Higher-Group Symmetry and Applications

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Strings 2020

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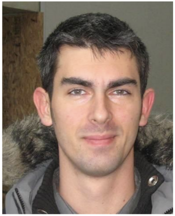
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References

Based on “Exploring 2-Group Global Symmetry”
[Córdova-Dumitrescu-Intriligator]

Based on “On 2-Group Global Symmetries and Their Anomalies”
[Benini-Córdova-Hsin]

Based on “Constraints on Axions and Emergent Symmetry”
[Brennan-Córdova]

Based on “2-Group Global Symmetries and Anomalies in Six-Dimensional Quantum Field Theories”
[Córdova-Dumitrescu-Intriligator]

Foundational work in this area was done in papers by:
[Kapustin-Thorngren]

Other closely related ideas have been explored in papers by:
[Barkeshli-Bonderson-Cheng-Wang], [Tachikawa]

Motivating Ideas

Symmetry is one of the few universally applicable tools to constrain QFTs and their renormalization group flows

We would therefore like to explore what symmetry principles exist and how they can be used to constrain dynamics

Generalized global symmetry is a powerful new organizing principle for thinking about QFT [Gaiotto-Kapustin-Seiberg-Willett]

- Ordinary (0-form) global symmetries act on local operators.
- Higher-form global symmetries act on extended defects

This Talk:

What possible mixings can occur when both types of symmetries are present? (analog of non-abelian structure)

Higher-Group Global Symmetry is one general possibility for mixing of form symmetries of different degrees.

Generalized Global Symmetry

A continuous q-form global symmetry is characterized by the existence of a $(q+1)$-form conserved current $J^{(q+1)}$

$$J^{(q+1)}_{A_1 \cdots A_{q+1}} = J^{(q+1)}_{[A_1 \cdots A_{q+1}]} , \qquad \partial^{A_1} J^{(q+1)}_{A_1 \cdots A_{q+1}} = 0 .$$

Charged objects are extended operators of dimension q.

- $q = 0 \longrightarrow$ point operators
- $q = 1 \longrightarrow$ line operators
- $q = 2 \longrightarrow$ surface operators

A basic example is 4d abelian gauge theory. The Bianchi identity and free equation of motion imply

$$\partial^A \epsilon_{ABCD} F^{CD} = 0 , \qquad \partial^A F_{AB} = 0 .$$

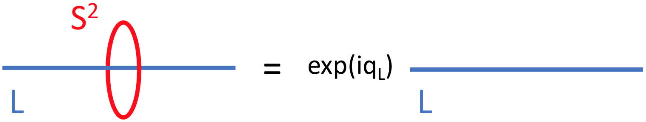
Thus free Maxwell theory has 1-form global symmetry $U(1) \times U(1)$

Charged Line Operators

The charged operators under these symmetries are Wilson and 't Hooft lines. To say that an operator is charged means that if S^2 is a 2-sphere surrounding the line L then

$$\exp\left(i\alpha \int_{S^2} *J^{(2)}\right) L = e^{i\alpha q_L} L$$

In pictures the geometry is



In Maxwell theory this is true since

$$\int_{S^2} *F \sim \text{electric charge}\ , \qquad \int_{S^2} F \sim \text{magnetic charge}$$

Background Fields

A tool for studying global symmetry is to couple to background gauge fields A, leading to a partition function $Z[A]$

A is a fixed classical source. For a continuous symmetry $Z[A]$ is a generating function of correlation functions for the current J

Higher-form symmetry currents couple to gauge fields with more indices. E.G. a 1-form symmetry couples to a 2-form $B^{(2)}$

$$\delta S \supset \int d^d x \; B^{CD} J_{CD}$$

Current conservation means that $Z[B]$ is invariant under background gauge transformations $B^{(2)} \to B^{(2)} + d\Lambda^{(1)}$.

(This invariance can be violated by 't Hooft anomalies)

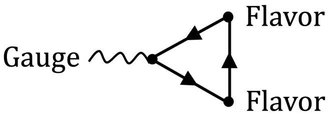
Symmetry of 4d QED

Consider $U(1)$ gauge theory with N_f fermions of charge Q. What is the symmetry now?

- There is a $SU(N_f)_L \times SU(N_f)_R$ ordinary global symmetry acting on left and right Weyl fermions
- The charged matter means that F is no longer conserved, but $*F$ is still conserved by the Bianchi identity. Thus the 1-form symmetry is $U(1)$ (magnetic charge).

Naively one might expect that ordinary global symmetry and 1-form global symmetry don't talk to each other. In fact they mix.

One way to diagnose the behavior is via a triangle diagram



Symmetry of 4d QED

Triangle diagrams have different interpretations depending on the whether the vertices couple dynamical gauge fields or currents for flavor symmetries

- $(\text{gauge})^3$: "Gauge anomaly" must vanish for the theory to be mathematically consistent
- $(\text{gauge})^2$flavor: "ABJ anomaly" non-zero implies that flavor symmetry is broken
- $(\text{flavor})^3$: "'t Hooft anomaly" flavor symmetry unbroken. Matches on RG flows
- $(\text{flavor})^2$gauge: flavor symmetry unbroken. deforms current algebra between the flavor symmetry and the 1-form symmetry

We refer to this current algebra as higher-group global symmetry. In this case it is called a 2-group because the highest form current has 2 indices, (and to match with mathematics literature)

Higher Group Global Symmetry: Current Algebra

Mixing between 0-form and 1-form symmetry encoded in $\langle J_A J_B J_{CD} \rangle$. (Analog: structure constants for non-abelian ordinary global symmetry are encoded in 3-point functions of J_A)

More precisely, we have Ward identities relating $\langle J_A J_B J_{CD} \rangle$ and $\langle J_{AB} J_{CD} \rangle$. On the locus in momentum space $p^2 = q^2 = (p+q)^2$, with M be some scale:

$$\langle J_{AB}(p) J_{CD}(-p) \rangle = \frac{1}{p^2}\, \mathtt{f}\left(\frac{p^2}{M^2}\right)(\text{tensor}_{ABCD})$$

$$\langle J_A^i(q) J_B^j(p) J_{CD}(-p-q) \rangle \supset \left(\frac{\kappa}{2\pi}\right)\frac{\delta^{ij}}{p^2}\, \mathtt{f}\left(\frac{p^2}{M^2}\right)(\text{tensor}'_{ABCD})$$

4d QED realizes these with $\kappa = Q$, J_A either of the chiral $SU(N_f)$ symmetries, and $J_{AB} = (*F)_{AB}$

Higher Group Global Symmetry: Current Algebra

We can think of these Ward identities as arising from a contact term in the OPE of two ordinary currents

$$\partial^A J_A(x) \cdot J_B(0) \sim \frac{\kappa}{2\pi} \partial^C \delta^{(d)}(x) J_{BC}(0)$$

The parameter κ is a structure constant (somewhat analagous to f^{abc} in a Lie algebra)

Note that contact terms in the OPE of a current are typically associated with charged operators

$$\partial^A J_A(x) \cdot \mathcal{O}(0) \sim i q_{\mathcal{O}} \delta^{(d)}(x) \mathcal{O}(0)$$

In the 2-group OPE above, the derivative on the delta function means that the global charge algebra is unmodified

2-Group Global Symmetry: Background Fields

Ward identities and OPEs can also be encoded in the properties of background fields. Appropriate backgrounds are (locally)

$$\text{1-form gauge field} \quad A^{(1)} \,, \qquad\qquad \text{2-form gauge field} \quad B^{(2)} \,.$$

Under gauge transformations these now mix as

$$A^{(1)} \longrightarrow A^{(1)} + d\lambda^{(0)} \,, \qquad\qquad B^{(2)} \longrightarrow B^{(2)} + d\Lambda^{(1)} + \frac{\kappa}{2\pi}\lambda^{(0)} dA^{(1)} \,.$$

This is a Green-Schwarz transformation for background fields.

With the modified transformations above the partition function $Z[A^{(1)}, B^{(2)}]$ is invariant (up to c-number anomalies)

Quantization of the Structure Constant κ

In mathematics the pair $(A^{(1)}, B^{(2)})$ together with the gluing rule specified via the gauge transformations above form a so-called 2-connection on a 2-group bundle.

The gauge transformations

$$A \longrightarrow A^{(1)} + d\lambda^{(0)} \ , \qquad B^{(2)} \longrightarrow B^{(2)} + d\Lambda^{(1)} + \frac{\kappa}{2\pi}\lambda^{(0)} dA^{(1)}$$

imply that κ must be quantized in integer units ($\lambda^{(0)}$ ambiguous up to 2π shifts)

When we generalize to other 0-form groups (G) and other 1-form groups ($\mathcal{A}$) we find that κ is a cohomology class in $H^3(G, \mathcal{A})$.

Higher-Group Symmetry Generalities

Higher-group symmetry is similar to ordinary global symmetry

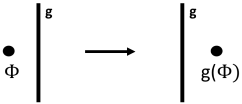
- Various groups are possible. The 0-form symmetry can be abelian or non-abelian, the higher-form symmetry is always abelian. Both can be discrete
- It can be emergent. Both the 0-form and higher-form part can be accidental at long distances though there are interesting constraints on energy scales
- It can be spontaneously broken with Goldstone bosons both for the 0-form part and the higher-form part
- It can have 't Hooft anomalies which match along RG flows and hence constrain dynamics

General Symmetries

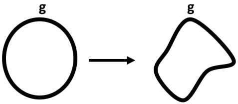
2-group symmetry common in models with discrete symmetry e.g. TQFTs and CSM theories [Benini-C-Hsin]

To characterize the global symmetry in these examples we use defects. For 0-form symmetry these are codimension 1 objects. For 1-form global symmetry they are codimension 2.

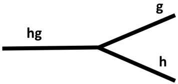
- motion of an operator across the defect implements the symmetry transformation



- the defects are topological



- their fusion determines the group multiplication law



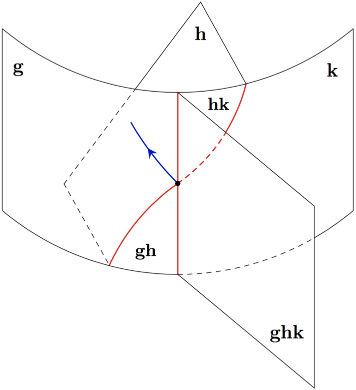
2-Group Global Symmetry via Defects

It is natural to look for the signature of 2-group global symmetry in the fusion of codimension-1 and codimension-2 defects

Let G be the 0-form symmetry, and $\mathcal{A}$ the 1-form symmetry

In 3d and above, triple fusion of G defects is generic

At such configurations a 1-form symmetry defect $a \in \mathcal{A}$ can appear (shown in blue)



The fusion is characterized by a cohomology class in $H^3(G, \mathcal{A})$

Examples: 3d TQFTs

3d TQFTs can in general exhibit 2-group global symmetry (discussed in [Barkeshli-Bonderson-Cheng-Wang]) Intrinsically such a system is described by a modular tensor category $\mathcal{C}$. The most natural setup is

$$G = \mathrm{Aut}(\mathcal{C}) \ , \qquad \mathcal{A} = \text{Abelian Anyons}$$

The action of the 0-form symmetry on the abelian anyons then encodes a class in $H^3(G, \mathcal{A})$

In some literature, this phenomenon was referred to as "an obstruction to symmetry fractionalization" however in our description it is viewed as a new kind of global symmetry

The defects also encode the anomaly of the 2-group symmetry which we classify in detail

Constraints on Emergent Symmetry

Suppose we have an RG flow where the UV does not have higher-group global symmetry, but the IR does

We can then define an energy scale E_q below which the q-form symmetry is effectively respected

One way to think about this is that in the EFT below the scale E_q, there are q-dimensional defects charged under the emergent symmetry. At the scale E_q these become the worldvolumes of new dynamical degrees of freedom

For instance if we are talking about a 2-form global symmetry, there are charged surface operators. At the scale E_2 we can resolve the thickness of these objects and they become the worldvolume of dynamical strings

Constraints on Emergent Symmetry

A non-trivial higher group (e.g. non-zero κ) then implies that the scales E_q obey the hierarchy [C-Dumitrescu-Intriligator]:

$$E_0 \leq E_1 \leq E_2 \leq \cdots$$

One way to understand this by examining the transformation rules for the background fields. For instance in a 2-group

$$A^{(1)} \longrightarrow A^{(1)} + d\lambda^{(0)} \, , \qquad B^{(2)} \longrightarrow B^{(2)} + d\Lambda^{(1)} + \frac{\kappa}{2\pi}\lambda^{(0)} dA^{(1)} \, .$$

We see that gauge transformations of the ordinary gauge field $A^{(1)}$ activate the background field $B^{(2)}$ of the 1-form symmetry

Thus if the 0-form symmetry is present so is the 1-form symmetry.

Axion Examples

Axion models illustrate these constraints on emergent symmetry [Brennan-C]

An axion is a periodic scalar field, a with periodicity $2\pi f$. Such theories have an interesting 2-form global symmetry with current

$$J^{(3)}_{ABC} = (*da)_{ABC}$$

The charged objects are surface defects with

$$\oint da = 2\pi n f \ , \qquad\qquad n \in \mathbb{Z}$$

At the scale E_2 the 2-form symmetry is broken and we expect new fields in the EFT. For instance, a may become the angular part of a complex scalar φ with radial part liberated at E_2

We expect $E_2 \sim f$, but these may differ by dimensionless couplings

Axion Examples

The key point is that in models where axions couple to gauge fields there is a higher-group global symmetry.
[Seiberg-Tachikawa-Yonekura, C-Freed-Lam-Seiberg]
Consider the action coupling a to $SU(N)$ gauge fields

$$S = \int d^4x \, \frac{1}{2g^2}\mathrm{Tr}(|F|^2) + \frac{1}{2}(\partial a)^2 + \frac{1}{8\pi^2 f} a\mathrm{Tr}(F \wedge F)$$

The $SU(N)$ gauge fields have a $\mathbb{Z}_N^{(1)}$ 1-form symmetry in a higher-group with the 2-form symmetry of the axion

Therefore we deduce the general inequality which applies to any UV completion

$$E_1 \leq E_2$$

Here E_1 is the scale associated to the 1-form symmetry, where charged fundamental matter appears

Axion Examples

Let's see how this is enforced in a concrete example. A typical KSVZ axion model arises from an action

$$S = \int d^4x \frac{1}{2g^2}\mathrm{Tr}(|F|^2) + i\bar{\psi}_\pm \not{D}\psi_\pm + |\partial_\mu \varphi|^2 - V(\varphi) + \lambda\bar{\varphi}\psi_+\psi_- + c.c.$$

Here $\psi_\pm$ are fundamentals of $SU(N)$ while φ is neutral complex scalar. We take the potential to be

$$V(\varphi) = m^2(|\varphi|^2 - f^2)^2$$

At weak coupling this flows to an axion model with a the angular part of φ and decay constant f

Comparing the mass of the fermions and the mass of $|\varphi|$ we see that in order to have the axion model be the EFT

$$\lambda \leq m$$

Axion Examples

How is this inequality $\lambda \leq m$ enforced?

One way to try to violate the inequality is to go to strong coupling. Of course then we lose control

Try to violate it at weak coupling with $1 >> \lambda >> m$. We find an important one-loop contribution to the potential for $|\varphi|$. This moves the minimum of the potential to the cutoff Λ_{UV}

$$|\varphi| \sim \Lambda_{UV}/\lambda$$

This means that at low-energies the angular mode a is effectively non-compact and no longer interacts with the gluons. Thus while there is no mathematical inconsistency with violating the inequality, it is necessary if we want a low-energy model of axion Yang-Mills

Applications of Higher-Group Symmetry in 6d

Higher-group symmetry can also be applied to 6d SCFTs and little string theories [C-Dumitrescu-Intriligator]

These theories typically look like gauge theories in the IR and have an interesting fixed point in the UV [Seiberg]. By now many (perhaps all!) constructed using F-theory [Vafa-...]

In the IR EFT there are interesting 1-form global symmetries

$$J^{(2)}_{AB} = (*\mathrm{Tr}(F \wedge F))_{AB}$$

So we can ask: are these 1-form global symmetries also present at the UV fixed point and do they form an interesting higher group?

Conformal Fixed Points

Suppose we look at models where the UV is an SCFT. There are dynamical tensor multiplets with bosonic fields a real scalar ρ and a two-form gauge field $b^{(2)}$. The action includes the couplings

$$S \supset \int d^6x \; \rho \mathrm{Tr}(|F|^2) + b^{AB} J_{AB}$$

So the 1-form symmetry is gauged and we expect that it disappears in the UV SCFT

In fact this can be demonstrated in a completely abstract way. There are no unitary representations of the superconformal group in 6d containing currents $J^{(2)}_{AB}$ [C-Dumitrescu-Intriligator]

Therefore in any 6d SCFT there are no (continuous) 1-form global symmetries and no higher-groups

Conformal Fixed Points

Absence of higher-group symmetries in 6d SCFTs sounds like a negative result but has interesting implications

We can fix the higher-group structure constants from the mixed anomaly terms involving flavor or R symmetries and gauge fields. Splitting into Green-Schwarz and matter contributions we find

$$0 = \mathcal{I}^{(8)}_{mixed,total} = \mathcal{I}^{(8)}_{mixed,GS} + \mathcal{I}^{(8)}_{mixed,matter}$$

Therefore we can use this to fix GS contributions purely from the IR matter content

Thus all 't Hooft anomalies of the UV SCFT are fixed by the IR moduli space gauge theory description. This matches an earlier proposal by [Ohmori-Shimizu-Tachikawa-Yonekura]

Conformal Fixed Points

We can use our new knowledge about the 't Hooft anomalies of these SCFTs to prove some general results

For instance using supersymmetry, we can determine all the the conformal anomalies of 6d SCFTs purely from their IR moduli space [C-Dumitrescu-Intriligator]. This means that e.g. that the 2 and 3-pt functions of T_{AB} at the fixed point are determined purely from the moduli space

We can also learn something about the a-type conformal anomaly that features in the a-theorem (still partly conjectural in 6d). Specifically, we can show that for all 6d SCFTs

$$a \geq 0$$

This dovetails nicely with the idea that a is a kind of measure of the degrees of freedom of the SCFT

Little String Theories

The situation for little string theories is different. For example consider the IIB (1,1) little strings that arise from the decoupling limit of NS5 branes

In this case the low-energy EFT is SYM with gauge group $SU(N)$. The instanton current $J^{(2)} = *\mathrm{Tr}(F \wedge F)$ measures the fundamental string charge so we expect it to be present along the entire RG flow

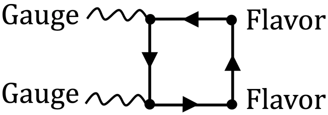
In particular, the background field $B^{(2)}$ that couples to $J^{(2)}$ can be interpreted as the decoupled NS-NS field

Since these theories have $SU(2) \times SU(2)$ R-symmetry it is natural to ask whether these symmetries are assembled into a higher-group

The answer of course is yes!

Little String Theories

Since there are no tensors in this example, there are no GS couplings. Therefore, we can read off the structure constants from box diagrams involving the IR fields



In this case we find terms

$$\mathcal{I}^{(8)}_{mixed} \sim N\left(c_2(R_1) - c_2(R_2)\right) c_2(F)$$

implying a non-trivial higher group and its Ward identities

This appears to be ubiquitous in LST. E.G. $SO(32)$ small instantons have a rich higher-group symmetry involving both the flavor, lorentz, and 1-form symmetry

Conclusions and Open Directions

Higher-group symmetry plays a prominent role in many QFTs, including 3d Chern-Simons-Matter theories and TQFTs, 4d gauge theories and models of axions, as well as SCFTs in d>4

Higher-group symmetry may also be a useful tool to organize little string theories

Many open directions to explore:

- Gauging higher-group symmetries (especially discrete groups) can lead to a rich class of TQFTs [Thorngren]
- In 4d there is a close connection between 2-group global symmetry and some aspects of the Callan-Rubakov effect. Work in progress [Brennan-C-Dumitrescu]

Thanks for Listening!