The status of the SANC project for polarized electron-positron beams

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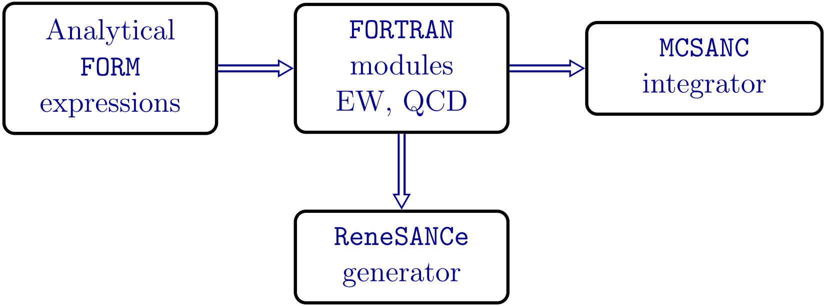
Outline

- Review of SANC products for e^+e^-
- Results for implemented processes at NLO EW
- Higher order improvements

The SANC/ARIeL framework and products family

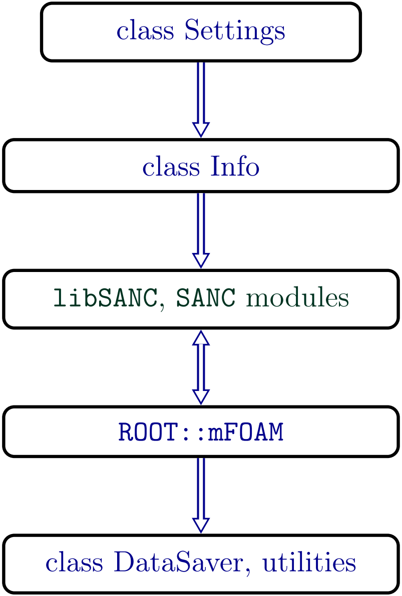


Publications:

SANC – Comput.Phys.Commun. 174 (2006), 481-517.
MCSANC (pp-mode) – Comput.Phys.Commun. 184 (2013), 2343-2350; JETP Letters 103 (2016), 131-136.
ReneSANCe – Comput.Phys.Commun. 256 (2020), 107445.

SANC products are available at http://sanc.jinr.ru/download.php.

Scheme of ReneSANCe event generator structure



read/validate preferences

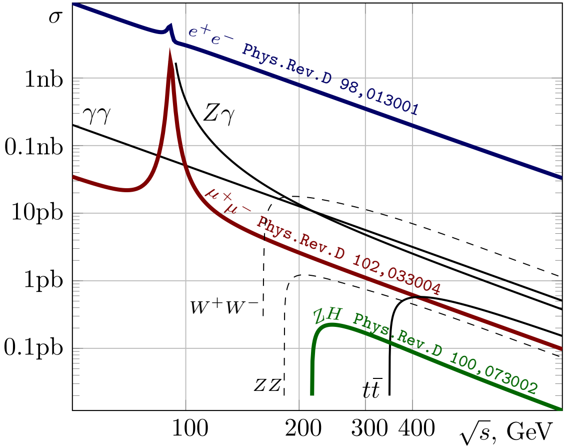
calculate derived parameters, store results

calculate matrix elements

grid construction & phase-space sampling

write events: root, LHEF, ... & analyse

Basic processes of SM for e^+e^- annihilation



The cross sections are given for polar angles between $10^o < \theta < 170^o$ in the final state.

Implemented processes

In the current version of the generator and the integrator for polarized e^+e^- physics implemented:

- Bhabha ($e^+e^- \to e^-e^+$), Phys. Rev. D 98 (2018), 013001.
- ZH ($e^+e^- \to ZH$), Phys. Rev. D 100 (2019), 073002.
- s-channel ($e^+e^- \to \mu^-\mu^+$, $e^+e^- \to \tau^-\tau^+$), Phys. Rev. D 102 (2020), 033004.

Based on the SANC (Support for Analytic and Numeric Calculations for experiments at colliders) modules the SANC tools take into account complete one-loop and some higher-order electroweak radiative corrections (RC), all the particle masses and polarizations.

Cross section structure

The cross section of processes at one-loop can be divided into:

$$\sigma^{\text{1-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

Contributions due to:
σ^{Born} — Born level cross section,
σ^{virt} — virtual(loop) corrections,
σ^{soft} — soft photon Bremsstrahlung,
σ^{hard} — hard photon Bremsstrahlung (with energy $E_\gamma > \omega$).
Auxiliary parameters λ ("photon mass") and ω cancel out after summation.

$$\delta = (\sigma^{\text{1-loop}} / \sigma^{\text{Born}} - 1) \cdot 100\%.$$

We use the helicity approach for all contributions.

Results for $e^+e^- \to ZH(\gamma)$

P_{e^-}, P_{e^+}	**0, 0**	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	**225.59(1)**	266.05(1)	127.42(1)	404.69(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	**206.77(1)**	223.33(2)	111.67(2)	334.99(1)
δ, %	**-8.3(1)**%	-16.1(1)%	-12.4(1)%	-17.2(1)%
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	**53.74(1)**	63.38(1)	30.35(1)	96.40(1)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	**62.42(1)**	68.31(1)	34.04(1)	102.58(1)
δ, %	**16.7(1)**%	7.8(1)%	12.1(1)%	6.4(1)%
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	**12.05(1)**	14.217(1)	6.809(1)	21.62(1)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	**14.56(1)**	15.80(1)	7.95(1)	23.66(1)
δ, %	**20.8(1)**%	11.1(1)%	16.7(1)%	9.4(1)%

Planned improvements for Higgs-strahlung

- Implementation of the final particle decay.
- Calculation of complete NLO EW matrix elements for $e^+e^- \to Hf\bar{f}$ process.

Results for $e^+e^- \to e^+e^-(\gamma)$ (Bhabha)

P_{e^-}, P_{e^+}	**0, 0**	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	**56.6763(1)**	57.7738(1)	56.2725(4)	59.2753(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	**61.731(6)**	62.587(6)	61.878(6)	63.287(7)
δ, %	**8.92(1)%**	8.33(1)%	9.96(1)%	6.77(1)%
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	**14.3789(1)**	15.0305(1)	12.7061(1)	17.3550(2)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	**15.465(2)**	15.870(2)	13.861(1)	17.884(2)
δ, %	**7.56(1)%**	5.59(1)%	9.09(1)%	3.05(1)%
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	**3.67921(1)**	3.90568(1)	3.03577(3)	4.77562(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	**3.8637(4)**	3.9445(4)	3.2332(3)	4.6542(7)
δ, %	**5.02(1)%**	0.99(1)%	6.50(1)%	-2.54(1)%

cuts are: $|\cos\theta_{e^-}| < 0.9, \quad |\cos\theta_{e^+}| < 0.9.$

Planned 2-loop improvements

Implementation of the 2-loop (NNLO) QED results on RC to Bhabha and annihilation cross sections and comparison with existing works:

- J. Blümlein et al., "The $O(\alpha^2)$ initial state QED corrections to $e^+e^- \to \gamma^*/Z_0^*$", Nucl. Phys. B 956 (2020), 115055
- F. Campanario et al., "Complete QED NLO contributions to the reaction $e^+e^- \to \mu^+\mu^-\gamma$ ...", JHEP 02 (2014), 114
- C.M. Carloni Calame et al., "NNLO massive corrections to Bhabha scattering and theoretical precision of BabaYaga@NLO", Nucl. Phys. B Proc. Suppl. 225-227 (2012), 293
- C. Carloni Calame et al., "NNLO leptonic and hadronic corrections to Bhabha scattering ...", JHEP 07 (2011), 126
- S. Actis et al., "Virtual hadronic and leptonic contributions to Bhabha scattering", Phys. Rev. Lett. 100 (2008), 131602
- A. Penin and G. Ryan, "Two-loop electroweak corrections to high energy large-angle Bhabha scattering", JHEP 11 (2011), 081

$e^+e^- \to l^+l^-(\gamma)$: HA for Born and Virtual parts

$$\mathcal{H}_{-++-} = -e^2 c_+ \Big(Q_e Q_l \mathcal{F}_\gamma(s) + \chi_Z^s \delta_e \Big[\beta^- I_l^{(3)} \mathcal{F}_{QL} + \delta_l \mathcal{F}_{QQ} \Big] \Big),$$

$$\mathcal{H}_{-+\pm\pm} = e^2 \frac{2m_l}{\sqrt{s}} \sin\theta \Big(Q_e Q_l \mathcal{F}_\gamma(s) + \chi_Z^s \delta_e \Big[I_l^{(3)} \mathcal{F}_{QL} + \delta_l \mathcal{F}_{QQ} + \frac{s}{2} \beta_l^2 I_l^{(3)} \mathcal{F}_{\mathbf{QD}} \Big] \Big),$$

$$\mathcal{H}_{+-\pm\pm} = -e^2 \frac{2m_l}{\sqrt{s}} \sin\theta \Big(Q_e Q_l \mathcal{F}_\gamma(s) + \chi_Z^s \Big[2I_e^{(3)} I_l^{(3)} \mathcal{F}_{LL} + 2I_e^{(3)} \delta_l \mathcal{F}_{LQ} + \delta_e I_l^{(3)} \mathcal{F}_{QL} + \delta_e \delta_l \mathcal{F}_{QQ} + \frac{s}{2} \beta_l^2 I_l^{(3)} \Big(2I_e^{(3)} \mathcal{F}_{\mathbf{LD}} + \delta_e \mathcal{F}_{\mathbf{QD}} \Big) \Big] \Big),$$

$$\mathcal{H}_{+--+} = -e^2 c_+ \Big(Q_e Q_l \mathcal{F}_\gamma(s) + \chi_Z^s \Big[\beta^+ I_l^{(3)} \Big(2I_e^{(3)} \mathcal{F}_{LL} + \delta_e \mathcal{F}_{QL} \Big) + \delta_l \Big(2I_e^{(3)} \mathcal{F}_{LQ} + \delta_e \mathcal{F}_{QQ} \Big) \Big] \Big),$$

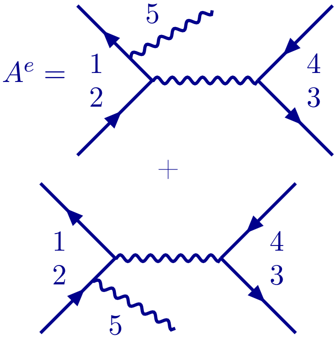
$$\mathcal{H}_{-+-+} = \mathcal{H}_{-++-}(c_+ \to c_-, \beta^- \to \beta^+),$$

$$\mathcal{H}_{+-+-} = \mathcal{H}_{+--+}(c_+ \to c_-, \beta^- \to \beta^+)$$

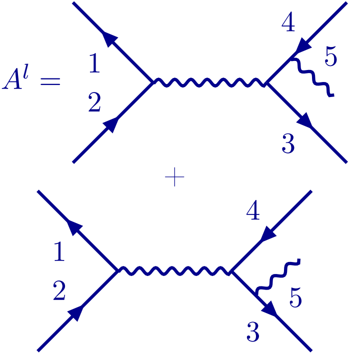
Gauge-invariant diagram sets

$\mathrm{A} = \mathrm{A}^e + \mathrm{A}^l$

ISR

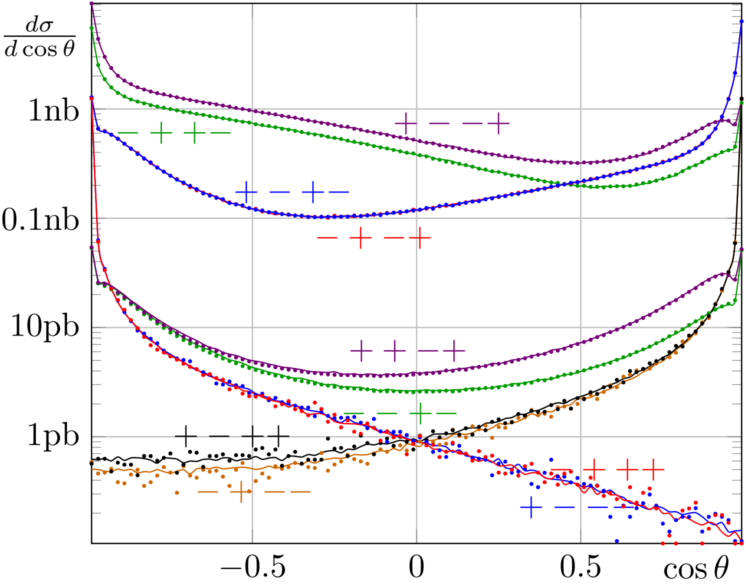


FSR

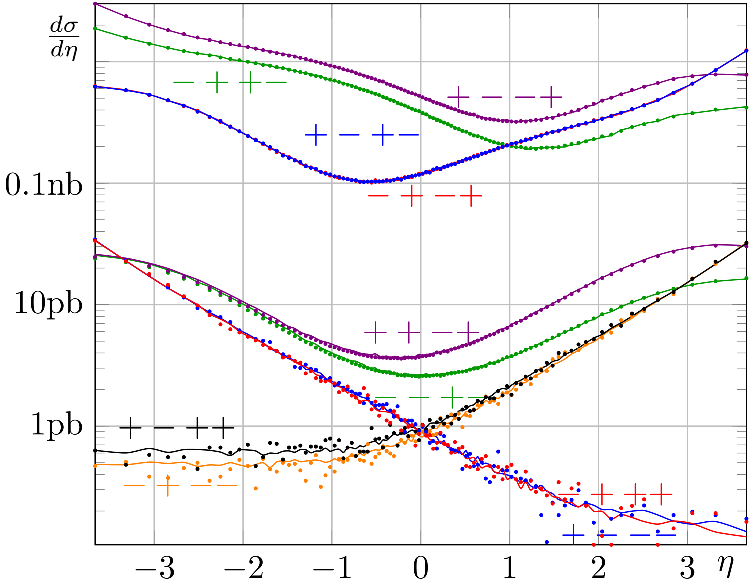


$$A^l{}_{\chi_1\chi_2\chi_3\chi_4\chi_5}(p_1, p_2, p_3, p_4, p_5) = A^e{}_{\chi_4\chi_3\chi_2\chi_1\chi_5}(p_4, p_3, p_2, p_1, p_5)|_{m_e \leftrightarrow m_l}.$$

SANC vs. WHIZARD (dots): all-polarized $e^+e^- \to \tau^+\tau^-\gamma$



SANC vs. WHIZARD (dots): all-polarized $e^+e^- \to \tau^+\tau^-\gamma$



$e^+e^- \to \mu^+\mu^-(\gamma)$: NLO EW results, no cuts

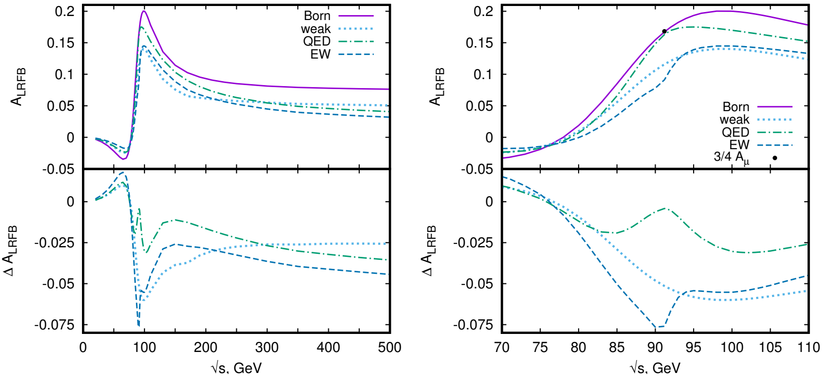
$\sqrt{s}$	P_{e^-}, P_{e^+}	**0, 0**	-0.8, 0	-0.8, 0.6	-0.8, -0.6
250 GeV	$\sigma^{\text{born}}_{\mu^+\mu^-}$, fb	**1653.7(1)**	1804.0(1)	2710.5(1)	897.5(1)
	$\sigma^{\text{ew}}_{\mu^+\mu^-}$, fb	**4526.3(2)**	4915.2(2)	7298.3(4)	2532.0(1)
	δ,%	**173.7(1)%**	172.4(1)%	169.3(1)%	182.1(1)%
500 GeV	$\sigma^{\text{born}}_{\mu^+\mu^-}$, fb	**400.85(1)**	433.51(1)	650.41(1)	216.61(1)
	$\sigma^{\text{ew}}_{\mu^+\mu^-}$, fb	**1138.9(1)**	1227.7(1)	1818.2(1)	637.2(1)
	δ,%	**184.1(1)%**	183.2(1)%	179.5(1)%	194.2(1)%
1000 GeV	$\sigma^{\text{born}}_{\mu^+\mu^-}$, fb	**99.57(1)**	107.47(1)	161.20(1)	53.75(1)
	$\sigma^{\text{ew}}_{\mu^+\mu^-}$, fb	**296.70(2)**	318.74(3)	471.61(4)	165.87(1)
	δ,%	**198.0(1)%**	196.6(1)%	192.6(1)%	208.6(1)%

$e^+e^- \to \mu^+\mu^-(\gamma)$: NLO EW results with cuts

$\sqrt{s}$	P_{e^-}, P_{e^+}	**0, 0**	-0.8, 0	-0.8, 0.6	-0.8, -0.6
250 GeV	$\sigma^{\text{born}}_{\mu^+\mu^-}$, fb	**1417.6(1)**	1546.5(1)	2323.5(1)	769.37(2)
	$\sigma^{\text{ew}}_{\mu^+\mu^-}$, fb	**2399(1)**	2614(1)	3909(1)	1318(1)
	δ,%	**69.2(1)**%	69.0(1)%	68.2(1) %	71.3(1)%
500 GeV	$\sigma^{\text{born}}_{\mu^+\mu^-}$, fb	**343.63(1)**	371.62(1)	557.56(1)	185.69(1)
	$\sigma^{\text{ew}}_{\mu^+\mu^-}$, fb	**469.8(4)**	495.4(5)	739.3(7)	251.5(2)
	δ,%	**36.7(1)**%	33.3(1) %	32.6(1) %	35.4(1)%
1000 GeV	$\sigma^{\text{born}}_{\mu^+\mu^-}$, fb	**85.355(3)**	92.131(5)	138.18(1)	46.079(2)
	$\sigma^{\text{ew}}_{\mu^+\mu^-}$, fb	**116.2(1)**	121.1(1)	180.3(1)	61.83(2)
	δ,%	**36.2(1)** %	31.4(1) %	30.5(1)%	34.2(1)%

cuts are: $|\cos\theta_{\mu^-}| < 0.9, \quad |\cos\theta_{\mu^+}| < 0.9.$

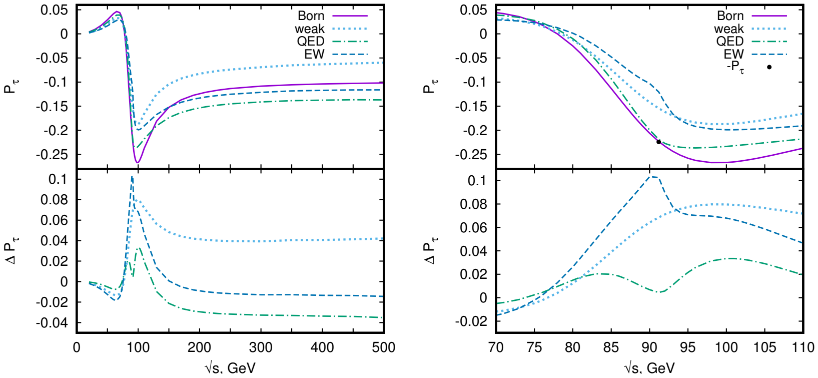
Left–Right Forward–Backward Asymmetry



(**Left**) The A_{LRFB} asymmetry in the Born and 1-loop (weak, QED, EW) approximations and ΔA_{LRFB} for c.m.s. energy range; (**Right**) the same for the Z peak region.

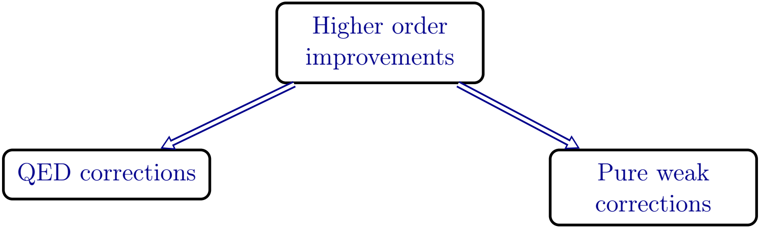
$$A_{LRFB} = \frac{(\sigma_{L_e} - \sigma_{R_e})_F - (\sigma_{L_e} - \sigma_{R_e})_B}{(\sigma_{L_e} + \sigma_{R_e})_F + (\sigma_{L_e} + \sigma_{R_e})_B},$$

Final-State Fermion Polarization



(**Left**) The P_τ polarization in the Born and 1-loop (weak, pure QED, and EW) approximations and ΔP_τ vs. c.m.s. energy in a wide range; (**Right**) the same for the Z peak region. The black dot indicates the Born value P_τ at the Z resonance.

Higher order improvements



- Leading logarithmic (LL) approximation.
- Corrections to $\Delta\alpha$.
- Shower with matching.

- Corrections to $\Delta\rho$.
- Leading Sudakov logarithms.

Higher order improvements, QED

The leading log in the annihilation channel is $L = \ln \frac{s}{m_l^2}$.

$O(1)$	1		
$O(\alpha)$	αL	α	
$O(\alpha^2)$	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
$O(\alpha^3)$	$\frac{1}{6}\alpha^3 L^3$	$\frac{1}{6}\alpha^3 L^2$	...

In the LL approximation we can separate pure photonic (marked "γ") and the rest corrections which include pure pair and mixed photon-pair effects (marked as "pair").

ISR corrections in LL approx. for $e^+e^- \to \mu^+\mu^-(\gamma)$

PRELIMINARY

$\sqrt{s} = 250$ GeV, Born cross section $\sigma_0 = 1417.6(1)$ fb.
$\sum \equiv \sum_{n=2}^{4} \mathcal{O}(\alpha^n L^n)$, $\delta = \delta_{\text{ISR LLA}} \equiv \delta\sigma_{\text{ISR LLA}}/\sigma_0$, $\bar{\delta}$ calculated at scale $= 4s$ and $\underline{\delta}$ at scale $= s/4$.

	$\mathcal{O}(\alpha^2 L^2)$		$\mathcal{O}(\alpha^3 L^3)$		$\mathcal{O}(\alpha^4 L^4)$	$\sum$
	$[\gamma]$	[pair]	$[\gamma]$	[pair]	$[\gamma]$	
Cuts: $\lvert\cos\theta_{\mu^\pm}\rvert < 0.9$, $M_{\mu^+\mu^-} > 10$ GeV.						
$\delta\sigma$, fb	108.2(1)	53.70(1)	−0.49(3)	3.47(1)	−0.23(1)	164.7(1)
δ, %	7.63(1)%	3.79(1)%	−0.035(2)%	0.245(1)%	−0.017(1)%	11.62(1)%
$\bar{\delta}$, %						13.91(1)%
$\underline{\delta}$, %						11.15(1)%
Cuts: $\lvert\cos\theta_{\mu^\pm}\rvert < 0.9$, $M_{\mu^+\mu^-} > 100$ GeV.						
$\delta\sigma$, fb	4.8(1)	5.9(1)	−0.76(2)	0.00(1)	0.00(1)	9.9(1)
δ, %	0.34(1)%	0.42(1)%	−0.053(1)%	0.00(1)%	0.00(1)%	0.70(1)%
$\bar{\delta}$, %						0.78(1)%
$\underline{\delta}$, %						0.63(1)%

FSR corrections in LL approx. for $e^+e^- \to \mu^+\mu^-(\gamma)$

PRELIMINARY

$\sqrt{s} = 250$ GeV, Born cross section $\sigma_0 = 1417.6(1)$ fb.
$\sum \equiv \sum_{n=2}^{4} \mathcal{O}(\alpha^n L^n)$, $\delta = \delta_{\text{FSR LLA}} \equiv \delta\sigma_{\text{FSR LLA}}/\sigma_0$, $\bar{\delta}$ calculated at scale $= 4s$ and $\underline{\delta}$ at scale $= s/4$.

	$\mathcal{O}(\alpha^2L^2)$		$\mathcal{O}(\alpha^3L^3)$		$\mathcal{O}(\alpha^4L^4)$	$\sum$
	$[\gamma]$	[pair]	$[\gamma]$	[pair]	$[\gamma]$	
Cuts: $\lvert\cos\theta_{\mu^\pm}\rvert < 0.9$, $M_{\mu^+\mu^-} > 10$ GeV.						
$\delta\sigma$, fb	0.00(1)	7.64(1)	0.00(1)	−0.129(1)	0.00(1)	7.50(1)
δ, %	0.00(1)%	0.539(1)%	0.00(1)%	−0.0091(1)%	0.00(1)%	0.529(1)%
$\bar{\delta}$, %						0.613(1)%
$\underline{\delta}$, %						0.450(1)%
Cuts: $\lvert\cos\theta_{\mu^\pm}\rvert < 0.9$, $M_{\mu^+\mu^-} > 100$ GeV.						
$\delta\sigma$, fb	−0.54(1)	0.87(1)	0.00(1)	−0.069(1)	0.00(1)	0.26(1)
δ, %	−0.038(1)%	0.061(1)%	0.00(1)%	−0.005(1)%	0.00(1)%	0.018(1)%
$\bar{\delta}$, %						0.019(1)%
$\underline{\delta}$, %						0.017(1)%

Higher order improvements, weak

Higher order improvements added throw $\Delta\rho$ parameter:
$s_W^2 \to \bar{s}_W^2 \equiv s_W^2 + \Delta\rho\, c_W^2$.

At the two-loop level, the quantity $\Delta\rho$ contains two contributions:

$$\Delta\rho = N_c X_t \left[1 + \rho^{(2)}\left(M_H^2/m_t^2\right) X_t\right]\left[1 - \frac{2\alpha_s(M_Z^2)}{9\pi}(\pi^2+3)\right],$$

where $X_t = \frac{\sqrt{2} G_F m_t^2}{16\pi^2}$.

Higher order improvements, weak for $e^+e^- \to \mu^+\mu^-$

P_{e^+}, P_{e^-}	0, 0	0,-0.8	0.3,-0.8	0,0.8	-0.3,0.8
$\sigma^{\rm Born}_{\alpha(0)}$, pb	1.41763(1)	1.54645(1)	1.93499(1)	1.28880(1)	1.58073(1)
$\sigma^{\rm Born}_{G_\mu}$, pb	1.50971(1)	1.64690(1)	2.06068(1)	1.37252(1)	1.68341(1)
$\sigma^{\rm Born}_{\alpha(M_Z^2)}$, pb	1.59923(1)	1.74456(1)	2.18287(1)	1.45391(1)	1.78323(1)
$\delta\sigma^{\rm weak}_{\alpha(0)}$, pb	0.15525(1)	0.11883(1)	0.14243(1)	0.19167(1)	0.242587(1)
$\delta\sigma^{\rm weak}_{G_\mu}$, pb	0.07911(1)	0.03249(1)	0.03400(1)	0.12574(1)	0.162206(1)
$\delta\sigma^{\rm weak}_{\alpha(M_Z^2)}$, pb	-0.01194(1)	-0.07003(1)	-0.09468(1)	0.46147(1)	0.06506(1)
$\delta\sigma^{\rm ho}_{\alpha(0)}$, pb	0.02122(1)	0.02304(1)	0.02882(1)	0.01940(1)	0.02380(1)
$\delta\sigma^{\rm ho}_{G_\mu}$, pb	-0.00555(1)	-0.00351(1)	-0.00407(1)	-0.00759(1)	-0.00969(1)
$\delta\sigma^{\rm ho}_{\alpha(M_Z^2)}$, pb	0.00387(1)	0.00898(1)	0.01183(1)	-0.00124(1)	-0.00222(1)

cuts are: $|\cos\theta_{\mu^-}| < 0.9, \quad |\cos\theta_{\mu^+}| < 0.9.$

Impact of all corrections for $e^+e^- \to \mu^+\mu^-(\gamma)$

	Born	+QED (1-loop)	+WEAK (1-loop)	+WEAK ($\Delta\rho$)	+QED (LL)
σ, pb	1.50971(1)	cut1: $+\,0.829(1)$	+0.07911(1)	−0.00555(1)	cut1: $+\,0.1837(1)$
		cut2: $+\,0.197(1)$			cut2: $+\,0.0108(1)$
δ, %	100%	cut1: $+\,54.9(1)\%$	+5.24(1)%	−0.37(1)%	cut1: $+\,12.17(1)\%$
		cut2: $+\,13.1(1)\%$			cut2: $+\,0.72(1)\%$

Calculated in G_μ EW scheme, $\sqrt{s} = 250$ GeV.
Cuts are: $|\cos\theta_{\mu^-}| < 0.9, \quad |\cos\theta_{\mu^+}| < 0.9$,
cut1: $M_{ll} > 10$ GeV,
cut2: $M_{ll} > 100$ GeV.

RESUME: SANC

• **Monte Carlo integrator MCSANCee and event generator ReneSANCe are being improved**

- **Own library of complete one-loop EW corrections**
- **Initial & final state polarization**
- **LL-accuracy QED improvements to cross section**
- **Higher order WEAK improvements throw $\Delta\rho$**
- **PROCESSES, DONE:** $e^+e^-, \mu^+\mu^-, \tau^+\tau^-, ZH$
- **PROCESSES, UNDERWAY:** $Z\gamma, \gamma\gamma, t\bar{t}$. **Next version of SANC.**
- **UNDERWAY: resonance approximation, showers, Sudakov logarithms.**

Numerical results: Setup for tuned comparison

We performed a tuned comparison of our results for polarized Born and hard Bremsstrahlung with the results WHIZARD and CalcHEP programs.

Initial parameters

$$
\begin{aligned}
\alpha^{-1}(0) &= 137.03599976, & M_W &= 80.451495\ \text{GeV}, & \Gamma_W &= 2.0836\ \text{GeV},\\
M_H &= 125.0\ \text{GeV}, & M_Z &= 91.1867\ \text{GeV}, & \Gamma_Z &= 2.49977\ \text{GeV},\\
m_e &= 0.5109990\ \text{MeV}, & m_\mu &= 0.105658\ \text{GeV}, & m_\tau &= 1.77705\ \text{GeV},\\
m_d &= 0.083\ \text{GeV}, & m_s &= 0.215\ \text{GeV}, & m_b &= 4.7\ \text{GeV},\\
m_u &= 0.062\ \text{GeV}, & m_c &= 1.5\ \text{GeV}, & m_t &= 173.8\ \text{GeV}.
\end{aligned}
$$

$$\text{with cuts } |\cos\theta| < 0.9, \quad E_\gamma > 1\ \text{GeV}$$

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