

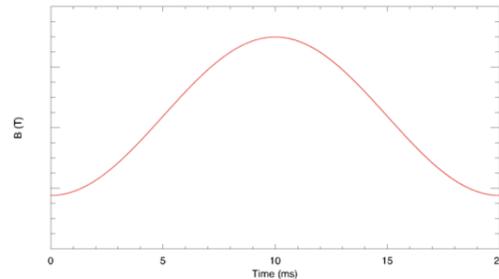
Longitudinal Problem Set

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(a) ISIS Synchrotron



(b) ISIS dipole field as a function of time

Problem 1.1. Warm up by deriving the following relativistic relationships from $E = \gamma mc^2$, $p = \gamma mv$, $\beta = v/c$ and $\gamma = 1/\sqrt{1 - \beta^2}$

- $dE = \beta c dp$
- $dE/E = \beta^2 dp/p$
- $d(\beta\gamma)/dt = c d\gamma/dz$

Problem 2.1. ISIS is a proton synchrotron that operates from 70 to 800 MeV on a 50 Hz sinusoidal main magnet field (see figure b) above). Given the main dipole field at 70 MeV is 0.17639 T calculate the magnetic rigidity, bending radius ρ and the dipole field at top energy.

Problem 2.2. What is the equation for the dipole field as a function of time? Plot out the dipole field over 20 ms.

Problem 2.3. Given the mean radius R of the synchrotron is 26 m, calculate and plot the revolution frequency as a function of time through the 10 ms acceleration.

Problem 2.4. Calculate the functional form, and plot as a function of time, the

- Momentum
- Kinetic Energy
- Relativistic parameters β , γ

What does γ_t have to be for ISIS to remain below transition throughout acceleration?

Problem 2.5. What is the minimum RF voltage required as a function of time to accelerate a proton at ISIS? Why do we need more?

Problem 2.6. Given a mean dispersion of 1 m, what is the γ_t ? What transition kinetic energy does that correspond to? Calculate and plot the slip factor η as a function of time over 10 ms.

From the longitudinal equation of motion on slide 29 one can derive the symplectic mapping equations

$$\Delta E_{n+1} = \Delta E_n + V_1(\sin \phi_n - \sin \phi_s) \tag{1}$$

$$\phi_{n+1} = \phi_n - \frac{2\pi h \eta}{E_0 \beta^2 \gamma} \Delta E_{n+1} \tag{2}$$

from the n^{th} to the $(n + 1)^{\text{th}}$ turn where V_1 is the peak gap voltage of the cavity and E_0 is the rest energy . These can be used to follow a particle’s trajectory in the longitudinal phase space $(\Delta E, \phi)$ on each turn.

Problem 3.1. Write a simulation program in the programming language of your choice that assumes several initial particle co-ordinates $(0, \phi)$ in the range $0 < \phi < \pi$ and calculates and applies the mapping equations over n turns. Assume a constant energy (70 MeV, $\phi_s = 0$) and use the parameters you calculated in exercise 1. (Other key parameters: $h = 2$, $V_1 = 19$ kV per turn and a sensible number for n). Track four more trajectories with start co-ordinates $(0.5 \text{ MeV}, \pi)$, $(-0.5 \text{ MeV}, -\pi)$, $(1 \text{ MeV}, \pi)$, $(-1 \text{ MeV}, -\pi)$ and plot out all the longitudinal phase space trajectories on one graph.

Problem 3.2. For the above problem pick out the three different types of trajectory and briefly describe the motion in each. Compare to the analogous situation of a simple pendulum.

Problem 3.3. Now assume we are half way through acceleration (5 ms). Using the parameters you calculated in exercise 1 and an RF voltage of 150 kV calculate ϕ_s and plot some more trajectories, this time with initial co-ordinates $(0, \phi)$ between $-\pi < \phi < \pi$. Ensure you track a particle with start co-ordinates $(0, \pi - \phi_s - 0.02)$ and thus trace out the maximum bucket size.

Problem 3.4. Calculate the RF bucket height (energy acceptance) with the equation on slide 47 for the two cases (2.1 and 2.3) and compare with the bucket size from your trajectories.

Problem 3.5. How does the RF bucket change, as a function of

- a. ϕ_s ?
- b. V ?

Hand in a copy of your program(s) with its (their) output.

OPTIONAL PROBLEM...if you’re interested

Problem 3.6. What if we now introduce a second harmonic RF system at twice the frequency of the first ($h = 4$) such that we now have the energy gain per turn

$$\Delta E_{\text{turn}} = e (V_1 \sin \phi_s + V_2 \sin(2\phi_s + \theta)), \tag{3}$$

where V_2 is the second harmonic RF voltage and θ is the phase difference between the two RF systems. How does this change our mapping equations? Try putting this into your tracking code with $V_2 = V_1$ for the case $\phi_s = 0$. How does the RF bucket change with

- a. $\theta = \pi$?
- b. $\theta = 0$?