Accelerator Physics Exercises No. 4

- Answers to be handed in on 26 November 2020

Question 4.1

Show that the RF power loss in the conducting walls of a cavity is given by the relation

$$\frac{R_{surf}}{2}\int_S |H|^2\, ds|$$

where R_{surf} is the surface resistance and S is the inner surface of the structure.

Question 4.2

Assuming a simple pill-box cavity, show that the Quality Factor is related to the surface S, volume V and skin depth δ

$$\frac{2}{\delta} K \frac{V}{S}$$

where K is a form factor.

Question 4.3

A new 50 GeV (kinetic energy) proton synchrotron, the PS2 accelerator, has been proposed to replace the CERN PS. The new PS2 will be in a new ring tunnel of mean radius 215 m. and will receive an injected beam at 4 GeV (kinetic energy) from a new linear accelerator - the Superconducting Proton Linac (SPL). The 1.8 T magnetic field of the bending magnets is excited by a sine wave oscillating between injection and top energy at a frequency of 0.3 Hz. Given that the mass of the proton is 0.9383 GeV:

a) What is the revolution frequency at 4 GeV, 20 GeV and 50 GeV?

b) Assuming the revolution frequency at 20 GeV, calculate the voltage per turn necessary to match the maximum rate of the rise of the field.

c) If $\sin \varphi_s = \sin 60^o$, what is the peak voltage necessary in the cavity? Note that $\varphi_s=0$ corresponds to the zero crossing of the accelerating voltage and the particle is not accelerated.

d) Given a harmonic number of 32, what are the RF frequencies at 4 GeV and 50 GeV for $\varphi_s=0$?

Question 4.4

Design a pill-box cavity 50 cm. long for a representative 10 MHz frequency for the PS2 accelerator. What is the resonant frequency for the next highest mode (TM_{011}) in the cavity? What is the transit-time factor for protons of 4 GeV (kinetic energy) for the PS2 accelerator?

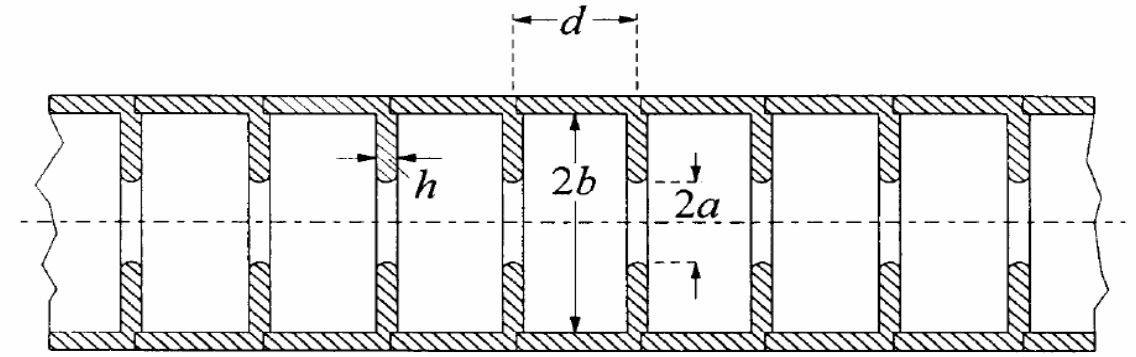
Question 4.5

How strong must the electric field intensity of a travelling plane wave be to accelerate electrons with an energy gradient of 10 MeV/m? (Hint: Use the Poynting vector).

Question 4.6

The SLAC accelerating linac structure has the design shown in the figure below with the following dimensions:

$2b$ = 82.474 mm.
$2a$ = 22.606 mm.
h = 5.842 mm.
d = 35.001 mm.



Assuming that the cavities are operated in the $2\pi/3$ mode with a phase velocity $\beta_z = 1$ and a supplied power of 35 MW, what is the total accelerating voltage and energy gain per metre for a SLAC structure of length l = 3 m.?