, Maxfield, in which a semiclassical analysis of an evaporating AdS black hole produces a unitary Page curve, and describe a newer model of an evaporation that exchanges the semiclassical analysis for a purely classical analysis at the cost of dynamical topology change Akers, NE, Harlow.

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- One upshot of both models as well as the recent developments on replica wormholes Penington et al; Almheiri et al; see Geoff's and Douglas's talks is that we really want to get a better understanding of the mechanism behind the surprisingly smart gravitational path integral. So I will also briefly advertise some upcoming work w/ Maloney and Fischetti on the calculation of free energy from the gravitational path integral and the role of replica wormholes.

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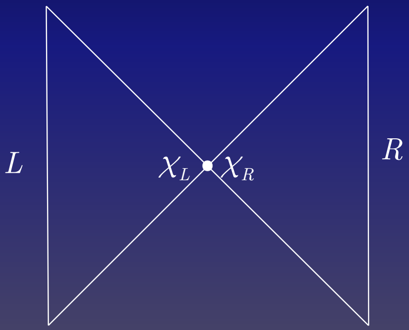
Modeling an Evaporating AdS BH Almheiri; Almheiri, NE, Marolf, Maxfield; Penington

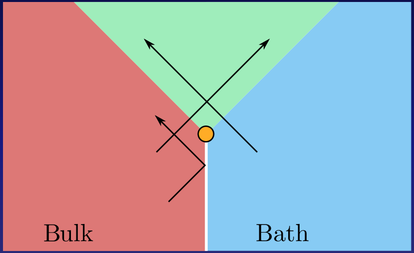
Qualitative setup: evaporate an AdS black hole by coupling it to an external bath.

In the 2-sided case, prior to evaporation, the bifurcation surface is the same for both ρ_L and ρ_R.



Method: Couple to a Reservoir

We impose transparent boundary conditions when the bath is coupled to our system:



To work in a setting where we can do the calculations directly, we work in JT gravity coupled to a CFT in the bulk Maldacena, Stanford, Yang; Engelsoy, Mertens, Verlinde; Almheiri-Polchinski; Jensen:. Similar results hold in higher dimensions Penington.

Evaporating the Black Hole

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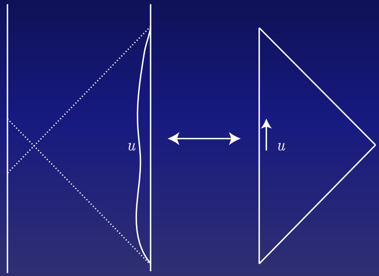

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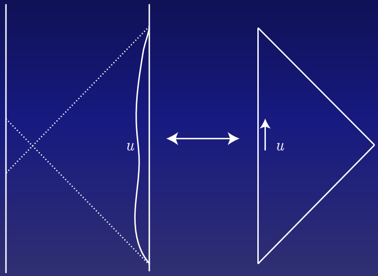


We couple the two systems (a quantum quench) at physical time u and then evolve them forwards in time. This results in a shockwave propagating into the bulk. Now we are ready to compute entanglement entropies.

Computing Entropy in AdS/CFT

To first order in $G\hbar$, we have the (H)RT/FLM proposal Ryu, Takayanagi; Hubeny, Rangamani, Takayangi; Faulkner, Lewkowycz, Maldacena

$$S_{vN}[\rho_R] = \frac{\text{Area}[X_R]}{4G\hbar} + S_{\text{vN}}[\rho_{\text{bulk}}]$$

where X_R is the minimal area (dilaton) surface which extremizes the area (dilaton). So long as the boundary system is evolving unitarily, there is no t-dependence.

But various consistency conditions can all be violated by the HRT prescription once we include quantum backreaction.

Generalizing to all orders

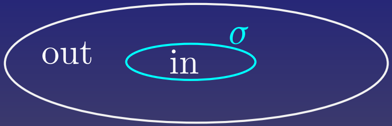
A similar problem occurs in BH thermo: the Hawking area theorem relies on $T_{kk} > 0$ and is violated by quantum backreaction.

Generalized Entropy

Replace area by "quantum corrected area" [Bekenstein]:

$$S_{\text{gen}}[\sigma] = \frac{\text{Area}[\sigma]}{4G\hbar} + S_{\text{out}}[\sigma]$$

Large body of evidence indicates that S_{gen} is UV finite.



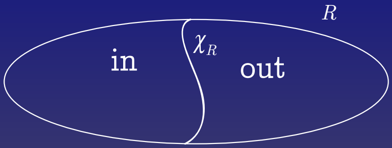
The QES Prescription

Holographic EE to all orders in $G\hbar$ [NE, Wall]:

$$S_{vN}[\rho_R] = \frac{\text{Area}[\chi_R]}{4G\hbar} + S_{\text{out}}[\chi_R] = S_{\text{gen}}[\chi_R]$$

where χ_R is the minimal-S_{gen} surface that extremizes S_{gen}.

For example, for a higher-dimensional system, the quantum extremal surface (QES) of a region R:



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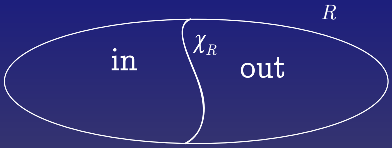
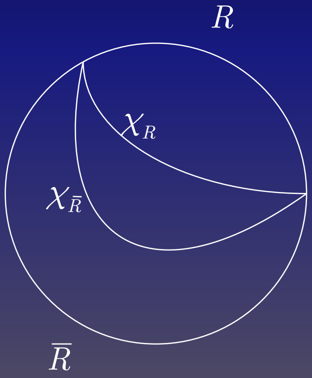


This satisfies the consistency conditions violated by HRT (assuming the quantum focusing conjecture [Bousso, Fisher, Leichenauer, Wall])

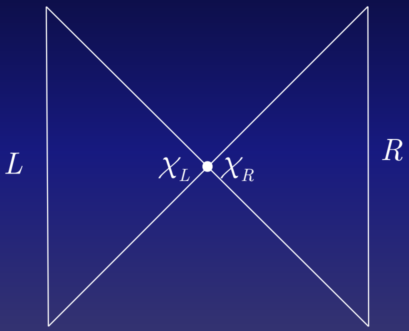
Aside: Complementary Recovery

If we have a system in a pure state, then the quantum (and also classical) extremal surface of a subsystem R is identical to the quantum extremal surface of its complement. In particular, if the bulk state is mixed – that is, there is an additional system that purifies the bulk – then if R is a subset of the boundary and $\overline{R}$ is the complement of the boundary, it must be true that $\chi_R \neq \chi_{\overline{R}}$. (A surface which is R-quantum-extremal need not be $\overline{R}$-quantum extremal if the bulk state is mixed.)



QESs in an Evaporating BH

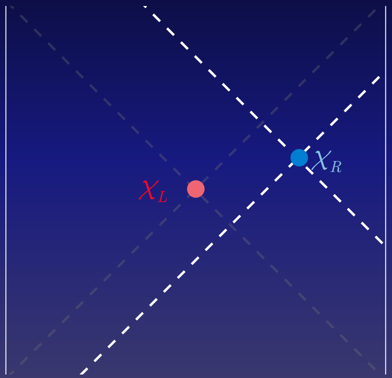
Before evaporating the black hole, the left and right QESs are just the bifurcation surface:

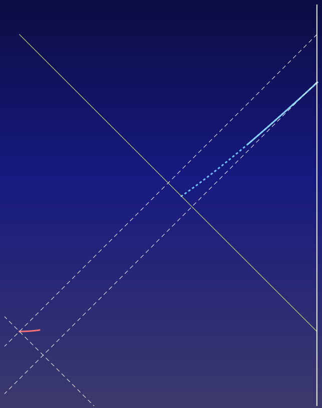


As we evaporate the black hole, initially χ_R moves continuously in a spacelike direction.

QESs in an Evaporating BH

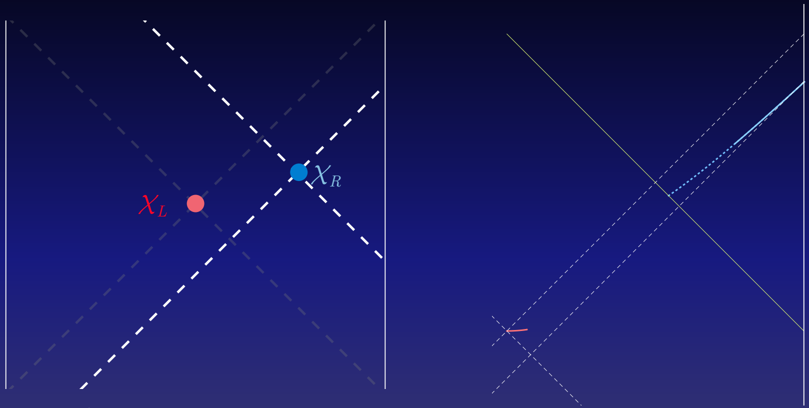
At late times, a branch of QESs with *no classical counterpart* begins to dominate:





After the transition, the dominant right QES is far from the left QES. The effect of the transition is a unitary Page curve in the bulk.

Quantum Extremal Islands



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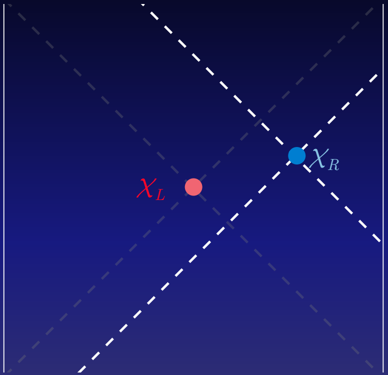

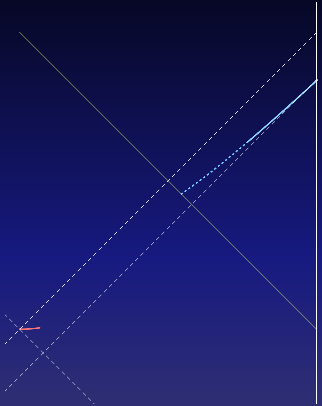

Hypothesis in Hayden, Penington, Almheiri, Mahajan, Maldacena, Zhao; see Geoff's and Douglas's talks on Friday: the gap between X_L and X_R belongs in the entanglement wedge of the purifier. The existence of this is synonymous with the bulk state being mixed: there is a purifier system (the bath) that purifies the left and right boundaries. The gap is the so-called "quantum extremal island".

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- This is hard to see in this model. A simpler model would be nice.
- What could be simpler than a semiclassical picture? A classical picture. An example would be Almheiri, Mahajan, Maldacena, Zhao, which gave a 3D holographic interpretation of the calculation above. We will look at a different model.

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A Toy Model of BH Evaporation without Bulk Entanglement

Akers, NE, Harlow

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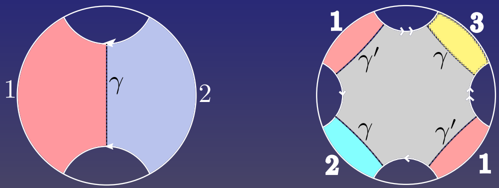
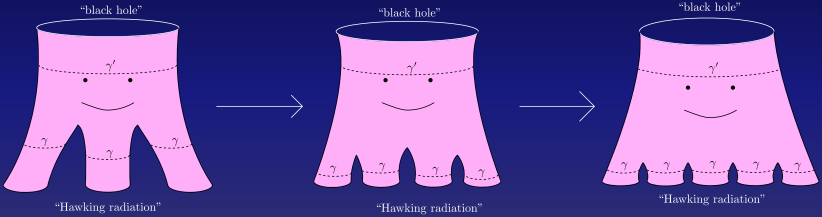
The emission of a black hole results in a wormhole with a new exit. This is inspired by ER=EPR and can be thought of as a classical version of the quantum octopus Van Raamsdonk; Maldacena, Susskind.

We can get time-symmetric geometries like this in 3D by different quotients of AdS_3 Krasnov; Skenderis, van Rees; Balasubramanian, Hayden, Marolf, Ross; Harlow, Ooguri.



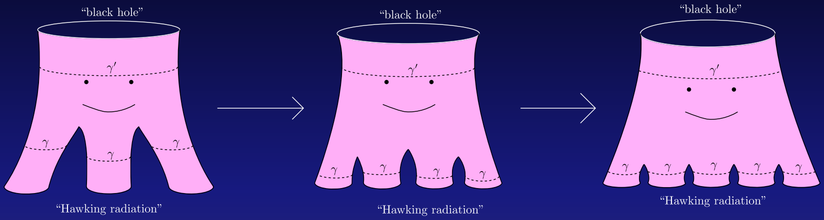
Dynamics

We consider discrete time steps, each bringing a new topology change. At any one of the fixed times, the geometry is time-symmetric, and the minimal surface on that time slice is the RT surface.



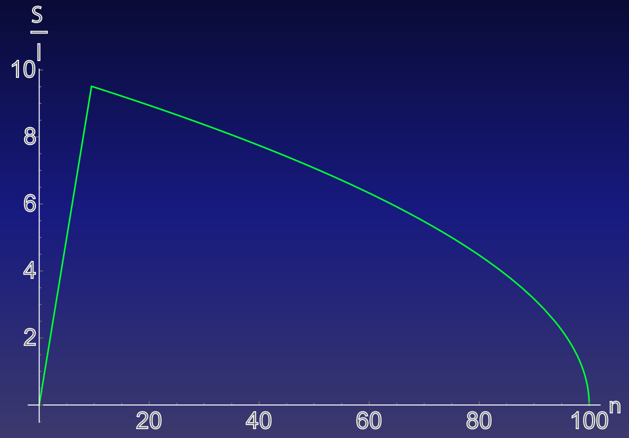
Again, caricature can be thought of as a classical version of the “quantum octopus” in ER=EPR.

Dynamics

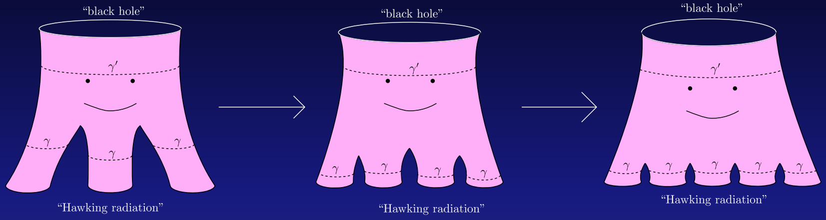


At early times, the minimal area surface is γ: the entanglement wedge of the black hole CFT is nearly the entire bulk. As the black hole evaporates, eventually there is a switchover and γ' becomes minimal: the entanglement wedge of the black hole now contains very little of the bulk.

The Page Curve for the Toy Model



The Island



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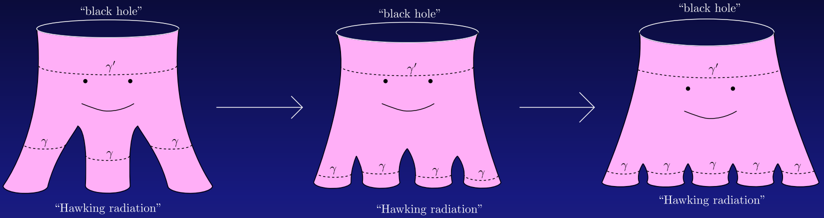

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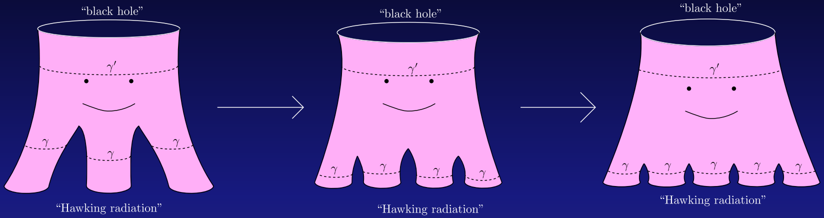
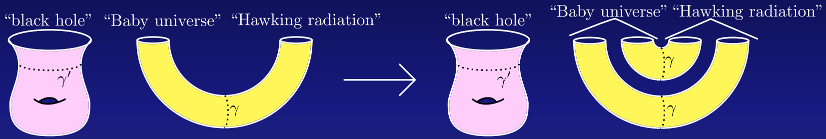


The island in this model is just the region between γ and γ'. A nice aspect of this model is that the island phenomenon doesn't seem so mysterious: it's just quantum error correction. Pick too few of the radiation CFTs and their entanglement wedge is given by the union of the γ's; add the black hole CFT (or, after the Page time, enough of the radiation CFTs), and the island is regained.

Information Loss

What would a holographic calculation of information loss (i.e. the Hawking calculation) look like?



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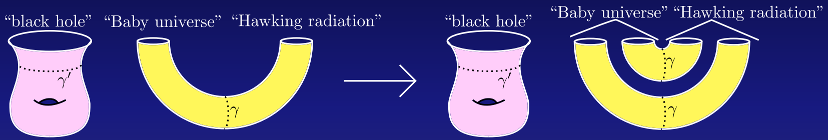
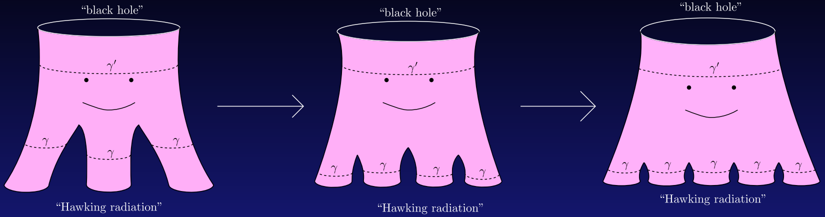
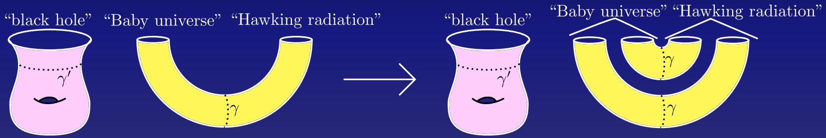


So it is possible to compute information loss with extremal surfaces: the HRRT/FLM/EW proposal doesn't just give a unitary answer. Of course, we really think that information is not lost in a realistic QG theory.

A Final Comparison





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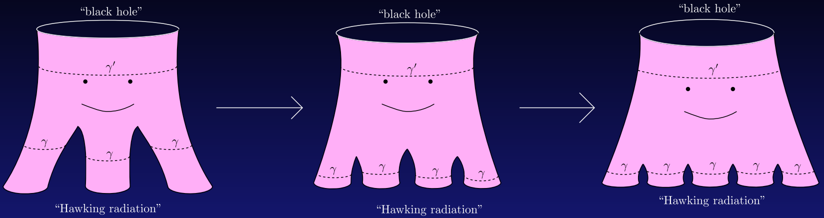

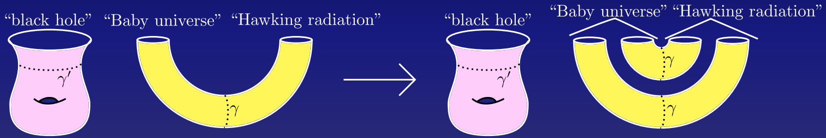

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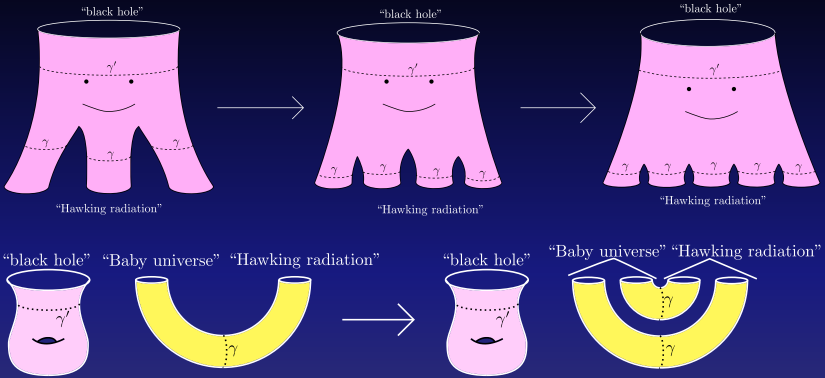

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Some Comments

- Takeaway: using the correct QES (for a given QG theory) is critical. The mechanism that identifies the correct QES is directly responsible for the unitary result in our calculation.

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- But ultimately to address the issue we need to have a better handle on the gravitational path integral itself. What entropy is it calculating? How does it know? And how can we compute the same entropy without using the path integral?
- Relatedly, the QES prescription can give as an answer both info loss and info conservation, depending on the dynamics of the theory and the fine-grained structure of the state. What is the fine-grained state whose entropy is computed by the path integral? What are more fine-grained quantities computed by the path integral telling us?

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Teaser Trailer: Free Energy from Replica Wormholes 2007.????? w/ A. Maloney and S. Fischetti

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- So this means that it is possible that not only is it possible, it happens in real life systems

$$\ln \overline{Z(B)} \neq \overline{\ln Z(B)}$$

To compute $\overline{\ln Z(B)}$, we can use a replica trick:

$$\overline{\ln Z(B)} = \lim_{m \to 0} \frac{1}{m} \left(\mathcal{P}(B^m) - 1 \right)$$

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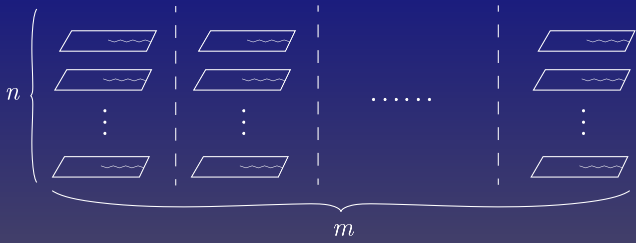
- This is different from the usual replica trick for the von Neumann entropy:

$$\overline{S_{\text{vN}}} = \lim_{n\to 1} \frac{1}{1-n}\left(\overline{\ln Z(B^n)} - n\overline{\ln Z(B)}\right)$$
$$= \lim_{n\to 1} \frac{1}{1-n}\left(\lim_{m\to 0}\frac{1}{m}\left(\mathcal{P}(B_n^m)-1\right) - n\lim_{m\to 0}\frac{1}{m}\left(\mathcal{P}(B^m)-1\right)\right)$$



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- Rather than focusing on entropy and the Page curve alone, let us ask a more general question: is there a dominant contribution (in some regime) from replica wormholes in any extensive quantity we compute from $\overline{\ln Z(B)}$?

- We find that in JT gravity and a variant of CGHS, there is a regime where replica wormholes dominate $\overline{\ln(Z)}$ as computed by the $m \to 0$ replica trick.

- We also find strong evidence for replica symmetry breaking in this regime. This bears conceptual similarity to behavior exhibited by spin glasses. Analogous behavior has been studied in certain limits of SYK e.g. [Aref'eva, Khramtsov, Tikhanovskaya, Volovich].

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- Thank you.