FCC-ee: the challenge for theory

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*My thanks to ~ 100 authors for excellent contributions to the recent FCC-ee theory CERN Yellow Reports "1" & "2"

4th FCC Physics and Experiments workshop

10-13 November 2020 (on-line)

Backup slides

FCC CDRs, Snowmass LoIs

FCC-ee CDRs:

1. FCC Physics Opportunities : Future Circular Collider Conceptual Design Report Volume 1, link
2. FCC-ee: The Lepton Collider : Future Circular Collider Conceptual Design Report Volume 2, link

Snowmass Letters of Intent, with Matthew Mccullough

1. Theory Needs for FCC-ee Part I: Towards high precision EWPO calculations https://snowmass21.org/docs/files/summaries/TF/SNOWMASS21-TF6_TF4-004.pdf
2. Theory Needs for FCC-ee Part II: New methods for SM calculations https://snowmass21.org/docs/files/summaries/TF/SNOWMASS21-TF6_TF4-005.pdf

Other related recent talks

- Ayres Freitas, "Thoughts on EW observables and calculations", Energy Frontier Workshop - Open Questions and New Ideas, Snowmass 2020
  https://indico.fnal.gov/event/43963/contributions/191333/attachments/131719/163400/ta
- Snowmass Theory Frontier Kick-Off Town Hall Meeting,
  "Mathematics of multi-scale loop integrals", Stefan Weinzierl
  https://indico.fnal.gov/event/44512/contributions/192852/attachments/132204/162329/W
- Snowmass Theory Frontier Kick-Off Town Hall Meeting,
  "The push to the N3LO frontier", Claude Duhr
  https://indico.fnal.gov/event/44512/contributions/192854/attachments/132324/162540/sn
- ICHEP 2020, Johannes Henn
  "Novel methods for calculating Feynman integrals and applications to QCD processes"
  https://indico.cern.ch/event/868940/contributions/3813254/attachments/2082330/349777

EWPOs 2020

Observable	present value ± error	FCC-ee **Stat.**	FCC-ee Syst.	Comment and leading exp. error
m_Z (keV)	91186700 ± 2200	**4**	100	From Z line shape scan Beam energy calibration
Γ_Z (keV)	2495200 ± 2300	**4**	25	From Z line shape scan Beam energy calibration
$\sin^2\theta_W^{eff}(\times 10^6)$	231480 ± 160	**2**	2.4	from $A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{QED}(m_Z^2)(\times 10^3)$	128952 ± 14	**3**	small	from $A_{FB}^{\mu\mu}$ off peak QED&EW errors dominate
R_ℓ^Z $(\times 10^3)$	20767 ± 25	**0.06**	0.2-1	ratio of hadrons to leptons **acceptance for leptons**
$\alpha_s(m_Z^2)$ $(\times 10^4)$	1196 ± 30	**0.1**	0.4-1.6	from R_ℓ^Z above
σ_{had}^0 $(\times 10^3)$ (nb)	41541 ± 37	**0.1**	4	peak hadronic cross section luminosity measurement
$N_\nu(\times 10^3)$	2996 ± 7	**0.005**	1	Z peak cross sections Luminosity measurement
R_b $(\times 10^6)$	216290 ± 660	**0.3**	< 60	ratio of $b\bar{b}$ to hadrons stat. extrapol. from SLD
$A_{FB}^b, 0$ $(\times 10^4)$	992 ± 16	**0.02**	1-3	b-quark asymmetry at Z pole from jet charge
$A_{FB}^{pol,\tau}$ $(\times 10^4)$	1498 ± 49	**0.15**	<2	τ polarization asymmetry τ decay physics
τ lifetime (fs)	290.3 ± 0.5	**0.001**	0.04	radial alignment
τ mass (MeV)	1776.86 ± 0.12	**0.004**	0.04	momentum scale
τ leptonic $(\mu\nu_\mu\nu_\tau)$ B.R. (%)	17.38 ± 0.04	**0.0001**	0.003	e/μ/hadron separation
m_W (MeV)	80350 ± 15	**0.25**	0.3	From WW threshold scan Beam energy calibration
Γ_W (MeV)	2085 ± 42	1.2	0.3	From WW threshold scan Beam energy calibration
$\alpha_s(m_W^2)(\times 10^4)$	1170 ± 420	**3**	small	from R_ℓ^W
$N_\nu(\times 10^3)$	2920 ± 50	**0.8**	small	ratio of invis. to leptonic in radiative Z returns
m_{top} (MeV/c^2)	172740 ± 500	**17**	small	From $t\bar{t}$ threshold scan QCD errors dominate
Γ_{top} (MeV/c^2)	1410 ± 190	45	small	From $t\bar{t}$ threshold scan QCD errors dominate
$\lambda_{top}/\lambda_{top}^{SM}$	1.2 ± 0.3	**0.10**	small	From $t\bar{t}$ threshold scan QCD errors dominate
ttZ couplings	± 30%	0.5 – 1.5%	small	From $\sqrt{s} = 365\,\mathrm{GeV}$ run

Estimated theoretical uncertainties from missing higher orders and the perturbative orders (QCD/elw.) of the results included in the analysis.

Partial Width	**QCD**	**Electroweak**	**Total**	**on-shell Higgs**
$H \to b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$\sim 0.5\%$	$\sim 0.5\%$	N^{4}LO / NLO
$H \to \tau^+\tau^-/\mu^+\mu^-$	—	$\sim 0.5\%$	$\sim 0.5\%$	— / NLO
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3\%$	N^{3}LO / NLO
$H \to \gamma\gamma$	$< 1\%$	$< 1\%$	$\sim 1\%$	NLO / NLO
$H \to Z\gamma$	$< 1\%$	$\sim 5\%$	$\sim 5\%$	LO / LO
$H \to WW/ZZ \to 4f$	$< 0.5\%$	$\sim 0.5\%$	$\sim 0.5\%$	NLO/NLO

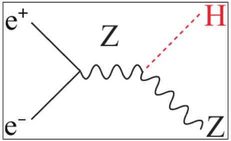
Projected intrinsic and parametric uncertainties for the partial and total Higgs-boson decay width predictions. The last column: the target of FCC-ee precisions.

decay	intrinsic	para. m_q	para. α_s	para. M_H	FCC-ee prec. on g^2_{HXX}
$H \to b\bar{b}$	$\sim 0.2\%$	0.6%	$< 0.1\%$	–	$\sim 0.8\%$
$H \to c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	$< 0.1\%$	–	$\sim 1.4\%$
$H \to \tau^+\tau^-$	$< 0.1\%$	–	–	–	$\sim 1.1\%$
$H \to \mu^+\mu^-$	$< 0.1\%$	–	–	–	$\sim 12\%$
$H \to gg$	$\sim 1\%$		0.5% (0.3%)	–	$\sim 1.6\%$
$H \to \gamma\gamma$	$< 1\%$	–	–	–	$\sim 3.0\%$
$H \to Z\gamma$	$\sim 1\%$	–	–	$\sim 0.1\%$	
$H \to WW$	$\lesssim 0.3\%$	–	–	$\sim 0.1\%$	$\sim 0.4\%$
$H \to ZZ$	$\lesssim 0.3\%^\dagger$	–	–	$\sim 0.1\%$	$\sim 0.3\%$
Γ_{tot}	$\sim 0.3\%$	$\sim 0.4\%$	$< 0.1\%$	$< 0.1\%$	$\sim 1\%$

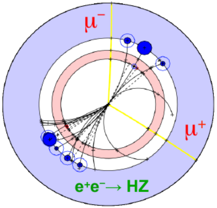
† From $e^+e^- \to HZ$ production

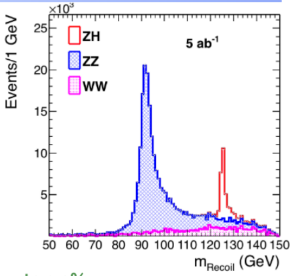
Absolute coupling and width measurement

- **Higgs tagged by a Z, Higgs mass from Z recoil**



$$m_H^2 = s + m_Z^2 - 2\sqrt{s}(E_+ + E_-)$$

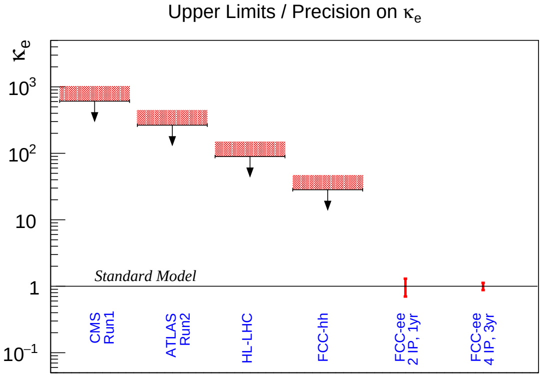




- Total rate $\propto g_{HZZ}^2$ → measure g_{HZZ} to 0.2%
- ZH → ZZZ final state $\propto g_{HZZ}^4 / \Gamma_H$ → measure Γ_H to a couple %
- ZH → ZXX final state $\propto g_{HXX}^2\, g_{HZZ}^2 / \Gamma_H$ → measure g_{HXX} to a few per-mil / per-cent
- Empty recoil = invisible Higgs width; Funny recoil = exotic Higgs decays

- **Note: The HL-LHC is a great Higgs factory (10^9 Higgs produced) but …**
  - $\sigma_{i\to f}^{(\mathrm{observed})} \propto \sigma_{\mathrm{prod}}\, (g_{Hi})^2\, (g_{Hf})^2 / \Gamma_H$
    - Difficult to extract the couplings : σ_{prod} is uncertain and Γ_H is largely unknown
      - ➨ Must do physics with ratios or with additional assumptions.

Sensitivity of FCC-ee, comparisons, Blondel & Janot inspires



Current upper limits on the Higgs boson coupling modifier to electrons, κ_e, from and ATLAS; projected κ_e upper limits at HL-LHC and FCC-hh; and projected κ_e precisions at FCC-ee in two different running configurations (one year with 2 IPs, or three years with 4 IPs).

FCC-ee fits, CDR link

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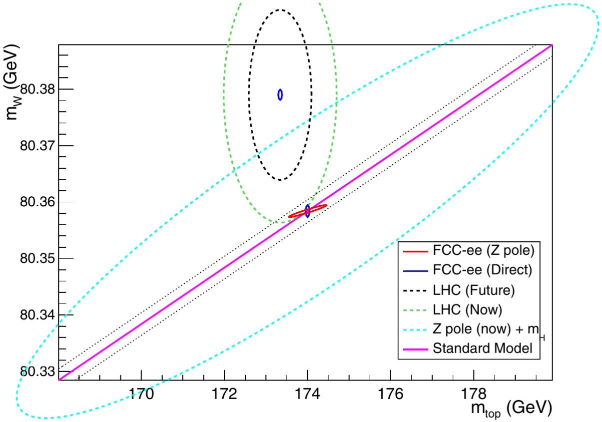


Flavour physics numbers for FCC-ee

Table 7.1: Expected production yields of heavy-flavoured particles at Belle II (50 ab^{-1}) and FCC-ee.

Particle production (10^9)	B^0 / $\overline{B}^0$	B^+ / B^-	B^0_s / $\overline{B}^0_s$	Λ_b / $\overline{\Lambda}_b$	$c\bar{c}$	$\tau^+\tau^-$
Belle II	27.5	27.5	n/a	n/a	65	45
FCC-*ee*	1000	1000	250	250	550	170

Table 7.2: Comparison of orders of magnitude for expected reconstructed yields of a selection of electroweak penguin and pure dileptonic decay modes in Belle II, LHCb upgrade and FCC-ee experiments. Standard model branching fractions are assumed. The yields for the electroweak penguin decay $\bar{B}^0 \to K^{*0}(892)e^+e^-$ are given in the low q^2 region.

Decay mode	$B^0 \to K^*(892)e^+e^-$	$B^0 \to K^*(892)\tau^+\tau^-$	$B_s(B^0) \to \mu^+\mu^-$
Belle II	$\sim$ 2 000	$\sim$ 10	n/a (5)
LHCb Run I	150	-	$\sim$ 15 (–)
LHCb Upgrade	$\sim$ 5000	-	$\sim$ 500 (50)
FCC-ee	$\sim$ 200000	$\sim$ 1000	$\sim$1000 (100)

E.g. effective weak mixing angle

The weak mixing angle $s_W^2 \equiv \sin^2\theta_W$ has three potential different meanings or functions in the model-building:

(i) It describes the ratio of the two gauge couplings,

$$g'/g = c_W/s_W, \tag{1}$$

usually in the $\overline{\text{MS}}$ scheme.

(ii) It describes the ratio of two gauge boson (on-shell) masses,

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}. \tag{2}$$

(iii) It describes the ratio of the vector and axial-vector couplings of an (on-shell) Z boson to fermions,

$$\frac{v_f}{a_f} = 1 - 4|Q_f|s_W^2. \tag{3}$$

This definition is called the effective weak mixing angle, denoted as $\sin^2\theta_W^{f,\text{eff}}$.

Z-resonance: QED and EW

1. Z-resonance and $\gamma, Z', \ldots \longrightarrow$ Laurent series,

$$\mathcal{M} = \frac{R}{s - s_0} + \sum_{n=0}^{\infty} (s - s_0)^n \; B^{(n)}, \quad s_0 = \bar{M}_Z^2 + i \bar{M}_Z \bar{\Gamma}_Z .$$

2. We want to extract EW Z-vertex couplings and definitions like $\sin^2 \theta_{\text{eff}}^f$, but in reality, we deal with complicated process

$$e^+ e^- \to f^+ f^- \quad + \text{invisible} \; (n \; \gamma + \mathrm{e}^+ \mathrm{e}^- \text{pairs} + \cdots)$$

$$\sigma^{e^+e^- \to f^+f^- + \cdots}(s) = \int dx \; \overbrace{f(x)} \; \underbrace{\sigma^{e^+e^- \to f^+f^-}(s')} \; \delta(x - s'/s)$$

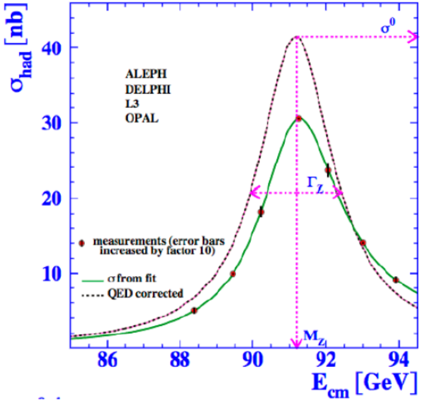
$\longrightarrow$ form factors, QED separation/deconvolution, non-factorizations, ...

To determine the structure function/flux function kernels and hard scattering ansatz for data preparation or for unfolding is one of the challenges of FCC-ee-Z physics.

QED unfolding

Altogether $17 \cdot 10^6$ Z-boson decays at LEP

- Cross section : Z mass and width



- ~30% QED corrections (ISR)

How to unfold - rough scheme

We have to describe

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f^+f^-(\gamma), \tag{4}$$

S-matrix Ansatz in the complex energy plane

$$\mathcal{A}^{e^+e^-\to b\bar{b}} = \underbrace{\frac{R_Z}{s-s_Z} + \overbrace{\frac{R_\gamma}{s}}_{\gamma-Z\ interference} + S + (s-s_Z)S' + \ldots}^{Background},$$

$$s_Z = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z$$

- $R, S, S', \ldots$ are individually gauge-invariant and UV-finite - unitarity and analyticity of the S-matrix. IR-finite, when soft and collinear real photon emission is added.

[Willenbrock, Valencia,1991] [Sirlin,1991] [Stuart,1991] [Riemann, 1991, 1992] [H. Veltman,1994] [Passera, Sirlin, 1998] [Gambino, Grassi, 2000] [Awramik, Czakon, Freitas, 2006].

The term $R_\gamma(s)/s$ is part of the the background

- The poles of $\mathcal{A}$ have complex residua R_Z and R_γ.
- There is only ONE pole in mathematics, while in physics we observe two of them: photon exchange at $s = 0$, Z exchange at $s_0 = s_Z$. Mathematicaly, the appearance of the photon pole is result of summing of part of background around Z pole, $s_0 = s_Z$

[T. Riemann, APPB 2015]

$$
\begin{aligned}
\frac{R_\gamma(s)}{s} &= \frac{\sum_{n=0}^{\infty} R_n (s-s_0)^n}{s} \\
&= \frac{\sum_{n=0}^{\infty} R_n (s-s_0)^n}{s_0 - (s_0 - s)} \\
&= \sum_{n=0}^{\infty} R_n (s-s_0)^n \, \frac{1}{s_0} \, \frac{1}{1 - \frac{s_0 - s}{s_0}} \\
&= \sum_{n=0}^{\infty} R_n (s-s_0)^n \, \frac{1}{s_0} \left[1 + \frac{s_0 - s}{s_0} + \left(\frac{s_0 - s}{s_0} \right)^2 \cdots \right];
\end{aligned}
$$

Consistent (gauge-invariant) theory setup:

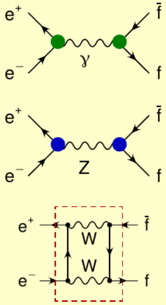
Expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_Z^2 - iM_Z\Gamma_Z$:

$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)T + \ldots$$

$$R = g_Z^e(s_0)g_Z^f(s_0)$$

$$S = \left[\frac{1}{M_Z^2} g_\gamma^e g_\gamma^f + g_Z^e g_Z^{f\,\prime} + g_Z^{e\,\prime} g_Z^f + S_{\text{box}}\right]_{s=s_0}$$

$g_V^f(s)$: effective $Vf\bar{f}$ couplings



At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

Current state of art: full one-loop for S, T

$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty within SM

(improvements may be needed)

see, e.g., Bardin, Grünewald, Passarino '99

QED unfolding, ISR, slide by A.Freitas, Snowmass 2020, pdf

Z lineshape

6/18

Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

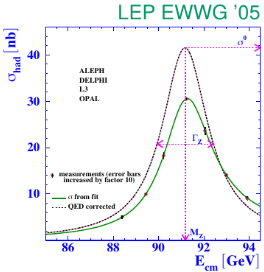
Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '91; Skrzypek '92
Montagna, Nicrosini, Piccinini '97

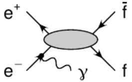
Soft photons (resummed) + collinear photons

$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^{n} h_{nm} \ln^m\left(\frac{s}{m_e^2}\right)$$

Universal ($m{=}n$) logs known to $n = 6$,
also some sub-leading terms

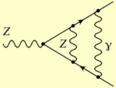
Ablinger, Blümlein, De Freitas, Schönwald '20





Factorization of massive and QED/QCD FSR:

$$\Gamma_f \approx \frac{N_c M_Z}{12\pi}\left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2\right)\frac{1}{1+\mathrm{Re}\,\Sigma'_Z}\right]_{s=M_Z^2}$$



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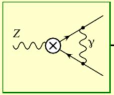
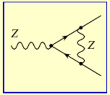
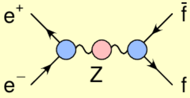


+ finite, with $\otimes$ =


$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;
known to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$

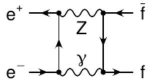
Kataev '92
Chetyrkin, Kühn, Kwiatkowski '96
Baikov, Chetyrkin, Kühn, Rittinger '12

g_V^f, g_A^f, Σ'_Z: Electroweak corrections

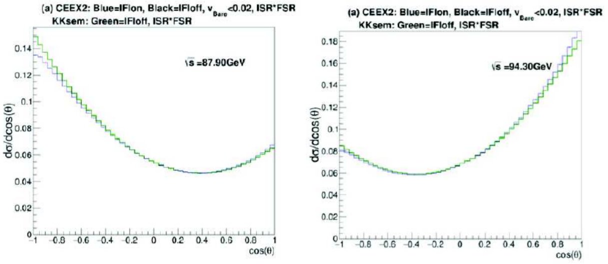


QED unfolding, IFI, slide by A.Freitas, Snowmass 2020, pdf

- Interference between ISR and FSR suppressed by Γ_Z/M_Z on Z resonance



- Still relevant for high precision an off-resonance



Jadach, Yost ’18

- Factorization from hard matrix element requires 4-variable convolution
- Soft-photon resummation can be included

Jadach, Yost ’18
Greco, Pancheri-Srivastava, Srivastava ’75

QED perturbative leading and subleading corrections, 1903.09895

$(a) \quad 0.5\%$

1				
$\alpha\mathsf{L}$	α			
$\alpha^2\mathsf{L}^2$	$\alpha^2\mathsf{L}$	α^2		
$\alpha^3\mathsf{L}^3$	$\alpha^3\mathsf{L}^2$	$\alpha^3\mathsf{L}$	α^3	
$\alpha^4\mathsf{L}^4$	$\alpha^4\mathsf{L}^3$	$\alpha^4\mathsf{L}^2$	$\alpha^4\mathsf{L}^1$	α^4
$\vdots$	$\vdots$	$\vdots$		

$(b) \quad 0.02\%$

1				
$\alpha\mathsf{L}$	α			
$\alpha^2\mathsf{L}^2$	$\alpha^2\mathsf{L}$	α^2		
$\alpha^3\mathsf{L}^3$	$\alpha^3\mathsf{L}^2$	$\alpha^3\mathsf{L}$	α^3	
$\alpha^4\mathsf{L}^4$	$\alpha^4\mathsf{L}^3$	$\alpha^4\mathsf{L}^2$	$\alpha^4\mathsf{L}^1$	α^4
$\vdots$	$\vdots$	$\vdots$		

$(c) \quad 0.001\%$

1				
$\alpha\mathsf{L}$	α			
$\alpha^2\mathsf{L}^2$	$\alpha^2\mathsf{L}$	α^2		
$\alpha^3\mathsf{L}^3$	$\alpha^3\mathsf{L}^2$	$\alpha^3\mathsf{L}$	α^3	
$\alpha^4\mathsf{L}^4$	$\alpha^4\mathsf{L}^3$	$\alpha^4\mathsf{L}^2$	$\alpha^4\mathsf{L}^1$	α^4
$\alpha^5\mathsf{L}^5$	$\vdots$	$\vdots$	$\vdots$	

ISR ($e^{\pm}$) and FSR ($\mu^{\pm}$) at the Z peak

$$\alpha \equiv \alpha_{QED}$$

$$L \equiv L_f = \ln(s/m_f^2),\ \ f = e, \mu$$

QED, LEP/FCC-ee, 1903.09895

Observable	Source LEP	Err.{QED} LEP	Stat[Syst] FCC-ee	$\frac{\text{LEP}}{\text{FCC-ee}}$	main development to be done
M_Z [MeV]	Z linesh.	$2.1\{0.3\}$	$0.005[0.1]$	$3{\times}3^{\star}$	light fermion pairs
Γ_Z [MeV]	Z linesh.	$2.1\{0.2\}$	$0.008[0.1]$	$2{\times}3^{\star}$	fermion pairs
$R_l^Z \times 10^3$	$\sigma(M_Z)$	$25\{12\}$	$0.06[1.0]$	$12{\times}3^{\star\star}$	better FSR
$\sigma^0_{\rm had}$ [pb]	$\sigma^0_{\rm had}$	$37\{25\}$	$0.1[4.0]$	$6{\times}3^{\star}$	better lumi MC
$N_\nu \times 10^3$	$\sigma(M_Z)$	$8\{6\}$	$0.005[1.0]$	$6{\times}3^{\star}$	CEEX in lumi MC
$N_\nu \times 10^3$	$Z\gamma$	$150\{60\}$	$0.8[<1]$	$60{\times}3^{\star\star}$	$\mathcal{O}(\alpha^2)$ for $Z\gamma$
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$	$53\{28\}$	$0.3[0.5]$	$55{\times}3^{\star\star}$	h.o. and EWPOs
$\sin^2\theta_W^{eff} \times 10^5$	$\langle\mathcal{P}_\tau\rangle, A_{\rm FB}^{pol,\tau}$	$41\{12\}$	$0.6[<0.6]$	$20{\times}3^{\star\star}$	better τ decay MC
M_W [MeV]	mass rec.	$33\{6\}$	$0.5[0.3]$	$12{\times}3^{\star\star\star}$	QED at threshold
$A_{FB,\mu}^{M_Z\pm3.5\rm GeV} \times 10^5$	$\frac{d\sigma}{d\cos\theta}$	$2000\{100\}$	$1.0[0.3]$	$100{\times}3^{\star\star\star}$	improved IFI

Rating from * to *** marks whether the needed improvement is relatively straightforward, difficult or very difficult to achieve.

EWPOs - refers to $|M|^2$; EWPPs - refers to M

Beyond Born level, one can write

$$\begin{aligned}
\mathcal{M}_\gamma^{(0)}(e^-e^+ \to f^-f^+) &= \frac{4\pi i\alpha_{em}(s)}{s}\, Q_e Q_f\, \gamma_\alpha \otimes \gamma^\alpha, \\
\mathcal{M}_Z^{(0)}(e^-e^+ \to f^-f^+) &= 4ie^2\frac{\chi_Z(s)}{s}\big[M_{vv}^{ef}\,\gamma_\alpha \otimes \gamma^\alpha - M_{av}^{ef}\,\gamma_\alpha\gamma_5 \otimes \gamma^\alpha \\
&\quad - M_{va}^{ef}\,\gamma_\alpha \times \gamma^\alpha\gamma_5 + M_{aa}^{ef}\,\gamma_\alpha\gamma_5 \otimes \gamma^\alpha\gamma_5\big].
\end{aligned}$$

In the pole scheme, where $\bar{M}_Z$ is defined as the real part of the pole of the S matrix, one has

$$\chi_Z(s) = \frac{G_F M_Z^2}{\sqrt{2}\; 8\pi\alpha_{em}}\; K_Z(s) \simeq \frac{1}{1 + i\frac{\bar{\Gamma}_Z}{\bar{M}_Z}}\,\frac{s}{s - \bar{M}_Z^2 + i\bar{M}_Z\bar{\Gamma}_Z} \simeq \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z(s)},$$

$$\Gamma_Z(s) = \frac{s}{M_Z^2}\;\Gamma_Z$$

EWPOs - refers to $|M|^2$; EWPPs - refers to M

Definitions are related:

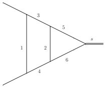
$$\bar{M}_Z \approx M_Z - \frac{1}{2}\frac{\Gamma_Z^2}{M_Z} \approx M_Z - 34 \text{ MeV},$$
$$\bar{\Gamma}_Z \approx \Gamma_Z - \frac{1}{2}\frac{\Gamma_Z^3}{M_Z^2} \approx \Gamma_Z - 0.9 \text{ MeV}.$$

- Known from LEP. One of examples why changing frameworks/assumptions/simplifications of calculations matter (!).
- However, at FCC-ee $\delta\Gamma_Z \sim 0.1$ MeV. Non-facotrization effects must be added properly beyond 1-loop.
- Is it necessary for FCC-ee accuracy to implement MC with radiative corrections calculated at the amplitudes level?
- At this precision it is important which parameters are taken as input parameters in schemes.

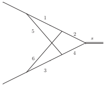
EWPOs and Form Factors

$$V_\mu^{Zb\bar{b}} = \gamma_\mu[v_b(s) + a_b(s)\gamma_5] = \cdots + \overbrace{\underbrace{\quad D_1 \quad + \quad D_2 \quad}_{\text{planar,non}-\text{planar}}}^{\text{fermionic,bosonic}} + \cdots$$

fermionic,bosonic



$+$



planar,non−planar

Note approximate factorization of weak couplings

$$A_{FB} = \frac{\left[\int_0^1 dcos\theta - \int_{-1}^0 dcos\theta\right] \frac{d\sigma}{dcos\theta}}{\sigma_T} \sim \overbrace{\frac{2a_e v_e}{a_e^2 + v_e^2}}^{A_e} \overbrace{\frac{2a_f v_f}{a_f^2 + v_f^2}}^{A_f} + \text{corrections}$$

$$A_f \quad = \quad \frac{2\Re e\frac{v_f}{a_f}}{1 + \left(\Re e\frac{v_f}{a_f}\right)^2} = \frac{1 - 4|Q_f|\sin^2\theta_{\text{eff}}^{\text{f}}}{1 - 4|Q_f|\sin^2\theta_{\text{eff}}^{\text{f}} + 8(Q_f\sin^2\theta_{\text{eff}}^{\text{f}})^2}, \quad \sin^2\theta_{\text{eff}}^{\text{f}} = F\left(\Re e\frac{v_f}{a_f}\right)$$

EWPOs, Z pole

$$
\begin{aligned}
\sigma^0_{\text{had}} &= \sigma[e^+e^- \to \text{hadrons}]_{s=M_Z^2}, \\
\Gamma_Z &= \sum_f \Gamma[Z \to f\bar{f}], \\
R_\ell &= \frac{\Gamma[Z \to \text{hadrons}]}{\Gamma[Z \to \ell^+\ell^-]}, \quad \ell = e, \mu, \tau, \\
R_q &= \frac{\Gamma[Z \to q\bar{q}]}{\Gamma[Z \to \text{hadrons}]}, \quad q = u, d, s, c, b.
\end{aligned}
$$

The remaining EWPOs are cross section asymmetries, measured at the Z pole, e.g., forward-backward asymmetry

$$
A^f_{\text{FB}} = \frac{\sigma_f\left[\theta < \frac{\pi}{2}\right] - \sigma_f\left[\theta > \frac{\pi}{2}\right]}{\sigma_f\left[\theta < \frac{\pi}{2}\right] + \sigma_f\left[\theta > \frac{\pi}{2}\right]},
$$

where θ is the scattering angle between the incoming e^- and the outgoing f.

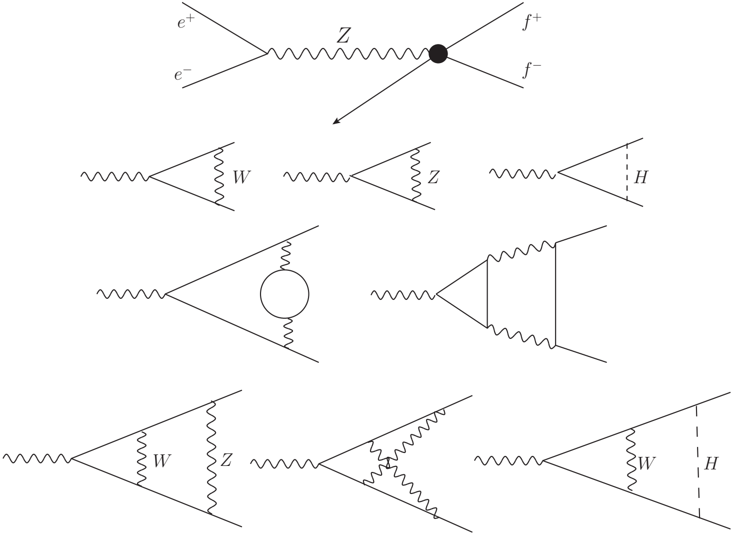
EW SM theory at loops, an example ($\Delta_{ef} \neq 0$)

$$\left\{\begin{array}{l} \Gamma_Z\,, \Gamma_{partial} \\ \\ A_{FB,peak}^{eff.,Born}\,, A_{LR,peak}^{eff.,Born} \\ \\ R_b, R_\ell, \ldots \end{array}\right. \longrightarrow \left\{\begin{array}{l} v_{\ell,\nu,u,d,b}^{eff} \\ \\ a_{\ell,\nu,u,d,b}^{eff} \\ \\ \sin^2\theta_{\rm eff}^{\rm b}, \sin^2\theta_{\rm eff}^{lept} \end{array}\right.$$

e.g. : improvements needed for subtle corrections $\Delta_{1,2}$ (e.g. boxes, **2L-boxes**)

$$\begin{aligned} A_{FB,peak}^{eff.,Born} &= \frac{2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\right]\, 2\Re e\left[\frac{v_f a_f^*}{|a_f|^2}\right]}{\left(1+\frac{|v_e|^2}{|a_e|^2}\right)\left(1+\frac{|v_f|^2}{|a_f|^2}\right)} + \Delta_1 - \Delta_2 \simeq \frac{3}{4} A_e A_f, \\ \Delta_1 &= 2\Re e\left[\Delta_{ef}\right],\ \Delta_2 = |\Delta_{ef}|^2 + 2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\frac{v_f a_f^*}{|a_f|^2}\Delta_{ef}^*\right], \\ \Delta_{ef} &= 16|Q_e Q_f| s_W^4 (\kappa_{ef} - \kappa_e\kappa_f) \end{aligned}$$

Rough scheme for extracting the $Zf\bar{f}$ vertex and EW corrections [loop corrections]



General remarks on usefulness of EWPOs

1. EWPOs encapsulate experimental data after extraction of well known and controllable QED and QCD effects, in a model-independent manner.
2. They provide a convenient bridge between real data and the predictions of the SM (or SM plus New Physics).
3. Contrary to raw experimental data (like differential crosssections), EWPOs are well suited for archiving and long term exploitation.
4. In particular archived EWPOscan be exploited over long periods of time for comparisons with steadily improving theoretical calculationsof the SM predictions, and for validations of the New Physics models beyond the SM.
5. They are also useful for comparison and combination of results from different experiments.

Complete NNLO results

- The standard model prediction for the effective weak mixing angle can be wri tten as

$$\sin^2\theta^{\rm b}_{\rm eff} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_{\rm b})$$

ions.

- The bosonic electroweak two-loop corrections amount to

$$\boxed{\Delta\kappa_{\rm b}^{(\alpha^2,bos)} = -0.9855 \times 10^{-4}}$$

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Collection of radiative corrections: Full stabilization at 10^{-4}!

$\pm 0.001 \xrightarrow{!}$

Order	Value $[10^{-4}]$	Order	Value $[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	-42.655	α_t^3	0.123
α^2_{ferm}	3.866	$\alpha_t\alpha_s^2$	-7.074
α^2_{bos}	**-0.9855**	$\alpha_t\alpha_s^3$	-1.196

Table: *Comparison of different orders of radiative corrections to* $\Delta\kappa_{\rm b}$.

Input Parameters: M_Z, Γ_Z, M_W, Γ_W, M_H, m_t, α_s *and* $\Delta\alpha$

- one-loop contributions [Akhundov, Bardin, Riemann, 1986] [Beenakker, Hollik, 1988]

Partial higher-order corrections

$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2)$ Avdeev: 1994,Chetyrkin: 1995

$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^3)$ Schroder: 2005,Chetyrkin: 2006,Boughezal: 2006

$\mathcal{O}(\alpha^2\alpha_{\rm t})$ and $\mathcal{O}(\alpha_{\rm t}^3)$ vanderBij: 2000,Faisst: 2003

The 2-loops EWPOs results for $\mathcal{O}(\alpha_{\rm bos}^2)$, https://doi.org/10.1016/j.physletb.2018.06.037

	$\Gamma_{\rm Z}$ [GeV]	$\sigma^0_{\rm had}$ [nb]
Born	2.53601	41.6171
$+\ \mathcal{O}(\alpha)$	2.49770	41.4687
$+\ \mathcal{O}(\alpha\alpha_{\rm s})$	2.49649	41.4758
$+\ \mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2,\ \alpha_{\rm t}\alpha_{\rm s}^3,\ \alpha_{\rm t}^2\alpha_{\rm s},\ \alpha_{\rm t}^3)$	2.49560	41.4770
$+\ \mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	2.49441	41.4883
$+\ \mathcal{O}(\alpha_{\rm bos}^2)$	**[+0.34 MeV]=**2.49475	**[+1.3 pb]=**41.4896

Results for $\Gamma_{\rm Z}$ and $\sigma^0_{\rm had}$, with $M_{\rm W}$ calculated from G_μ using the same order of perturbation theory as indicated in each line.

The 2-loops EWPOs results for $\mathcal{O}(\alpha_{\mathrm{bos}}^2)$, https://doi.org/10.1016/j.physletb.2018.06.037

	R_ℓ	R_c	R_b
Born	21.0272	0.17306	0.21733
$+\ \mathcal{O}(\alpha)$	20.8031	0.17230	0.21558
$+\ \mathcal{O}(\alpha\alpha_{\mathrm{s}})$	20.7963	0.17222	0.21593
$+\ \mathcal{O}(\alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^2,\ \alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^3,\ \alpha_{\mathrm{t}}^2\alpha_{\mathrm{s}},\ \alpha_{\mathrm{t}}^3)$	20.7943	0.17222	0.21593
$+\ \mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	20.7512	0.17223	0.21580
$+\ \mathcal{O}(\alpha_{\mathrm{bos}}^2)$	20.7516	0.17222	0.21585

Results for the ratios R_ℓ, R_c and R_b, with M_{W} calculat ed from G_μ to the same order as indicated in each line.

The 2-loops EWPOs results for $\mathcal{O}(\alpha_{\mathrm{bos}}^2)$, https://doi.org/10.1016/j.physletb.2018.06.037

Γ_i [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	Γ_d, Γ_s	Γ_u, Γ_c	Γ_b	Γ_Z
Born	81.142	160.096	371.141	292.445	369.56	2420.2
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_{\mathrm{s}})$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha_{\mathrm{bos}}^2)$	**0.017**	**0.019**	**0.058**	**0.057**	**0.167**	**0.505**
$\mathcal{O}(\alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^2,\ \alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^3,\ \alpha_{\mathrm{t}}^2\alpha_{\mathrm{s}},\ \alpha_{\mathrm{t}}^3)$	0.038	0.059	0.191	0.170	**0.190**	1.20

1. 2016, estimation, bosonic NNLO $\sim 0 \pm 0.1$ MeV
   2018, exact result: 0.505 MeV

* Fixed values of M_W

Currently most precise prediction for $\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}}$

$$
\begin{aligned}
\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}} =& s_0 + d_1\,L_H + d_2\,L_H^2 + d_3\,\Delta_\alpha + d_4\,\Delta_t + d_5\,\Delta_t^2 \\
&+ d_6\,\Delta_t\,L_H + d_7\,\Delta_{\alpha_{\mathrm{s}}} + d_8\,\Delta_t\,\Delta_{\alpha_{\mathrm{s}}} + d_9\,\Delta_Z
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
L_H &= \log\left(\frac{M_{\mathrm{H}}}{125.7\mathrm{GeV}}\right), \quad \Delta_t = \left(\frac{m_{\mathrm{t}}}{173.2\mathrm{GeV}}\right)^2 - 1, \quad \Delta_Z = \frac{M_{\mathrm{Z}}}{91.1876\mathrm{GeV}} - 1, \\
\Delta_\alpha &= \frac{\Delta\alpha}{0.0059} - 1, \quad \Delta_{\alpha_{\mathrm{s}}} = \frac{\alpha_{\mathrm{s}}}{0.1184} - 1.
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
s_0 &= 0.232704, \quad d_1 = 4.723\times10^{-4}, \quad d_2 = 1.97\times10^{-4}, \quad d_3 = 2.07\times10^{-2}, \\
d_4 &= -9.733\times10^{-4}, \quad d_5 = 3.93\times10^{-4}, \quad d_6 = -1.38\times10^{-4}, \\
d_7 &= 2.42\times10^{-4}, \quad d_8 = -8.10\times10^{-4}, \quad d_9 = -0.664.
\end{aligned} \tag{7}
$$

- M_{W} is calculated from the Fermi constant G_μ [Awramik, et al., 2004]
- The deviations to the full calculation amount to average (maximal) 2×10^{-7} (1.3×10^{-6}), in the input parameter ranges.

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Decreasing theoretical errors

Complicated subject, theoretical, parametric errors.
See:
1. "Report 1"
2. "Report 4"

Errors, a simple observation:

1. Lack of knowledge about HO corrections is a real pain, estimates even in the perturbative regime can differ substantially from concrete results.
2. Estimations for each next piece of HO take into account **AMOUNT** of the correction
3. Real calculation gives a **CONCRETE** number, with an error which is at least 2 digits.

These points are essential when we are at the level of accuracy which approaches experimental precision.

Updates for error estimations

- Theory error estimate is not well defined, ideally $\Delta_{\mathrm{th}} \ll \Delta_{\mathrm{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs (m_{t}, m_b, α_{s}, $\Delta\alpha_{\mathrm{had}}$, ...)

see, Ayres Freitas: 1604.00406

E.g.: Intrinsic theory error estimation for Γ_Z, 1804.10236 [1604.00406]

1. Geometric series

$$\delta_1 : \mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.20 \text{ MeV } [0.26 \text{ MeV}]$$

$$\delta_2 : \mathcal{O}(\alpha^2 \alpha_s) - \mathcal{O}(\alpha_t^2 \alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.21 \text{ MeV } [0.3 \text{ MeV}]$$

$$\delta_3 : \mathcal{O}(\alpha \alpha_s^2) - \mathcal{O}(\alpha_t \alpha_s^2) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.23 \text{ MeV}$$

$$\delta_4 : \mathcal{O}(\alpha \alpha_s^3) - \mathcal{O}(\alpha_t \alpha_s^3) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\delta_5 : \mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1} \text{ MeV } \underline{[\text{Now we know it!}]}$$

$$\text{Total: } \delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} \delta_i^2} \sim \mathbf{0.4} \ \ \textbf{MeV} \quad [0.5 \ \ \text{MeV}]$$

Accuracy of calculations

For 2-loops we maintained 4 digits for EWPOs.

A calculation of the radiative corrections $\delta_1 \div \delta_4$ and $\delta'_1 \div \delta'_3$ with a 10% accuracy (corresponding to two significant digits) should suffice to meet future experimental demands.

Minimal precision of 3-loop EW calculations, an example.

- Calculating N^3LO with 10% accuracy (two digits), we can replace intrinsic error estimation $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} \delta_i^2} \sim 0.4$ MeV by $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} (\delta_i/10)^2} \sim 0.04$ MeV.

- The requirement of FCC-ee$^{exper.\ error}(\Gamma_Z) \sim 0.1$ MeV can be met and the condition

$$\delta[\mathrm{FCCee}^{\mathrm{theor.}}(\Gamma_\mathrm{Z})] \sim 0.04 \text{ MeV} < \delta[\mathrm{FCCee}^{exper.}(\Gamma_Z)] \sim 0.1 \text{ MeV}$$

will be fullfilled.

Summary: estimations for higher order EW and QCD corrections

δ_1 :	δ_2 :	δ_3 :	δ_4 :	δ_5 :	$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(\alpha^3)$	$\mathcal{O}(\alpha^2\alpha_s)$	$\mathcal{O}(\alpha\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$	$\mathcal{O}(\alpha_{bos}^2)$	$=\sqrt{\sum_{i=1}^5 \delta_i^2}$
TH1 (estimated error limits from geometric series of perturbation)					
0.26	0.3	0.23	0.035	0.1	0.5
TH1-new (estimated error limits from geometric series of perturbation)					
0.2	0.21	0.23	0.035	$< 10^{-4}$	**0.4**

δ_1' :	δ_2' :	δ_3' :	δ_4 :		$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(N_f^{\leq 1}\alpha^3)$	$\mathcal{O}(\alpha^3\alpha_s)$	$\mathcal{O}(\alpha^2\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$		$\sqrt{\delta_1'^2+\delta_2'^2+\delta_2'^3+\delta_4^2}$
TH2 (extrapolation through prefactor scaling)					
0.04	0.1	0.1	0.035	10^{-4}	**0.15**

"Report 2"

More EWPOs, taken from ESPPU "Report 3"

	$\delta\Gamma_Z$ [MeV]	δR_l $[10^{-4}]$	δR_b $[10^{-5}]$	$\delta \sin^{2,l}_{eff}\theta$ $[10^{-6}]$
Present EWPO theoretical uncertainties				
EXP-2018	2.3	250	66	160
TH-2018	0.4	60	10	45
EWPO theoretical uncertainties when FCC-ee will start				
EXP-FCC-ee	0.1	10	$2 \div 6$	6
TH-FCC-ee	0.07	7	3	7

Table: *Comparison for selected precision observables of present experimental measurements (EXP-2018), current theory errors (TH-2018), FCC-ee precision goals at the end of the Tera-Z run (EXP-FCC-ee) and rough estimates of the theory errors assuming that electroweak 3-loop corrections and the dominant 4-loop EW-QCD corrections* $\mathcal{O}(\alpha\alpha_s^2), \mathcal{O}(N_f\alpha^2\alpha_s), \mathcal{O}(N_f^2\alpha^3)$ *are available at the start of FCC-ee (TH-FCC-ee). Based on discussion in 1809.01830.*

For more details, see Executive Summary and Chapter 2 in "Report 1".

EW corrections will become more important also at HL-LHC

HL-LHC will be tangible to EW physics, see this month "LHC EW Precision sub-group workshop", `https://indico.cern.ch/event/801961/`.
or

- Standard Model Physics at the HL-LHC and HE-LHC
  HL-LHC and HE-LHC Working Group, e-Print: arXiv:1902.04070

and ESPPU contribution `https://indico.cern.ch/event/765096`, e.g.:

100. Precision calculations for high-energy collider processes (Charalampos Anastasiou, Stefan Dittmaier, Thomas Gehrmann, Nigel Glover, Massimiliano Grazzini, Michelangelo Mangano, Stefano Pozzorini, Gavin Salam, Giulia Zanderighi)

Input and renormalization schemes

- E.g. the bosonic 2-loop corrections shift the value of Γ_Z by 0.51 MeV when using M_W as input and 0.34 MeV when using G_μ as input. $\delta\Gamma_{FCC-ee} = 0.1$ MeV (Dubovyk et al, http://arxiv.org/pdf/1804.10236.pdf)
- In general, there are many different approaches. Which measured parameters to choose as an independent input parameters? E.g. recently Piccinini et al, Durham talk

  https://indico.cern.ch/event/801961/contributions/3361495/attachments/1823019/2982558/piccinini.pdf

  are proposing to take for LHC $(\alpha/G_\mu, \sin^2\theta^f_{\text{eff}}, M_Z)$

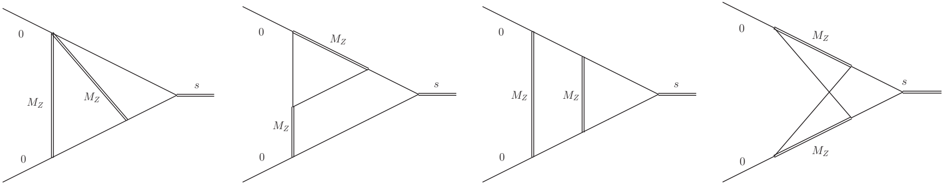
  $\sin^2\theta^f_{\text{eff}}$ fixed at measured leptonic $\sin^2\theta^f_{\text{eff}}$ requiring v_l/a_l does not get radiative corrections. Procedure independent of QED corrections (both couplings get the same QED corrections and we have a ratio).

Mellin-Barnes and Sector Decomposition methodsare very much complementary!

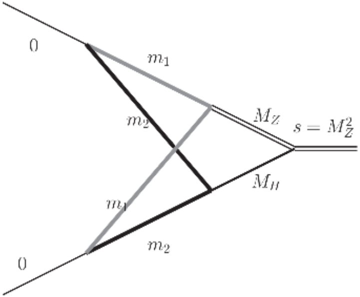
- MB works well for hard threshold, on-shell cases, not many internal masses (more IR); SD more useful for integrals with many internal masses

10^{-8} accuracy achieved for **any** self-energy and vertex Feynman integral with one of the methods - in **Minkowskian region**.

Available for several years!



2-loops $\longrightarrow$ 3-loops



$$m_1 = M_t, m_2 = M_W$$

The integrals contain up to three dimensionless parameters

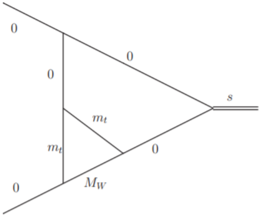
$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

Towards 3-loop results (Report "1")

$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	$14 \to^{(\mathbf{A})} 7 \to^{(\mathbf{B})}$ **5**	$211 \to^{(\mathbf{A})} 84 \to^{(\mathbf{B})}$ **50**
Number of diagrams	15	$2383 \to^{(\mathbf{A},\mathbf{B})}$ **1114**	$490387 \to^{(\mathbf{A},\mathbf{B})}$ **120187**
Fermionic loops	0	150	**17580**
Bosonic loops	15	**964**	102607
Planar diagrams	1T/15D	4T/981D	35T/84059D
Non-planar diagrams	0	1T/133D	15T/36128D

Some statistical overview for $Z \to b\bar{b}$ multiloop studies. At 3 loops there are in total almost half a million of diagrams present. After basic refinements (A) and (B) about 10^5 genuine 3-loop vertex diagrams remain. In (A) tadpoles and products of lower loops are excluded, in (B) symmetries of topologies are taken into account.

soft7 ϵ^0:[MB - 3 dim] [SD - 5 dim], ϵ^{-1}:[MB - 2 dim] [SD - 4 dim], ϵ^{-2}:[MB - 1 dim] [SD - 3 dim]



MB	$0.06026648655769\boldsymbol{9}\,\epsilon^{-2}$	
SD - 90 Mio	$0.060266486\boldsymbol{5}\,\epsilon^{-2}$	
MB	$(-0.031512489\boldsymbol{0}3$	$+0.189332751\boldsymbol{4}2i)\,\epsilon^{-1}$
SD - 90 Mio	$(-0.03151248\boldsymbol{1}6$	$+0.18933271\boldsymbol{6}96i)\,\epsilon^{-1}$
MB 1	$(-0.228231867\boldsymbol{5}1$	$-0.0882479456\boldsymbol{9}1i) + \mathcal{O}(\epsilon)$
MB 2	$(-0.228231867\boldsymbol{5}1$	$-0.0882479457\boldsymbol{3}9i) + \mathcal{O}(\epsilon)$
SD - 90 Mio	$(-0.2282265\boldsymbol{5}3$	$-0.088245\boldsymbol{9}6i) + \mathcal{O}(\epsilon)$
SD - 15 Mio	$(-0.2281\boldsymbol{6}2$	$-0.0882\boldsymbol{0}9i) + \mathcal{O}(\epsilon)$

1. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics: besides top Yukawa y_t and Higgs self-coupling λ

α , G_μ, M_Z **most precise input parameters** $\Rightarrow$ **precision predictions**

50% non-perturbative $\qquad\qquad$ $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$\begin{array}{lcccll}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} & \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} & \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} & \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 0.9 \div 1.6 & \times & 10^{-4} & (\mathrm{present:\ lost\ }10^5\ \mathrm{in\ precision!}) \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 5.3 & \times & 10^{-5} & (\mathtt{FCC-ee/ILC\ requirement})
\end{array}$$

LEP/SLD: $\sin^2\Theta_{\mathrm{eff}} = (1 - v_l/a_l)/4 = 0.23148 \pm 0.00017$

$\delta\Delta\alpha(M_Z) = 0.00020 \quad \Rightarrow \quad \delta\sin^2\Theta_{\mathrm{eff}} = 0.00007$; $\delta M_W/M_W \sim 4.3 \times 10^{-5}$

affects most precision tests and new physics searches!!!

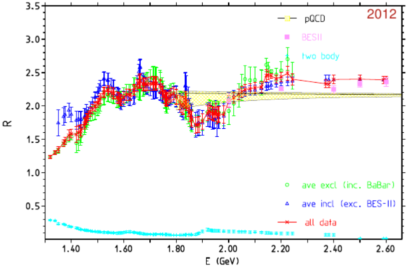
$$\frac{\delta M_W}{M_W} \sim 1.5 \times 10^{-4}, \quad \frac{\delta M_H}{M_H} \sim 1.3 \times 10^{-3}, \quad \frac{\delta M_t}{M_t} \sim 2.3 \times 10^{-3}$$

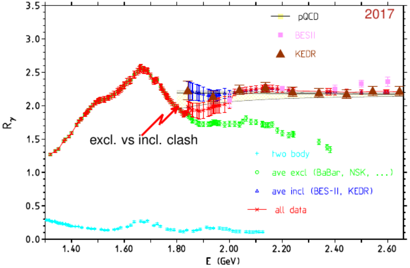
For pQCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t \Rightarrow$ **Lattice-QCD**

SM precision parameters determination: $\alpha(M_Z^2)$

Still an issue in HVP

- region 1.2 to 2 GeV data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty





- illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

Three approaches should be further explored for better error estimate

Note: theory-driven standard analyses ($R(s)$ integral) using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

precision in α:	present	direct	1.7×10^{-4}
		Adler	1.2×10^{-4}
	future	Adler QCD 0.2%	5.4×10^{-5}
		Adler QCD 0.1%	3.9×10^{-5}
	future	via $A_{\rm FB}^{\mu\mu}$ off Z	3×10^{-5}

- Adler function method is competitive with Patrick Janot's direct near Z pole determination via forward backward asymmetry in $e^+e^- \to \mu^+\mu^-$

$$A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3\,a^2}{4\,v^2}\frac{\mathcal{I}}{\mathcal{Z}+\mathcal{G}}$$

where

$\gamma - Z$ interference term	$\mathcal{I} \propto \alpha(s)\,G_\mu$
Z alone	$\mathcal{Z} \propto G_\mu^2$
γ only	$\mathcal{G} \propto \alpha^2(s)$
v vector Z coupling	also depends on $\alpha(s \sim M_Z^2)$ and $\sin^2\Theta_f(s \sim M_Z^2)$
a axial Z coupling	sensitive to ρ-parameter (strong M_t dependence)

❏ using v, a as measured at Z-peak

$e^+e^- \to \mu^+\mu^-$ and $\alpha^2(s)$

$\sigma_{\mu\mu}$:

1. the photon-exchange term, $\mathcal{G}$, proportional to $\alpha^2(s)$;
2. the Z-exchange term, $\mathcal{Z}$, proportional to G_F^2 (where G_F is the Fermi constant);
3. the Z-photon interference term, $\mathcal{I}$, proportional to $\alpha(s) \times G_F$

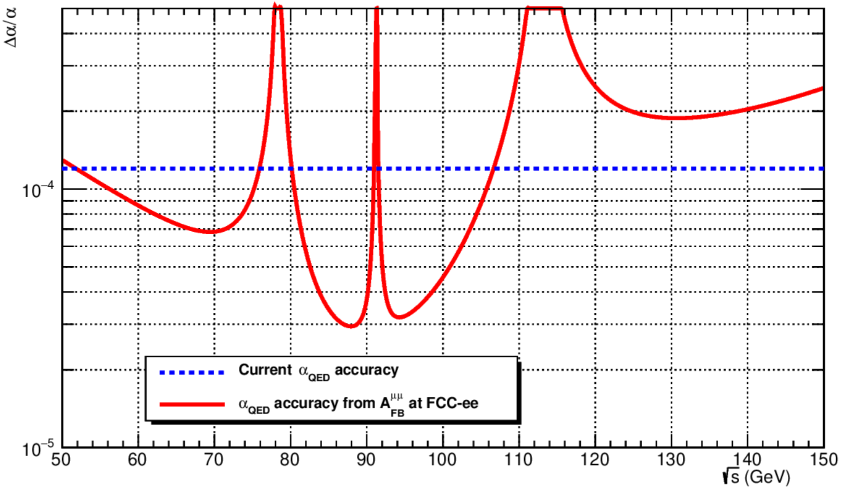
The muon forward-backward asymmetry, $A_{\mathrm{FB}}^{\mu\mu}$, is maximally dependent on the interference term

$$A_{\mathrm{FB}}^{\mu\mu} = A_{\mathrm{FB},0}^{\mu\mu} + \frac{3}{4}\frac{2}{2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}},$$

varies with $\alpha_{\mathrm{QED}}(s)$ as follows:

$$\Delta A_{\mathrm{FB}}^{\mu\mu} = \left(A_{\mathrm{FB}}^{\mu\mu} - A_{\mathrm{FB},0}^{\mu\mu}\right) \times \frac{\mathcal{Z}-\mathcal{G}}{\mathcal{Z}+\mathcal{G}} \times \frac{\Delta\alpha}{\alpha}.$$

$e^+e^- \to \mu^+\mu^-$ and $\alpha^2(s)$



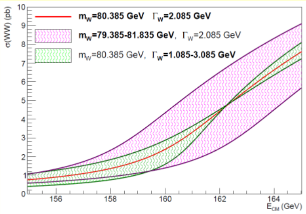
The best accuracy is obtained for one year of running either just below or just above the Z pole, at 87.9 and 94.3 GeV, respectively.

W-mass, slide by A.Freitas, Snowmass 2020, pdf

- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold
- a) Corrections near threshold enhanced by $1/\beta$ and $\ln\beta$

  $$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$

  b) Non-resonant contributions are important

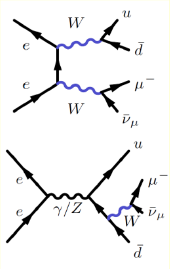


- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$

  Denner, Dittmaier, Roth, Wieders '05
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$

  Beneke, Falgari, Schwinn, Signer, Zanderighi '07
  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{\rm th}M_W \sim 3$ MeV

    Actis, Beneke, Falgari, Schwinn '08
  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{\rm th}M_W \lesssim 0.6$ MeV



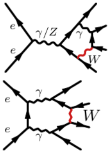
SM W-physics, FCC-ee-W, 11th FCC-ee workshop 2019, pdf

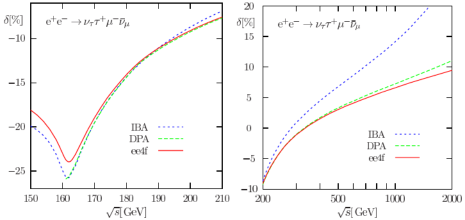
Full NLO calculation for $e^+e^- \to 4f$ (Denner, Dittmaier, Roth, Wieders 05)

- More than 1000 1-loop diagrams, 5, 6-point loop integrals

⇒ pioneering methods for six-point diagrams
now automated for LHC: `RECOLA`, `OpenLoops`, `MadLoops`

- complex mass scheme for W decay width
- fully differential calculation
- not easy to incorporate higher-order effects
- DPA not sufficient at threshold and for $\sqrt{s} > 500$ GeV





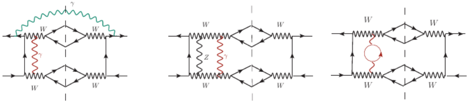
SM W-physics, FCC-ee-W

EFT expansion in $\alpha \sim \frac{\Gamma_W}{M_W} \sim \beta^2$ (Beneke/Falgari/CS/Signer/Zanderighi 07)

- systematically possible to include higher-order corrections
- limited to total cross section near threshold

Leading NNLO corrections

- 2nd Coulomb correction $\sim \alpha^2/\beta^2 \sim \alpha$ (Fadin et al. 95)
- Coulomb-enhanced corrections $\sim \alpha^2/\beta \sim \alpha^{3/2}$ (Actis et al. 08)



- Numerical effect: $\Delta\sigma_{WW} \sim 5‰$; $[\delta M_W] \lesssim 3\,\mathrm{MeV}$

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d})$(fb)			
$\sqrt{s}$ [GeV]	$\mathrm{NLO}_{\mathrm{EFT}}$	$\mathrm{NLO}_{\mathrm{ee4f}}$ [DDRW]	$\Delta_{\mathrm{NNLO}}(\alpha^2/\beta^2)$	$\Delta_{\mathrm{NNLO}}(\alpha^2/\beta)$
161	117.5	118.77	0.44 (3.7‰)	0.15 (1.3‰)
170	397.8	404.5	0.25 (0.6‰)	1.6 (3.9‰)

SM W-physics, FCC-ee-W

Future improvements of theory predictions?

Implementation of state-of-the art calculations in public tools?

- NLO-EW $e^-e^+ \to 4f$ now possible with standard tools
  (`RECOLA`, `OpenLoops`, `MadLoops + SHERPA`, `MadGraph`, `WHIZARD...`)
  but not (yet) optimized for e^-e^+ (ISR, Beamstrahlung)
- Two-loop Coulomb-enhanced corrections for differential observables doable; (related: $t\bar{t}$ with Coulomb resummation in WHIZARD)
  (no guarantee of formal accuracy for general distributions)

Full NNLO in EFT for total cross section

- Soft $\log\beta$ terms can be adapted from QCD results
- NNLO $\log(m_e/M_W)$ terms doable (c.f. Bhabha scattering)
- two-loop hard non-logarithmic corrections
  (from amplitudes for $e^+e^- \to W^+W^-$ at threshold: border of current capabilities)
  resulting uncertainty from cross-section calculation

$$\Delta\sigma^{(2}_{\text{hard}} = \left(\frac{\alpha}{2\pi}\right)^2 c^{(2)}\sigma^{(0)} \sim (1-2)\text{‰ for estimate } c^{(2)} = (c^{(1)})^2$$

Full NNLO for $e^+e^- \to 4f$: completely new methods needed

SM W-physics, M. Skrzypek, FCC-ee-W: hybrid approach, 11th FCC-ee workshop 2019, pdf

Conclusions and outlook

- KoralW+YFSWW3: LEP2 precision is 0.5%.
  Factor of 20 $\div$ 50 improvement is needed for FCCee
- Lesson from LEP2: be pragmatic, split into Double- and Single-Pole, pick only numerically dominant terms:
  - $\mathcal{O}(\alpha^1)$ for $e^- e^+ \to 4f$ must be implemented in MC with explicit split into Double Pole and Single Pole. Calculations exist
  - $\mathcal{O}(\alpha^2)_{DP}$ calculations for the Double-Pole production and decay parts are needed! Feasible?
  - $\mathcal{O}(\alpha^2)_{SP}$ and $\mathcal{O}(\alpha^3)$ seem to be negligible
- More detailed analysis at the threshold may be instrumental
  - EFT methods promising, but for now inclusive results only
  - Non-factorizable soft interferences can be exponentiated within YFS scheme. How much of the higher order corrs. would be reproduced this way?

The overall precision tag $\sim 2 \times 10^{-4}$ feasible (?)

YFSWW3$\oplus$KoralW with new exponentiation
look like a good starting point

QED unfolding, ISR, slide by A.Freitas, Snowmass 2020, pdf

- M_Z, Γ_Z: From $\sigma(\sqrt{s})$ lineshape
  $\longrightarrow$ Main uncertainties: B-field calibration, QED
  $\longrightarrow$ δM_Z, $\delta\Gamma_Z \sim 0.1$ MeV could be achievable

- m_t: Current status $\delta m_t \sim 0.4$ GeV at LHC — PDG '18
  $\longrightarrow$ Additional theory uncertainties? — Butenschoen et al. '16; Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:	**future:**	
$\delta m_t^{\overline{\mathrm{MS}}} = [\quad]_{\mathrm{exp}}$	$[20\ \mathrm{MeV}]_{\mathrm{exp}}$	
$\oplus\ [50\ \mathrm{MeV}]_{\mathrm{QCD}}$	$\oplus\ [30\ \mathrm{MeV}]_{\mathrm{QCD}}$	(h.o. resummation)
$\oplus\ [10\ \mathrm{MeV}]_{\mathrm{mass\ def.}}$	$\oplus\ [10\ \mathrm{MeV}]_{\mathrm{mass\ def.}}$	
$\oplus\ [70\ \mathrm{MeV}]_{\alpha_s}$	$\oplus\ [15\ \mathrm{MeV}]_{\alpha_s}$	($\delta\alpha_s \lesssim 0.0002$)
> 100 MeV	$\lesssim 50$ MeV	

Conclusions

- Top pair threshold scan allows precise mass determination

$$\Delta m_t < 100\,\text{MeV}$$

- Theory-dominated error, $\sim 3\%$ QCD scale uncertainty
- Known corrections:
  - N^3LO QCD + Higgs
  - N^2LO electroweak + non-resonant
  - LL initial state radiation
- All corrections included in version 2 of `QQbar_threshold`
  `https://qqbarthreshold.hepforge.org/`

MC Top Quark Mass Parameter

Why is there a non-trivial issue in the interpretation of $\mathbf{m_t^{MC}}$?

- picture of "top quark particle" does not apply (non-zero color charge)
- m_t is a scheme-dependent parameter of a perturbative computation
  $\rightarrow$ in which scheme do MC event generators calculate?
- relation of m_t^{MC} to any field theory mass definition can be affected by different contributions (let's consider pole mass just for convention)

$$m_t^{\mathrm{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\mathrm{MC}}$$

pQCD contribution:

- perturbative corrections
- depends on MC parton shower setup

this talk

non-perturbative contribution:

- effects of hadronization model
- may depend on parton shower setup

Monte Carlo shift:

- contribution arising from systematic MC uncertainties
- e.g. color reconstruction, b-jet modelling, finite width,...
- should be covered by "MC uncertainty" or better negligible