Muonic hydrogen Lamb shift

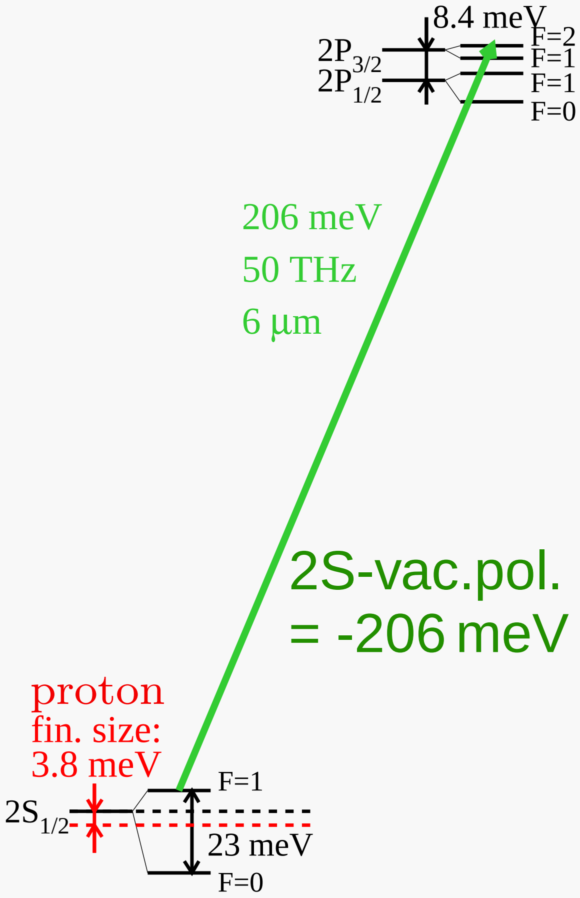
F. Kottmann, IPP at ETH Zürich, Switzerland

1947	Lamb shift = ΔE(2S$_{1/2}$-2P$_{1/2}$) in H → QED
1969	Di Giacomo calculates ΔE(2S-2P) = 0.2 eV in μp → $\lambda = 6\,\mu$m ! V. Hughes, V. Telegdi, E. Zavattini consider μp(2S-2P) → $\tau_{2\mathrm{S}}$ = ?
1979	Proposal for μp(2S-2P) at SIN [H. Hofer et al.] (0.3 mbar)
1981	SIN: no long-lived μp(2S) at $\sim$ mbar; problems with laser development
mid-80's	no motivation for "test of vac.pol." at 100 ppm-level

mid-90's	• big progress in H-spectroscopy [Haensch et al.] → new motivation: determine r_p precisely (2 % → 0.1 %) • new μ^--beams, new ideas for μp(2S-2P) [L. Simons, D. Taqqu, F.K.]
1998	new Proposal for μp(2S-2P) at PSI [new collaboration: MPQ, Paris, Coimbra, FR...]
2009	2S-2P resonance found, 5σ off ! (nothing in 2003, 2007) → unexpected new situation, new motivation: solve problem !

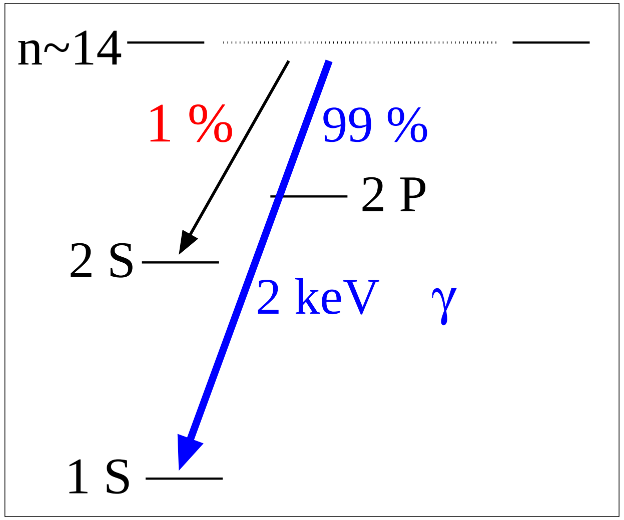
Principle of μp(2S-2P) experiment

- special low-energy μ^- beam-line at PSI (unpulsed !)
- μ^- detected in-flight → trigger of laser system
- μ^-p atoms formed in 1 mbar H_2 gas
- laser pulse excites the 2S-2P transition ($\lambda \approx 6\,\mu$m)
- delayed 2P-1S X-ray detected: signature



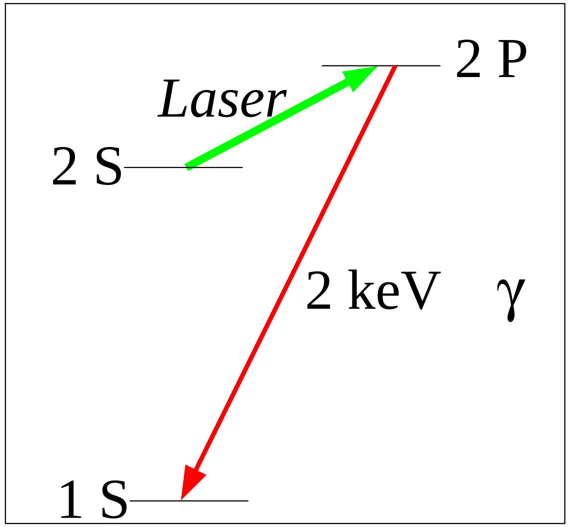
2S-vac.pol.
= -206 meV

“prompt” ($t \sim 0$)



1% with $\tau_{2S} = 1\,\mu$s

“delayed” ($t \sim 1\,\mu$s)

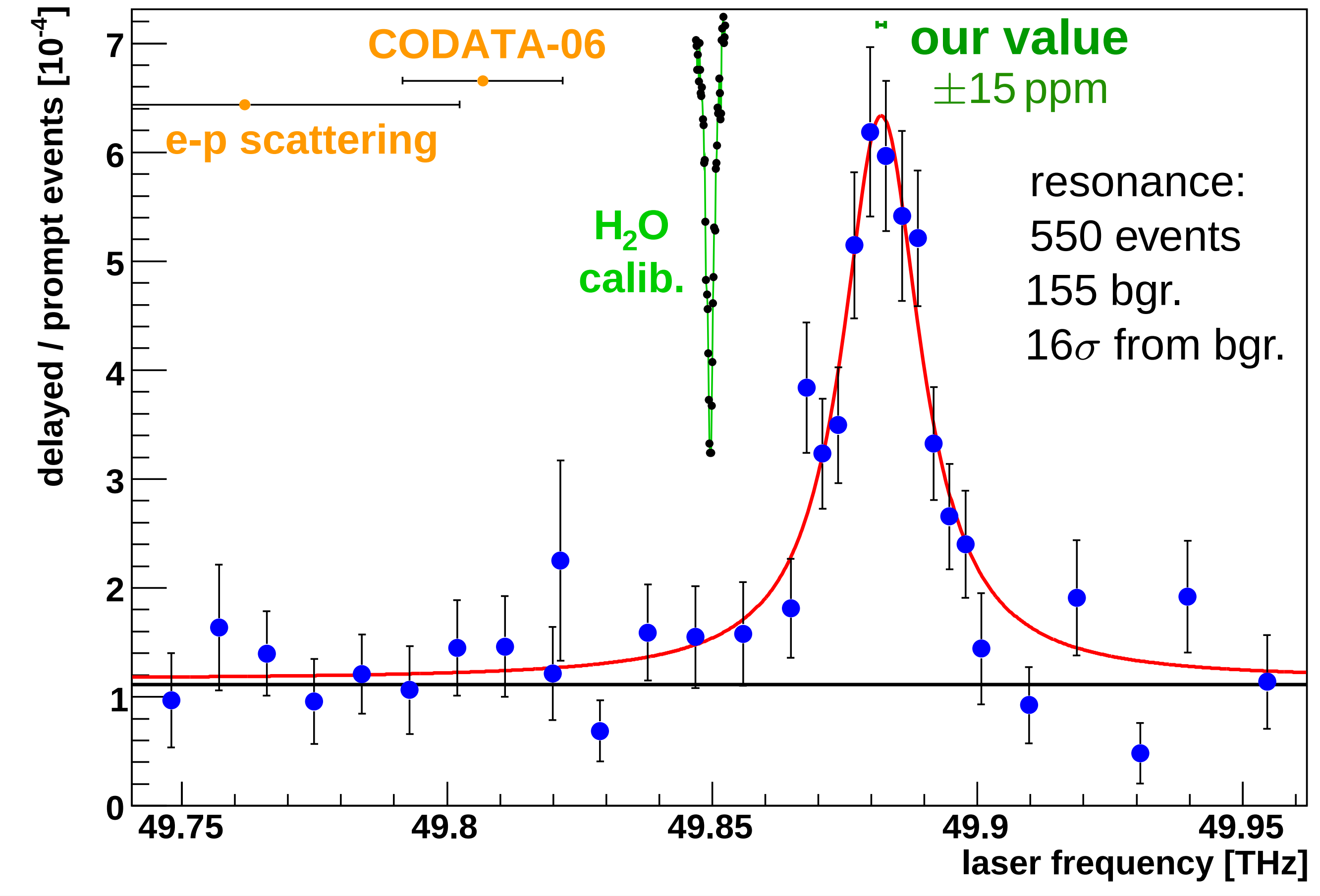


normalize delayed/prompt

linewidth = Γ_{2P} = 18.6 GHz

→ 6 transitions separated !

Measured resonance $\mu p(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2})$



Reference:
R. Pohl, A. Antognini,
F. Nez, D. Taqqu, et al.,
Nature **466**, 213 (2010)

Collaboration:

- MPQ Garching
- LKB Paris
- Coimbra and Aveiro
- Stuttgart
- Fribourg
- Yale
- PSI - ETHZ - ...

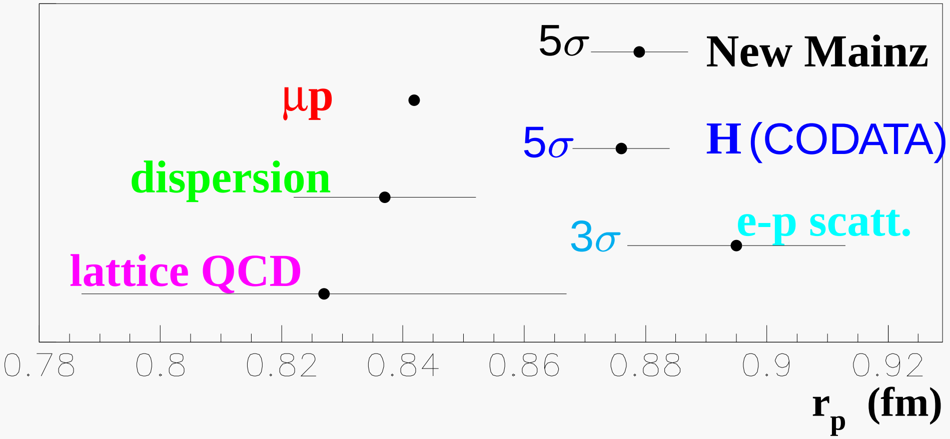
Statistics: ± 0.70 GHz
Systematics: ± 0.30 GHz (laser calibration)

Discrepancy (to CODATA-06):
$\sim 75\ \mathrm{GHz} \ \leftrightarrow\ 5.0\,\sigma \ \leftrightarrow\ \delta\nu/\nu = 1.5\times10^{-3}$

Proton radius puzzle

Measured μp resonance: $\nu(2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}) = 49881.88(76)$ GHz

$$\left.\begin{array}{l} \rightarrow L^{\text{exp.}} = 206.2949(32)\ \text{meV} \\ L^{\text{th.}} = 209.9779(49) - 5.2262\, r_p^2 + 0.0347\, r_p^3\ \text{meV} \end{array}\right\} \Rightarrow \boxed{r_p = 0.84184(36)^{\text{exp}}(56)^{\text{th}}\ \text{fm}}$$



- Bernauer et al., arXiv nucl-ex, 1007.5076 (2010)
- Pohl et al., Nature **466**, 213 (2010)
- Mohr at al., Rev. Mod. Phys. **80**, 633 (2008)
- Belushkin et al., Phys. Rev. C **75**, 035202 (2007)
- Sick, Phys. Lett. B **576**, 62 (2003)
- Wang et al., Phys. Rev. D **79**, 094001 (2009)

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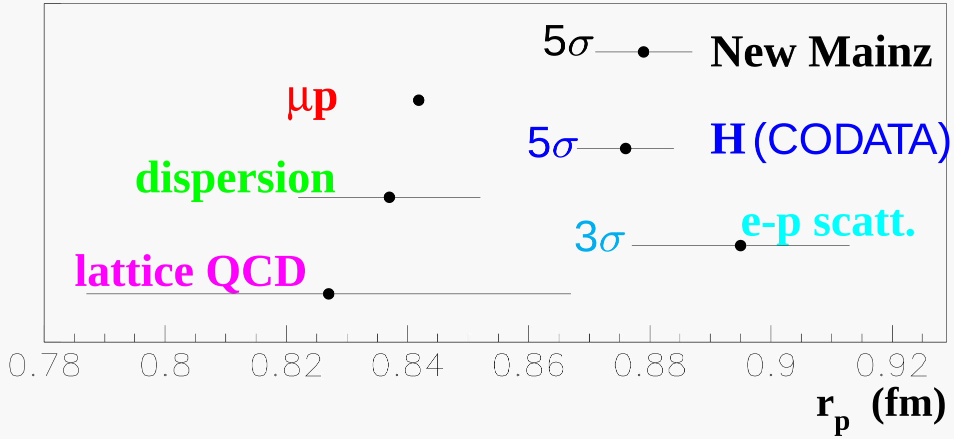

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What is wrong,
by ~75 GHz ?

μp, systematics ?

• laser calibration	0.300 GHz
• Zeeman effect ($B = 5$ T)	0.030 GHz
• AC-Stark, DC-Stark shift	< 0.001 GHz
• Doppler shift	< 0.001 GHz
• pressure shift (1 mbar)	0.001 GHz
• molecules {pμp*-p-e}	< 0.001 GHz

→ NO !

What may be wrong?

Discrepancy: $$L^{\text{th.}}_{\mu p}(r_{\text{p}}^{\text{CODATA}}) - L^{\text{exp.}}_{\mu p} = \begin{cases} 75\ \text{GHz} \\ 0.31\ \text{meV} \\ 0.15\ \% \end{cases}$$

μp exp. wrong ? but

- good statistics
- linewidth $\sim$ 19 GHz $\ll$ discrepancy
- several methods for frequency calibration
- another μp(2S-2P) measured !

μp theory wrong ? but

- mainly pure QED (vac.pol., etc.)
- 'huge' relative discrepancy
- hadronic terms small
- pol. term = 0.015(4) meV
- weak interactions very small

H spectroscopy wrong ? but

- 2S-8S, 2S-8D, 2S-12S, etc. all consistent ...

H theory wrong ? but

- uncertainties 10$\times$ smaller than discrepancy ...

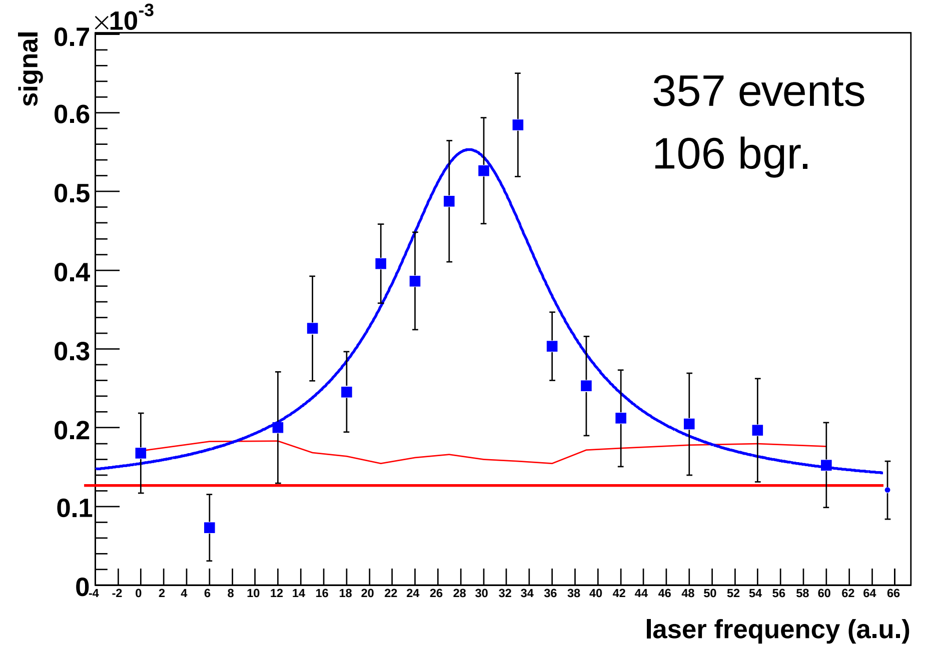
e-p scattering wrong ? but

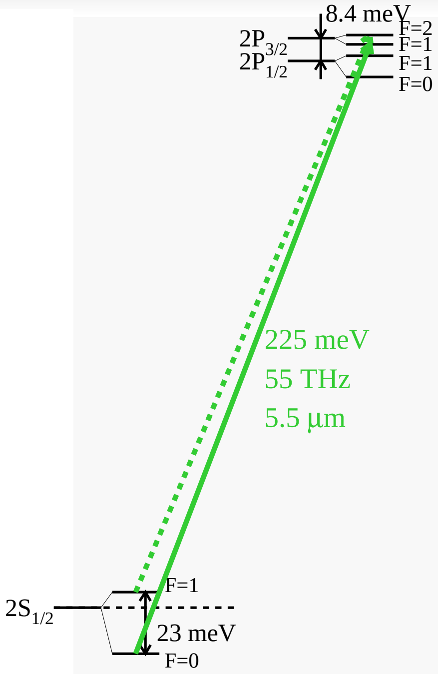
- new Mainz result ...

Second Resonance, Conclusions, Outlook

Second resonance measured at $\lambda = 5.5\,\mu$m: μp($2S_{1/2}^{F=0} \rightarrow 2P_{3/2}^{F=1}$)

- prelimary analysis
- position in agreement with first resonance
- extract HFS and r_{Zemach}





Second Resonance, Conclusions, Outlook

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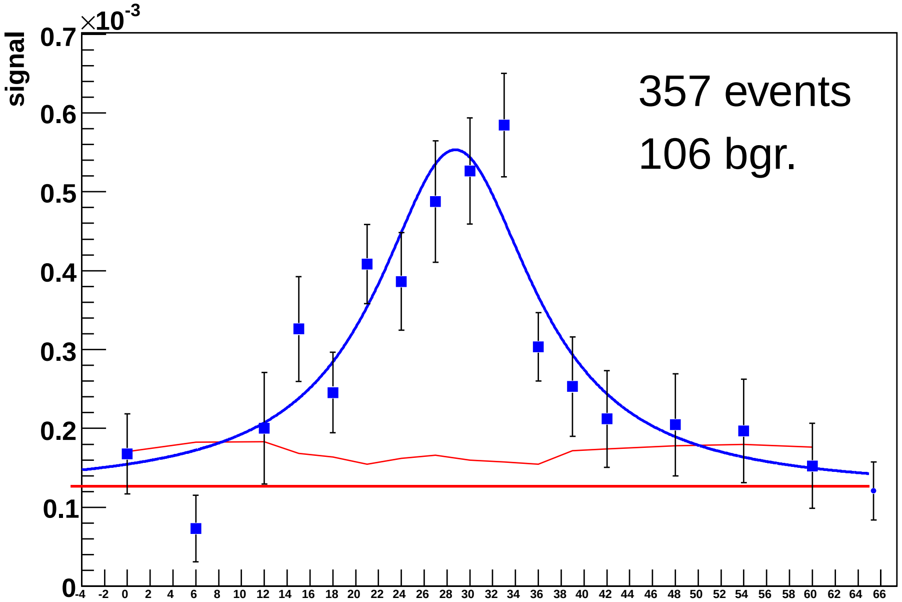

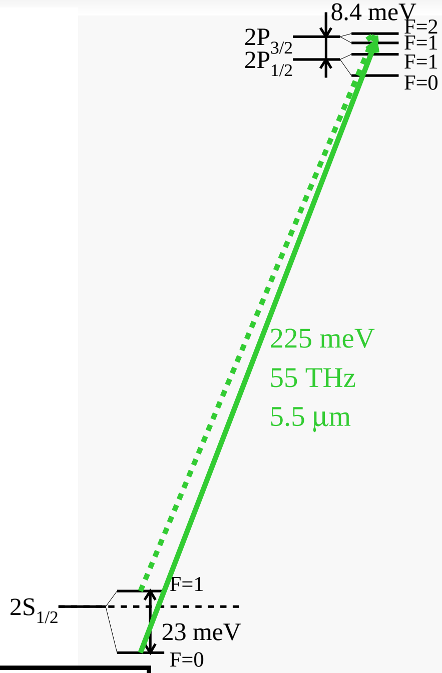

Conclusion from μp(2S-2P): r_{p}-discrepancy

→ μp theory wrong ? H spectroscopy wrong ? e-p wrong ?

when solved:
- best test of bound-state QED (combine μp and H)
- fundamental constants (R_∞)
- test of lattice QCD for p, few-nucleon theory for d

2009: 3 resonances measured in μd(2S-2P) → r_{d}, d-polarizabilities

future: μHe$^+$(2S-2P)
- more sensitivity to QED-effects, less to R_∞ (in He$^+$)
- test of few-nucleon theory for ^{3}He, ^{4}He

There are surprises in physics.

Collaboration

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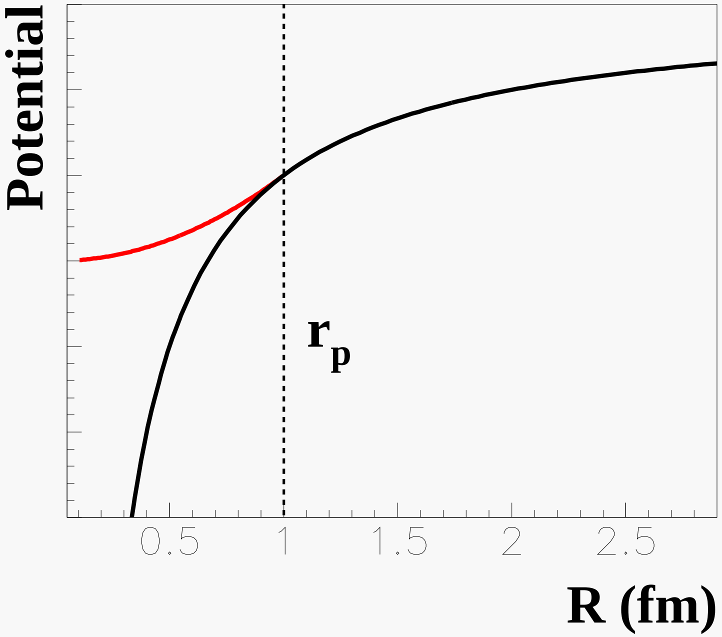
D. Taqqu	PSI, Switzerland
F. Kottmann	IPP, ETH Zürich, Switzerland

The leading proton finite size contribution



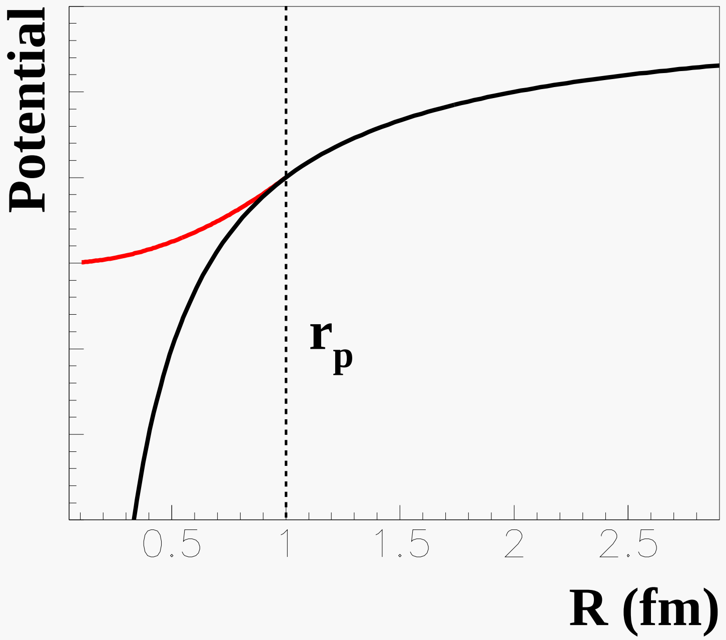
Maxwell equation: $\nabla E = 4\pi\rho$

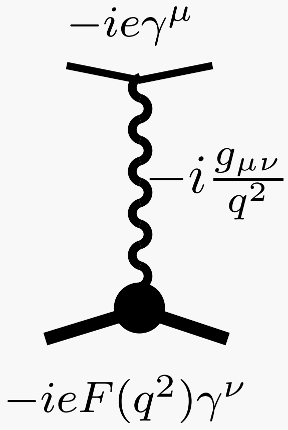
$$V = \begin{cases} -\frac{Z\alpha}{2r_{\mathrm{p}}}\left(3-(\frac{r}{r_{\mathrm{p}}})^2\right) & (r < r_{\mathrm{p}}) \\ -\frac{Z\alpha}{r} & (r > r_{\mathrm{p}}) \end{cases}$$

$$\Delta V = \begin{cases} -\frac{Ze^2}{2r_{\mathrm{p}}}\left(3-(\frac{r}{r_{\mathrm{p}}})^2 - \frac{2r_{\mathrm{p}}}{r}\right) \\ 0 \end{cases}$$

$$\Delta E^{FS} = \langle\bar{\Psi}|\Delta V|\Psi\rangle$$

The leading proton finite size contribution





$$\frac{1}{q^2} \longrightarrow \frac{F(q^2)}{q^2}$$

$$\boxed{r_{\mathrm{p}}^2 \equiv \int d^3r\, \rho(\mathbf{r}) r^2}$$

$$F(\mathbf{q^2}) = \int d^3r\, \rho(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}} \simeq Z(1 - \frac{\mathbf{q^2}}{6} r_{\mathrm{p}}^2 + \cdots)$$

Maxwell equation: $\nabla E = 4\pi\rho$

$$V = \begin{cases} -\frac{Z\alpha}{2r_{\mathrm{p}}}\left(3 - (\frac{r}{r_{\mathrm{p}}})^2\right) & (r < r_{\mathrm{p}}) \\ -\frac{Z\alpha}{r} & (r > r_{\mathrm{p}}) \end{cases}$$

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$$\Delta E^{FS} = \langle\bar{\Psi}|\Delta V|\Psi\rangle$$

$$\Delta V(r) = V(r) - \left(-\frac{Z\alpha}{r}\right)$$

$$\Delta V(\mathbf{q}) = \frac{4\pi Z\alpha}{\mathbf{q}^2}\,(1 - F(\mathbf{q})) \simeq \frac{2\pi(Z\alpha)}{3} r_{\mathrm{p}}^2$$

$$\Delta V(r) = \frac{2\pi(Z\alpha)}{3}\; r_{\mathrm{p}}^2\; \delta(r)$$

$$\boxed{\begin{aligned} \Delta E^{FS} &= \frac{2\pi(Z\alpha)}{3}\; r_{\mathrm{p}}^2\; |\Psi_n(0)|^2 \\ &= \frac{2(Z\alpha)^4}{3n^3} m_r^3\; r_{\mathrm{p}}^2\; \delta_{l0} \end{aligned}}$$

Free and bound-state QED

- Free QED

$g-2$ → electron anomaly: test of QED, determination of α, NP

$$a_e = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \Delta(\mathrm{had.}, \mathrm{NP})$$

$$u[a_e^{\mathrm{exp}}] = 6.6\times 10^{-10}, \quad u[a_e^{\mathrm{th}}] = 2.4\times 10^{-10}, \qquad u[\mathrm{QED\ test}] = 7\times 10^{-9}$$

- Bound-state QED in Hydrogen

  - Binding effects ($Z\alpha$) bad convergence, all-order approach/expansion
  - Radiative corrections (α and $Z\alpha$)
  - Recoil corrections (m/M and $Z\alpha$) relativity $\not\Leftrightarrow$ two-body system
  - Radiative–recoil corrections (α, m/M and $Z\alpha$)
  - Proton structure corrections (r_{p}, r_{Zemach} and $Z\alpha$)

The role of nuclear physics in atomic physics

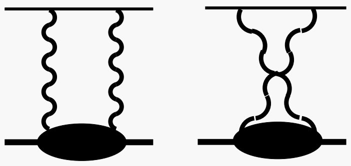
Atomic physics means high-precison measurements.
However their interpertations are usually limited by nuclear-physics effects

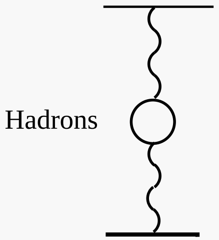
Interpertation of H, D, ^{3,4}He$^+$, μp, μd, $\mu^{3,4}$He$^+$:

- Lamb shifts limited by: r^2, shape, nucl. pol., hadronic VP pol.
- HFS limited by: Zemach radius, shape, nucl. pol., hadronic VP pol.

muonic atoms

nucl. physics





Zemach radius $\sim \int_0^\infty \frac{dQ}{Q^2}\left[G_E(-Q^2)\frac{G_M(-Q^2)}{1+\kappa_p}-1\right]$

Nucl. Pol. $\sim T^A_{\mu\nu} = \frac{i}{m_p}\epsilon_{\nu\mu\alpha\beta}\left[(G_1(\nu,q^2)+G_2(\nu,q^2))S^\beta - G_2(\nu,q^2)\frac{S\cdot q^2\, p^\beta}{p\cdot q}\right] \quad \mathrm{Im}G_1 = \frac{1}{\nu}g_1(\nu,q^2)\ldots$

ab-inition calcultion for d and He very promising

Contributions to the μp Lamb shift

P. Indelicato, 2010

#	Contribution	Value	Unc.
3	Relativistic one loop VP	205.0282	
4	NR two-loop electron VP	1.5081	
5	Polarization insertion in two Coulomb lines	0.1509	
6	NR three-loop electron VP	0.00529	
7	Polarisation insertion in two and three Coulomb lines (corrected)	0.00223	
8	Three-loop VP (total, uncorrected)		
9	Wichmann-Kroll	-0.00103	
10	Light by light electron loop ((Virtual Delbrück)	0.00135	0.00135
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	-0.00500	0.0010
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	-0.00150	
13	Mixed electron and muon loops	0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	0.01077	0.00038
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	0.000047	
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	-0.000015	
17	Recoil contribution	0.05750	
18	Recoil finite size	0.01300	0.001
19	Recoil correction to VP	-0.00410	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	-0.66770	
21	Muon Lamb shift 4th order	-0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	-0.04497	
23	Recoil of order α^6	0.00030	
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	-0.00960	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability)	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	0.00019	
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	-0.00001	
	Sum	206.0573	0.0045

Contributions to the μp Lamb shift

Contribution	our selection		Pachucki	Borie
Leading nuclear size contribution	-5.19745	$< r_{\rm p}^2 >$	-5.1974	-5.1971
Radiative corrections to nuclear finite size effect	-0.0275	$< r_{\rm p}^2 >$	-0.0282	-0.0273
Nuclear size correction of order $(Z\alpha)^6 < r_{\rm p}^2 >$	-0.001243	$< r_{\rm p}^2 >$		
Total $< r_{\rm p}^2 >$ contribution	-5.22619	$< r_{\rm p}^2 >$	-5.2256	-5.2244
Nuclear size correction of order $(Z\alpha)^5$	0.0347	$< r_{\rm p}^3 >$	0.0363	0.0347
Nuclear size correction of order $(Z\alpha)^6 < r_{\rm p}^4 >$	-0.000043	$< r_{\rm p}^2 >^2$		

P. Indelicato, 2010

Contributions to the μp Lamb shift

Lamb shift: $\Delta E_{LS} = 206.0573(45) - 5.2262\, r_{\mathrm{p}}^2 + 0.0347\, r_{\mathrm{p}}^3$ meV

$u = 0.0045$ meV dominated by proton polarizability

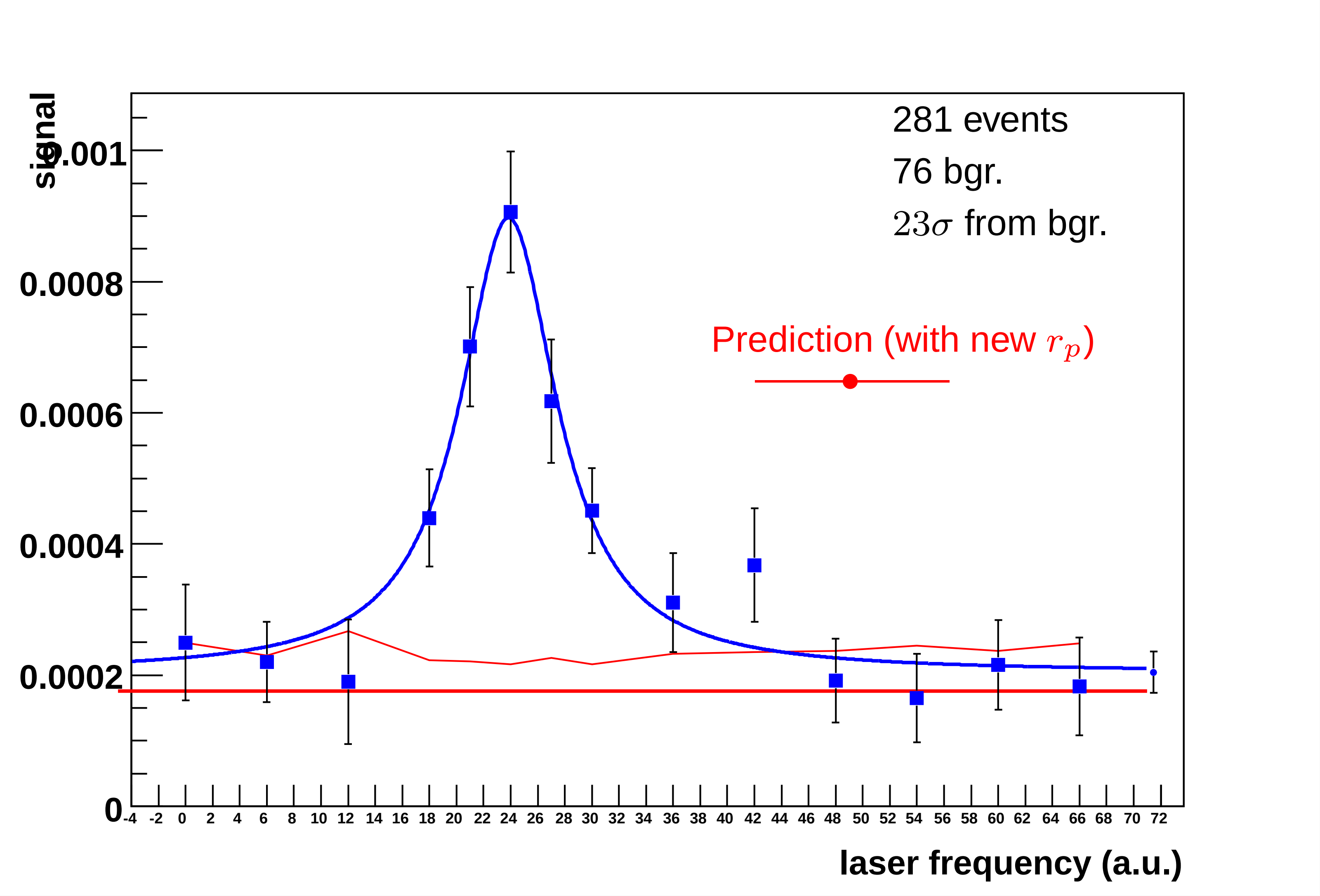
$2S$ Hyperfine structure: $\Delta E_{HFS}^{2S} = 22.8148\,(78)$ meV

using $R_Z = 1.022$ fm and scatter.

Fine structure: $\Delta E_{FS} = 8.352082$ meV

$2P_{3/2}$ Hyperfine structure: $\Delta E_{HFS}^{2P_{3/2}} = 3.392588$ meV

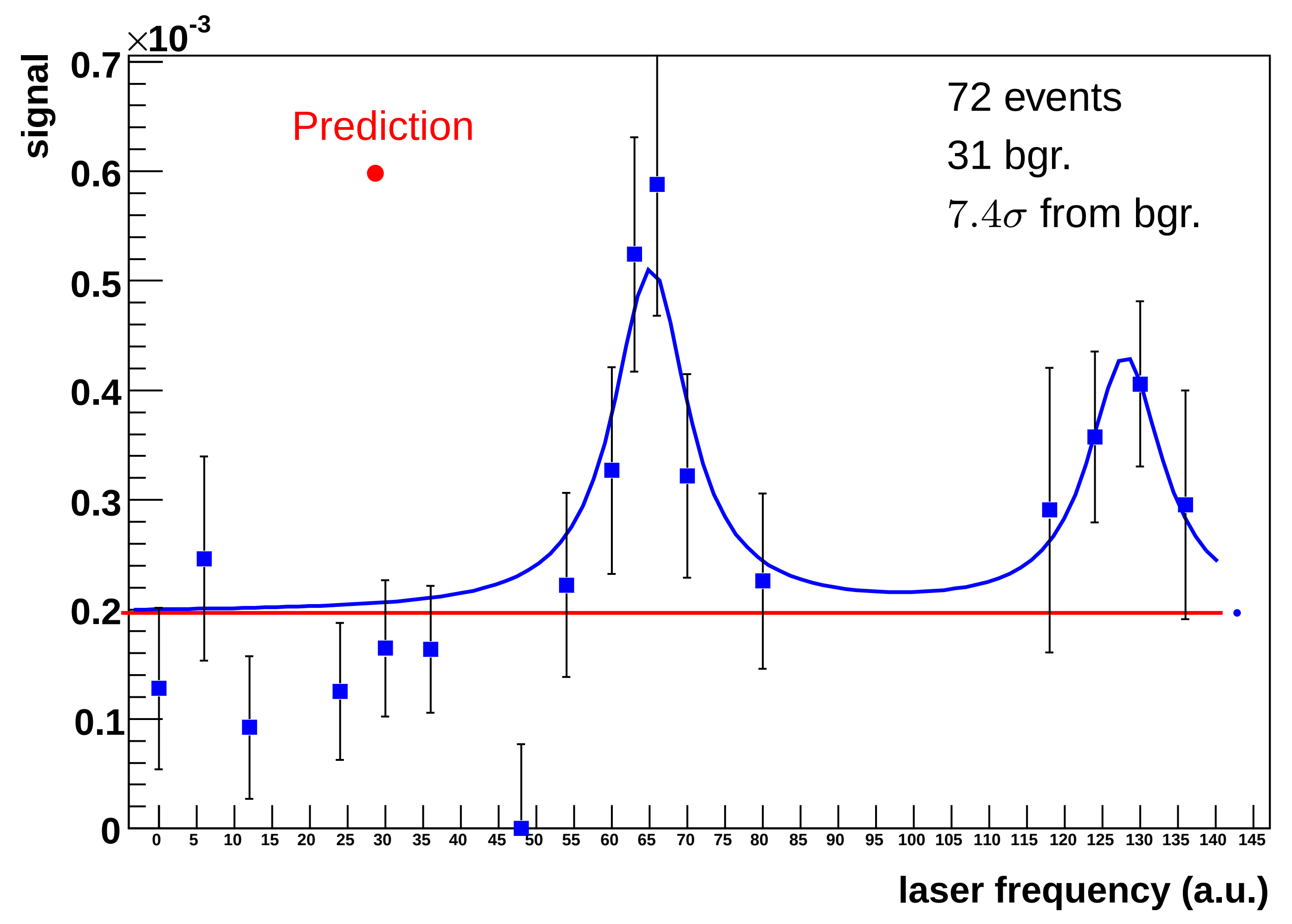
μd ($2S_{1/2}$(F=3/2) $\rightarrow$ $2P_{3/2}$(F=5/2))



- $\sigma_{\text{position}} = 880$ MHz $\Longleftrightarrow$ 17 ppm ($\Gamma = 19$ GHz)
- Position does not fit with prediction: 3.5σ deviation

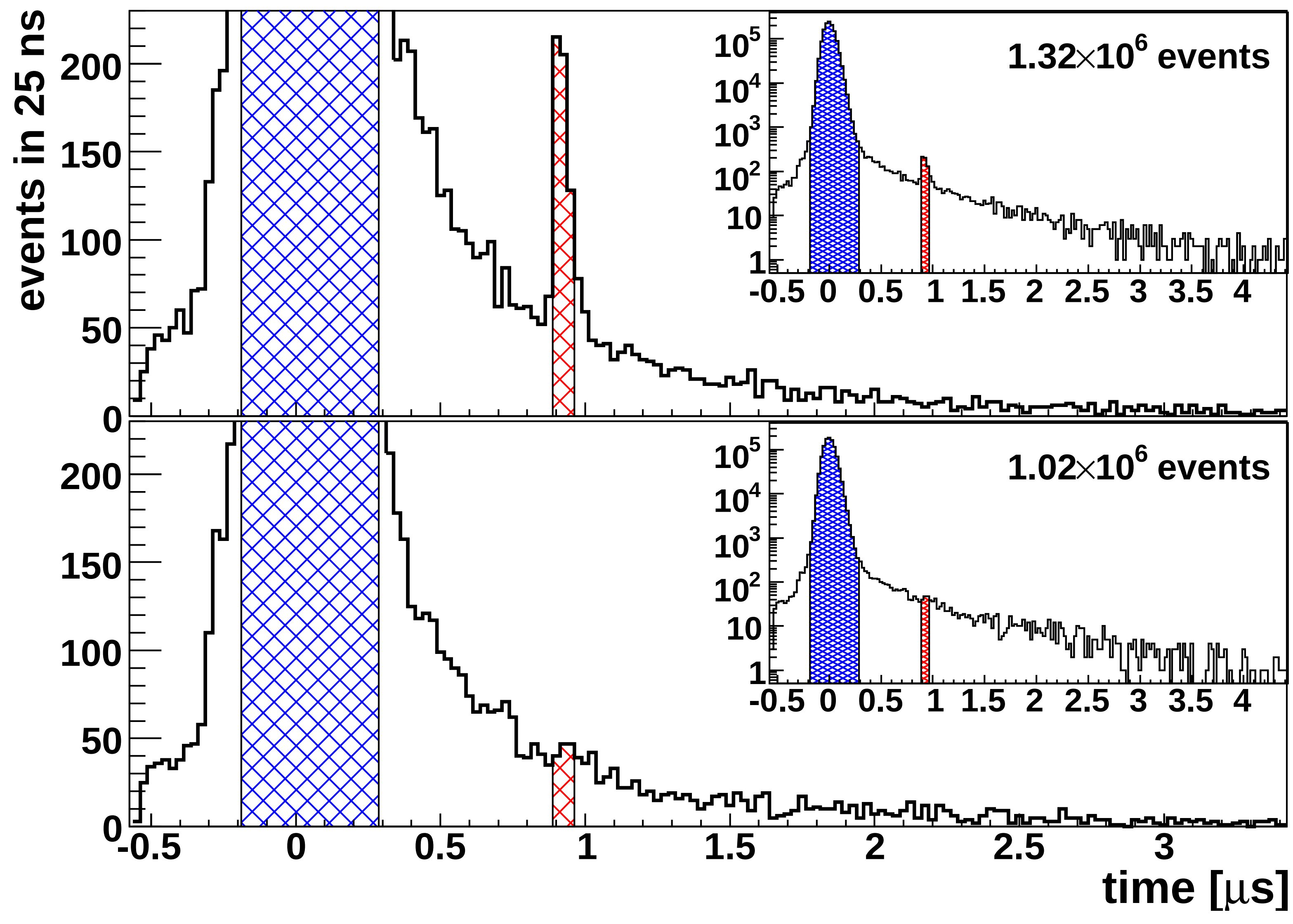
Extract r_{d} and d. pol.

μd ($2S_{1/2}$(F=1/2) $\rightarrow$ $2P_{3/2}$(F=3/2 and 1/2))



- $\sigma_{\text{position}} = 2.2$ GHz $\Longleftrightarrow$ 43 ppm ($\Gamma = 19$ GHz)
- Relative pos. fit to each others but not with the first μd line
- Background well know from previous μd line

. . . and the time spectrum

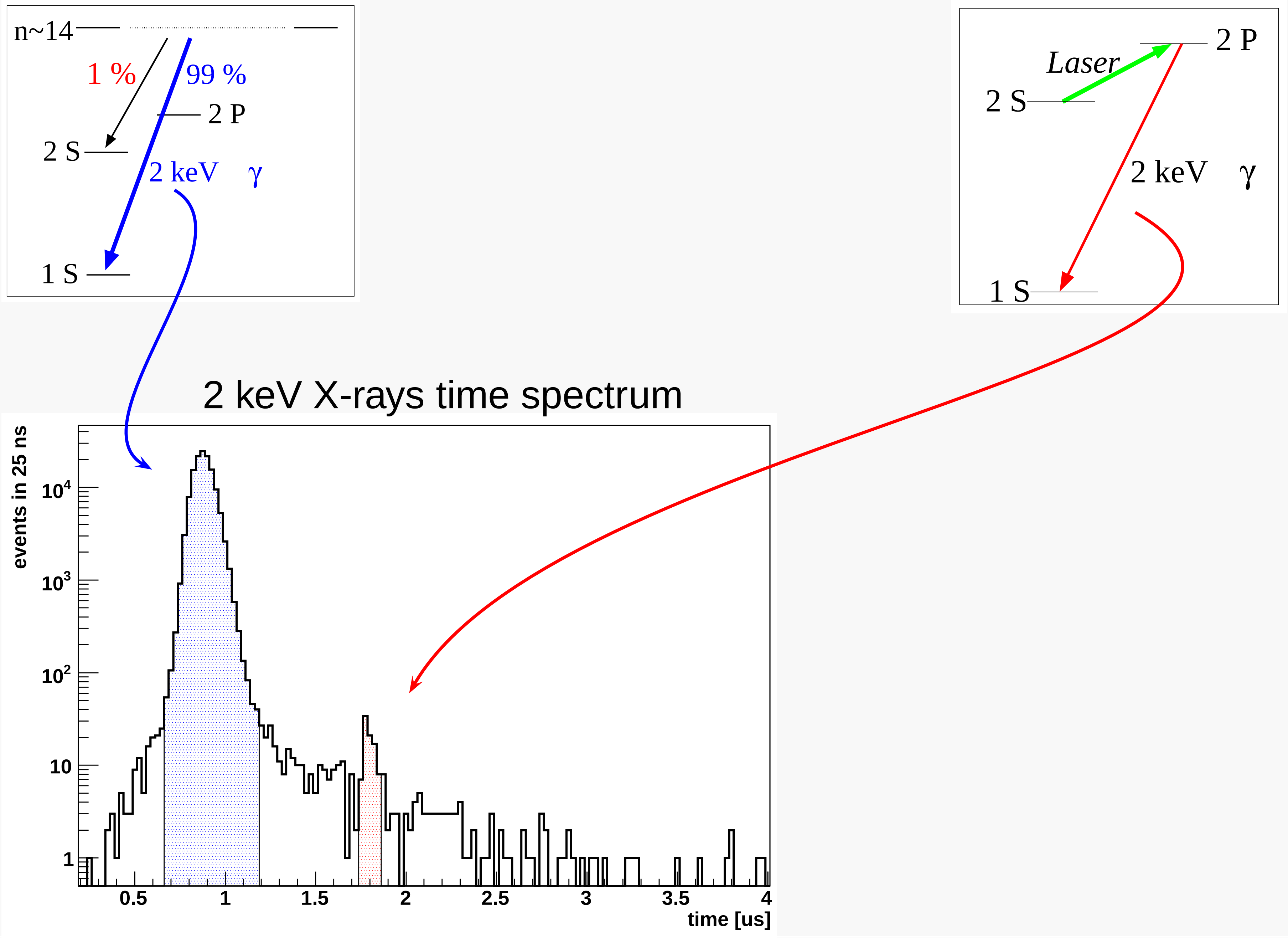


Laser
ON
resonance

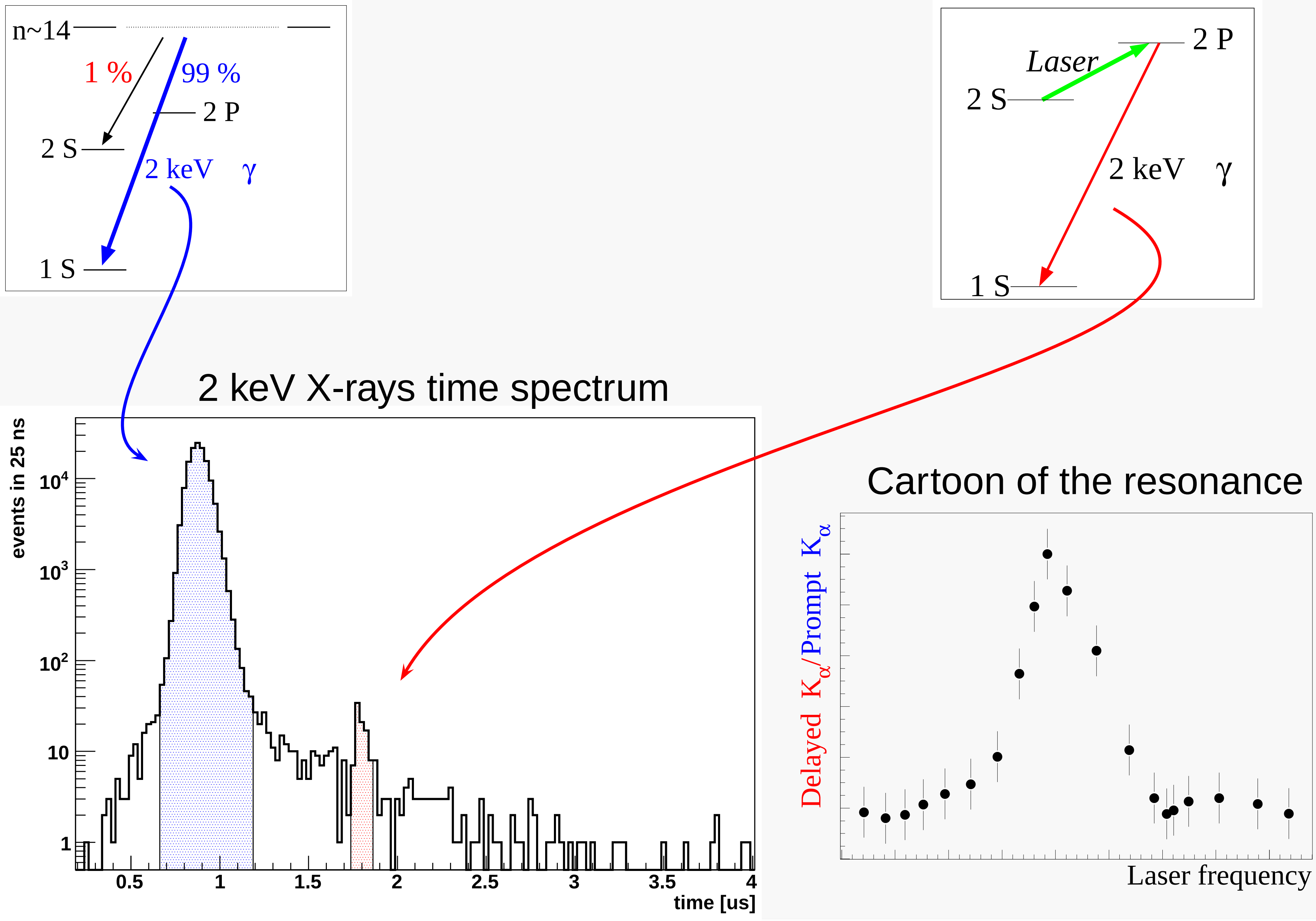
Laser
OFF
resonance

Time-spectrum fit around laser time ⇒ Extract precise bgr. value

Principle of the experiment



Principle of the experiment



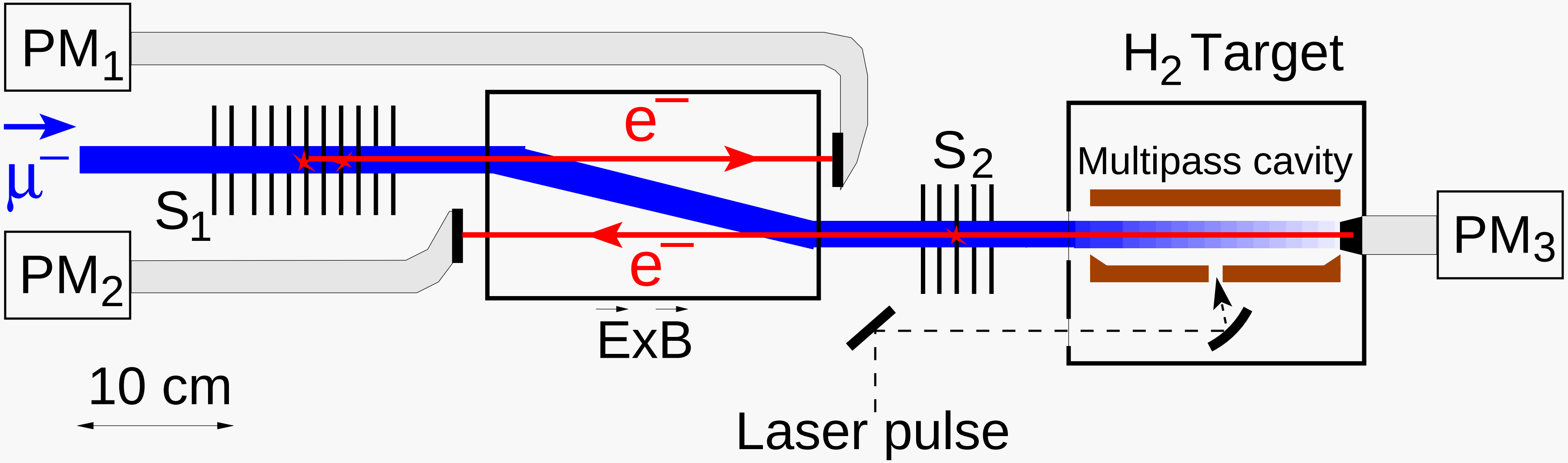
2 keV X-rays time spectrum

Cartoon of the resonance

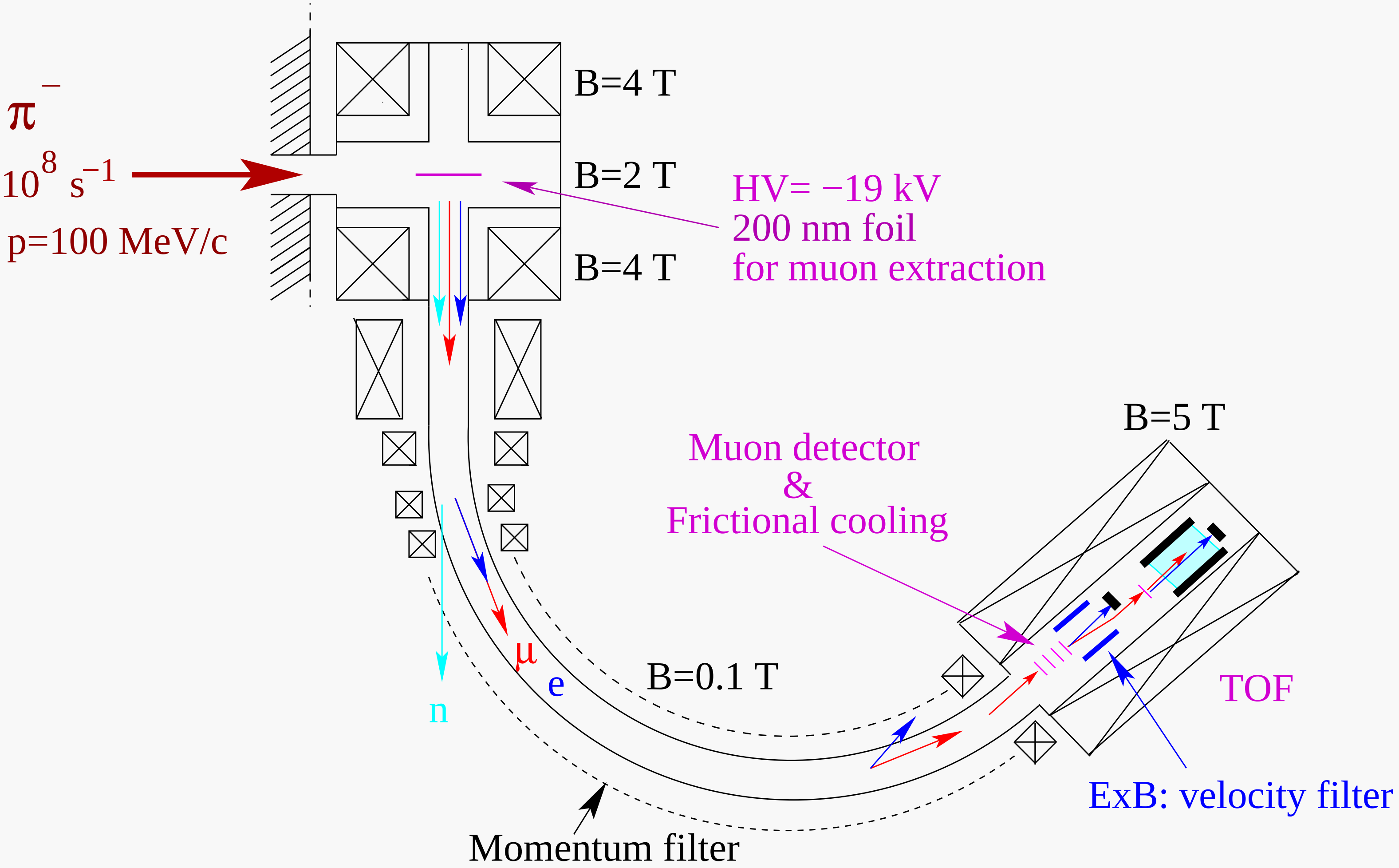
Muon beam line

Target entrance: 5 keV μ^-, 400 s^{-1}

- From the muon extraction channel: 20-50 keV μ^-
  slowing down + frictional cooling + e^- emission + $E \times B$ + TOF + trigger
- $\epsilon_{S_1} = 85\%$, $\epsilon_{S_2} = 35\%$, $\epsilon_{S_3} = 55\%$
- Stopping volume in 1 hPa H_2: $= 5 \times 15 \times 190$ mm^3

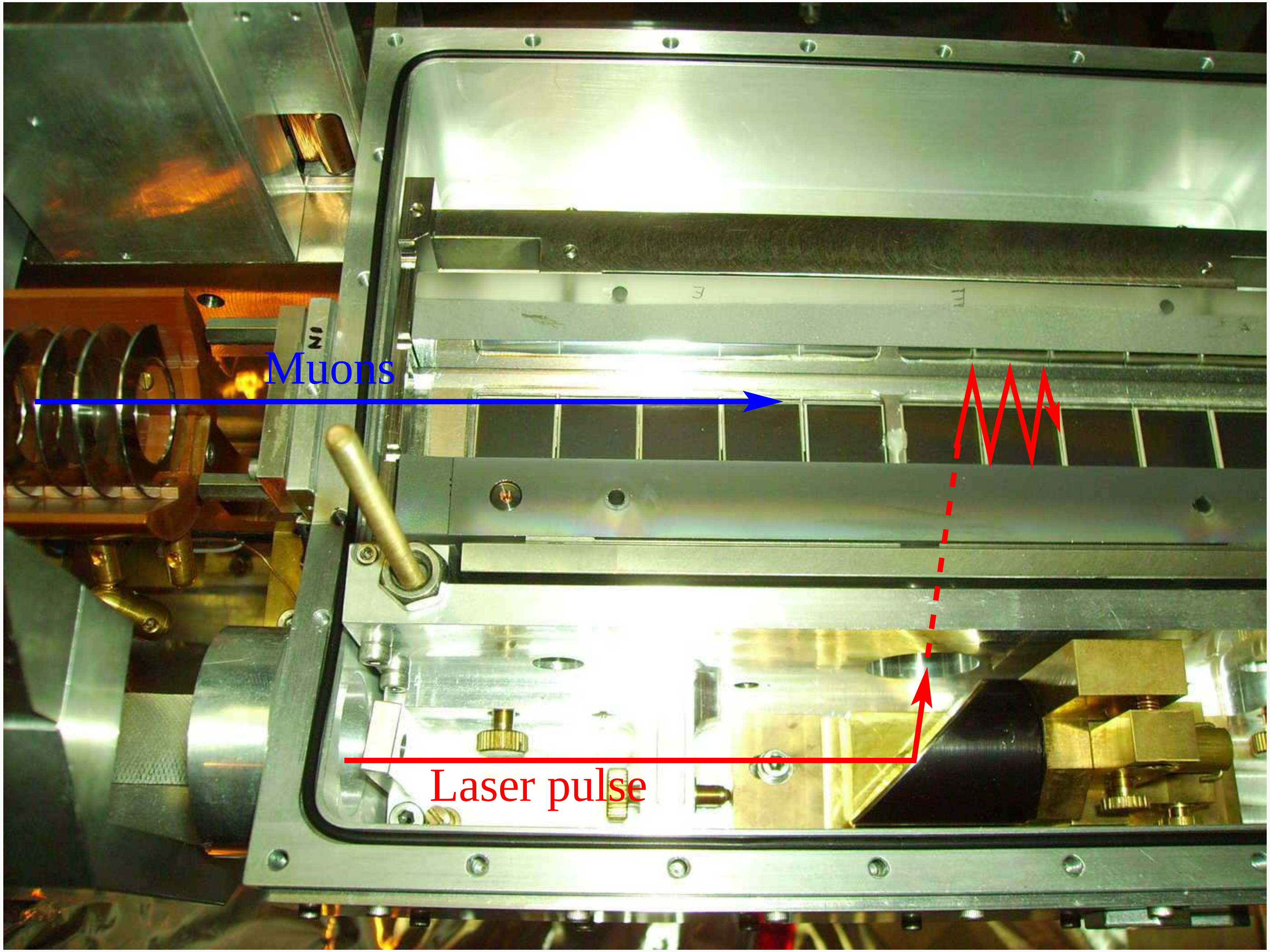


Animation muon beam

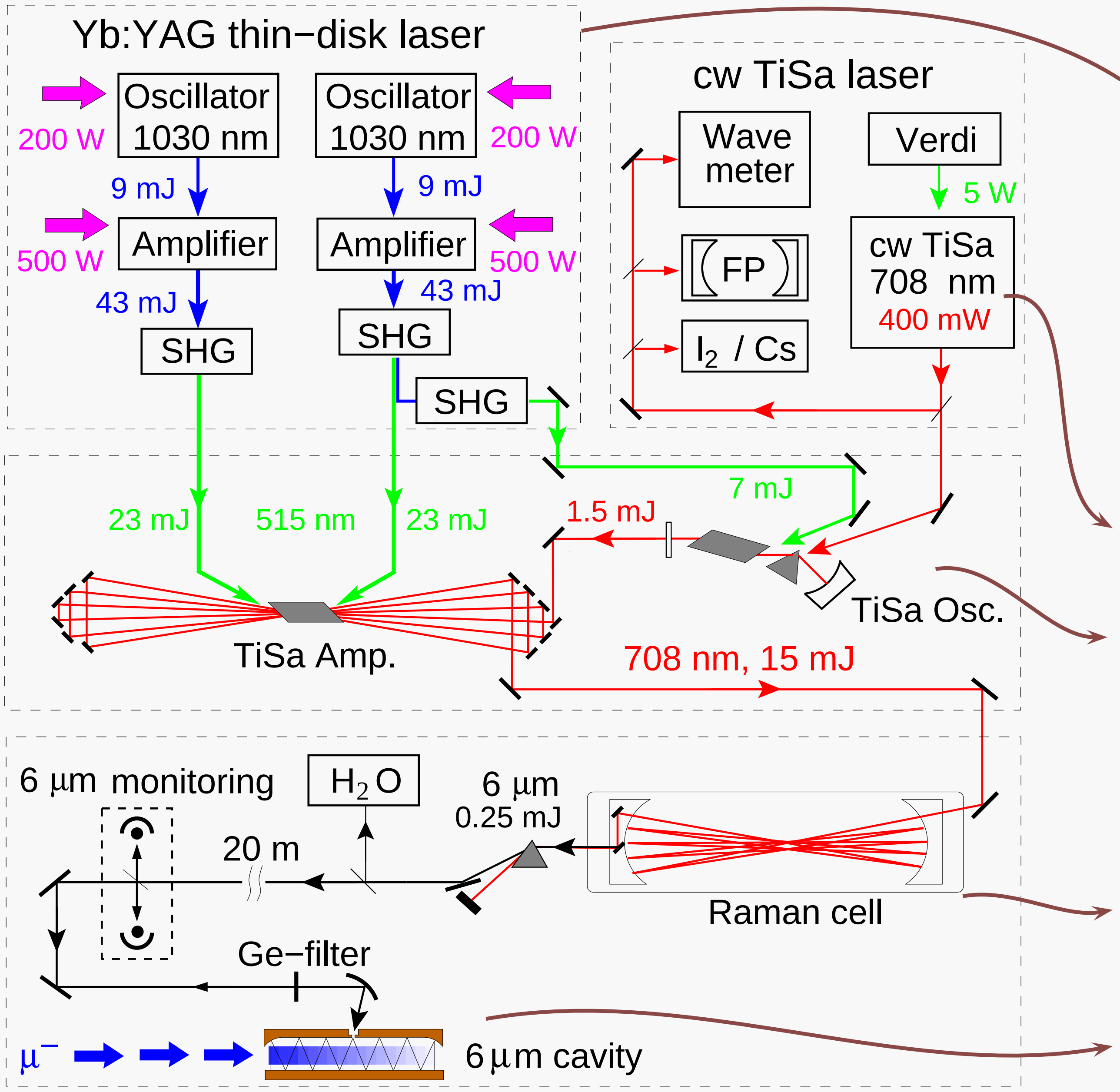


(T.W. Hänsch)

Open target



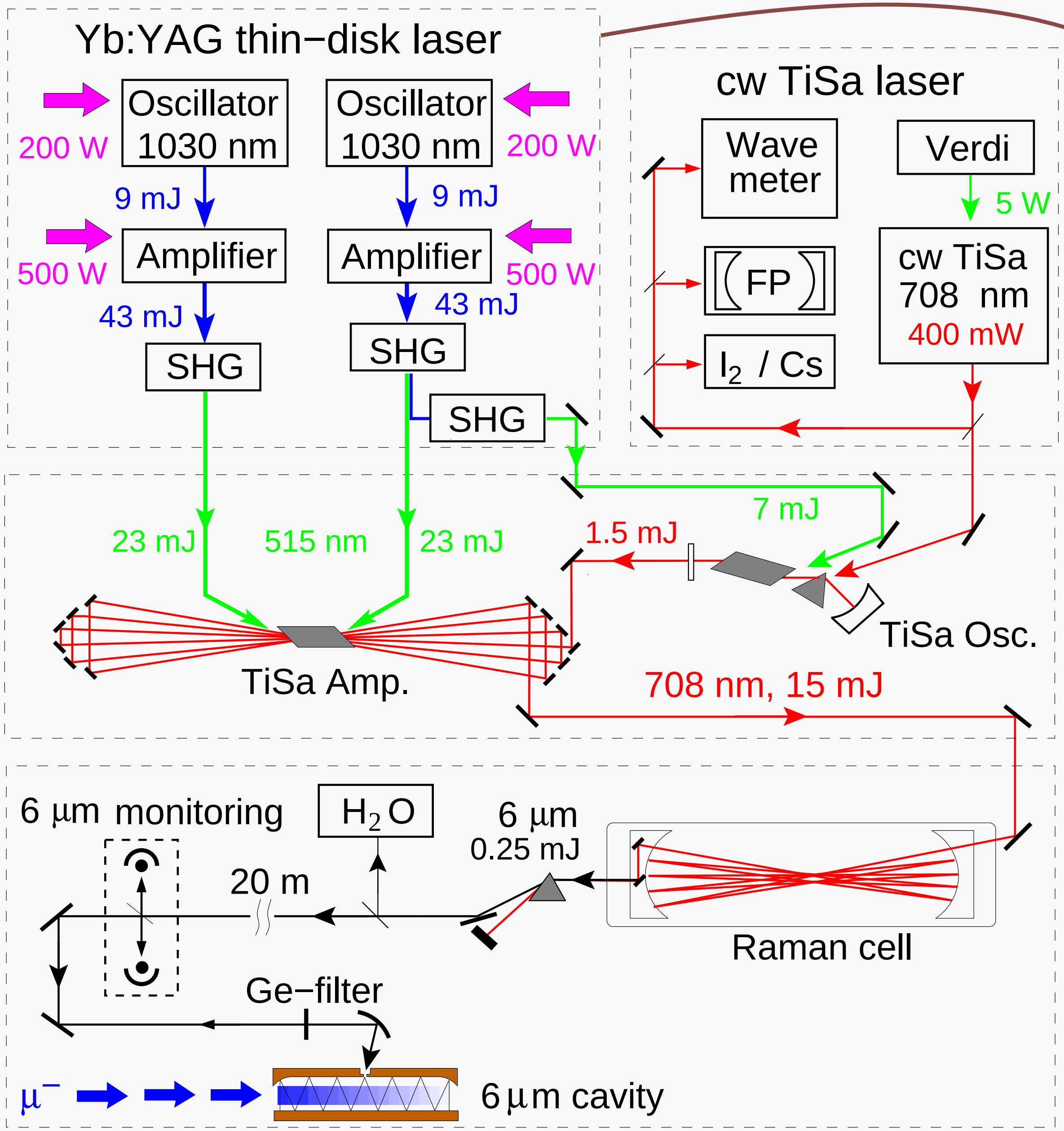
The laser system



Main components:

- Thin-disk laser
- Frequency doubling
- TiSa laser:
  - cw frequency stabilized laser
  - injected seeded oscillator
  - multipass amplifier
- Raman cell
- Target cavity

The laser system



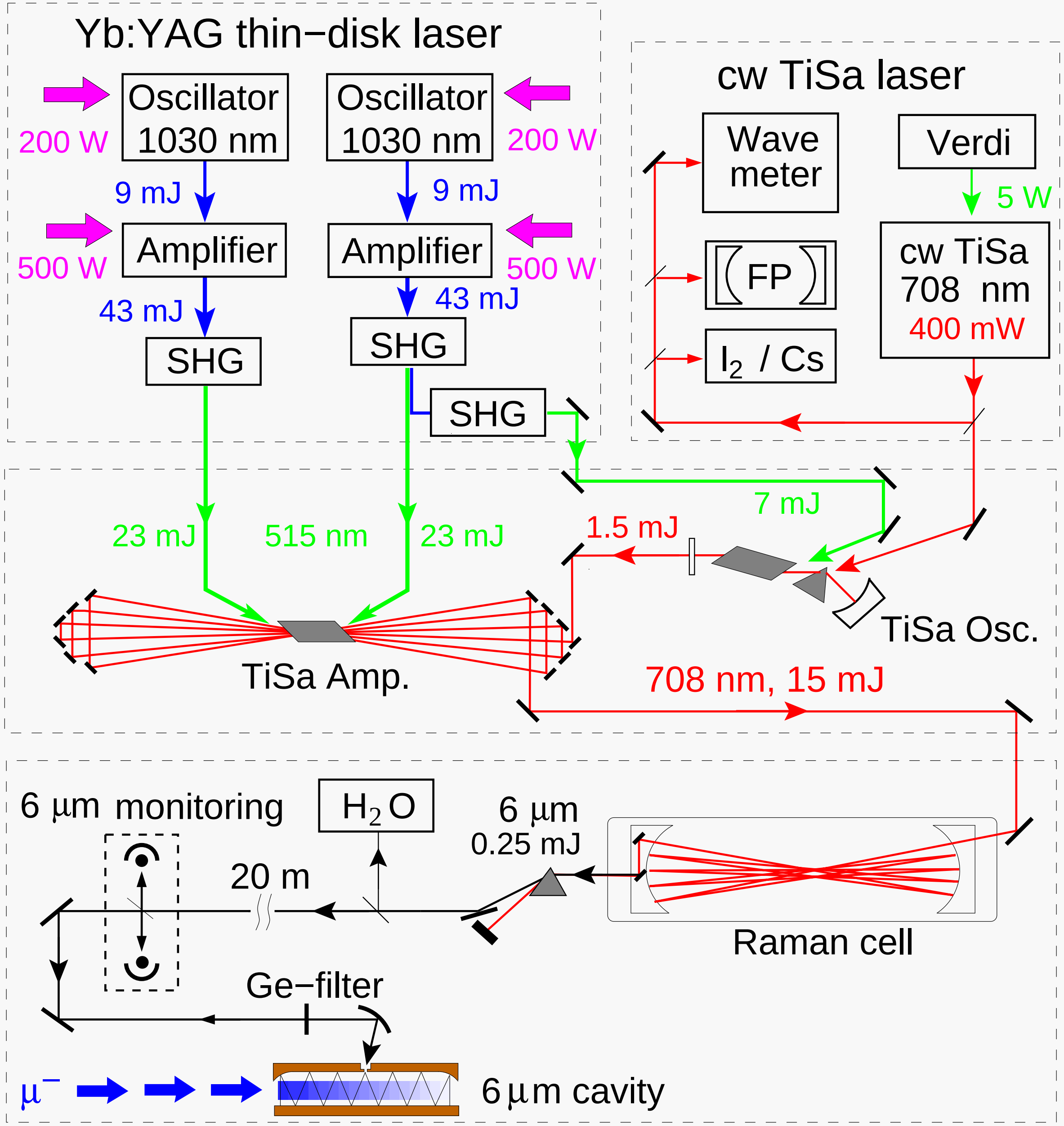
Thin-disk laser

- Large pulse energy: 85 (160) mJ
- Short trigger-to-pulse delay: $\lesssim 400$ ns
- Random trigger
- Pulse-to-pulse delays down to 2 ms
  (rep. rate $\gtrsim 500$ Hz)

- Each single μ^- triggers the laser system
- $2S$ lifetime $\approx 1\ \mu$s → short laser delay

A. Antognini *et. al.*, IEEE J. Quant. Electr. Vol. 45, No. 8, 993-1005 (2009).

The laser system



MOPA TiSa laser:

Cw frequency stabilized laser

- referenced to a stable FP cavity
- FP cavity calibrated with I_2, Rb, Cs lines

$\nu_{\mathrm{FP}} = N \cdot FRS$

$FRS = 1497.344(6)$ MHz, $N \approx 2 \times 10^5$.

$\nu_{\mathrm{TiSa}}^{\mathrm{cw}}$ absolutely know with $\sigma = 30$ MHz

$\Gamma_{\mathrm{2P-2S}} = 18.6$ GHz

Seeded oscillator

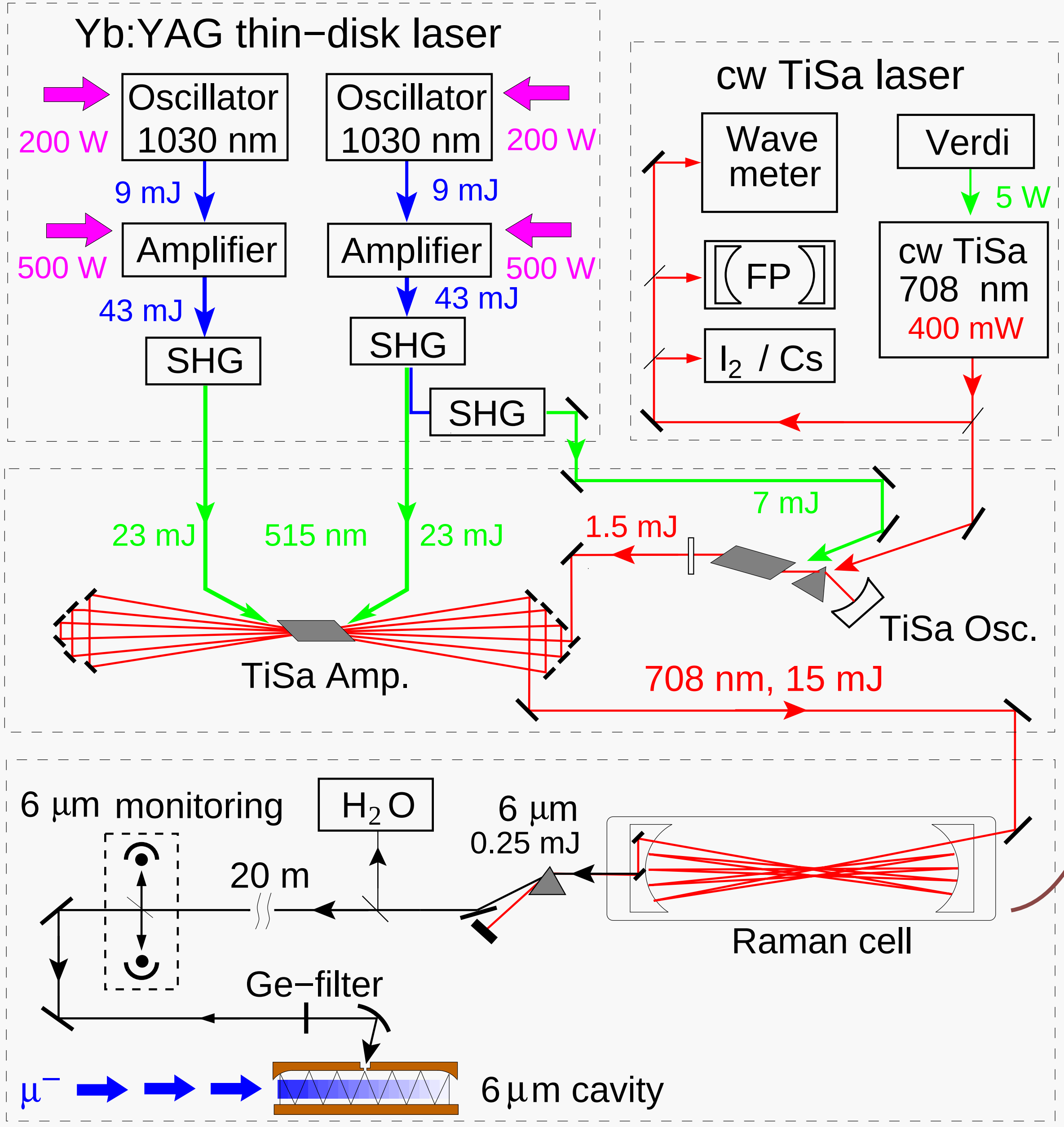
$\rightarrow \nu_{\mathrm{TiSa}}^{\mathrm{pulsed}} = \nu_{\mathrm{TiSa}}^{\mathrm{cw}}$

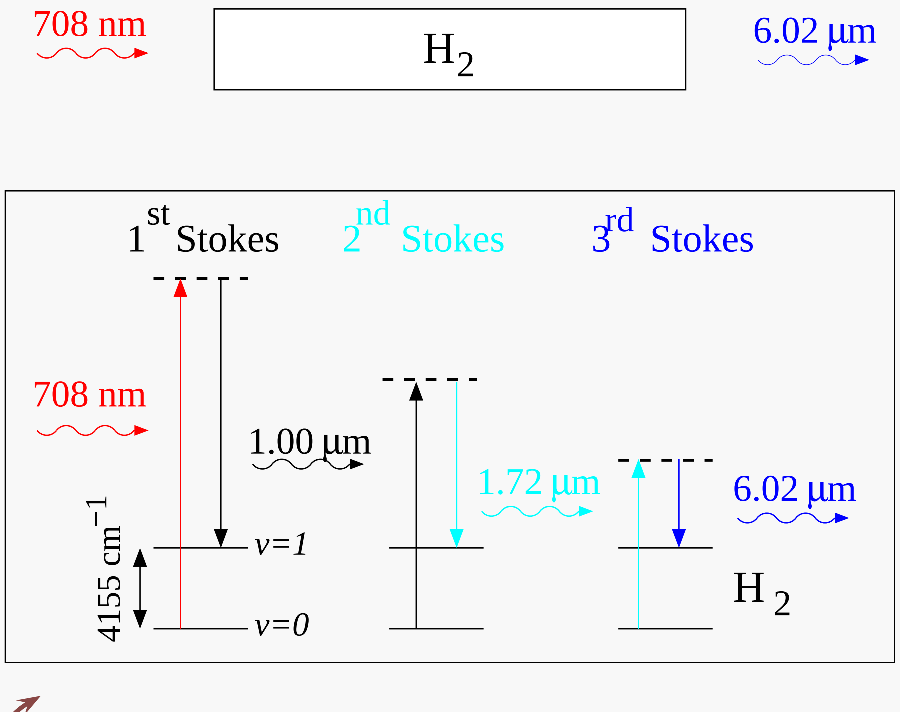
(frequency chirp ≤ 100 MHz)

Multipass amplifier (2f- configuration)

gain=10

The laser system



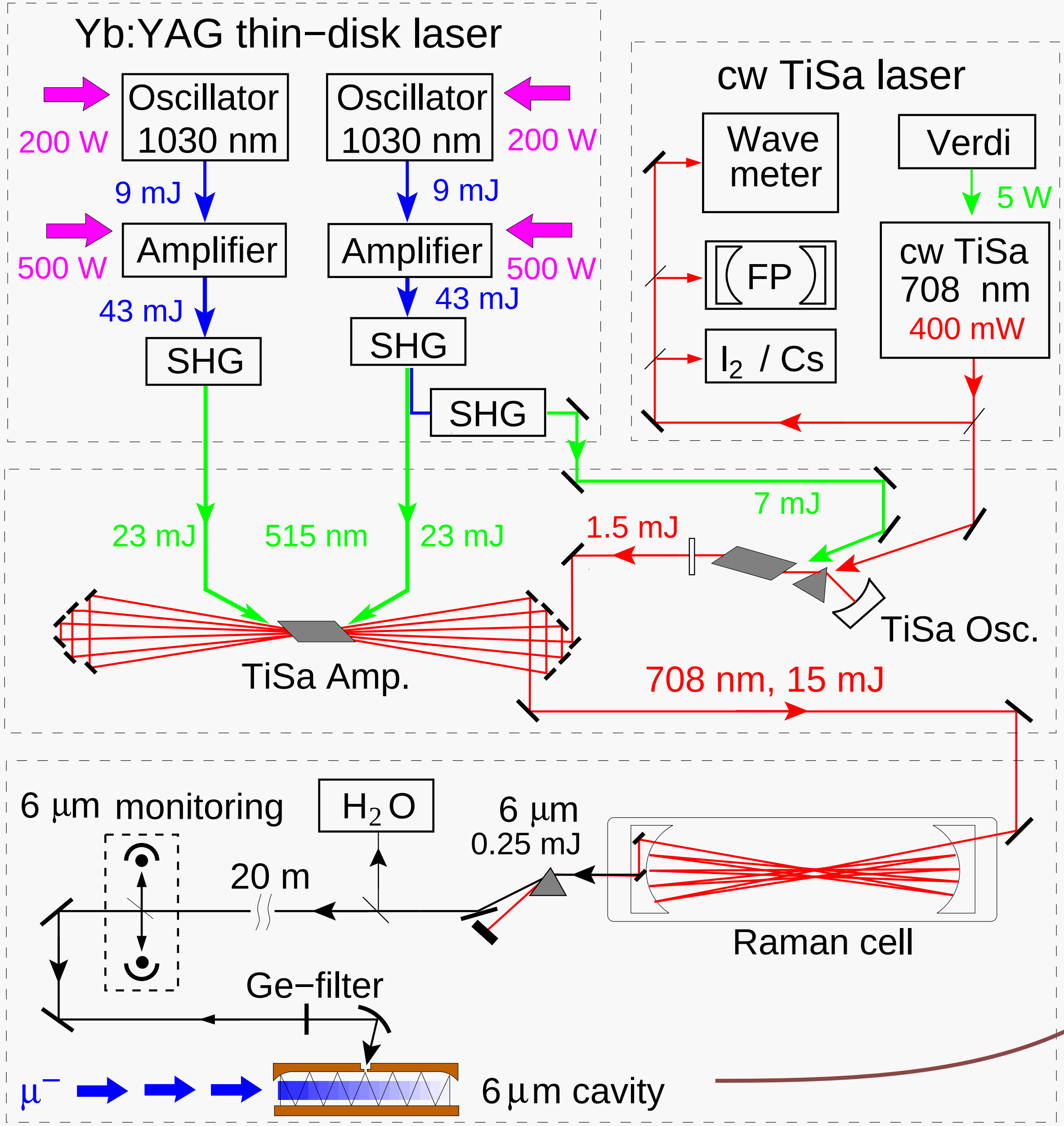


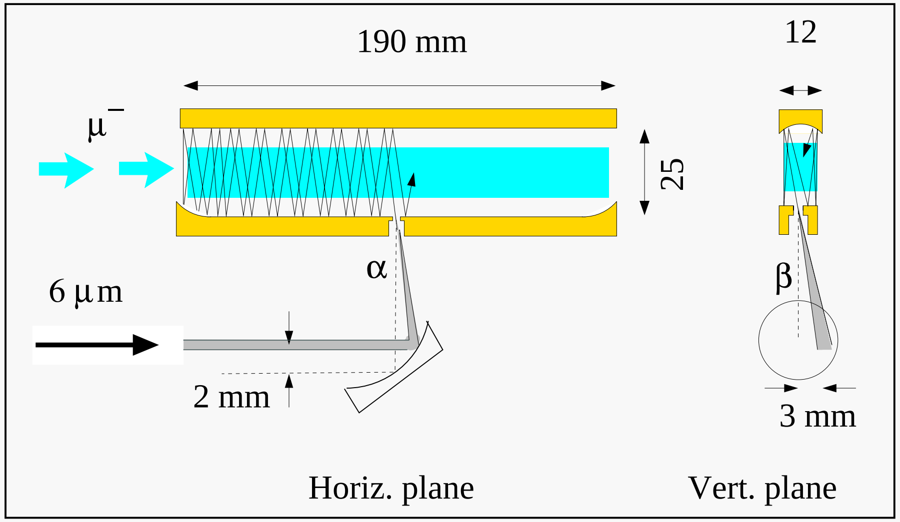
$$\nu^{6\mu m} = \nu^{708\text{nm}} - 3 \cdot \hbar\omega_{\text{vib}}$$

tunable

$\omega_{\text{vib}}(p,T) =$ const

The laser system





Design: insensitive to misalignment

Transverse illumination

Large volume

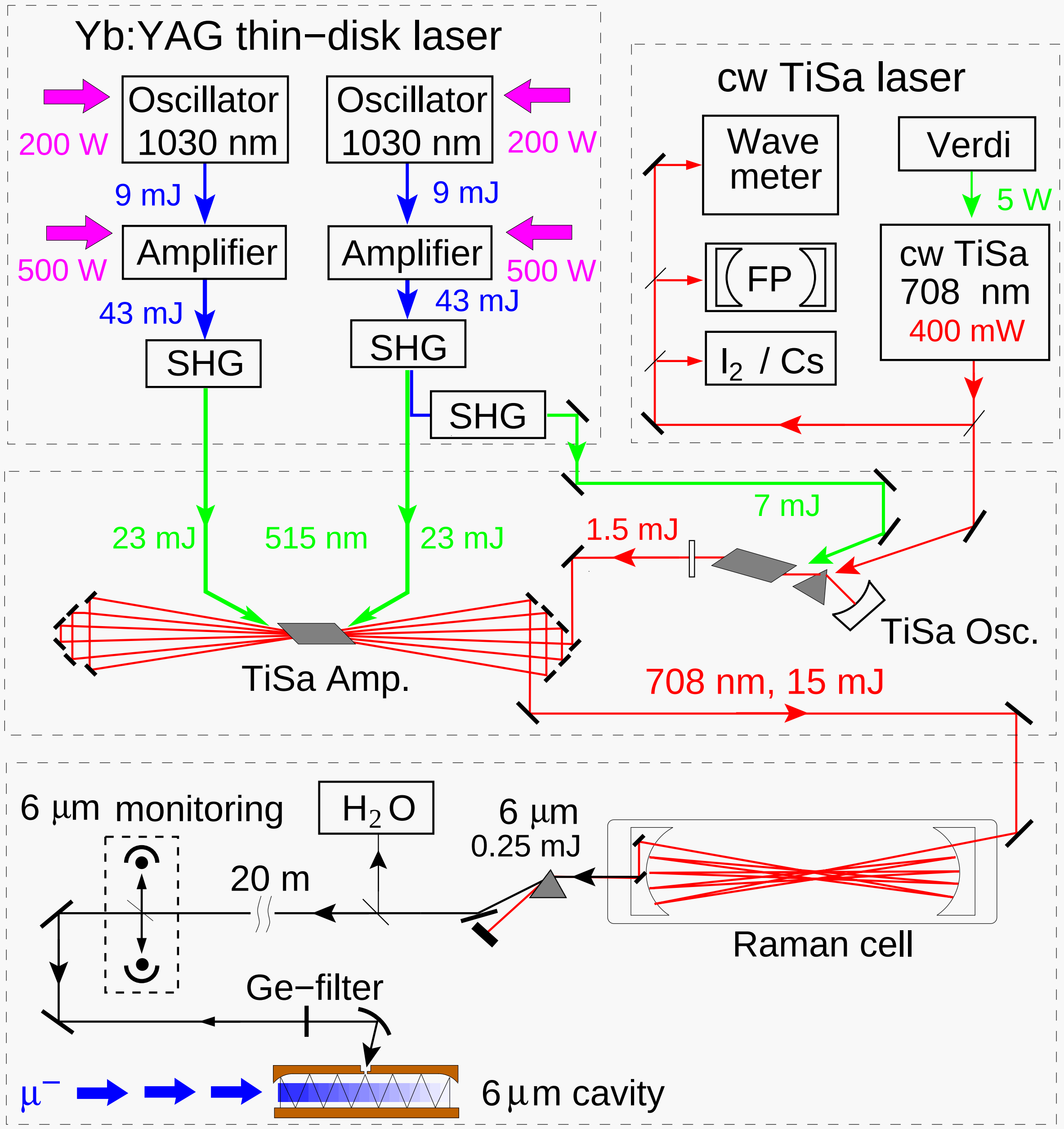
Dielectric coating with $R \geq 99.9\%$ (at 6 μm)

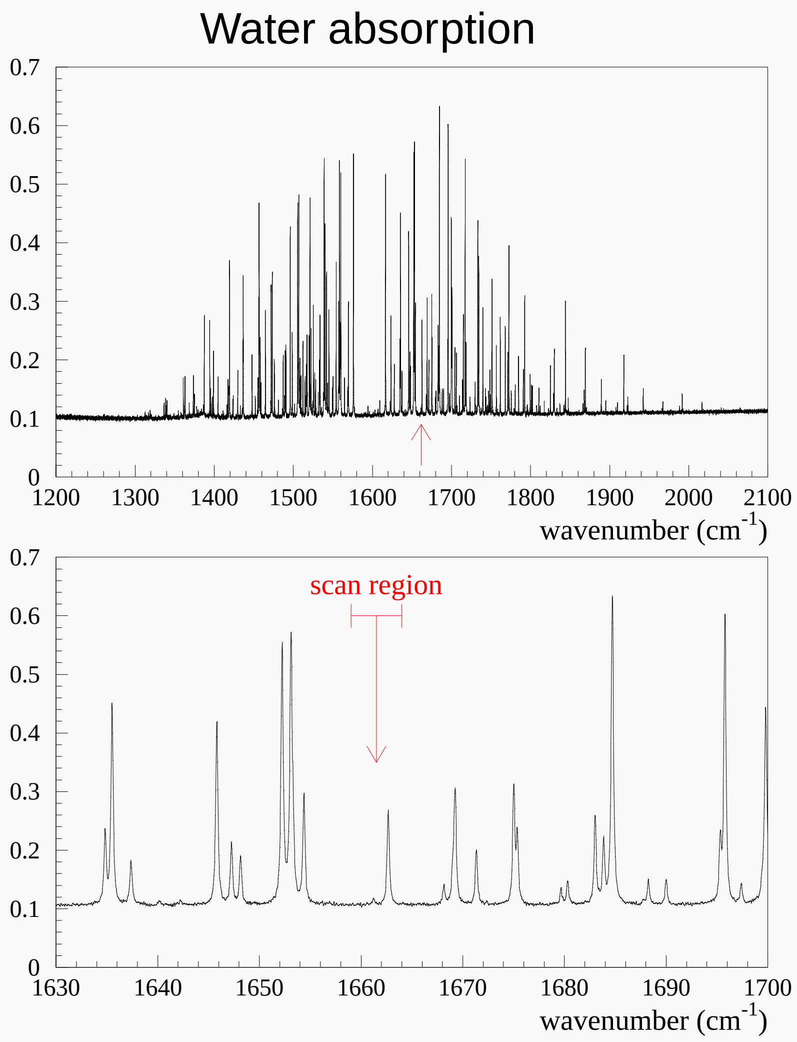
→ Light makes 1000 reflections

→ Light is confined for 50 ns

→ 0.15 mJ saturates the $2S - 2P$ transition

The laser system

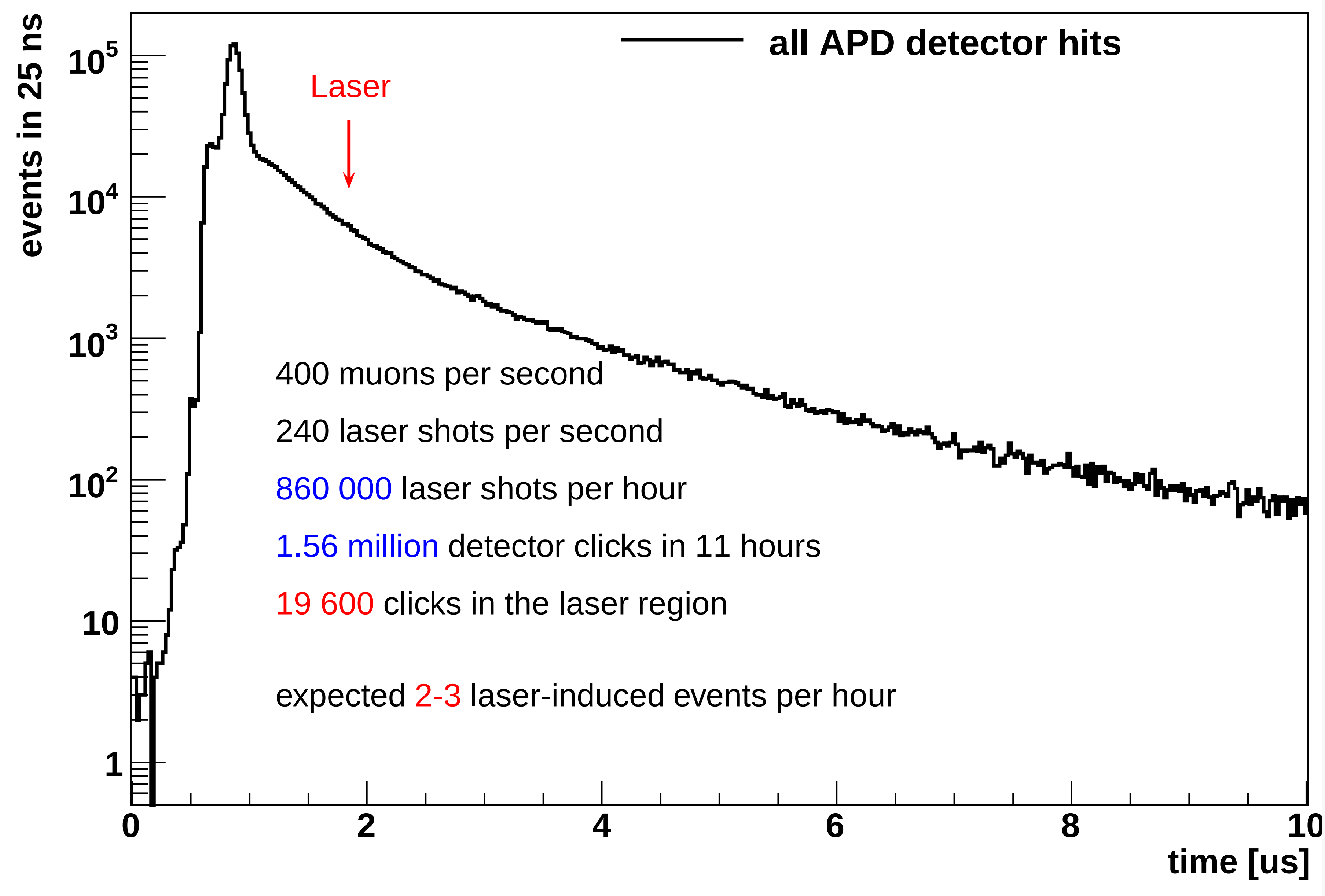




- Vacuum tube for 6 μm beam transport.
- Direct frequency calibration at 6 μm.

Data analysis: time spectra

FP 900, 11 hours measurement



400 muons per second

240 laser shots per second

860 000 laser shots per hour

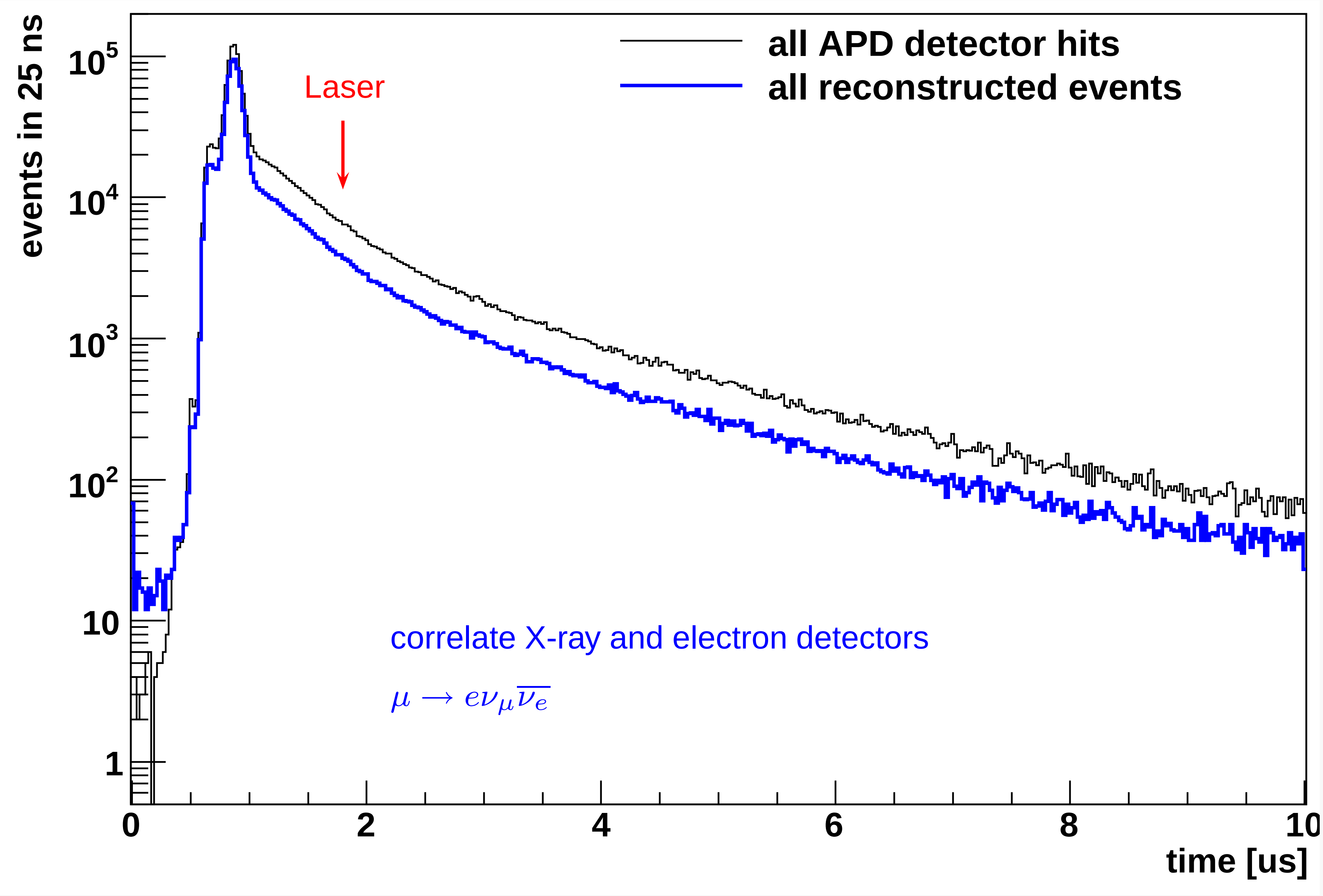
1.56 million detector clicks in 11 hours

19 600 clicks in the laser region

expected 2-3 laser-induced events per hour

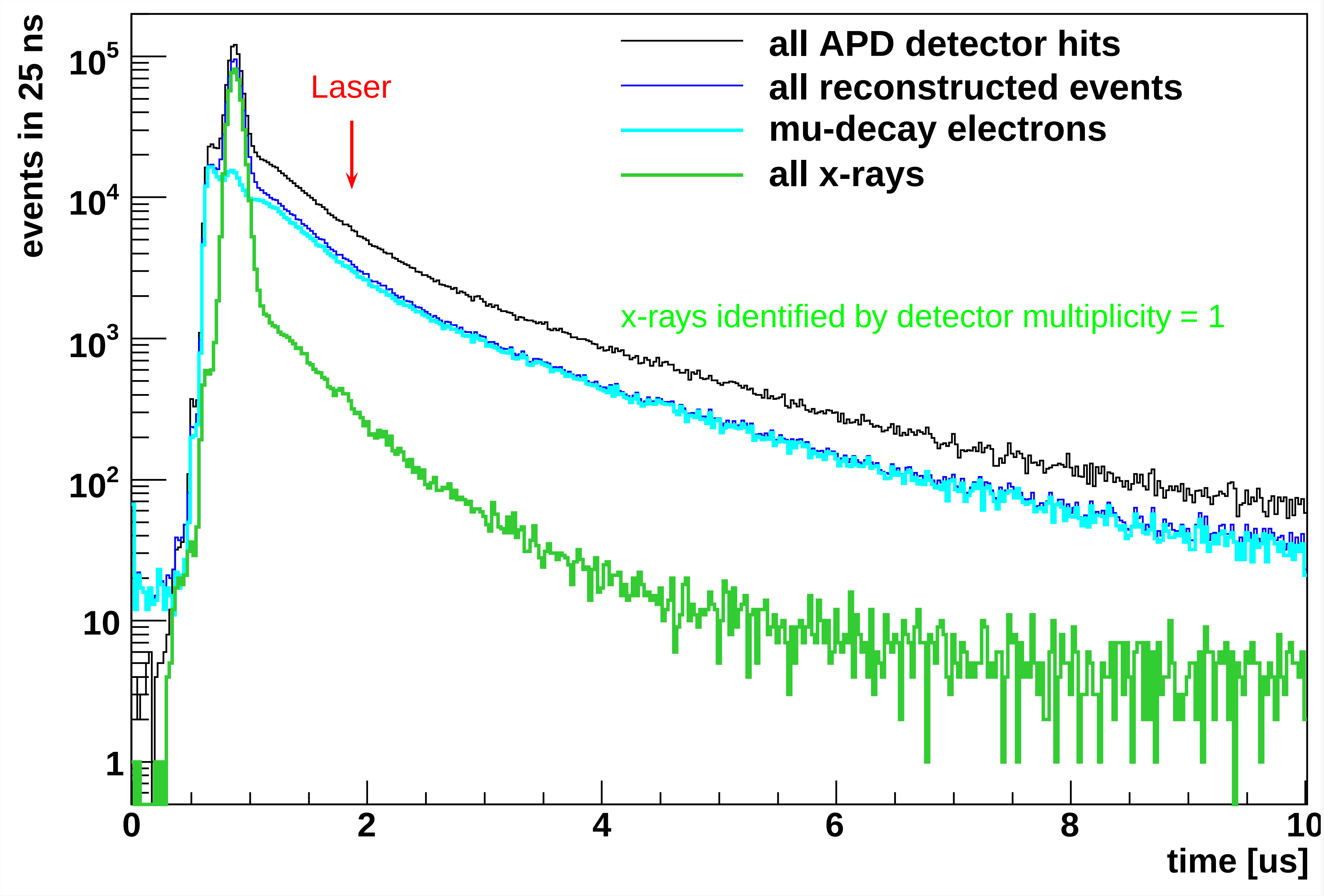
Data analysis: time spectra

FP 900, 11 hours measurement



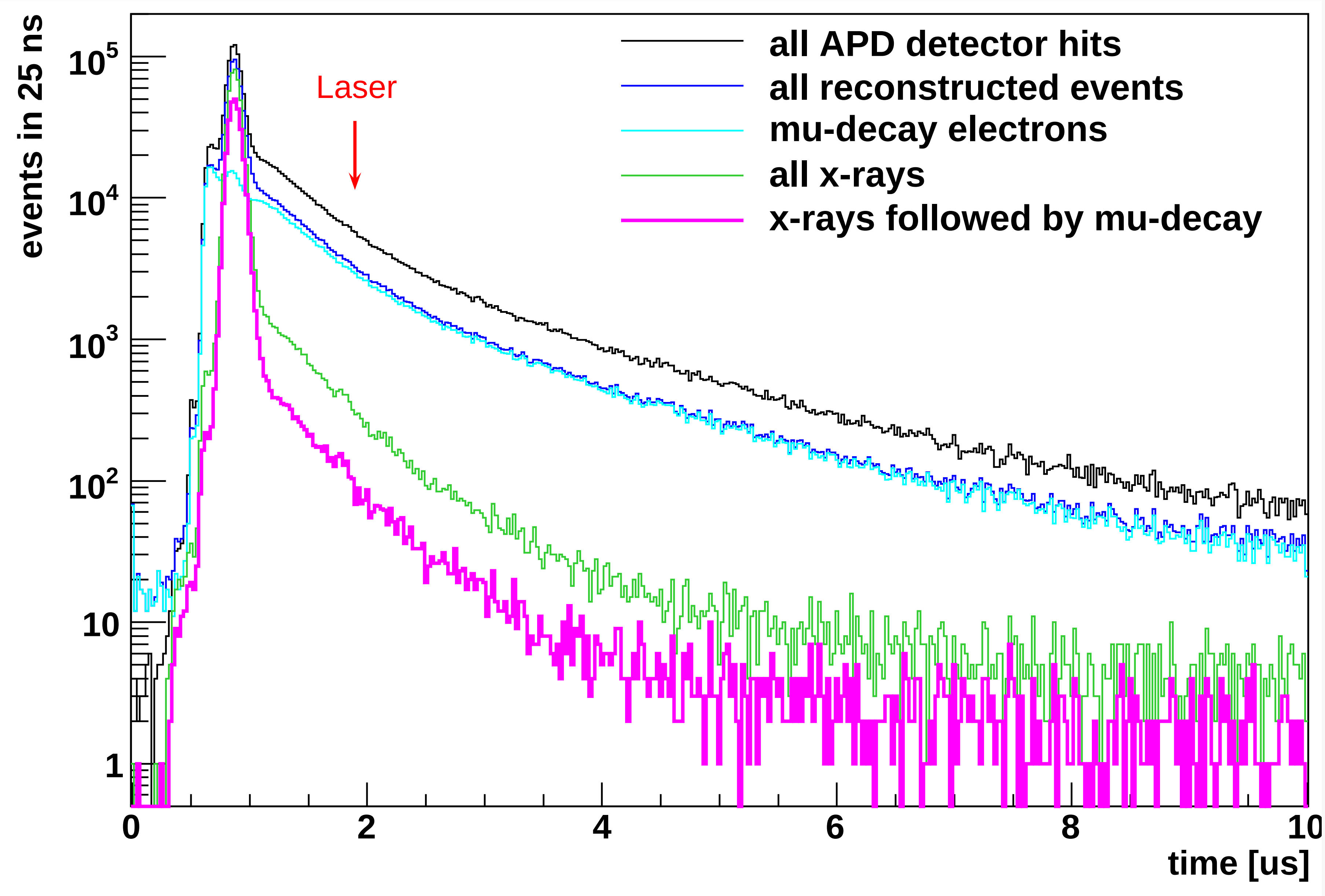
Data analysis: time spectra

FP 900, 11 hours measurement



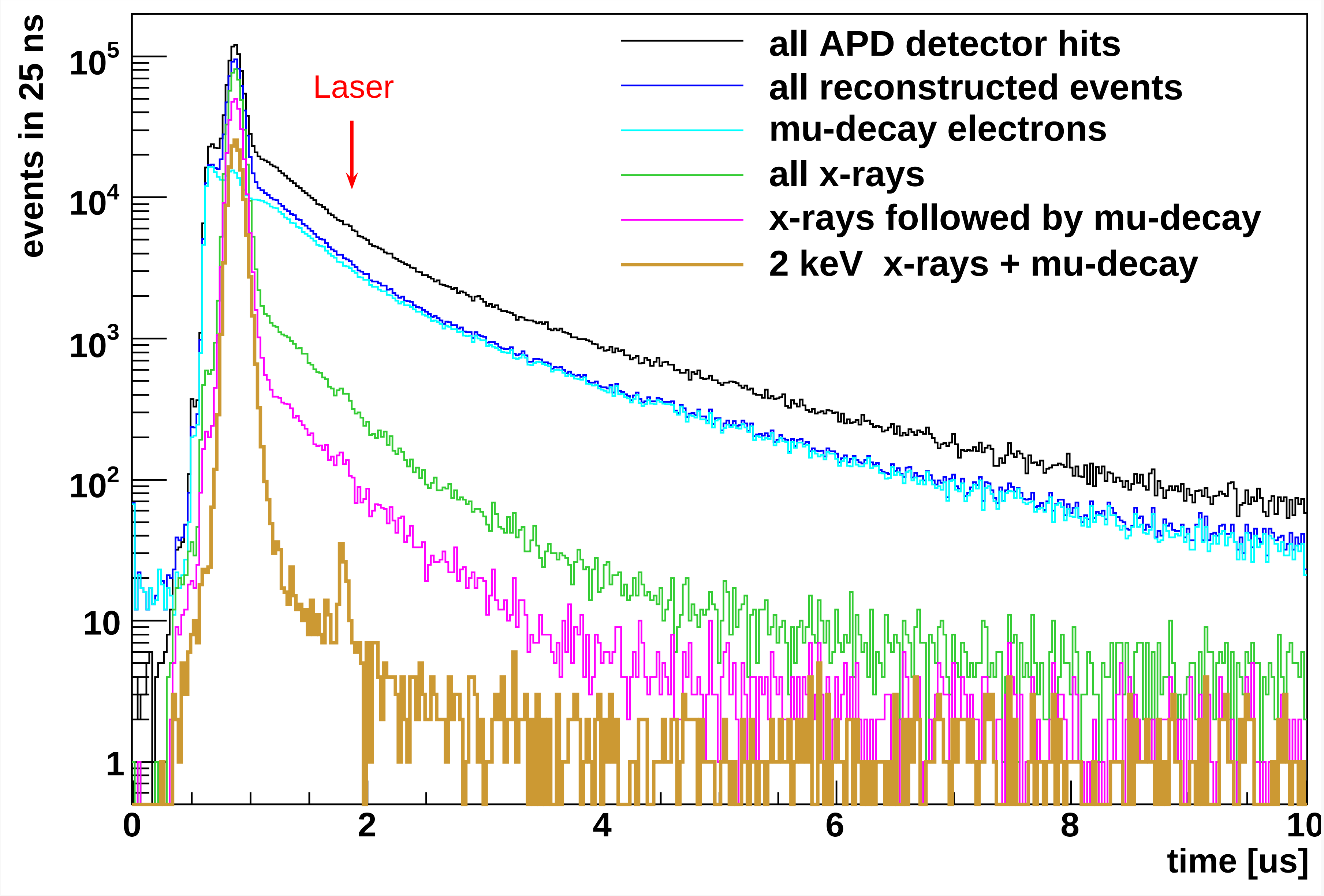
Data analysis: time spectra

FP 900, 11 hours measurement



Data analysis: time spectra

FP 900, 11 hours measurement



Data analysis: time spectra

FP 900, 11 hours measurement

7 events per hour!

1 bgr. event/hour