Automation of one-loop EW Sudakov logarithms

Mainly based on
Pagani, Zaro, *JHEP* 02 (2022) 161 (arXiv:2110.03714)

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SEZIONE DI BOLOGNA

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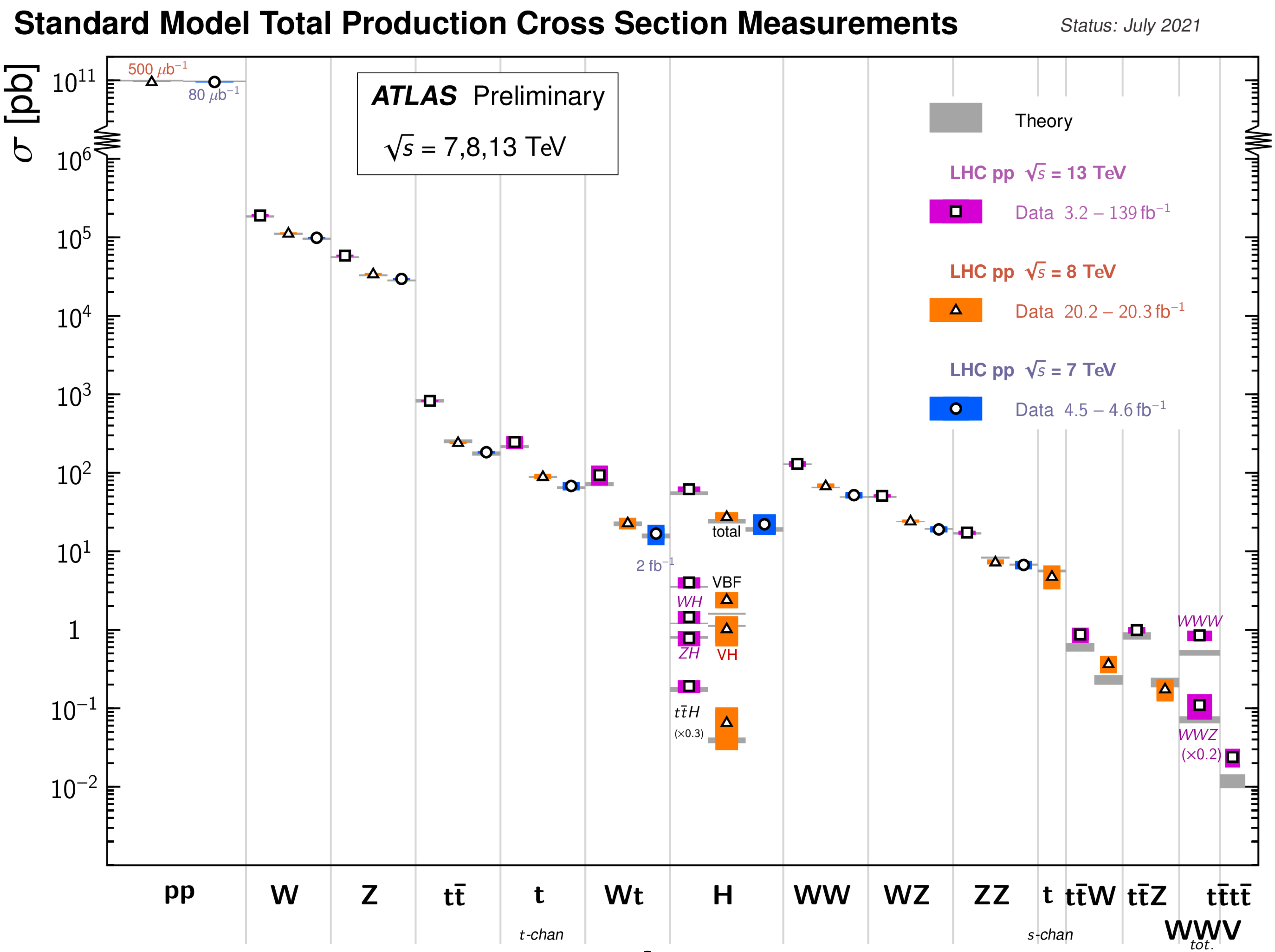
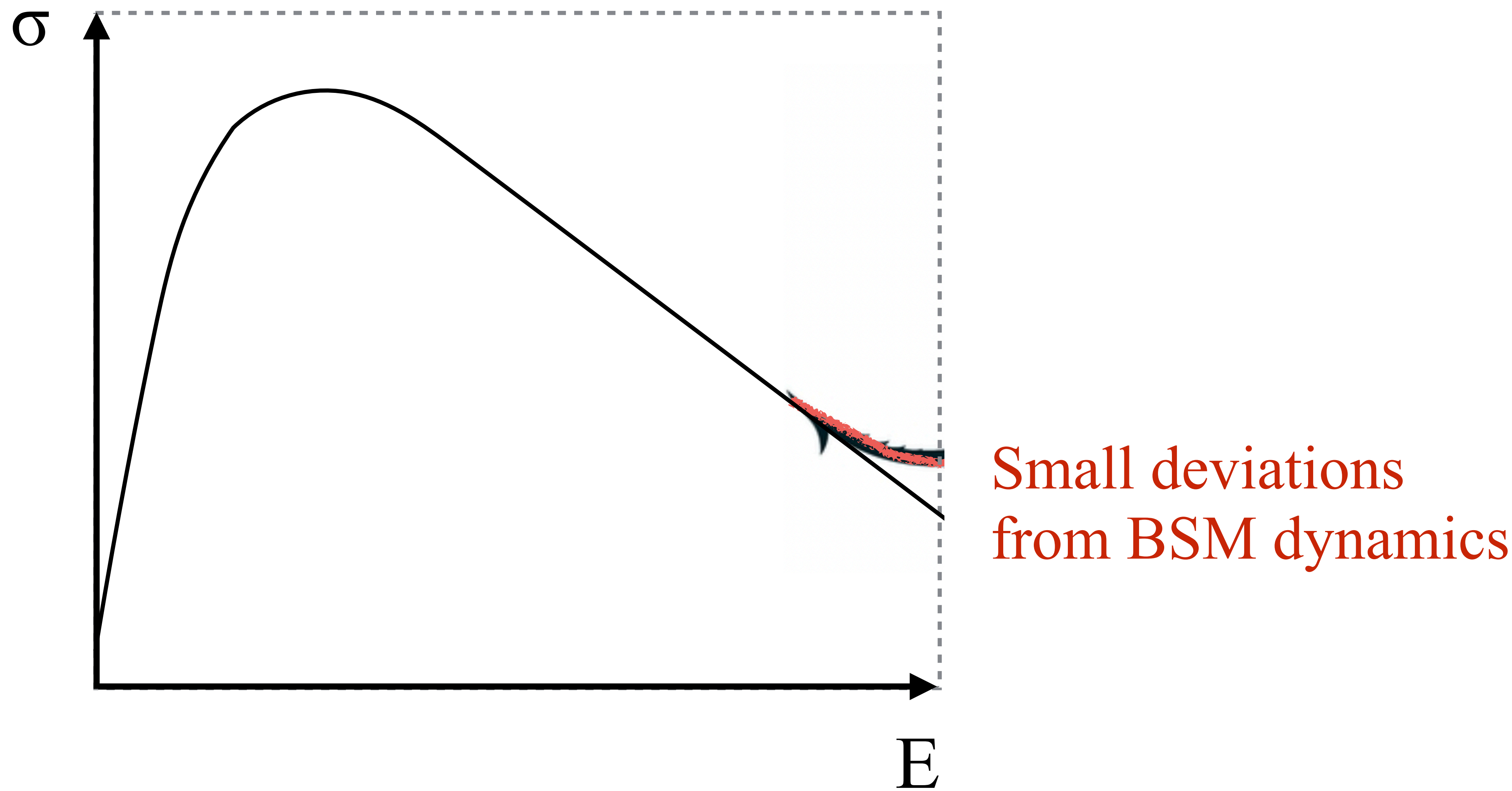
SM at the LHC (all good, too good)

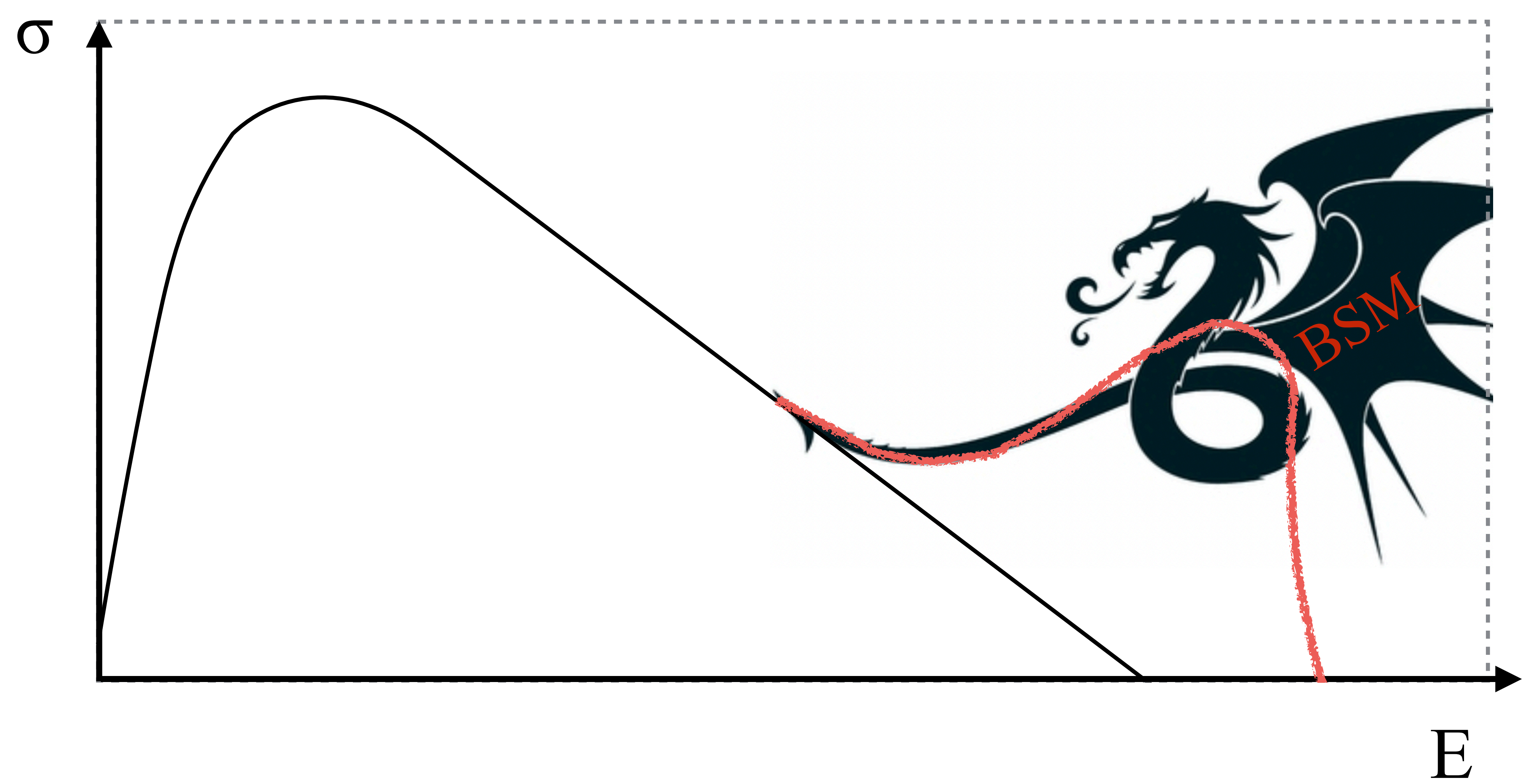


Differential distributions: let's look at the tails



With higher luminosity (and higher energy), at the LHC the accuracy of all measurements will in general increase, especially in the tail of distributions.

Differential distributions: let's look at the tails

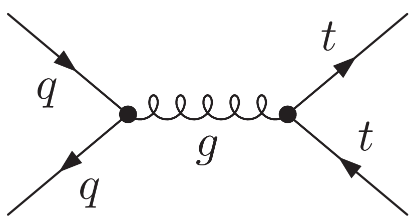


With higher luminosity (and higher energy), at the LHC the accuracy of all measurements will in general increase, especially in the tail of distributions.

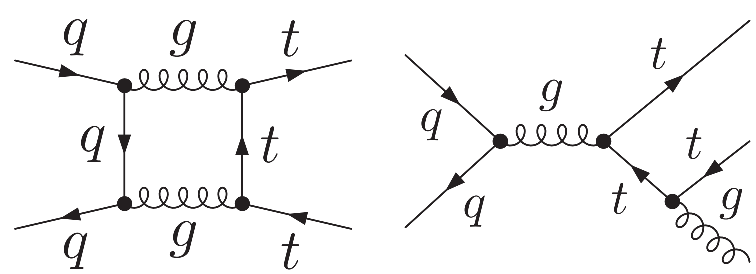
Precise predictions are necessary for the current and future measurements at the LHC. In order to match the experimental precision, NLO EW corrections and especially their Sudakov-logarithm components are essential.

Improving SM predictions: fixed-order calculations

In the SM, contributions to the partonic cross section can be organised according to the powers of α_s and α (number of loop corrections and real emissions).

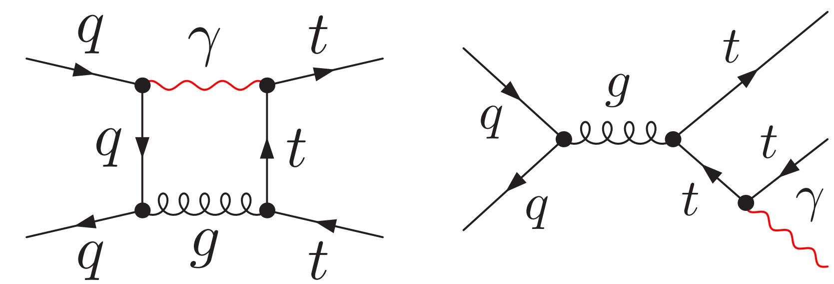


Born LO



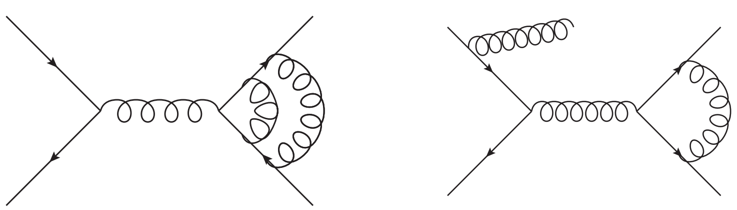
NLO QCD

$\mathcal{O}(\alpha_s)$ corrections



NLO EW

$\mathcal{O}(\alpha)$ corrections



NNLO QCD

$\mathcal{O}(\alpha_s^2)$ corrections

NNLO EW,

NNNLO QCD

.....

At the LHC, QCD is everywhere.
Nowadays, a "standard" prediction in the SM is at least at NLO QCD accuracy.

NNLO QCD is expected to be of the same order of NLO EW $\alpha_s^2 \sim \alpha$.

EW corrections are not flat. Large and negative for large pt ***(Sudakov logs)***. Moreover they in general involve all the SM masses and couplings.

NLO EW Automated results with MadGraph

Process	Syntax	Cross section (in pb) LO	Cross section (in pb) NLO	Correction (in %)
$pp \to e^+\nu_e$	`p p > e+ ve QCD=0 QED=2 [QED]`	$5.2498 \pm 0.0005 \cdot 10^{3}$	$5.2113 \pm 0.0006 \cdot 10^{3}$	-0.73 ± 0.01
$pp \to e^+\nu_e j$	`p p > e+ ve j QCD=1 QED=2 [QED]`	$9.1468 \pm 0.0012 \cdot 10^{2}$	$9.0449 \pm 0.0014 \cdot 10^{2}$	-1.11 ± 0.02
$pp \to e^+\nu_e jj$	`p p > e+ ve j j QCD=2 QED=2 [QED]`	$3.1562 \pm 0.0003 \cdot 10^{2}$	$3.0985 \pm 0.0005 \cdot 10^{2}$	-1.83 ± 0.02
$pp \to e^+e^-$	`p p > e+ e- QCD=0 QED=2 [QED]`	$7.5367 \pm 0.0008 \cdot 10^{2}$	$7.4997 \pm 0.0010 \cdot 10^{2}$	-0.49 ± 0.02
$pp \to e^+e^- j$	`p p > e+ e- j QCD=1 QED=2 [QED]`	$1.5059 \pm 0.0001 \cdot 10^{2}$	$1.4909 \pm 0.0002 \cdot 10^{2}$	-1.00 ± 0.02
$pp \to e^+e^- jj$	`p p > e+ e- j j QCD=2 QED=2 [QED]`	$5.1424 \pm 0.0004 \cdot 10^{1}$	$5.0410 \pm 0.0007 \cdot 10^{1}$	-1.97 ± 0.02
$pp \to e^+e^-\mu^+\mu^-$	`p p > e+ e- mu+ mu- QCD=0 QED=4 [QED]`	$1.2750 \pm 0.0000 \cdot 10^{-2}$	$1.2083 \pm 0.0001 \cdot 10^{-2}$	-5.23 ± 0.01
$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$	`p p > e+ ve mu- vm~ QCD=0 QED=4 [QED]`	$5.1144 \pm 0.0007 \cdot 10^{-1}$	$5.3019 \pm 0.0009 \cdot 10^{-1}$	$+3.67 \pm 0.02$
$pp \to He^+\nu_e$	`p p > h e+ ve QCD=0 QED=3 [QED]`	$6.7643 \pm 0.0001 \cdot 10^{-2}$	$6.4914 \pm 0.0012 \cdot 10^{-2}$	-4.03 ± 0.02
$pp \to He^+e^-$	`p p > h e+ e- QCD=0 QED=3 [QED]`	$1.4554 \pm 0.0001 \cdot 10^{-2}$	$1.3700 \pm 0.0002 \cdot 10^{-2}$	-5.87 ± 0.02
$pp \to Hjj$	`p p > h j j QCD=0 QED=3 [QED]`	$2.8268 \pm 0.0002 \cdot 10^{0}$	$2.7075 \pm 0.0003 \cdot 10^{0}$	-4.22 ± 0.01
$pp \to W^+W^-W^+$	`p p > w+ w- w+ QCD=0 QED=3 [QED]`	$8.2874 \pm 0.0004 \cdot 10^{-2}$	$8.8017 \pm 0.0012 \cdot 10^{-2}$	$+6.21 \pm 0.02$
$pp \to ZZW^+$	`p p > z z w+ QCD=0 QED=3 [QED]`	$1.9874 \pm 0.0001 \cdot 10^{-2}$	$2.0189 \pm 0.0003 \cdot 10^{-2}$	$+1.58 \pm 0.02$
$pp \to ZZZ$	`p p > z z z QCD=0 QED=3 [QED]`	$1.0761 \pm 0.0001 \cdot 10^{-2}$	$0.9741 \pm 0.0001 \cdot 10^{-2}$	-9.47 ± 0.02
$pp \to HZZ$	`p p > h z z QCD=0 QED=3 [QED]`	$2.1005 \pm 0.0003 \cdot 10^{-3}$	$1.9155 \pm 0.0003 \cdot 10^{-3}$	-8.81 ± 0.02
$pp \to HZW^+$	`p p > h z w+ QCD=0 QED=3 [QED]`	$2.4408 \pm 0.0000 \cdot 10^{-3}$	$2.4809 \pm 0.0005 \cdot 10^{-3}$	$+1.64 \pm 0.02$
$pp \to HHW^+$	`p p > h h w+ QCD=0 QED=3 [QED]`	$2.7827 \pm 0.0001 \cdot 10^{-4}$	$2.4259 \pm 0.0027 \cdot 10^{-4}$	-12.82 ± 0.10
$pp \to HHZ$	`p p > h h z QCD=0 QED=3 [QED]`	$2.6914 \pm 0.0003 \cdot 10^{-4}$	$2.3926 \pm 0.0003 \cdot 10^{-4}$	-11.10 ± 0.02
$pp \to t\bar{t}W^+$	`p p > t t~ w+ QCD=2 QED=1 [QED]`	$2.4119 \pm 0.0003 \cdot 10^{-1}$	$2.3025 \pm 0.0003 \cdot 10^{-1}$	-4.54 ± 0.02
$pp \to t\bar{t}Z$	`p p > t t~ z QCD=2 QED=1 [QED]`	$5.0456 \pm 0.0006 \cdot 10^{-1}$	$5.0033 \pm 0.0007 \cdot 10^{-1}$	-0.84 ± 0.02
$pp \to t\bar{t}H$	`p p > t t~ h QCD=2 QED=1 [QED]`	$3.4480 \pm 0.0004 \cdot 10^{-1}$	$3.5102 \pm 0.0005 \cdot 10^{-1}$	$+1.81 \pm 0.02$
$pp \to t\bar{t}j$	`p p > t t j QCD=3 QED=0 [QED]`	$3.0277 \pm 0.0003 \cdot 10^{2}$	$2.9683 \pm 0.0004 \cdot 10^{2}$	-1.96 ± 0.02
$pp \to jjj$	`p p > j j j QCD=3 QED=0 [QED]`	$7.9639 \pm 0.0010 \cdot 10^{6}$	$7.9472 \pm 0.0011 \cdot 10^{6}$	-0.21 ± 0.02
$pp \to tj$	`p p > t j QCD=0 QED=2 [QED]`	$1.0613 \pm 0.0001 \cdot 10^{2}$	$1.0539 \pm 0.0001 \cdot 10^{2}$	-0.70 ± 0.02

first time in the literature

Frederix, Frixione, Hirschi, DP, Shao, Zaro '18

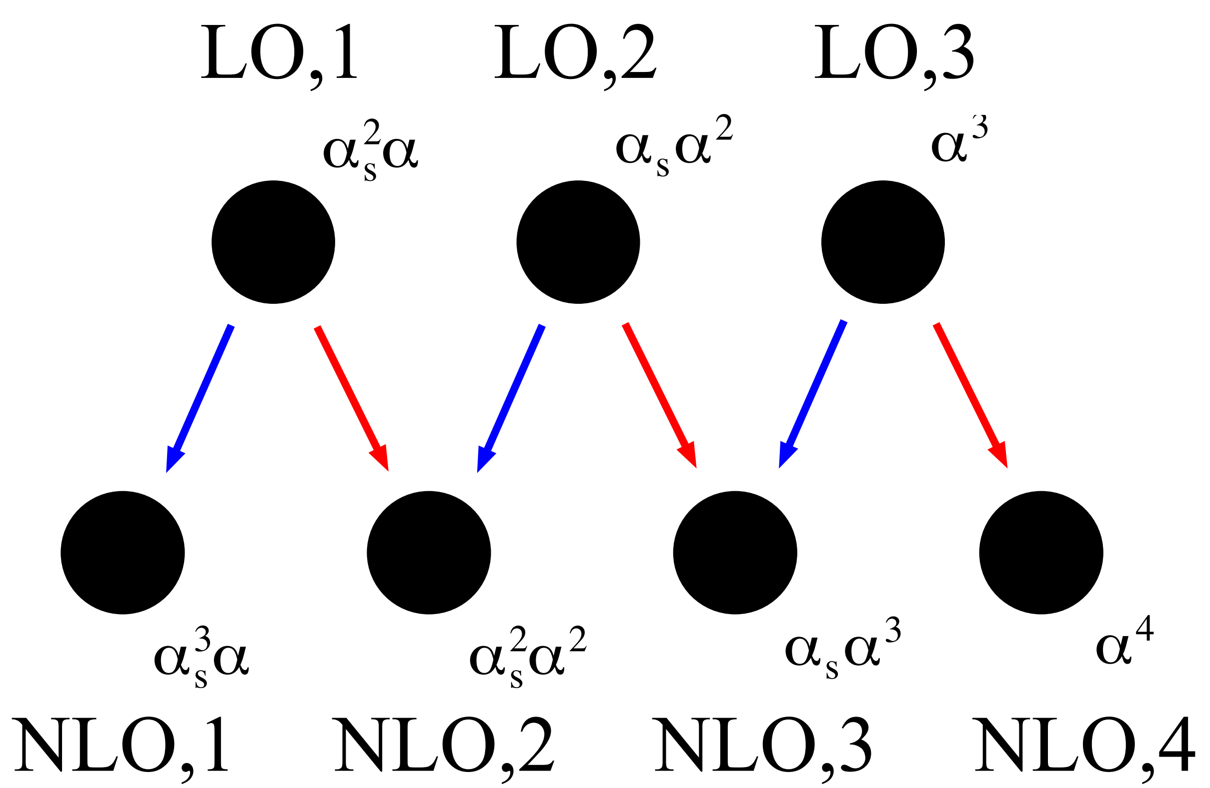
$$\delta_{\rm EW} = \frac{\Sigma_{\rm NLO_2}}{\Sigma_{\rm LO_1}} = \frac{\rm NLO}{\rm LO} - 1\,.$$

Structure of NLO EW-QCD corrections

$t\bar{t}H$

as example

All the LO,i and NLO,i can be calculated in a completely automated way. We denote the complete set of LO,i and NLO,i as **“Complete NLO”**.



NLO,1 = NLO QCD
NLO,2 = NLO EW

In general, NLO,3 and NLO,4 sizes are negligible, but there are exceptions.

Automated Results: Complete NLO

	$pp \to t\bar{t}$	$pp \to t\bar{t}Z$	$pp \to t\bar{t}W^+$	$pp \to t\bar{t}H$	$pp \to t\bar{t}j$
LO_1	$4.3803 \pm 0.0005 \cdot 10^2$ pb	$5.0463 \pm 0.0003 \cdot 10^{-1}$ pb	$2.4116 \pm 0.0001 \cdot 10^{-1}$ pb	$3.4483 \pm 0.0003 \cdot 10^{-1}$ pb	$3.0278 \pm 0.0003 \cdot 10^2$ pb
LO_2	$+0.405 \pm 0.001$ %	-0.691 ± 0.001 %	$+0.000 \pm 0.000$ %	$+0.406 \pm 0.001$ %	$+0.525 \pm 0.001$ %
LO_3	$+0.630 \pm 0.001$ %	$+2.259 \pm 0.001$ %	$+0.962 \pm 0.000$ %	$+0.702 \pm 0.001$ %	$+1.208 \pm 0.001$ %
LO_4					$+0.006 \pm 0.000$ %
NLO_1	$+46.164 \pm 0.022$ %	$+44.809 \pm 0.028$ %	$+49.504 \pm 0.015$ %	$+28.847 \pm 0.020$ %	$+26.571 \pm 0.063$ %
NLO_2	-1.075 ± 0.003 %	-0.846 ± 0.004 %	-4.541 ± 0.003 %	$+1.794 \pm 0.005$ %	-1.971 ± 0.022 %
NLO_3	$+0.552 \pm 0.002$ %	$+0.845 \pm 0.003$ %	$+12.242 \pm 0.014$ %	$+0.483 \pm 0.008$ %	$+0.292 \pm 0.007$ %
NLO_4	$+0.005 \pm 0.000$ %	-0.082 ± 0.000 %	$+0.017 \pm 0.003$ %	$+0.044 \pm 0.000$ %	$+0.009 \pm 0.000$ %
NLO_5					$+0.005 \pm 0.000$ %

$$\frac{\Sigma_{\mathrm{LO}_i}}{\Sigma_{\mathrm{LO}_1}}\,, \qquad i = 2,3,4\,,$$
$$\frac{\Sigma_{\mathrm{NLO}_i}}{\Sigma_{\mathrm{LO}_1}}\,, \qquad i = 1,\ldots 5\,;$$

NLO3 in ttW is ~12%:

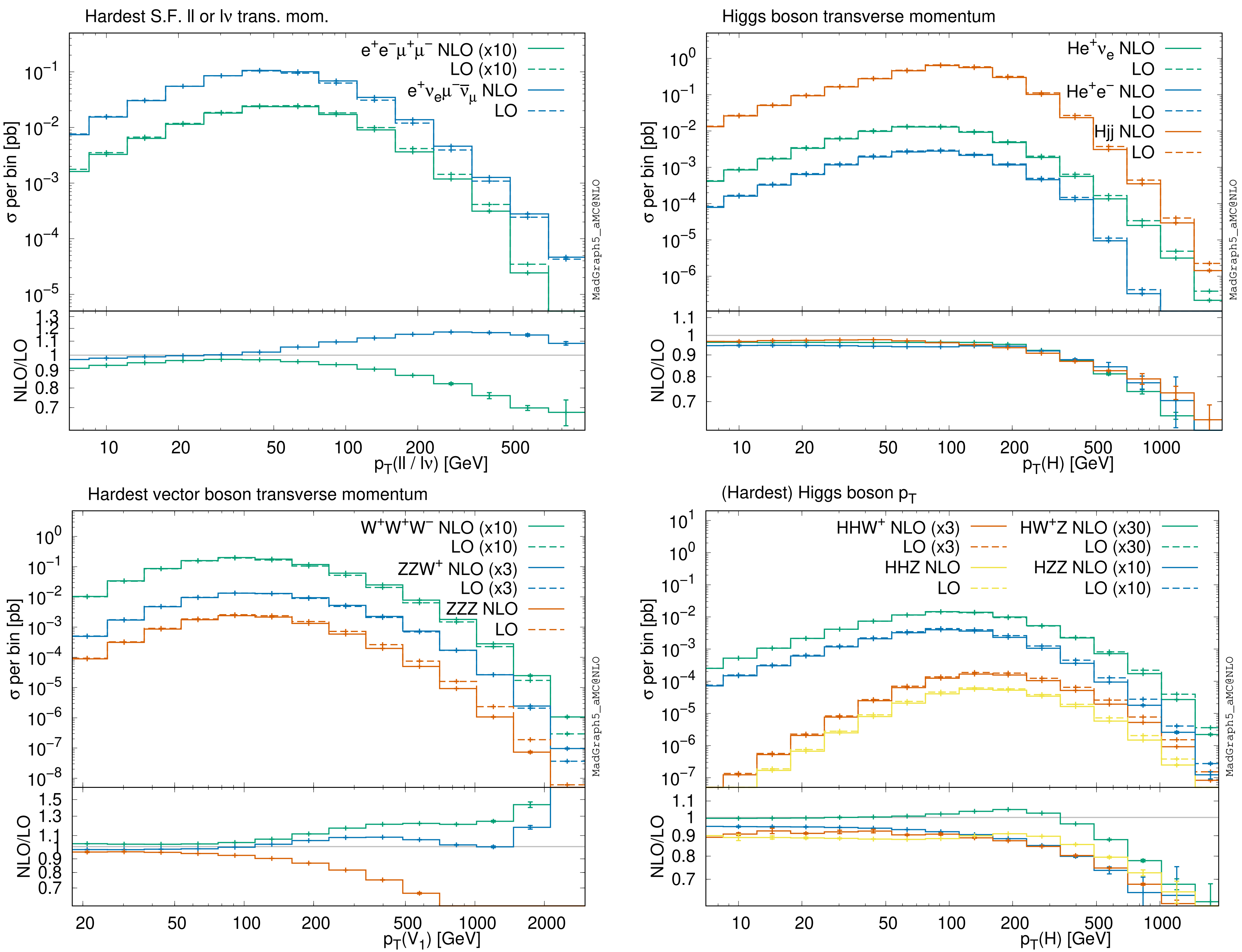
Frederix, Frixione, Hirschi, DP, Shao, Zaro ‘18

NLO_3 and Complete-NLO studied in detail in *Frederix, DP, Zaro ‘17* and several subsequent publications.

Other processes show this kind of enhancements (VBS: see talk of Mathieu Pellen), but they are exceptions.

Differential distributions: Sudakov enhancements

Sudakov enhancements are NOT exceptions and involve at NLO corrections of order $-\alpha \log^k(s/m_W^2)$ with $k = 1{,}2$.



Frederix, Frixione, Hirschi, DP, Shao, Zaro '18

What are EW Sudakov logarithms?

QCD: virtual and real terms are separately IR divergent ($1/\epsilon$ poles). In physical cross sections the contributions are combined and poles cancel.

QED: same story, but I can also regularise IR divergencies via a photon-mass λ. So $1/\epsilon$ poles $\to \log(Q^2/\lambda^2)$, where Q is a generic scale.

EW: with weak interactions $\lambda \to m_W, m_Z$ and W and Z radiation are typically not taken into account, which is anyway IR-safe.

Therefore, at high energies EW loops induce corrections of order

$$-\alpha^k \log^n(s/m_W^2)$$

where k is the number of loops and $n \leq 2k$. These logs are physical. Even including the real radiation of W and Z, there is not the full cancellation of this kind of logarithms.

Previous works

EW Sudakov logarithms have already been extensively studied in the literature.

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+ other ~ 30 works

Automation

However, **only two tools** are able to compute them for a generic SM process. Both have appeared in the literature in the last two years.

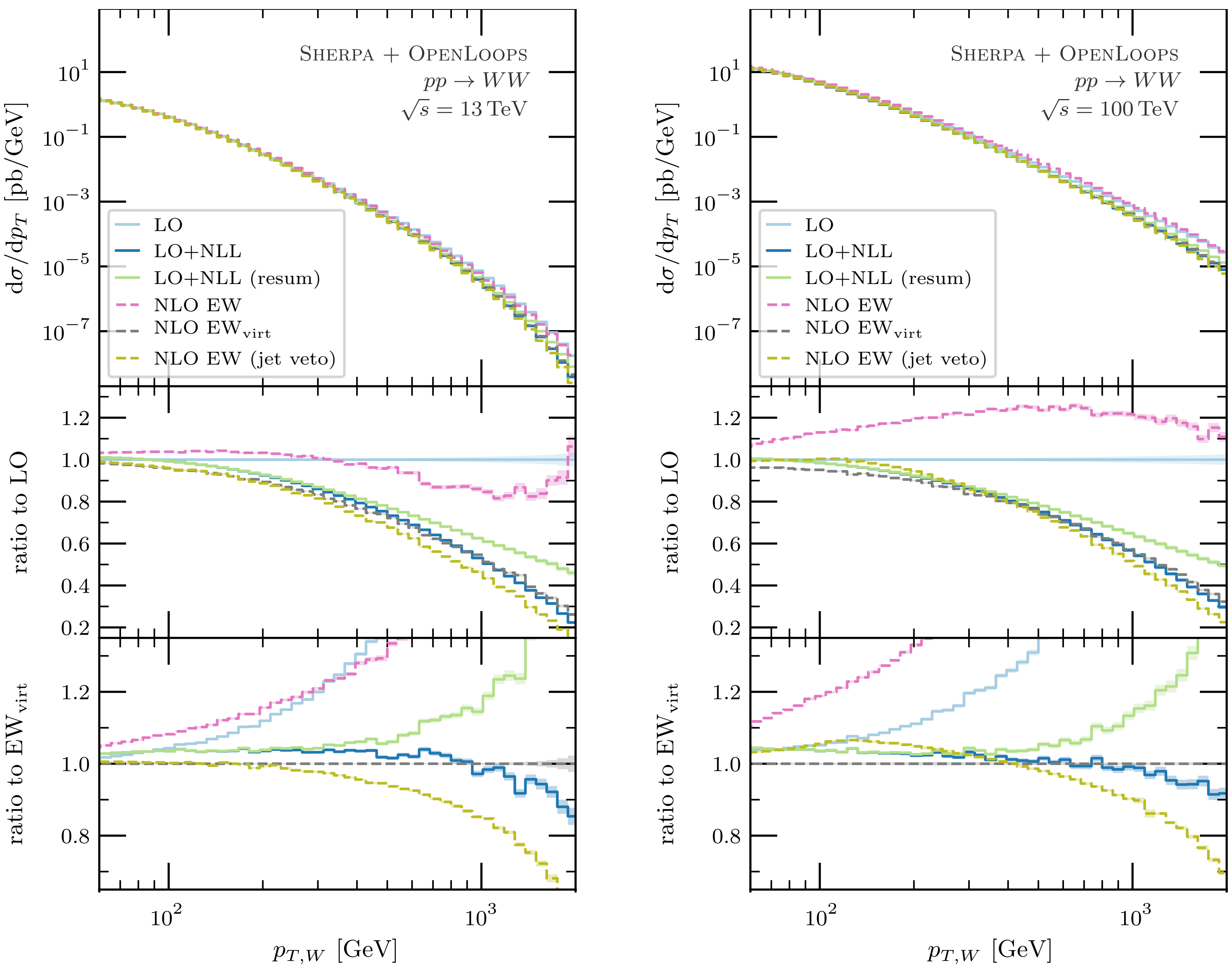
- **SHERPA** (*Bothmann, Napoletano ’20*)
- **MadGraph5_aMC@NLO** (*DP, Zaro ‘21*)

For specific classes of processes this has been done in the past also in **ALPGEN** (*Chiesa et al. ’13*).

All these works are based on the **Denner&Pozzorini algorithm** for one-loop EW Sudakov corrections (*Denner, Pozzorini **’01***).

SHERPA

The **Denner&Pozzorini algorithm** for one-loop EW Sudakov corrections has been implemented in its original form.



Comparisons with **SHERPA + OpenLoops** results at NLO EW accuracy have been performed.

Sudakov approximation works **quite well** for WW when the **jet veto** is applied.

Resummation of EW Sudakov logarithms is **essential** for precision.

Bothmann, Napoletano '20

already discussed in this conference …

Multi-jet merging

Higher-order EW corrections in ZZ and ZZj production at the LHC

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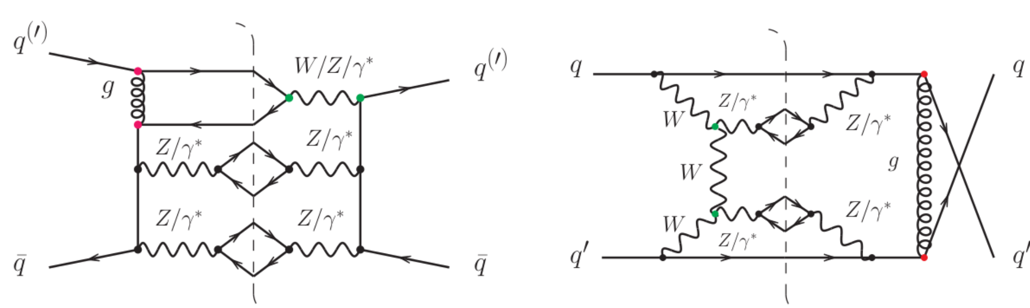
Motivation: include EW corrections along with QCD corrections

- Non trivial for QCD multi-merged samples
  → Extension of [Bräuer, Denner, MP, Schönherr, Schumann; 2005.12128] for WWj
- Complicated task due to interferences
- CPU costly for high multiplicity



Automation of EW Sudakov logarithms in MadGraph5_aMC@NLO

DP, Zaro '21

Why automate Sudakov in Madgraph5_aMC@NLO?

NLO EW corrections already fully include one-loop EW Sudakov logarithms $(n = 1,\ k = 1{,}2)$, why automate them?

- They can be calculated **analytically via tree-level** amplitudes only. They are a very good approximation of NLO EW at high energy and they can be computed much **faster**. No cancellations among virtual and real, so very stable results.

- When **NLO EW** becomes **large and negative**, **Sudakov logarithms** have to be **resummed**. Having in one tool separately the exact NLO EW and its Sudakov component will allow **matching of NLO EW and EW LL resummed.**

- They **depend only on** properties of the **external particles**: masses, momenta, ***helicities***, charges, SU(2) components, hypercharges. The **generalisation to** the **BSM** case is therefore much **easier** than the NLO EW case.

Our revisitation and automation: Amplitude level

We have **revisited** and **automated** in aMG5 the **Denner&Pozzorini algorithm** for the evaluation of one-loop EW Sudakov corrections to amplitudes (*Denner, Pozzorini '01*). In particular we have introduced the **following novelties**.

- **IR QED** divergencies are dealt with **via D**imensional **R**egularisation, with strictly massless photons and light fermions.

- **Additional logarithms** that involve ratios between invariants, and therefore **angular** dependences, are taken into account.

- We correctly take into account an **imaginary term** that was **previously omitted** in the literature. Relevant for $2 \to n$ processes with $n > 2$

- Moving to the level of interferences of tree and one-loop amplitudes, we take into account NLO EW contributions originating from **QCD loops on top of subleading LO terms**.

Master formula (Denner&Pozzorini)

Born amplitude: $\mathcal{M}_0^{i_1 \dots i_n}(p_1, \dots, p_n)$

One-loop EW Sudakov corrections: $\delta \mathcal{M}^{i_1 \dots i_n}(p_1, \dots, p_n) = \mathcal{M}_0^{i'_1 \dots i'_n}(p_1, \dots, p_n) \delta_{i'_1 i_1 \dots i'_n i_n}$

other tree-level amplitudes

the logs

$$\delta = \delta^{\mathrm{LSC}} + \delta^{\mathrm{SSC}} + \delta^{\mathrm{C}} + \delta^{\mathrm{PR}}$$

eikonal approximation of soft EW boson exchange ($\delta^{\mathrm{LSC}} + \delta^{\mathrm{SSC}}$)

Leading Soft-Collinear: It depends only on s and it is the only term involving double logarithms.

Subleading Soft-Collinear: The only one involving ratios of s with other invariants and also angular dependences.

Collinear / **Parameter renormalis.:** In an on-shell scheme, the dependence on the UV regularisation scale cancels. No μ_r dependence is left.

The logs inside the δ^i have always the form.

$$L(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi} \log^2 \frac{|r_{kl}|}{M^2}$$

$$l(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi} \log \frac{|r_{kl}|}{M^2}$$

$$M = M_W, M_Z, m_f, \lambda, \dots$$
$$r_{kl} \equiv (p_k + p_l)^2$$

ASSUMPTIONS:

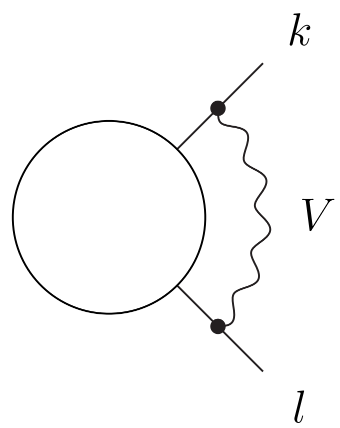
$$r_{kl} \equiv (p_k + p_l)^2 \simeq 2 p_k p_l \gg M_W^2 \simeq M_H^2, m_t^2, M_W^2, M_Z^2$$

the high-energy limit

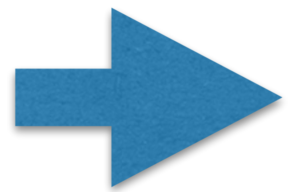
$$r_{kl}/r_{k'l'} \simeq 1$$

All invariants $\simeq s$. Reasonable, but $r_{kl} = s$ is impossible.

Derivation of LSC and SSC



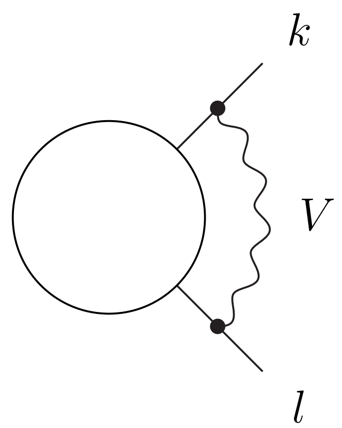
Denner&Pozzorini



$$L(|r_{kl}|, M^2) = L(s, M^2) + 2l(s, M^2) \log \frac{|r_{kl}|}{s} + L(|r_{kl}|, s)$$

$$L(s) \equiv L(s, M_W^2) \qquad \text{and} \qquad l(s) \equiv l(s, M_W^2)$$

Derivation of LSC and SSC



Denner&Pozzorini

$$L(|r_{kl}|, M^2) = L(s, M^2) + 2l(s, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s)$$

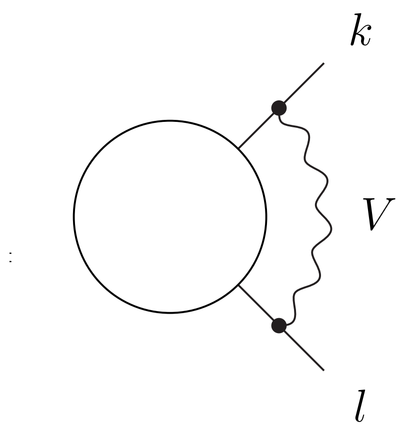
$$= \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\text{LSC}} + \underbrace{2l(s)\log\frac{|r_{kl}|}{s}}_{\text{SSC}} + \cdots$$

LSC **SSC**

$$L(s) \equiv L(s, M_W^2) \quad \text{and} \quad l(s) \equiv l(s, M_W^2)$$

The relation $r_{kl} = r_{k'l'} = s$ is used in all logs, unless they multiply $l(s)$.

Derivation of LSC and SSC



Denner&Pozzorini

$$L(|r_{kl}|, M^2) = L(s, M^2) + 2l(s, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s)$$

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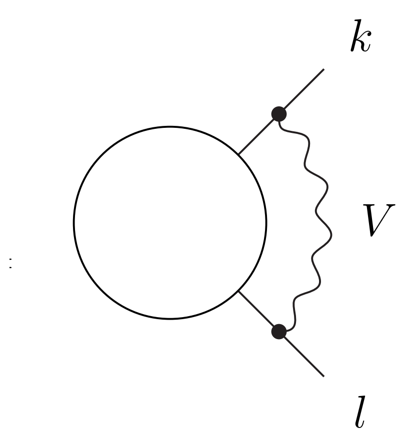


Our approach:

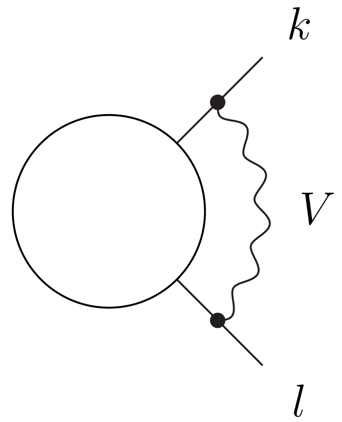
$$C_0(p_k, p_l, M, M_k, M_l) \quad \longrightarrow \quad L(|r_{kl}|, M^2) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2)$$

Previously omitted imaginary term

$$L(|r_{kl}|, M^2) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2) =$$

$$= L(s, M^2) + 2l(s, M^2)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right) + L(|r_{kl}|, s) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, s)$$

Derivation of LSC and SSC



Denner&Pozzorini

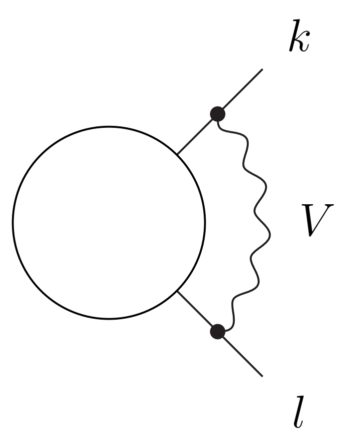
$$L(|r_{kl}|, M^2) = L(s, M^2) + 2l(s, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s)$$

$$= \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\text{LSC}} + \underbrace{2l(s)\log\frac{|r_{kl}|}{s}}_{\text{SSC}} + \cdots$$

LSC **SSC**

$$L(s) \equiv L(s, M_W^2) \quad \text{and} \quad l(s) \equiv l(s, M_W^2)$$

The relation $r_{kl} = r_{k'l'} = s$ is used in all logs, unless they multiply $l(s)$.



Our approach: ~~$r_{kl} = r_{k'l'} = s$~~ **in the expressions**

$$C_0(p_k, p_l, M, M_k, M_l) \Rightarrow L(|r_{kl}|, M^2) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2)$$

Previously omitted imaginary term

$$L(|r_{kl}|, M^2) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2) =$$

$$= L(s, M^2) + 2l(s, M^2)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right) + L(|r_{kl}|, s) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, s) =$$

$$= \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\text{LSC}} + \underbrace{2l(s)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right)}_{\text{SSC}} +$$

$$\underbrace{2l(M_W^2, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, s)}_{\mathrm{SSC}^{s\to r_{kl}}} + \cdots$$

The conceptual derivation relies on the assumption $s = r_{kl}$, but is not actually used in the expressions. Therefore, further angular dependencies are taken into account.

New angular dependences via ratios among invariants

Implementation

Born amplitude: $\mathcal{M}_0^{i_1\ldots i_n}(p_1,\ldots,p_n)$

One-loop EW Sudakov corrections: $\delta\mathcal{M}^{i_1\ldots i_n}(p_1,\ldots,p_n) = \mathcal{M}_0^{i'_1\ldots i'_n}(p_1,\ldots,p_n)\delta_{i'_1 i_1 \ldots i'_n i_n}$

other tree-level amplitudes — the logs

Born process: $\varphi_{i_1}(p_1)\ldots\varphi_{i_n}(p_n) \to 0$

$$Z \longrightarrow \chi\,,$$
$$W^\pm \longrightarrow \phi^\pm\,,$$

GBE theorem for longitudinal W and Z bosons.

$$\varphi_{i_1}(p_1)\ldots\varphi_{i'_k}\ldots\varphi_{i_n}(p_n) \to 0\,,$$

$$Z \longleftrightarrow A\,,$$
$$H \longleftrightarrow \chi\,.$$

Relevant for LSC and C contributions.

Amplitudes with one or 2 different external particles w.r.t. the Born have to be generated.

$$\varphi_{i_1}(p_1)\ldots\varphi_{i'_k}\ldots\varphi_{i'_l}\ldots\varphi_{i_n}(p_n) \to 0$$

$$f_\sigma \longleftrightarrow f_{-\sigma}\,,$$
$$H \longleftrightarrow \phi^\pm\,,$$
$$\chi \longleftrightarrow \phi^\pm\,,$$
$$A \longleftrightarrow W^\pm\,,$$
$$Z \longleftrightarrow W^\pm\,.$$

Relevant for SSC charged contributions.

one-loop EW virtual corrections $\mathcal{O}(\alpha)$

=

α **[Sudakov Logs** $\mathcal{O}(-\log^k(s/m_W^2), k = 1,2)$ **+**

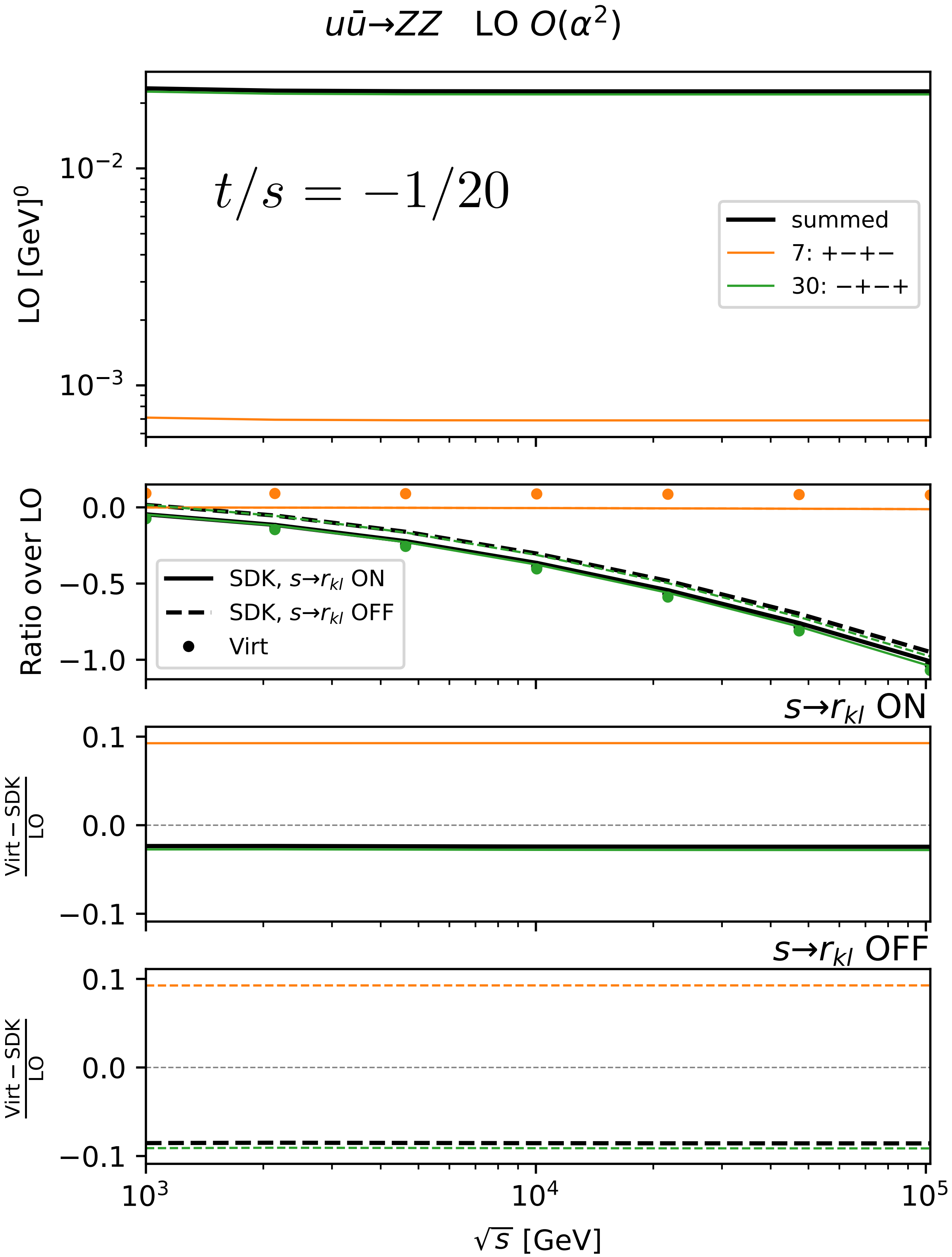
constant term $\mathcal{O}(1)$ **+**

mass-suppressed terms $\mathcal{O}(m_W^2/s)$**]**

We can **validate** the new implementation, and check the **relevance of the novelties** introduced **comparing** the results with the **MadLoop** output.

We consider only the **finite part of** the **MadLoop** output setting $Q^2 = s$. Due to the QED component, the **virtual** contribution **is** total **IR divergent** and therefore **not physical**.

Example $(2 \to 2)$: $u\bar{u} \to ZZ$ scan in s



Denner&Pozzorini algorithm works only with non mass-suppressed LO processes: we select only helicity configurations > 10^(-3) of the dominant one.

Dots: NLO EW (MadLoop). **Lines =** Sudakov.
Dashed: standard approach.
Solid: our formulation (more angular information)

Dots-Solid/LO: horizontal, the correct Log dependence is captured.

Dots-Dashed/LO: horizontal, the correct Log dependence is captured.

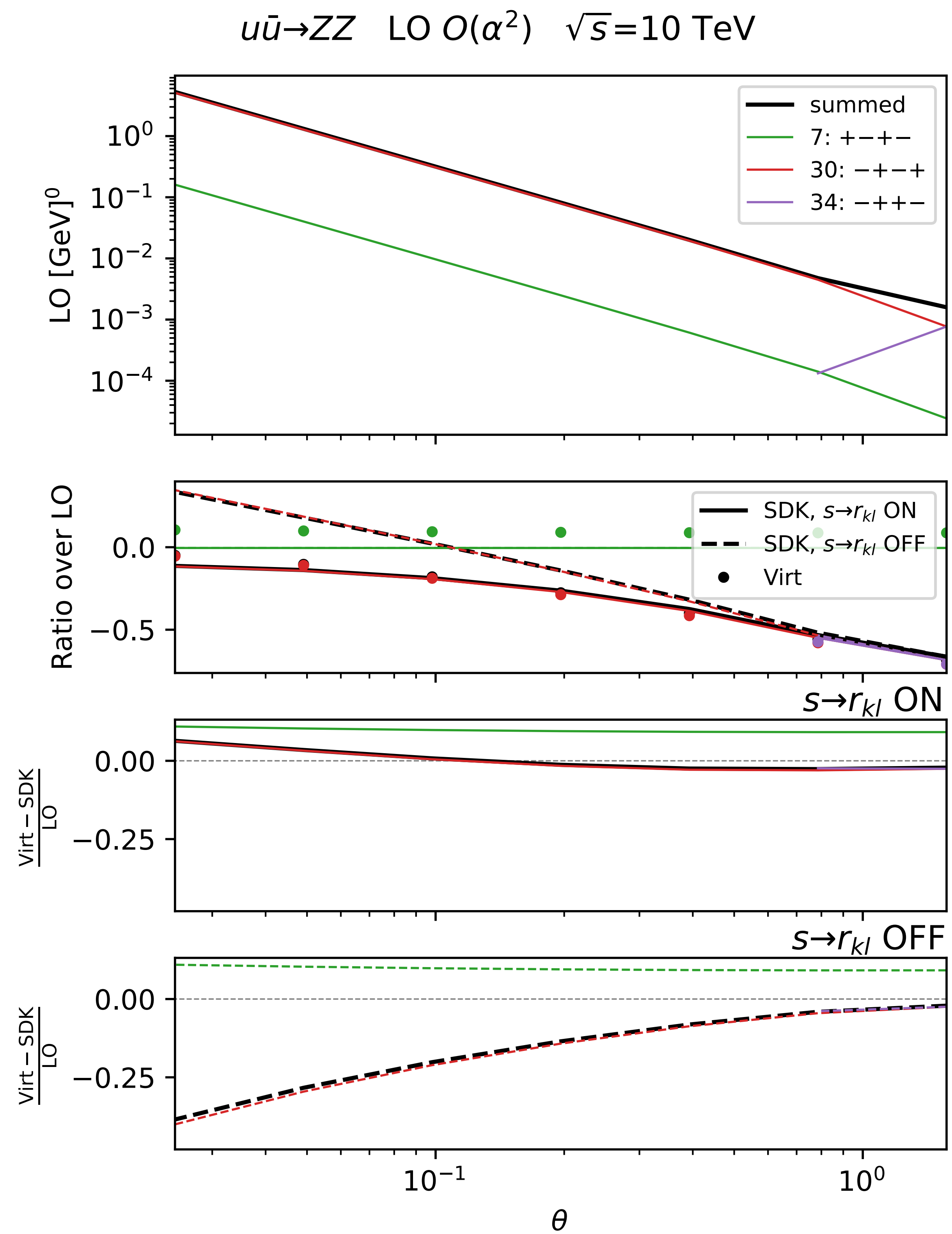
Example ($2 \to 2$): $u\bar{u} \to ZZ$ scan in θ

Denner&Pozzorini algorithm works only with non mass-suppressed LO processes: we select only helicity configurations > 10^(-3) of the dominant one.

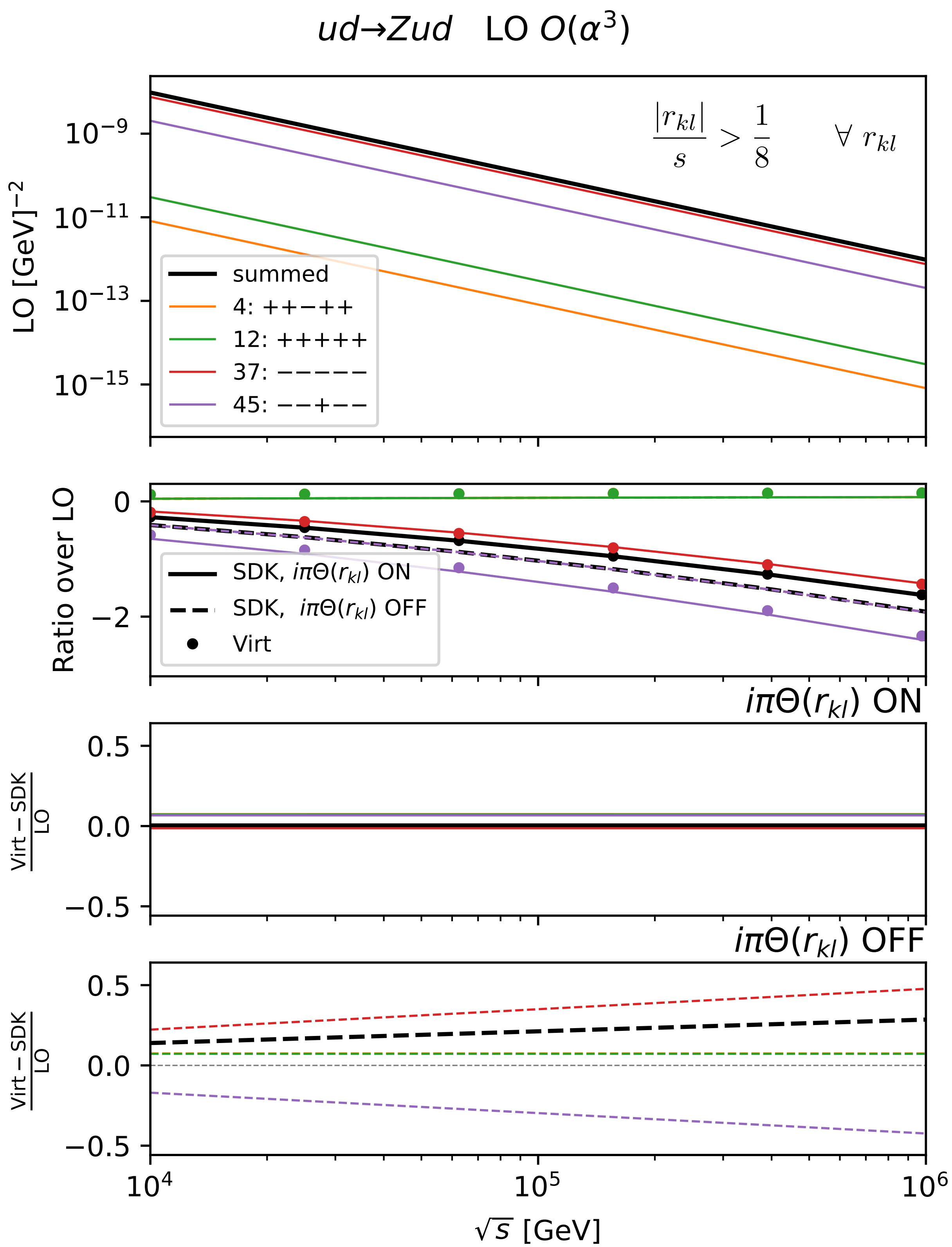
Dots: NLO EW (MadLoop). **Lines =** Sudakov.
Dashed: standard approach.
Solid: our formulation (more angular information)

Dots-Solid/LO: quite horizontal, the correct Log dependence is very-well approximated.

Dots-Dashed/LO: not horizontal, the correct Log dependence is lost.



Example $(2 \to 3)$: Imaginary term and scan in s



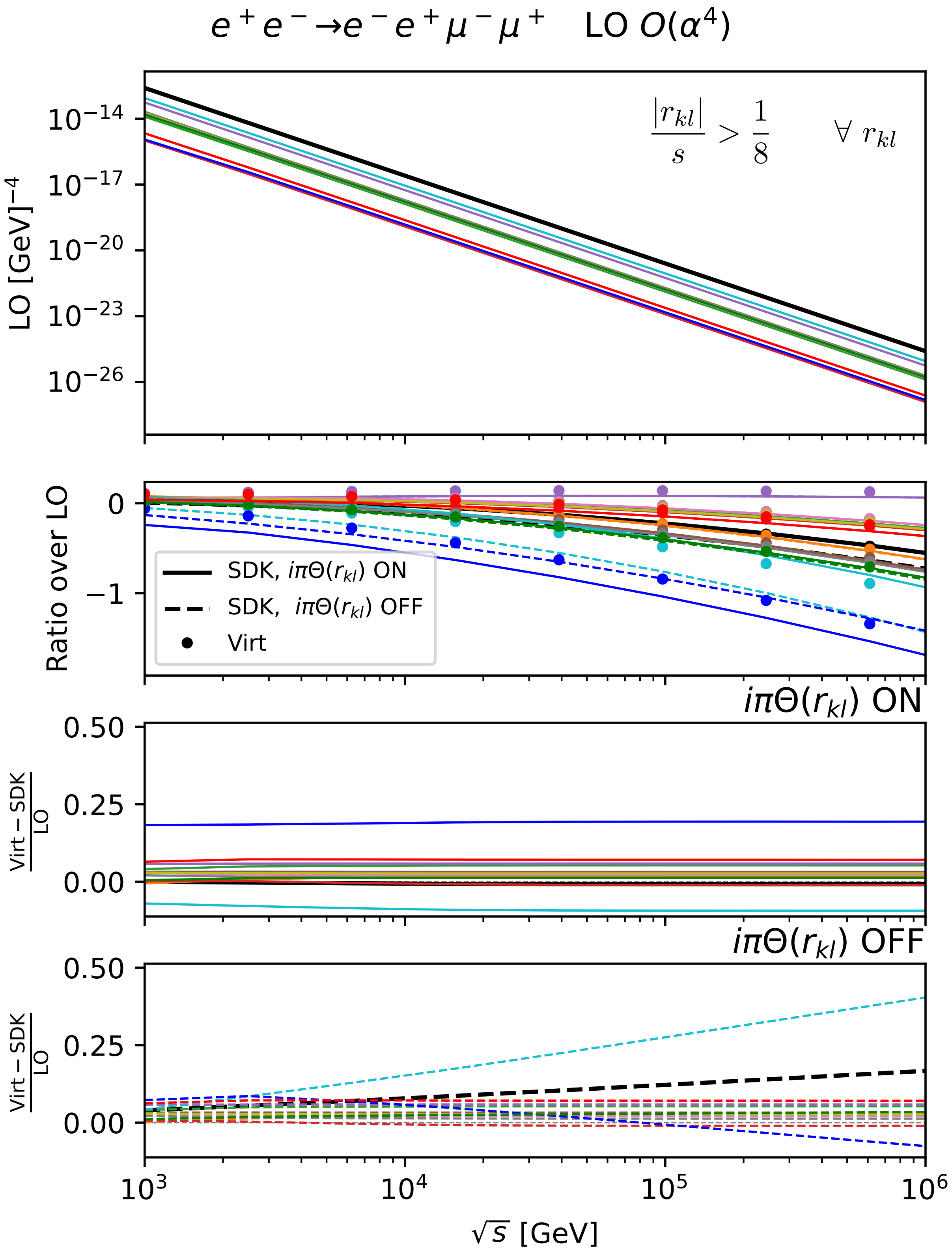
Denner&Pozzorini algorithm works only with non mass-suppressed LO processes: we select only helicity configurations > 10^(-3) of the dominant one.

Dots: NLO EW (MadLoop). **Lines =** Sudakov.
Dashed: standard approach, $i\pi\Theta(r_{kl})$ omitted
Solid: our formulation, $i\pi\Theta(r_{kl})$ included

Dots-Solid/LO: horizontal, the correct Log dependence is captured.

Dots-Dashed/LO: not horizontal, the correct Log dependence is lost.

Example $(2 \to 4)$: Imaginary term and scan in s



Denner&Pozzorini algorithm works only with non mass-suppressed LO processes: we select only helicity configurations > 10^(-3) of the dominant one.

Dots: NLO EW (MadLoop). **Lines =** Sudakov.
Dashed: standard approach, $i\pi\Theta(r_{kl})$ omitted
Solid: our formulation, $i\pi\Theta(r_{kl})$ included

Dots-Solid/LO: horizontal, the correct Log dependence is captured.

Dots-Dashed/LO: not horizontal, the correct Log dependence is lost.

Moving from IR-divergent virtual contributions to physical cross sections

The QED part is IR divergent, regardless if it is regularised in DR of with a fictitious λ mass for the photon.

Cross-sections: our approach.

FOR WHAT WILL EW SUDAKOV BE USEFUL?
For providing a very **good approximation of NLO EW** in the **high-energy** limit.

HOW SHOULD ONE PERFORM THE CALCULATION IN THE HIGH-ENERGY LIMIT?
Photons have to be **always clustered with massless charged particle for IR**-safety reasons. But from an experimental point of view, **at high energy also clustering tops and W bosons with photons** is very reasonable, either if you imagine to tag heavy object directly or via their massless decay products.

The **QED Logs,** involving s and λ^2 (or Q^2), **cancel against their real-emission counterparts and PDF counterterms**. The only one surviving are those from tops in vacuum polarisation for external (not tagged) photons, both in the initial and final state:

$$\mathrm{SDK}_{\mathrm{weak}}$$

Almost all the contributions of QED are removed (e.g. $C_{\mathrm{EW}}(k) \to C_{\mathrm{EW}}(k) - Q_k^2$, $Q_k^2 = 0$), but NOT in the parameter renormalisation δ^{PR}.

The same applies for the case QCD contributions to NLO EW ($\delta_{\mathrm{LA}}^{\mathrm{QCD}}$ multiplying the LO,2).

Cross-sections: standard approach in the literature

SDK_0

Two examples: LSC and C for fermions

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}_{i'_k i_k}(k) L(s) - 2(I^Z(k))^2_{i'_k i_k} \log\frac{M_Z^2}{M_W^2}\, l(s) + \cancel{\delta_{i'_k i_k} Q_k^2 L^{\mathrm{em}}(s, \lambda^2, m_k^2)} \right]$$

Casimir for the entire $SU(2)_L \times U(1)_B$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2} C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8 s_{\mathrm{w}}^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m^2_{f_\sigma}}{M_{\mathrm{W}}^2} + \delta_{\kappa\mathrm{L}}\frac{m^2_{f_{-\sigma}}}{M_{\mathrm{W}}^2}\right)\right] l(s) + \cancel{Q^2_{f_\sigma} l^{\mathrm{em}}(s, m^2_{f_\sigma})}\right\}$$

$L(s) \equiv L(s, M_W^2)$ and $l(s) \equiv l(s, M_W^2)$

The logarithms between M_W^2 and the infrared scale are simply removed. Equivalently in the case of DR, logarithms involving M_W^2 and the IR regulator Q^2.

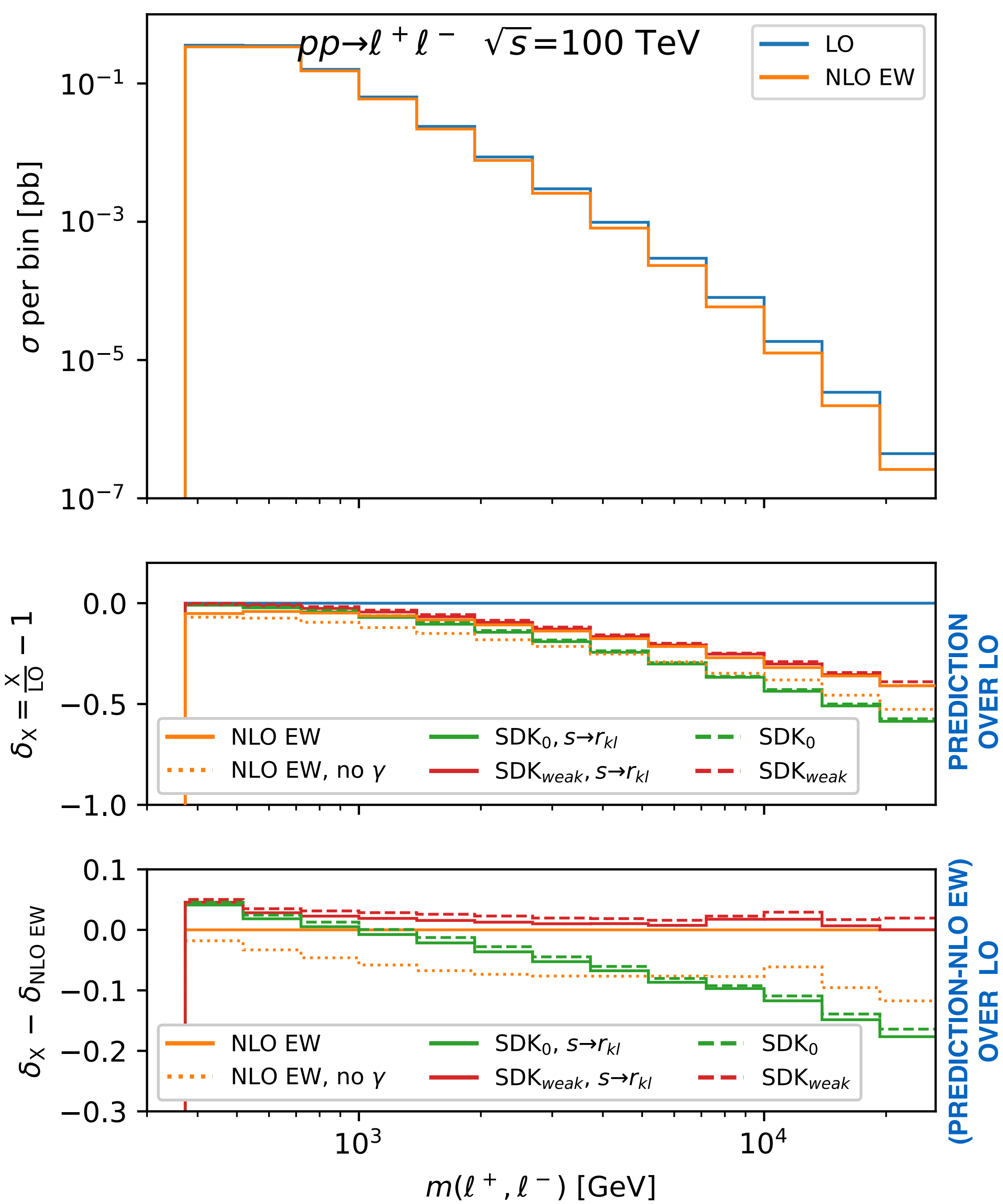
Easy, but not very well motivated.

We will denote in the following this approach as SDK_0.

SOME EXAMPLES AT 100 TeV

e^+e^- production at 100 TeV

$$p_T(\ell^\pm) > 200\ \text{GeV}, \qquad |\eta(\ell^\pm)| < 2.5, \qquad m(\ell^+,\ell^-) > 400\ \text{GeV}, \qquad \Delta R(\ell^+,\ell^-) > 0.5\,.$$



Orange: NLO EW, (**dotted**: NLO EW no γ PDF)
Green = SDK_0, **Red** = $\mathrm{SDK}_{\mathrm{weak}}$
Dashed: standard approach for amplitudes.
Solid: our formulation (more angular information)

Reference Prediction: Red-solid line

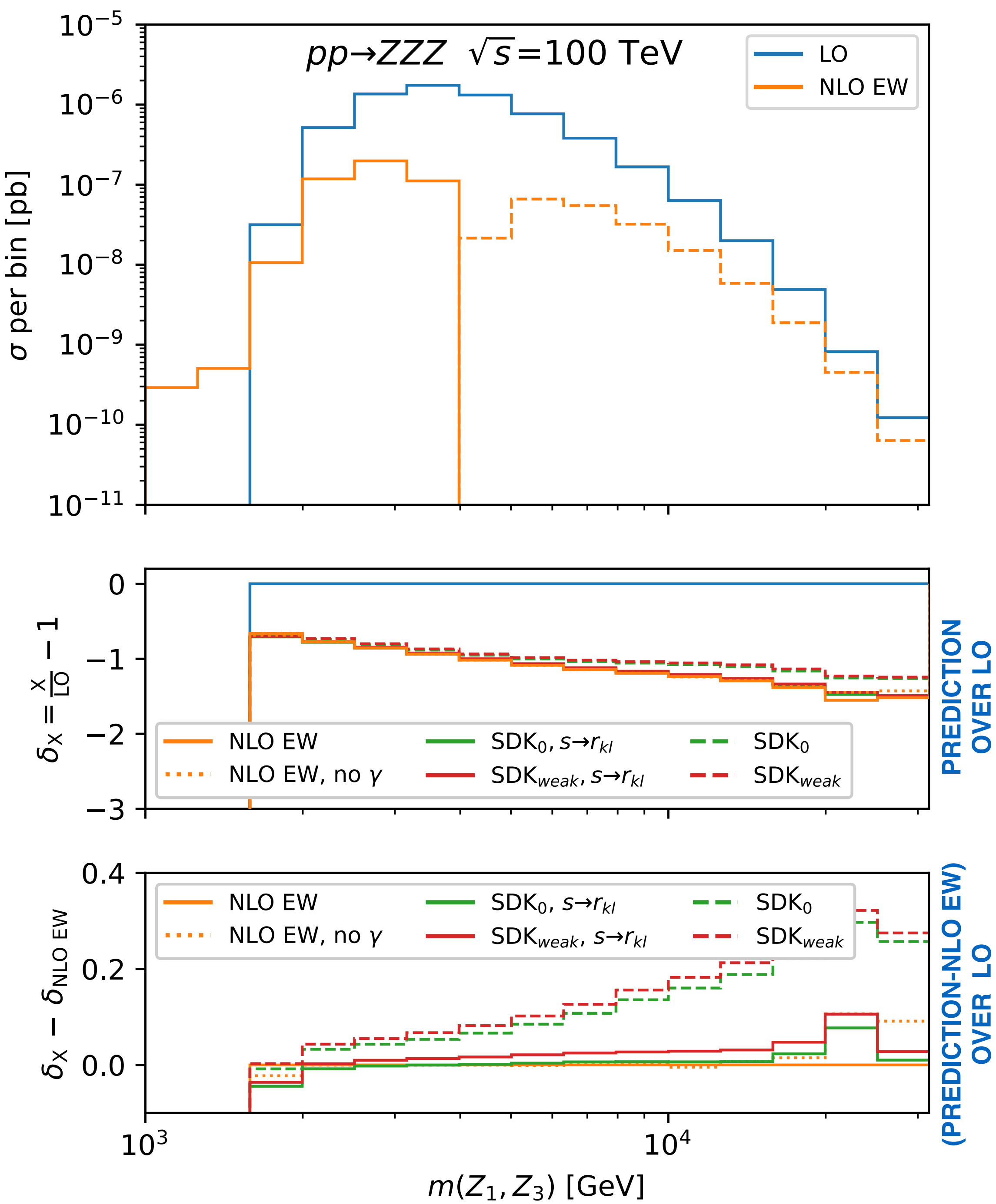
Only the $\mathrm{SDK}_{\mathrm{weak}}$ approach correctly captures the NLO EW prediction.

Solid and dashed very similar.

Photon PDF cannot be ignored.

ZZZ production at 100 TeV

$$p_T(Z_i) > 1\ \text{TeV}\,, \qquad |\eta(Z_i)| < 2.5\,, \qquad m(Z_i, Z_j) > 1\ \text{TeV}\,, \qquad \Delta R(Z_i, Z_j) > 0.5\,.$$



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Reference Prediction: Red-solid line

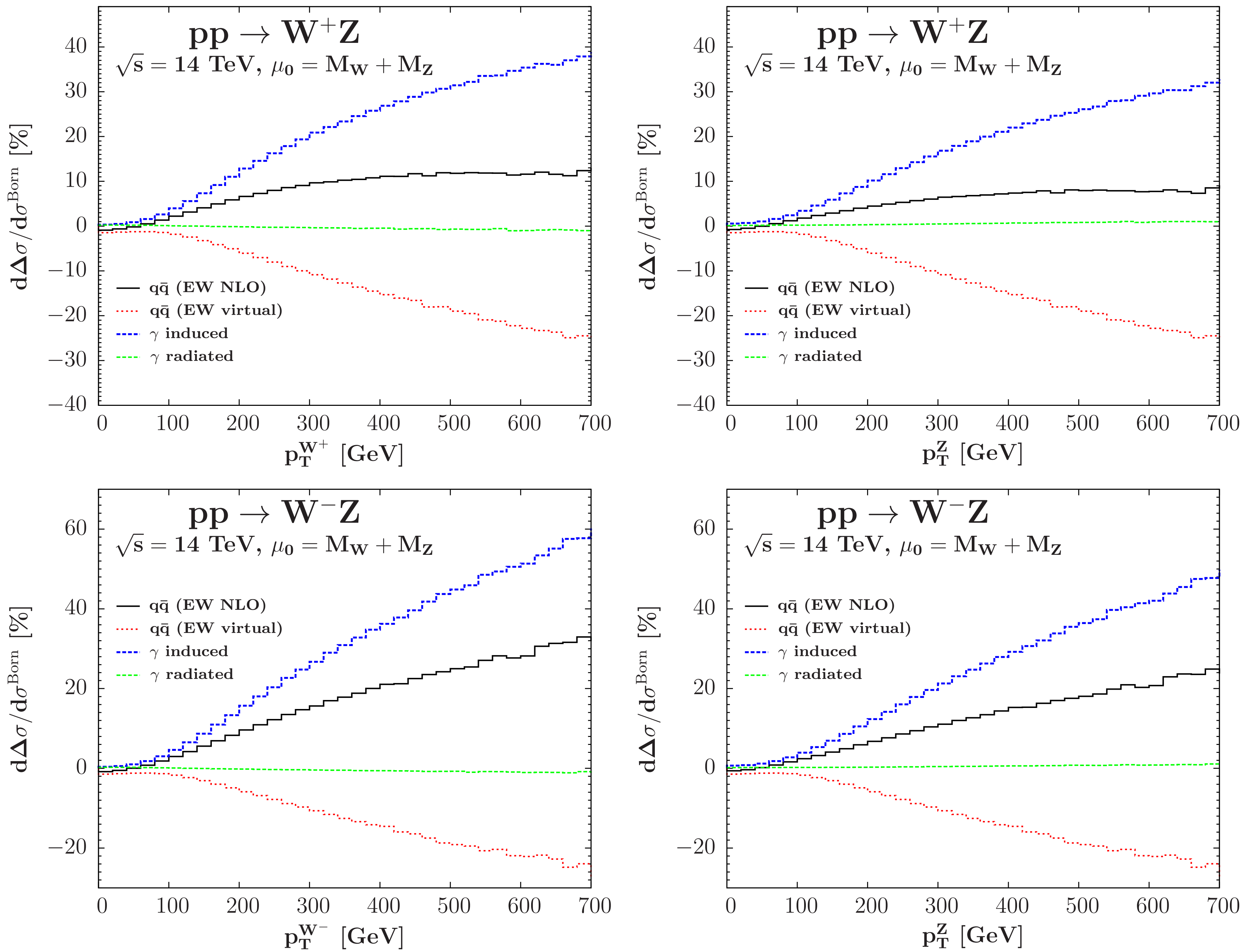
$\mathrm{SDK}_{\mathrm{weak}}$ **and** SDK_0 **approaches return very similar results (neutral final state).**

Only the solid lines, having more angular information, correctly capture NLO EW.

Sometimes things get more complicated

some preliminary considerations about WZ

WZ



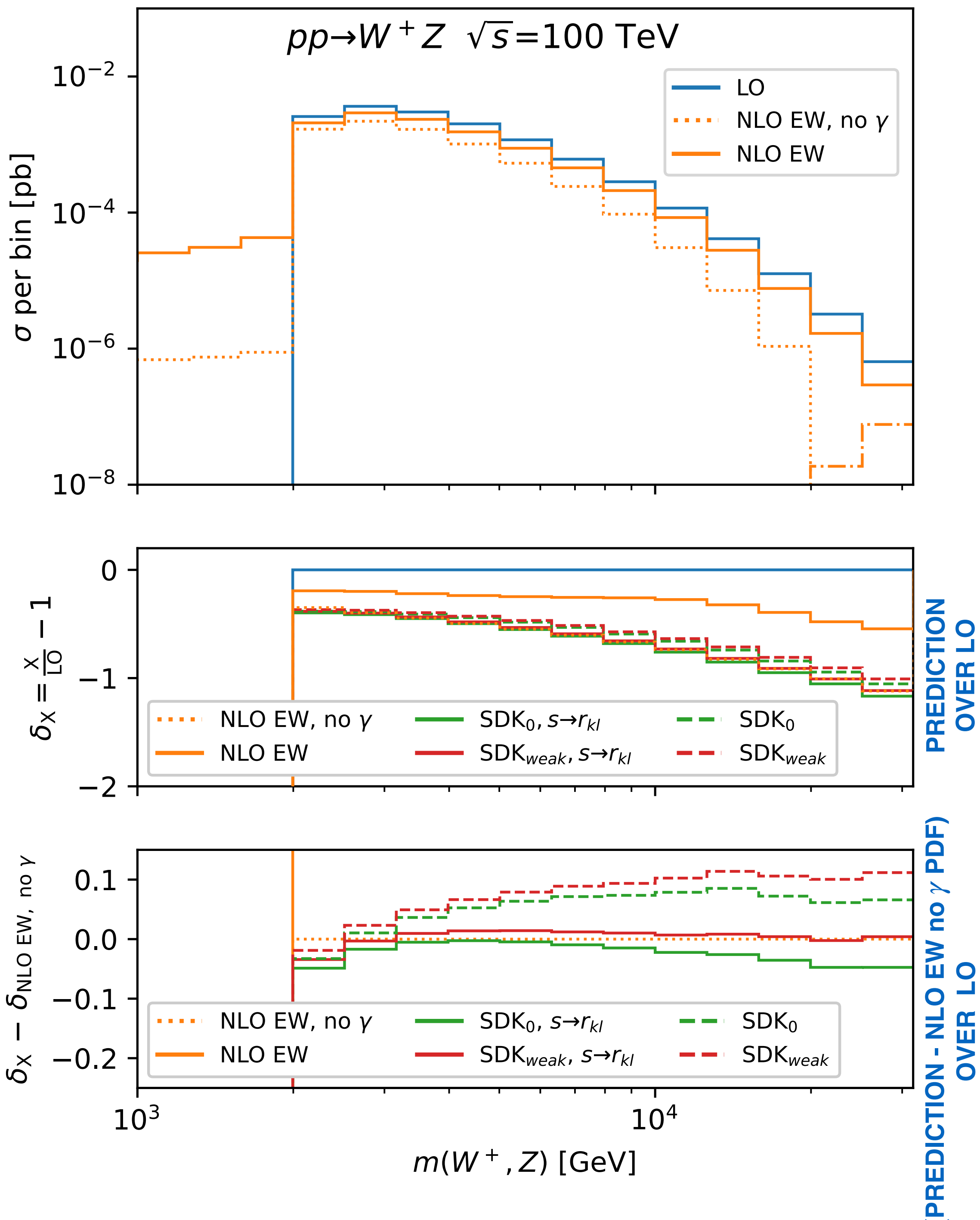
Baglio et al ‘13

NLO EW:

Sudakov logs are present, but the γq initial-state contribution is huge and overcompensates them.

WZ production at 100 TeV

$$p_T(V_i) > 1 \text{ TeV}\,, \qquad |\eta(V_i)| < 2.5\,, \qquad m(W^+, Z) > 1 \text{ TeV}\,, \qquad \Delta R(W^+, Z) > 0.5\,.$$



Orange: NLO EW, (**dotted**: NLO EW no γ PDF)
Green = SDK_0, **Red** = $\mathrm{SDK}_{\mathrm{weak}}$
Dashed: standard approach for amplitudes.
Solid: our formulation (more angular information)

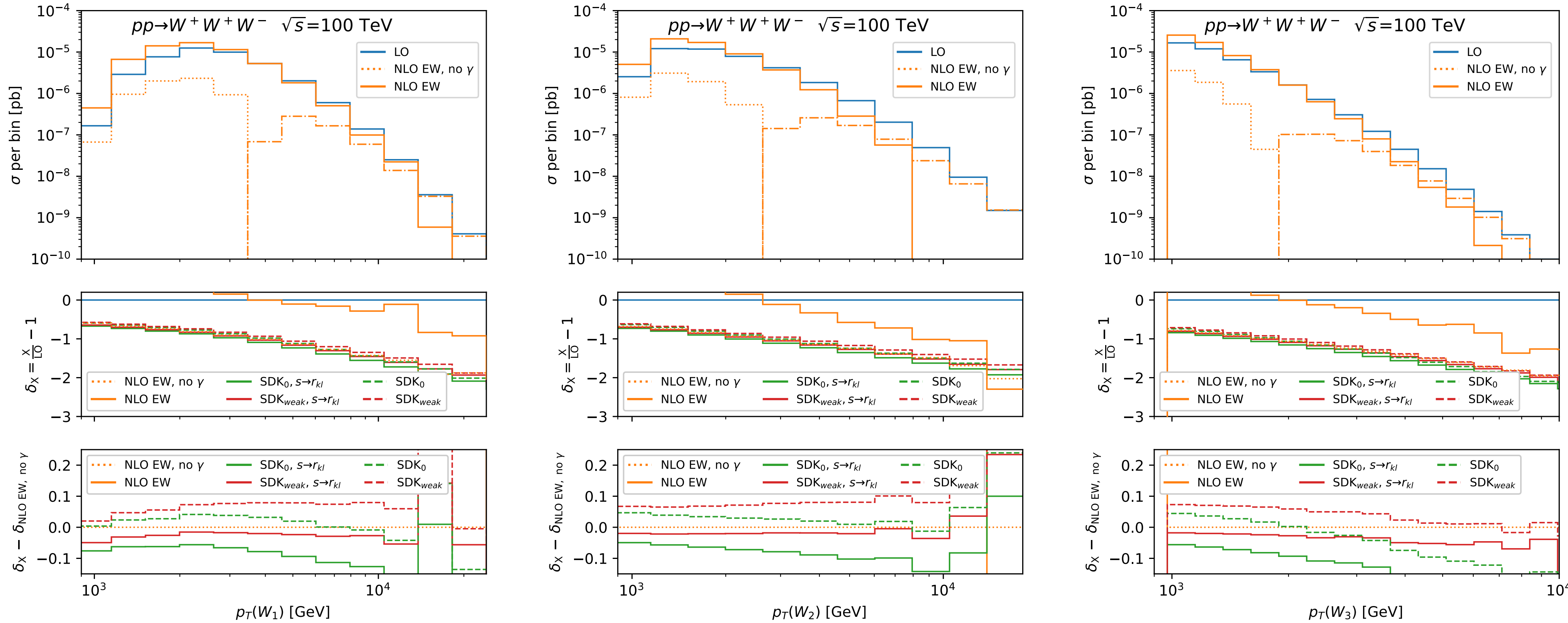
Reference Prediction: Red-solid line

The Sudakov approximation cannot approximate large logs from the opening of new channels. The fair comparison is with NLO EW no γ PDF.

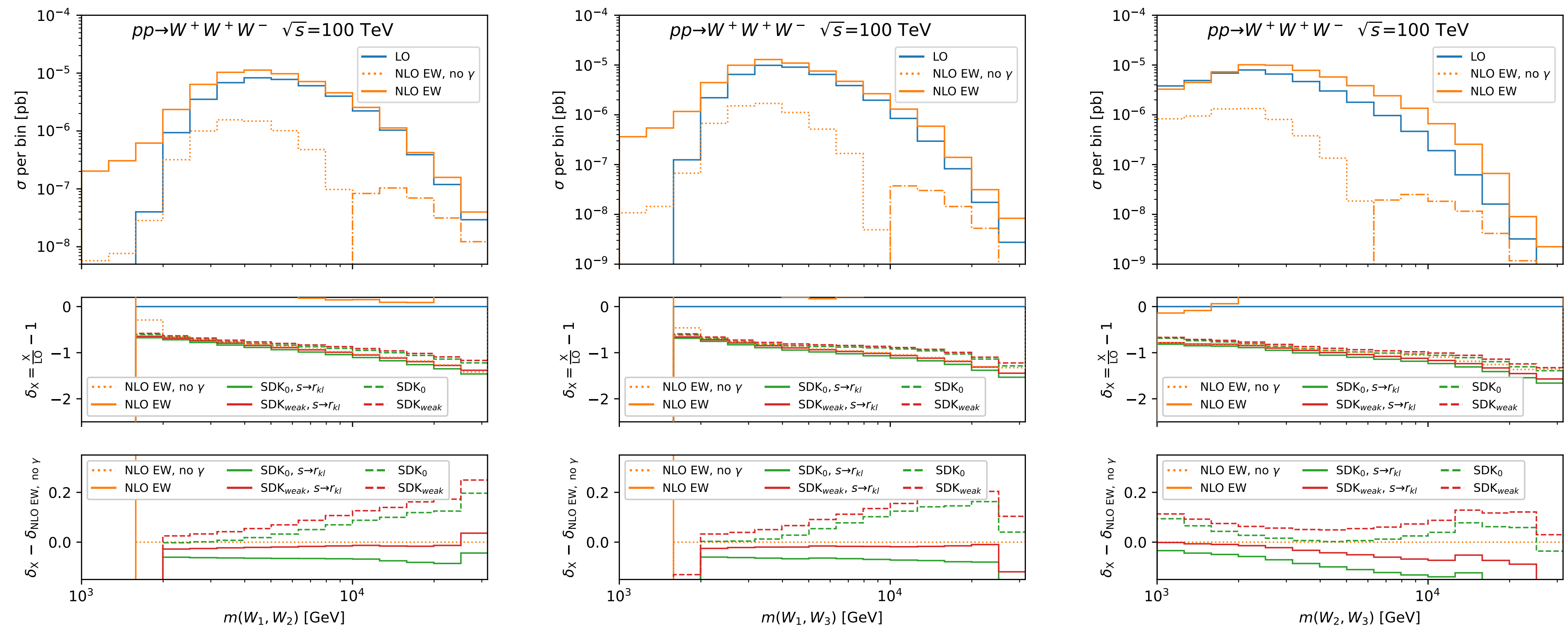
Only the $\mathrm{SDK}_{\mathrm{weak}}$ approach correctly captures the NLO EW prediction.

Only the solid lines, having more angular information, correctly capture NLO EW.

WWW production at 100 TeV



Similar to the WZ case. The Red-solid line is always superior, but at different levels.



OUTLOOK rather than CONCLUSION

Why did we automate Sudakov in aMG5?

- Fast evaluation of leading one-loop NLO EW corrections to amplitudes at high energies.
- Fast evaluation of leading NLO EW corrections at high energies for physical observables.
- Combination of leading NLO EW corrections at high energies with QCD+shower. (reweighting?)
- Combination in FxFx (merging at NLO) with recent advancements for weak radiation.
- Application to both hadron and lepton colliders. Combination with new W PDF implementation.
- Automation of matching of NLO EW and resummed EW LL
- Extending UFO information, generalisation to BSM case.
- DM production/annihilation including EW Sudakov effects (MadDM).
- Can we use this approach with SMEFT? ($s < \Lambda^2$)
- …..

EXTRA SLIDES

Automation of NLO corrections in Madgraph5_aMC@NLO

What do we mean with automation of EW corrections?

The possibility of calculating QCD and EW corrections for SM processes (matched to shower effects) with a process-independent approach.

```
generate process [QCD]
output process_QCD
```

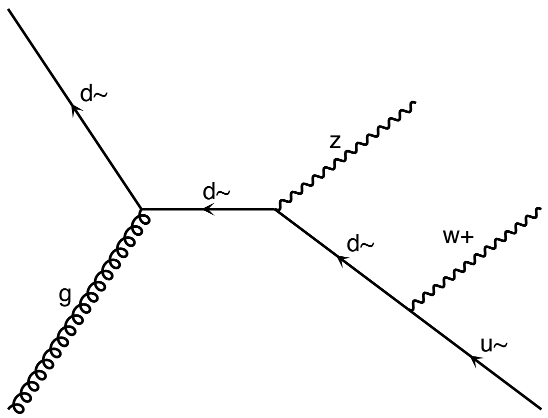
```
generate process [QCD EW]
output process_QCD_EW
```

- NNLO QCD complete automation is out of our theoretical capabilities at the moment.
- NLO EW and NNLO QCD corrections are of the same order ($\alpha_s^2 \sim \alpha$), but NLO EW corrections **can be automated.** Moreover effects such as Sudakov logarithms or photon FSR can enhance their size.

The automation of NLO QCD + EW corrections has already been achieved in the Madgraph5_aMC@NLO framework.

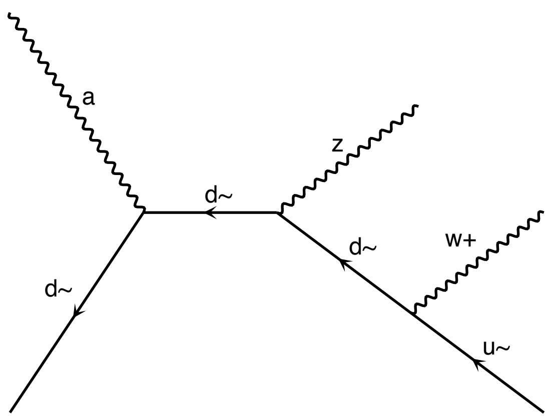
WZ

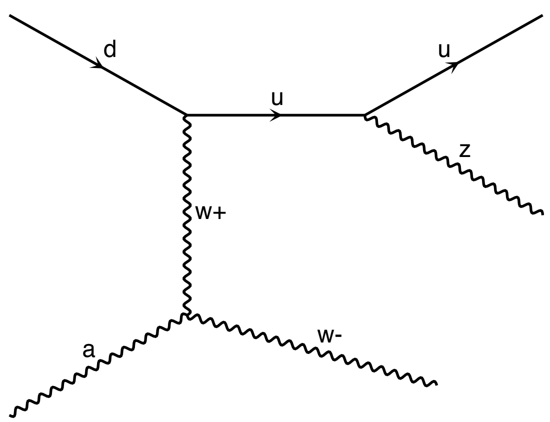
what's going on?



$$d\sigma^{dg\to W^-Zu} = c^d_{WZ}\, d\sigma_L^{dg\to Zd}\frac{\alpha}{2\pi}\log^2\left[\frac{(p_T^Z)^2}{M_W^2}\right]$$

Zj +
soft and collinear W





$$d\sigma^{d\gamma\to W^-Zu} = \frac{c^2_{L,d}c^d_{WZ}}{a_W^2}\, d\sigma_L^{d\gamma\to W^-u}\frac{\alpha}{2\pi}\log^2\left[\frac{(p_T^{W^-})^2}{M_Z^2}\right]$$

See also Baglio et al ‘13

- The large growth at high pt in NLO EW has similar origin of the case of NLO QCD corrections: giant K-factors *Frixione et al. ’92*.

- The photon couples to the W, originating new t-channel configurations that enhance the relative size of photon-quark contributions in NLO EW. NLO QCD corrections do not exhibit similar features.

Massless photons and light quarks

Representative expressions with IR divergent QED contributions in the Denner&Pozzorini algorithm.

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}_{i'_k i_k}(k)L(s) - 2(I^Z(k))^2_{i'_k i_k}\log\frac{M_Z^2}{M_W^2}\,l(s) + \delta_{i'_k i_k}Q_k^2 \boxed{L^{\mathrm{em}}(s,\lambda^2,m_k^2)}\right]$$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8s_{\mathrm{w}}^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m_{f_\sigma}^2}{M_W^2} + \delta_{\kappa\mathrm{L}}\frac{m_{f_{-\sigma}}^2}{M_W^2}\right)\right]l(s) + Q_{f_\sigma}^2 \boxed{l^{\mathrm{em}}(m_{f_\sigma}^2)}\right\},$$

$$\boxed{l^{\mathrm{em}}(m_f^2)} := \frac{1}{2}l(M_W^2,m_f^2) + l(M_W^2,\lambda^2) \qquad \boxed{L^{\mathrm{em}}(s,\lambda^2,m_k^2)} := 2l(s)\log\left(\frac{M_W^2}{\lambda^2}\right) + L(M_W^2,\lambda^2) - L(m_k^2,\lambda^2)$$

Massless photons and light quarks

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$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8s_{\mathrm{w}}^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m_{f_\sigma}^2}{M_W^2} + \delta_{\kappa\mathrm{L}}\frac{m_{f_{-\sigma}}^2}{M_W^2}\right)\right]l(s) + Q_{f_\sigma}^2 l^{\mathrm{em}}(m_{f_\sigma}^2)\right\},$$

$$l^{\mathrm{em}}(m_f^2) := \frac{1}{2}l(M_W^2, m_f^2) + l(M_W^2,\lambda^2)$$

$$L^{\mathrm{em}}(s,\lambda^2,m_k^2) := 2l(s)\log\left(\frac{M_W^2}{\lambda^2}\right) + L(M_W^2,\lambda^2) - L(m_k^2,\lambda^2)$$

$$l^{\mathrm{em}}(m_f^2) \equiv \frac{1}{2}l^{\mathrm{reg}}(M_W^2, m_f^2) + l(M_W^2, Q^2)\,,$$

$$l^{\mathrm{reg}}(M_W^2, m_f^2) \equiv \begin{cases} l(M_W^2, Q^2) & \text{if } m_f^2 = 0 \\ l(M_W^2, m_f^2) & \text{otherwise} \end{cases},\ .$$

$$L^{\mathrm{em}}(s, Q^2, m_k^2) \equiv 2l(s)\log\left(\frac{M_W^2}{Q^2}\right) + L(M_W^2, Q^2) - L^{\mathrm{reg}}(m_k^2, Q^2)\,,$$

$$L^{\mathrm{reg}}(m_k^2, Q^2) \equiv \begin{cases} 0 & \text{if } m_k^2 = 0 \\ L(m_k^2, Q^2) & \text{otherwise} \end{cases},\ .$$

Q is the IR regularisation scale in DR and the expressions have been obtained simply via the following substitutions in the expressions.

$$\log(\lambda^2) \to \log(Q^2)\,, \qquad \log(m_{f\neq t}^2) \to \log(Q^2)$$

Organisation of the logs in the algorithm

Two examples: LSC and C for fermions

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}_{i'_k i_k}(k) L(s) - 2(I^Z(k))^2_{i'_k i_k}\log\frac{M_Z^2}{M_W^2}\, l(s) + \delta_{i'_k i_k} Q_k^2 L^{\mathrm{em}}(s,\lambda^2,m_k^2)\right]$$

Casimir for the entire $SU(2)_L \times U(1)_B$

Charge for $U(1)_{QED}$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8s_{\mathrm{w}}^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m^2_{f_\sigma}}{M_{\mathrm{W}}^2} + \delta_{\kappa\mathrm{L}}\frac{m^2_{f_{-\sigma}}}{M_{\mathrm{W}}^2}\right)\right] l(s) + Q^2_{f_\sigma} l^{\mathrm{em}}(m^2_{f_\sigma})\right\},$$

$$L(s) \equiv L(s, M_W^2) \qquad \text{and} \qquad l(s) \equiv l(s, M_W^2)$$

$$l^{\mathrm{em}}(m_f^2) := \frac{1}{2} l(M_{\mathrm{W}}^2, m_f^2) + l(M_{\mathrm{W}}^2, \lambda^2) \qquad L^{\mathrm{em}}(s,\lambda^2,m_k^2) := 2l(s)\log\left(\frac{M_{\mathrm{W}}^2}{\lambda^2}\right) + L(M_{\mathrm{W}}^2,\lambda^2) - L(m_k^2,\lambda^2)$$

The full EW is present between s and M_W^2, while only QED is present between M_W^2 and λ^2.

So the QED contribution is split between the intervals $(s, M_W^2) + (M_W^2, \lambda^2)$. But the division at M_W^2 is simply determined by convenience, in parallel with the weak case. In this case M_W^2 is just a technical parameter and not a physical quantity.

Interference tree-level and one-loop

$$\delta_{\mathrm{LA}}^{\mathrm{EW}} \equiv \frac{2\Re(\mathcal{M}_0 \delta\mathcal{M}^*)}{|\mathcal{M}_0|^2}$$

As shown, the two amplitudes have different external states.

As shown, the two amplitudes have **different external states**.
The same **STR** (Simplified Treatment of Resonances) techniques used for subtractions of resonances in MSSM calculations. *Frixione et al. '19*
However, in this case STR are applied:

- to the kinematic mass of a single particle,
- also in the initial state,
- once or twice per amplitude,
- in the interference only for one of the two amplitudes

$$\delta^{\mathrm{PR}}\mathcal{M} = \left(\frac{\delta\mathcal{M}_0}{\delta\alpha}\delta\alpha + \frac{\delta\mathcal{M}_0}{\delta M_W^2}\delta M_W^2 + \frac{\delta\mathcal{M}_0}{\delta M_Z^2}\delta M_Z^2 + \frac{\delta\mathcal{M}_0}{\delta m_t}\delta m_t + \frac{\delta\mathcal{M}_0}{\delta n_{\mathrm{tad}}}\delta n_{\mathrm{tad}} + \frac{\delta\mathcal{M}_0}{\delta M_H}\delta M_H \right)\Big|_{\mu^2=s}$$

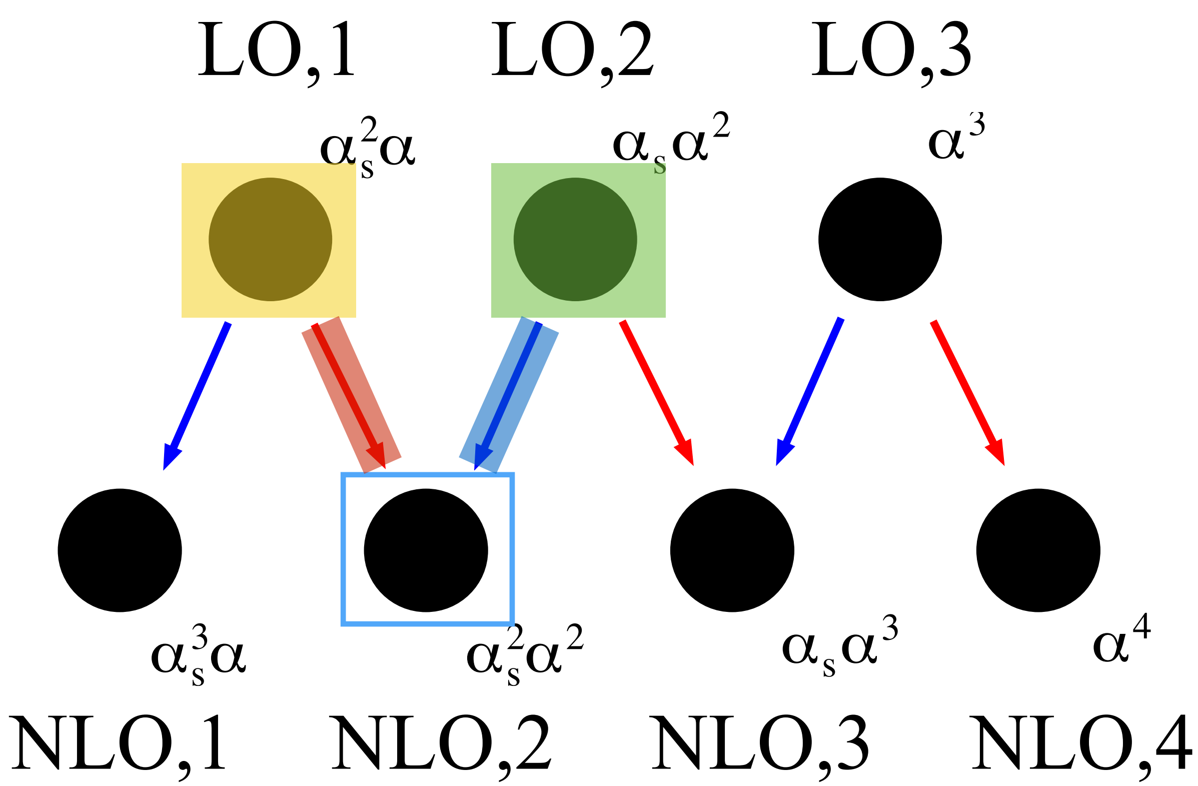
Derivatives implemented **numerically**, point by point.

$$\left.\frac{\delta\mathcal{M}}{\delta x}\right|_{x=\bar{x}} \equiv \frac{\left(\mathcal{M}|_{x=\bar{x}(1+\delta_x)} - \mathcal{M}|_{x=\bar{x}(1-\delta_x)}\right)}{2\delta_x}$$

Not the **fastest** approach, but **very flexible.**

In the SHERPA automation, higher-order terms of the form $l^2(s) = \alpha^2 \log(s/M_W^2)$ are also present.

QCD in the NLO EW corrections



$$\left(\Sigma^{\rm virt}_{{\rm NLO}_i}\right)\Big|_{\rm LA} = \Sigma_{{\rm LO}_{i-1}}\delta^{\rm EW}_{\rm LA} + \Sigma_{{\rm LO}_i}\delta^{\rm QCD}_{\rm LA}$$

Also QCD leads to Sudakov logarithms in the virtual amplitude.

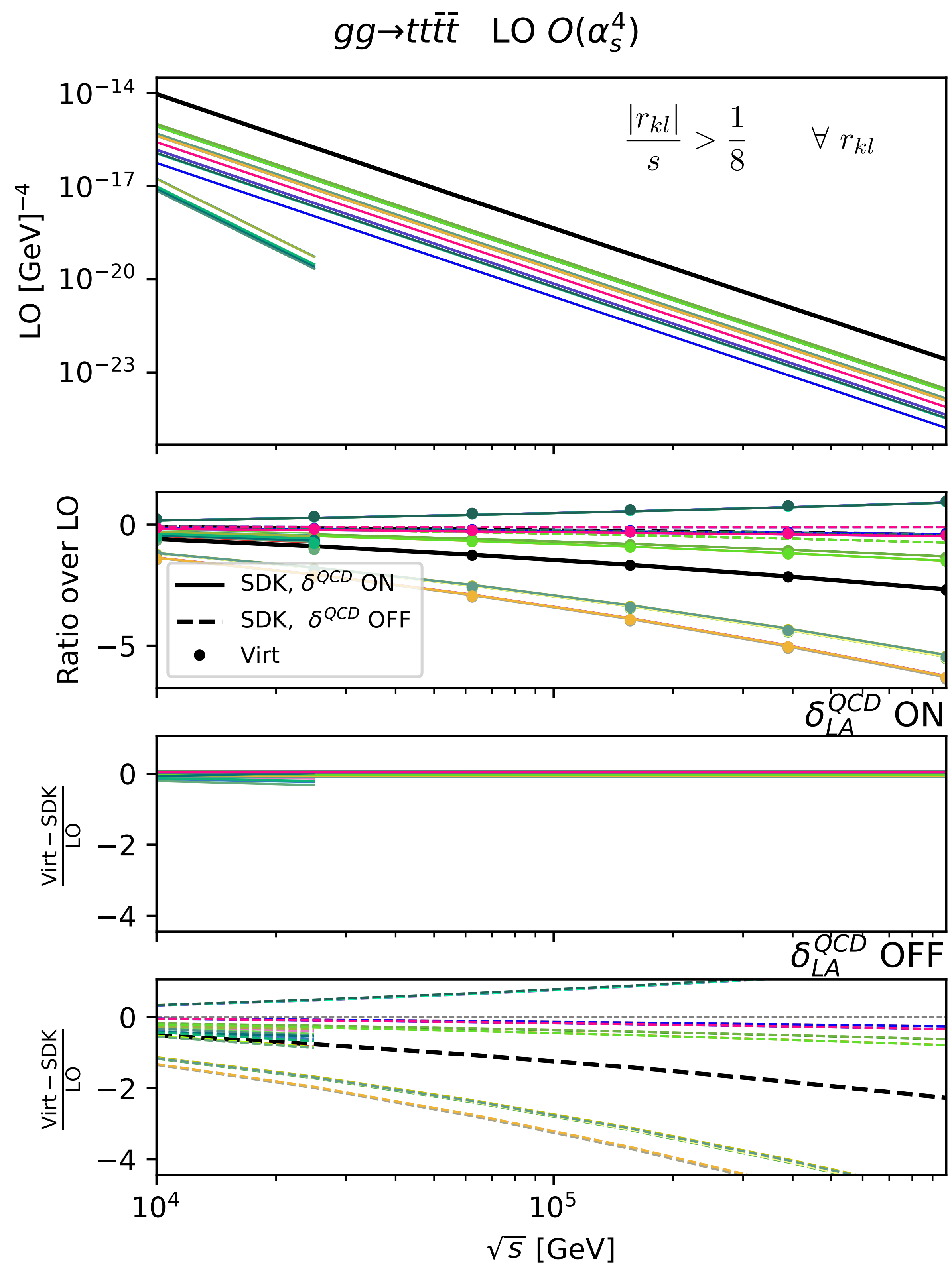
If $r_{kl} = r_{k'l'} = s$ is assumed, the logs have a very simple form in QCD.

$$\delta\tilde{\mathcal{M}} \equiv \tilde{\mathcal{M}}_0 \left[\left(n_t\, L^t(s) + n_{\alpha_S}\, l^{\alpha_S}(\mu_R^2) - n_g\, l^{\alpha_S}(s) \right) + \frac{\delta\tilde{\mathcal{M}}_0}{\delta m_t}(\delta m_t)^{\rm QCD} \right]$$

tops in the external legs — powers of α_s — gluons in the external legs

$$L^t(s) \equiv \frac{C_F}{2}\frac{\alpha_S}{4\pi}\left(\log^2\frac{s}{m_t^2} + \log\frac{s}{m_t^2}\right) \qquad l^{\alpha_S}(\mu^2) \equiv \frac{1}{3}\frac{\alpha_S}{4\pi}\log\frac{\mu^2}{m_t^2} \qquad (\delta m_t)^{\rm QCD} \equiv -3C_F\frac{\alpha_S}{4\pi}\log\frac{s}{m_t^2}$$

Example $(2 \to 4)$: QCD in NLO EW and scan in s



Denner&Pozzorini algorithm works only with non mass-suppressed LO processes: we select only helicity configurations > 10^(-3) of the dominant one.

Dots: NLO EW (MadLoop). **Lines =** Sudakov.

Dashed: $\delta_{\rm LA}^{\rm QCD}$ omitted

Solid: $\delta_{\rm LA}^{\rm QCD}$ included

Dots-Solid/LO: horizontal, the correct Log dependence is captured.

Dots-Dashed/LO: not horizontal, the correct Log dependence is lost.

Example $(2 \to 4)$: QCD in NLO EW and scan in s

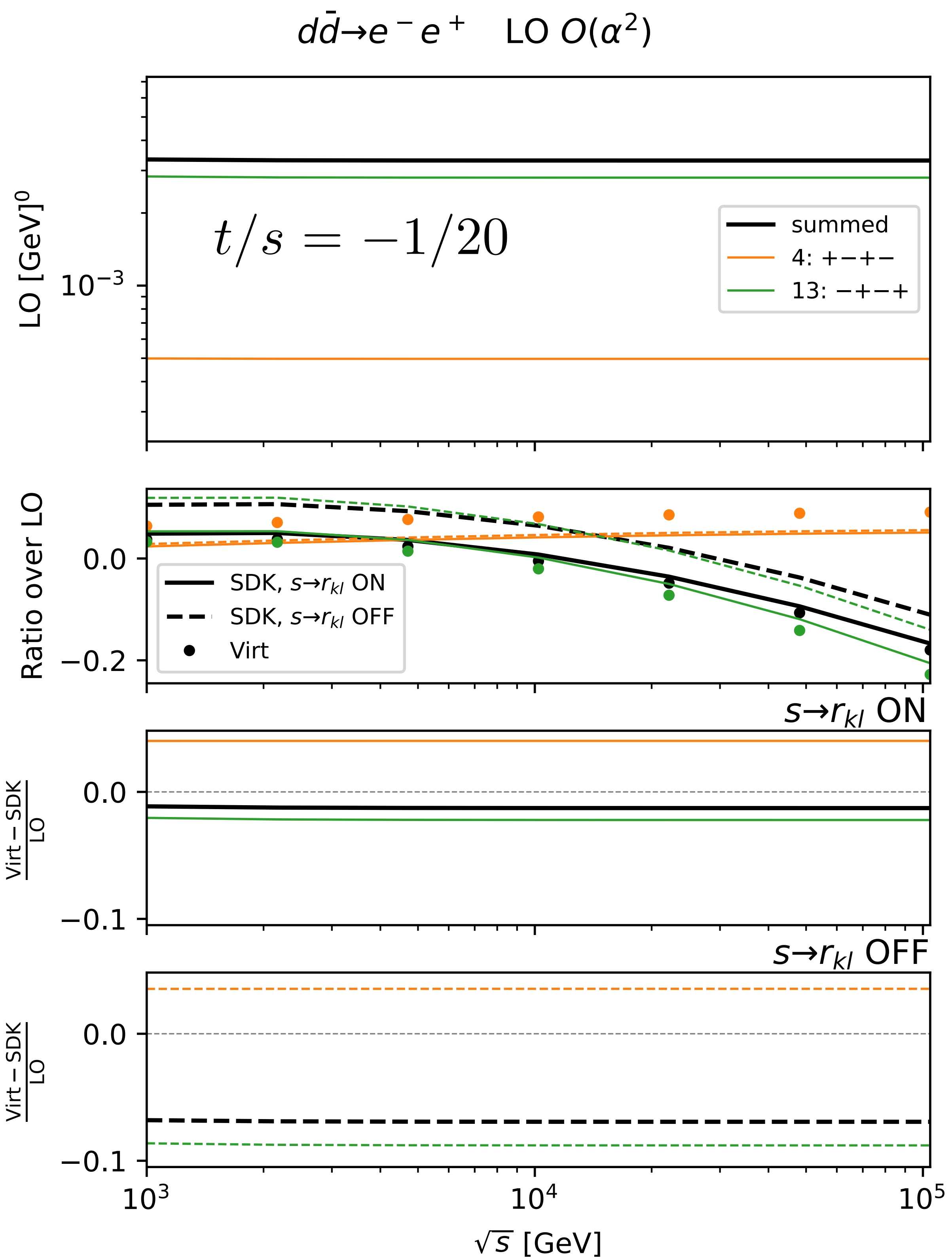
	$\delta_{\mathrm{LA}}^{\mathrm{QCD}}$ ON		$\delta_{\mathrm{LA}}^{\mathrm{QCD}}$ OFF	
Helicity	A	B	A	B
summed	$(9.9 \pm 8.4) \cdot 10^{-4}$	$(-2.7 \pm 0.4) \cdot 10^{-2}$	$(-8.9 \pm 1.2) \cdot 10^{-1}$	$(3.1 \pm 0.6) \cdot 10^{0}$
$1: - - - - ++$	$(2.7 \pm 1.6) \cdot 10^{-4}$	$(-2.56 \pm 0.08) \cdot 10^{-2}$	$(-2.9 \pm 0.4) \cdot 10^{-1}$	$(9.9 \pm 1.8) \cdot 10^{-1}$
$6: - - - + + -$	$(-5.5 \pm 5.4) \cdot 10^{-3}$	$(6.6 \pm 2.7) \cdot 10^{-2}$	$(5.3 \pm 0.7) \cdot 10^{-1}$	$(-1.8 \pm 0.4) \cdot 10^{0}$
$7: - - - + - +$	$(4.3 \pm 6.7) \cdot 10^{-4}$	$(5.8 \pm 0.3) \cdot 10^{-2}$	$(5.3 \pm 0.7) \cdot 10^{-1}$	$(-1.8 \pm 0.3) \cdot 10^{0}$
$10: - - + - + -$	$(6.5 \pm 6.1) \cdot 10^{-4}$	$(5.3 \pm 0.3) \cdot 10^{-2}$	$(5.3 \pm 0.7) \cdot 10^{-1}$	$(-1.8 \pm 0.3) \cdot 10^{0}$
$11: - - + - - +$	$(-5.9 \pm 5.8) \cdot 10^{-3}$	$(8.2 \pm 2.9) \cdot 10^{-2}$	$(5.3 \pm 0.7) \cdot 10^{-1}$	$(-1.8 \pm 0.4) \cdot 10^{0}$
$16: - - + + - -$	$(5 \pm 14) \cdot 10^{-5}$	$(3.53 \pm 0.07) \cdot 10^{-2}$	$(-1.4 \pm 0.2) \cdot 10^{-1}$	$(5.4 \pm 0.9) \cdot 10^{-1}$
$17: - + - - ++$	$(4.3 \pm 6.3) \cdot 10^{-4}$	$(-4.6 \pm 0.3) \cdot 10^{-2}$	$(-2.2 \pm 0.3) \cdot 10^{-1}$	$(7.3 \pm 1.4) \cdot 10^{-1}$
$22: - + - + + -$	$(-5.5 \pm 3.5) \cdot 10^{-4}$	$(-6.4 \pm 0.2) \cdot 10^{-2}$	$(-2.3 \pm 0.3) \cdot 10^{0}$	$(8.1 \pm 1.5) \cdot 10^{0}$
$23: - + - + - +$	$(1.2 \pm 0.6) \cdot 10^{-3}$	$(-6.3 \pm 0.3) \cdot 10^{-2}$	$(-1.9 \pm 0.3) \cdot 10^{0}$	$(6.9 \pm 1.3) \cdot 10^{0}$
$26: - + + - + -$	$(1.2 \pm 0.9) \cdot 10^{-3}$	$(-5.9 \pm 0.4) \cdot 10^{-2}$	$(-2.3 \pm 0.3) \cdot 10^{0}$	$(8.2 \pm 1.5) \cdot 10^{0}$
$27: - + + - - +$	$(-3.7 \pm 5.0) \cdot 10^{-4}$	$(-8.1 \pm 0.3) \cdot 10^{-2}$	$(-2.0 \pm 0.3) \cdot 10^{0}$	$(7.0 \pm 1.3) \cdot 10^{0}$
$32: - + + + - -$	$(3.1 \pm 3.1) \cdot 10^{-4}$	$(2.7 \pm 0.2) \cdot 10^{-2}$	$(-1.1 \pm 0.1) \cdot 10^{-1}$	$(4.1 \pm 0.6) \cdot 10^{-1}$
$33: + - - - ++$	$(1.5 \pm 1.2) \cdot 10^{-3}$	$(-4.5 \pm 0.6) \cdot 10^{-2}$	$(-2.2 \pm 0.3) \cdot 10^{-1}$	$(7.3 \pm 1.4) \cdot 10^{-1}$
$38: + - - + + -$	$(1.4 \pm 1.0) \cdot 10^{-3}$	$(-7.1 \pm 0.5) \cdot 10^{-2}$	$(-2.0 \pm 0.3) \cdot 10^{0}$	$(7.0 \pm 1.3) \cdot 10^{0}$
$39: + - - + - +$	$(-1.9 \pm 3.7) \cdot 10^{-4}$	$(-7.4 \pm 0.2) \cdot 10^{-2}$	$(-2.3 \pm 0.3) \cdot 10^{0}$	$(8.2 \pm 1.5) \cdot 10^{0}$
$42: + - + - + -$	$(-2.7 \pm 2.5) \cdot 10^{-4}$	$(-8.3 \pm 0.1) \cdot 10^{-2}$	$(-2.0 \pm 0.3) \cdot 10^{0}$	$(6.8 \pm 1.3) \cdot 10^{0}$
$43: + - + - - +$	$(1.5 \pm 0.7) \cdot 10^{-3}$	$(-6.6 \pm 0.4) \cdot 10^{-2}$	$(-2.3 \pm 0.3) \cdot 10^{0}$	$(8.1 \pm 1.5) \cdot 10^{0}$
$48: + - + + - -$	$(4.0 \pm 2.3) \cdot 10^{-4}$	$(2.1 \pm 0.1) \cdot 10^{-2}$	$(-1.1 \pm 0.1) \cdot 10^{-1}$	$(4.1 \pm 0.6) \cdot 10^{-1}$
$49: + + - - ++$	$(4.0 \pm 2.5) \cdot 10^{-4}$	$(-2.9 \pm 0.1) \cdot 10^{-2}$	$(-2.9 \pm 0.4) \cdot 10^{-1}$	$(9.9 \pm 1.8) \cdot 10^{-1}$
$54: + + - + + -$	$(7.4 \pm 7.8) \cdot 10^{-4}$	$(4.9 \pm 0.4) \cdot 10^{-2}$	$(5.3 \pm 0.7) \cdot 10^{-1}$	$(-1.8 \pm 0.3) \cdot 10^{0}$
$55: + + - + - +$	$(-5.4 \pm 5.2) \cdot 10^{-3}$	$(7.4 \pm 2.6) \cdot 10^{-2}$	$(5.3 \pm 0.7) \cdot 10^{-1}$	$(-1.8 \pm 0.4) \cdot 10^{0}$
$58: + + + - + -$	$(-6.0 \pm 6.0) \cdot 10^{-3}$	$(7.7 \pm 3.0) \cdot 10^{-2}$	$(5.3 \pm 0.7) \cdot 10^{-1}$	$(-1.8 \pm 0.4) \cdot 10^{0}$
$59: + + + - - +$	$(3.7 \pm 4.6) \cdot 10^{-4}$	$(6.1 \pm 0.2) \cdot 10^{-2}$	$(5.3 \pm 0.7) \cdot 10^{-1}$	$(-1.8 \pm 0.3) \cdot 10^{0}$
$64: + + + + - -$	$(10 \pm 141) \cdot 10^{-6}$	$(3.85 \pm 0.07) \cdot 10^{-2}$	$(-1.4 \pm 0.2) \cdot 10^{-1}$	$(5.4 \pm 0.9) \cdot 10^{-1}$

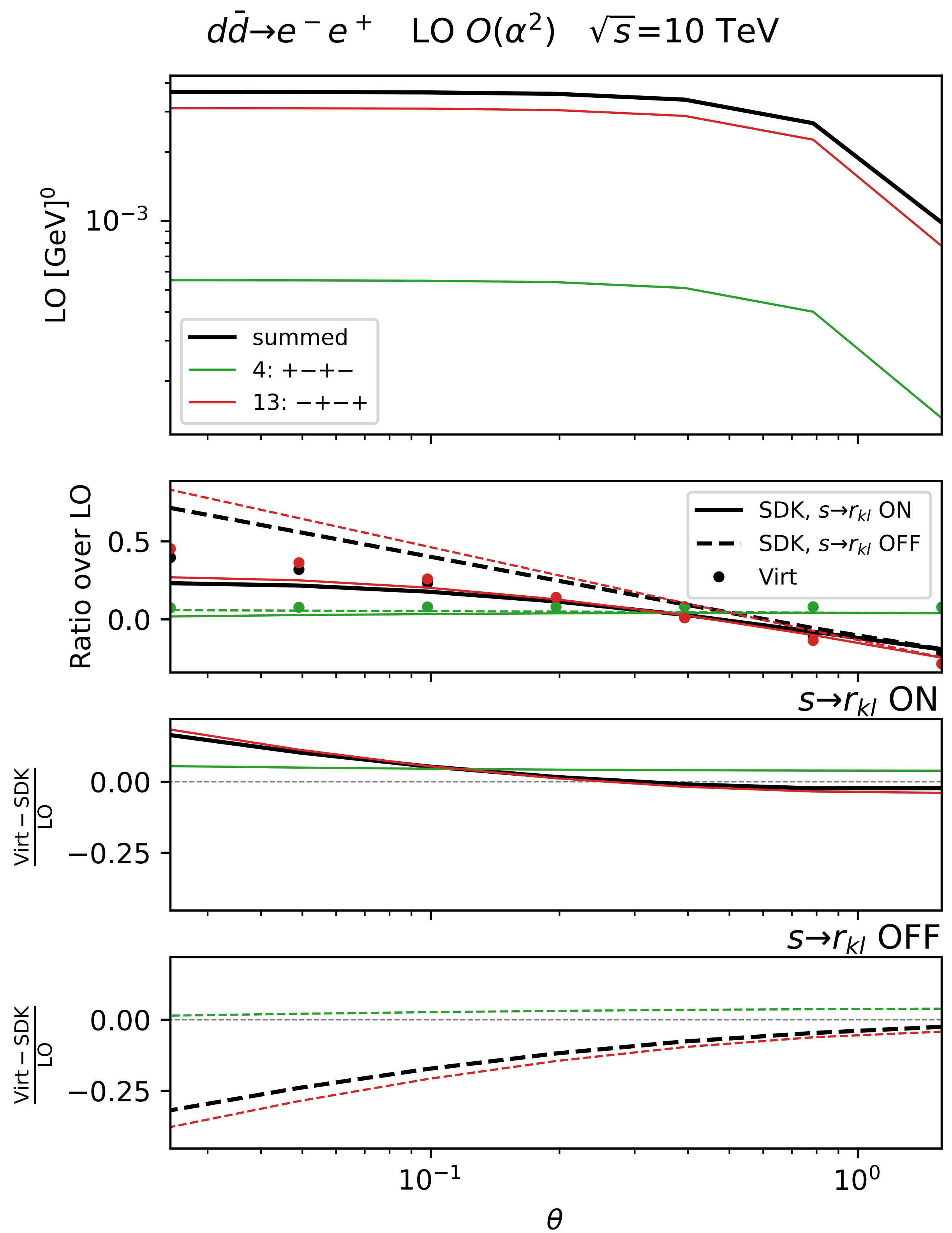
Compatible with 0 | % | $\neq 0$

Fit of (Virt-Sudakov)/LO via

$$A \log_{10}(\sqrt{s}/[1 \ \mathrm{GeV}]) + B$$

Example $(2 \to 2)$: $d\bar{d} \to e^+e^-$





Formulas

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}_{i'_k i_k}(k)L(s) - 2(I^Z(k))^2_{i'_k i_k}\log\frac{M_Z^2}{M_W^2}\,l(s) + \delta_{i'_k i_k}Q_k^2 L^{\mathrm{em}}(s,Q^2,m_k^2)\right].$$

$$C^{\mathrm{ew}} := \sum_{V_a=A,Z,W^\pm} I^{V_a}I^{\bar{V}_a} = \frac{1}{c_{\mathrm{w}}^2}\left(\frac{Y}{2}\right)^2 + \frac{1}{s_{\mathrm{w}}^2}C. \qquad I^A = -Q, \qquad I^Z = \frac{T^3 - s_{\mathrm{w}}^2 Q}{s_{\mathrm{w}}c_{\mathrm{w}}}, \qquad I^\pm = \frac{1}{s_{\mathrm{w}}}T^\pm = \frac{1}{s_{\mathrm{w}}}\frac{T^1 \pm \mathrm{i}T^2}{\sqrt{2}}$$

$$2\Re\left(\mathcal{M}_0^{i_1\dots i_n}(\delta\mathcal{M}^{i_1\dots i_n})^*\right) \supset 2\Re\left(\mathcal{M}_0^{i_1\dots i_n}\left(\delta^{V_a,\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l)\mathcal{M}_0^{i_1\dots i'_k\dots i'_l\dots i_n}\right)^*\right)$$

Unlike the LSC terms, the SSC ones remain a sum over pairs of external legs of the form

$$\delta^{\mathrm{SSC}}\mathcal{M}^{i_1\dots i_n} = \sum_{k=1}^{n}\sum_{l<k}\sum_{V_a=A,Z,W^\pm}\delta^{V_a,\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l)\mathcal{M}_0^{i_1\dots i'_k\dots i'_l\dots i_n}\,. \qquad (2.22)$$

This part is the one with the largest differences w.r.t. Ref. [39]. The exchange of soft neutral gauge bosons contributes with

$$\delta^{A,\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l) = \left[2\left(l(s) + l(M_W^2,Q^2)\right)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right) + \Delta^{s\to r_{kl}}(r_{kl},M_W^2)\right]I^A_{i'_k i_k}(k)I^A_{i'_l i_l}(l),$$

$$\delta^{Z,\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l) = \left[2l(s)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right) + \Delta^{s\to r_{kl}}(r_{kl},M_Z^2)\right]I^Z_{i'_k i_k}(k)I^Z_{i'_l i_l}(l), \qquad (2.23)$$

and charged gauge bosons yields

$$\delta^{W^\pm,\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l) = \left[2l(s)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right) + \Delta^{s\to r_{kl}}(r_{kl},M_W^2)\right]I^\pm_{i'_k i_k}(k)I^\mp_{i'_l i_l}(l), \qquad (2.24)$$

The quantity $\Delta^{s\to r_{kl}}(r_{kl},M^2)$ is set equal to zero when the condition (2.4) is assumed and the LA is applied in a strict sense, as done in Ref. [39]. Taking instead into account the fact that $s \gg r_{kl} \gg M^2$, this quantity reads

$$\Delta^{s\to r_{kl}}(r_{kl},M^2) \equiv L(|r_{kl}|,s) + 2l(M_W^2,M^2)\log\frac{|r_{kl}|}{s} - 2i\pi\Theta(r_{kl})l(|r_{kl}|,s)\,, \qquad (2.25)$$

and precisely corresponds to the SSC$^{s\to r_{kl}}$ logarithms of eq. (2.17).

Formulas

2.5 C: Collinear and soft single logarithms

In this section we provide the results obtained in Ref. [39], adapting them for the case with massless light-fermions and photons. The formula for the collinear and soft single logarithms can be written as a sum over the external particles and polarisations,

$$\delta^{\mathrm{C}}\mathcal{M}^{i_1\dots i_n} = \sum_{k=1}^{n} \delta^{\mathrm{C}}_{i'_k i_k}(k)\mathcal{M}_0^{i_1\dots i'_k\dots i_n}\,, \tag{2.27}$$

with $\delta^{\mathrm{C}}_{i'_k i_k}(k)$ that depends on the external particle and polarisation φ_{i_k}. We provide the results in the following. The expressions for all the new terms introduced in the formulas can be found in Ref. [39].

Chiral fermions

Considering fermions f^κ_σ with chirality $\kappa = \mathrm{R},\mathrm{L}$ and isospin indices $\sigma = \pm$, the result is

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8s_{\mathrm{w}}^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m^2_{f_\sigma}}{M_W^2} + \delta_{\kappa\mathrm{L}}\frac{m^2_{f_{-\sigma}}}{M_W^2}\right)\right] l(s) + Q^2_{f_\sigma} l^{\mathrm{em}}(m^2_{f_\sigma})\right\}, \tag{2.28}$$

where the pure electromagnetic logarithms reads

$$l^{\mathrm{em}}(m_f^2) \equiv \frac{1}{2} l^{\mathrm{reg}}(M_W^2, m_f^2) + l(M_W^2, Q^2)\,, \tag{2.29}$$

with

$$l^{\mathrm{reg}}(M_W^2, m_f^2) \equiv \begin{cases} l(M_W^2, Q^2) & \text{if } m_f^2 = 0\,, \\ l(M_W^2, m_f^2) & \text{otherwise}\,. \end{cases} \tag{2.30}$$

Transverse charged gauge bosons W

The result is

$$\delta^{\mathrm{C}}_{W^\sigma W^{\sigma'}}(V_{\mathrm{T}}) = \delta_{\sigma\sigma'}\left[\frac{1}{2} b^{\mathrm{ew}}_W l(s) + Q_W^2 l^{\mathrm{em}}(M_W^2)\right], \tag{2.31}$$

where b^{ew}_W is a coefficient of the β-function.

Transverse neutral gauge bosons A,Z

The results for symmetric and antisymmetric parts are expressed in terms of the coefficients $b^{\mathrm{ew}}_{NN'}$ of the β-function. The result is

$$\delta^{\mathrm{C}}_{N'N}(V_{\mathrm{T}}) = \frac{1}{2}\left[E_{N'N} b^{\mathrm{ew}}_{AZ} + b^{\mathrm{ew}}_{N'N}\right] l(s) + \frac{1}{2}\delta_{NA}\delta_{N'A}\delta Z^{\mathrm{em}}_{AA}. \tag{2.32}$$

where the non-diagonal β-function $b^{\mathrm{ew}}_{N'N}$ coefficient is entering the expression. Since $E_{AZ} = -E_{ZA} = 1$ the non-diagonal components read

$$\delta^{\mathrm{C}}_{AZ}(V_{\mathrm{T}}) = b^{\mathrm{ew}}_{AZ} l(s), \qquad \delta^{\mathrm{C}}_{ZA}(V_{\mathrm{T}}) = 0. \tag{2.33}$$

The quantity Z^{em}_{AA} in DR reads

$$\delta Z^{\mathrm{em}}_{AA} = -\frac{4}{3}\sum_{f,i,\sigma\neq t} N_{\mathrm{C}}^f Q^2_{f_\sigma} l(M_W^2, Q^2)\,. \tag{2.34}$$

Longitudinally polarised gauge bosons

By means of amplitudes involving Goldstone bosons, the complete collinear corrections (2.27) for longitudinal gauge bosons is

$$\begin{aligned} \delta^{\mathrm{C}}\mathcal{M}^{\dots W^\pm_{\mathrm{L}}\dots} &= \delta^{\mathrm{C}}_{\phi^\pm\phi^\pm}(\Phi)\,\mathcal{M}_0^{\dots\phi^\pm\dots} = \delta^{\mathrm{C}}_{\phi^\pm\phi^\pm}(\Phi)\,\mathcal{M}_0^{\dots W^\pm_{\mathrm{L}}\dots}, \\ \delta^{\mathrm{C}}\mathcal{M}^{\dots Z_{\mathrm{L}}\dots} &= \mathrm{i}\delta^{\mathrm{C}}_{\chi\chi}(\Phi)\,\mathcal{M}_0^{\dots\chi\dots} = \delta^{\mathrm{C}}_{\chi\chi}(\Phi)\,\mathcal{M}_0^{\dots Z_{\mathrm{L}}\dots}\,, \end{aligned} \tag{2.35}$$

with

$$\begin{aligned} \delta^{\mathrm{C}}_{\phi^\pm\phi^\pm}(\Phi) &= \left[2C^{\mathrm{ew}}_\Phi - \frac{N_{\mathrm{C}}^t}{4s_{\mathrm{w}}^2}\frac{m_t^2}{M_W^2}\right] l(s) + Q_W^2 l^{\mathrm{em}}(M_W^2), \\ \delta^{\mathrm{C}}_{\chi\chi}(\Phi) &= \left[2C^{\mathrm{ew}}_\Phi - \frac{N_{\mathrm{C}}^t}{4s_{\mathrm{w}}^2}\frac{m_t^2}{M_W^2}\right] l(s)\,. \end{aligned} \tag{2.36}$$

Higgs boson

The complete correction is

$$\delta^{\mathrm{C}}_{HH}(\Phi) = \left[2C^{\mathrm{ew}}_\Phi - \frac{N_{\mathrm{C}}^t}{4s_{\mathrm{w}}^2}\frac{m_t^2}{M_W^2}\right] l(s)\,. \tag{2.37}$$

2.6 PR: Logarithms connected to the parameter renormalisation

The last ingredient is the logarithms related to the UV renormalisation. In Ref. [39] they have been identified via the formula

$$\delta^{\mathrm{PR}}\mathcal{M} = \left(\frac{\delta\mathcal{M}_0}{\delta e}\delta e + \frac{\delta\mathcal{M}_0}{\delta c_{\mathrm{w}}}\delta c_{\mathrm{w}} + \frac{\delta\mathcal{M}_0}{\delta h_t}\delta h_t + \frac{\delta\mathcal{M}_0}{\delta h_H}\delta h_H^{\mathrm{eff}}\right)\Big|_{\mu^2=s}, \tag{2.38}$$

where the quantities

$$h_t = \frac{m_t}{M_W}\,, \qquad h_H = \frac{M_H^2}{M_W^2}\,, \tag{2.39}$$

Formulas

We use the following formulas:

$$\frac{\delta M_W^2}{M_W^2} = -\left[b_W^{\rm ew} - 4C_\Phi^{\rm ew}\right] l(\mu^2) - \frac{N_{\rm C}^t}{2s_{\rm w}^2}\frac{m_t^2}{M_W^2} l(\mu^2),$$
$$\frac{\delta M_Z^2}{M_Z^2} = -\left[b_{ZZ}^{\rm ew} - 4C_\Phi^{\rm ew}\right] l(\mu^2) - \frac{N_{\rm C}^t}{2s_{\rm w}^2}\frac{m_t^2}{M_W^2} l(\mu^2), \tag{2.41}$$

and

$$\delta\alpha = \frac{2\delta Z_e}{4\pi} = \frac{1}{4\pi}\left(-b_{AA}^{\rm ew} l(\mu^2) + 2\delta Z_e^{\rm em}\right), \tag{2.42}$$

where the pure electromagnetic part reads

$$\delta Z_e^{\rm em} \equiv \begin{cases} -\frac{1}{2}\delta Z_{AA}^{\rm em} = \frac{2}{3}\sum_{f,i,\sigma\neq t} N_{\rm C}^f Q_{f_\sigma}^2 l(M_W^2, Q^2) & \text{in the } \alpha(0) \text{ scheme },\\ 0 & \text{in the } G_\mu \text{ or } \alpha(M_Z) \text{ scheme.} \end{cases} \tag{2.43}$$

In this work, all the results are presented by adopting the G_μ scheme, where in the place of α the input parameter is G_μ, which is related to α via the tree-level relation $G_\mu = \pi\alpha/(\sqrt{2}M_Z^2 c_{\rm w}^2 s_{\rm w}^2)$. This translates into the substitution

$$\frac{\delta\mathcal{M}_0}{\delta\alpha}\delta\alpha \to \frac{\delta\mathcal{M}_0}{\delta G_\mu}\delta G_\mu \quad \text{with} \quad \delta G_\mu = \frac{\delta G_\mu}{\delta\alpha}\delta\alpha + \frac{\delta G_\mu}{\delta M_Z^2}\delta M_Z^2 + \frac{\delta G_\mu}{\delta M_W^2}\delta M_W^2\,, \tag{2.44}$$

in eq. (2.40).

The remaining terms are

$$\frac{\delta m_t}{m_t} = \left[\frac{1}{4s_{\rm w}^2} + \frac{1}{8s_{\rm w}^2 c_{\rm w}^2} + \frac{3}{2c_{\rm w}^2}Q_t - \frac{3}{c_{\rm w}^2}Q_t^2 + \frac{3}{8s_{\rm w}^2}\frac{m_t^2}{M_W^2}\right] l(\mu^2)\,, \tag{2.45}$$

where on-shell renormalisation for the mass is assumed, and finally

$$\delta n_{\rm tad} = \frac{e}{2s_{\rm w}}\frac{\delta t}{M_W M_H^2}\,, \tag{2.46}$$

with the contribution from the tadpole renormalisation reading

$$\delta t = -T = \frac{1}{e s_{\rm w} M_W}\left[-\frac{3}{2}M_W^2\left(\frac{M_Z^2}{c_{\rm w}^2} + 2M_W^2\right) - \frac{M_H^2}{4}(2M_W^2 + M_Z^2 + 3M_H^2) + 2N_{\rm C}^t m_t^4\right] l(\mu^2)\,. \tag{2.47}$$

Purely Weak

1. Calculate the δ^{PR} in eq. (2.12) as in the standard SDK approach.

2. For each external particle φ_{i_k} in (2.9), set

$$Q_k = I^A(k) = 0\,. \tag{4.1}$$

   This step alone has the effect of eliminating all the terms tagged as "em", with the exception of $\delta Z^{\mathrm{em}}_{AA}$. It also eliminates all the SSC terms and C terms that lead to SL originating from photons, with the exception of those related to transverse W bosons.

3. For each external particle φ_{i_k} in (2.9), perform the replacement

$$C^{\mathrm{ew}}_{i'_k i_k}(k) \longrightarrow C^{\mathrm{ew}}_{i'_k i_k}(k) - Q_k^2\,, \tag{4.2}$$

   with the value of Q_k^2 before enforcing eq. (4.1). This, in combination with eq. (4.1), has the effect of eliminating the DL due to photons.

4. Perform the replacement

$$b^{\mathrm{ew}}_W \longrightarrow b^{\mathrm{ew}}_W - 11/3\,. \tag{4.3}$$

   This has the effect of eliminating for the transverse W bosons the C terms that lead to SL originating from photons.

5. Set

$$\delta Z^{\mathrm{em}}_{AA} = 0\,, \tag{4.4}$$

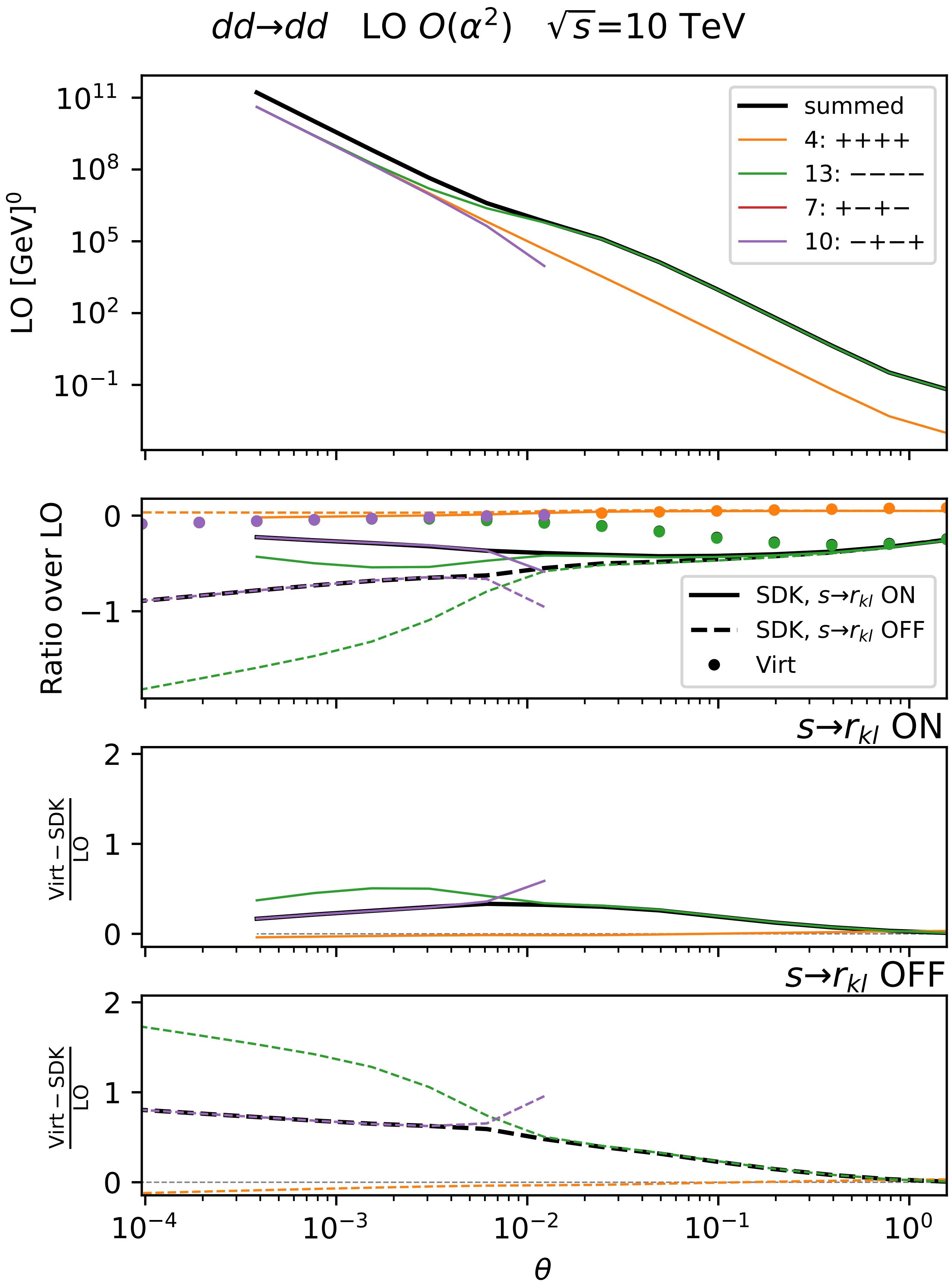
   and perform the replacement

$$b^{\mathrm{ew}}_{AA} \longrightarrow b^{\mathrm{ew}}_{AA} + \frac{4}{3}\sum_{f,i,\sigma\neq t} N_{\mathrm{C}}^f Q^2_{f_\sigma} = b^{\mathrm{ew}}_{AA} + 80/9\,. \tag{4.5}$$

   This has the effect of eliminating, for the photons, the C terms that lead to SL originating from light fermions.

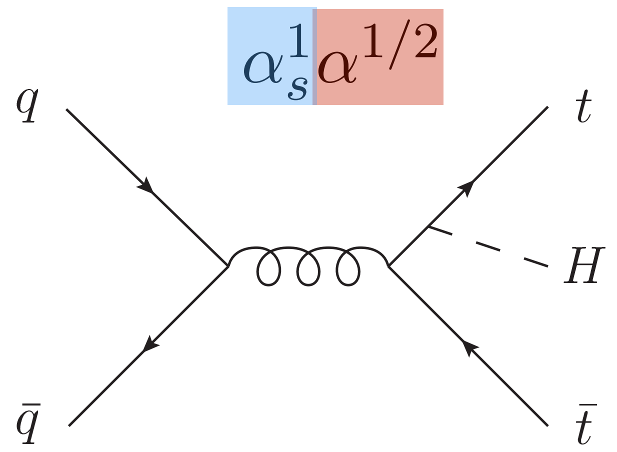
6. Calculate the remaining terms in eq. (2.12) with the new redefinitions of steps 2–5.

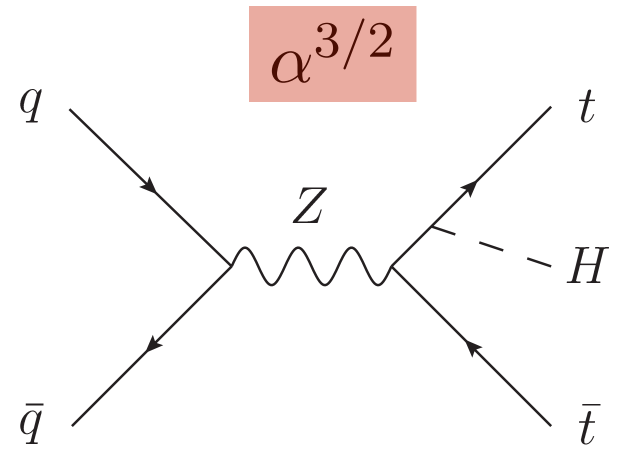
Another test of the angle dependency



Structure of NLO EW-QCD corrections

$t\bar{t}H$
as example





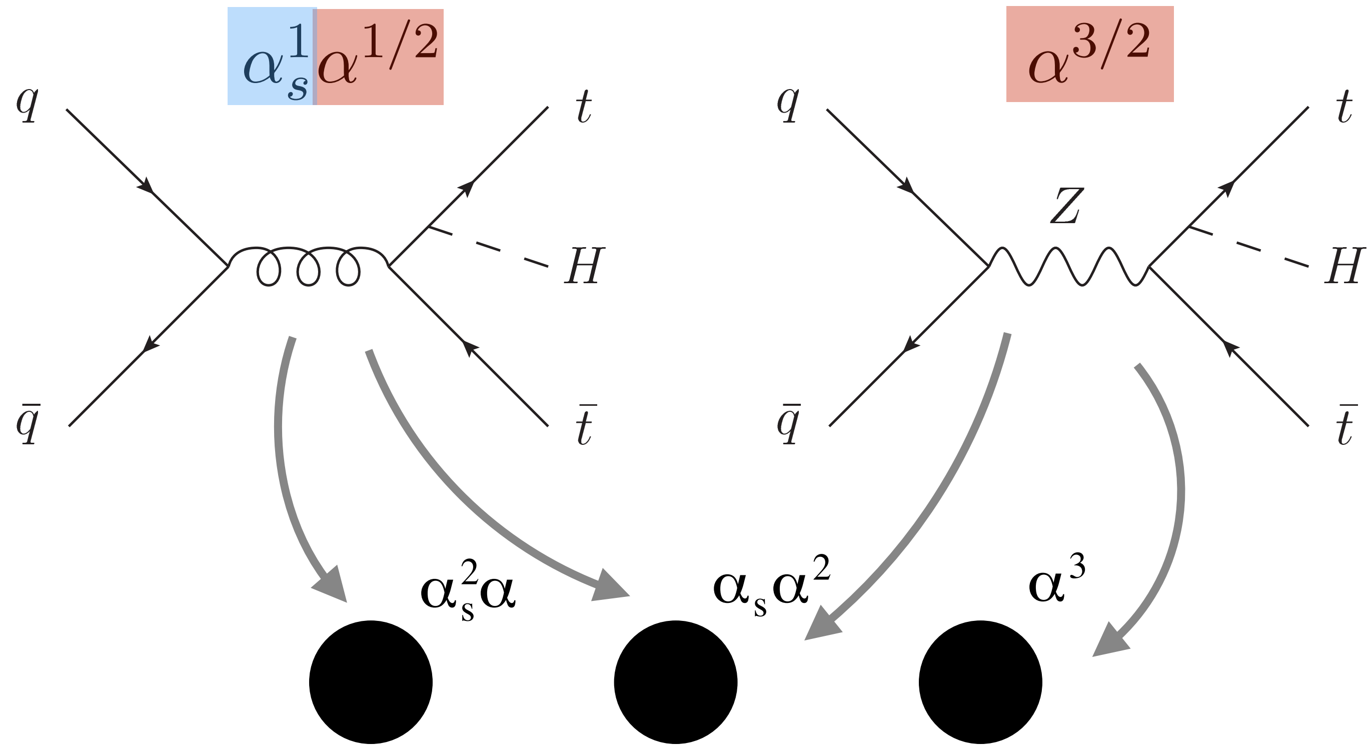
LO

$\alpha_s^2\alpha$ $\alpha_s\alpha^2$ α^3

Structure of NLO EW-QCD corrections

$t\bar{t}H$
as example

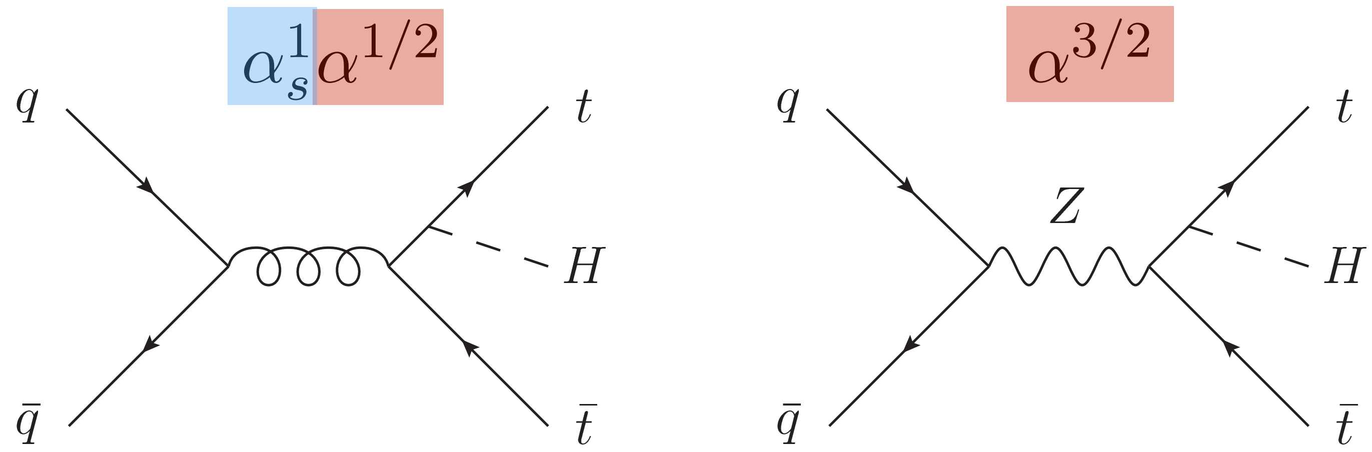
LO

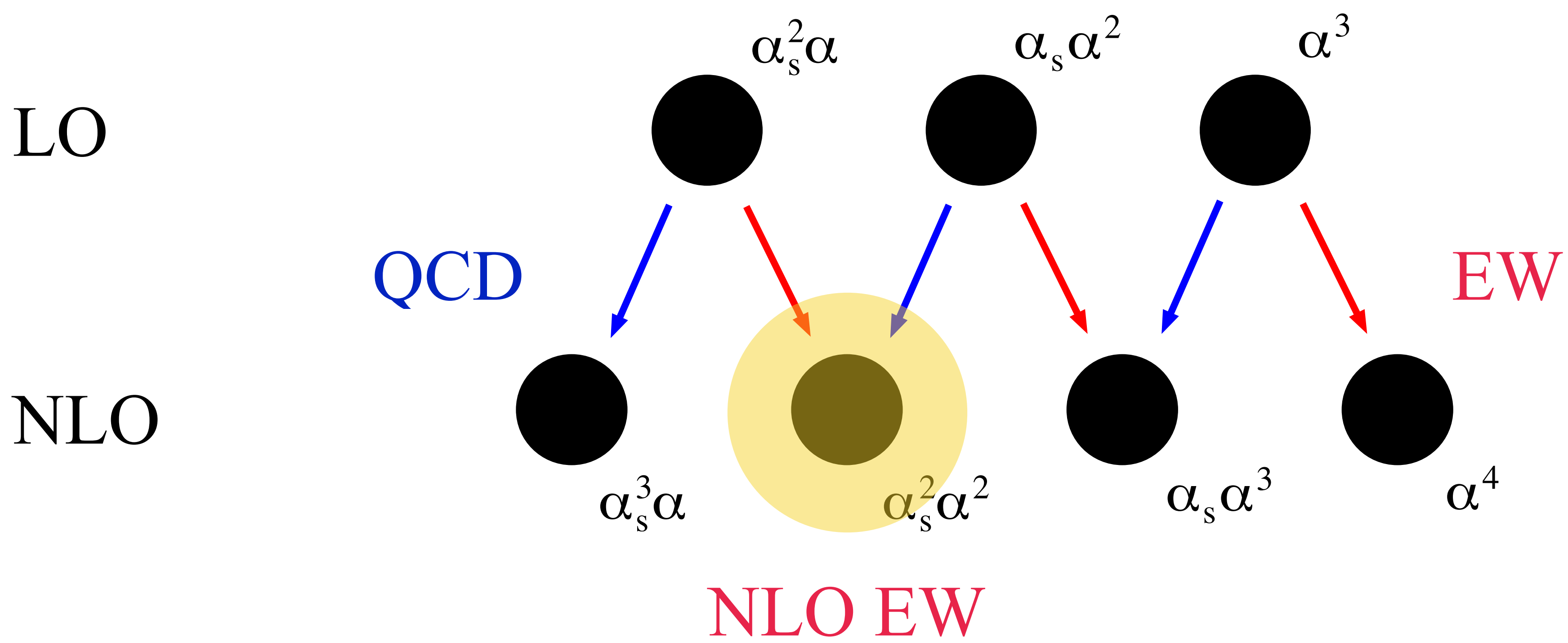


Structure of NLO EW-QCD corrections

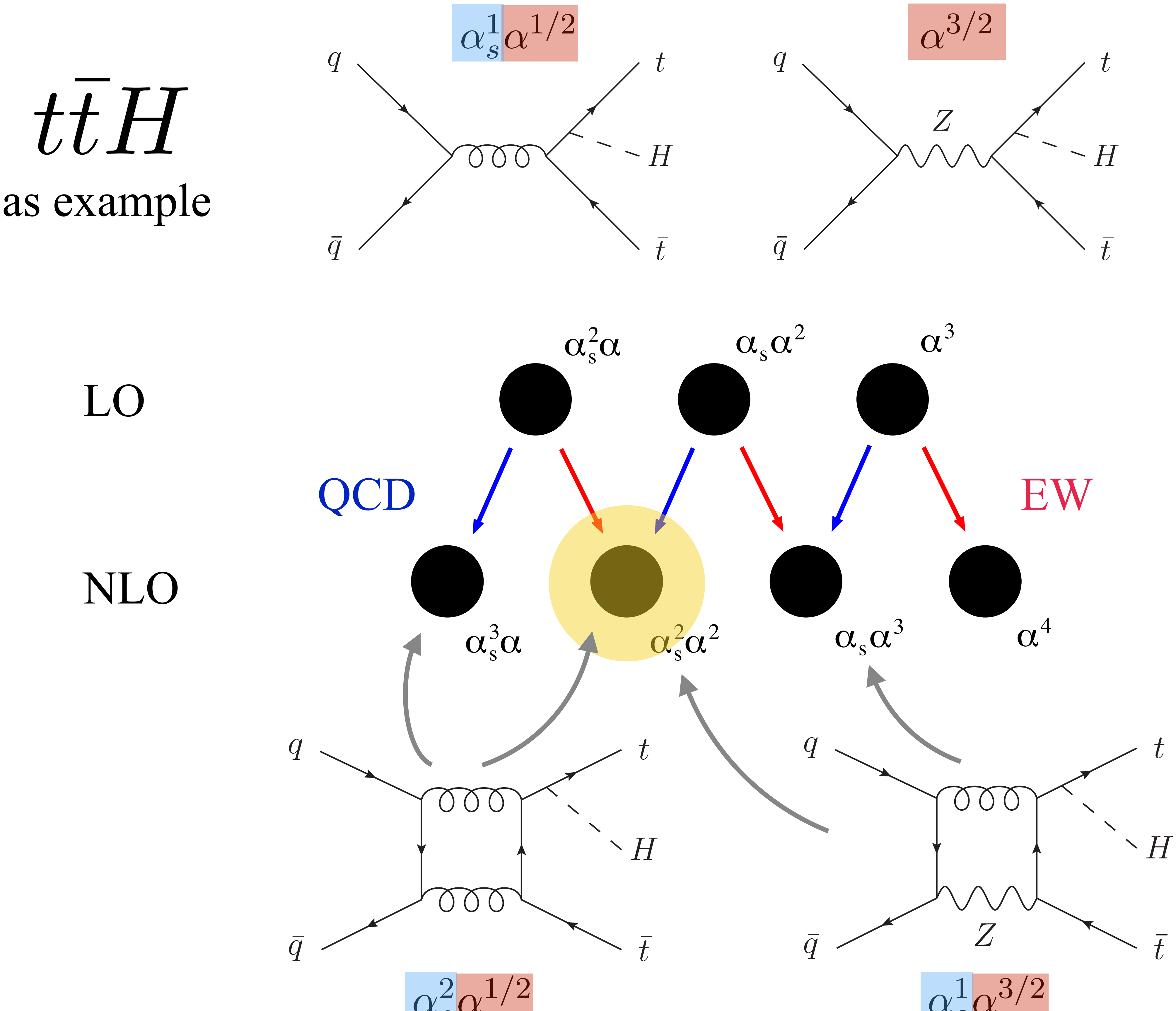
$t\bar{t}H$

as example

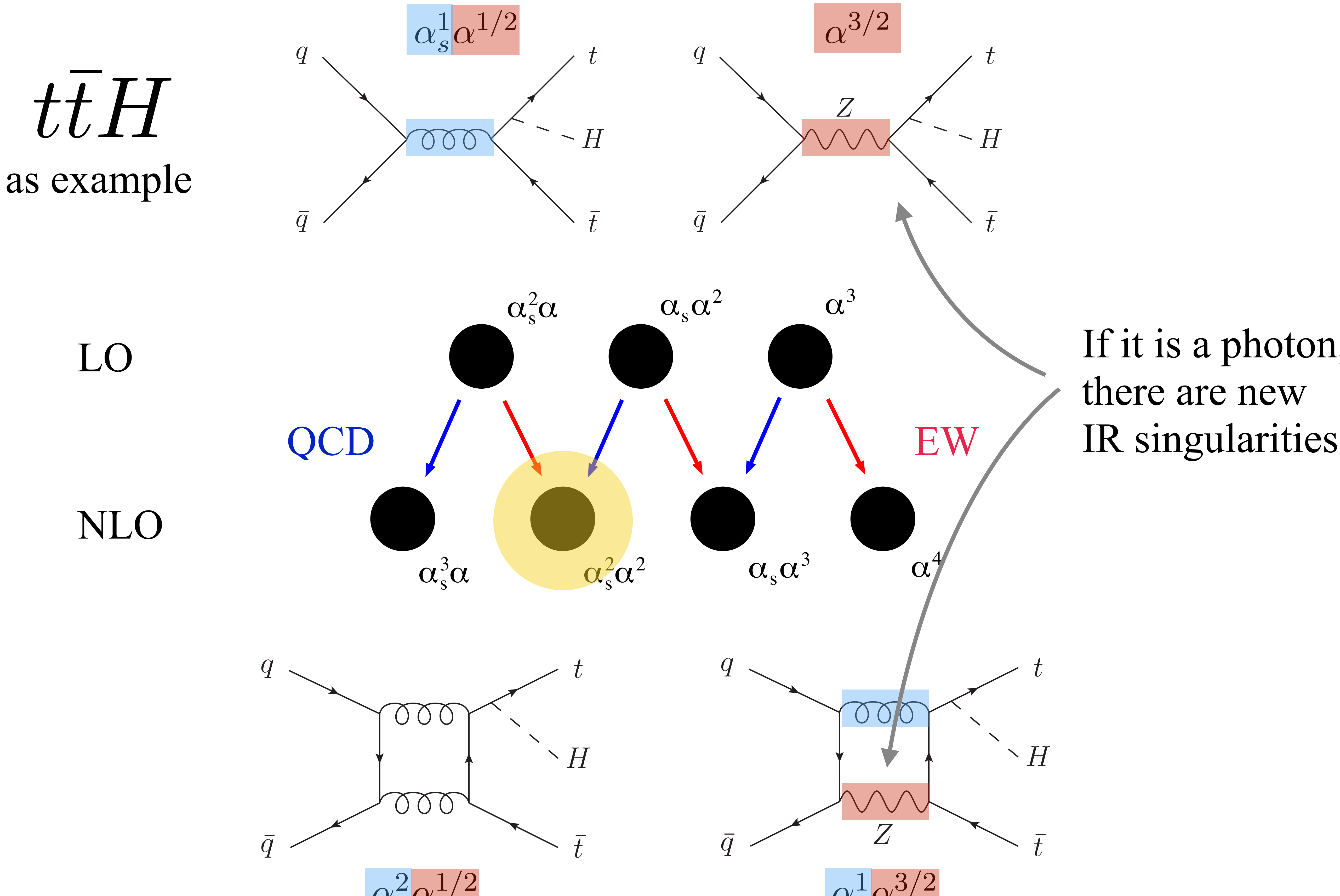




Structure of NLO EW-QCD corrections



Structure of NLO EW-QCD corrections



Complete-NLO

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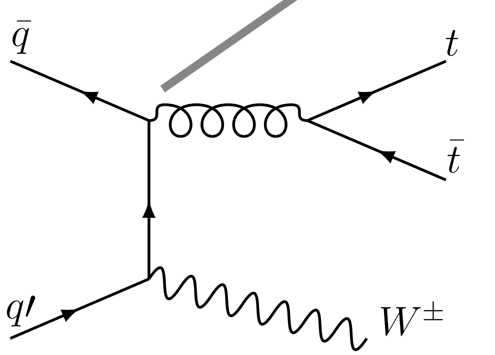
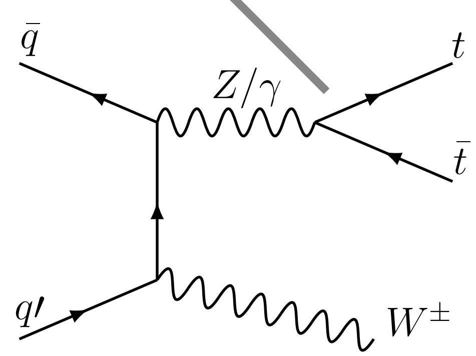
$$\Sigma_{\mathrm{LO}}^{t\bar{t}W^{\pm}}(\alpha_s,\alpha) = \alpha_s^2\alpha\Sigma_{3,0}^{t\bar{t}W^{\pm}} + \alpha_s\alpha\Sigma_{3,1}^{t\bar{t}W^{\pm}} + \alpha^2\Sigma_{3,2}^{t\bar{t}W^{\pm}}$$
$$\equiv \Sigma_{\mathrm{LO}_1} + \cancel{\Sigma_{\mathrm{LO}_2}} + \Sigma_{\mathrm{LO}_3}\,,$$

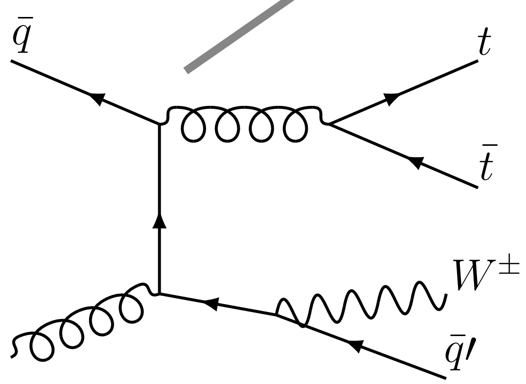


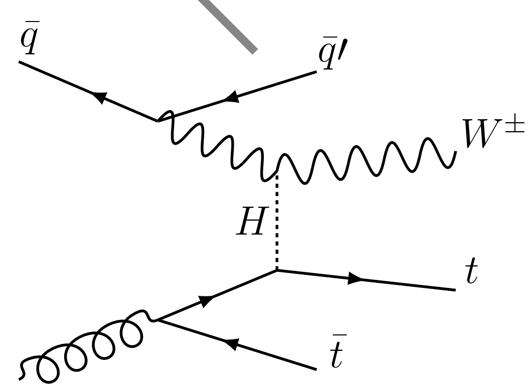


Only initial states without gluons are present.

$\Sigma_{\mathrm{LO}_1} \longrightarrow \mathrm{LO}_{\mathrm{QCD}}$

$$\Sigma_{\mathrm{NLO}}^{t\bar{t}W^{\pm}}(\alpha_s,\alpha) = \alpha_s^3\alpha\Sigma_{4,0}^{t\bar{t}W^{\pm}} + \alpha_s^2\alpha^2\Sigma_{4,1}^{t\bar{t}W^{\pm}} + \alpha_s\alpha^3\Sigma_{4,2}^{t\bar{t}W^{\pm}} + \alpha^4\Sigma_{4,3}^{t\bar{t}W^{\pm}}$$
$$\equiv \Sigma_{\mathrm{NLO}_1} + \Sigma_{\mathrm{NLO}_2} + \Sigma_{\mathrm{NLO}_3} + \Sigma_{\mathrm{NLO}_4}\,,$$





$\Sigma_{\mathrm{NLO}_1} \longrightarrow \mathrm{NLO}_{\mathrm{QCD}}$

$\Sigma_{\mathrm{NLO}_2} \longrightarrow \mathrm{NLO}_{\mathrm{EW}}$

MadGraph5_aMC@NLO

Cross sections: order by order

$$\delta_{(\mathrm{N})\mathrm{LO}_i}(\mu) = \frac{\Sigma_{(\mathrm{N})\mathrm{LO}_i}(\mu)}{\Sigma_{\mathrm{LO}_{\mathrm{QCD}}}(\mu)}$$

Numbers in parentheses refer to the case of a jet veto $p_T(j) > 100$ GeV and $|y(j)| < 2.5$ applied

13 TeV

$\delta[\%]$	$\mu = H_T/4$	$\mu = H_T/2$	$\mu = H_T$
LO_2	-	-	-
LO_3	0.8	0.9	1.1
NLO_1	34.8 (7.0)	50.0 (25.7)	63.4 (42.0)
NLO_2	$-4.4\,(-4.8)$	$-4.2\,(-4.6)$	$-4.0\,(-4.4)$
NLO_3	11.9 (8.9)	12.2 (9.1)	12.5 (9.3)
NLO_4	0.02 (-0.02)	0.04 (-0.02)	0.05 (-0.01)

Naive estimate

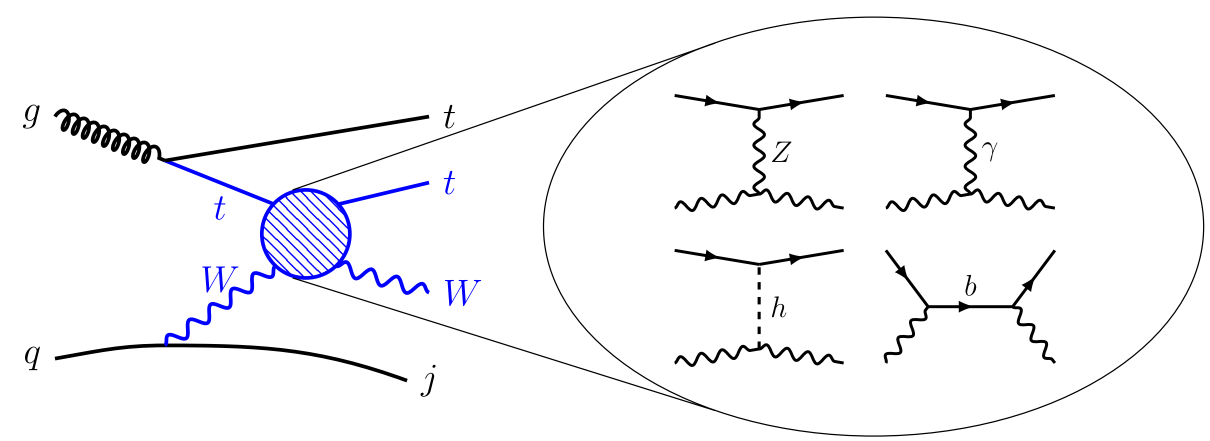
LO_2	10
LO_3	1
NLO_1	10
NLO_2	1
NLO_3	0.1
NLO_4	0.01

100 TeV

$\delta[\%]$	$\mu = H_T/4$	$\mu = H_T/2$	$\mu = H_T$
LO_2	-	-	-
LO_3	0.9	1.1	1.3
NLO_1	159.5 (69.8)	149.5 (71.1)	142.7 (73.4)
NLO_2	$-5.8\,(-6.4)$	$-5.6\,(-6.2)$	$-5.4\,(-6.1)$
NLO_3	67.5 (55.6)	68.8 (56.6)	70.0 (57.6)
NLO_4	0.2 (0.1)	0.2 (0.2)	0.3 (0.2)

NLO3 is large and it is not suppressed by the jet veto (numbers in parentheses) as much as NLO QCD corrections.

NLO QCD corrections depend on the scale, while NLO EW and NLO3 do not.



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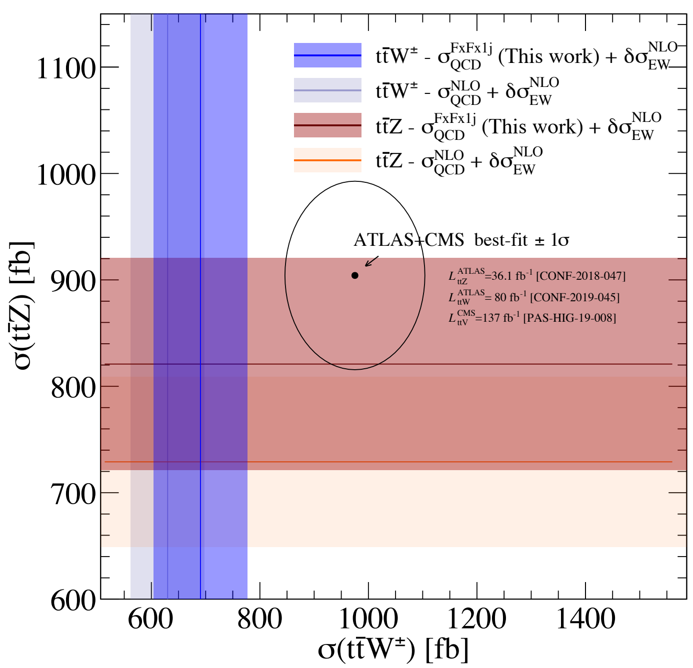
NLO QCD + Jet merging +EW

[Buddenbrock, Ruiz, Mellado '20]

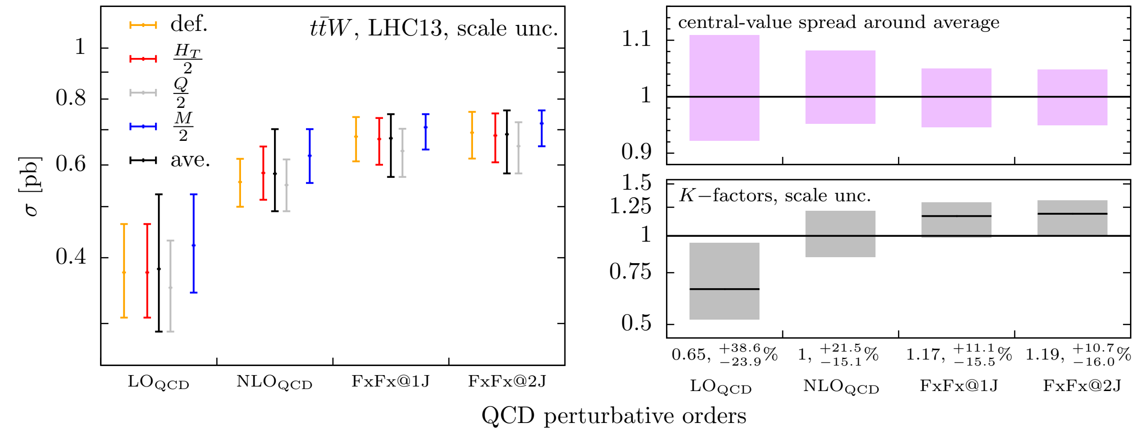


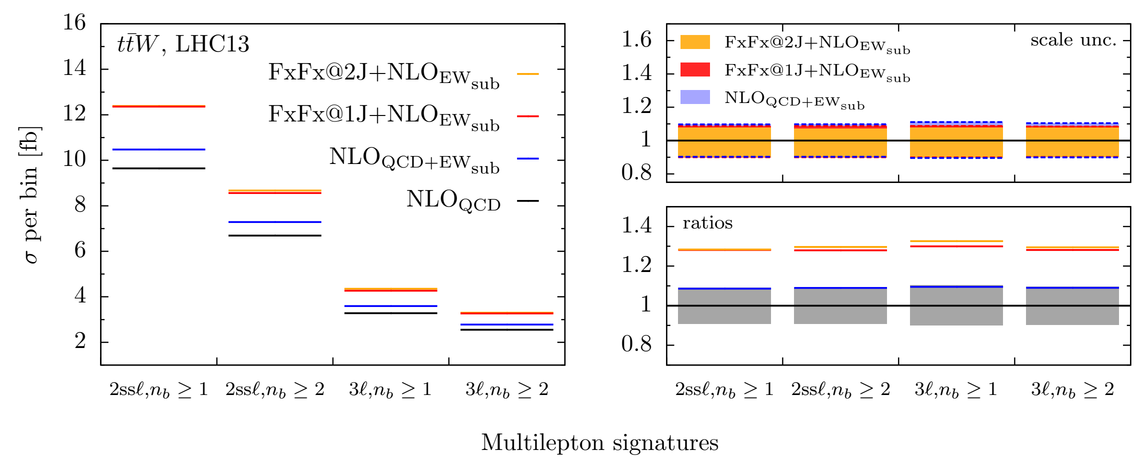
$\sqrt{s} = 13$ TeV

Light: NLO QCD+EW
Dark: NLO QCD+FxFX1j+ EW

↪ Moving in the right direction but still tension wrt ATLAS+CMS results.

[Tsnikos, Rikkert '21]



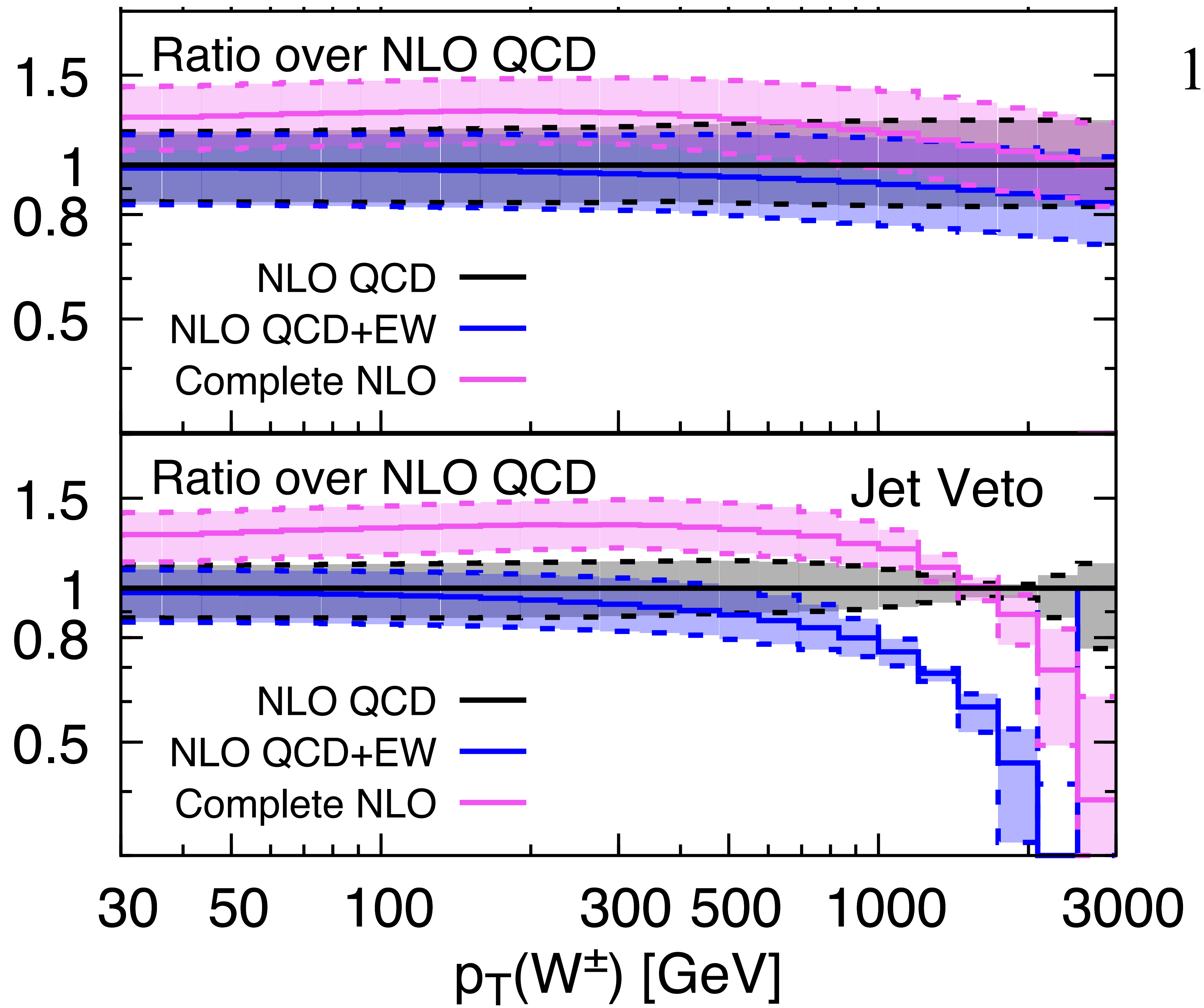


↪ Tension partially resolved
↪ Improved scale behavior

Strong indication that NNLO QCD corrections will bring better agreement with SM predictions.

Slides taken from L.Reina's talk at WG1 fall meeting 2021

Distributions



100 TeV

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