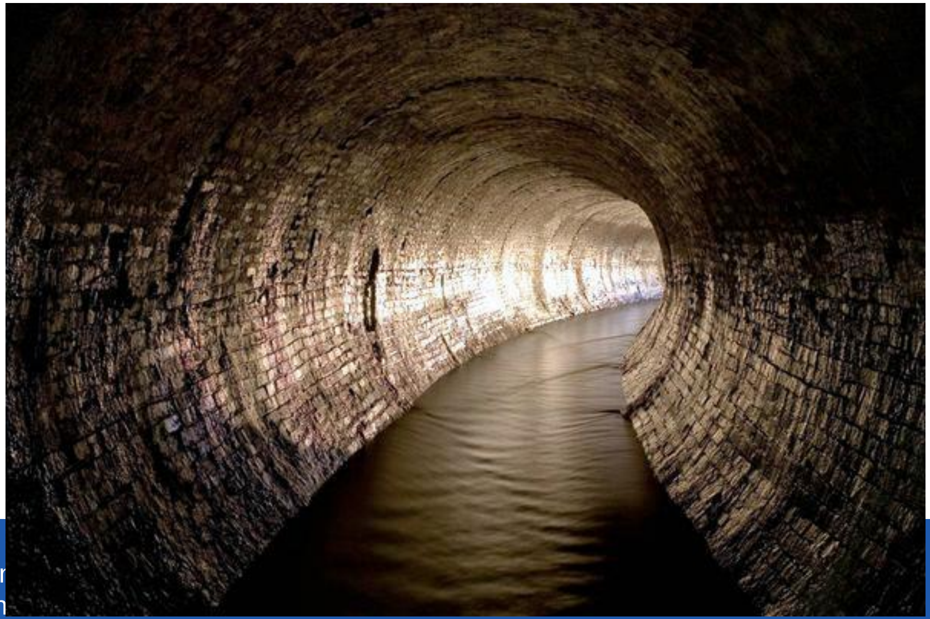
Building Blocks of an accelerator

1) A particle source

2) An accelerating system

3) A series of guiding and storage devices



Everything under vacuum

Synchrotron (1952, 3 GeV, BNL)

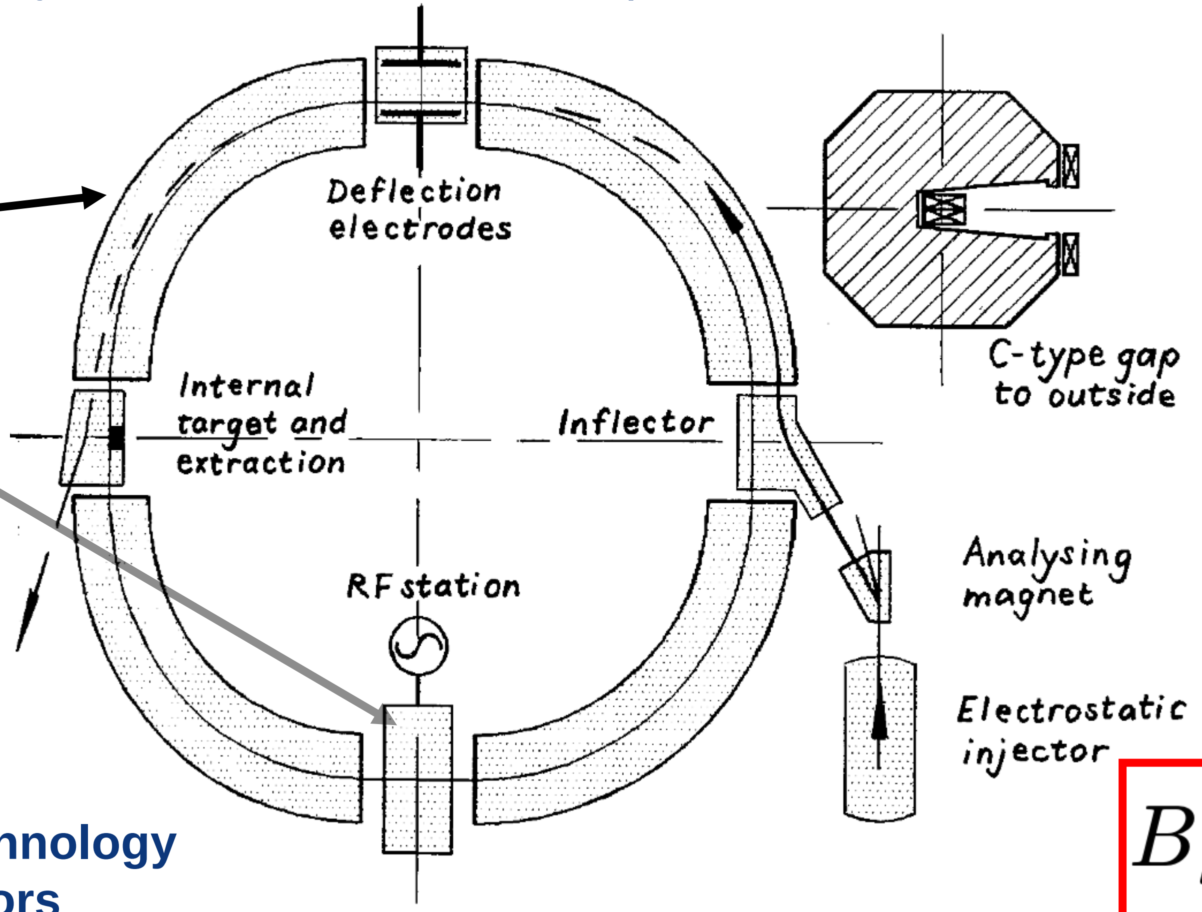
New concept of circular accelerator. The magnetic field of the bending magnet varies with time.
As particles accelerate, the B field is increased proportionally.
The frequency of the accelerating cavity, used to accelerate the particles, has also to change.

B = B(t) magnetic field from the bending magnets

p = p(t) particle momentum varies by the RF cavity

e electric charge

ρ constant radius of curvature



Bending strength limited by used technology to max ~ 1 T for room temperature conductors

Particle rigidity:

$$B\rho = \frac{p}{e}$$

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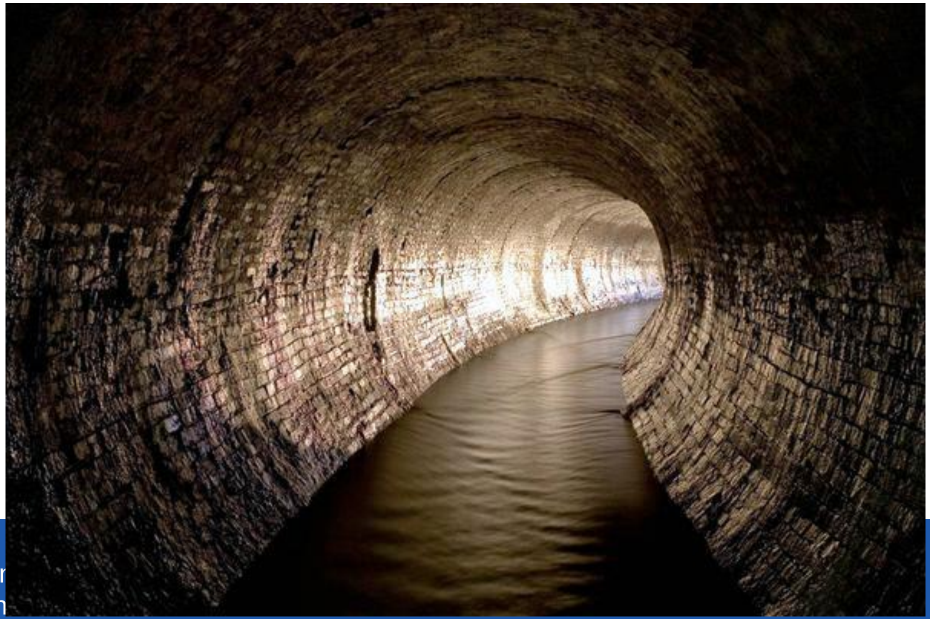
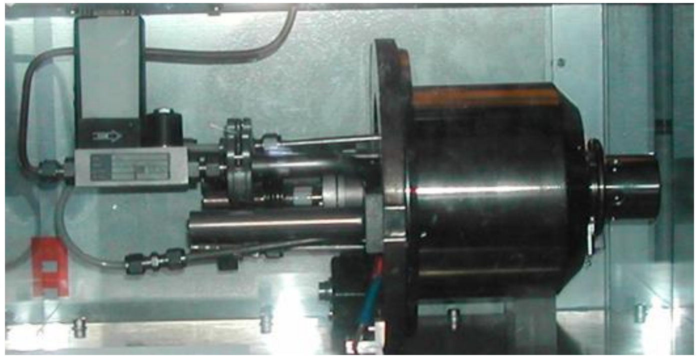


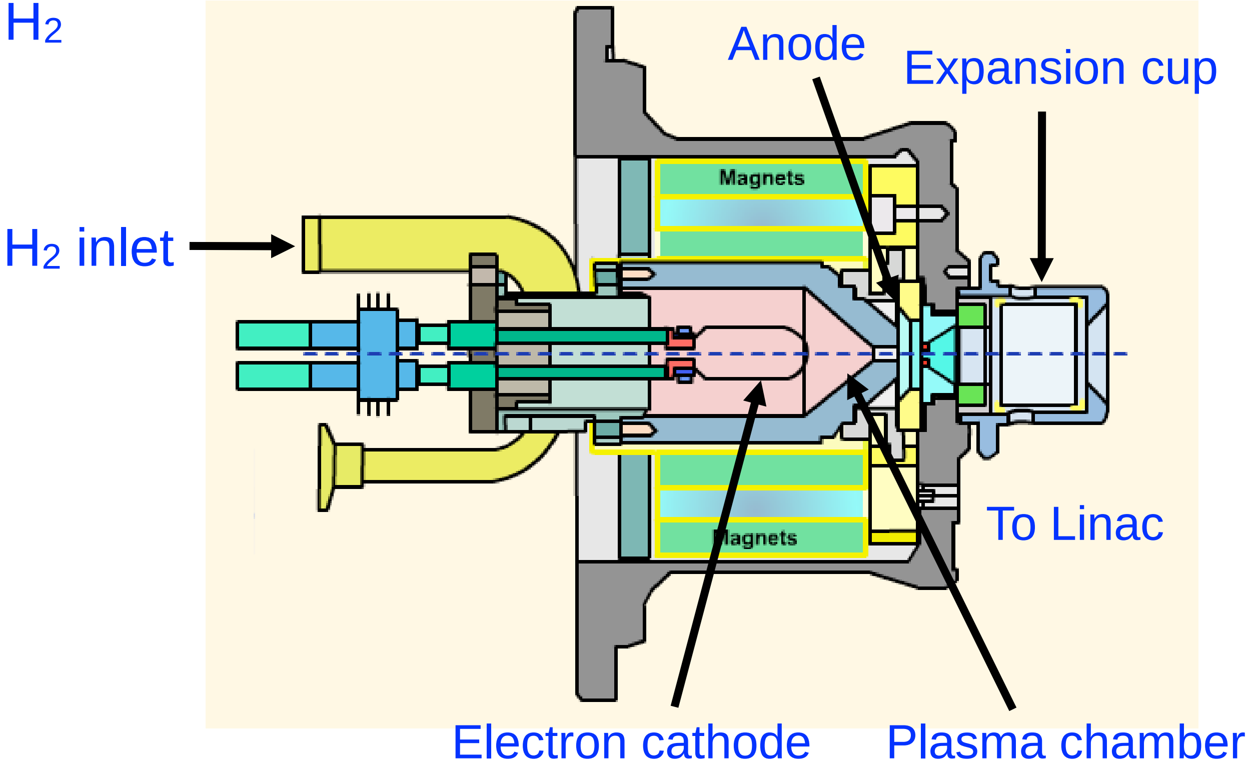
Everything under vacuum

How to get protons: duoplasmatron source

Protons are produced by the ionization of H_2 plasma enhanced by an electron beam



Hydrogen supply (one lasts for 6 months)

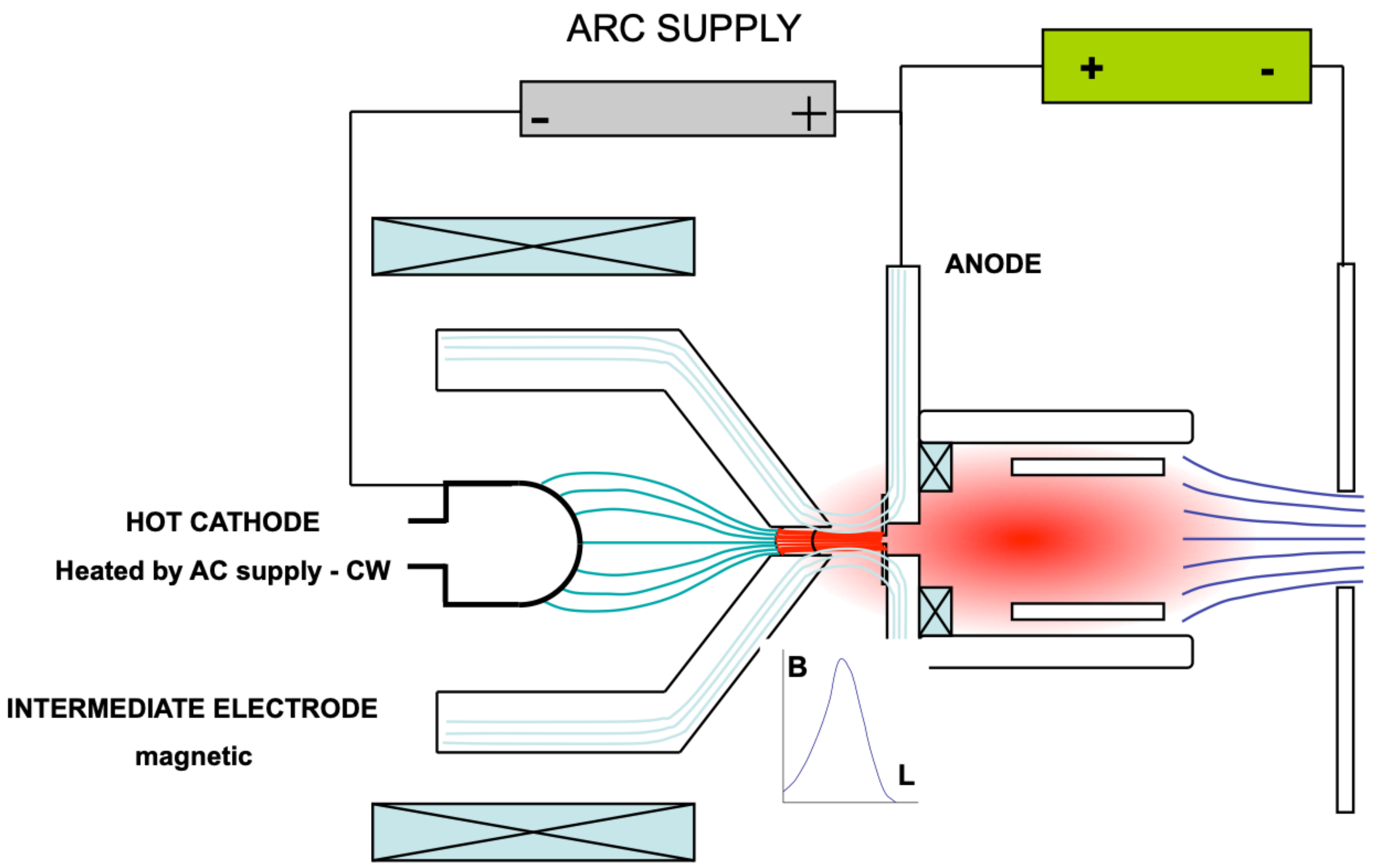


Proton exiting from the about 1 mm^2 hole have a speed of 1.4 % c, v ≈4000 km/s

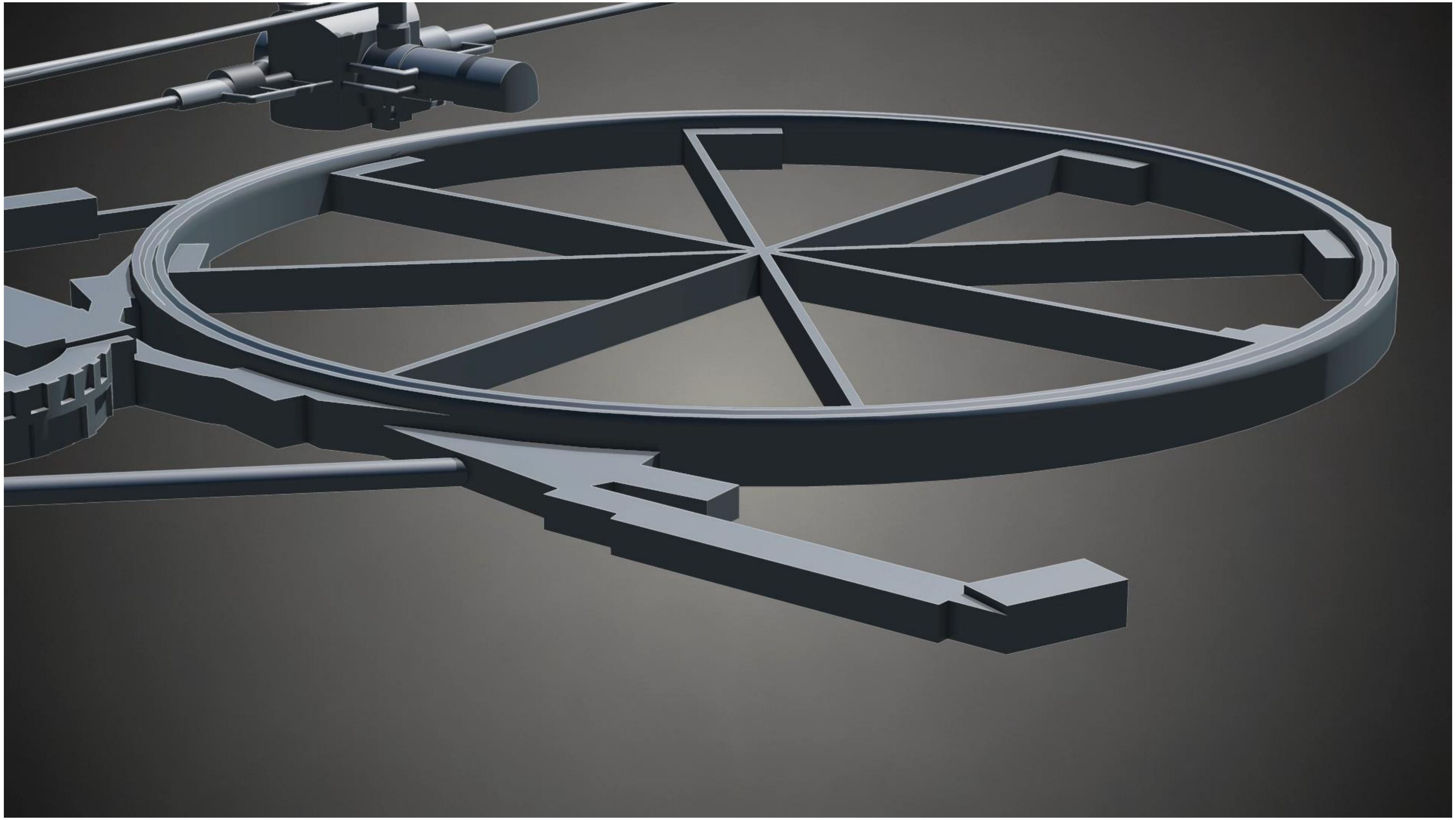
The SPACE SHUTTLE goes only up to 8 km/s

Today we have an H^- source, 2nd level lectures

Source electrical scheme



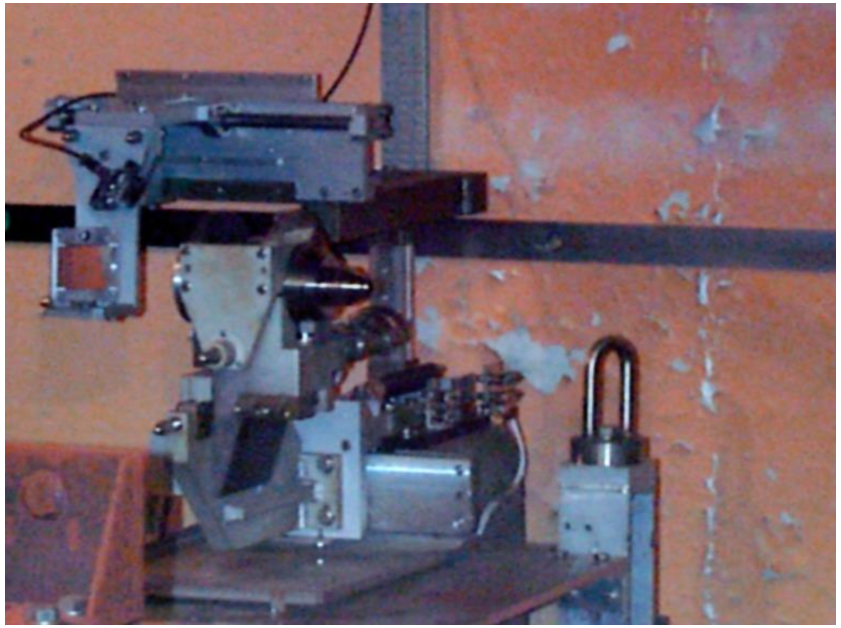
Courtesy R. Scrivens

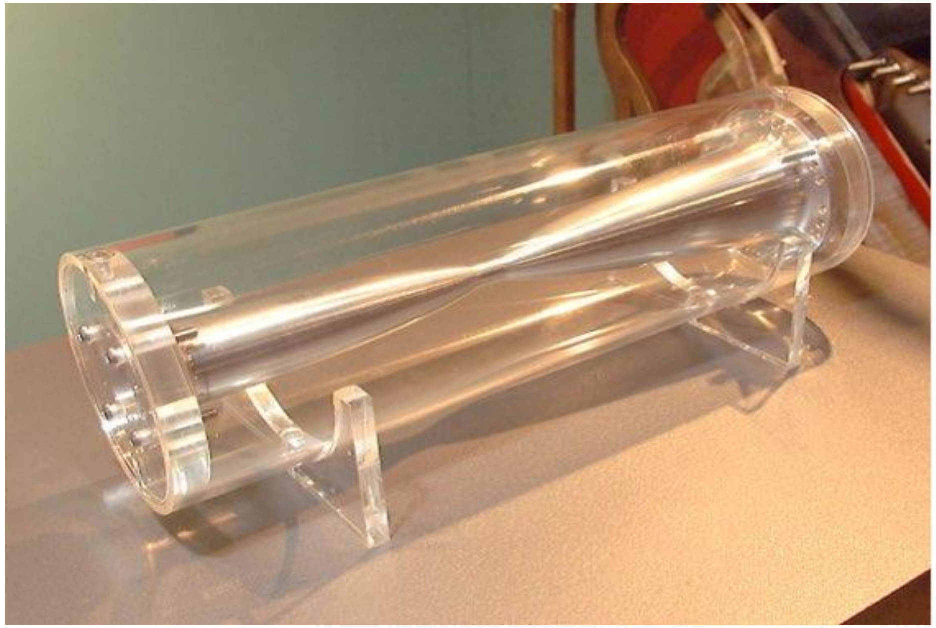


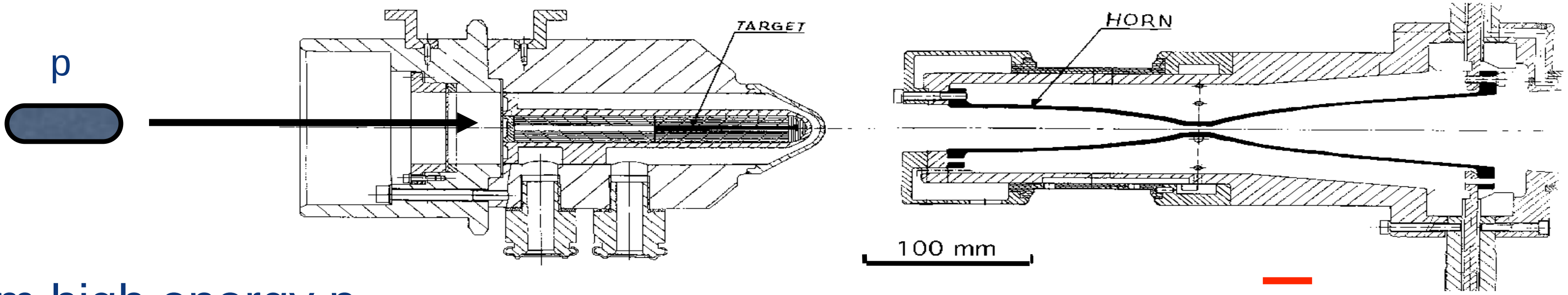
http://cern60.web.cern.ch/fr/exhibitions/duoplasmatron

Cern Control Center: first LHC day

How to get antiprotons







Starting from high energy p
and with a very low efficiency

$p + p \longrightarrow p + p + p + \bar{p}$

10^{13} p to have about 10^{7} antiprotons

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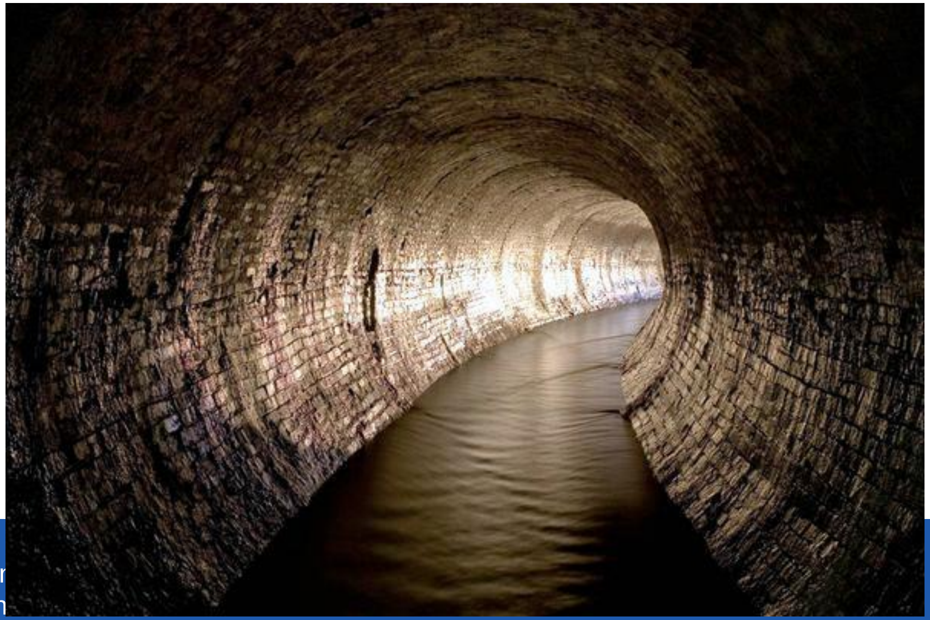
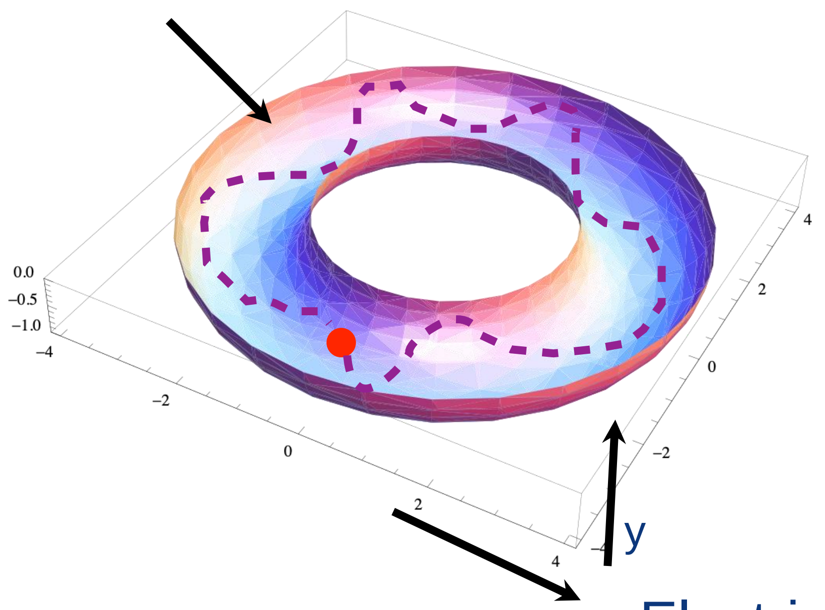


Everything under vacuum

How an accelerator works ?

Accelerator



Goal: keep enough CHARGED particles confined in a well defined volume to accelerate them for a sufficiently long time (ms - hours)

How ? Lorentz Force!

$$\overline{F(t)} = q\left(\overline{E(t)} + \overline{v(t)} \otimes \overline{B(t)}\right)$$

Electric field accelerates particles

Particles of different energy (speed) behave differently

Magnetic field confines particles on a given trajectory

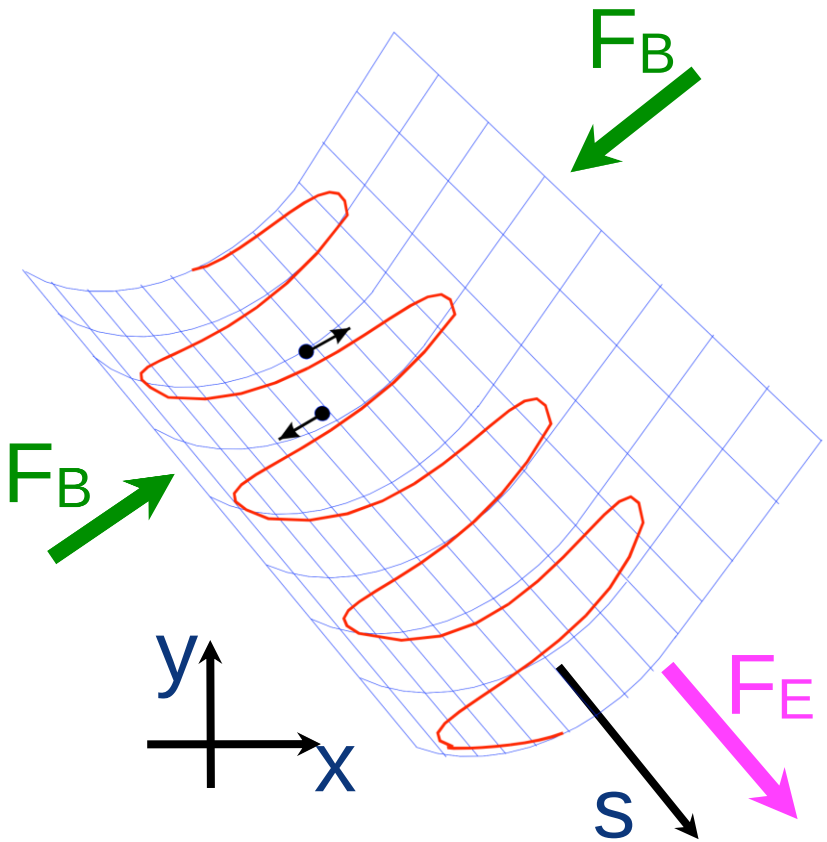
*An **accelerator** is formed by a sequence (called **lattice**) of:*

a) Magnets → Magnetic Field

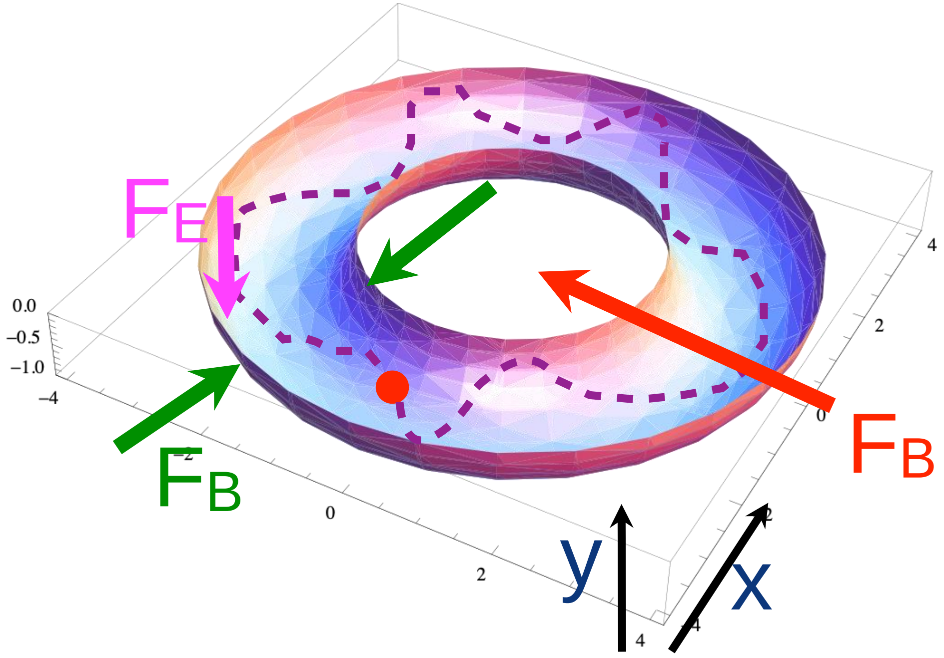
b) Accelerating Cavity → Electric Field

$$\overline{F(t)} = q\left(\underbrace{\overline{E(t)}}_{F_E} + \underbrace{\overline{v(t)} \otimes \overline{B(t)}}_{F_B}\right)$$

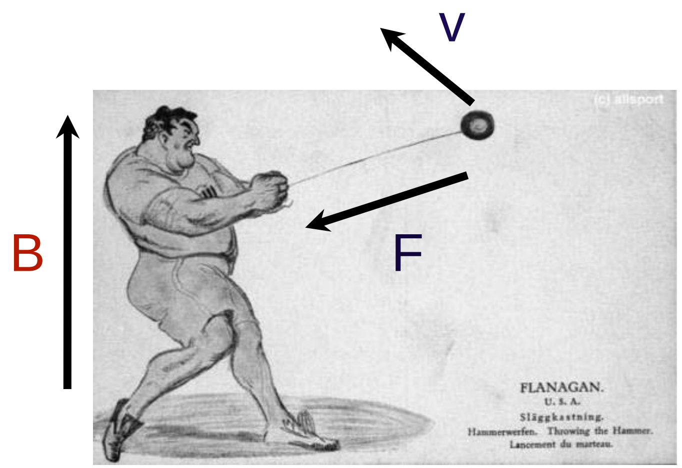
Linear Accelerator

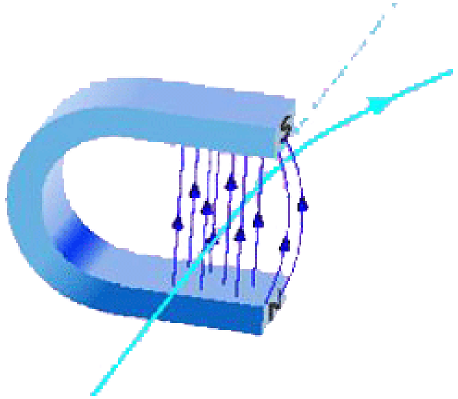


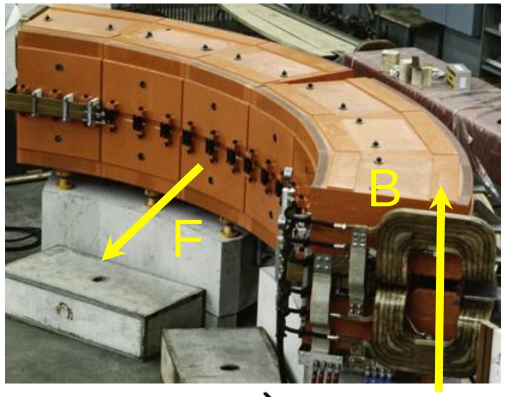
Circular Accelerator



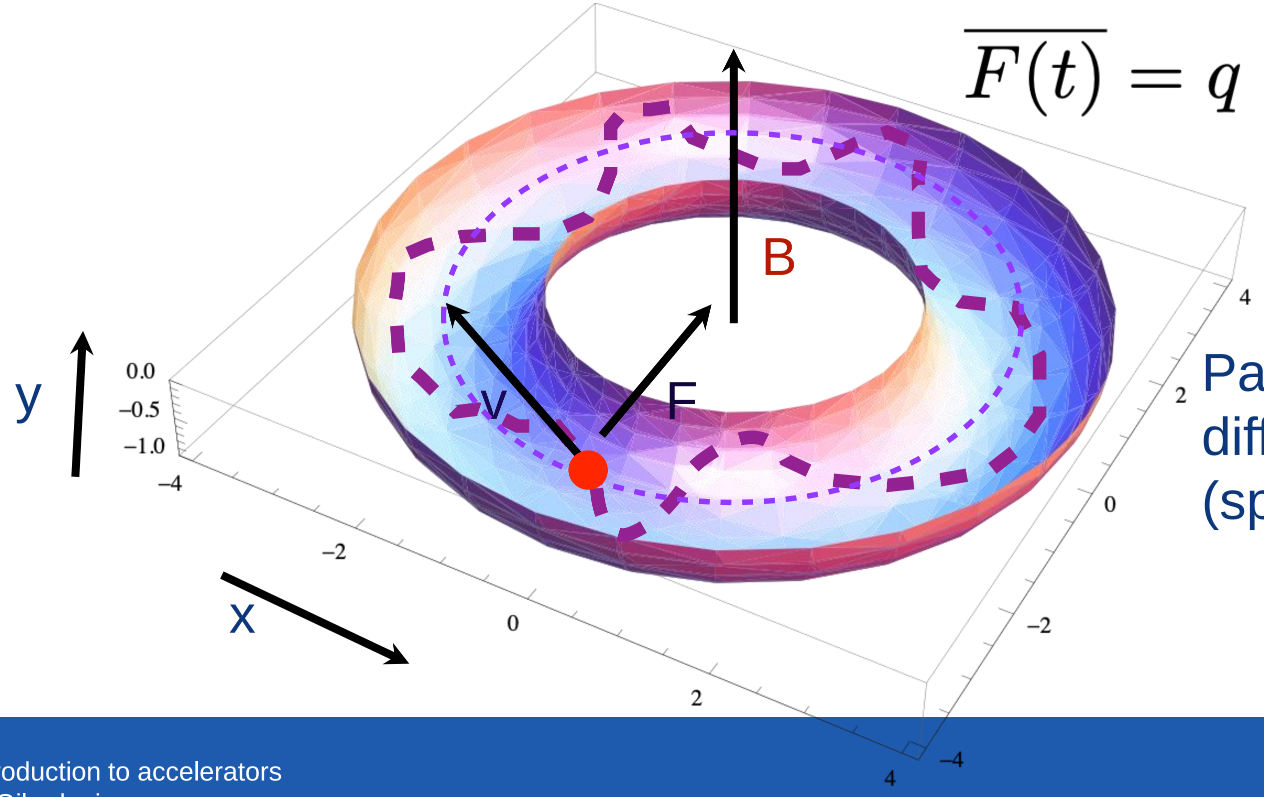
How an accelerator works ? A dipole







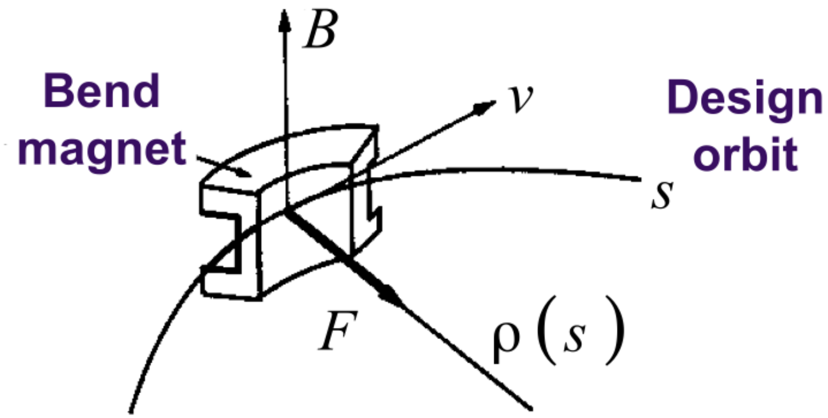
$$\overline{F(t)} = q\left(\overline{E(t)} + \overline{v(t)} \otimes \overline{B(t)}\right)$$

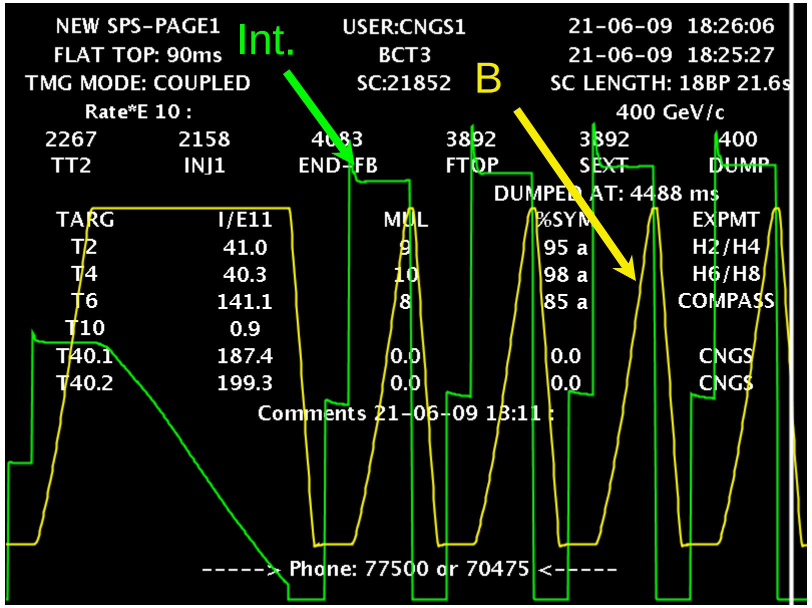


Particles of different energy (speed) behave differently

Magnetic field confines particles on a given trajectory

Dipoles



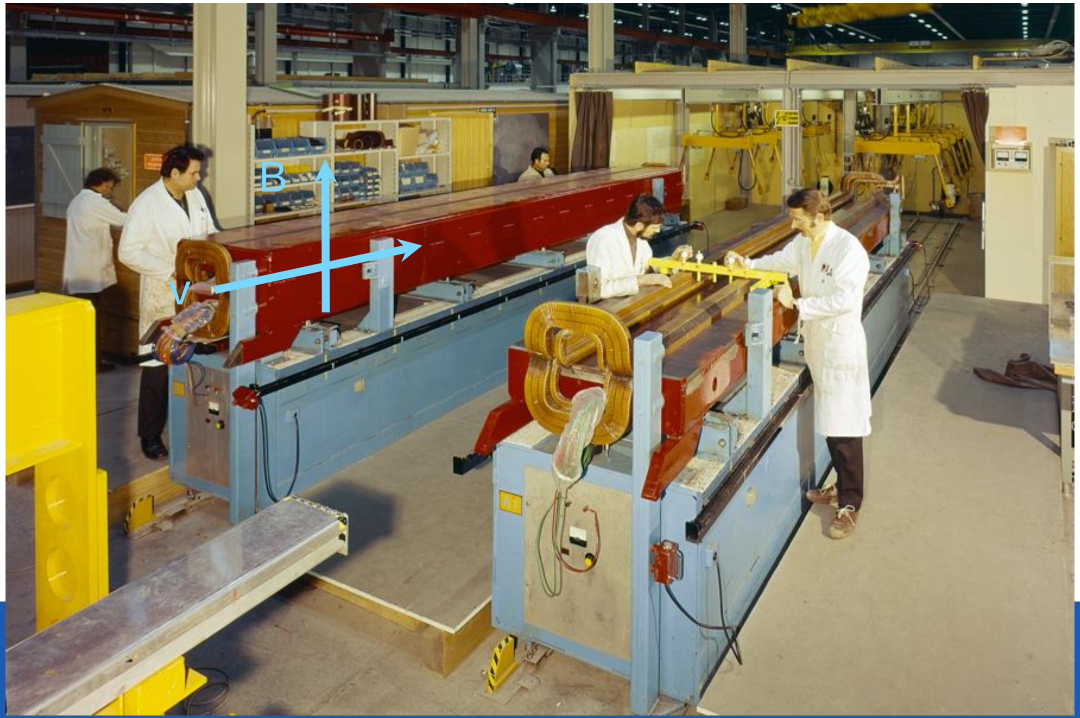


time (s) [21.6 s]

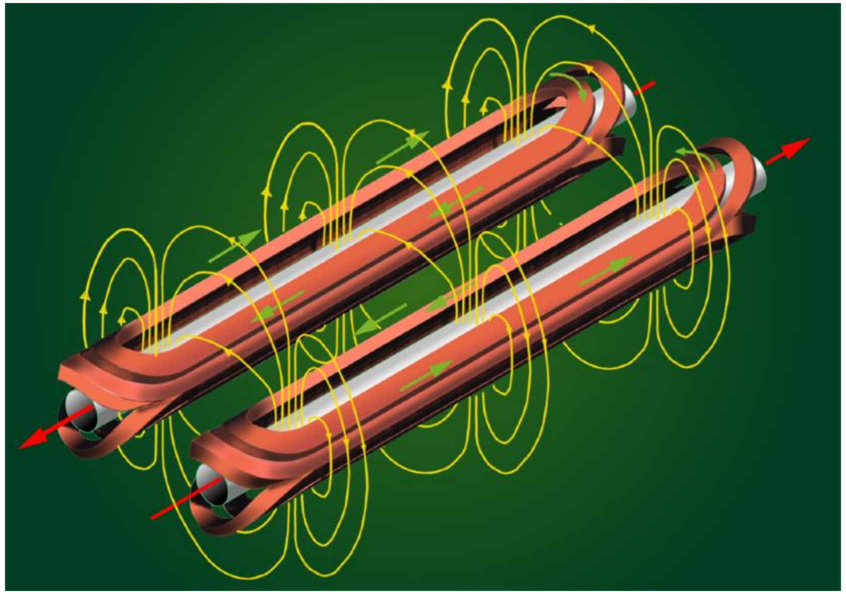
Force given by the vertical magnetic field compensates the centrifugal force to keep the particles on the central trajectory,
i.e. in the center of the beam pipe.

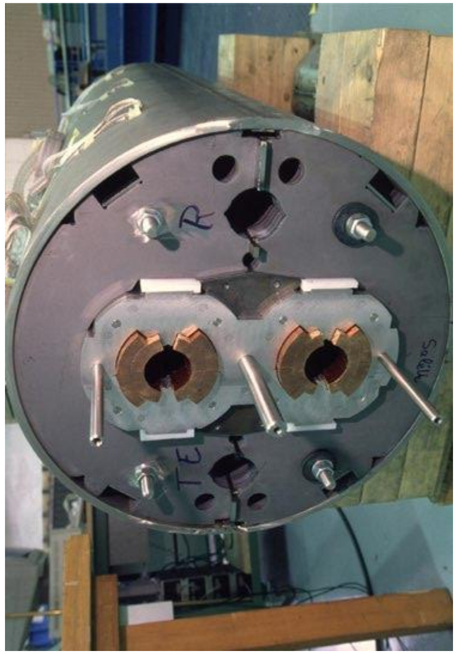
A fast dipole, able to deflect the beam in few μs is called **kicker**. A kicker is used to extract the beam from the machine.

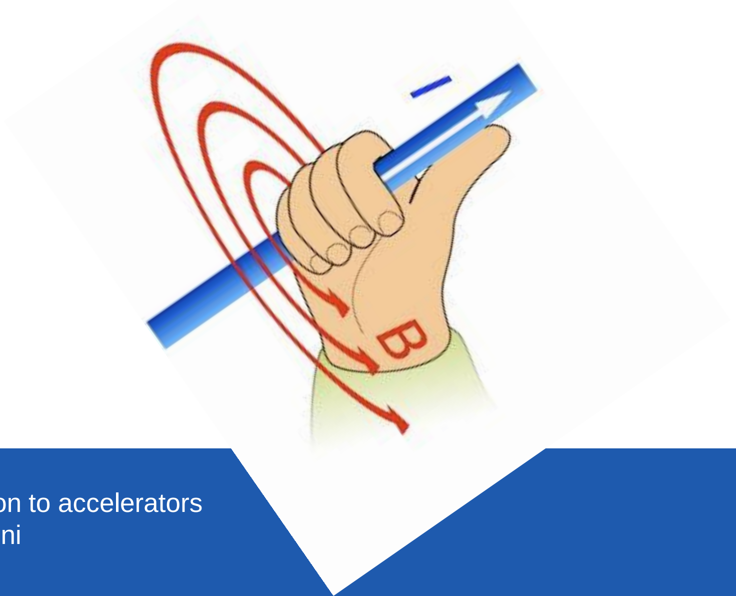
CERN-SPS dipoles, in total about 500

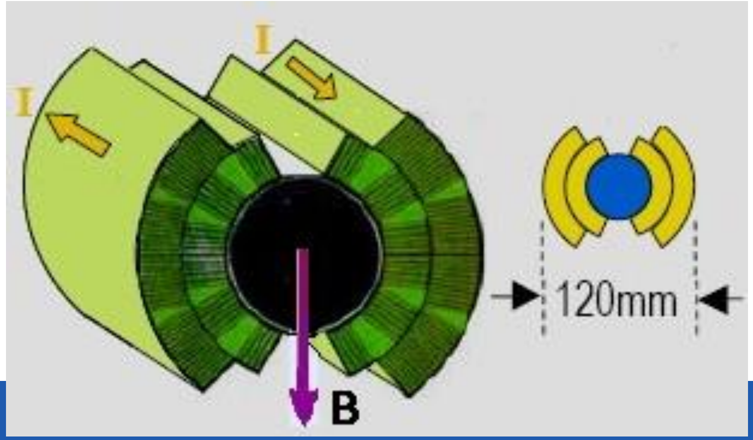


B-field right hand rule

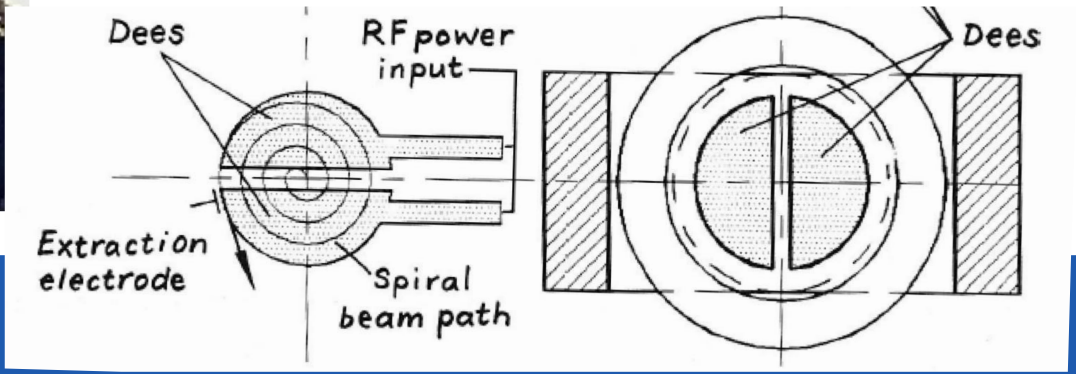




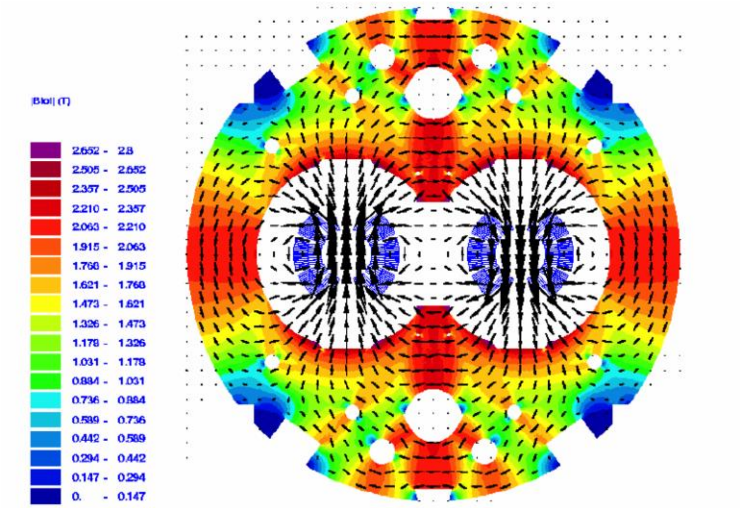


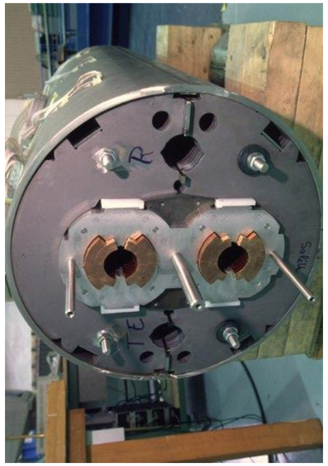


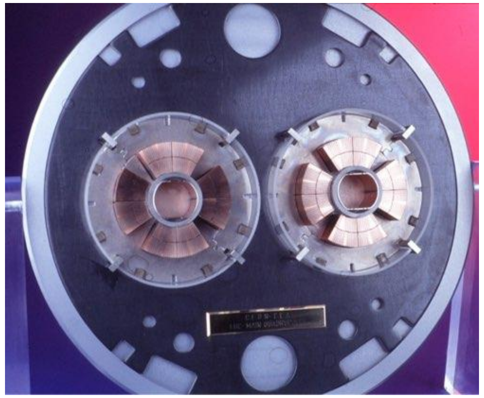
The first cyclotron and the Berkeley one



Two-in-one magnet design





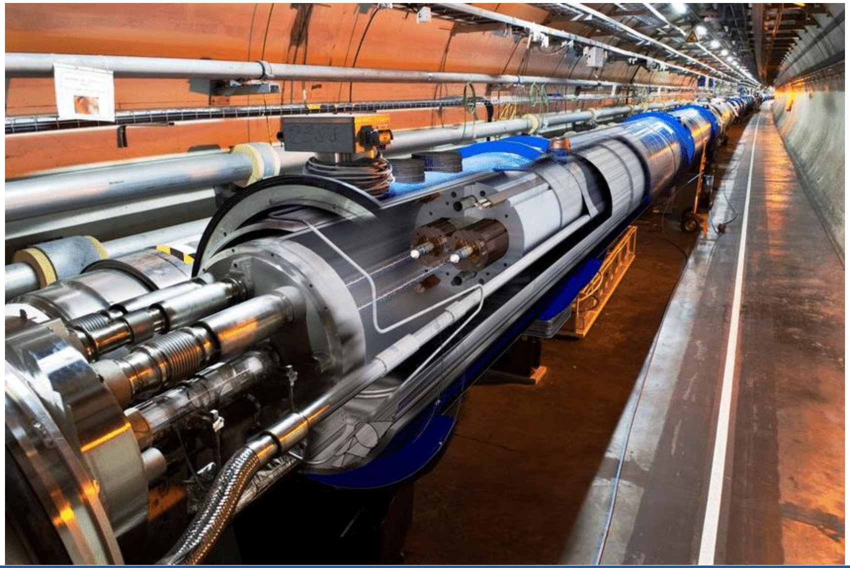


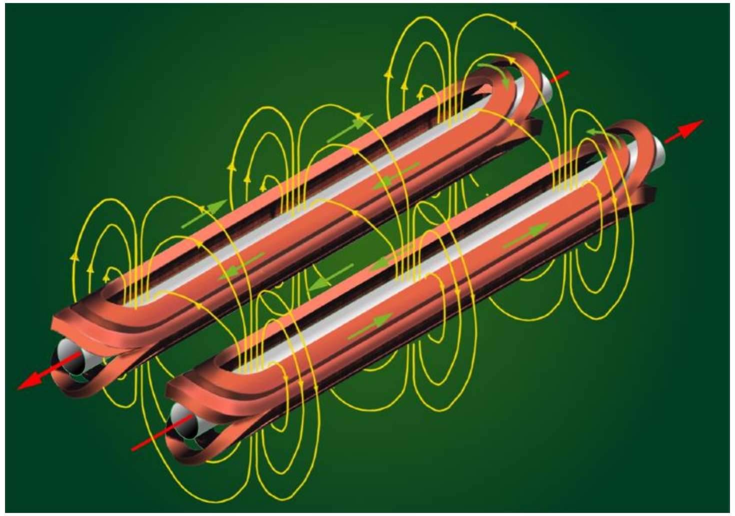
The LHC is one ring where two accelerators are coupled by the magnetic elements.

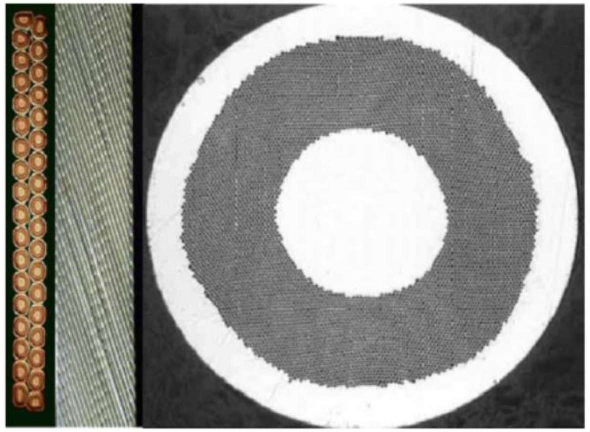
1232 dipoles
15 m long
11800 A

Nb –Ti **superconducting** cable in a Cu matrix

LHC lives at 1.9 - 2 K

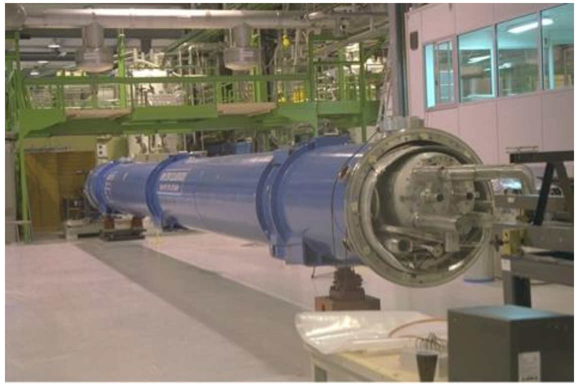




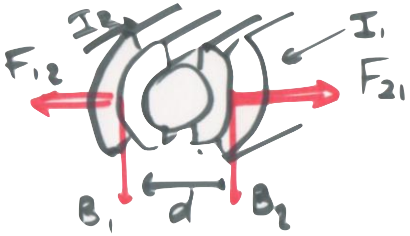


The force generated by this at liftoff

LHC dipoles



FORCE BETWEEN CABLES



$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

$I_1 = I_2 = 11800A$

$d = 90\ mm$

$$F_L/L = 310\ N/m$$

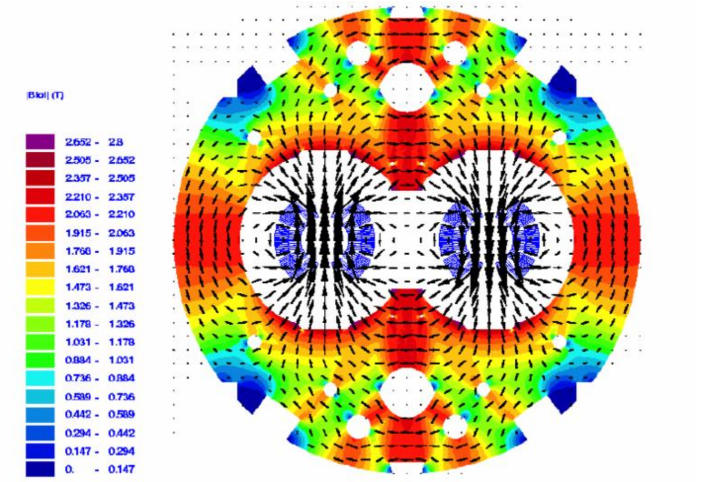
2 SETS OF 80 CABLES

$$F_{TOT} = 80 \cdot 80 \cdot 310 \sim 2 \cdot 10^6\ N/m$$

At every meter of this between the cables….

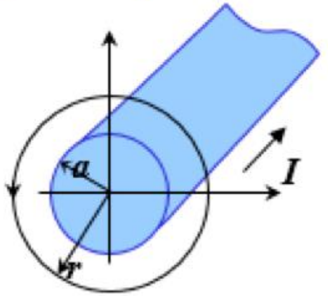
Max Current: ~ 11800 A
Max Field (7 TeV) : 8.33 T
Temperature: ~ 2 K
Tot. Energy stored: ~10 GJ

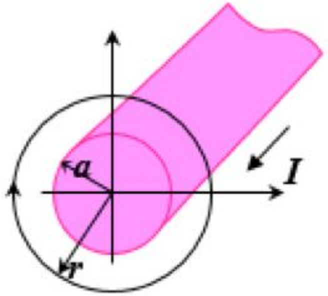
To compare
Current at home : 15 A
Earth magnetic Field: 25-65 μT
Temperature : 297.15 K (24 C)

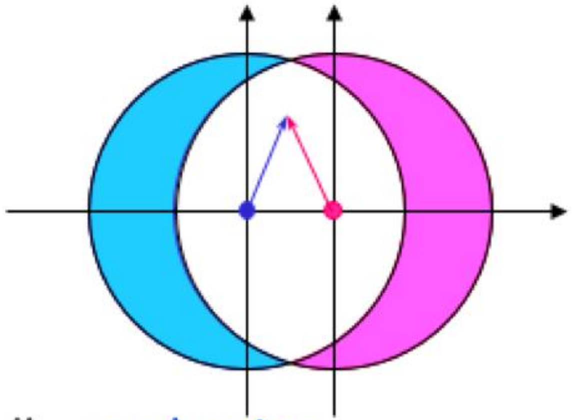


The cos ϕ coil

- Consider now the field generated by two wires
  - They carry equal currents in opposite directions







- Now consider the geometry with the two wire partially overlapping
  - The current in the overlap is zero
  - The magnetic field in the overlap is uniform and directed along y

- c $B_x = B_{1x} + B_{2x}$

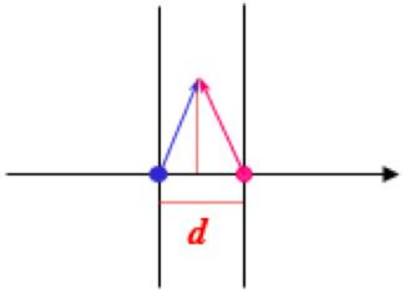
$$B_{1x} = -\frac{m_0}{2} J_1 r_1 \sin f_1$$

$$B_{2x} = -\frac{m_0}{2} J_2 r_2 \sin f_2$$

$$B_x = m_0 \frac{|J|}{2}(r_1 \sin f_1 - r_2 \sin f_2)$$

$$r_1 \sin f_1 = r_2 \sin f_2$$

$$\boxed{B_x = 0}$$



$$\boxed{B_y = m_0 \frac{|J| d}{2}}$$

$$B_y = B_{1y} + B_{2y}$$

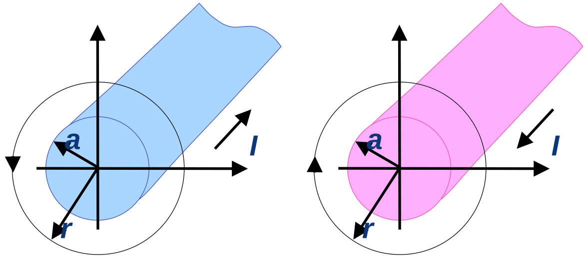
$$B_{1y} = \frac{m_0}{2} J_1 r_1 \cos f_1$$

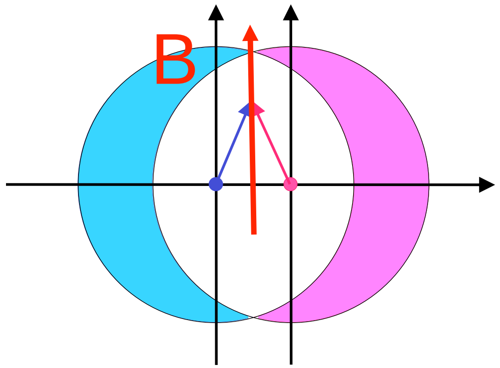
$$B_{2y} = \frac{m_0}{2} J_2 r_2 \cos f_2$$

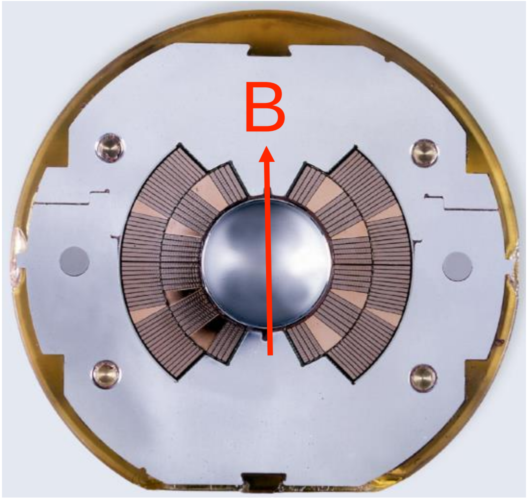
$$B_y = -m_0 \frac{|J|}{2}(r_1 \cos f_1 - r_2 \cos f_2)$$

$$r_1 \cos f_1 - r_2 \cos f_2 = d$$

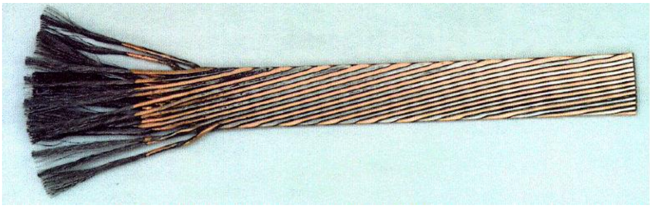
Cosθ coil of main dipoles







A 2D cosθ current distribution generates a quasi-perfect vertical field in the aperture between the two conductors.

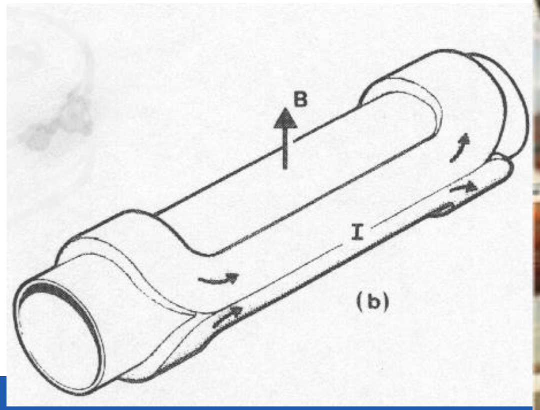


$$I = I_o \cos\vartheta$$

$$B_\vartheta = \frac{\mu o \; I_o}{2 \; r_o} \cos\vartheta \qquad B_x = o$$

$$B_\vartheta = \frac{\mu o \; I_o}{2 \; r_o} \sin\vartheta \qquad B_y = \frac{\mu o \; I_o}{2 \; r_o}$$

Dipolar Vertical field



CENTRIPETAL FORCE

$v \sim c$

$$F_c = \frac{mc^2}{r}$$

$$F_c = E/r$$

$N_{DIPOLES} = 1232$

$L_{DIPOLES} = 14,3\ m$

$L_{TOT,DIP} = 1232 \times 14,3 = 17618\ m$

BENDING RADIUS $\rightarrow$ $r_b = \frac{17618\ m}{2\pi}$

$r_b = 2804\ m$

$F_c = 7\ TeV / 2804\ m$ $\Rightarrow$ $F_c = \frac{1,12 \cdot 10^{-6}\ J}{2804\ m}$

$$F_c = 4 \cdot 10^{-10}\ N$$

Force scale (from Wikipedia)

Factor (N)	Value	Item
10^{-47}	3.6×10^{-47} N	Gravitational attraction of the proton and the electron in hydrogen atom[1]
10^{-30}	8.9×10^{-30} N	Weight of an electron[1]
10^{-26}	1.6×10^{-26} N	Weight of a hydrogen atom[1]
10^{-24} yoctonewton (yN)	5 yN	Force necessary to synchronize the motion of a single trapped ion with an external signal measured in a 2010 experiment[2][3]
10^{-22}	170 yN	Force measured in a 2010 experiment by perturbing 60 beryllium-9 ions[4][5]
10^{-15} femtonewton (fN)		
10^{-14}	~10 fN	Brownian motion force on an *E. coli* bacterium averaged over 1 second[6]
	~10 fN	Weight of an *E. coli* bacterium[7][8]
10^{-13}	~100 fN	Force to stretch double-stranded DNA to 50% relative extension[6]
10^{-12} piconewton (pN)	~4 pN	Force to break a hydrogen bond[6]
	~5 pN	Maximum force of a molecular motor[6]
10^{-11}		
10^{-10}	~160 pN	Force to break a typical noncovalent bond[6]
10^{-9} nanonewton (nN)	~1.6 nN	Force to break a typical covalent bond[6]
10^{-8}	8.2×10^{-8} N	Force on an electron in a hydrogen atom[1]
10^{-7}	2×10^{-7} N	Force between two 1 meter long conductors, 1 meter apart by the definition of one ampere
10^{-6} micronewton (µN)	1–150 µN	Output of FEEP ion thrusters used in NASA's Laser Interferometer Space Antenna [9]
10^{-4}		
10^{-3} millinewton (mN)		
10^{-2}	19-92 mN	Thrust of the NSTAR ion engine tested on NASA's space probe Deep Space 1[10]
10^{-1}		

$$\vec{F_c} = q \cdot \vec{v} \wedge \vec{B}$$

$$B = F_c / (q \cdot v)$$

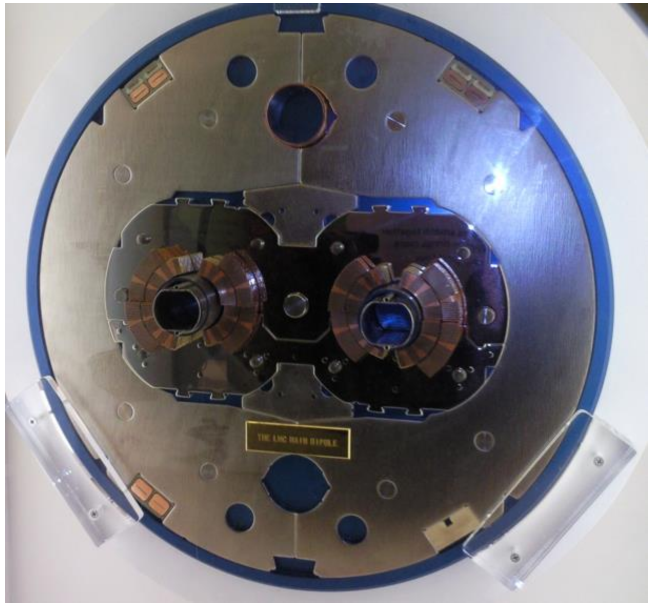
$$\vec{F_c} = 4 \cdot 10^{-10}\,N$$

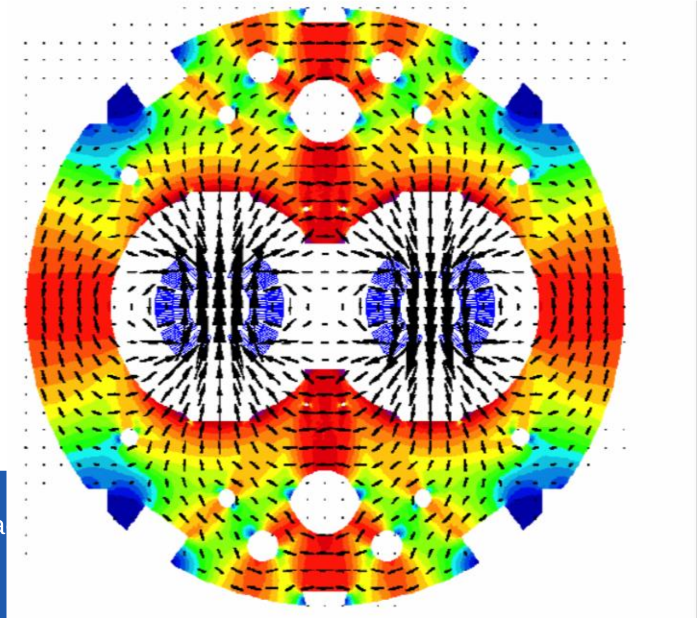
$$q = 1{,}602 \cdot 10^{-19}\,C$$

$$v \sim c$$

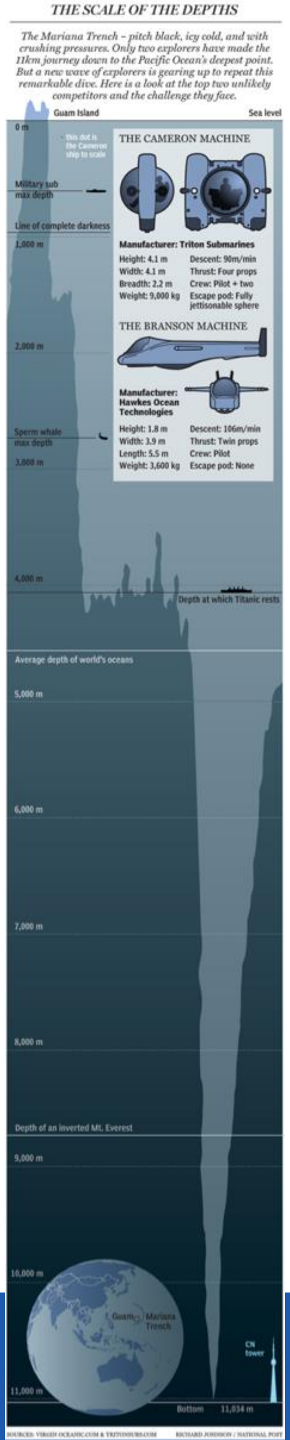
$$\boxed{B \sim 8{,}33\,T}$$

Pressure on conductors ~ 110 MPa





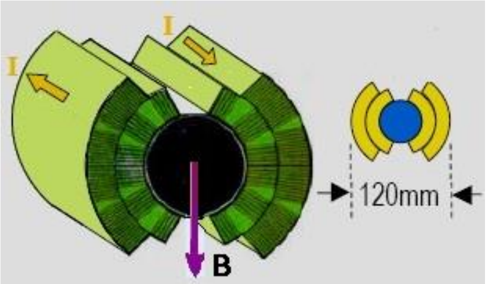
About same pressure at the bottom of Mariana trench:
11 000 m of water



DIPOLES

$$B = \frac{\mu_0 I}{2\pi d}$$

BIOT - SAVART



Average distance between cables and beam pipe center

$d = 2 \cdot r \qquad r = 45\ mm$

$I \sim 11800\ A \Rightarrow B_c \sim 0{,}05\ T$ per cable

$$B_{DIPOLE} = 8{,}33\ T \Rightarrow N_{CABLE} = \frac{B_{DIPOLE}}{B_{CABLE}}$$

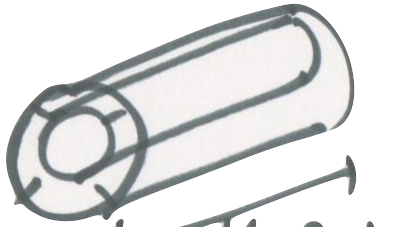
$$N_{CABLE} = \frac{8{,}33\ T}{0{,}05\ T} \sim 160$$

DIPOLE INDUCTANCE

$$\Phi = N \cdot B \cdot S$$

N → TURN

B → FIELD



$$\Phi = 80 \cdot 8{,}33\,T \cdot (14{,}3\,m \cdot 0{,}09\,m) \simeq 1000\,Wb$$

$$\Phi = L \cdot I$$

$$L = \frac{1000\,Wb}{11000\,A} \sim 0{,}1\,H$$

STORED ENERGY

$$E_D = \frac{1}{2} L I^2$$

PER DIPOLE

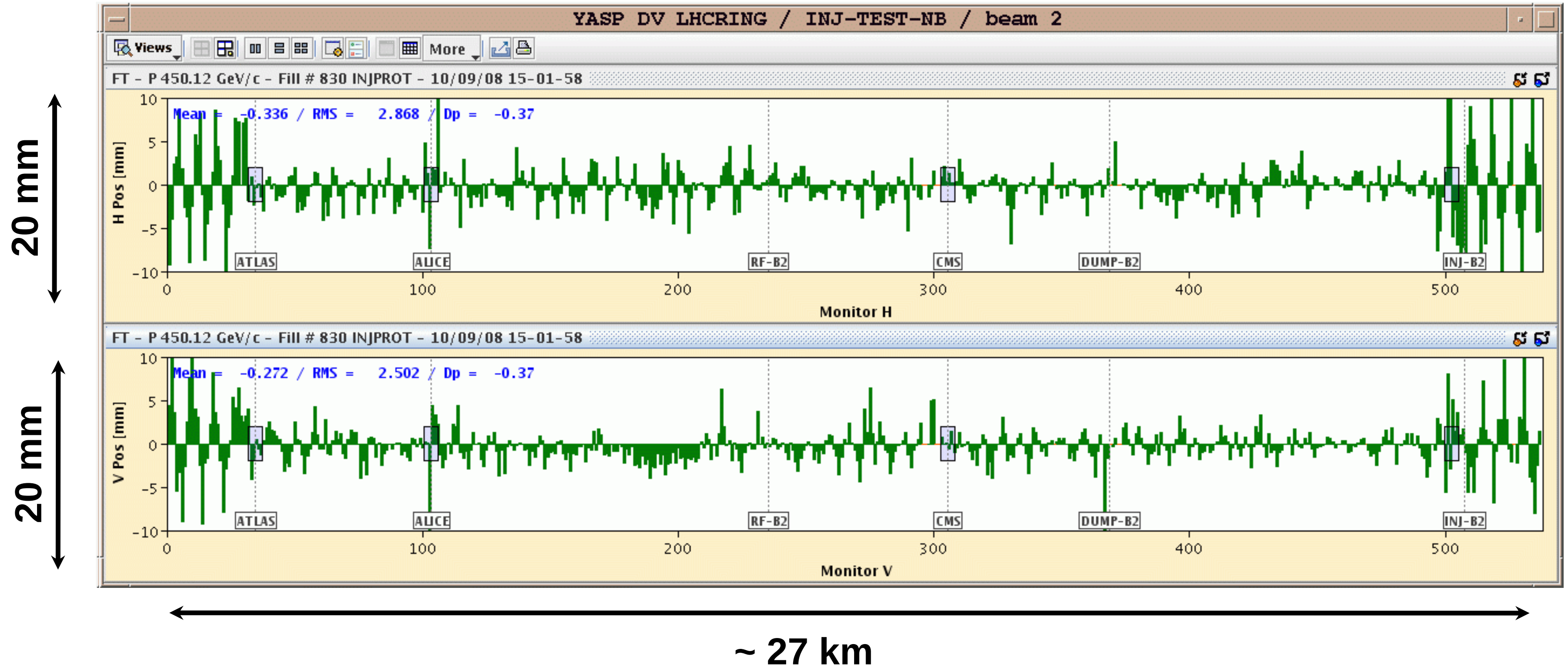
$$E_d \simeq 7\,MJ$$

$$N_d = 1232$$

$$E_{TOT} \simeq 9\,GJ$$

Real LHC orbit - correction of dipolar error

Real orbit taken the 1st day of the LHC

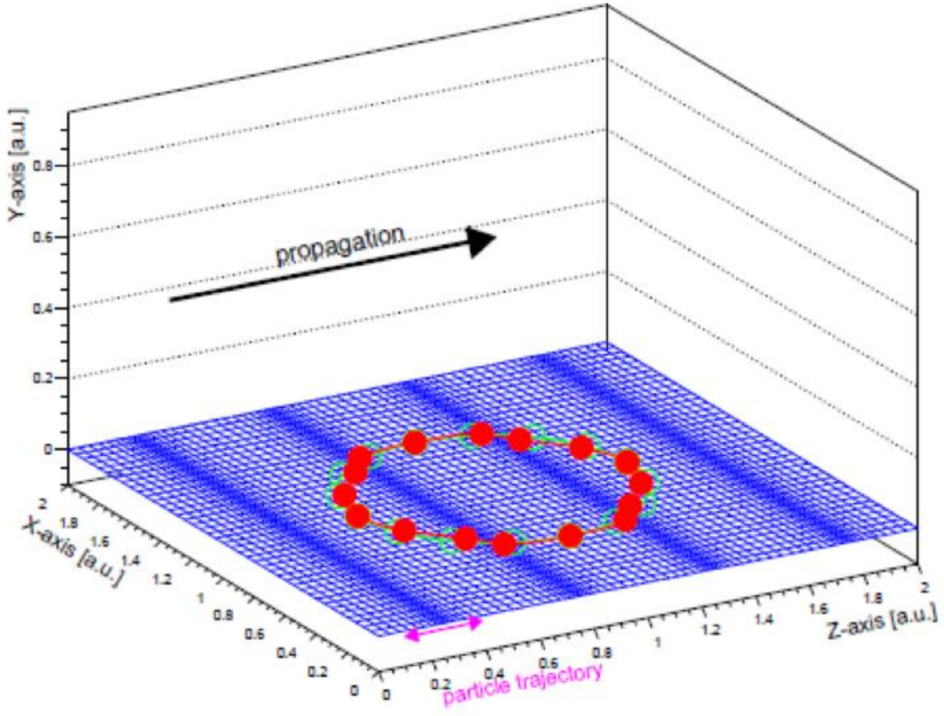


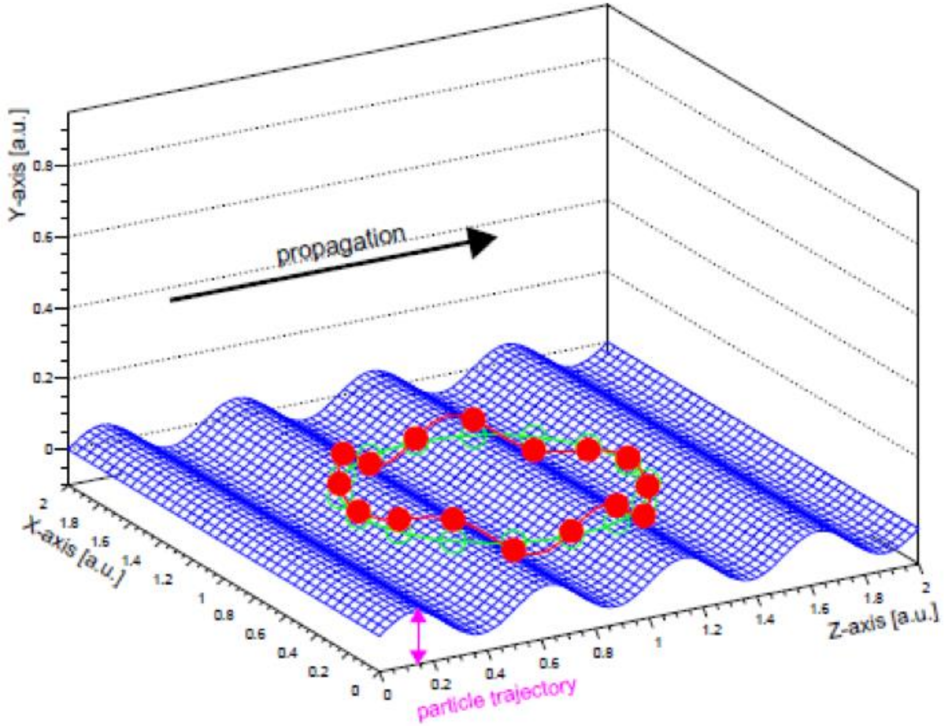
Please notice:
Horizontal and vertical scale are different by 6 orders of magnitude

Courtesy of J. Wenninger

Impact of Earthquakes on LHC (not only)

The impact on LHC of seismic waves depends on amplitude, wavelength (lattice resonances), wave type (longitudinal, transverse)

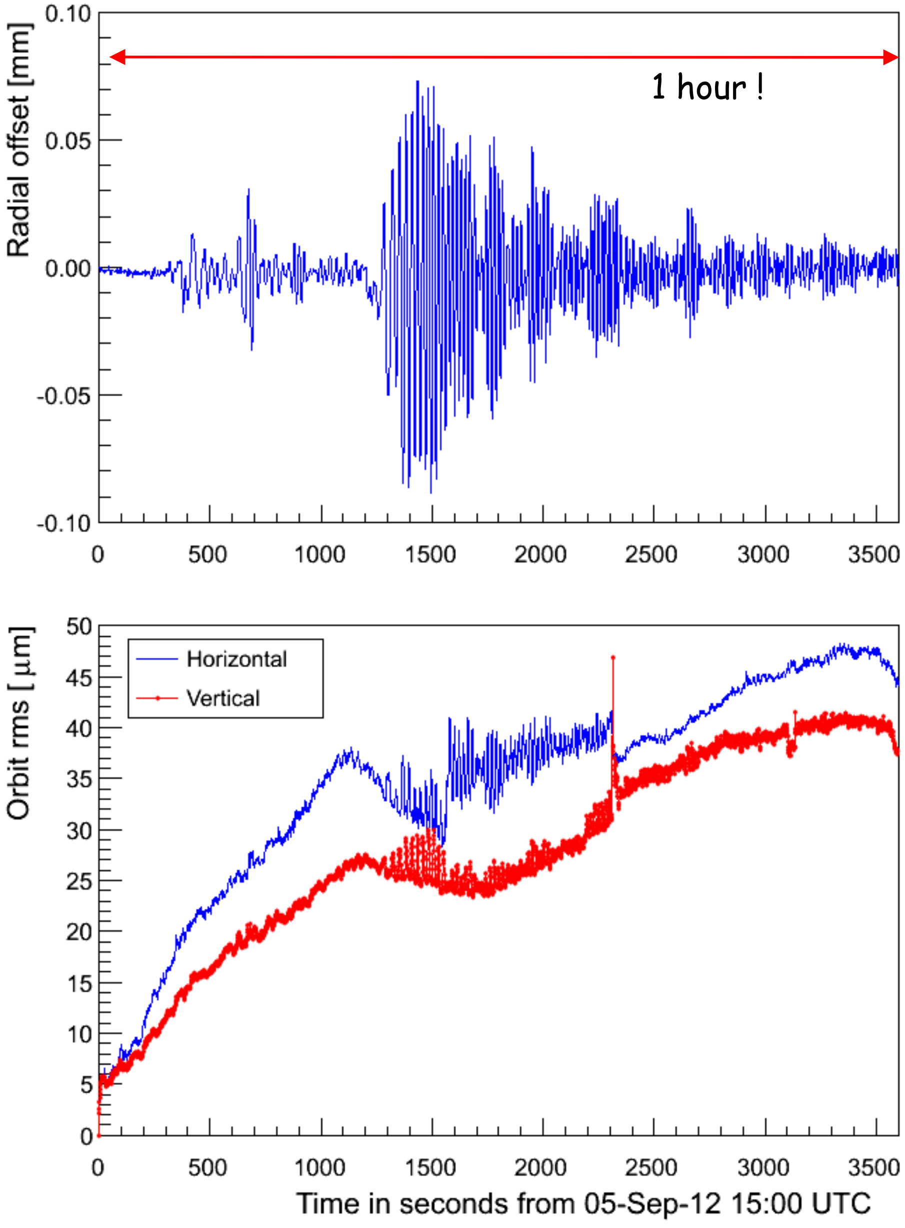




An earthquake in Costa Rica

- By scanning the logging data M. Fitterer found a candidate earthquake of magnitude 7.6 that occurred in Costa Rica during **fill 3032**.
- UTC time of the earthquake :
  - 05/09/2012   14:42:10
- Arrival of the first waves at CERN ~15:06 UTC.

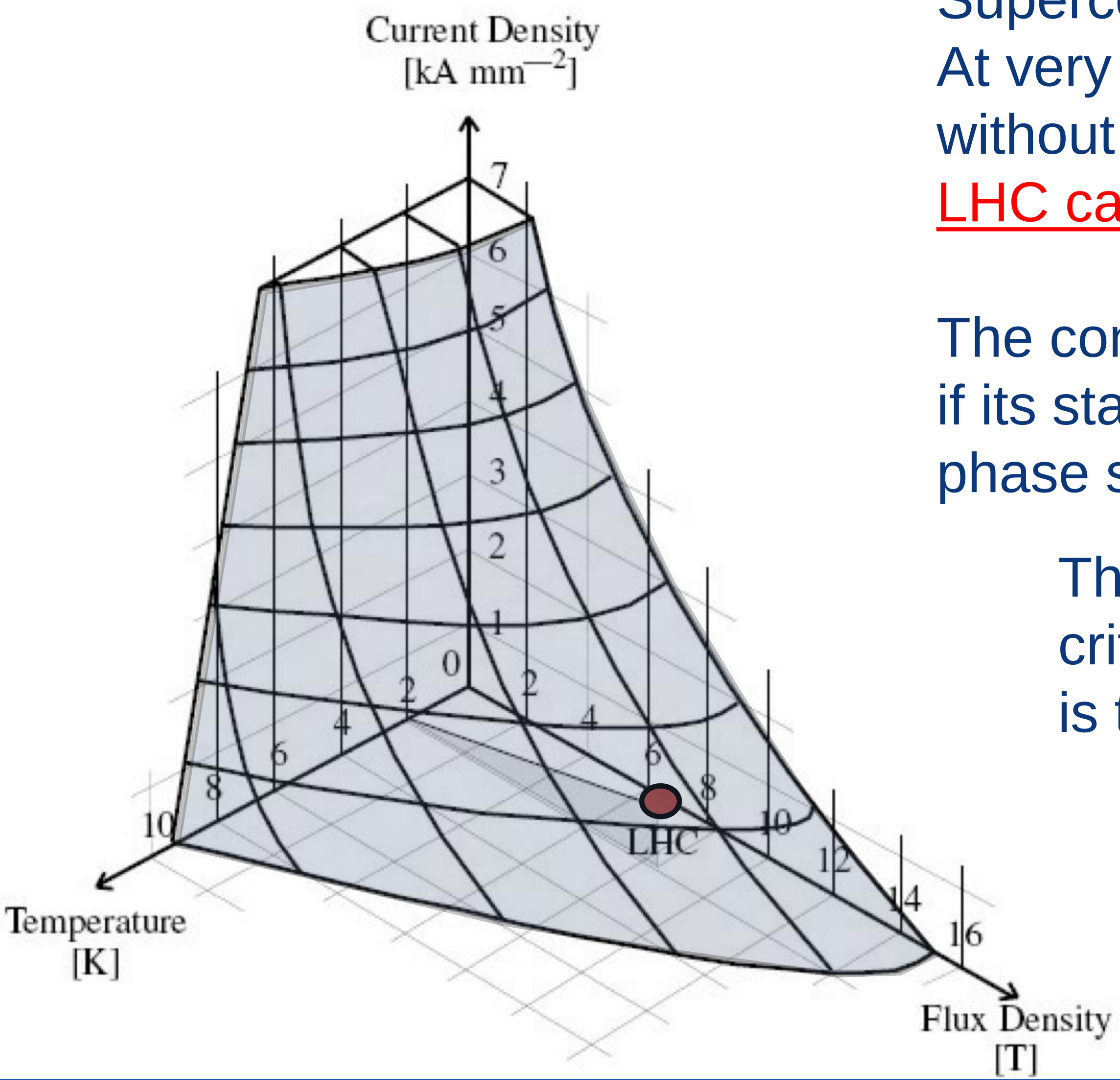
Costa Rica earthquake – orbit response



- Earthquake visible on the ring radius for over 1 hour.
- The first waves (6 km/s) seemed to affect the LHC mainly radially – but it is also weaker.
- The second type of waves (4 km/s) is visible in radial and transverse.
  - *Radial amplitude is larger than for Italy Earthquake,* ***equivalent to strongest tides****.*

From J. Wenninger

Very, very short introduction to Superconductivity for accelerators



Superconductivity is a property of some materials.
At very low temperature they can carry currents without voltage drop, i.e. their resistivity goes to zero.
LHC cables: Nb-Ti working at 1.9 K

The conductor remains Superconductor if its status in Current Density, Temperature, B field phase space is below the Critical Surface

The distance between the working point and the critical surface for a fixed B field and Current Density is the temperature margin (critical temperature)

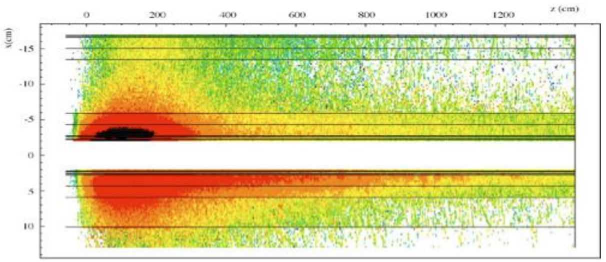
Transition to a normal conducting state is called magnet quench

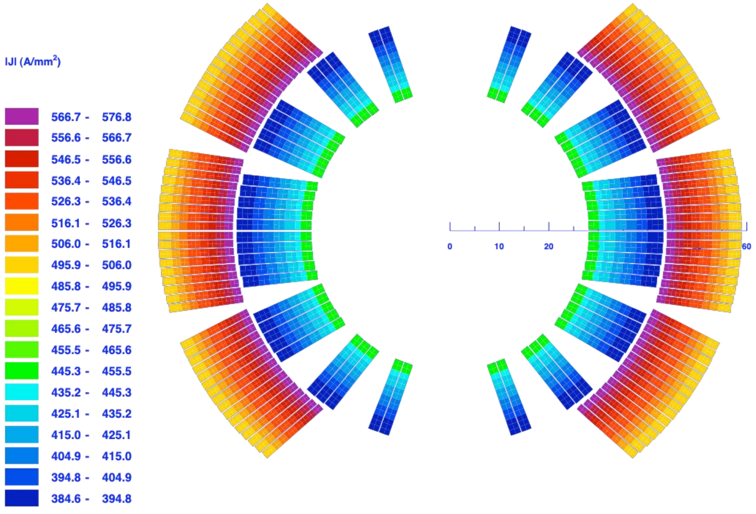
What can increase the temperature in a magnet ?

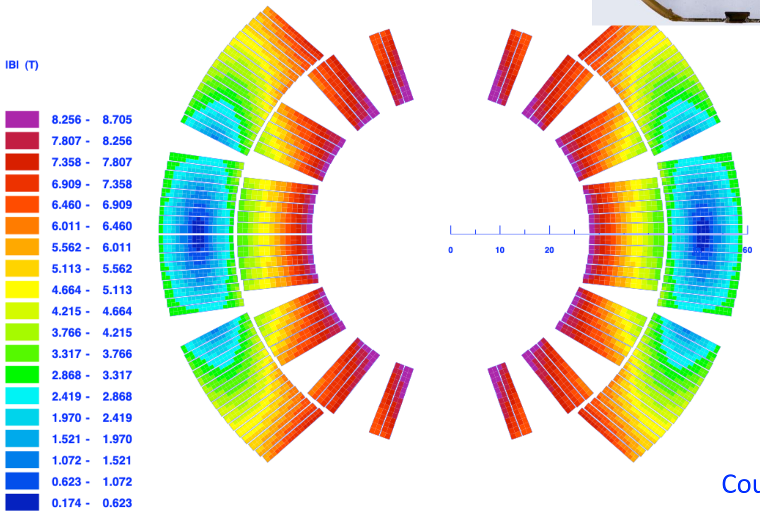
V. V. S. Introduction to Superconductivity II

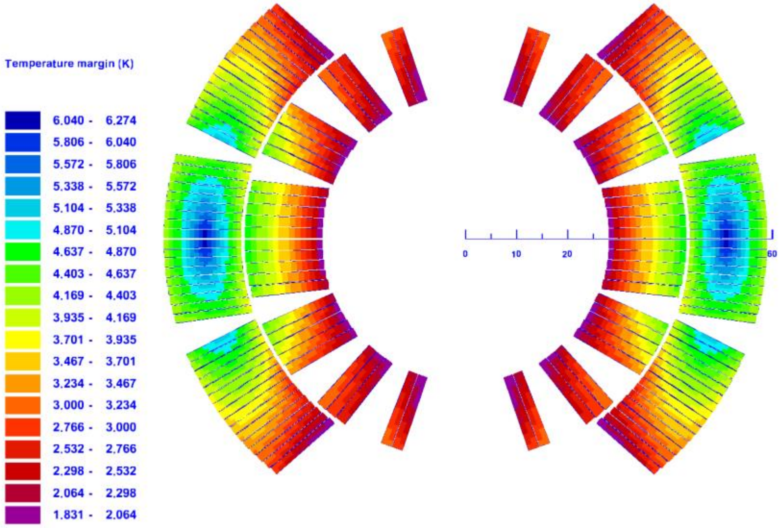
Beam losses can eat the temperature margin because of energy deposition

Limit of accepted losses: ~ <u>**10 mW/cm^3**</u>
to avoid ΔT> 2 K, the temperature margin









Courtesy of Stephan Russenschuck

How much is 10 mW/cm^3 ?

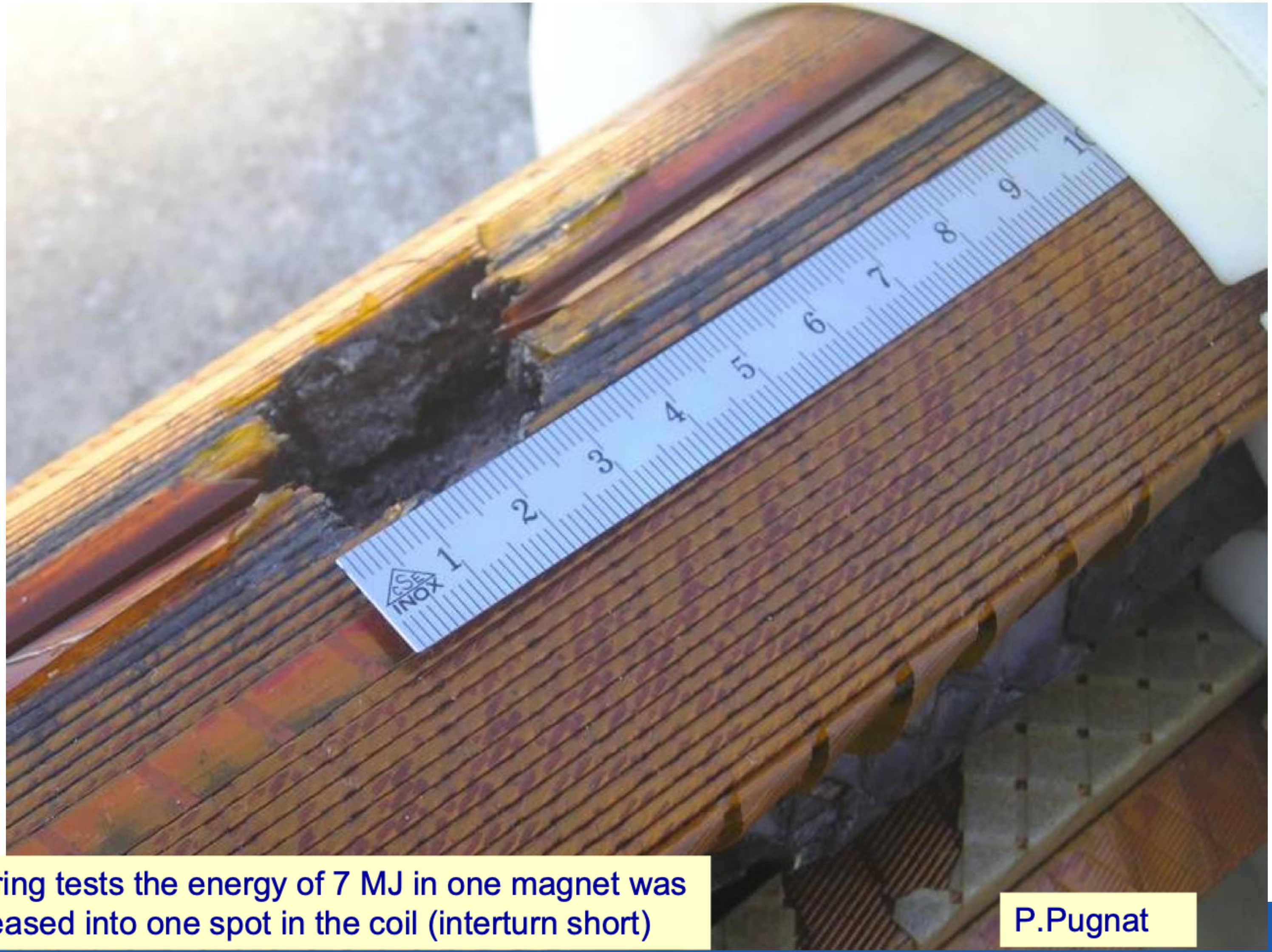
A fluorescent (known as neon) tube can be typically 1.2 m long with a diameter of 26 mm, with an input power of 36 W.

This makes a power density of about 56 mW/cm^3.

The power of a neon tube can quench about 5 LHC dipoles at collision energy.... because one does not need 10 mW/cm^3 for the entire volume of a magnet, but for about 1 cm^3.

If you do the same basic computation with a normal 100 W resistive bulbs is even worst

When something goes wrong.... bad quench...



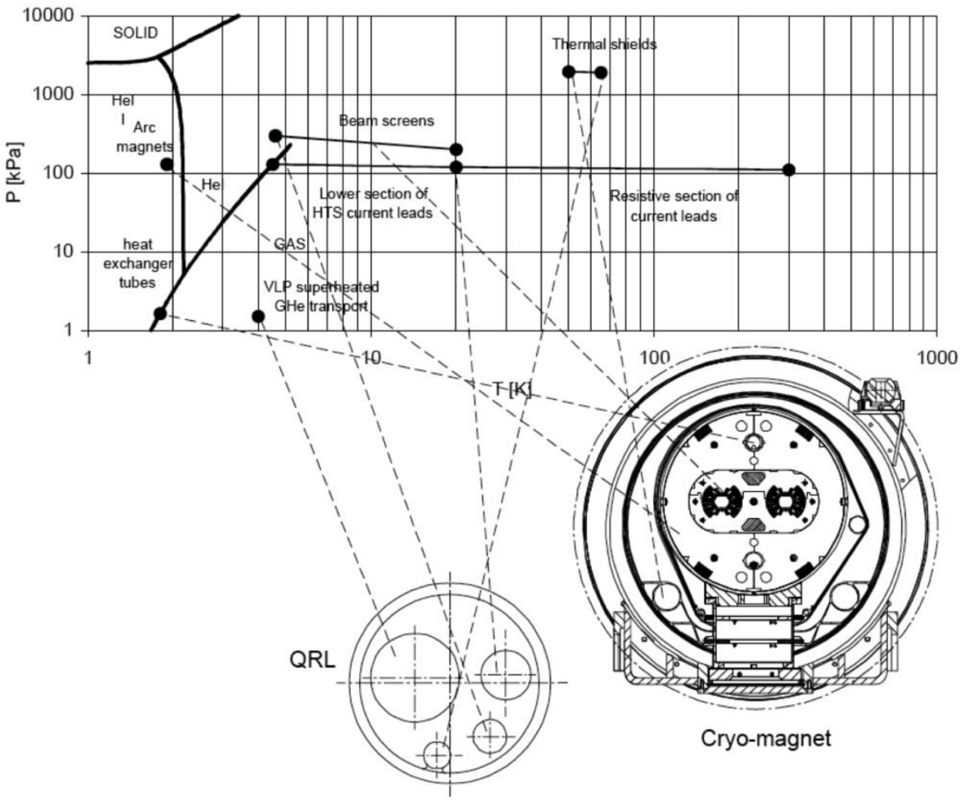
During tests the energy of 7 MJ in one magnet was released into one spot in the coil (interturn short)

P.Pugnat

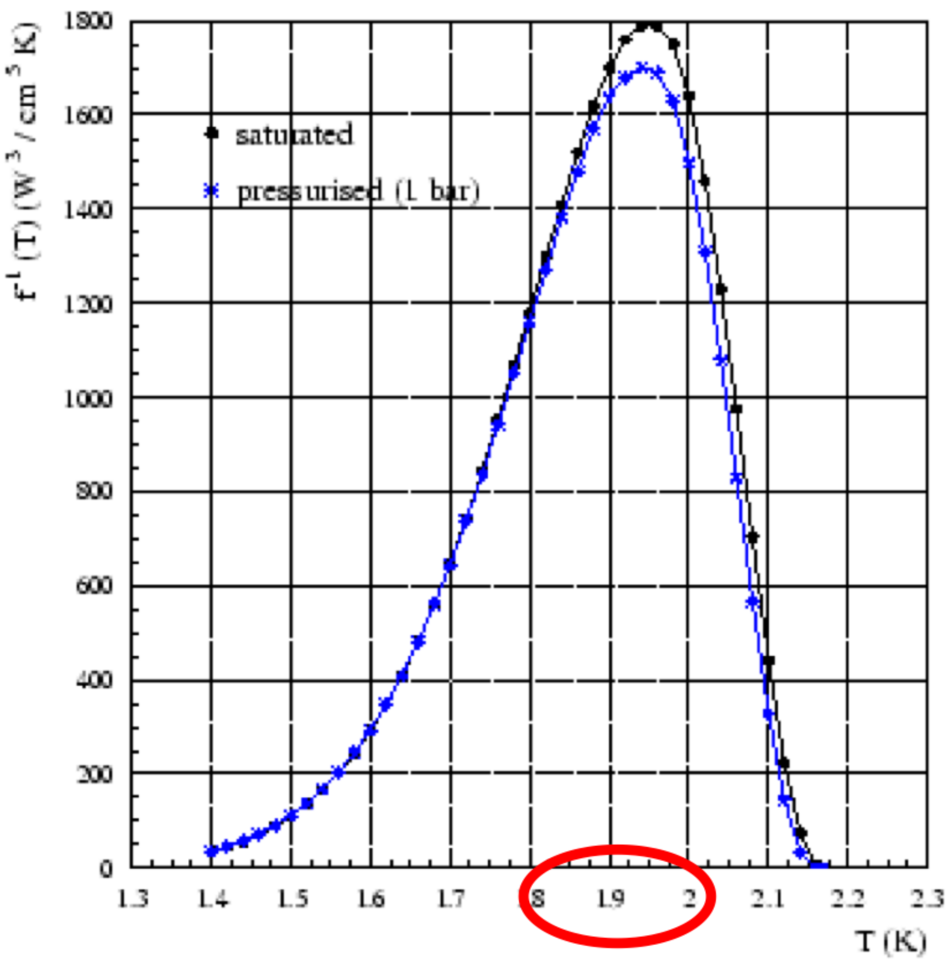
Second, one has to cool the LHC ….

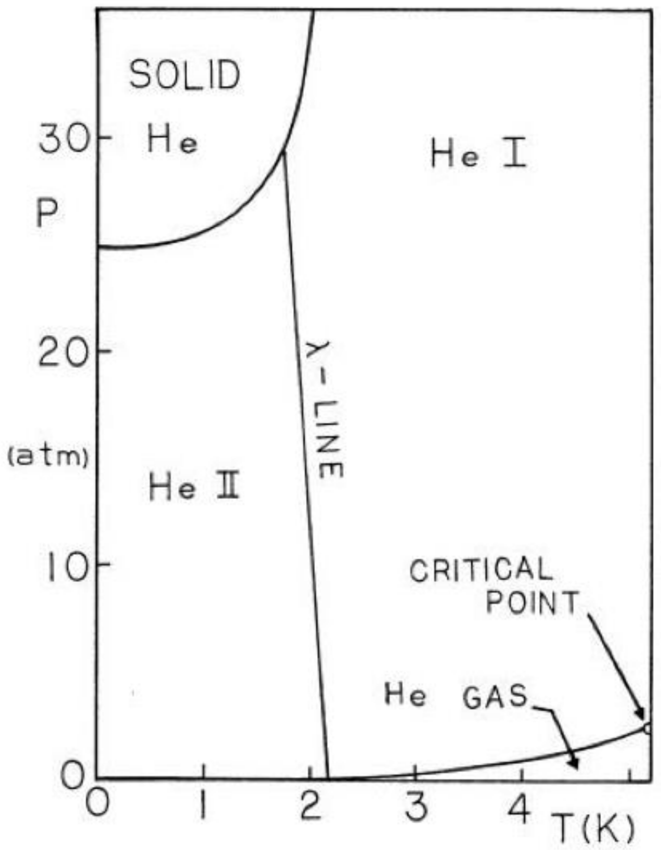
LHC cryogenics needs 40,000 leak-tight pipe junctions.

12 million litres of liquid nitrogen are vaporised during the initial cooldown of 31,000 tons of material and the total inventory of liquid helium is 700,000 l (about 100 tonnes).



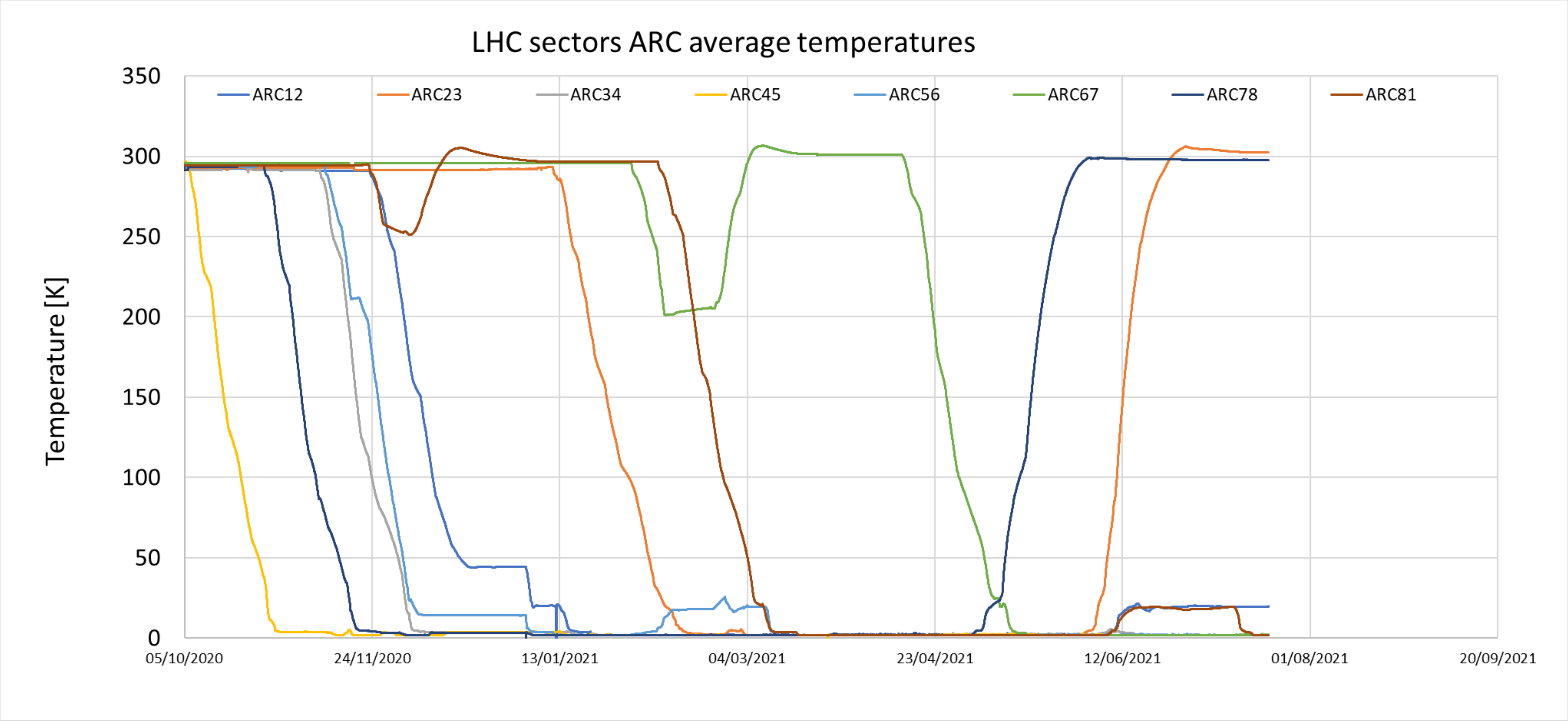
Why helium ?





Helium at 1.8-2 K has a very large thermal conductivity and very low viscosity

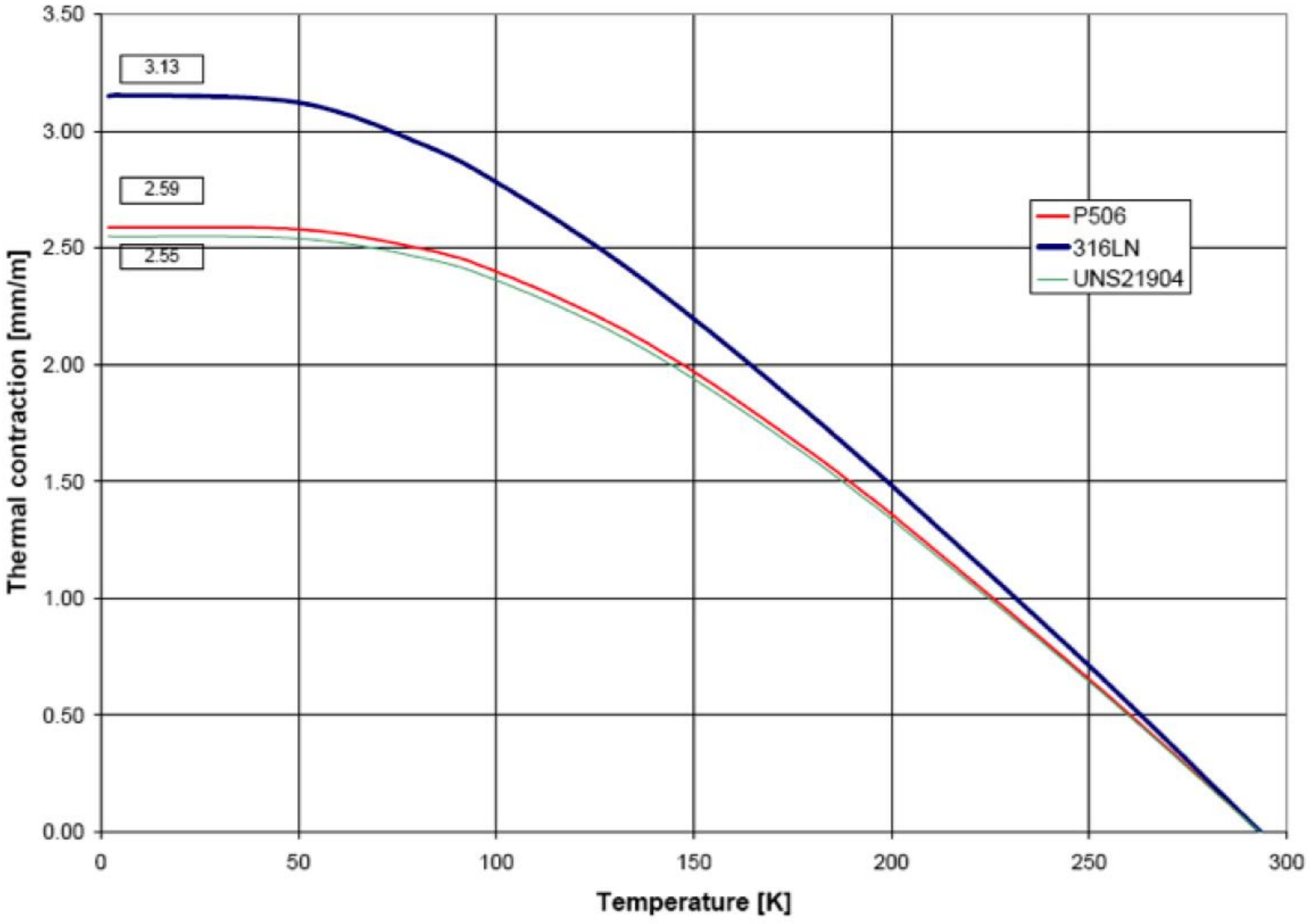
Cooling the LHC (and re-warming due to some surprises…)



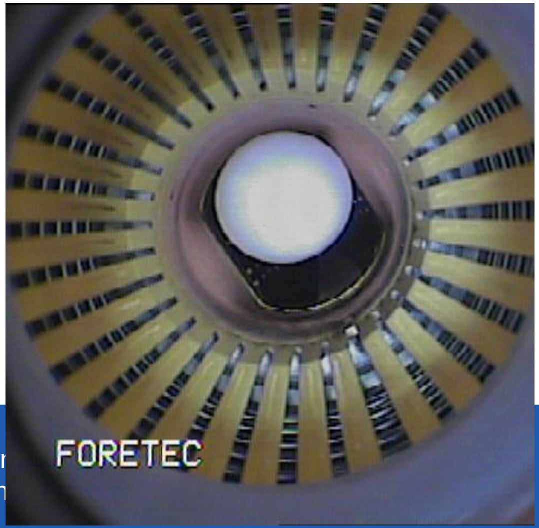
Different working places, similar temperatures..

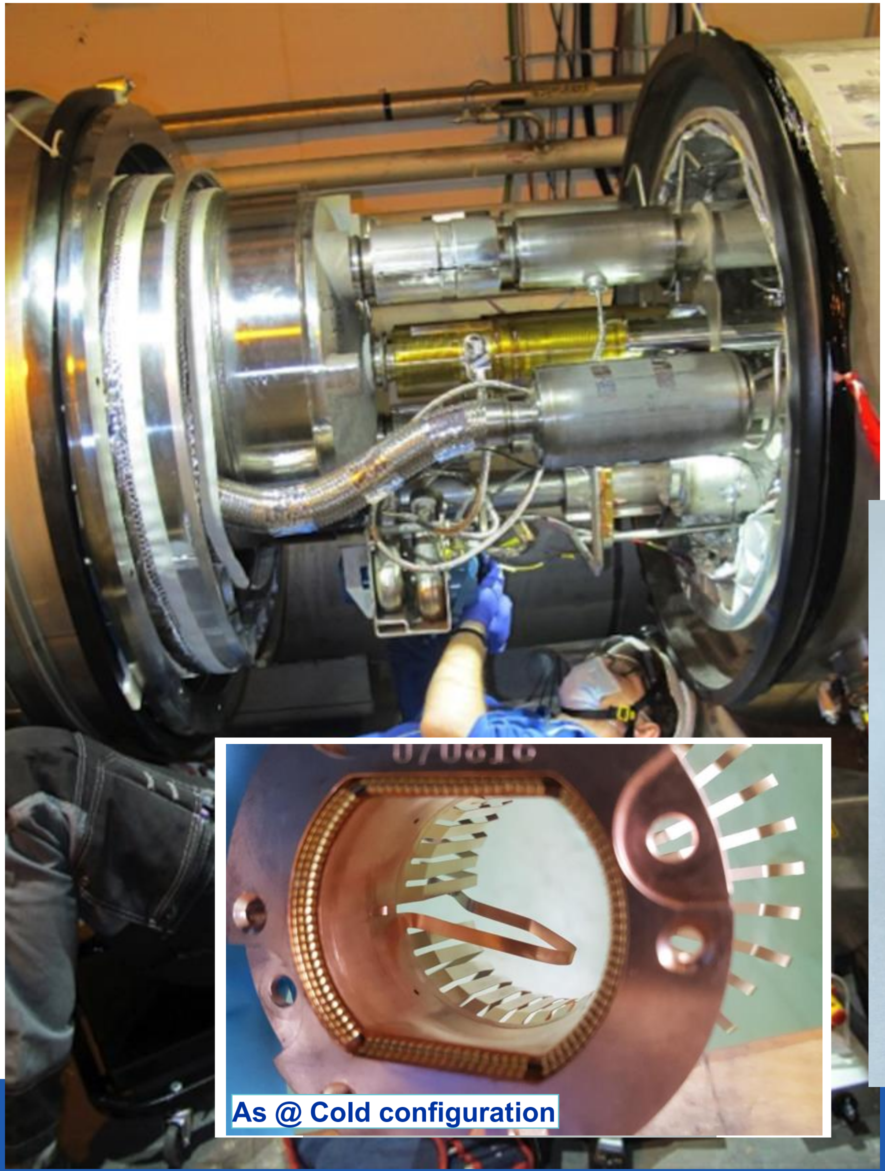
Credits: NASA

The magnets shrinks due to temperature…

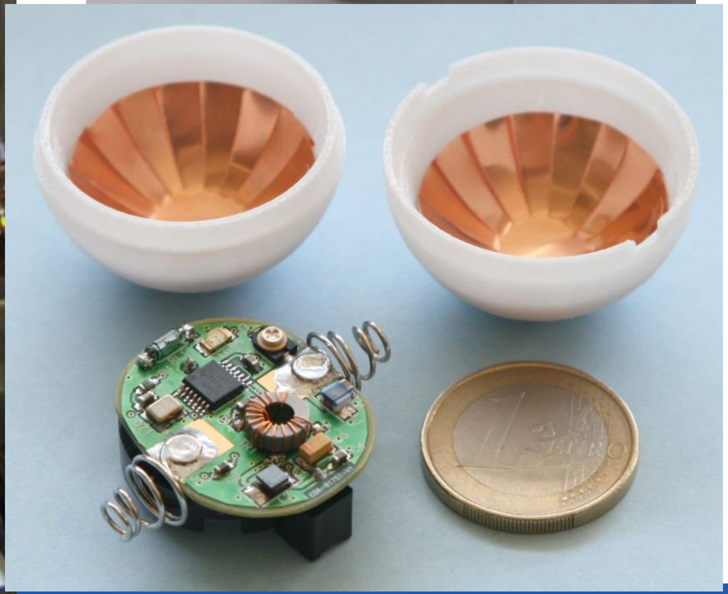


Anything inside? “Ping-pong” 40 MHz RF ball

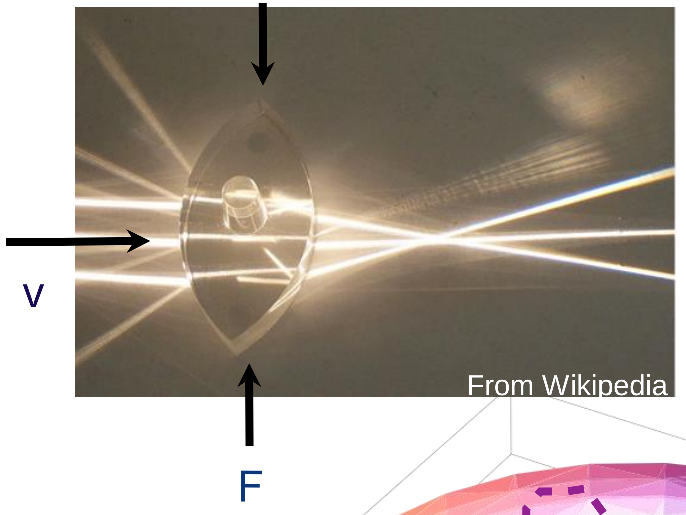




40 MHz transmitter



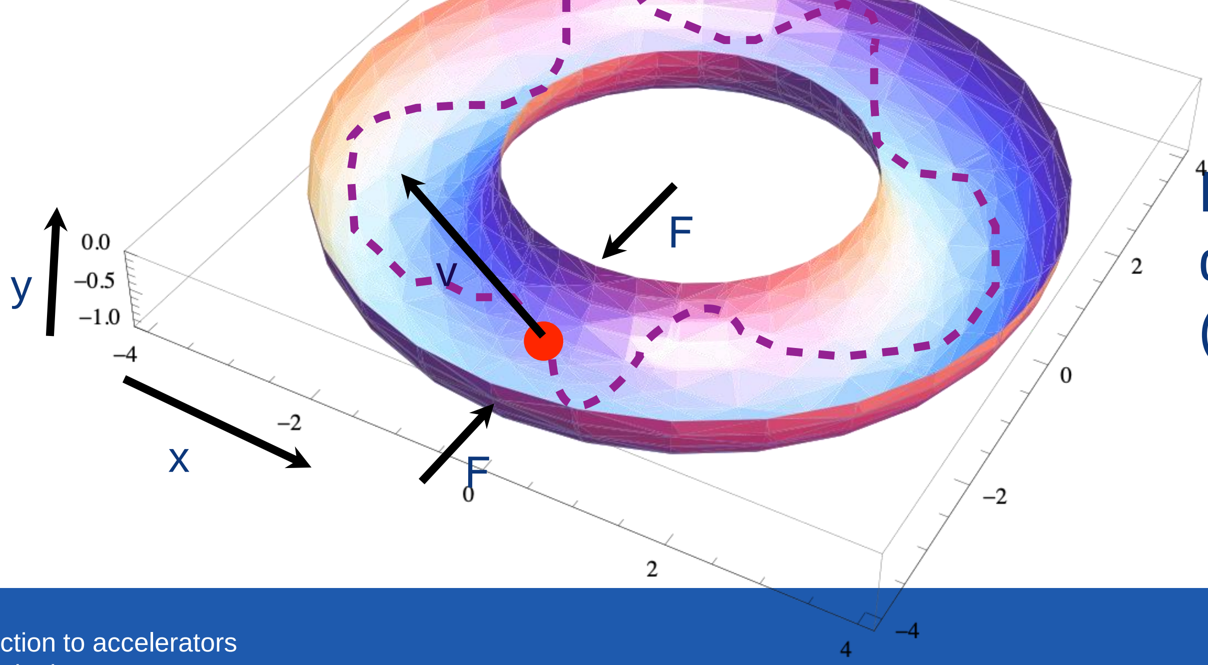
How an accelerator works ?



Goal: keep enough particles confined in ***a well defined volume*** *to accelerate them.*

How ? Lorentz Force!

$$\overline{F(t)} = q\left(\cancel{\overline{E(t)}} + \overline{v(t)} \otimes \overline{B(t)}\right)$$



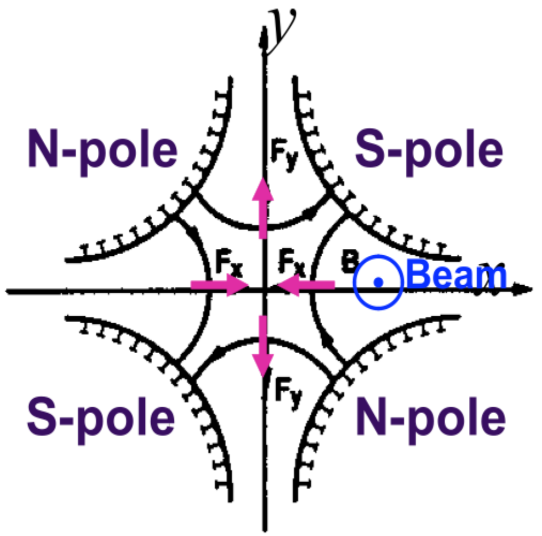
Particles of different energy (speed) behave differently

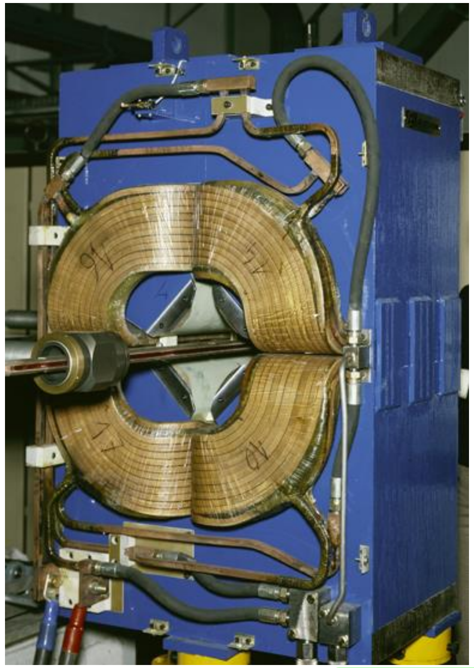
Magnetic field confines particles on a given trajectory

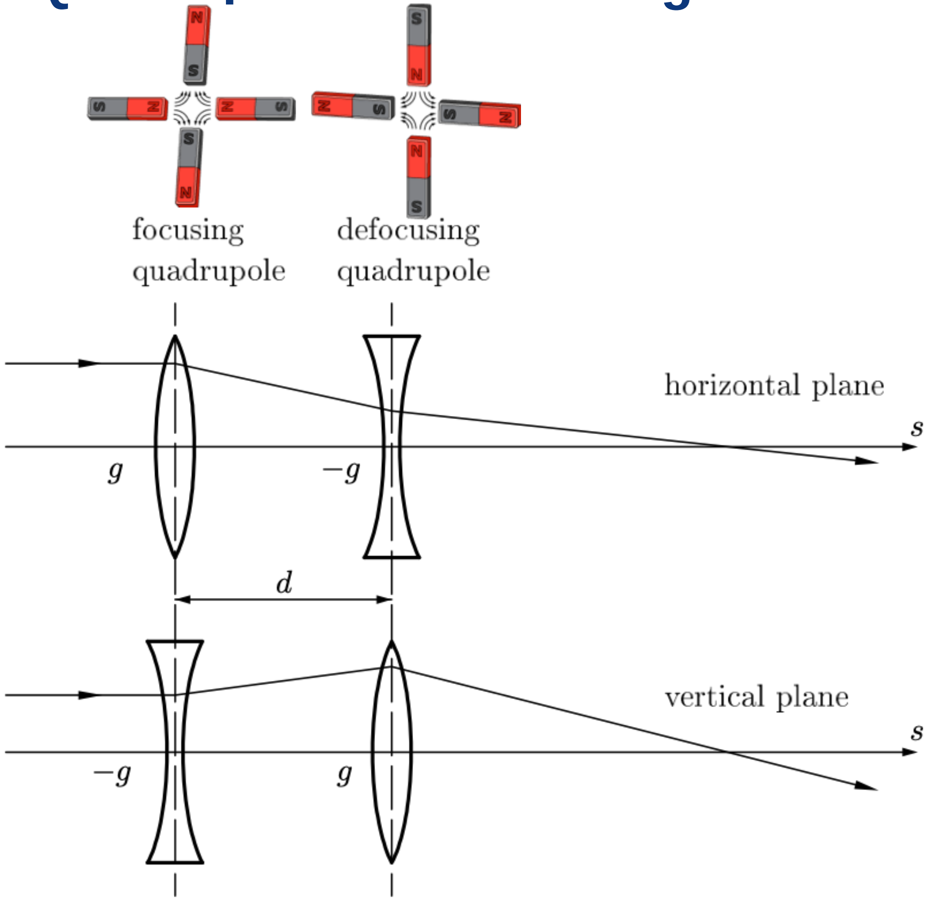
Synchrotrons: strong focusing machine

Dipoles are interleaved with quadrupoles to focus the beam. Quadrupoles act on charged particles as lens for light. By alternating focusing and defocusing lens (Alternating Grandient quadrupoles) the beam dimension is kept small (even few mum^2).

QUADRUPOLE







B field is focusing in one plane but defocusing in the other.

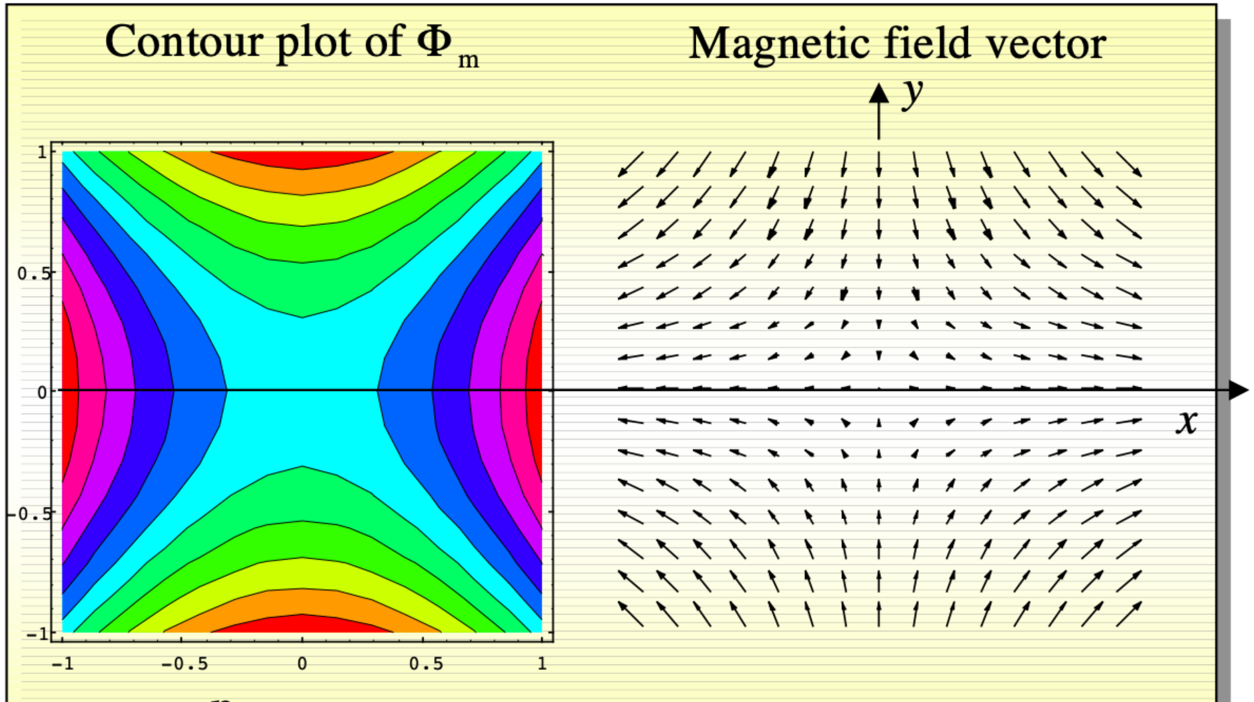
Typical lattice is FODO, focusing-drift-defocusing

Quadrupole field

$$B = -\nabla \Phi_m$$

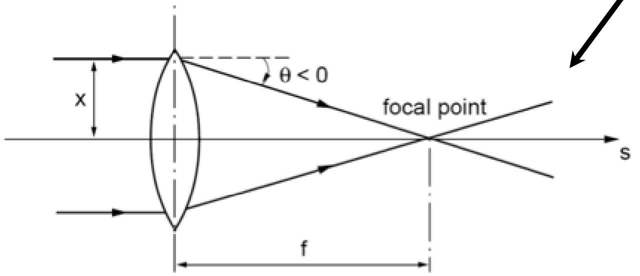
Φ_m = Vector potential

The field increases linearly with the distance from the center of the magnet
Obviously, K, the gradient, has a sign.
By convention + means focusing quadrupole in the horizontal plane.



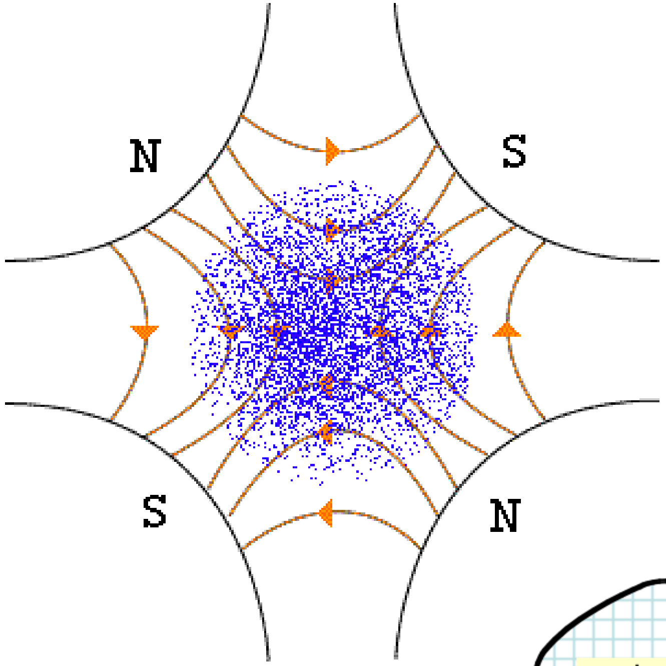
$$\Phi_m = -\frac{p}{e} K_1 (x^2 - y^2), \text{ for quadrupole magnet}$$

$$l\,K = -\frac{l}{B\rho}\frac{\partial B}{\partial x} = \frac{1}{f}$$

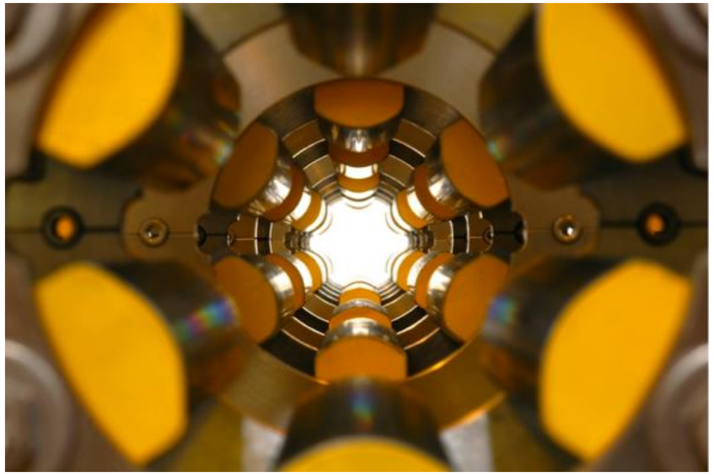


$$\tan\theta = -\frac{x}{f}$$

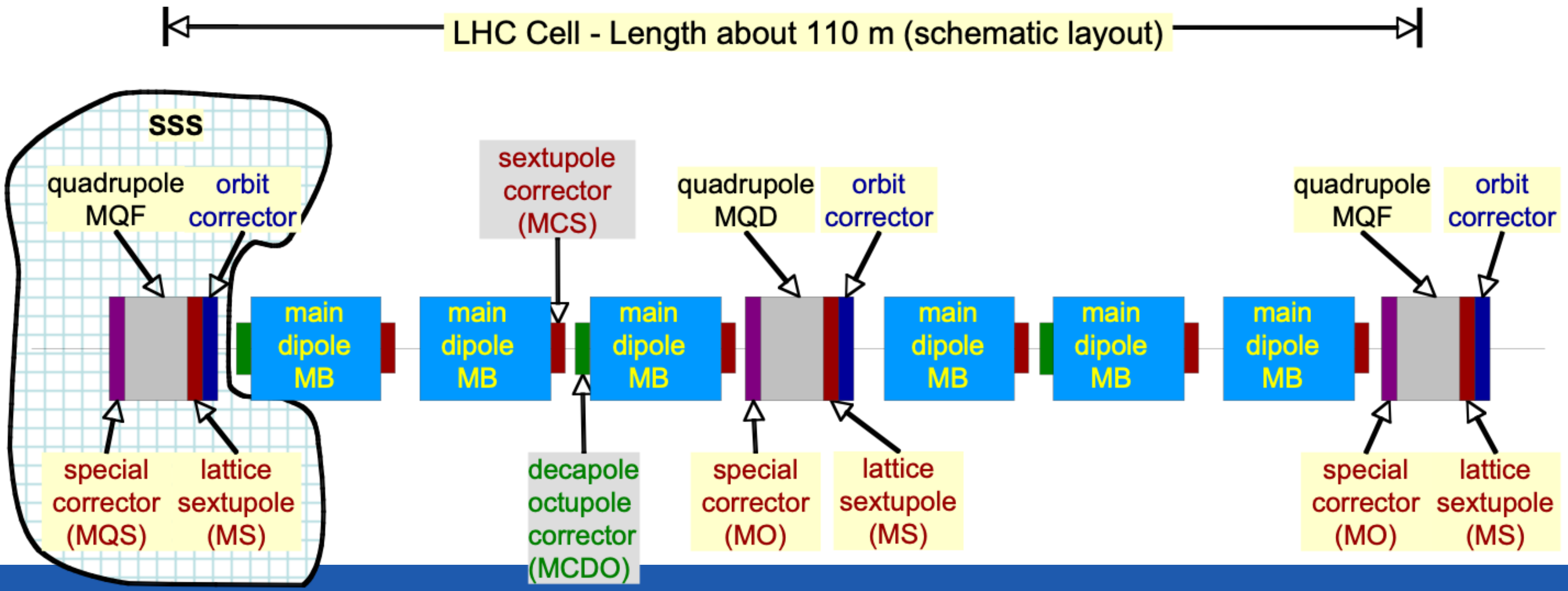
Example of FODO lattice



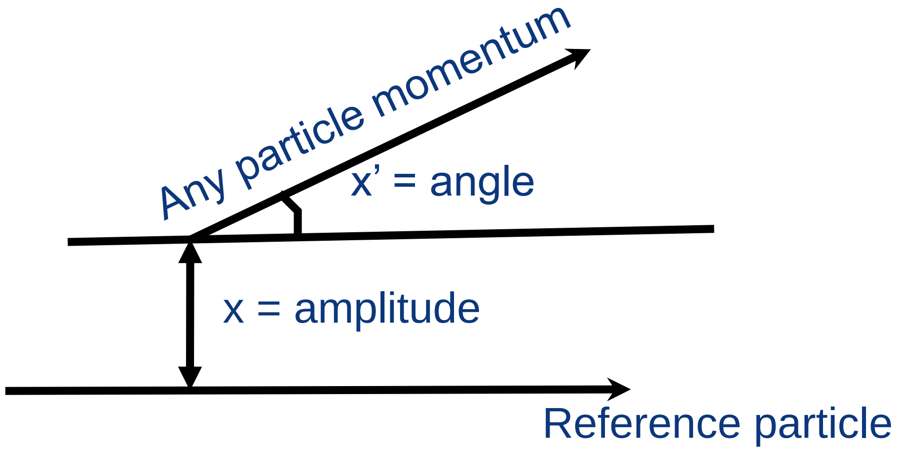
The beam point of view - Those are sextupoles - Six poles

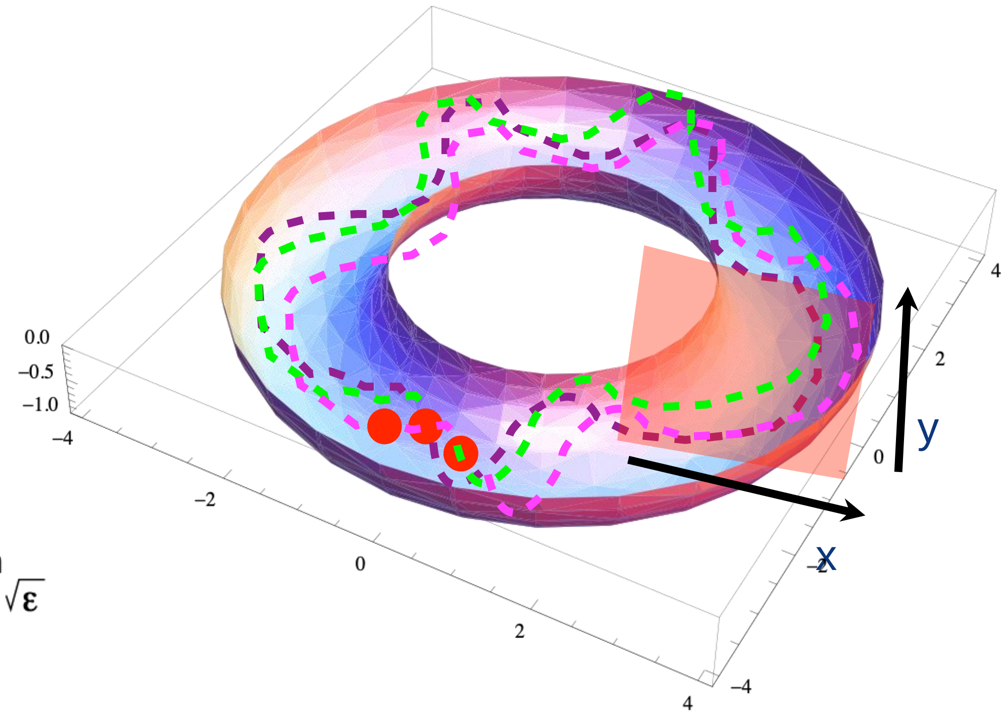


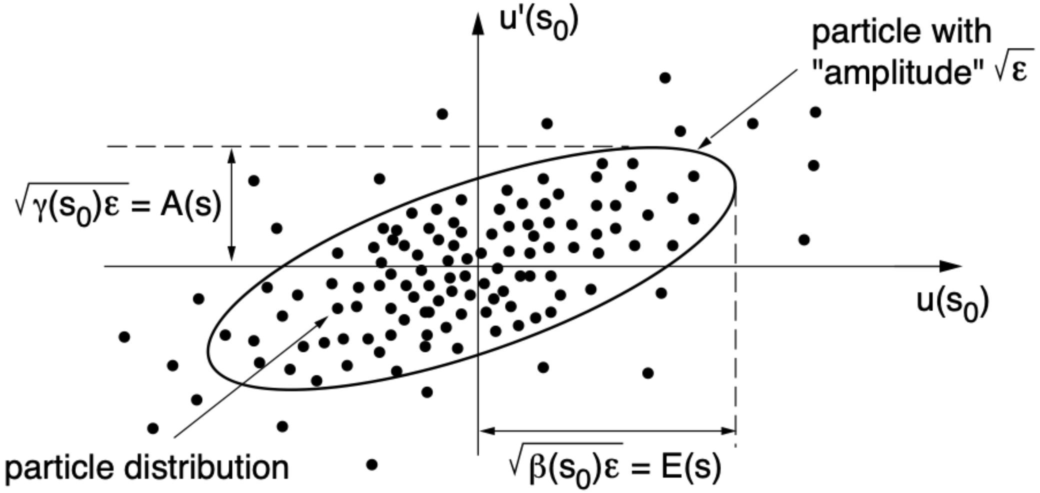
Diamond light source - UK



Our reference frame: xx', the phase space





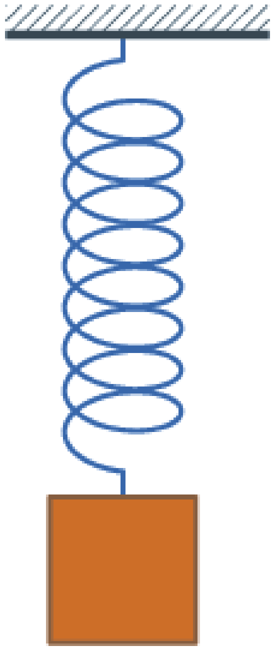


The space occupied in the xx' (or yy') plane by the beam at a given position in the machine is defined as Emittance

Classical mechanics.... spring with a mass

$$F = ma = m\frac{d^2x}{dt^2} = -kx$$

with *k* the spring constant and *m* the mass



Solution of the equation of motion is a periodic function:

$$x(t) = A\cos(2\pi f\, t + \phi)$$

with 1/period equals to

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Equation of motion, not too in details

Equation of motion of a particle in an accelerator composed by a sequence of elements, each one eventually with a *k* at a position *s* of the ring, repeated at every *C*

***Hill's equation: pendulum-like with non-constant spring force wrt to *s*.**

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

beer **= solution** →

$$x(s) = a\sqrt{\beta(s)}\cos(\phi(s) + \phi_0)$$

Local force at a position ***s*** in the ***ring***

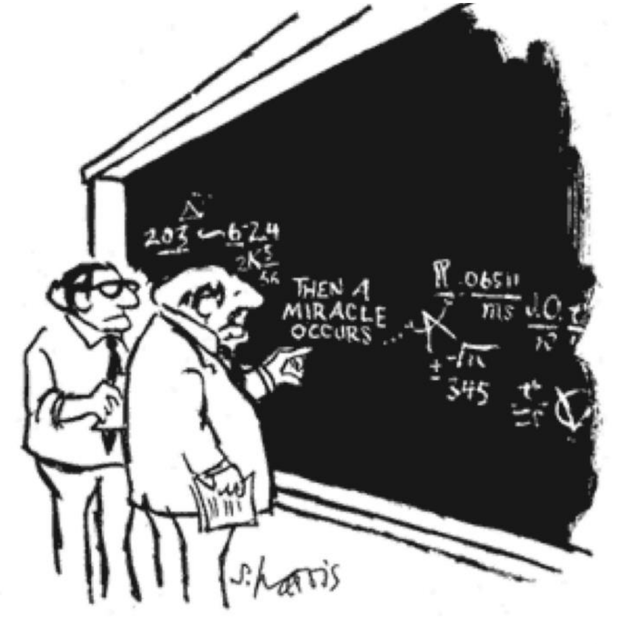
$$K(s) = 1/\rho^2 + k(s)$$

$$K(s) = K(s + C)$$

Dipoles → **forget them for a moment**

Quadrupoles

This imply periodicity of the solution



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

***there was a Mr. Hill, an astronomer**

me too... in a moment...

Solution of Hill's equation

$$x(s) = a\sqrt{\beta(s)}\cos(\phi(s) + \phi_0)$$

this "probably" contains k

Spring solution

$$x(t) = A\cos(2\pi f\, t + \phi)$$

this contains k and m

This actually... look alike should not be there...

The beta function is a product of the locally changing force in the accelerator, i.e., of the quadrupoles.

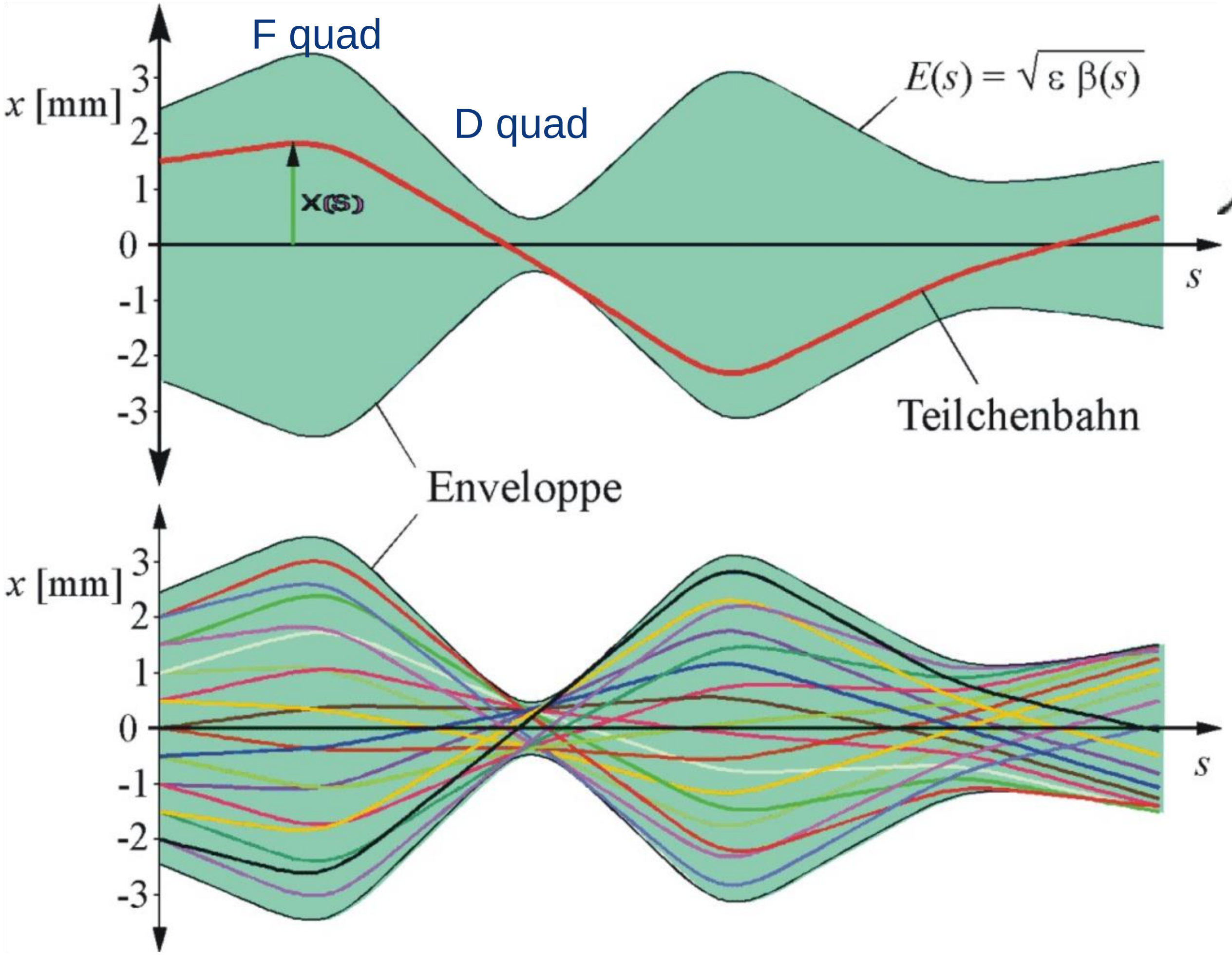
Every section of an accelerator has a constant k, so alone would be similar to an harmonic oscillator

By definition (*ipse dixit...*): $\phi(s) = \int \frac{1}{\beta(s)} ds$

is called the *phase advance*

Definition of envelope

Beam physical dimension



$$E(s) = \sqrt{\varepsilon\,\beta(s)}$$

The envelope is defined as the maximum amplitude for which the particle remains in the machine vacuum chamber.

Relationship between beam ellipse and beta

Nearly no beer ... full proof ...

if the emittance is a surface this can be an amplitude (I am cheating... I know)

$$x''(s)+K(s)\cdot x(s)=0$$

1) Let's suppose

$$x(s)=\sqrt{\varepsilon}\cdot u(s)\cdot\cos(\phi(s)+\varphi_0)$$

What is this???

2) Let's apply

$$\left[u''-u\cdot\phi'^2+K\cdot u\right]\cdot\cos(\phi+\varphi_0)-\left[2\cdot u'\cdot\phi'+u\cdot\phi''\right]\sin(\phi+\varphi_0)=0$$

beer + trick. Coeffs in front of sin et cos should be zero and $\phi(s)=\int_0^s\frac{d\tilde{s}}{u^2(\tilde{s})}$

$$u''-\frac{1}{u^3}+K\cdot u=0$$

def.

$$\beta(s):=u^2(s)$$

$$x(s)=\sqrt{\varepsilon}\cdot\sqrt{\beta(s)}\cdot\cos(\phi(s)+\varphi_0)$$

def.

$$\alpha(s):=-\frac{\beta'(s)}{2}$$

$$x'(s)=-\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\left\{\alpha(s)\cdot\cos(\phi(s)+\varphi_0)+\sin(\phi(s)+\varphi_0)\right\}$$

beer

$$\sin^2(\phi+\varphi_0)=\left(\sqrt{\frac{\beta}{\varepsilon}}\cdot x'+\frac{\alpha}{\sqrt{\varepsilon\beta}}\cdot x\right)^2$$

def.

$$\gamma(s):=\frac{1+\alpha^2(s)}{\beta(s)}$$

We brilliantly find...

….. what we wanted…

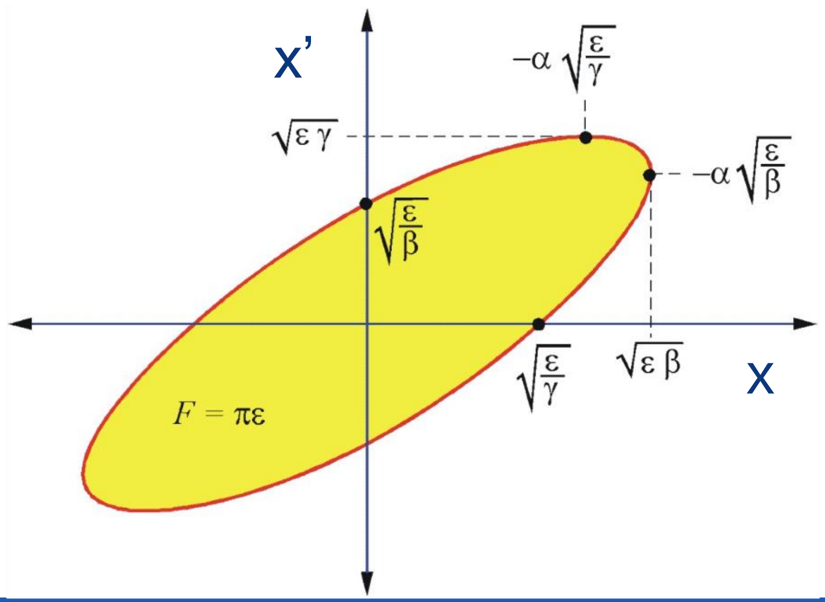
oh surprise... →

$$\gamma x^2 + 2\alpha\, xx' + \beta x'^{\,2} = \epsilon$$

Learned:

a) definition of Twiss parameters comes from the equation of motion and beta function

b) The dynamics is really on/within an ellipse



Twiss parameters:

$$\alpha(s) := -\frac{\beta'(s)}{2}$$

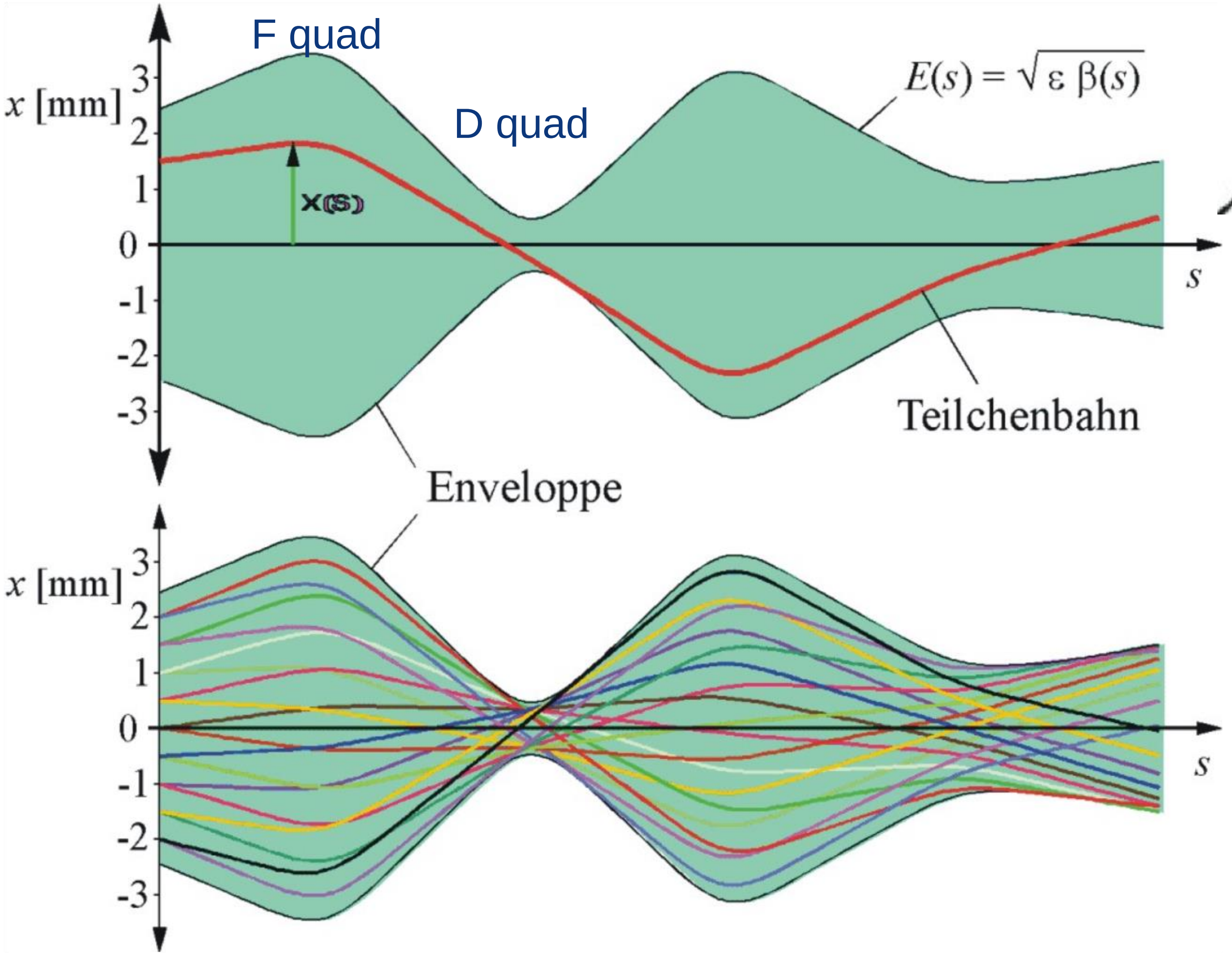
$$\gamma(s) := \frac{1+\alpha^2(s)}{\beta(s)}$$

$$\beta(s)$$

Those are not the relativistic homonyms

Definition of envelope

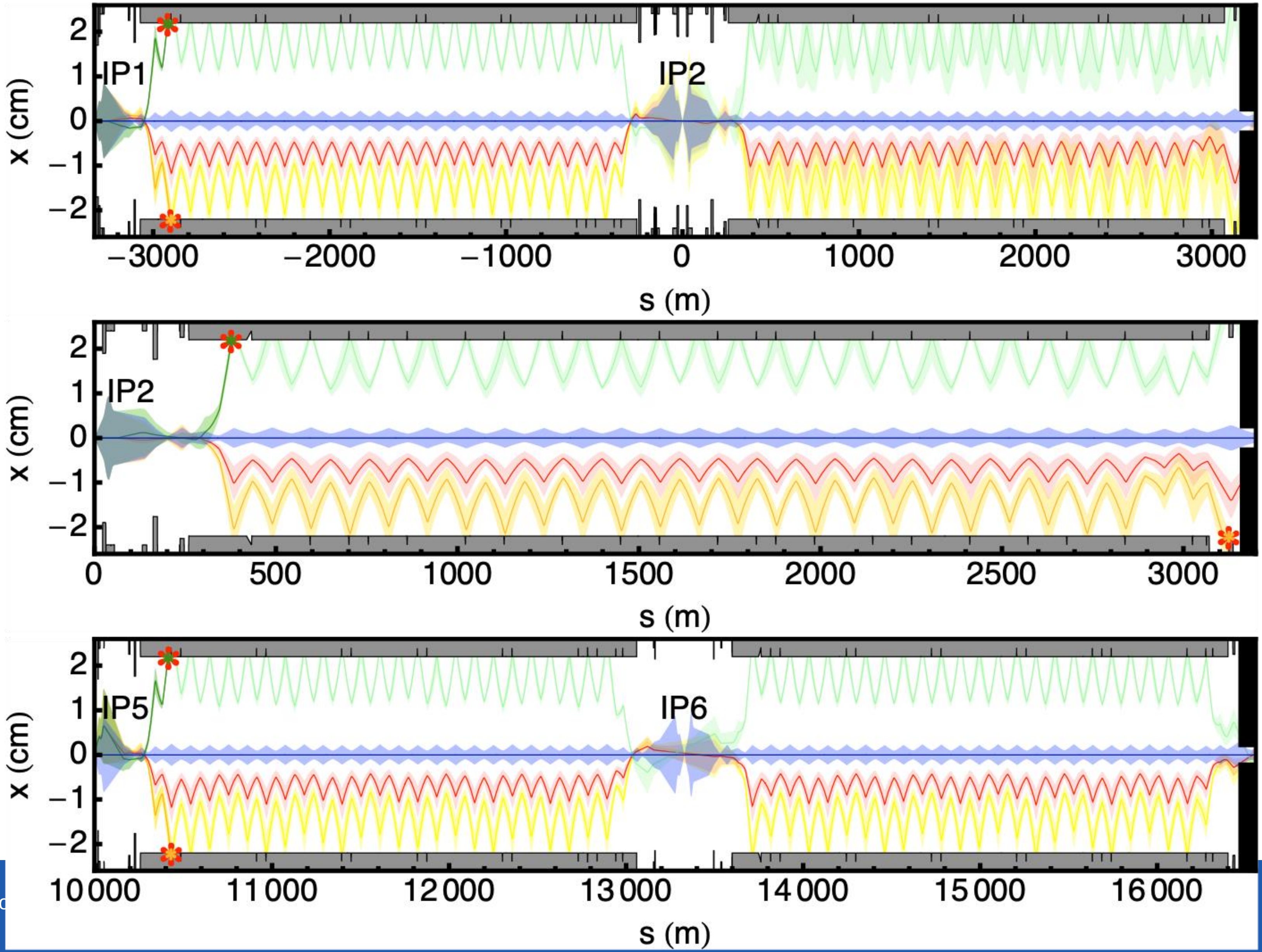
Beam physical dimension



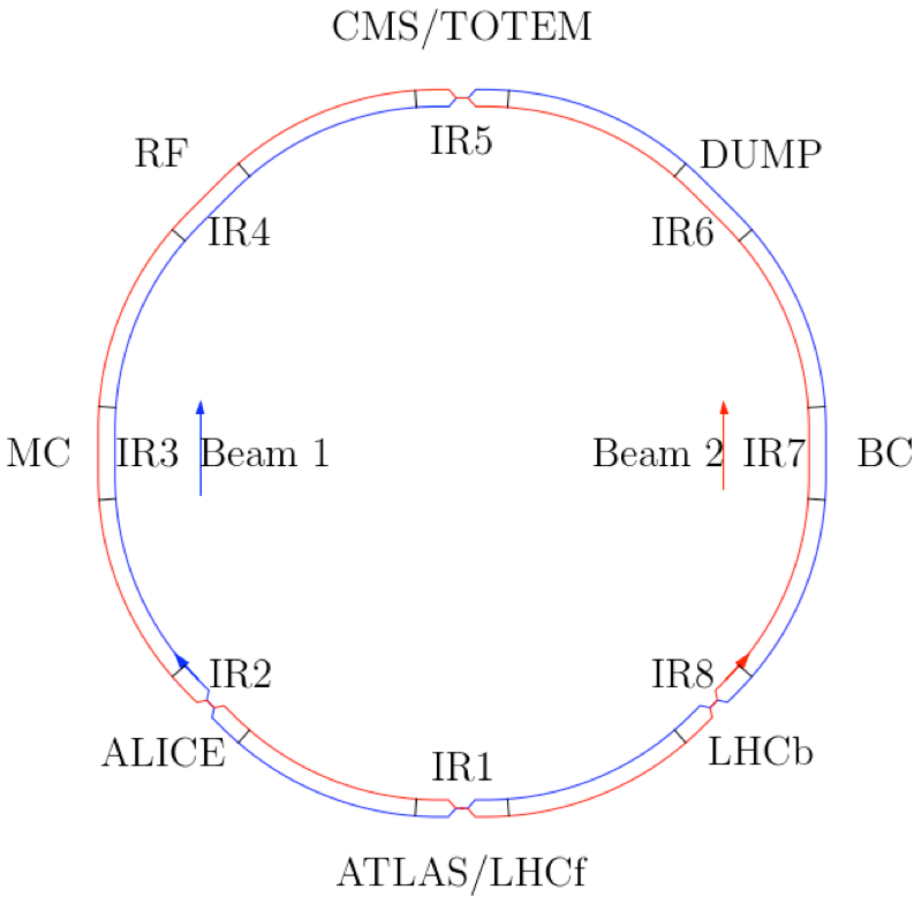
$$E(s) = \sqrt{\varepsilon\,\beta(s)}$$

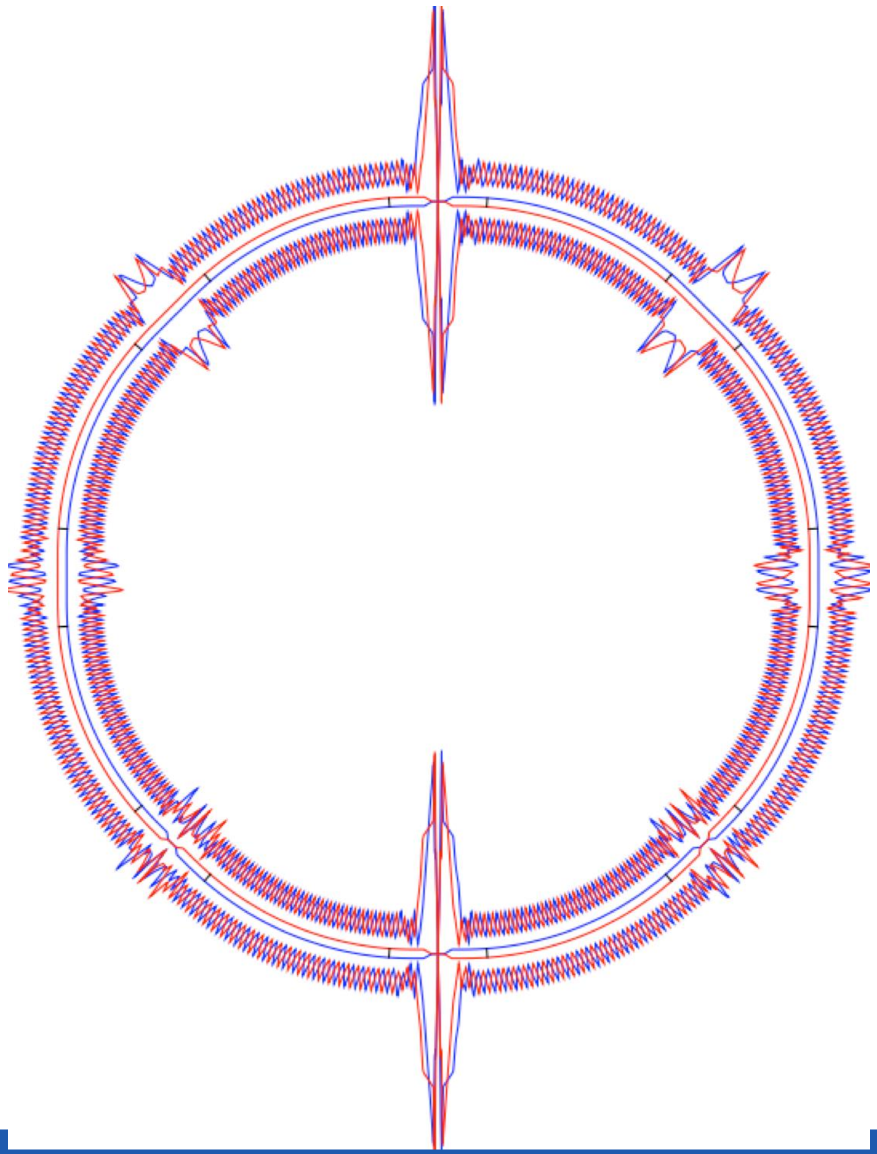
The envelope is defined as the maximum amplitude for which the particle remains in the machine vacuum chamber.

Envelope around the LHC

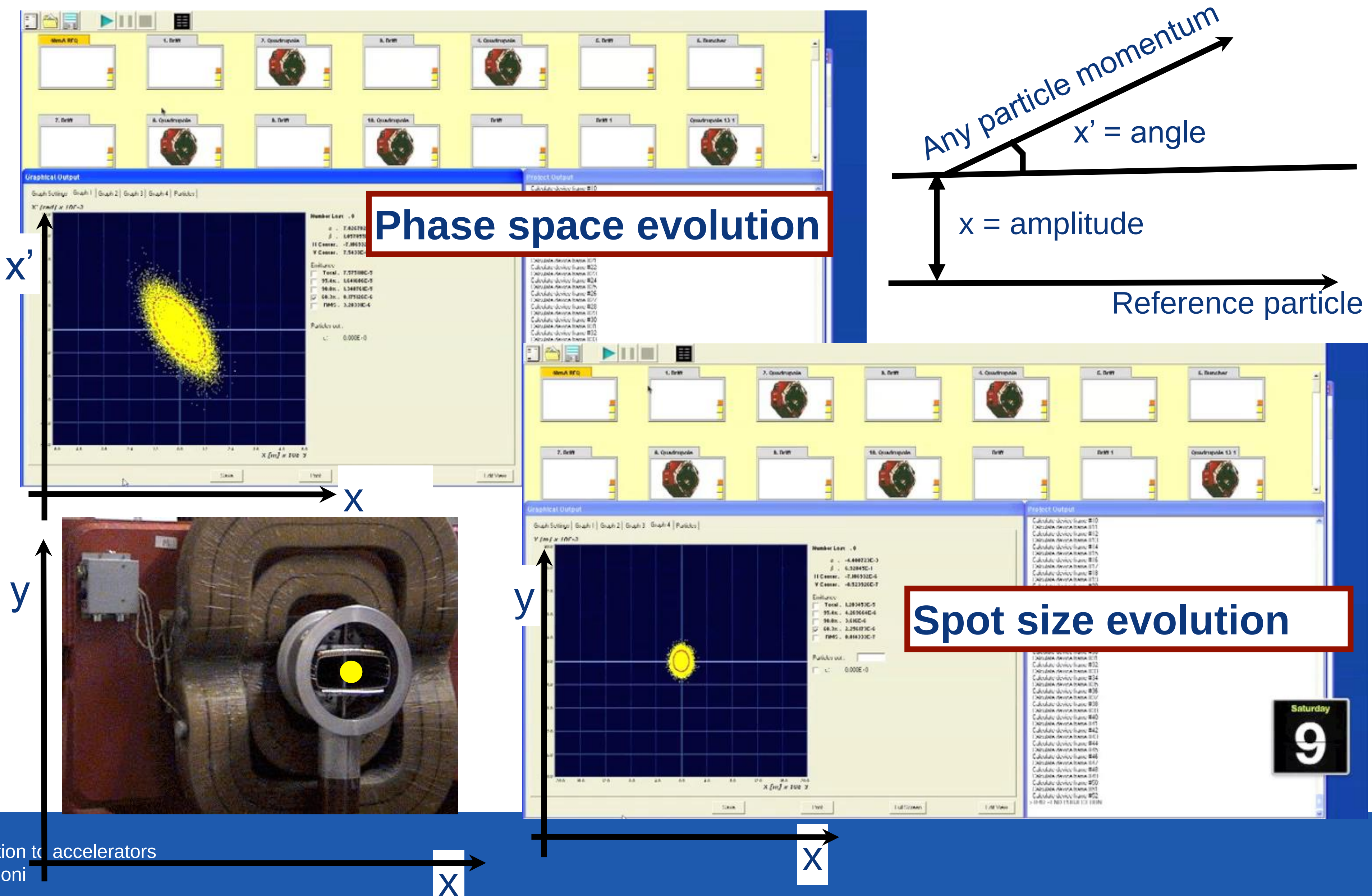


The first LHC collision optics in one slide





Particle transport in a lattice

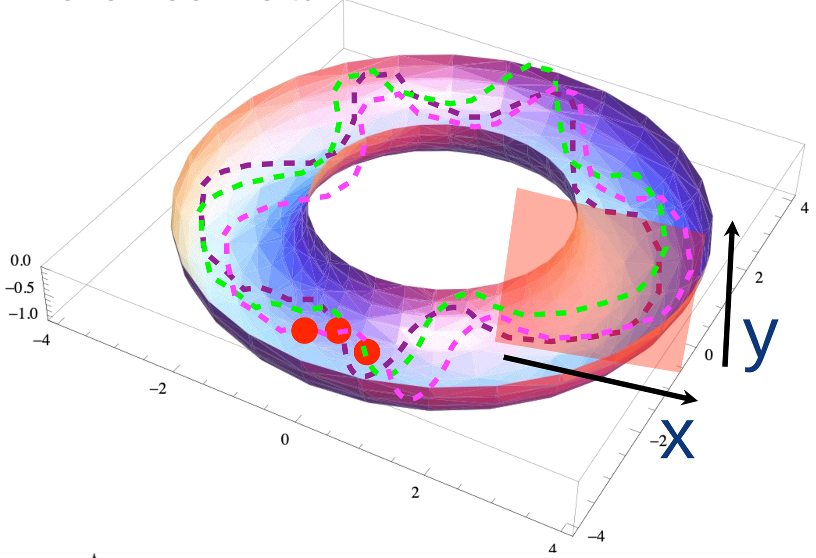


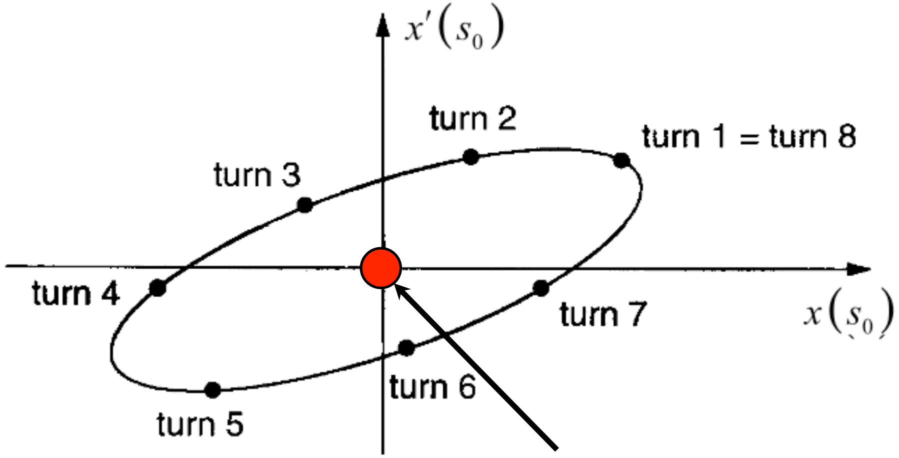
Tune

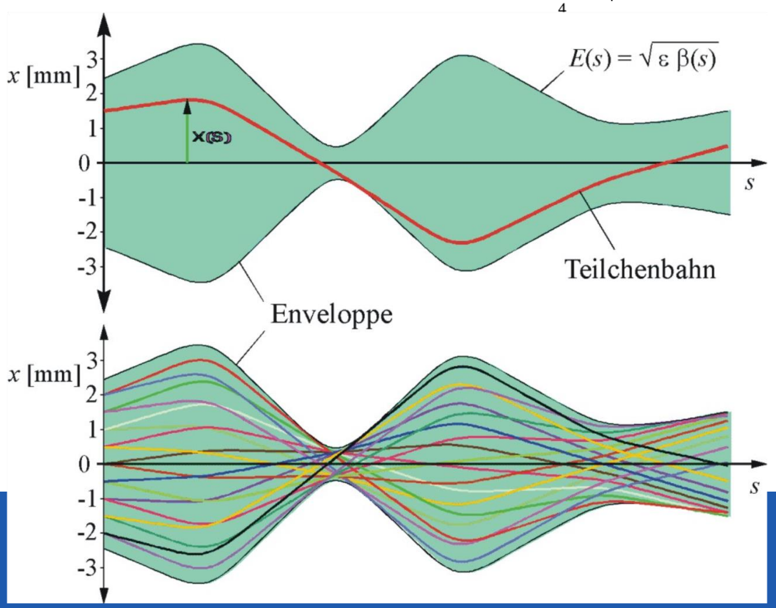
Tune:
number of oscillations (called betatronic) in the xx‘ plane a particle does in one machine turn.

The tune depends on the quadrupoles settings and is the integral of the phase advance on one machine turn

$$Q_x = \frac{1}{2\pi} \oint \frac{ds}{\beta_x(s)}$$





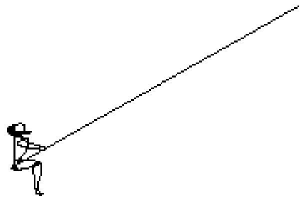


Reference particle

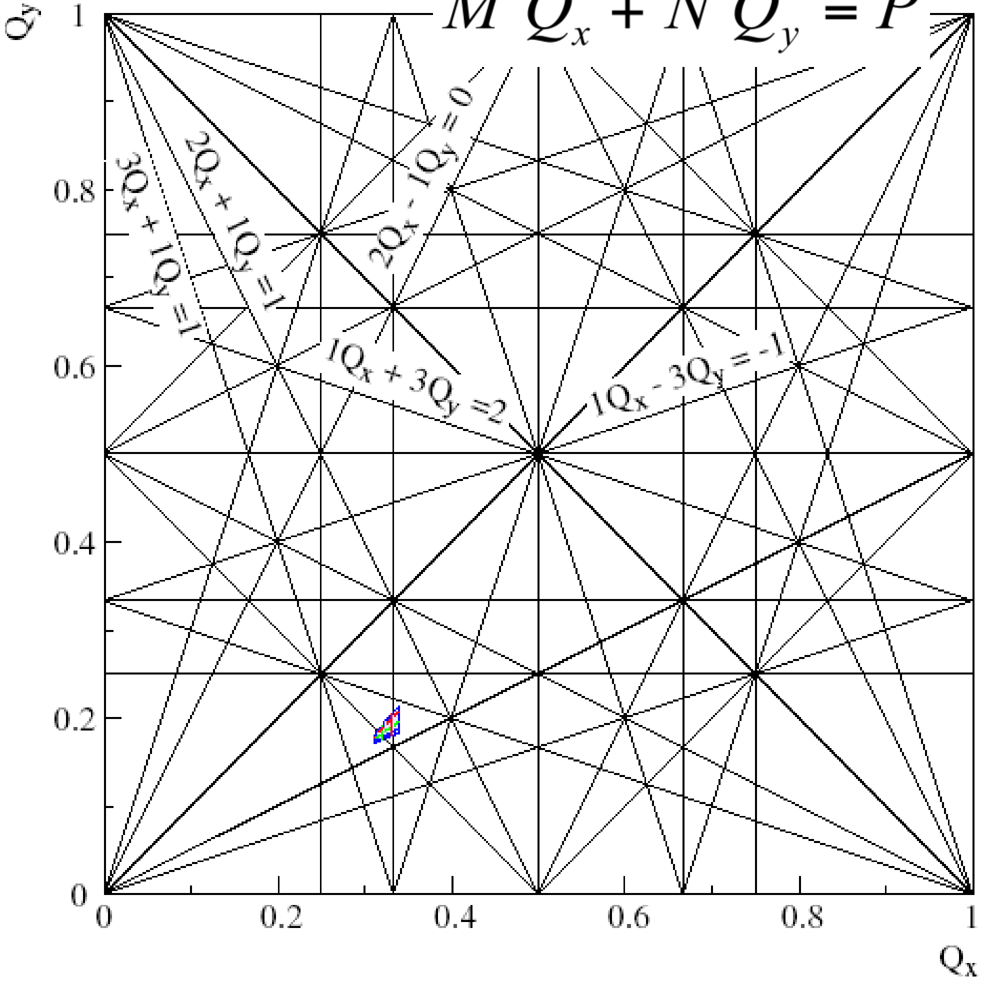
Tune and resonances

Like on a swing, to keep the oscillations bounded in amplitude, one has to avoid to excite the beam in a resonant way.

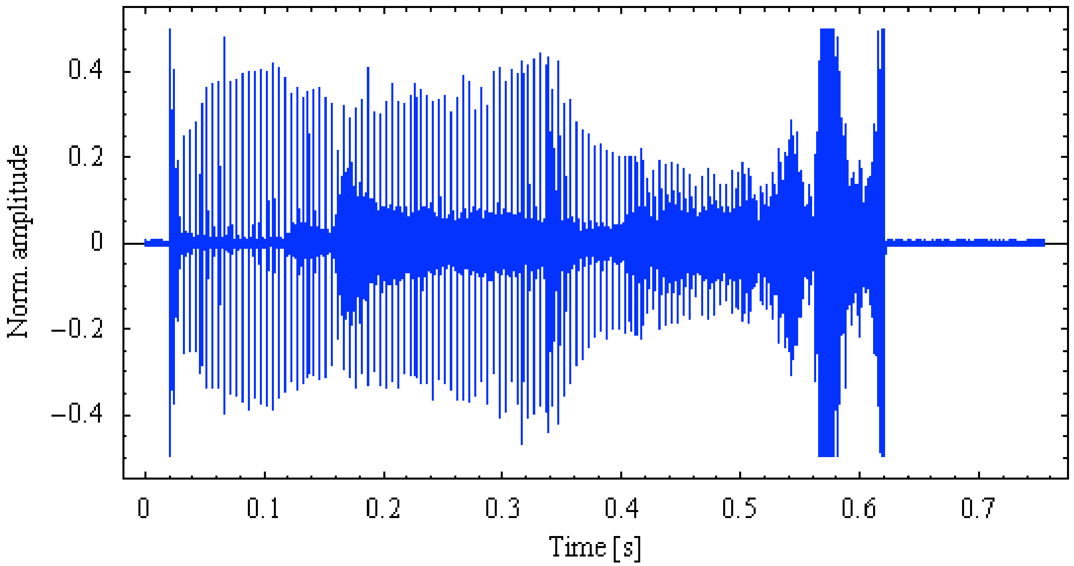
The tune has to be far away from some values, like exciting the beam with the same force at each turn

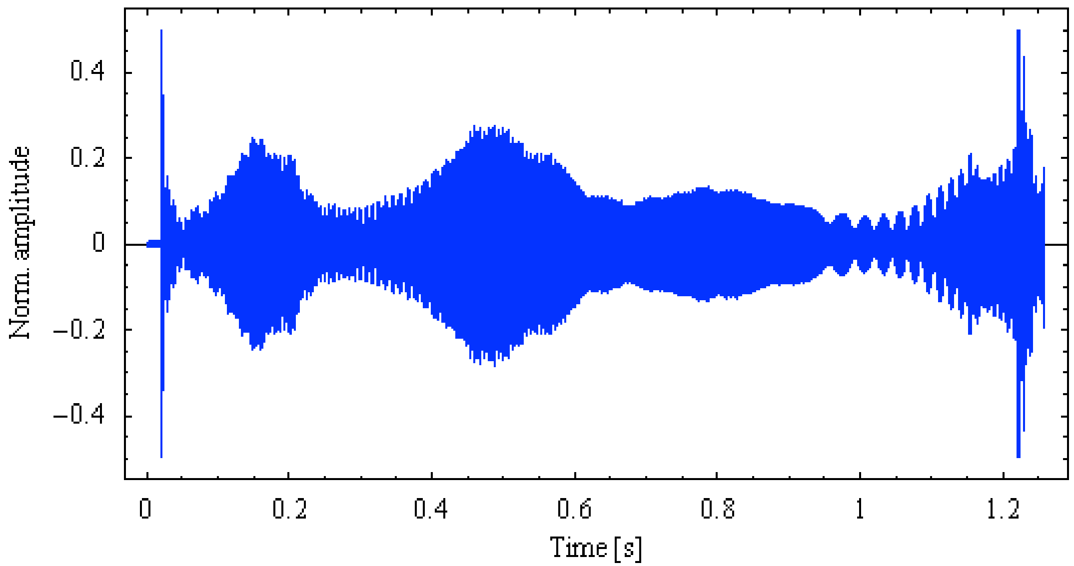


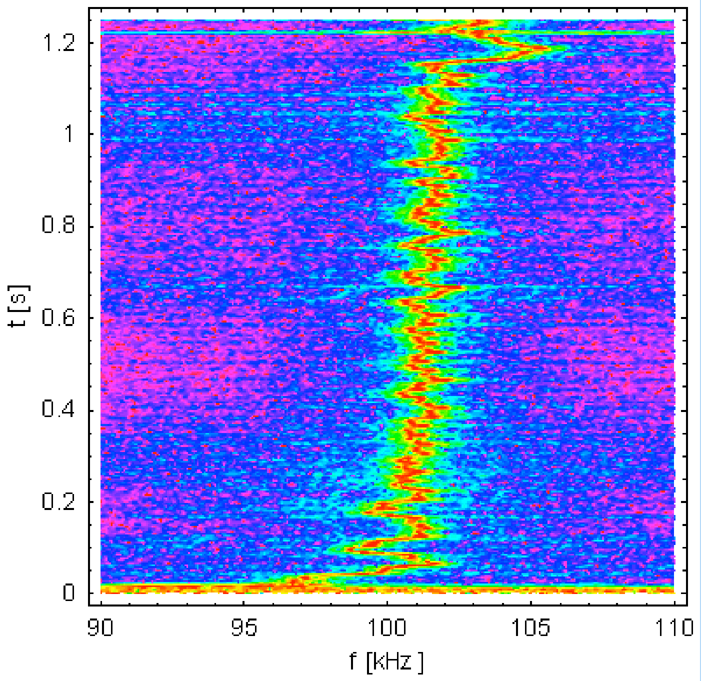
To avoid $M\,Q_x + N\,Q_y = P$



Tune: number of betatron oscillation in the transverse plane







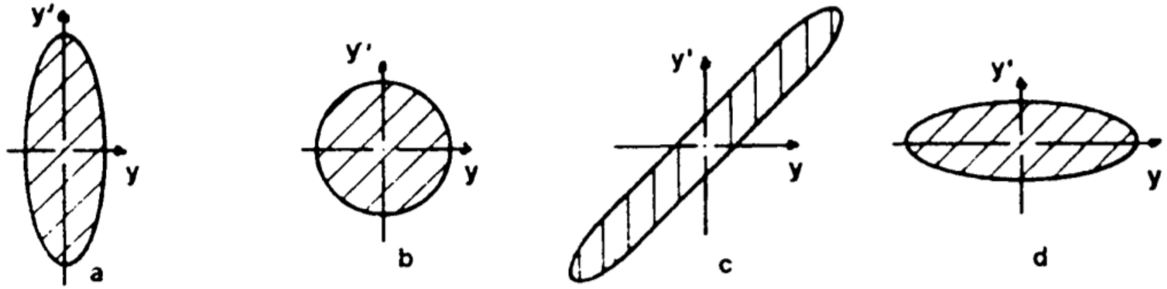
http://mgasior.web.cern.ch/mgasior/pro/3D-BBQ/ps.html

THE LAW: Lioville theorem

Theorem: In the vicinity of a particle, the particle density in phase space is a constant if the particle move in an external magnetic field or in a general field which the force do not depend upon velocity (*ipse dixit*...), i.e., **the beam is like an incompressible fluid in phase space**

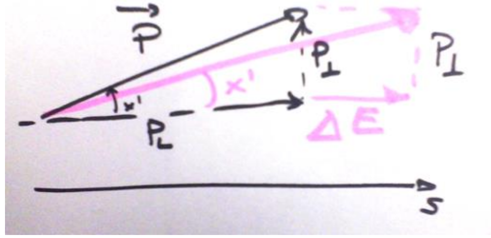
Implications:

a) the emittance is conserved when the beam is transported via a magnetic system



The ellipse is distorted/stretched but the surface is conserved.

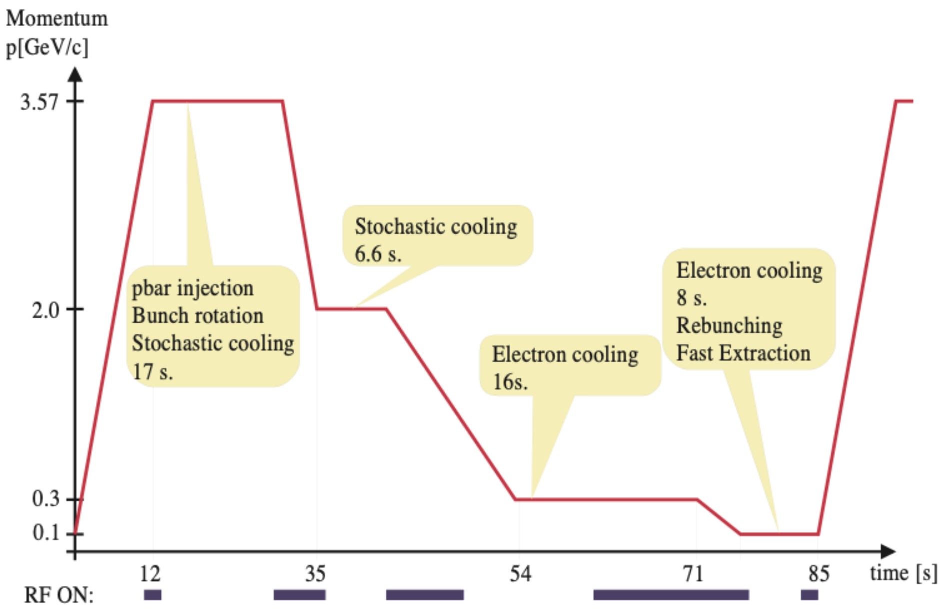
b) the emittance is **NOT** conserved if we accelerate, except if we normalize the emittance wrt to βγ (relativistic). **x' is reduced by the acceleration.**



$$\epsilon_{norm} = \epsilon_{phys} * \beta_{rel} * \gamma_{rel}$$

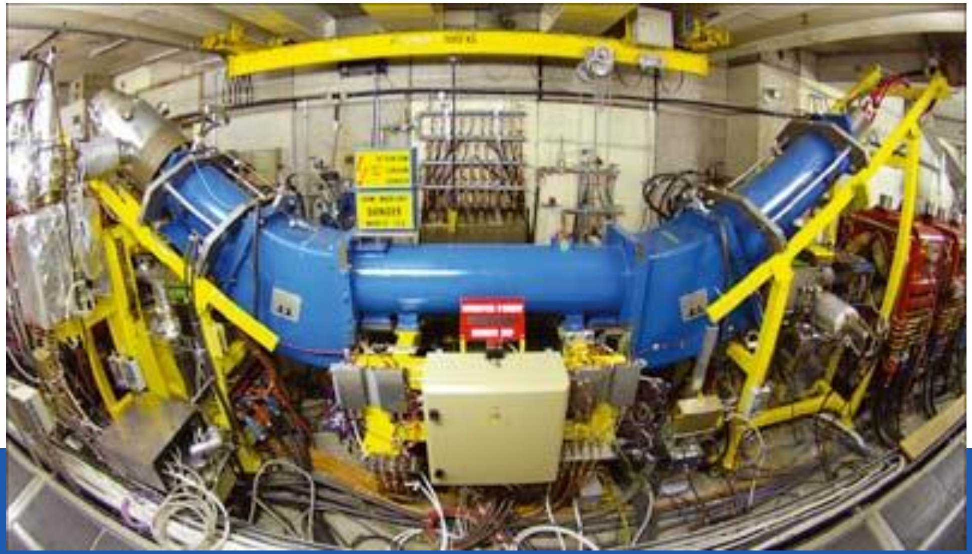
c) if we want to reduce emittance at constant energy, we have to "cheat": **BEAM COOLING**

AD (Antiproton decelerator)

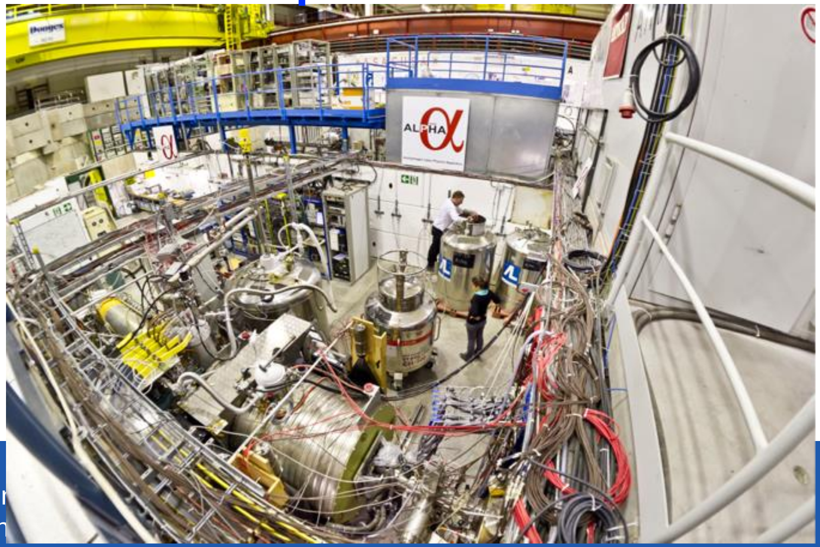


Lattice quadrupoles

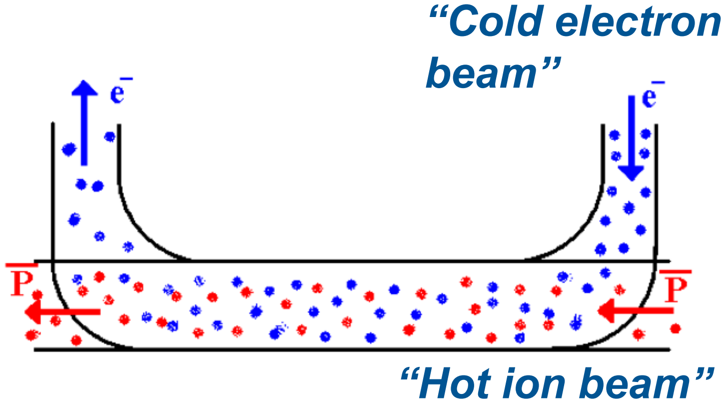
Electron cooler

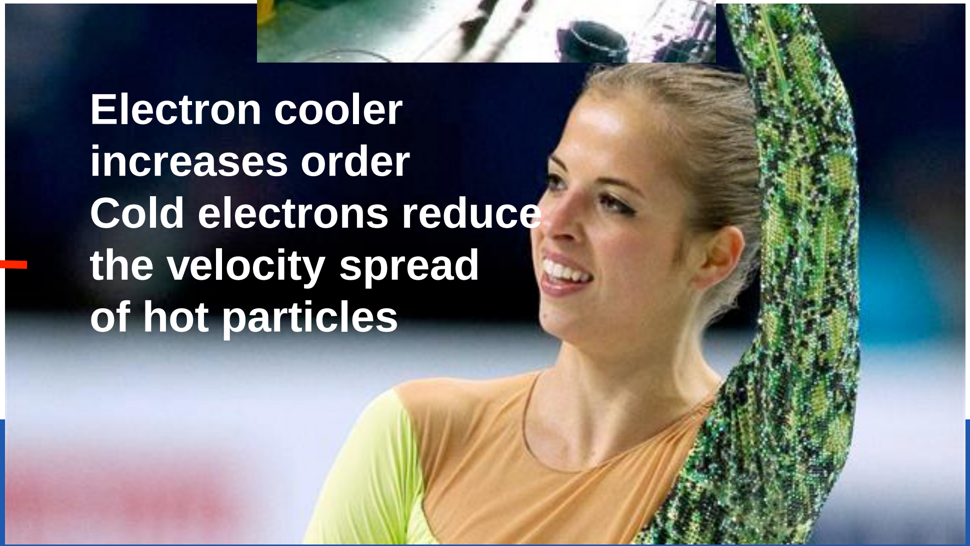


Experiments

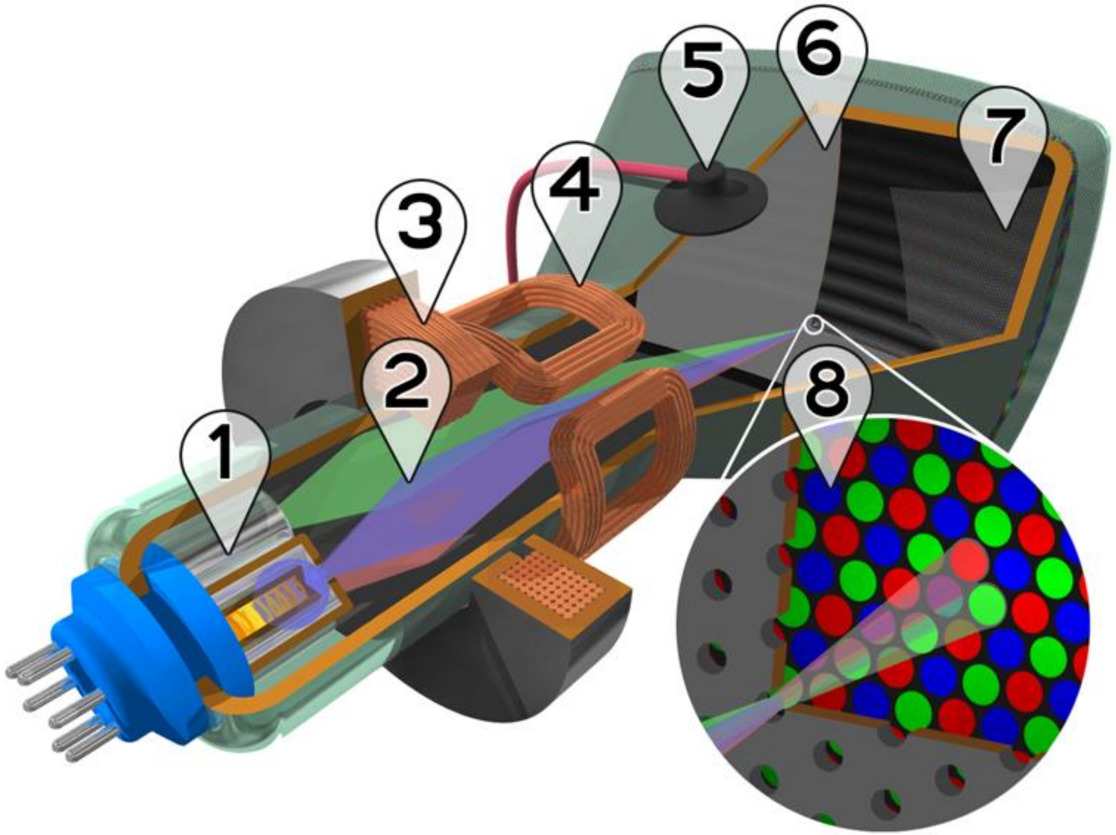


Electron cooling





Summary: an accelerator that you know very well

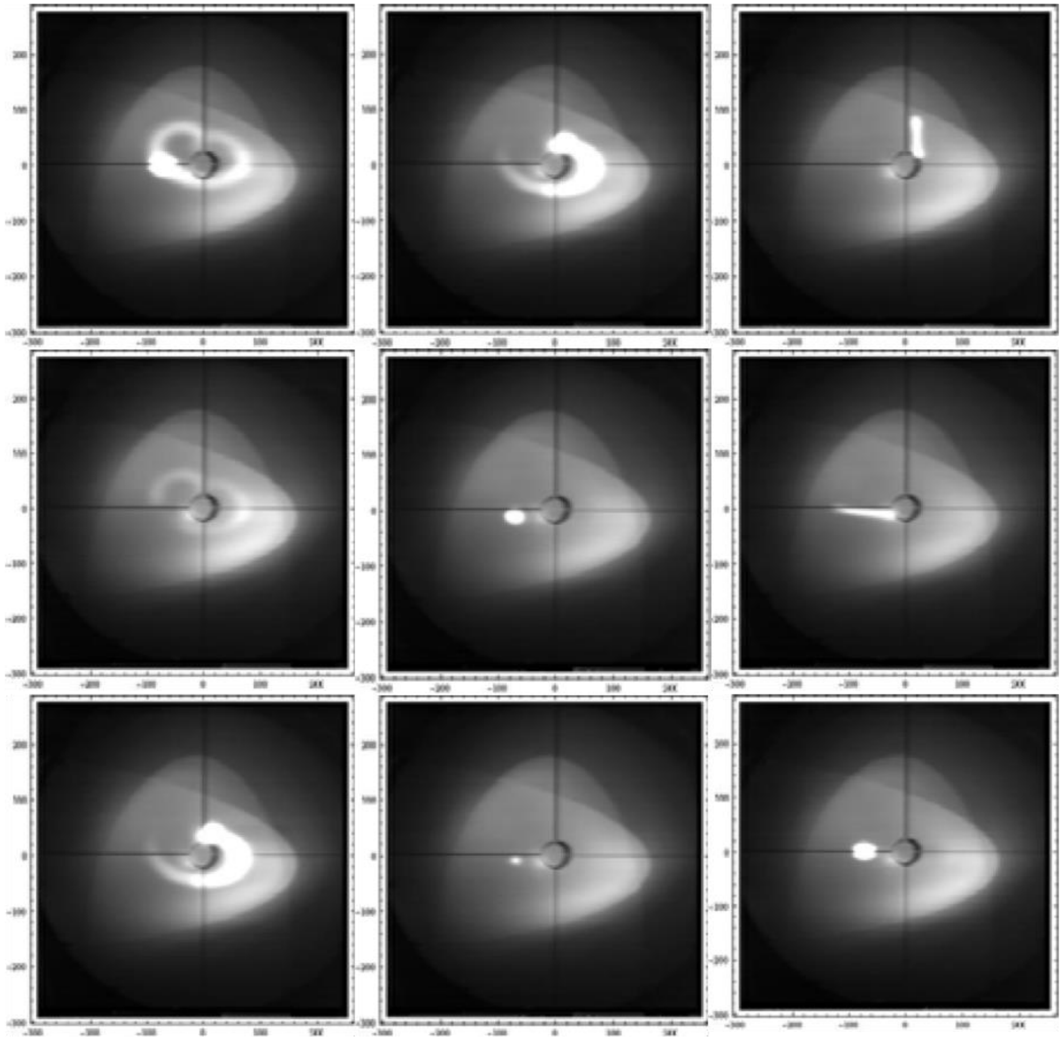


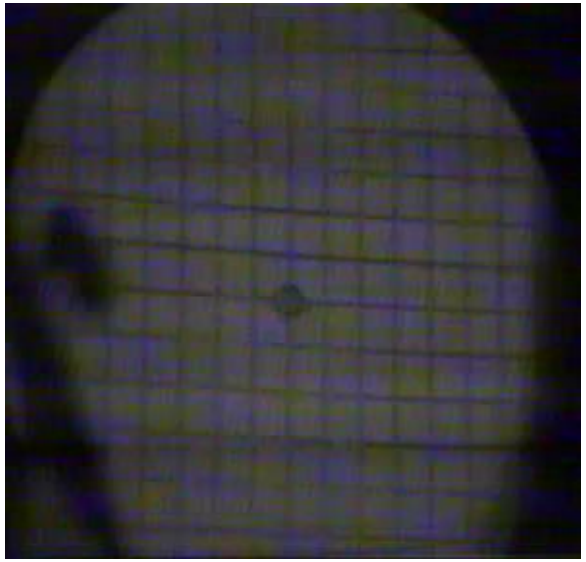
1. Three Electron guns (for red, green, and blue phosphor dots)
2. Electron beams
3. Focusing coils
4. Deflection coils
5. Anode connection
6. Mask for separating beams for red, green, and blue part of displayed image
7. Phosphor layer with red, green, and blue zones
8. Close-up of the phosphor-coated inner side of the screen

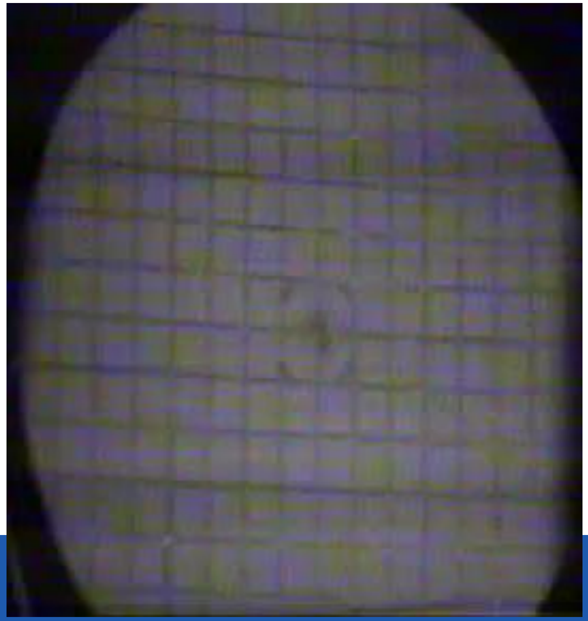
From Wikipedia

Real beam images

Courtesy of B. Goddard







Apples vs Antiapples: protons vs antiprotons (matter vs antimatter)

Does matter fall?

And what about antimatter?

We still not not fully understand matter vs. antimmater in the universe, and by the way gravity neither …

First part summary

- **Dipoles** bend charged particles in the accelerator
- **Quadrupoles** focus particles and define the beam **tune**

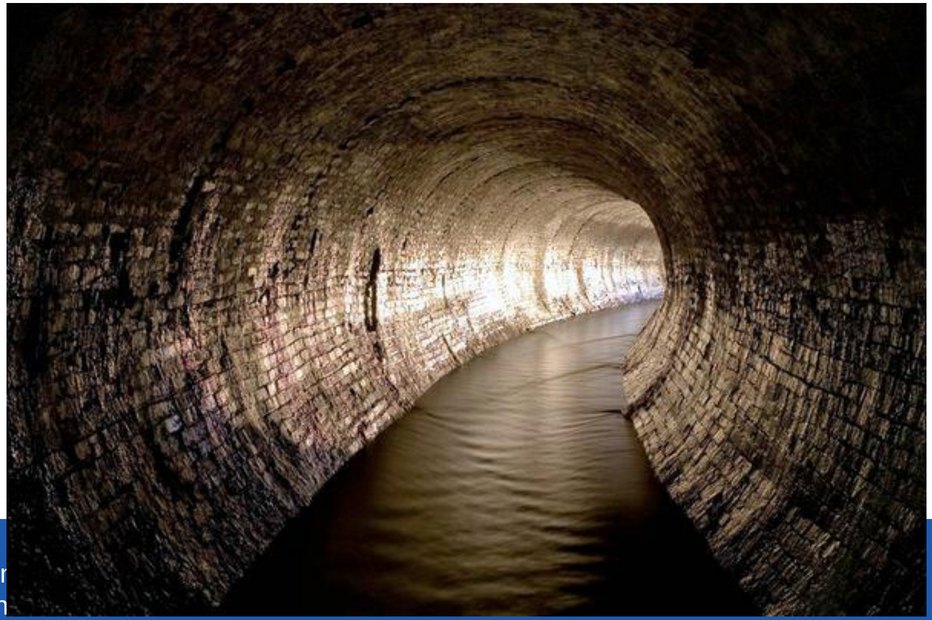
- The emittance is the space occupied by the particles in the xx’ plane
- The envelope is defined by the quadrupoles via the beta function

Building Blocks of an accelerator

1) A particle source

2) An accelerating system

3) A series of guiding and storage devices

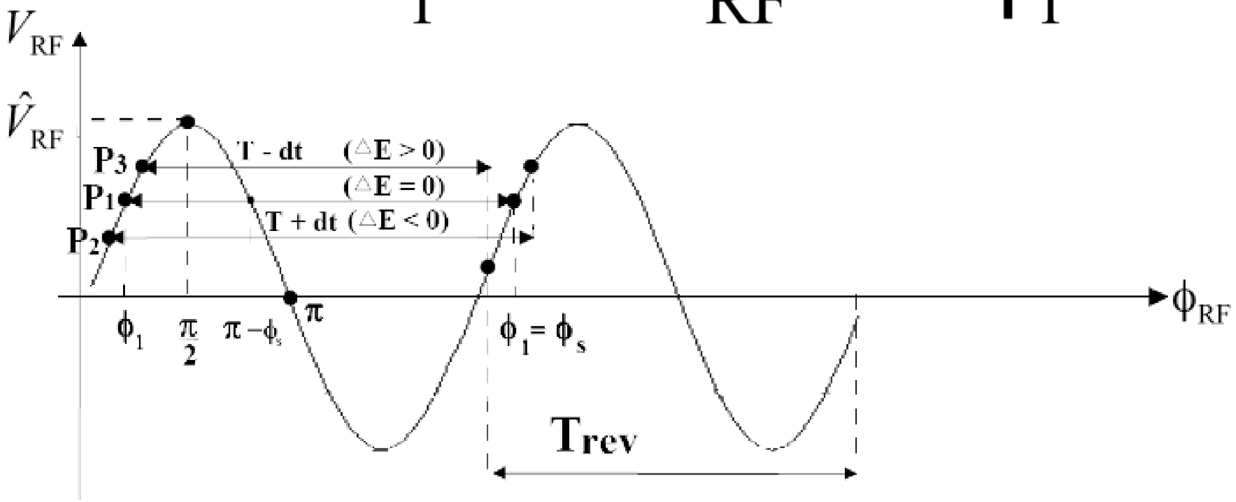


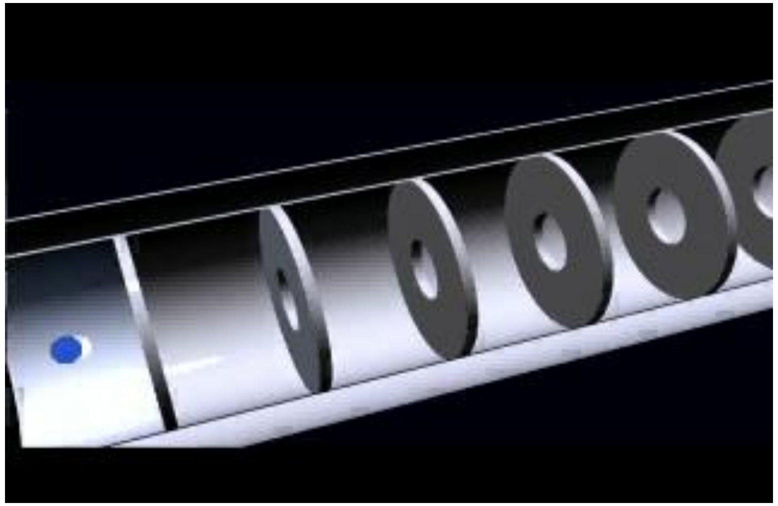
Everything under vacuum

Acceleration

- Particles are accelerated by an **RF (radio frequency) electric field which is confined in cavities.**
- **The electric field varies in time as a sinus wave in such a way, that at each revolution, the particle comes back at the RF to see the acceleration.**

$$\Rightarrow \Delta E_1 = e\,\hat{V}_{RF}\,\sin\phi_1$$

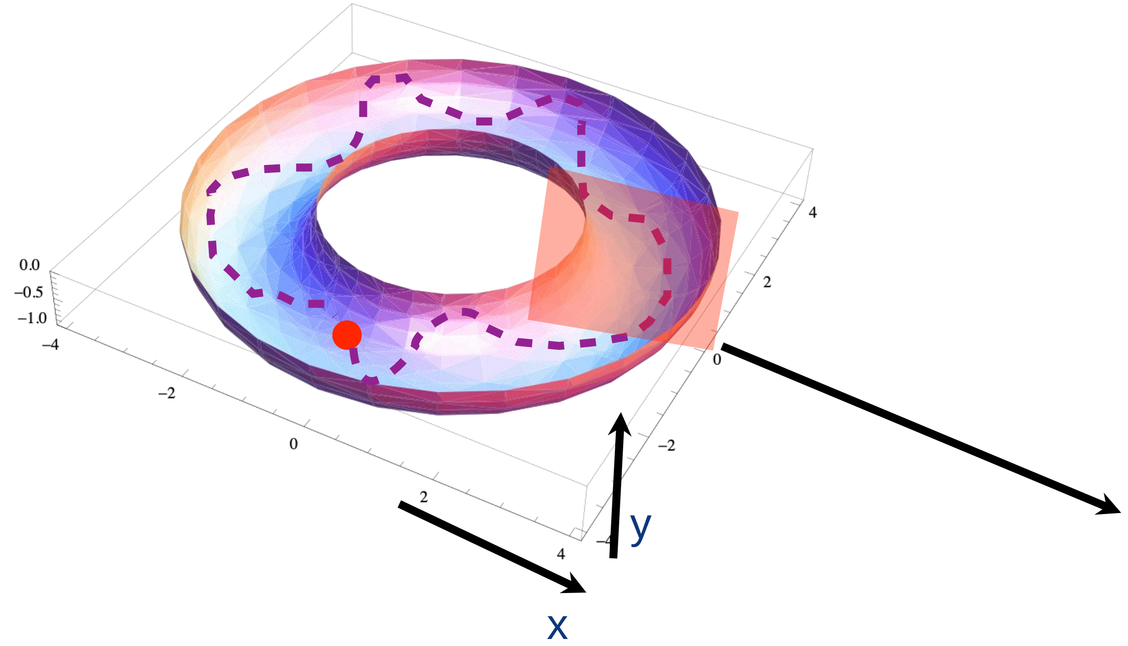


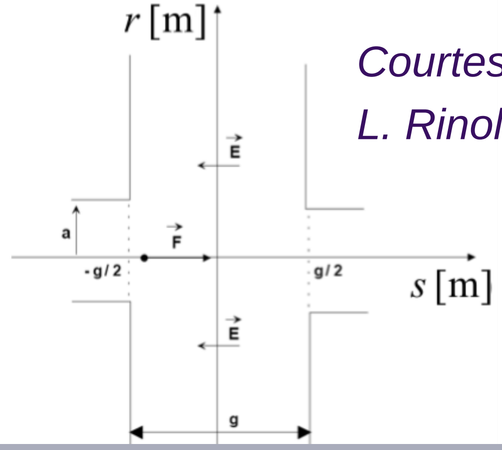


Acceleration I

Acceleration again with Lorentz force:

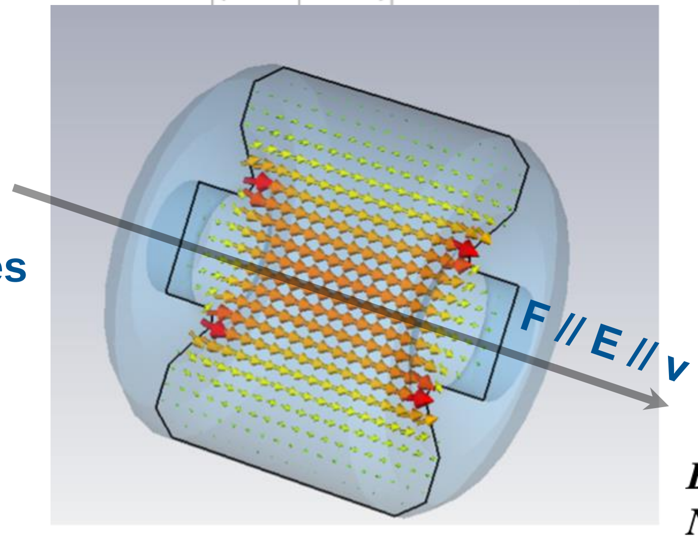
$$\overline{F(t)} = q\left(\overline{E(t)} + \overline{v(t)} \otimes \overline{B(t)}\right)$$





Courtesy
L. Rinolfi

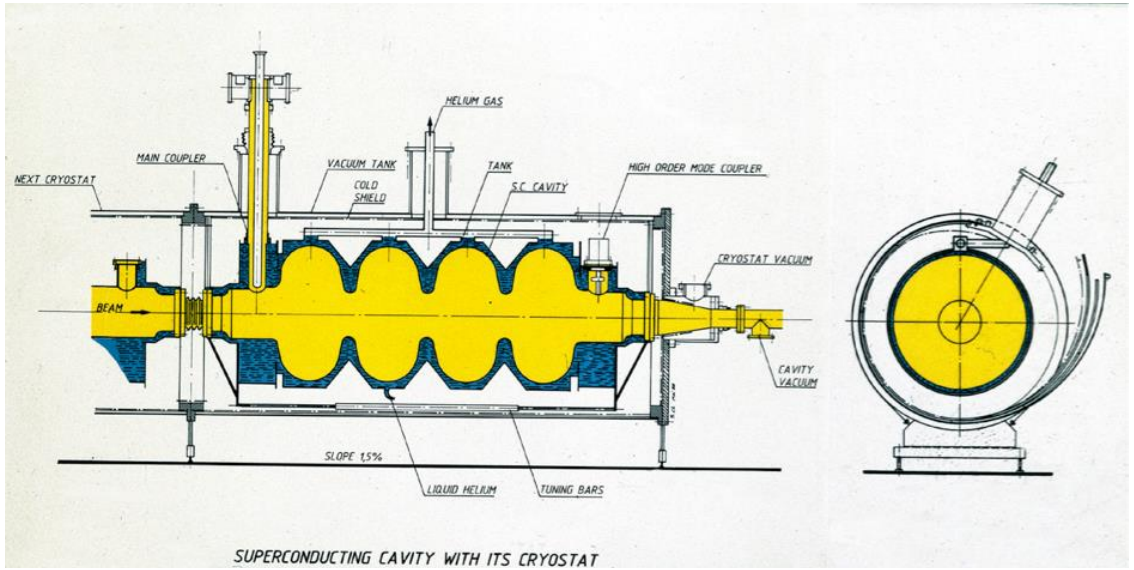
In a well defined part of the accelerator, **a RF (radio frequency) cavity generates an electric field parallel to the velocity of a zero divergence particle**.
The cavity itself acts as a resonator.



Obs: The magnetic field associated to the RF wave is negligible (for us).

B. Salvant
N. Biancacci

RF systems, LEP, LHC



A typical cavity can provide from few kV/m few MV/m

Example for LEP:

120 cavities (room temperature) at 352 MHz, provided over **300 MV circumferential voltage** (! that's why we do not bend with E fields...)

Then, the new superconducting RF provided **2000 MV circumferential voltage (LEP was 27 km circumference, basically filled by RF cavities)**

Example for LHC:

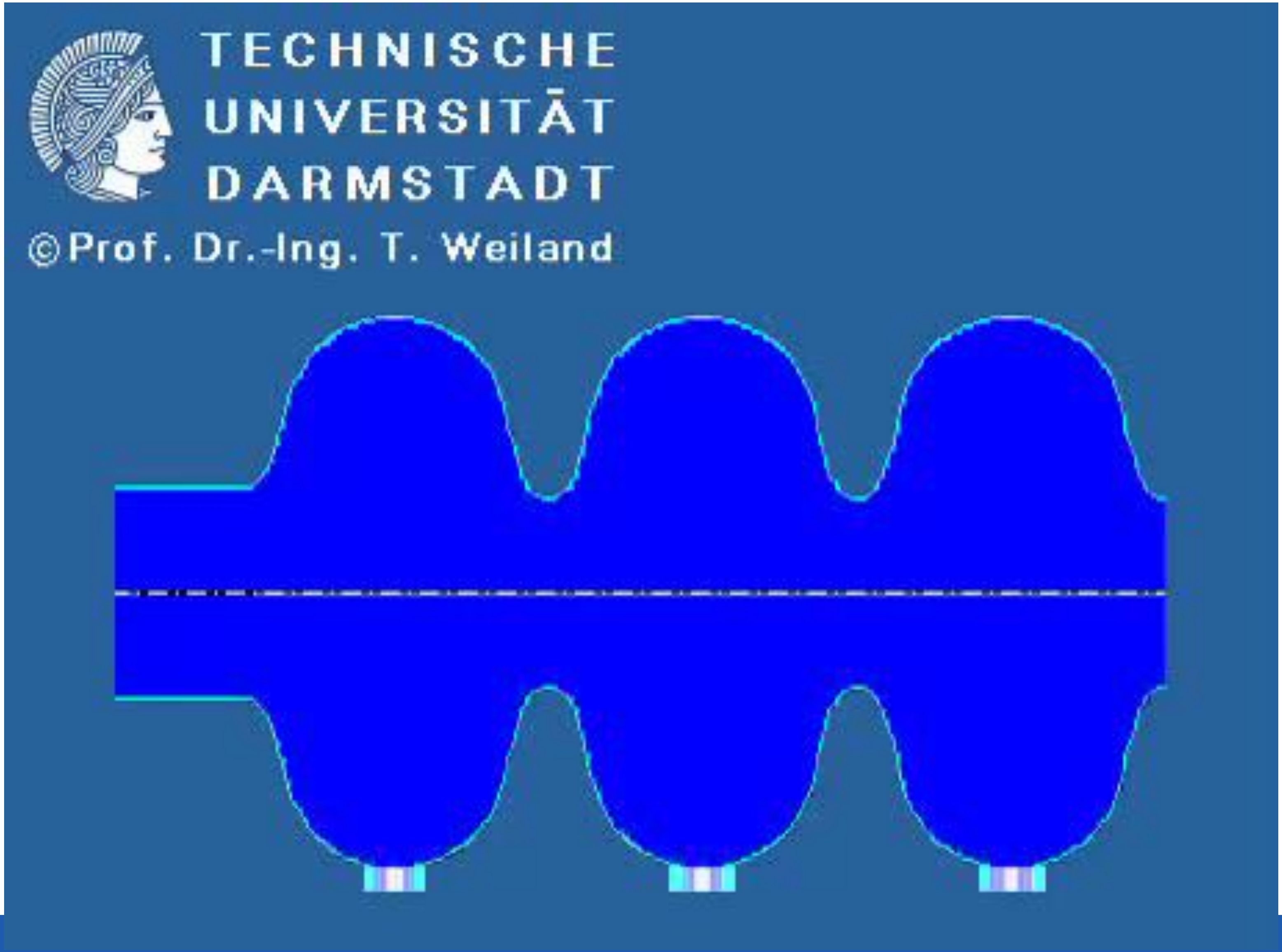
485 keV gain per turn
ACCELERATION TAKES TIME

How long is a wave?
fcav= 400 MHz

λ= c / fcav ~ 75 cm

RF Cavity 2013

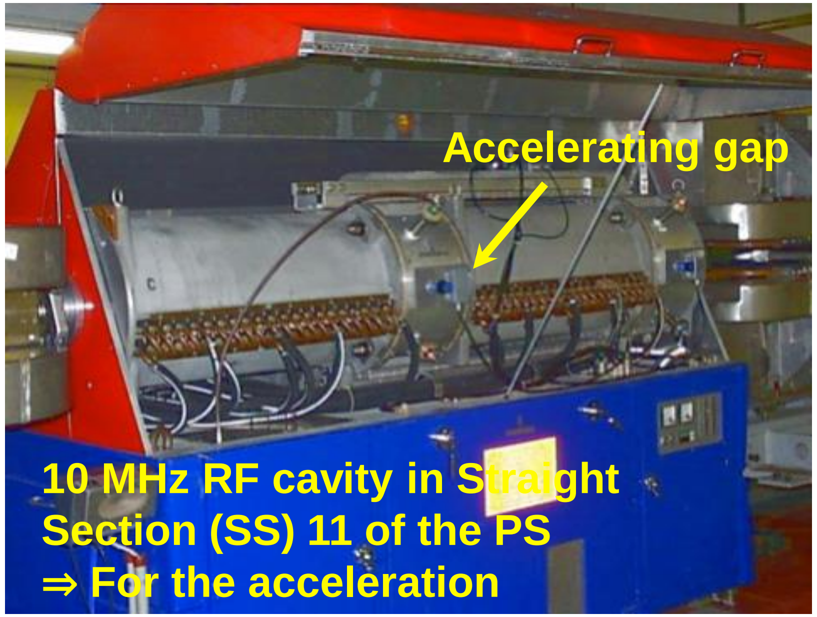
Electromagnetic field of bunch in a cavity



https://www.youtube.com/watch?v=t5hnM1gsgfo

Example of RF cavities in the PS

The dimension of the cavity changes with the RF wave length



World Radio Switzerland: 88.4 MHz

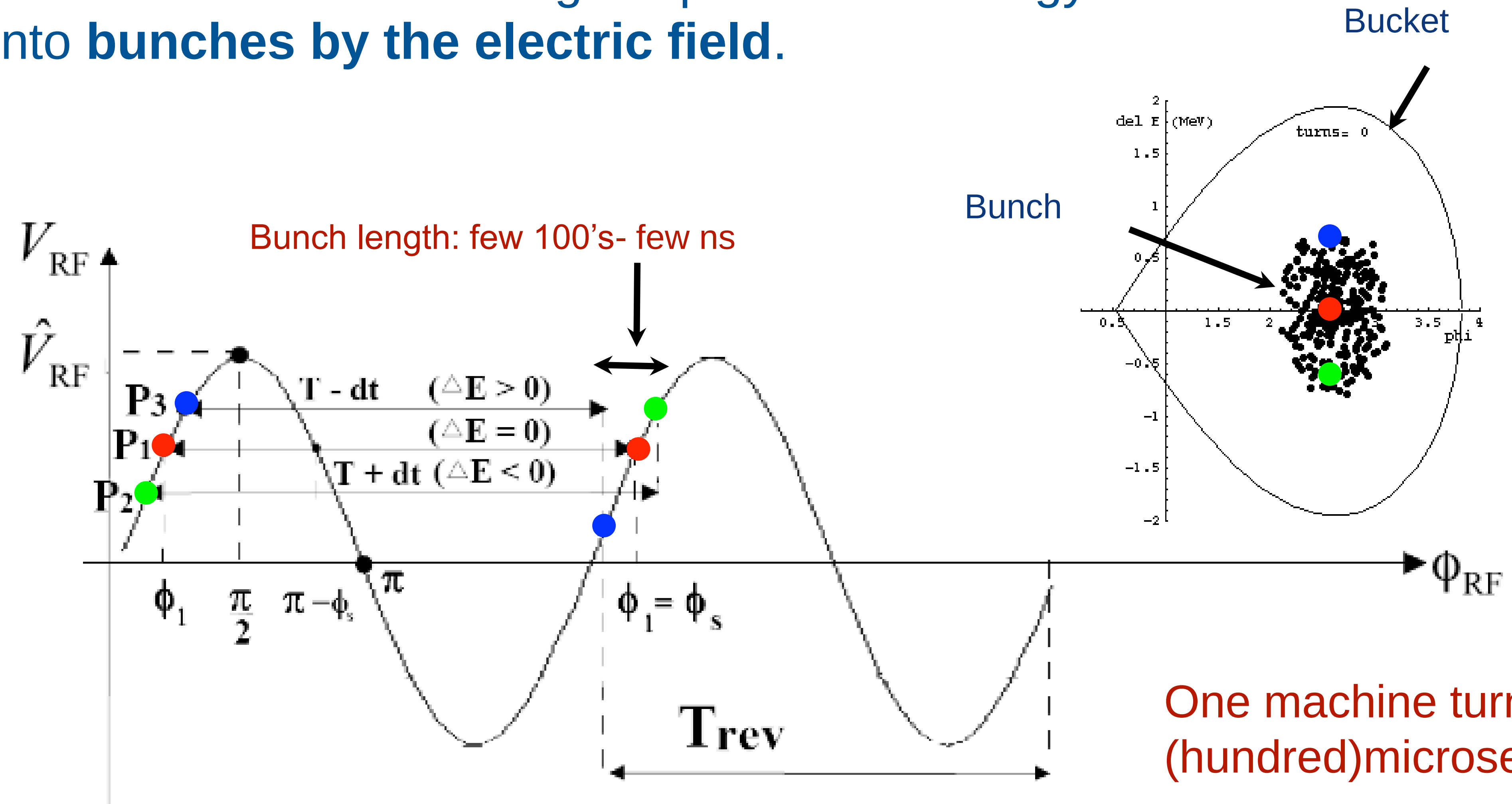
Some italian radios (Provincia di Vicenza)

(Mhz)	nominativo
87.60	EASY NETWORK
87.85	RADIO CAPITAL
88.10	RAI, RADIO UNO
88.40	RADIO PADOVA
88.70	RADIO RICERCA REALTA' (CIRC. MARCONI)
89.00	RAI, RADIO DUE
89.30	RADIO DEEJAY
89.60	BELLISSIMA FM
89.90	RAI, RADIO TRE
90.20	RADIO OREB (CIRCUITO MARCONI)
90.40	BUM BUM ENERGY
90.65	RADIO PICO
90.80	RADIO RICERCA REALTA' (CIRC. MARCONI)
90.90	RADIO COMPANY
91.10	RADIO SOLE
91.30	RADIO SORRRISO
91.60	RADIO PITERPAN
91.60	RADIO BIRIKINA

(Mhz)	nominativo
97.70	RADIO COLLINA STUDIO UNO
97.95	RADIO FOLLIA
98.20	RADIO CAPITAL
98.45	BUM BUM NETWORK
98.60	RAI, RADIO TRE
98.70	EASY NETWORK
99.00	TRV TELE RADIO VENETA
99.30	RADIO PITERPAN
99.55	RADIO PRINCIPESSA
99.80	RDS, RADIO DIMENSIONE SUONO
100.05	RSB RADIO SAN BONIFACIO
100.25	RCA - RADIO CITY ANTENNA UNO
100.50	RADIO COMPANY
100.80	RMC, MONTECARLO
101.00	RADIO BLU
101.30	RCA - RADIO CITY ANTENNA UNO
101.50	RADIO ITALIA SOLO MUSICA ITALIANA

Longitudinal focusing, a pendulum ...

Particles are confined within a range in phase and energy called **BUCKET** and are grouped into **bunches by the electric field**.



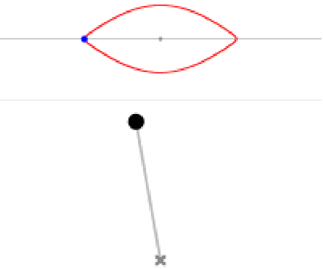
One machine turn ~ some (hundred)microseconds

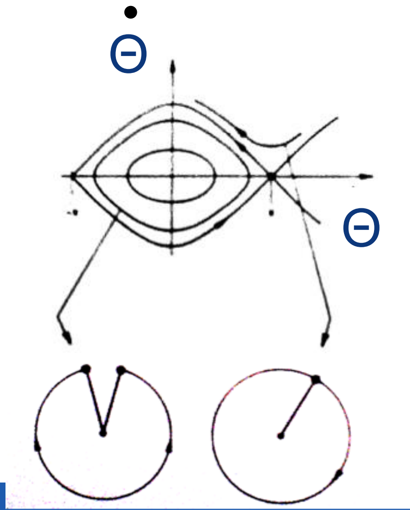
Longitudinal dynamics
Classical mechanics.... oscillating pendulum

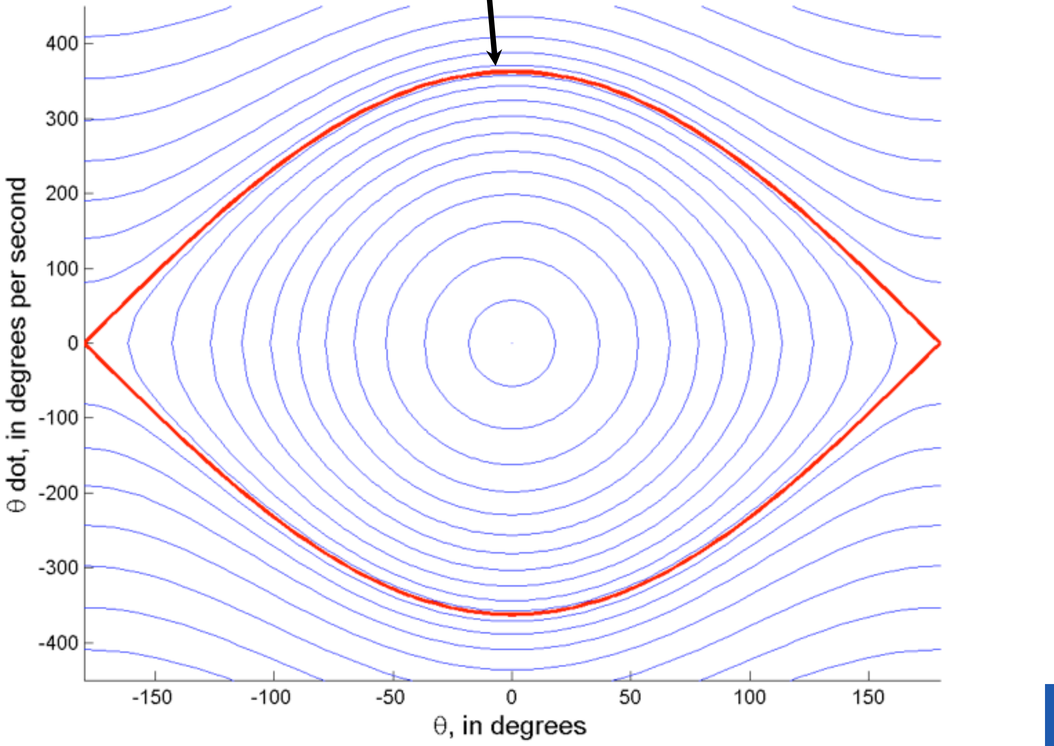
$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

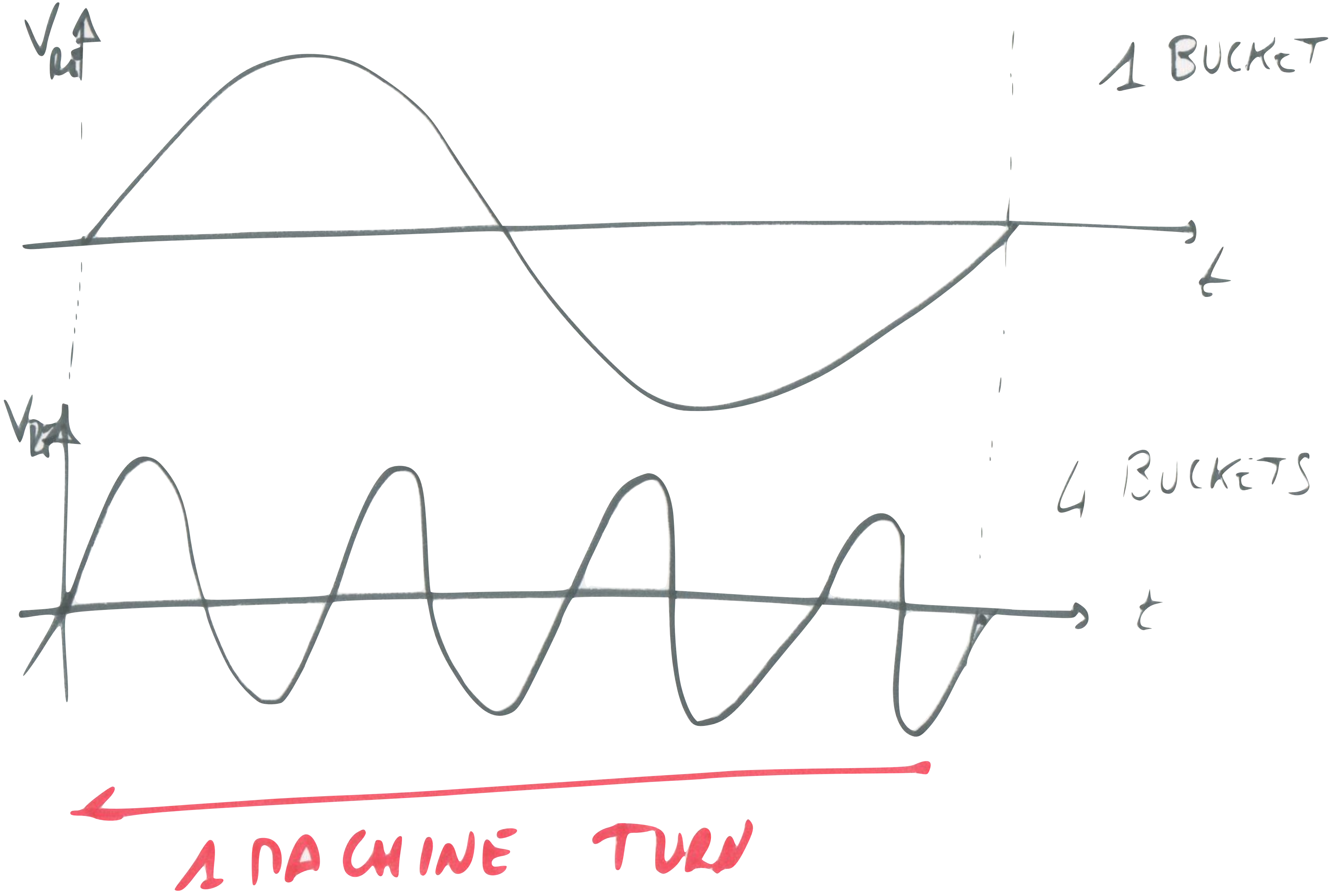
The movement is stable only inside the separatrix

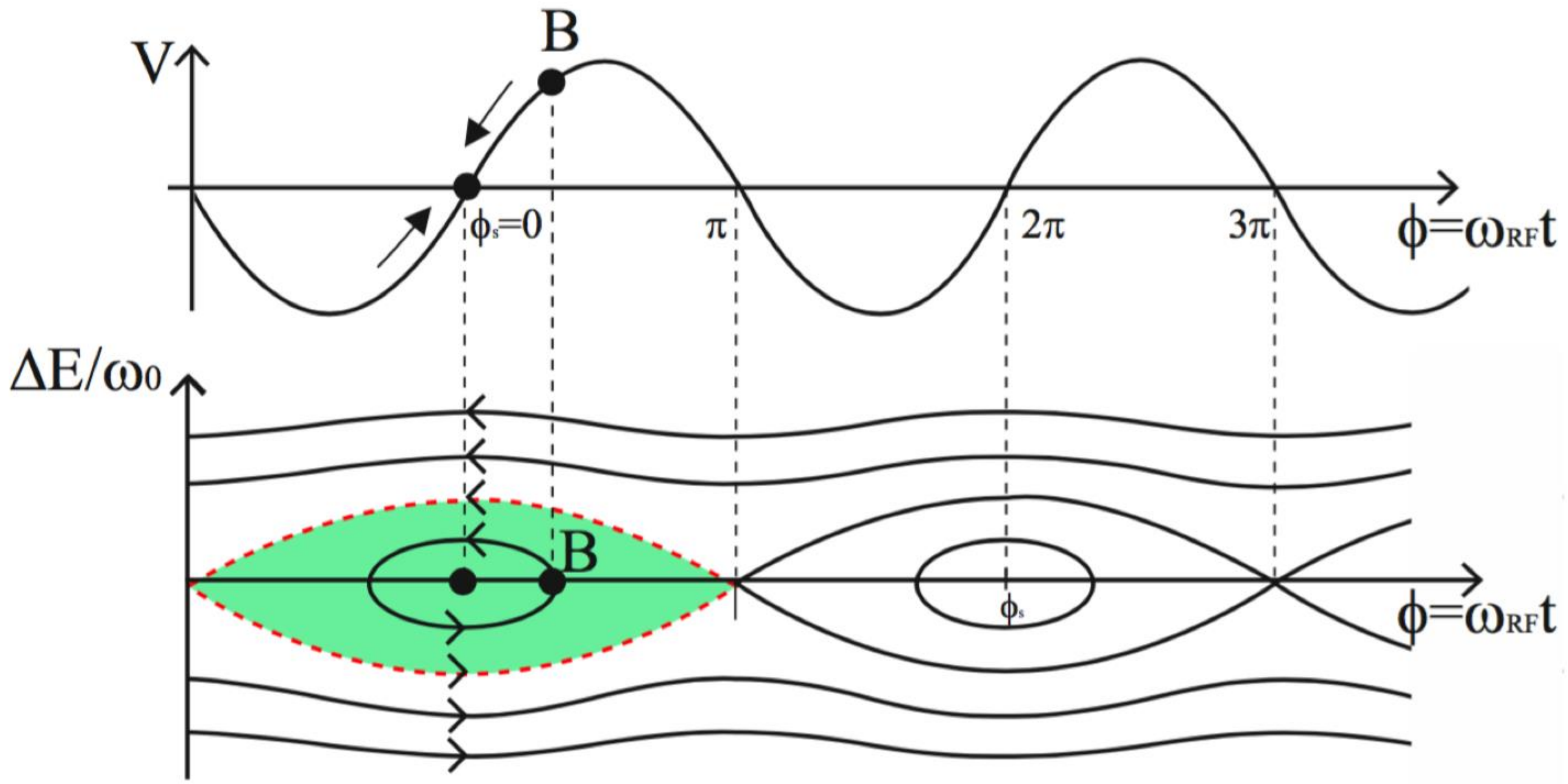
Constant force:
g = acceleration of gravity
l = pendulum length



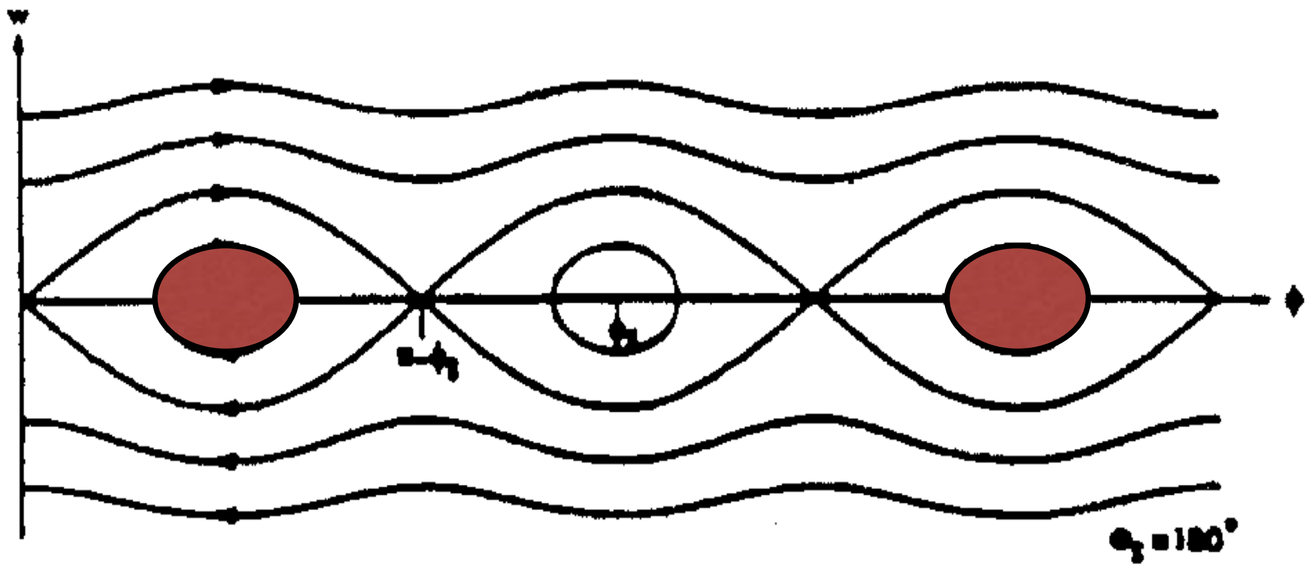





V_RF
1 BUCKET
t
V_RF
4 BUCKETS
t
1 MACHINE TURN


B
V
ϕ_s=0
π
2π
3π
$\phi=\omega_{RF}t$
ΔE/ω_0
B
ϕ_s
$\phi=\omega_{RF}t$

A chain of buckets



Courtesy
E. Wilson

Number of buckets:

possible positions along the machine circumference where ***there could be a bunch.***

In the example: 3 buckets and 2 bunches

From wavelength to meters

Frequency	Wavelength
1 MHz	300 meters
10 MHz	30 meters
50 MHZ	6.0 meters
100 MHz	3.0 meters
200 MHz	1.5 meters
300 MHz	1.0 meter
400 MHz	0.75 meters
500 MHz	0.6 meters
600 MHz	0.5 meters
700 MHz	42.9 cm
800 MHz	37.5 cm
900 MHz	33.3 cm
1.0 GHz	30 cm

PS

SPS

LHC

Circumference increases

Bunches are shorter

More bunches

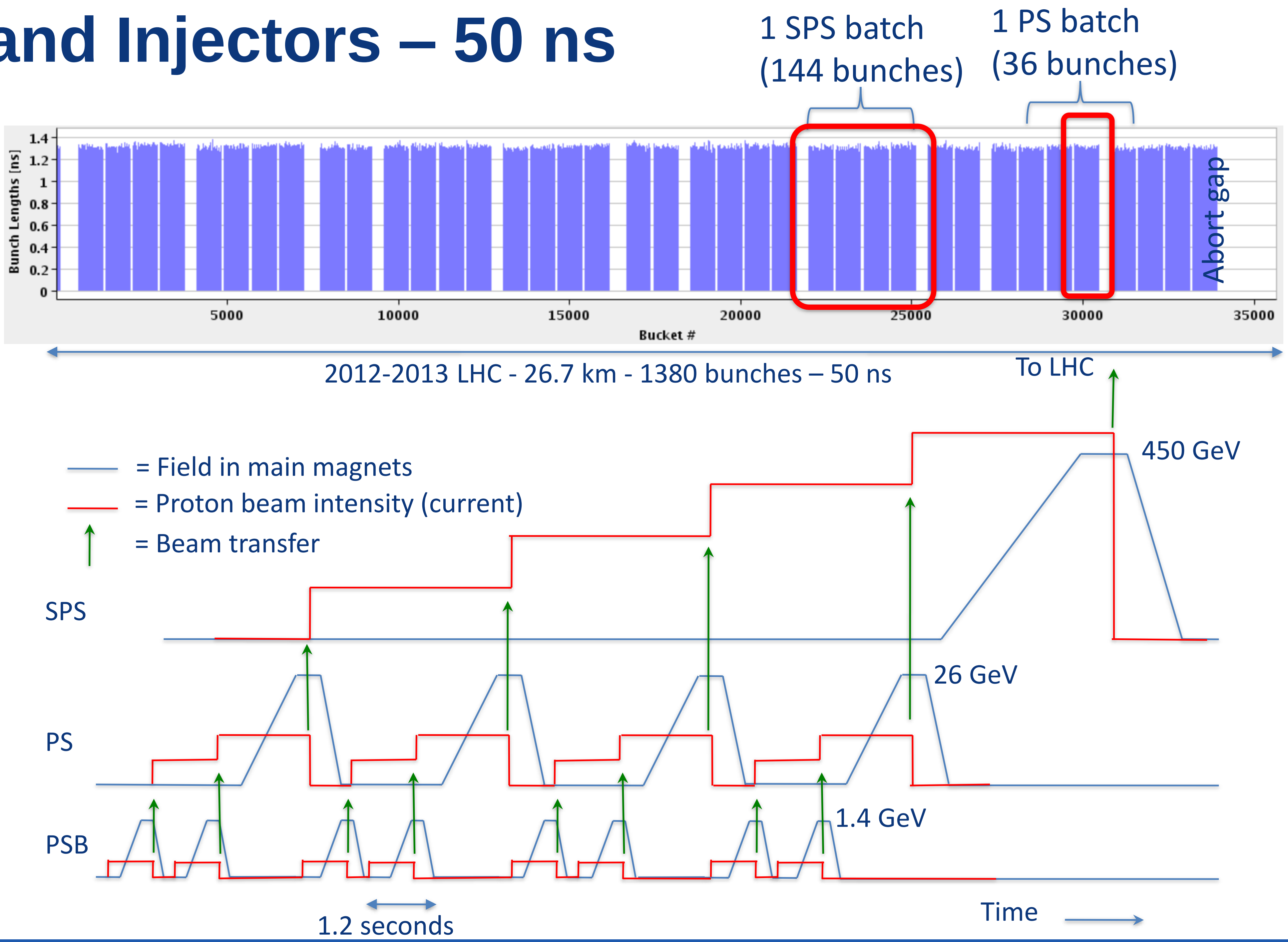
Q: Why I want a lot of bunches in the LHC?

How many buckets are in the LHC?

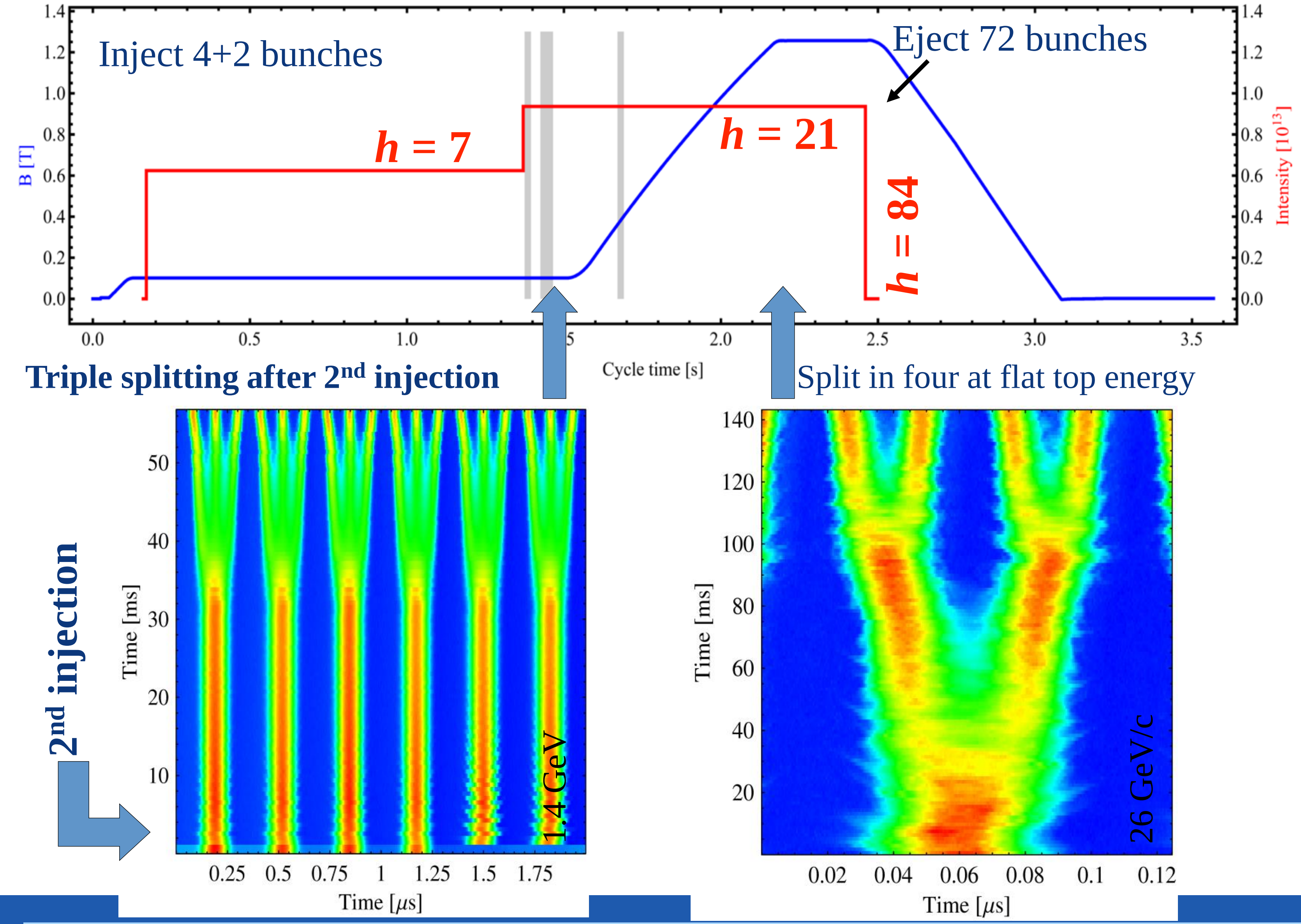
- $f_{RF} = 400$ MHz $\qquad f_{RF} = h \times f_{rev}$
- $f_{rev} = \beta c/(2\pi R)$
- $2\pi R = 26659$ m
- Harmonic Number: $h = f_{RF}/f_{rev}$
- $h = (400 \cdot 10^6)/(c/26659)$
- Harmonic Number ≈ 35640 — number of buckets

- Q: why we have only 2808 bunches ?
- Q: why bunches spaced by 25 ns?
- Q: why bunches at all ?

LHC and Injectors – 50 ns



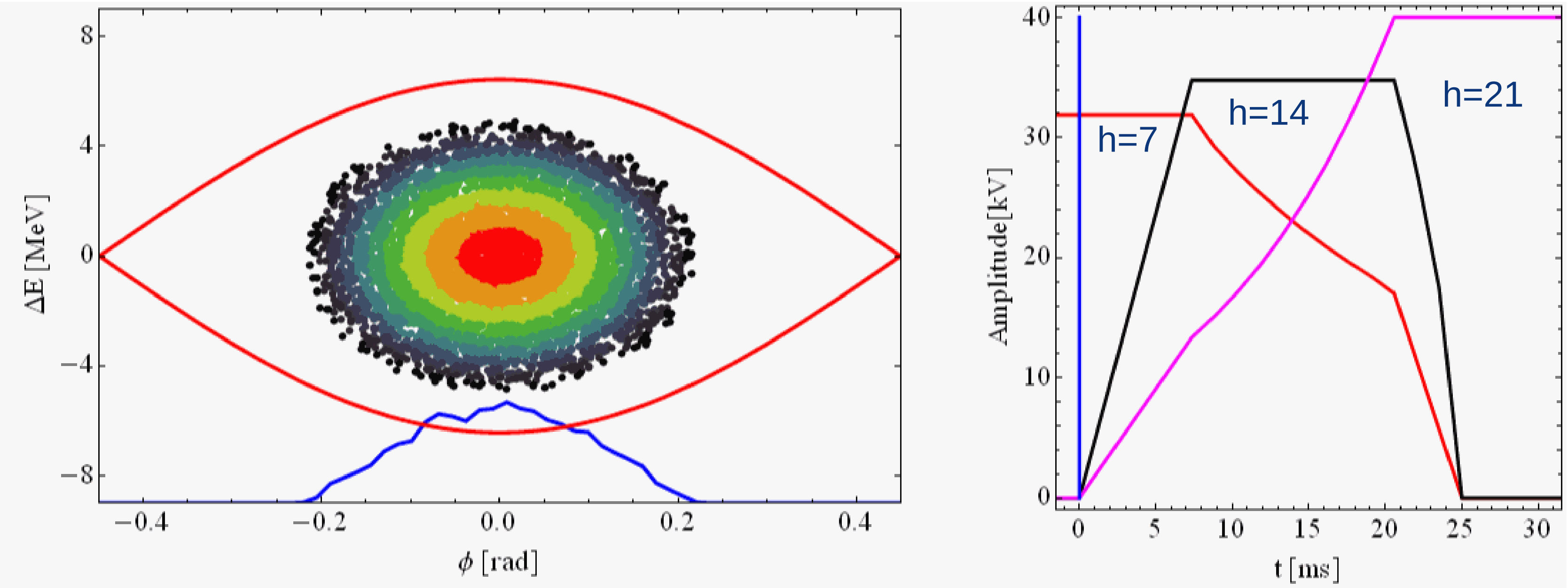
The LHC25 (ns) cycle in the PS. Double batch



→ Each bunch from the Booster divided by 12 → 6 × 3 × 2 × 2 = 72

Example of RF gymnastics: Triple bunch-splitting (LHC25, LHC50)

Split bunches in three similar parts ($h7 \rightarrow h21$)

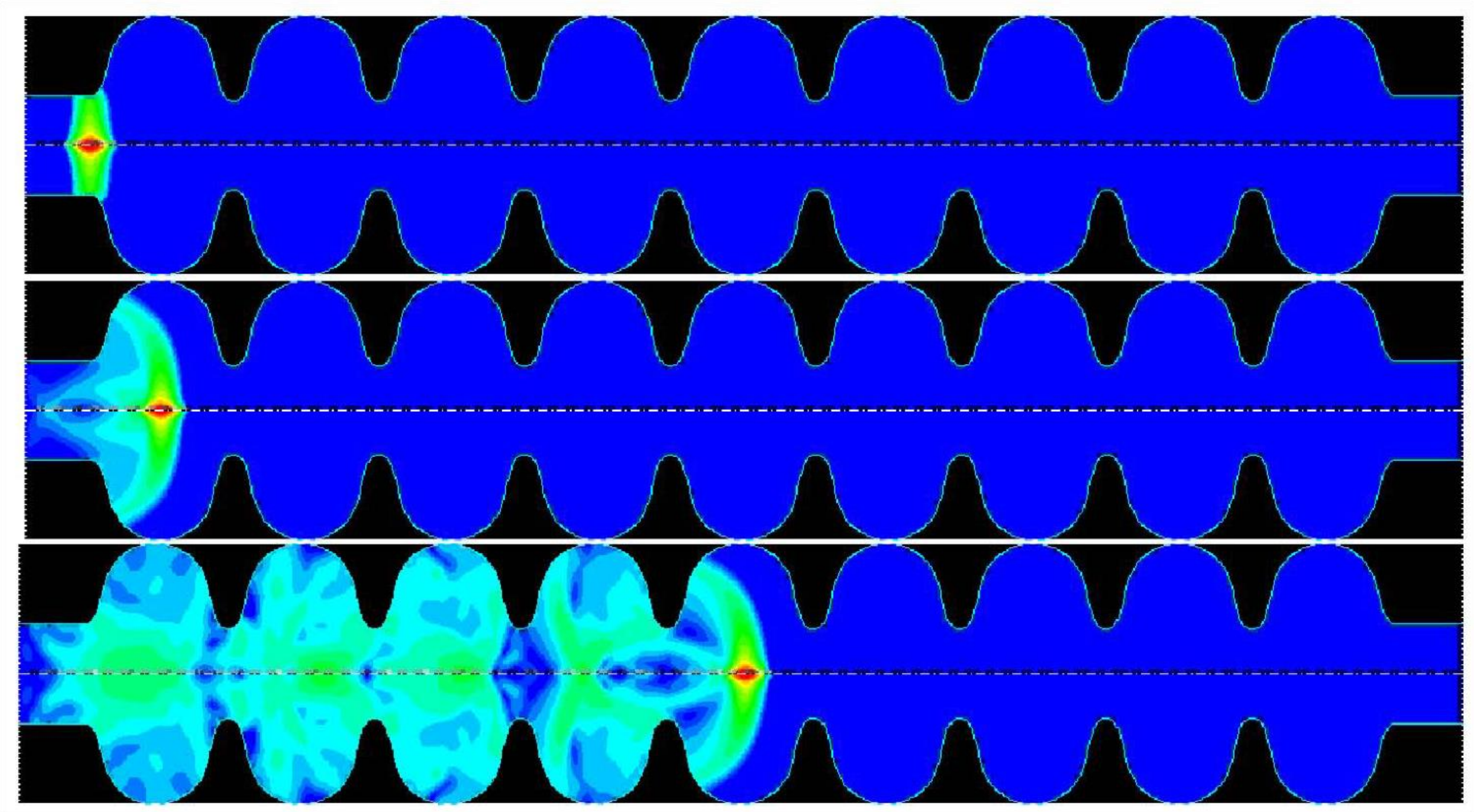


Three RF harmonics at the same time h=7→ h=14→ h=21.

The Voltage of different group of cavities at different harmonics are "adiabatically" increases and decreased to obtain a bunch splitting with stationary buckets

If we would be looking to the Hamiltonian of the system, we would be creating extra fix points by introducing the high field harmonics *(ipse dixit…)*

Induced field inside a cavity from a bunch passage

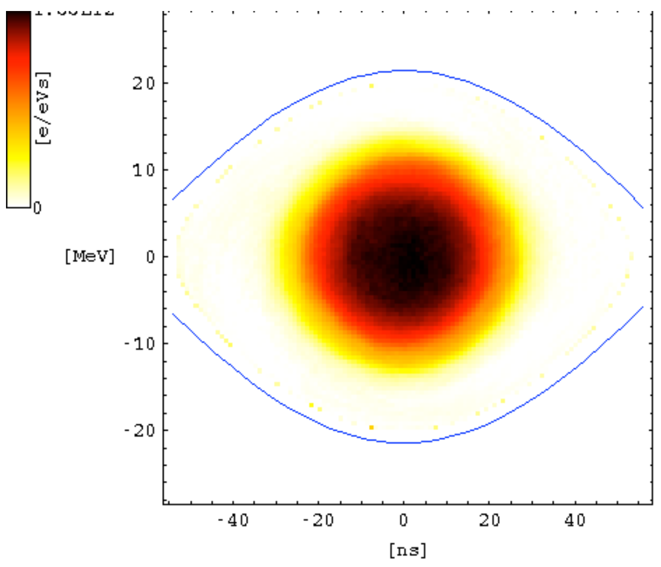


D. Trines, Bodrum 2007

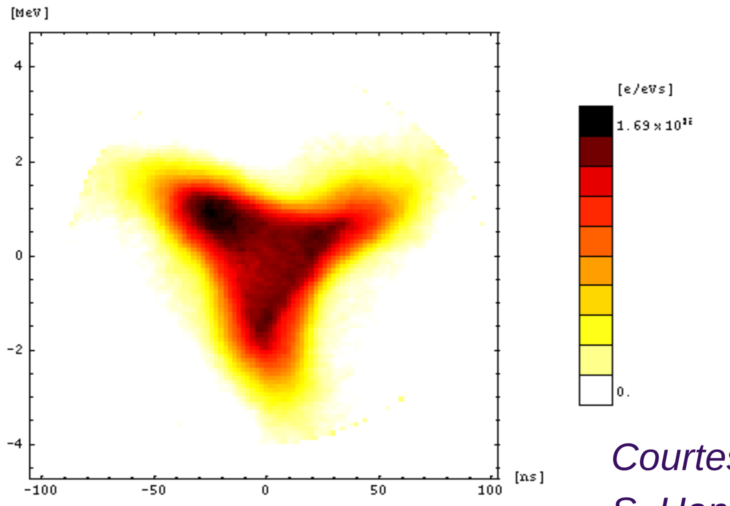
Following bunch “feels” the presence of the previous one due to induced electromagnetic field

How to completely screw up a bunch

a) This is a normal bunch



b) This is an unstable bunch (sextupolar instability)



Courtesy S. Hancock

Everyone can understand that b) risks to be more difficult to threat than a).....

Summary

- **Dipoles** bend charged particles in the accelerator
- **Quadrupoles** focus particles and define the beam **tune**
- **RF cavities** accelerate the beam

- The emittance is the space occupied by the particles in the xx' plane
- The envelope is defined by the quadrupoles via the beta function