Celestial Holography

Strings2020 Capetown, South Africa

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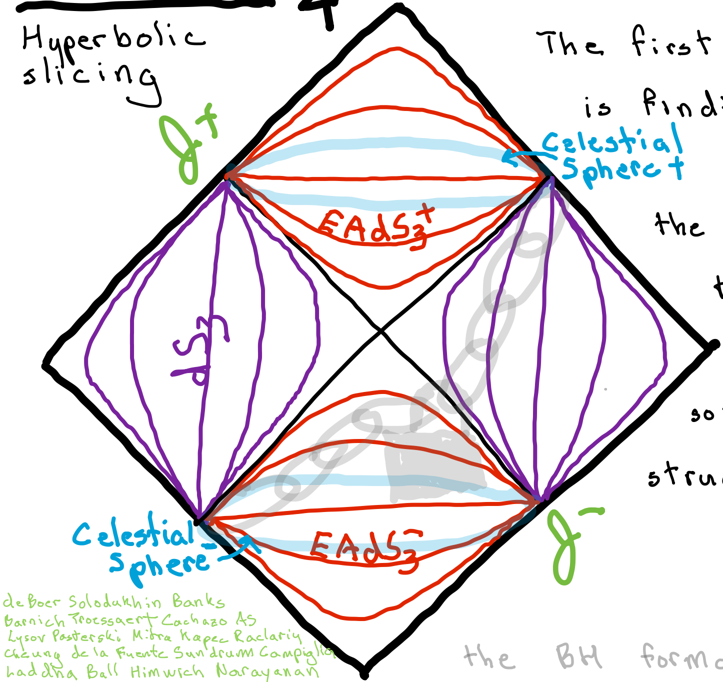
The black hole area-entropy law

$$S_{BH} = \frac{\text{Area (horizon)}}{4 G_N \hbar}$$

is the primeval holographic relationship: $N_{dof} \propto \partial V$
Understanding it for BHs with near-horizon AdS regions went hand-in-hand with understanding AdS/CFT holography.
One might guess that understanding black holes in flat space — where the information paradox takes its most striking form — will go hand-in-hand with understanding holography for quantum gravity in asymptotically flat spacetimes.

$\mathrm{Minkowski}_4$



deBoer Solodukhin Banks
Barnich Troessaert Cachazo AS
Lysov Pasterski Mitra Kapec Raclariu
Cheung de la Fuente Sundrum Campiglia
Laddha Ball Himwich Narayanan
....

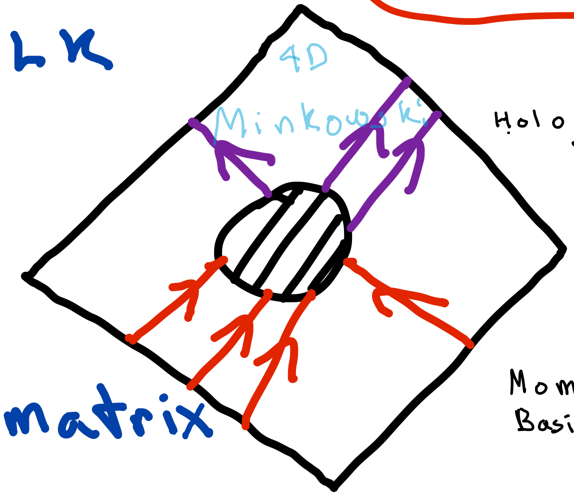
The first step in finding a holographically dual pair is finding the symmetries both sides obey. Thanks to the subleading soft graviton thm, we now know the ∞-dimensional 2D conformal symmetry is uplifted to the celestial sphere. But there are ∞-ly many more from supertranslations, subsubleading soft gravitons, subleading soft gluons... a rich & beautiful structure to explore.

These symmetries greatly constrain the BH formation/evaporation S-matrix as partially analyzed in Hawking Perry AS Zhiboedov Flanagan Nichols Bousso Porrati Semenoff Neunfeld......

c/f Raju

Basic Idea

BULK



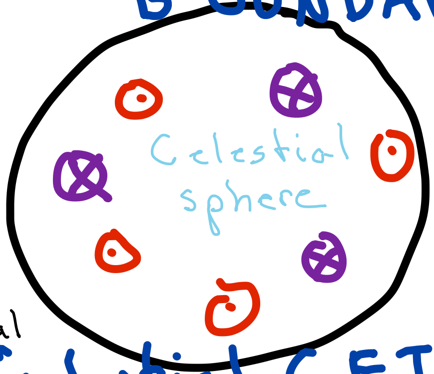
S-matrix

Holographic Duality

=

Momentum Basis $\Longleftrightarrow$ Conformal Basis

BOUNDARY



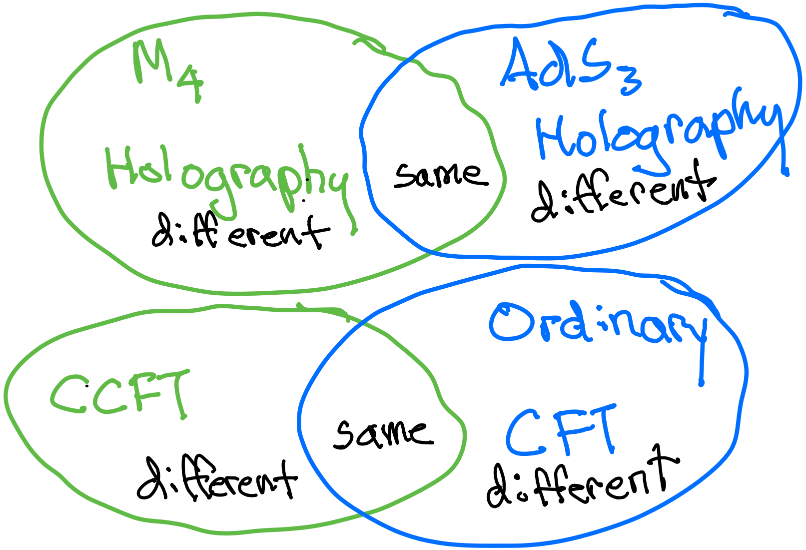
Celestial CFT correlator

$SL(2,\mathbb{C})$ Lorentz = global 2D conformal

Superrotations = local 2D conformal

Supertranslations $\mathcal{O}_{\Delta}(z,\bar{z}) \to \mathcal{O}_{\Delta+1}(z,\bar{z})$!

.... more ...



SO WHAT ARE THE PROPERTIES OF A CCFT?

So far, mostly BOTTOM→UP. LHS defines RHS.
c.f. Brown-Henneaux.

TOP→DOWN approach from string theory
not ruled out.

String WS CFT $\xrightarrow{\Delta \to \infty}$ CCFT

Stieberger
Taylor
c.f. Gopakumar talk

CHY WS CFT $\xrightarrow{?}$ CCFT

Intrinsic construction of a CCFT = Major Breakthrough!

Many string-like structures encountered.
Strings without strings?

CCFT

Two Virasoros
Current Algebras
Collinear Singularities

Continuous Complex
Highest Weight Reps
Single valued

$$S^\dagger S = 1$$

Spacetime crossing

∞-dim supertranslation
& subleading soft
symmetry

New!

USUAL CFT

Two Virasoros
Current Algebras
OPE Singularities

Discrete Highest
Weight Reps

Locality

???

???

No analog

Old!

A lot to understand

Interesting developments in the last years indicate a rich & fascinating structure. But a coherent overarching picture has not yet emerged - so I can't give you one! Instead I will share some tantalizing tidbits.....

For massless particles, the map between momentum-space and conformal basis amplitudes is a Mellin transform

$$\tilde{A}(\lambda_1, z_1, \ldots, \lambda_n, z_n) \equiv \int d\omega_1 \ldots d\omega_n \, \omega_1^{h_1+\bar{h}_1-1} \ldots \omega_n^{h_n+\bar{h}_n-1} A(p_1, \ldots, p_n)$$

$$= \langle \mathcal{O}_1(z_1) \ldots \mathcal{O}_n(z_n) \rangle_{CCFT}$$

$$h = \tfrac{1}{2}(1 + \overset{\text{spin}}{s} + i\lambda)$$

$$\bar{h} = \tfrac{1}{2}(1 - s + i\lambda)$$

$$z_k = \frac{p_k^1 + i p_k^2}{p_k^3 + p_k^0}$$

$\lambda \in \mathbb{R}$ (subtleties range, $\lambda = 0$) Donnay, Puhm, AS

‖ Pasterski & Shao

complete basis of states

Full CCFT operator content is known! $\Longleftrightarrow$ Spectrum of stable particles.

Conformal basis for massive particles involves bulk-to-boundary propagator on H_3.

$A(\lambda_1, z_1, \ldots \lambda_n, z_n)$ has been computed in a number of examples, sometimes via AdS-Witten diagrams:

① Scalars: massive, massless, 1-loop

Pasterski Shao AS Huang Cardona Lam

② All tree-level n-point YM

Pasterski Shao AS Cheung de la Fuente Sundrum Schreiber Volovich Zlotnik

Kinematic singularities @ 3 & 4 pt, 5+ are smooth.

MARGINALLY CONVERGENT

Good UV behavior "Baked-in"!

DEFINES

③ All n-point Einstein gravity

∞ !!!

④ String theory up to 4pt gauge + gravity

Stieberger Taylor

CONVERGENT

CONFORMALLY SOFT THEOREM

cf. Andrea

A conformally soft operator is one for which

Donnay Puhm AS

$$(h,\bar{h}) = (n,0) \quad (\text{or } (0,n))$$

It is not energetically soft! Nevertheless it has been recently shown in gauge theory

$$\lim_{\lambda_n \to 0} A(\lambda_1 z_1, \ldots, \lambda_n z_n) = -\frac{i}{\lambda_n} \sum_k \frac{1}{z_n - z_k} A(\lambda_1 z_1, \ldots, \lambda_k z_k, \ldots \lambda_{n-1} z_{n-1})$$

Fan Fotopoulos Taylor, Puhm

Pate Raclariu AS

using UV convergence & usual soft thm.

Expected in conformal picture, surprising in momentum space picture. Subleading soft graviton $\Rightarrow$ $(2,0)$ celestial stress tensor T_{ww} as a further exampl

SUPERTRANSLATIONS are more subtle. For gravity:

$$(h,\bar{h}) = \left(\frac{3}{2}+\frac{i\lambda}{2}, -\frac{1}{2}+\frac{i\lambda}{2}\right)$$

for $\lambda \to 0$

$$(h,\bar{h}) = \left(\frac{3}{2}, -\frac{1}{2}\right)$$

The associated operator P_z obeys

$$P_z(z)\,\mathcal{O}_{h,\bar{h}}(w) \sim \frac{1}{z-w}\,\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}} \;!$$

∞ constraints on amplitudes/OPEs correlators. Even global translations nontrivial. Illustration....

YM +++ OPE from asymptotic symmetries

M. Pate, A. Raclariu, A.S., E. Yuan

Fan Fotopoulos Stieberger Taylor

collinear singularity

$$\mathcal{O}^{+a}_{h_1}(z_1)\,\mathcal{O}^{+b}_{h_2}(z_2) \sim \frac{B(h_1,h_2)}{z_1-z_2} f^{abc}\mathcal{O}^{+c}_{h_1+h_2-1}(z_2)$$

Global translation invariance requires

$$B(h_1,h_2) = B(h_1+\tfrac{1}{2},h_2) + B(h_1,h_2+\tfrac{1}{2})$$

Subleading soft symmetry requires ($h_1=0$ pole)

$$(2h_1-3)B(h_1-\tfrac{1}{2},h_2) = (2h_2-3)B(h_1,h_2-\tfrac{1}{2})$$

These two recursion relations, together with $h \to \infty$ falloff, imply the OPE coefficient

$$B(h_1, h_2) = B(2h_1 - 2, 2h_2 - 2) \qquad B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

in agreement w/ direct Mellin transform of of collinear singularities. Fan Fotopoulos Taylor

In fact, all the leading graviton/gluon OPE/collinear sing are so-determined by symmetry! Full implications of constraints TBD

The mixed helicity gluon OPEs are

$$
\begin{aligned}
O^{+a,\epsilon}_{\Delta_1}(z_1,\bar z_1)O^{-b,\epsilon}_{\Delta_2}(z_2,\bar z_2) &\sim \frac{-if^{ab}{}_c}{z_{12}}\epsilon B(\Delta_1-1,\Delta_2+1)O^{-c,\epsilon}_{\Delta_1+\Delta_2-1}(z_2,\bar z_2) \\
&\quad + \frac{\kappa}{2}\frac{\bar z_{12}}{z_{12}}\delta^{ab}B(\Delta_1,\Delta_2+2)G^{-,\epsilon}_{\Delta_1+\Delta_2}(z_2,\bar z_2), \\
O^{+a,\epsilon}_{\Delta_1}(z_1,\bar z_1)O^{-b,-\epsilon}_{\Delta_2}(z_2,\bar z_2) &\sim \frac{-if^{ab}{}_c}{z_{12}}\epsilon\big[-B(\Delta_2+1,1-\Delta_1-\Delta_2)O^{-c,\epsilon}_{\Delta_1+\Delta_2-1}(z_2,\bar z_2) \\
&\quad +B(\Delta_1-1,1-\Delta_1-\Delta_2)O^{-c,-\epsilon}_{\Delta_1+\Delta_2-1}(z_2,\bar z_2)\big] \\
&\quad + \frac{\kappa}{2}\frac{\bar z_{12}}{z_{12}}\delta^{ab}\left[B(\Delta_2+2,-1-\Delta_1-\Delta_2)G^{-,\epsilon}_{\Delta_1+\Delta_2}(z_2,\bar z_2)\right. \\
&\quad \left.+B(\Delta_1,-1-\Delta_1-\Delta_2)G^{-,-\epsilon}_{\Delta_1+\Delta_2}(z_2,\bar z_2)\right].
\end{aligned}
\tag{106}
$$

The graviton OPEs are

$$
\begin{aligned}
G^{+,\epsilon}_{\Delta_1}(z_1,\bar z_1)G^{\pm,\epsilon}_{\Delta_2}(z_2,\bar z_2) &\sim -\frac{\kappa}{2}\frac{\bar z_{12}}{z_{12}}B(\Delta_1-1,\Delta_2+1\mp 2)G^{\pm,\epsilon}_{\Delta_1+\Delta_2}(z_2,\bar z_2), \\
G^{+,\epsilon}_{\Delta_1}(z_1,\bar z_1)G^{\pm,-\epsilon}_{\Delta_2}(z_2,\bar z_2) &\sim \frac{\kappa}{2}\frac{\bar z_{12}}{z_{12}}\left[B(\Delta_2+1\mp 2,1\pm 2-\Delta_1-\Delta_2)G^{\pm,\epsilon}_{\Delta_1+\Delta_2}(z_2,\bar z_2)\right. \\
&\quad \left.+B(\Delta_1-1,1\pm 2-\Delta_1-\Delta_2)G^{\pm,-\epsilon}_{\Delta_1+\Delta_2}(z_2,\bar z_2)\right].
\end{aligned}
\tag{107}
$$

The gluon-graviton OPEs are

$$
\begin{aligned}
G^{+,\epsilon}_{\Delta_1}(z_1,\bar z_1)O^{\pm a,\epsilon}_{\Delta_2}(z_2,\bar z_2) &\sim -\frac{\kappa}{2}\frac{\bar z_{12}}{z_{12}}B(\Delta_1-1,\Delta_2+1\mp 1)O^{\pm a,\epsilon}_{\Delta_1+\Delta_2}(z_2,\bar z_2), \\
G^{+,\epsilon}_{\Delta_1}(z_1,\bar z_1)O^{\pm a,-\epsilon}_{\Delta_2}(z_2,\bar z_2) &\sim -\frac{\kappa}{2}\frac{\bar z_{12}}{z_{12}}\left[B(\Delta_2+1\mp 1,1\pm 1-\Delta_1-\Delta_2)O^{\pm a,\epsilon}_{\Delta_1+\Delta_2}(z_2,\bar z_2)\right. \\
&\quad \left.-B(\Delta_1-1,1\pm 1-\Delta_1-\Delta_2)O^{\pm a,-\epsilon}_{\Delta_1+\Delta_2}(z_2,\bar z_2)\right].
\end{aligned}
\tag{108}
$$

From (8), we recall a factor of g_{YM} is absorbed in $f^{ab}{}_c$. The $\bar z_{12}\to 0$ celestial OPEs are obtained in a similar way by imposing the δ symmetry instead.

The presence of higher-dimension operators due to quantum, stringy or other corrections is expected to augment this list with the finite number of additions allowed by the general formula (13). A list of all possible corrections in theories with only gluons and gravitons is given in appendix B.

Acknowledgements

Pate, Raclariu, AS Yuan

1910.07424

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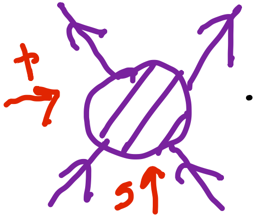
Analyticity, Factorization & IR Safety on the Celestial Sphere

Arkani-Hamed, Pate, Raclariu & AS, to appear

Start w/ tree 4-pt:



$$A(s,t) \xrightarrow{s\to\infty} e^{-S_{BH}} \sim e^{-s}$$ exponentially soft due to BHs

$$\underset{s\to 0}{\simeq} \sum_p s^p a_p(z) \; ; \; z = \frac{t}{s}$$ = conformal cross ratio

$\mathcal{L}_{eff}$, EFThedron coefficients

Arkani-Hamed, Huang & Huang

$$\tilde{A}(\Delta,z) = \int ds\, s^{\frac{\Delta}{2}-3} A(s,z) \sim \sum_p \frac{a_p(z)}{\Delta-4+2p} + \text{finite}$$

Celestial tree amplitudes are meromorphic in Δ with poles at even negative integers! Residues = coefficients of $\mathcal{L}_{eff}$/EFThedron with rich positivity constraints.

Quantum Theory

i) UV behavior still soft, divergences in $\tilde{A}(\Delta, z)$ all from IR – still meromorphic

ii) IR divergences w/ logs in exclusive $A(s,t) \sim \frac{s^2}{t} \ln \frac{s}{\Lambda_{IR}} + \ldots$ ← regulator required

iii) logs controlled by soft theorem. For QED/photons:

$$A(p_i, Q_i) = e^{-\Gamma \sum_{i<j} Q_i Q_j \ln(p_i \cdot p_j)} A_{hard} \; ; \quad \Gamma = \frac{e^2}{4\pi} \ln \frac{\Lambda_{UV}}{\Lambda_{IR}}$$ cusp anomalous dimension

iv) Transforming to celestial sphere

shift in dimension

$$\tilde{A}(z_i, \Delta_i) = \tilde{A}_{soft} \, \tilde{A}_{hard}(z_i, \Delta_i - \Gamma Q_i^2) \; ; \quad \tilde{A}_{soft} = \prod_{i<j} |z_i - z_j|^{-\Gamma Q_i Q_j}$$

reveals exact current algebra factorization

v) $\tilde{A}_{hard}$ are nothing but the IR safe scattering amplitudes w/ unique conformal primary Fadeev-Kulish dressings!!

SIM. FOR GRAVITY, WITH $Q_i \to p_i$, $\tilde{A}_{soft} \to$ operator!

In order to ask if BH S-matrix is unitary, must have an S-matrix!

MORE COOL STUFF!

Arkani-Hamed, Pate, Raclariu AS; Himwich, Narayanan, Nisarga, Pate, AS

To add to/echo Shiraz, Nima,...

What is the most general form of the graviton 4pt amplitude consistent with

0. Shiraz's constraints
1. Conformal invariance
2. Supertranslation invariance
3. Causality
4. Crossing symmetry
5. Subsubleading soft symmetries ?
6. .....

Does it look like string scattering?

CONCLUSION

The reformulation of Minkowski4 scattering as correlators on the celestial sphere is leading to novel insights. There is much to understand.


Thank you
for listening!