Adam Falkowski

Summary of the Higgs basis parametrization of the SMEFT

VR, July 8, 2020
SMEFT = minimal EFT above the weak scale

\[ \mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{D=5} + \mathcal{L}_{D=6} + \mathcal{L}_{D=7} + \mathcal{L}_{D=8} + \mathcal{L}_{D=9} + \ldots \]

Known SM Lagrangian

Higher-dimensional SU(3)_c \times SU(2)_L \times U(1)_Y invariant interactions added to the SM

Assumption: at energies of interest no other relevant degrees of freedom than those of the SM
SMEFT = minimal EFT above the weak scale

\[ \mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{D=5} + \mathcal{L}_{D=6} + \mathcal{L}_{D=7} + \mathcal{L}_{D=8} + \mathcal{L}_{D=9} + \ldots \]

Known SM Lagrangian

In this talk only dimension-6 operators are taken into account

\[ \mathcal{L}_{D=6} = \sum_i C_i \mathcal{Q}_i \]

Wilson coefficients

Gauge-invariant dimension-6 operators
equations of motion, integration by parts, field redefinitions, or Fierz transformations. This is the case if the operators can be related by using recognized long ago, see e.g. [21]. Their importance for characterizing low-energy e

2.2 Dimension-6 operators

We turn to discussing operators with canonical dimensions. Bosonic CP-even

<table>
<thead>
<tr>
<th>Bosonic CP-even</th>
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<tr>
<td>$O_H$</td>
<td>$(H^\dagger H)^3$</td>
</tr>
<tr>
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| Warsaw basis |
|-----------------|----------------|
| $O_{H_{I,J}}$   | $(\tilde{H}^I \tilde{H}^J)_I J$ |
| $O_{H_{E,J}}$   | $(\tilde{H}^I \tilde{H}^J)_E J$ |
| $O_{H_{B,J}}$   | $(\tilde{H}^I \tilde{H}^J)_B J$ |
| $O_{H_{W,J}}$   | $(\tilde{H}^I \tilde{H}^J)_W J$ |
| $O_{H_{D,J}}$   | $(\tilde{H}^I \tilde{H}^J)_D J$ |
| $O_{H_{\mu}}$   | $(\tilde{H}^I \tilde{H}^J)_{\mu}$ |
| $O_{H_{\nu}}$   | $(\tilde{H}^I \tilde{H}^J)_{\nu}$ |
| $O_{H_{\mu \nu}}$ | $(\tilde{H}^I \tilde{H}^J)_{\mu \nu}$ |
| $O_{H_{\mu \nu \rho \mu \nu}}$ | $(\tilde{H}^I \tilde{H}^J)_{\mu \nu \rho \mu \nu}$ |

Table 2.2: Bosonic $D=6$ operators in the Warsaw basis.

Wilson coefficient of these operators can be connected (now semi-automatically) to fundamental parameters of BSM models like SUSY, composite Higgs, etc.

Grzadkowski et al. [1008.4884]

Wilson coefficient of these operators can be connected (now semi-automatically) to fundamental parameters of BSM models like SUSY, composite Higgs, etc.

Alonso et al 1312.2014,
Henning et al 1512.03433
Warsaw Basis

- Warsaw basis is now commonly used in the literature
- Convenient for many theory applications
- Implemented in many numerical tools
- Interpretation of various Wilson coefficients not always intuitive
- O(200) distinct operators affecting LHC Higgs physics
- Larger correlations between Higgs and other precision measurements

### Table 2.2: Bosonic \( D=6 \) operators in the Warsaw basis.

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For example, operator \( O_{HWB} \) affects

- Higgs to \( \gamma\gamma, Z\gamma, ZZ \) decays
- \( Z \) pole measurements
- Triple gauge couplings
- \( W \) boson mass
Higgs Basis

- Higgs basis was conceived in 2015 to facilitate practical applications of the SMEFT for LHC Higgs analyses
- The construction closely follows the idea introduced by Gupta, Pomarol, and Riva in [arXiv:1405.0181]

The goals of the Higgs basis

1. Each Wilson coefficient has a simple physical interpretation
2. Higgs observables at leading order are affected by a minimal set of Wilson coefficients
3. Large correlations between the Higgs and electroweak constraints are avoided

In other words, the point is to amend certain impractical features of the Warsaw basis
Higgs Basis - definition

\[ \vec{c}_{\text{HB}} = M_{W \to H} \vec{c}_{\text{WB}} \]

Vector of 2499 Wilson coefficients in Higgs basis

2499x2499 dimensional numerical invertible matrix

Vector of 2499 Wilson coefficients in Warsaw basis

Alternatively, the same transformation can be defined at the level of gauge-invariant operators

\[ Q_{\text{HB}} = M_{W \to H}^{-1} T Q_{\text{WB}} \]
\[ v^{-2}\delta c_z = C_\varphi - \frac{1}{4} C_\varphi D - \frac{3}{2} \Delta_{GF}, \quad \Delta_{GF} = [C^{(3)}_{\varphi l}]_{11} + [C^{(3)}_{\varphi l}]_{22} - \frac{1}{2} [C_L]_{1221} \]

\[ v^{-2}c_{z\square} = \frac{1}{2g_L^2} (C_\varphi D + 2 \Delta_{GF}), \]

\[ v^{-2}c_{gg} = \frac{4}{g_s^2} C_\varphi G, \]

\[ v^{-2}c_{\gamma\gamma} = 4 \left( \frac{1}{g_L^2} C_\varphi W + \frac{1}{g_Y^2} C_\varphi B - \frac{1}{g_L g_Y} C_\varphi WB \right), \]

\[ v^{-2}c_{zz} = 4 \left( \frac{g_L^2 C_\varphi W + g_Y^2 C_\varphi B + g_L g_Y C_\varphi WB}{(g_L^2 + g_Y^2)^2} \right), \]

\[ v^{-2}c_{z\gamma} = 4 \left( \frac{C_\varphi W - C_\varphi B - \frac{g_L^2 - g_Y^2}{2g_L g_Y} C_\varphi WB}{g_L^2 + g_Y^2} \right), \]

\[ v^{-2}\tilde{c}_{gg} = \frac{4}{g_s^2} C_\varphi \tilde{g}, \]

\[ v^{-2}\tilde{c}_{\gamma\gamma} = 4 \left( \frac{1}{g_L^2} C_\varphi \tilde{W} + \frac{1}{g_Y^2} C_\varphi \tilde{B} - \frac{1}{g_L g_Y} C_\varphi \tilde{W} \tilde{B} \right), \]

\[ v^{-2}\tilde{c}_{zz} = 4 \left( \frac{g_L^2 C_\varphi \tilde{W} + g_Y^2 C_\varphi \tilde{B} + g_L g_Y C_\varphi \tilde{W} \tilde{B}}{(g_L^2 + g_Y^2)^2} \right), \]

\[ v^{-2}\tilde{c}_{z\gamma} = 4 \left( \frac{C_\varphi \tilde{W} - C_\varphi \tilde{B} - \frac{g_L^2 - g_Y^2}{2g_L g_Y} C_\varphi \tilde{W} \tilde{B}}{g_L^2 + g_Y^2} \right), \]

\[ v^{-2}\delta\lambda_3 = -\frac{1}{\lambda} C_\varphi + 3C_\varphi \square - \frac{3}{4} C_\varphi D - \frac{1}{2} \Delta_{GF}, \]

\[ v^{-2}[\delta g_f]_{JK} = -\frac{v}{\sqrt{2m_{fJ} m_{fK}}} [C^\dagger_{f\varphi}]_{JK} + \delta_{JK} \left( C_\varphi \square - \frac{1}{4} C_\varphi D - \frac{1}{2} \Delta_{GF} \right) \]
Higgs Basis - interpretation part 1

First group of Wilson coefficients corresponds has a simple interpretation of certain Higgs couplings

\[
\mathcal{L} \supset \frac{h}{v} \left[ (1 + \delta c_w) \frac{g_L^2 v^2}{2} W^+_{\mu} W^-_\mu + (1 + \delta c_z) \frac{(g_L^2 + g_Y^2) v^2}{4} Z_\mu Z_\mu \right] - \sum_{f \in u, d, e} \sum_{IJ} \sqrt{m_{f_I} m_{f_J}} \left[(\delta_{IJ} + [\delta yf]_{IJ}) \tilde{f}_L f_R + \text{h.c.}\right]
\]

\[
+ c_{ww} \frac{g_L^2}{2} W^+_{\mu \nu} W^-_{\mu \nu} + \tilde{c}_{ww} \frac{g_L^2}{2} W^+_{\mu \nu} \tilde{W}^-_{\mu \nu} + c_{\square} g_L^2 (W^-_\mu \partial_\nu W^+_{\mu \nu} + \text{h.c.})
\]

\[
+ c_{gg} \frac{g_s^2}{4} G^a_{\mu \nu} G^a_{\mu \nu} + c_{\gamma \gamma} \frac{g_s^2 g_Y^2}{4(g_L^2 + g_Y^2)} A_{\mu \nu} A_{\mu \nu} + c_{z \gamma} \frac{g_L g_Y}{2} Z_\mu A_{\mu \nu} + c_{zz} \frac{g_L^2 + g_Y^2}{4} Z_\mu Z_\mu
\]

\[
+ c_{z \square} g_L^2 Z_\mu \partial_\nu Z_\mu + c_{\gamma \square} g_L g_Y Z_\mu \partial_\nu A_{\mu \nu}
\]

\[
+ \tilde{c}_{gg} \frac{g_s^2}{4} G^a_{\mu \nu} \tilde{G}^a_{\mu \nu} + \tilde{c}_{\gamma \gamma} \frac{g_L^2 g_Y^2}{4(g_L^2 + g_Y^2)} A_{\mu \nu} \tilde{A}_{\mu \nu} + \tilde{c}_{z \gamma} \frac{g_L g_Y}{2} Z_\mu \tilde{A}_{\mu \nu} + \tilde{c}_{zz} \frac{g_L^2 + g_Y^2}{4} Z_\mu \tilde{Z}_{\mu \nu}
\]

\[
\delta c_w = \delta c_z + 4 \delta m_w, \quad \delta m_w \equiv \frac{1}{2} g_L W^e + \frac{1}{2} g_L W^\mu - \frac{1}{4} [c_{\ell}]_{121},
\]

\[
c_{ww} = c_{zz} + \frac{2 g_Y^2}{g_L^2 + g_Y^2} c_{\gamma \gamma} + \frac{g_Y^4}{(g_L^2 + g_Y^2)^2} c_{\gamma \gamma},
\]

\[
\tilde{c}_{ww} = \tilde{c}_{zz} + \frac{2 g_Y^2}{g_L^2 + g_Y^2} \tilde{c}_{\gamma \gamma} + \frac{g_Y^4}{(g_L^2 + g_Y^2)^2} \tilde{c}_{\gamma \gamma},
\]

\[
c_{\square} = \frac{1}{g_L^2 - g_Y^2} \left[ g_L^2 c_{\square} + g_Y^2 c_{zz} - \frac{g_Y^4 (g_L^2 + g_Y^2)}{g_L^2 + g_Y^2} c_{\gamma \gamma} - \frac{g_L^2 g_Y^4}{(g_L^2 + g_Y^2)^2} c_{\gamma \gamma} \right],
\]

\[
c_{\gamma \square} = \frac{1}{g_L^2 - g_Y^2} \left[ 2 g_L^2 c_{\square} + (g_L^2 + g_Y^2) c_{zz} - (g_L^2 - g_Y^2) c_{\gamma \gamma} - \frac{g_L^2 g_Y^2}{g_L^2 + g_Y^2} c_{\gamma \gamma} \right],
\]

These Wilson coefficients at leading order are probed only by Higgs and diboson processes. They were largely unconstrained before LHC, and remain weakly constrained nowadays!
Higgs Basis - map part 2

\[
v^{-2} \delta g_{W}^{\ell} = C_{\varphi \ell}^{(3)} + f(1/2, 0) - f(-1/2, -1),
\]
\[
v^{-2} \delta g_{L}^{Z \ell} = -\frac{1}{2} C_{\varphi \ell}^{(3)} - \frac{1}{2} C_{\varphi \ell}^{(1)} + f(-1/2, -1),
\]
\[
v^{-2} \delta g_{R}^{Z \ell} = -\frac{1}{2} C_{\varphi e}^{(1)} + f(0, -1),
\]
\[
v^{-2} \delta g_{L}^{Z u} = \frac{1}{2} C_{\varphi q}^{(3)} - \frac{1}{2} C_{\varphi q}^{(1)} + f(1/2, 2/3),
\]
\[
v^{-2} \delta g_{L}^{Z d} = -\frac{1}{2} C_{\varphi q}^{(3)} - \frac{1}{2} C_{\varphi q}^{(1)} + f(-1/2, -1/3),
\]
\[
v^{-2} \delta g_{R}^{Z u} = -\frac{1}{2} C_{\varphi u} + f(0, 2/3),
\]
\[
v^{-2} \delta g_{R}^{Z d} = -\frac{1}{2} C_{\varphi d} + f(0, -1/3),
\]
\[
v^{-2} \delta g_{R}^{W q} = \frac{1}{2} C_{\varphi ud},
\]

\[
f(T^3, Q) \equiv \left\{ - Q \frac{g_{l} g_{Y}}{g_{l}^{2} - g_{Y}^{2}} C_{\varphi WB} - \left( \frac{1}{4} C_{\varphi D} + \frac{1}{2} \Delta_{G}^{(1)} \right) \left( T^3 + Q \frac{g_{Y}^{2}}{g_{l}^{2} - g_{Y}^{2}} \right) \right\}^1
\]
The Wilson coefficients $\delta g$ are interpreted as vertex correction to electroweak gauge boson couplings to matter.

$$\mathcal{L} \supset - \frac{g_L g_Y}{\sqrt{g_L^2 + g_Y^2}} A_\mu \sum_{f=u,d,e} Q_f \bar{f} \gamma_\mu f - g_s G^a_\mu \sum_{f \in u,d} \bar{f} \gamma_\mu T^a f,$$

$$- \frac{g_L}{\sqrt{2}} \left( W^+_{\mu} \bar{\nu}_L \gamma_\mu (I + \delta g^W_{L}) e_L + W^+_{\mu} \bar{u}_L \gamma_\mu (V_{CKM} + \delta g^W_q) d_L + W^+_{\mu} \bar{u}_R \gamma_\mu \delta g^W_R d_R + \text{h.c.} \right),$$

$$- \sqrt{g_L^2 + g_Y^2} Z_\mu \left[ \sum_{f \in u,d,e,\nu} \bar{f}_L \gamma_\mu (T^3 - s^2_\theta Q_f + \delta g^Z_{L} f_L + \sum_{f \in u,d,e} \bar{f}_R \gamma_\mu (-s^2_\theta Q_f + \delta g^Z_{R}) f_R \right]$$

These parameters are strongly constrained, in a model-independent way, by LEP-1 precision measurements.

$$\delta g^Z_{\nu} = \delta g^W_{\ell} + \delta g^Z_{e},$$
$$\delta g^W_{q} = V_{CKM}^\dagger \delta g^Z_{u} V_{CKM} - \delta g^Z_{d}.$$
The remaining (more trivial) part of the map

\[ v^{-2} \lambda_z = \frac{3}{2} g_L C_W, \quad v^{-2} \tilde{\lambda}_z = \frac{3}{2} g_L C_{\tilde{W}}, \]

\[ v^{-2} \lambda_g = \frac{C_G}{g_s^3}, \quad v^{-2} \tilde{\lambda}_g = \frac{C_{\tilde{G}}}{g_s^3}. \]

\[ v^{-2} d_{Gf} = -\frac{16}{g_s^2} C_{fG}^*, \]

\[ v^{-2} d_{Af} = -\frac{16}{g_L^2} \left( \eta_f C_{fW}^* + C_{fB}^* \right), \]

\[ v^{-2} d_{Zf} = -16 \left( \eta_f \frac{1}{g_L^2 + g_Y^2} C_{fW}^* - \frac{g_Y^2}{g_L^2 (g_L^2 + g_Y^2)} C_{fB}^* \right), \quad \text{dipoles} \]

\[ v^{-2} d_{Wf} = -\frac{16}{g_L^2} C_{fW}^*, \]

\[ v^{-2} c_i = C_i. \quad \text{4-fermion} \]
\[ \vec{c}_{HB} = \begin{pmatrix} \vec{c}_{\text{Higgs}} \\ \delta g_{\text{vertex}} \\ \vec{d}_{\text{dipole}} \\ \vec{c}_{F^3} \\ \vec{c}_{4\text{fermion}} \end{pmatrix} \]

- At LO, probed only by Higgs and diboson processes
  - Most relevant for typical LHC Higgs analyses

- Affecting Higgs observables, but almost all strongly constrained by other precision measurements
  - Irrelevant for typical LHC Higgs analyses

- Affecting Higgs observables, but most are constrained by other precision measurements
  - Irrelevant for many LHC Higgs analyses

- At LO, probed only by diboson processes
  - Most relevant for typical LHC Higgs analyses

- At LO, probed e.g. by Drell-Yan processes
  - Irrelevant for typical LHC Higgs analyses
Higgs Basis - conclusions

For typical LHC Higgs analyses, in the Higgs basis only a limited set of Wilson coefficients is relevant

\[ \delta c_z, c_z\Box, c_{zz}, c_{\gamma\gamma}, c_{z\gamma}, c_{gg}, \delta \lambda_3, \tilde{c}_{zz}, \tilde{c}_{\gamma\gamma}, \tilde{c}_{z\gamma}, \tilde{c}_{gg}, \delta y_u, \delta y_d, \delta y_e; \]

(plus eventually a handful of dipole and vertex corrections)

They are interpreted as certain Higgs couplings in the SMEFT Lagrangian

Likelihood obtained for these parameters can be translated to any other SMEFT basis, in particular to the Warsaw Basis
References


2. My HDR (available online)

3. Summary note prepared for the Offshell WG