Transverse Beam Dynamics - Tutorial

JAI lectures 2020 - Michaelmas Term

1 Preliminary exercices

1. Watch this Iron Man clip and discuss the main accelerator physics concepts involved either if they are properly represented or not in the movie.
2. Go through the short questions posted during lectures and try to answer them.

2 To think about

1. How can we measure β^*(β-function at the IP) in the LHC?
2. What are the possible effects of ground motion in the beam?
3. What can we do if there is a small object partially blocking the beam aperture?

3 Exercise: Understanding the phase space concept

1. Phase Space Representation of a Particle Source:
   - Consider a source at position s_0 with radius w emitting particles. Make a drawing of this setup in the configuration space and in the phase space. Which part of the phase space can be occupied by the emitted particles?
   - Any real beam emerging from a source like the one above will be collimated. This can be modelled by assuming that a distance d away from the source there is an iris with opening radius $R = w$. Draw this setup in the configuration space and in the phase space. Which part of the phase space is occupied by the beam, right after the collimator?
2. Sketch the emittance ellipse of a particle beam in:

   (I) horizontal x-x' phase space at the position of a transverse waist,

   (II) when the beam is divergent, and

   (III) when the beam is convergent.

4 Exercise: Stability condition

Consider a lattice composed by a single 2 meters long quadrupole, with $f = 1$ m

- Prove that if the quadrupole is defocusing, then a lattice is not stable
- Prove that if the quadrupole is focusing, then the lattice is stable

5 Twiss functions evolution

Which of the optics parameters can be constant

1. In a drift.
2. In a quadrupole with constant strength K.

Justify the response.

Hint: The differential equation representing the evolution of the β-function reads,

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + \beta K = 1$$

6 Exercise: Bump and Orbit Control

Given two kickers located at the two ends of a FODO cell with phase advance 45 degrees (the two kickers are located at L_{cell} distance from each other), compute the strengths of such kickers (in radians) in order to give the beam, initially at $(x_i, x_i') = (0, 0)$, an arbitrary offset at the end of the cell while preserving its angle, $\left(x_f, x_f'\right) = (x_{\text{arbitrary}}, 0)$.

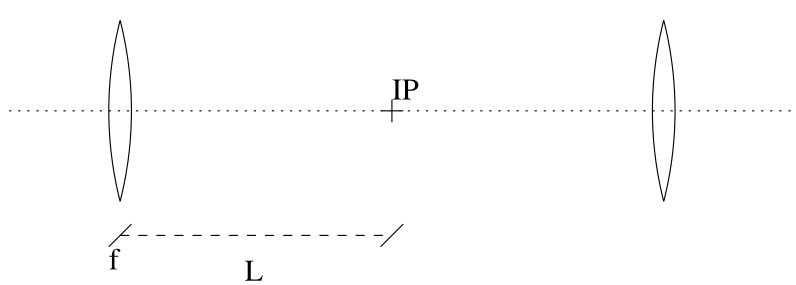
7 Exercise: Chromaticity in a FODO cell

Consider a ring made of N_{cell} identical FODO cells with equally spaced quadrupoles. Assume that the two quadrupoles are both of length l_q, but their strengths may differ.

1. Calculate the maximum and the minimum betatron function in the FODO cell. (*Use the thin-lens approximations*)
2. Calculate the natural chromaticities for this ring.
3. Design the FODO cell such that it has: phase advance $\mu = 90$ degrees, a total length of 10 m, and a total bending angle of 5 degrees. What are β_{max}, β_{min}, D_{max}, D_{min}?
4. Add two sextupoles at appropriate locations to correct horizontal and vertical chromaticities. (hints: use 1 sextupole for the horizontal plane and 1 for the vertical plane; do not consider geometric aberrations).
5. If the gradient of all focusing quadrupoles in the ring is wrong by $+10\%$, how much is the tune-shift with and without sextupoles?

8 Exercise: Low-Beta Insertion

Consider the following low-beta insertion around an interaction point (IP). The quadrupoles are placed with mirror-symmetry with respect to the IP:



The beam enters the quadrupole with Twiss parameters $\beta_0 = 20$ m and $\alpha_0 = 0$. The drift space has length $L = 10$ m.

(i) Determine the focal length of the quadrupole in order to locate the waist at the IP.

(ii) What is the value of $\beta^\star$?

(iii) What is the phase advance between the quadrupole and the IP?